11.1

GIVEN: $x = 4t^4 - 6t^3 + 2t - 1 \quad x \sim m,\ t \sim s$

FIND: x, v, AND a AT $t = 2\,s$

HAVE.. $x = 4t^4 - 6t^3 + 2t - 1$

THEN $v = \frac{dx}{dt} = 16t^3 - 18t^2 + 2$

AND $a = \frac{dv}{dt} = 48t^2 - 36t$

AT $t = 2\,s$: $x_2 = 4(2)^4 - 6(2)^3 + 2(2) - 1$ OR $x_2 = 19\ m$ ◄

$v_2 = 16(2)^3 - 18(2)^2 + 2$ OR $v_2 = 58\ \frac{m}{s}$ ◄

$a_2 = 48(2)^2 - 36(2)$ OR $a_2 = 120\ \frac{m}{s^2}$ ◄

11.2

GIVEN: $x = 3t^4 + 4t^3 - 7t^2 - 5t + 8 \quad x \sim mm,\ t \sim s$

FIND: x, v, AND a AT $t = 3\,s$

HAVE.. $x = 3t^4 + 4t^3 - 7t^2 - 5t + 8$

THEN $v = \frac{dx}{dt} = 12t^3 + 12t^2 - 14t - 5$

AND $a = \frac{dv}{dt} = 36t^2 + 24t - 14$

AT $t = 3\,s$: $x_3 = 3(3)^4 + 4(3)^3 - 7(3)^2 - 5(3) + 8$ OR $x_3 = 281\ mm$ ◄

$v_3 = 12(3)^3 + 12(3)^2 - 14(3) - 5$ OR $v_3 = 385\ \frac{mm}{s}$ ◄

$a_3 = 36(3)^2 + 24(3) - 14$ OR $a_3 = 382\ \frac{mm}{s^2}$ ◄

11.3

GIVEN: $x = 6t^2 - 8 + 40\cos\pi t \quad x \sim in.,\ t \sim s$

FIND: x, v, AND a AT $t = 6\,s$

HAVE.. $x = 6t^2 - 8 + 40\cos\pi t$

THEN $v = \frac{dx}{dt} = 12t - 40\pi\sin\pi t$

AND $a = \frac{dv}{dt} = 12 - 40\pi^2\cos\pi t$

AT $t = 6\,s$: $x_6 = 6(6)^2 - 8 + 40\cos 6\pi$ OR $x_6 = 248\ in.$ ◄

$v_6 = 12(6) - 40\pi\sin 6\pi$ OR $v_6 = 72\ \frac{in.}{s}$ ◄

$a_6 = 12 - 40\pi^2\cos 6\pi$ OR $a_6 = -383\ \frac{in.}{s^2}$ ◄

11.4

GIVEN: $x = \frac{5}{3}t^3 - \frac{5}{2}t^2 - 30t + 8 \quad x \sim ft,\ t \sim s$

FIND: t, x, AND a WHEN $v = 0$

HAVE.. $x = \frac{5}{3}t^3 - \frac{5}{2}t^2 - 30t + 8$

THEN $v = \frac{dx}{dt} = 5t^2 - 5t - 30$

AND $a = \frac{dv}{dt} = 10t - 5$

WHEN $v = 0$: $5t^2 - 5t - 30 = 5(t^2 - t - 6) = 0$

OR $t = 3\,s$ AND $t = -2\,s$ (REJECT) $\therefore\ t = 3\,s$ ◄

AT $t = 3\,s$: $x_3 = \frac{5}{3}(3)^3 - \frac{5}{2}(3)^2 - 30(3) + 8$ OR $x_3 = -59.5\ ft$ ◄

$a_3 = 10(3) - 5$ OR $a_3 = 25\ \frac{ft}{s^2}$ ◄

11.5

GIVEN: $x = 6t^4 - 2t^3 - 12t^2 + 3t + 3 \quad x \sim m,\ t \sim s$

FIND: t, x, AND v WHEN $a = 0$

HAVE.. $x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$

THEN $v = \frac{dx}{dt} = 24t^3 - 6t^2 - 24t + 3$

AND $a = \frac{dv}{dt} = 72t^2 - 12t - 24$

WHEN $a = 0$: $72t^2 - 12t - 24 = 12(6t^2 - t - 2) = 0$

OR $(3t - 2)(2t + 1) = 0$

OR $t = \frac{2}{3}\,s$ AND $t = -\frac{1}{2}\,s$ (REJECT) $\therefore\ t = 0.667\,s$ ◄

AT $t = \frac{2}{3}\,s$: $x_{2/3} = 6(\frac{2}{3})^4 - 2(\frac{2}{3})^3 - 12(\frac{2}{3})^2 + 3(\frac{2}{3}) + 3$ OR $x_{2/3} = 0.259\ m$ ◄

$v_{2/3} = 24(\frac{2}{3})^3 - 6(\frac{2}{3})^2 - 24(\frac{2}{3}) + 3$ OR $v_{2/3} = -8.56\ \frac{m}{s}$ ◄

11.6

GIVEN: $x = 3t^3 - 6t^2 - 12t + 5 \quad x \sim m,\ t \sim s$

FIND: (a) t WHEN $v = 0$

(b) x, a, TOTAL DISTANCED TRAVELED WHEN $t = 4\,s$

HAVE.. $x = 3t^3 - 6t^2 - 12t + 5$

THEN $v = \frac{dx}{dt} = 9t^2 - 12t - 12$

AND $a = \frac{dv}{dt} = 18t - 12$

(a) WHEN $v = 0$: $9t^2 - 12t - 12 = 3(3t^2 - 4t - 4) = 0$

OR $(3t + 2)(t - 2) = 0$

OR $t = 2\,s$ AND $t = -\frac{2}{3}\,s$ (REJECT) $\therefore\ t = 2\,s$ ◄

(b) AT $t = 4\,s$: $x = 3(4)^3 - 6(4)^2 - 12(4) + 5$ OR $x_4 = 53\ m$ ◄

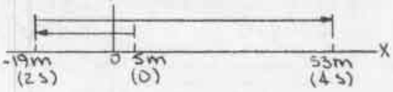
$a = 18(4) - 12$ OR $a_4 = 60\ \frac{m}{s^2}$ ◄

FIRST OBSERVE THAT.. $0 \le t < 2\,s$: $v < 0$

$t > 2\,s$: $v > 0$

NOW.. AT $t = 0$: $x_0 = 5\ m$

$t = 2\,s$: $x_2 = 3(2)^3 - 6(2)^2 - 12(2) + 5 = -19\ m$



THEN $|x_2 - x_0| = |-19 - 5| = 24\ m$

$x_4 - x_2 = 53 - (-19) = 72\ m$

$\therefore$ TOTAL DISTANCE TRAVELED $= (24 + 72)\ m = 96\ m$ ◄

11.7

GIVEN: $x = t^3 - 9t^2 + 24t - 8 \quad x \sim in.,\ t \sim s$

FIND: (a) t WHEN $v = 0$

(b) x AND TOTAL DISTANCE TRAVELED WHEN $a = 0$

HAVE.. $x = t^3 - 9t^2 + 24t - 8$

THEN $v = \frac{dx}{dt} = 3t^2 - 18t + 24$

AND $a = \frac{dv}{dt} = 6t - 18$

(a) WHEN $v = 0$: $3t^2 - 18t + 24 = 3(t^2 - 6t + 8) = 0$

OR $(t - 2)(t - 4) = 0$

OR $t = 2\,s$ AND $t = 4\,s$ ◄

(b) WHEN $a = 0$: $6t - 18 = 0$ OR $t = 3\,s$

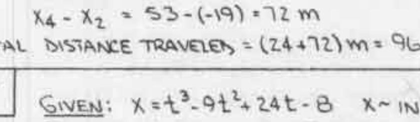
AT $t = 3\,s$: $x_3 = (3)^3 - 9(3)^2 + 24(3) - 8$ OR $x_3 = 10\ in.$ ◄

FIRST OBSERVE THAT.. $0 \le t < 2\,s$: $v > 0$

$2\,s < t \le 3\,s$: $v < 0$

NOW.. AT $t = 0$: $x_0 = -8\ in.$

AT $t = 2\,s$: $x_2 = (2)^3 - 9(2)^2 + 24(2) - 8 = 12\ in.$



THEN $x_2 - x_0 = 12 - (-8) = 20\ in.$

$|x_3 - x_2| = |10 - 12| = 2\ in.$

$\therefore$ TOTAL DISTANCE TRAVELED $= (20 + 2)\ in. = 22\ in.$ ◄

11.8

GIVEN: $x = t^3 - 6t^2 - 36t - 40$ $x \sim \text{ft},\ t \sim \text{s}$

FIND: (a) t WHEN $v = 0$

(b) v, a, AND TOTAL DISTANCE TRAVELED WHEN $x = 0$

HAVE... $x = t^3 - 6t^2 - 36t - 40$

THEN $v = \frac{dx}{dt} = 3t^2 - 12t - 36$

AND $a = \frac{dv}{dt} = 6t - 12$

(a) WHEN $v = 0$: $3t^2 - 12t - 36 = 3(t^2 - 4t - 12) = 0$

OR $(t+2)(t-6) = 0$

OR $t = -2\ \text{s}$ (REJECT) AND $t = 6\ \text{s}$ $\quad \therefore t = 6\ \text{s}$ ◀

(b) WHEN $x = 0$: $t^3 - 6t^2 - 36t - 40 = 0$

FACTORING... $(t-10)(t+2)(t+2) = 0$ OR $t = 10\ \text{s}$

NOW OBSERVE THAT... $0 \le t < 6\ \text{s}$: $v < 0$

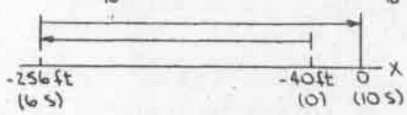
$6\ \text{s} < t \le 10\ \text{s}$: $v > 0$

AND AT $t = 0$: $x_0 = -40\ \text{ft}$

$t = 6\ \text{s}$: $x_6 = (6)^3 - 6(6)^2 - 36(6) - 40 = -256\ \text{ft}$

$t = 10\ \text{s}$: $v_{10} = 3(10)^2 - 12(10) - 36$ OR $v_{10} = 144\ \frac{\text{ft}}{\text{s}}$ ◀

$a_{10} = 6(10) - 12$ OR $a_{10} = 48\ \frac{\text{ft}}{\text{s}^2}$ ◀



THEN $|x_6 - x_0| = |-256 - (-40)| = 216\ \text{ft}$

$x_{10} - x_6 = 0 - (-256) = 256\ \text{ft}$

$\therefore$ TOTAL DISTANCE TRAVELED $= (216 + 256)\ \text{ft} = 472\ \text{ft}$ ◀

11.9

GIVEN: $a = 6\ \frac{\text{ft}}{\text{s}^2}$; AT $t = 0$, $x = -32\ \text{ft}$;

AT $t = 2\ \text{s}$, $v = -6\ \frac{\text{ft}}{\text{s}}$

FIND: v, x, AND TOTAL DISTANCE TRAVELED AT $t = 5\ \text{s}$

HAVE... $\frac{dv}{dt} = a = 6\ \frac{\text{ft}}{\text{s}^2}$

AT $t = 2\ \text{s}$, $v = -6\ \frac{\text{ft}}{\text{s}}$: $\int_{-6}^{v} dv = \int_2^t 6\, dt$

OR $v - (-6) = 6(t - 2)$

OR $v = 6t - 18$ $\left(\frac{\text{ft}}{\text{s}}\right)$

ALSO... $\frac{dx}{dt} = v = 6t - 18$

AT $t = 0$, $x = -32\ \text{ft}$: $\int_{-32}^{x} dx = \int_0^t (6t - 18)\, dt$

OR $x - (-32) = 3t^2 - 18t$

OR $x = 3t^2 - 18t - 32$ (ft)

AT $t = 5\ \text{s}$: $v_5 = 6(5) - 18$ OR $v_5 = 12\ \frac{\text{ft}}{\text{s}}$ ◀

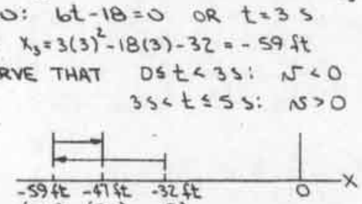
$x_5 = 3(5)^2 - 18(5) - 32$ OR $x_5 = -47\ \text{ft}$ ◀

WHEN $v = 0$: $6t - 18 = 0$ OR $t = 3\ \text{s}$

AT $t = 3\ \text{s}$: $x_3 = 3(3)^2 - 18(3) - 32 = -59\ \text{ft}$

NOW OBSERVE THAT $0 \le t < 3\ \text{s}$: $v < 0$

$3\ \text{s} < t \le 5\ \text{s}$: $v > 0$



THEN $|x_3 - x_0| = |-59 - (-32)| = 27\ \text{ft}$

$|x_5 - x_3| = |-47 - (-59)| = 12\ \text{ft}$

$\therefore$ TOTAL DISTANCE TRAVELED $= (27 + 12)\ \text{ft} = 39\ \text{ft}$ ◀

11.10

GIVEN: $a \propto t$; AT $t = 0$, $v = 16\ \text{in./s}$; AT $t = 1\ \text{s}$,

$v = 15\ \text{in./s}$, $x = 20\ \text{in.}$

FIND: v, x, AND TOTAL DISTANCE TRAVELED AT $t = 7\ \text{s}$

HAVE... $a = kt$ $\quad k \sim$ CONSTANT

NOW $\frac{dv}{dt} = a = kt$

AT $t = 0$, $v = 16\ \frac{\text{in.}}{\text{s}}$: $\int_{16}^{v} dv = \int_0^t kt\, dt$

OR $v - 16 = \frac{1}{2}kt^2$

OR $v = 16 + \frac{1}{2}kt^2$ $\left(\frac{\text{in.}}{\text{s}}\right)$

AT $t = 1\ \text{s}$, $v = 15\ \frac{\text{in.}}{\text{s}}$: $15\ \frac{\text{in.}}{\text{s}} = 16\ \frac{\text{in.}}{\text{s}} + \frac{1}{2}k(1\ \text{s})^2$

OR $k = -2\ \frac{\text{in.}}{\text{s}^3}$ AND $v = 16 - t^2$

ALSO $\frac{dx}{dt} = v = 16 - t^2$

AT $t = 1\ \text{s}$, $x = 20\ \text{in.}$: $\int_{20}^{x} dx = \int_1^t (16 - t^2)\, dt$

OR $x - 20 = \left[16t - \frac{1}{3}t^3\right]_1^t$

OR $x = -\frac{1}{3}t^3 + 16t + \frac{13}{3}$ (in.)

THEN... AT $t = 7\ \text{s}$: $v_7 = 16 - (7)^2$ OR $v_7 = -33\ \frac{\text{in.}}{\text{s}}$ ◀

$x_7 = -\frac{1}{3}(7)^3 + 16(7) + \frac{13}{3}$ OR $x_7 = 2.00\ \text{in.}$ ◀

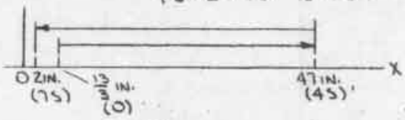
WHEN $v = 0$: $16 - t^2 = 0$ OR $t = 4\ \text{s}$

AT $t = 0$: $x_0 = \frac{13}{3}$

$t = 4\ \text{s}$: $x_4 = -\frac{1}{3}(4)^3 + 16(4) + \frac{13}{3} = 47\ \text{in.}$

NOW OBSERVE THAT... $0 \le t < 4\ \text{s}$: $v > 0$

$4\ \text{s} < t \le 7\ \text{s}$: $v < 0$



THEN $x_4 - x_0 = 47 - \frac{13}{3} = 42.67\ \text{in.}$

$|x_7 - x_4| = |2 - 47| = 45\ \text{in.}$

$\therefore$ TOTAL DISTANCE TRAVELED $= (42.67 + 45)\ \text{in.} = 87.7\ \text{in.}$ ◀

11.11

GIVEN: $a = A - 6t^2$; AT $t = 0$, $x = 8\ \text{m}$, $v = 0$;

AT $t = 1\ \text{s}$, $v = 30\ \frac{\text{m}}{\text{s}}$

FIND: (a) t WHEN $v = 0$

(b) TOTAL DISTANCE TRAVELED WHEN $t = 5\ \text{s}$

HAVE... $a = A - 6t^2$ $\quad A \sim$ CONSTANT

NOW $\frac{dv}{dt} = a = A - 6t^2$

AT $t = 0$, $v = 0$: $\int_0^{v} dv = \int_0^t (A - 6t^2)\, dt$

OR $v = At - 2t^3$ $\left(\frac{\text{m}}{\text{s}}\right)$

AT $t = 1\ \text{s}$, $v = 30\ \frac{\text{m}}{\text{s}}$: $30 = A(1) - 2(1)^3$

OR $A = 32\ \frac{\text{m}}{\text{s}^2}$ AND $v = 32t - 2t^3$

ALSO $\frac{dx}{dt} = v = 32t - 2t^3$

AT $t = 0$, $x = 8\ \text{m}$: $\int_8^{x} dx = \int_0^t (32t - 2t^3)\, dt$

OR $x = 8 + 16t^2 - \frac{1}{2}t^4$ (m)

(a) WHEN $v = 0$: $32t - 2t^3 = 2t(16 - t^2) = 0$

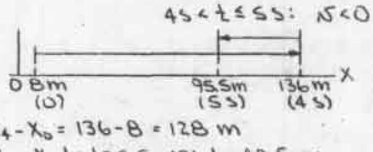
OR $t = 0$ AND $t = 4\ \text{s}$ ◀

(b) AT $t = 4\ \text{s}$: $x_4 = 8 + 16(4)^2 - \frac{1}{2}(4)^4 = 136\ \text{m}$

$t = 5\ \text{s}$: $x_5 = 8 + 16(5)^2 - \frac{1}{2}(5)^4 = 95.5\ \text{m}$

NOW OBSERVE THAT $0 < t < 4\ \text{s}$: $v > 0$

$4\ \text{s} < t \le 5\ \text{s}$: $v < 0$



THEN $x_4 - x_0 = 136 - 8 = 128\ \text{m}$

$|x_5 - x_4| = |95.5 - 136| = 40.5\ \text{m}$

$\therefore$ TOTAL DISTANCE TRAVELED $= (128 + 40.5)\ \text{m} = 168.5\ \text{m}$ ◀

11.12

GIVEN: $a \propto t^2$; AT $t = 0$, $x = 24$ m; AT $t = 6$ s, $x = 96$ m, $v = 18$ m/s

FIND: $x(t)$ AND $v(t)$

HAVE.. $a = kt^2$ $\quad k \sim$ CONSTANT

NOW. $\frac{dv}{dt} = a = kt^2$

AT $t = 6$ s, $v = 18\,\frac{m}{s}$: $\int_{18}^{v} dv = \int_{6}^{t} kt^2 dt$

OR $v - 18 = \frac{1}{3}k(t^3 - 216)$

OR $v = 18 + \frac{1}{3}k(t^3 - 216)$ $\quad (\frac{m}{s})$

ALSO $\frac{dx}{dt} = v = 18 + \frac{1}{3}k(t^3 - 216)$

AT $t = 0$, $x = 24$ m: $\int_{24}^{x} dx = \int_{0}^{t} [18 + \frac{1}{3}k(t^3 - 216)]dt$

OR $x - 24 = 18t + \frac{1}{3}k(\frac{1}{4}t^4 - 216t)$

NOW.. AT $t = 6$ s, $x = 96$ m: $96 - 24 = 18(6) + \frac{1}{3}k[\frac{1}{4}(6)^4 - 216(6)]$

OR $k = \frac{1}{9}\,\frac{m}{s^4}$

THEN.. $x - 24 = 18t + \frac{1}{3}(\frac{1}{9})(\frac{1}{4}t^4 - 216t)$

OR $x(t) = \frac{1}{108}t^4 + 10t + 24$ ◀

AND $v = 18 + \frac{1}{3}(\frac{1}{9})(t^3 - 216)$

OR $v(t) = \frac{1}{27}t^3 + 10$ ◀

11.13

GIVEN: FOR $2\text{ s} \le t \le 10\text{ s}$, $a \propto \frac{1}{t^3}$; AT $t = 2$ s, $v = -15$ m/s; AT $t = 10$ s, $v = 0.36$ m/s; $|x_2| = 2|x_{10}|$

FIND: (a) x AT $t = 2$ s AND AT $t = 10$ s

(b) TOTAL DISTANCE TRAVELED FROM $t = 2$ s TO $t = 10$ s

HAVE.. $a = \frac{k}{t^3}$ $\quad k \sim$ CONSTANT

NOW $\frac{dv}{dt} = a = \frac{k}{t^3}$

AT $t = 2$ s, $v = -15\,\frac{m}{s}$: $\int_{-15}^{v} dv = \int_{2}^{t} \frac{k}{t^3}dt$

OR $v - (-15) = -\frac{k}{2}\left[\frac{1}{t^2} - \frac{1}{(2)^2}\right]$

OR $v = \frac{k}{2}\left(\frac{1}{4} - \frac{1}{t^2}\right) - 15$ $\quad (\frac{m}{s})$

AT $t = 10$ s, $v = 0.36\,\frac{m}{s}$: $0.36 = \frac{k}{2}\left(\frac{1}{4} - \frac{1}{10^2}\right) - 15$

OR $k = 128$ m·s

AND $v = 1 - \frac{64}{t^2}$ $\quad (\frac{m}{s})$

(a) HAVE $\frac{dx}{dt} = v = 1 - \frac{64}{t^2}$

THEN $\int dx = \int (1 - \frac{64}{t^2})dt + C$ $\quad C \sim$ CONSTANT

OR $x = t + \frac{64}{t} + C$ $\quad$ (m)

NOW $x_2 = 2x_{10}$: $2 + \frac{64}{2} + C = 2(10 + \frac{64}{10} + C)$

OR $C = 1.2$ m

AND $x = t + \frac{64}{t} + 1.2$ $\quad$ (m)

∴ AT $t = 2$ s: $x_2 = 2 + \frac{64}{2} + 1.2$ $\quad$ OR $x_2 = 35.2$ m ◀

$t = 10$ s: $x_{10} = 10 + \frac{64}{10} + 1.2$ $\quad$ OR $x_{10} = 17.6$ m ◀

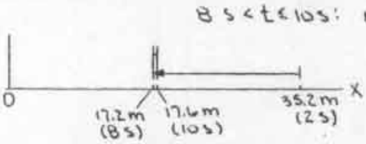
NOTE: A SECOND SOLUTION EXISTS FOR THE CASE $x_2 > 0$, $x_{10} < 0$. FOR THIS CASE $C = -22\frac{4}{15}$ m AND $x_2 = 11\frac{11}{15}$ m, $x_{10} = -5\frac{13}{15}$ m

(b) WHEN $v = 0$: $1 - \frac{64}{t^2} = 0$ $\quad$ OR $t = 8$ s

AT $t = 8$ s: $x_8 = 8 + \frac{64}{8} + 1.2 = 17.2$ m

NOW OBSERVE THAT $2\text{ s} \le t < 8\text{ s}$: $v < 0$

$8\text{ s} < t \le 10\text{ s}$: $v > 0$



THEN $|x_8 - x_2| = |17.2 - 35.2| = 18$ m

$x_{10} - x_8 = 17.6 - 17.2 = 0.4$ m

∴ TOTAL DISTANCE TRAVELED = $(18 + 0.4)$ m = 18.4 m ◀

NOTE: THE TOTAL DISTANCE TRAVELED IS THE SAME FOR BOTH CASES.

11.14

GIVEN: $a = -8$ m/s²; AT $t = 4$ s, $x = 20$ m; WHEN $v = 16$ m/s, $x = 4$ m

FIND: (a) t WHEN $v = 0$

(b) v AND TOTAL DISTANCE TRAVELED AT $t = 11$ s

HAVE $\frac{dv}{dt} = a = -8\,\frac{m}{s^2}$

THEN $\int dv = \int -8\,dt + C$ $\quad C \sim$ CONSTANT

OR $v = -8t + C$ $\quad (\frac{m}{s})$

ALSO $\frac{dx}{dt} = v = -8t + C$

AT $t = 4$ s, $x = 20$ m: $\int_{20}^{x} dx = \int_{4}^{t}(-8t + C)dt$

OR $x - 20 = [-4t^2 + Ct]_4^t$

OR $x = -4t^2 + C(t - 4) + 84$ $\quad$ (m)

WHEN $v = 16\,\frac{m}{s}$, $x = 4$ m: $16 = -8t + C \Rightarrow C = 16 + 8t$

$4 = -4t^2 + C(t - 4) + 84$

COMBINING.. $0 = -4t^2 + (16 + 8t)(t - 4) + 80$

SIMPLIFYING.. $t^2 - 4t + 4 = 0$

OR $t = 2$ s

AND $C = 32\,\frac{m}{s}$ $\quad v = -8t + 32$ $\quad (\frac{m}{s})$

$x = -4t^2 + 32t - 44$ $\quad$ (m)

(a) WHEN $v = 0$: $-8t + 32 = 0$ $\quad$ OR $t = 4$ s ◀

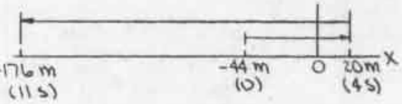
(b) AT $t = 0$: $x_0 = -44$ m

$t = 4$ s: $x_4 = 20$ m

$t = 11$ s: $x_{11} = -4(11)^2 + 32(11) - 44 = -176$ m

NOW OBSERVE THAT $0 \le t < 4\text{ s}$: $v > 0$

$4\text{ s} < t \le 11\text{ s}$: $v < 0$



THEN $x_4 - x_0 = 20 - (-44) = 64$ m

$|x_{11} - x_4| = |-176 - 20| = 196$ m

∴ TOTAL DISTANCE TRAVELED = $(64 + 196)$ m = 260 m ◀

11.15

GIVEN: $a = k(100 - x)$, $k \sim$ CONSTANT; $v = 0$ AT $x = 40$ mm, $x = 160$ mm; WHEN $x = 100$ mm, $v = 18$ mm/s

FIND: (a) k

(b) v WHEN $x = 120$ mm

(a) HAVE $v\frac{dv}{dx} = a = k(100 - x)$

WHEN $x = 40$ mm, $v = 0$: $\int_{0}^{v} v\,dv = \int_{40}^{x} k(100 - x)dx$

OR $\frac{1}{2}v^2 = k[100x - \frac{1}{2}x^2]_{40}^{x}$

OR $\frac{1}{2}v^2 = k(100x - \frac{1}{2}x^2 - 3200)$

WHEN $x = 100$ mm, $v = 18\,\frac{mm}{s}$:

$\frac{1}{2}(18)^2 = k[100(100) - \frac{1}{2}(100)^2 - 3200]$

OR $k = 0.09\,\frac{1}{s^2}$ ◀

(b) WHEN $x = 120$ mm: $\frac{1}{2}v^2 = 0.09[100(120) - \frac{1}{2}(120)^2 - 3200]$

$= 144$

OR $v = \pm 16.97\,\frac{mm}{s}$ ◀

11.16

GIVEN: $a = k/(x+4)^2$, $k \sim$ CONSTANT; WHEN $x=0$, $v=0$; WHEN $x=8$ m, $v=4$ m/s

FIND: (a) k

(b) x WHEN $v=4.5$ m/s

(c) v_{MAX}

(a) HAVE $v\frac{dv}{dx} = a = \frac{k}{(x+4)^2}$

WHEN $x=0$, $v=0$: $\int_0^v v\,dv = \int_0^x \frac{k}{(x+4)^2}dx$

OR $\frac{1}{2}v^2 = -k\left(\frac{1}{x+4} - \frac{1}{4}\right)$

WHEN $x=8$ m, $v=4\frac{m}{s}$: $\frac{1}{2}(4)^2 = -k\left(\frac{1}{8+4} - \frac{1}{4}\right)$

OR $k = 48\frac{m^3}{s^2}$

(b) WHEN $v=4.5\frac{m}{s}$: $\frac{1}{2}(4.5)^2 = -48\left(\frac{1}{x+4} - \frac{1}{4}\right)$

OR $x = 21.6$ m

(c) NOTE THAT WHEN $v=v_{MAX}$, $a=0$. NOW.. $a \to 0$ AS $x \to \infty$ SO THAT

$\frac{1}{2}v_{MAX}^2 = 48\lim_{x\to\infty}\left(\frac{1}{4} - \frac{1}{x+4}\right) = 48\left(\frac{1}{4}\right)$

OR $v_{MAX} = 4.90\frac{m}{s}$

11.17

GIVEN: $a = 6x-14$, $a \sim$ ft/s², $x \sim$ ft; WHEN $x=0$, $v=4$ ft/s

FIND: (a) x_{MAX}

(b) v WHEN TOTAL DISTANCE TRAVELED = 1 ft

HAVE $v\frac{dv}{dx} = a = 6x-14$

WHEN $x=0$, $v=4\frac{ft}{s}$: $\int_4^v v\,dv = \int_0^x (6x-14)dx$

OR $\left[\frac{1}{2}v^2\right]_4^v = \left[3x^2-14x\right]_0^x$

OR $\frac{1}{2}v^2 = 3x^2-14x+8$

(a) FIRST DETERMINE WHERE $v=0$..

$3x^2-14x+8 = (3x-2)(x-4) = 0$

OR $x = \frac{2}{3}$ ft AND $x = 4$ ft

NOW OBSERVE THAT AS THE PARTICLE PASSES THROUGH $x=0$, $v>0$ AND $a<0$ AND THAT AT $x=\frac{2}{3}$ ft, $v=0$ AND $a<0$. THUS, THE PARTICLE WILL NEVER REACH $x=4$ ft AND, THEREFORE,

$x_{MAX} = 0.667$ ft

(b) THE PARTICLE WILL HAVE TRAVELED A TOTAL DISTANCE OF 1 ft WHEN IT PASSES THROUGH $x=\frac{1}{3}$ ft FOR THE SECOND TIME AND IS MOVING TO THE LEFT. THEN--

AT $x=\frac{1}{3}$ ft: $\frac{1}{2}v^2 = 3\left(\frac{1}{3}\right)^2 - 14\left(\frac{1}{3}\right) + 8 = \frac{11}{3}$

OR $v = 2.71\frac{ft}{s} \leftarrow$

11.18

GIVEN: $a = k\left(x - \frac{A}{x}\right)$, k AND A ARE CONSTANTS; AT $t=0$, $x=1$ ft, $v=0$; WHEN $x=16$ ft, $v=29$ ft/s; $v(x=8\text{ ft}) = 2[v(x=2\text{ ft})]$

FIND: A AND k

HAVE $v\frac{dv}{dx} = a = k\left(x - \frac{A}{x}\right)$

WHEN $x=1$ ft, $v=0$: $\int_0^v v\,dv = \int_1^x k\left(x-\frac{A}{x}\right)dx$

OR $\frac{1}{2}v^2 = k\left[\frac{1}{2}x^2 - A\ln x\right]_1^x$

$= k\left(\frac{1}{2}x^2 - A\ln x - \frac{1}{2}\right)$

AT $x=2$ ft: $\frac{1}{2}v_2^2 = k\left[\frac{1}{2}(2)^2 - A\ln 2 - \frac{1}{2}\right] = k\left(\frac{3}{2} - A\ln 2\right)$

$x=8$ ft: $\frac{1}{2}v_8^2 = k\left[\frac{1}{2}(8)^2 - A\ln 8 - \frac{1}{2}\right] = k(31.5 - A\ln 8)$

(CONTINUED)

11.18 CONTINUED

NOW.. $\frac{v_8}{v_2} = 2$: $\frac{\frac{1}{2}v_8^2}{\frac{1}{2}v_2^2} = (2)^2 = \frac{k(31.5 - A\ln 8)}{k\left(\frac{3}{2} - A\ln 2\right)}$

OR $6 - 4A\ln 2 = 31.5 - A\ln 8$

OR $25.5 = A(\ln 8 - 4\ln 2) = A(\ln 8 - \ln 2^4) = A\ln\left(\frac{1}{2}\right)$

OR $A = -36.8$ ft²

WHEN $x=16$ ft, $v=29\frac{ft}{s}$: $\frac{1}{2}(29)^2 = k\left[\frac{1}{2}(16)^2 - \frac{25.5}{\ln\left(\frac{1}{2}\right)}\ln(16) - \frac{1}{2}\right]$

NOTING THAT $\ln(16) = 4\ln 2$ AND $\ln\left(\frac{1}{2}\right) = -\ln(2)$

HAVE.. $841 = k\left[256 - \frac{2\times 25.5}{-\ln(2)}\cdot 4\ln(2) - 1\right]$

OR $k = 1.832\frac{1}{s^2}$

11.19

GIVEN: $a = k(1-e^{-x})$, $k \sim$ CONSTANT; WHEN $x=-2$ m, $v=6\frac{m}{s}$; WHEN $x=0$, $v=0$

FIND: (a) k

(b) v WHEN $x=-1$ m

(a) HAVE $v\frac{dv}{dx} = a = k(1-e^{-x})$

WHEN $x=-2$ m, $v=6\frac{m}{s}$: $\int_6^v v\,dv = \int_{-2}^x k(1-e^{-x})dx$

OR $\frac{1}{2}(v^2-36) = k\left[x+e^{-x}\right]_{-2}^x$

OR $\frac{1}{2}v^2 = k(x+e^{-x}+2-e^2)+18$

WHEN $x=0$, $v=0$: $0 = k(1+2-e^2)+18$

OR $k = 4.1011\frac{m}{s^2}$ $\quad k = 4.10\frac{m}{s^2}$

(b) WHEN $x=-1$ m: $\frac{1}{2}v^2 = 4.1011(-1+e^1+2-e^2)+18$

OR $v = 2.43\frac{m}{s}$

11.20

GIVEN: $a = -\left(0.1 + \sin\frac{x}{b}\right)$, $a \sim$ m/s², $x \sim$ m; $b = 0.8$ m; WHEN $x=0$, $v=1$ m/s

FIND: (a) v WHEN $x=-1$ m

(b) x WHERE $v=v_{MAX}$

(c) v_{MAX}

HAVE $v\frac{dv}{dx} = a = -\left(0.1 + \sin\frac{x}{0.8}\right)$

WHEN $x=0$, $v=1\frac{m}{s}$: $\int_1^v v\,dv = \int_0^x -\left(0.1+\sin\frac{x}{0.8}\right)dx$

OR $\frac{1}{2}(v^2-1) = -\left[0.1x - 0.8\cos\frac{x}{0.8}\right]_0^x$

OR $\frac{1}{2}v^2 = -0.1x + 0.8\cos\frac{x}{0.8} - 0.3$

(a) WHEN $x=-1$ m: $\frac{1}{2}v^2 = -0.1(-1) + 0.8\cos\frac{-1}{0.8} - 0.3$

OR $v = \pm 0.323\frac{m}{s}$

(b) WHEN $v=v_{MAX}$, $a=0$: $-\left(0.1+\sin\frac{x}{0.8}\right) = 0$

OR $x = -0.080\,134$ m $\quad x = -0.0801$ m

(c) WHEN $x = -0.080\,134$ m:

$\frac{1}{2}v_{MAX}^2 = -0.1(-0.080\,134) + 0.8\cos\frac{-0.080\,134}{0.8} - 0.3$

OR $v_{MAX} = 1.004\frac{m}{s}$

11.21

GIVEN: $a = 0.8\sqrt{v^2+49}$, $a \sim \mathrm{m/s^2}$, $v \sim \mathrm{m/s}$

WHEN $x=0$, $v=0$

FIND: (a) x WHEN $v = 24\ \mathrm{m/s}$

(b) v WHEN $x = 40\ \mathrm{m}$

HAVE $v\dfrac{dv}{dx} = a = 0.8\sqrt{v^2+49}$

WHEN $x=0$, $v=0$: $\displaystyle\int_0^v \frac{v\,dv}{\sqrt{v^2+49}} = \int_0^x 0.8\,dx$

OR $\left[\sqrt{v^2+49}\right]_0^v = 0.8x$

OR $\sqrt{v^2+49} - 7 = 0.8x$

(a) WHEN $v = 24\ \frac{\mathrm{m}}{\mathrm{s}}$: $\sqrt{24^2+49} - 7 = 0.8x$

OR $x = 22.5\ \mathrm{m}$ ◀

(b) WHEN $x = 40\ \mathrm{m}$: $\sqrt{v^2+49} - 7 = 0.8(40)$

OR $v = 38.4\ \frac{\mathrm{m}}{\mathrm{s}}$ ◀

11.22

GIVEN: $a = -k\sqrt{v}$, $k \sim$ CONSTANT; AT $t=0$, $x=0$, $v = 81\ \mathrm{m/s}$; WHEN $x = 18\ \mathrm{m}$, $v = 36\ \mathrm{m/s}$

FIND: (a) v WHEN $x = 20\ \mathrm{m}$

(b) t WHEN $v = 0$

(a) HAVE $v\dfrac{dv}{dx} = a = -k\sqrt{v}$

SO THAT $\sqrt{v}\,dv = -k\,dx$

WHEN $x=0$, $v = 81\ \frac{\mathrm{m}}{\mathrm{s}}$: $\displaystyle\int_{81}^v \sqrt{v}\,dv = \int_0^x -k\,dx$

OR $\frac{2}{3}\left[v^{3/2}\right]_{81}^v = -kx$

OR $\frac{2}{3}(v^{3/2} - 729) = -kx$

WHEN $x = 18\ \mathrm{m}$, $v = 36\ \frac{\mathrm{m}}{\mathrm{s}}$: $\frac{2}{3}(36^{3/2} - 729) = -k(18)$

OR $k = 19\sqrt{\frac{\mathrm{m}}{\mathrm{s}^3}}$

FINALLY.. WHEN $x = 20\ \mathrm{m}$: $\frac{2}{3}(v^{3/2} - 729) = -19(20)$

OR $v^{3/2} = 159$ $\qquad v = 29.3\ \frac{\mathrm{m}}{\mathrm{s}}$ ◀

(b) HAVE $\dfrac{dv}{dt} = a = -19\sqrt{v}$

AT $t=0$, $v = 81\ \frac{\mathrm{m}}{\mathrm{s}}$: $\displaystyle\int_{81}^v \frac{dv}{\sqrt{v}} = \int_0^t -19\,dt$

OR $2\left[\sqrt{v}\right]_{81}^v = -19t$

OR $2(\sqrt{v} - 9) = -19t$

WHEN $v=0$: $2(-9) = -19t$

OR $t = 0.947\ \mathrm{s}$ ◀

11.23

GIVEN: $a = -kv^{2.5}$, $k \sim$ CONSTANT; AT $t=0$, $x=0$, $v = 16\ \mathrm{in./s}$; WHEN $x = 6$ IN., $v = 4\ \mathrm{in./s}$

FIND: (a) v WHEN $x = 5$ IN.

(b) t WHEN $v = 9\ \mathrm{in./s}$

(a) HAVE $v\dfrac{dv}{dx} = a = -kv^{2.5}$

SO THAT $v^{1.5}\,dv = -k\,dx$

WHEN $x=0$, $v = 16\ \frac{\mathrm{in.}}{\mathrm{s}}$: $\displaystyle\int_{16}^v v^{-1.5}\,dv = \int_0^x -k\,dx$

OR $-2\left[v^{-\frac{1}{2}}\right]_{16}^v = -kx$

OR $2\left(\frac{1}{\sqrt{v}} - \frac{1}{4}\right) = kx$

WHEN $x = 6$ IN., $v = 4\ \mathrm{in./s}$: $2\left(\frac{1}{\sqrt{4}} - \frac{1}{4}\right) = k(6)$

OR $k = \frac{1}{12}\sqrt{\frac{\mathrm{s}}{\mathrm{in}^3}}$

FINALLY -- WHEN $x = 5$ IN.: $2\left(\frac{1}{\sqrt{v}} - \frac{1}{4}\right) = \frac{1}{12}(5)$

OR $\frac{1}{\sqrt{v}} = \frac{11}{24}$ $\qquad v = 4.76\ \frac{\mathrm{in.}}{\mathrm{s}}$ ◀

(CONTINUED)

11.23 CONTINUED

(b) HAVE $\dfrac{dv}{dt} = a = -\frac{1}{12}v^{2.5}$

AT $t=0$, $v = 16\ \frac{\mathrm{in.}}{\mathrm{s}}$: $\displaystyle\int_{16}^v v^{-2.5}\,dv = \int_0^t -\frac{1}{12}\,dt$

OR $-\frac{2}{3}\left[v^{-3/2}\right]_{16}^v = -\frac{1}{12}t$

OR $\frac{2}{3}\left(\frac{1}{v^{3/2}} - \frac{1}{64}\right) = \frac{t}{12}$

WHEN $v = 9\ \frac{\mathrm{in.}}{\mathrm{s}}$: $\frac{2}{3}\left(\frac{1}{9^{3/2}} - \frac{1}{64}\right) = \frac{t}{12}$

OR $t = 0.1713\ \mathrm{s}$ ◀

11.24

GIVEN: $a = -5/(2v_0 - v)$ $\quad a \sim \mathrm{ft/s^2}$, $v \sim \mathrm{ft/s}$; AT $t=0$, $x=0$, $v = v_0$; AT $t = 2\ \mathrm{s}$, $v = 0.5v_0$

FIND: (a) v_0

(b) t WHEN $v=0$

(c) x WHEN $v = 1\ \mathrm{ft/s}$

(a) HAVE $\dfrac{dv}{dt} = a = -\dfrac{5}{2v_0 - v}$

AT $t=0$, $v = v_0$: $\displaystyle\int_{v_0}^v (2v_0 - v)\,dv = \int_0^t -5\,dt$

OR $-\frac{1}{2}\left[(2v_0 - v)^2\right]_{v_0}^v = -5t$

OR $(2v_0 - v)^2 - v_0^2 = 10t$

AT $t = 2\ \mathrm{s}$, $v = 0.5v_0$: $(2v_0 - 0.5v_0)^2 - v_0^2 = 10(2)$

OR $\frac{5}{4}v_0^2 = 20$ $\qquad v_0 = 4\ \frac{\mathrm{ft}}{\mathrm{s}}$ ◀

(b) HAVE $(8 - v)^2 - 16 = 10t$

WHEN $v=0$: $(8)^2 - 16 = 10t$

OR $t = 4.8\ \mathrm{s}$ ◀

(c) HAVE $v\dfrac{dv}{dx} = a = -\dfrac{5}{2v_0 - v} = -\dfrac{5}{8 - v}$

WHEN $x=0$, $v = v_0 = 4\ \frac{\mathrm{ft}}{\mathrm{s}}$: $\displaystyle\int_4^v v(8 - v)\,dv = \int_0^x -5\,dx$

OR $\left[4v^2 - \frac{1}{3}v^3\right]_4^v = -5x$

OR $\left(4v^2 - \frac{1}{3}v^3\right) - \left[4(4)^2 - \frac{1}{3}(4)^3\right] = -5x$

OR $\left(4v^2 - \frac{1}{3}v^3\right) - \frac{128}{3} = -5x$

WHEN $v = 1\ \frac{\mathrm{ft}}{\mathrm{s}}$: $\left[4(1)^2 - \frac{1}{3}(1)^3\right] - \frac{128}{3} = -5x$

OR $x = 7.80\ \mathrm{ft}$ ◀

11.25

GIVEN: $a = 0.4(1 - kv)$, $k \sim$ CONSTANT; AT $t=0$, $x = 4\ \mathrm{m}$, $v=0$; AT $t = 15\ \mathrm{s}$, $v = 4\ \mathrm{m/s}$

FIND: (a) k

(b) x WHEN $v = 6\ \mathrm{m/s}$

(c) v_{MAX}

(a) HAVE $\dfrac{dv}{dt} = a = 0.4(1 - kv)$

AT $t=0$, $v=0$: $\displaystyle\int_0^v \frac{dv}{1 - kv} = \int_0^t 0.4\,dt$

OR $-\frac{1}{k}\left[\ln(1 - kv)\right]_0^v = 0.4t$

OR $\ln(1 - kv) = -0.4kt \qquad (1)$

AT $t = 15\ \mathrm{s}$, $v = 4\ \frac{\mathrm{m}}{\mathrm{s}}$: $\ln(1 - 4k) = -0.4k(15) = -6k$

SOLVING YIELDS $k = 0.145703\ \frac{\mathrm{s}}{\mathrm{m}}$

OR $k = 0.1457\ \frac{\mathrm{s}}{\mathrm{m}}$ ◀

(b) HAVE $v\dfrac{dv}{dx} = a = 0.4(1 - kv)$

WHEN $x = 4\ \mathrm{m}$, $v=0$: $\displaystyle\int_0^v \frac{v\,dv}{1 - kv} = \int_4^x 0.4\,dx$

NOW -- $\dfrac{v}{1 - kv} = -\dfrac{1}{k} + \dfrac{1/k}{1 - kv}$

THEN $\displaystyle\int_0^v \left[-\frac{1}{k} + \frac{1}{k(1 - kv)}\right]dv = \int_4^x 0.4\,dx$

OR $\left[-\frac{v}{k} - \frac{1}{k^2}\ln(1 - kv)\right]_0^v = 0.4\left[x\right]_4^x$

(CONTINUED)

11.25 CONTINUED

OR $-\left[\frac{v}{k} + \frac{1}{k^2}\ln(1-kv)\right] = 0.4(x-4)$

WHEN $v = 6\ \frac{m}{s}$:

$-\left[\frac{6}{0.145\,703} + \frac{1}{(0.145\,703)^2}\ln(1-0.145\,703\times 6)\right] = 0.4(x-4)$

OR $0.4(x-4) = 56.4778$

OR $x = 145.2$ m ◀

(c) THE MAXIMUM VELOCITY OCCURS WHEN $a=0$.

$\therefore\ a=0$: $0.4(1-kv_{MAX}) = 0$

OR $v_{MAX} = \frac{1}{0.145703}$

OR $v_{MAX} = 6.86\ \frac{m}{s}$ ◀

AN ALTERNATIVE SOLUTION IS TO BEGIN WITH EQ.(1).

$\ln(1-kv) = -0.4kt$

THEN $v = \frac{1}{k}(1-e^{-0.4kt})$

THUS, v_{MAX} IS ATTAINED AS $t \to \infty$ ...

$v_{MAX} = \frac{1}{k}$ ... AS ABOVE

11.26

GIVEN: $a = -0.6v^{3/2}$ $a \sim \text{m/s}^2$, $v \sim \text{m/s}$;
AT $t=0$, $x=0$, $v = 9$ m/s

FIND: (a) x WHEN $v = 4$ m/s
(b) t WHEN $v = 1$ m/s
(c) t WHEN $x = 6$ m

(a) HAVE $v\frac{dv}{dx} = a = -0.6v^{3/2}$

WHEN $x=0$, $v = 9\ \frac{m}{s}$: $\int_9^v v^{-\frac{1}{2}}dv = \int_0^x -0.6\,dx$

OR $2\left[v^{\frac{1}{2}}\right]_9^v = -0.6x$

OR $x = \frac{1}{0.3}(3 - v^{1/2})$ (1)

WHEN $v = 4\ \frac{m}{s}$: $x = \frac{1}{0.3}(3 - 4^{1/2})$

OR $x = 3.33$ m ◀

(b) HAVE $\frac{dv}{dt} = a = -0.6v^{3/2}$

WHEN $t=0$, $v = 9\ \frac{m}{s}$: $\int_9^v v^{-3/2}dv = \int_0^t -0.6\,dt$

OR $-2\left[v^{-1/2}\right]_9^v = -0.6t$

OR $\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$

WHEN $v = 1\ \frac{m}{s}$: $\frac{1}{\sqrt{1}} - \frac{1}{3} = 0.3t$

OR $t = 2.22$ s ◀

(c) HAVE $\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$

OR $v = \left(\frac{3}{1+0.9t}\right)^2 = \frac{9}{(1+0.9t)^2}$

NOW.. $\frac{dx}{dt} = v = \frac{9}{(1+0.9t)^2}$

AT $t=0$, $x=0$: $\int_0^x dx = \int_0^t \frac{9}{(1+0.9t)^2}dt$

OR $x = 9\left[-\frac{1}{0.9}\frac{1}{1+0.9t}\right]_0^t$

$= 10\left(1 - \frac{1}{1+0.9t}\right)$

$= \frac{9t}{1+0.9t}$

WHEN $x = 6$ m: $6 = \frac{9t}{1+0.9t}$

OR $t = 1.667$ s ◀

AN ALTERNATIVE SOLUTION IS TO BEGIN WITH EQ. (1).

$x = \frac{1}{0.3}(3 - v^{1/2})$

THEN $\frac{dx}{dt} = v = (3 - 0.3x)^2$

NOW.. AT $t=0$, $x=0$: $\int_0^x \frac{dx}{(3-0.3x)^2} = \int_0^t dt$

OR $t = \frac{1}{0.3}\left[\frac{1}{3-0.3x}\right]_0^x = \frac{x}{9-0.9x}$

WHICH LEADS TO THE SAME EQUATION AS ABOVE.

11.27

GIVEN: $v = 7.5(1-0.04x)^{0.3}$ $v \sim$ mi/h,
$x \sim$ mi; AT $t=0$, $x=0$

FIND: (a) x AT $t = 1$ h
(b) a (ft/s²) AT $t=0$
(c) t WHEN $x = 6$ mi

(a) HAVE $\frac{dx}{dt} = v = 7.5(1-0.04x)^{0.3}$

AT $t=0$, $x=0$: $\int_0^x \frac{dx}{(1-0.04x)^{0.3}} = \int_0^t 7.5\,dt$

OR $\frac{1}{0.7}\left(-\frac{1}{0.04}\right)\left[(1-0.04x)^{0.7}\right]_0^x = 7.5t$

OR $1 - (1-0.04x)^{0.7} = 0.21t$ (1)

OR $x = \frac{1}{0.04}\left[1 - (1-0.21t)^{1/0.7}\right]$

AT $t = 1$ h: $x = \frac{1}{0.04}\left\{1 - [1 - 0.21(1)]^{1/0.7}\right\}$

OR $x = 7.15$ mi ◀

(b) HAVE $a = v\frac{dv}{dx}$

$= 7.5(1-0.04x)^{0.3}\frac{d}{dx}\left[7.5(1-0.04x)^{0.3}\right]$

$= 7.5^2(1-0.04x)^{0.3}\left[(0.3)(-0.04)(1-0.04x)^{-0.7}\right]$

$= -0.675(1-0.04x)^{-0.4}$

AT $t=0$, $x=0$: $a_0 = -0.675\ \frac{mi}{h^2} \times \frac{5280\ ft}{1\ mi} \times \left(\frac{1\ h}{3600\ s}\right)^2$

OR $a_0 = -275 \times 10^{-6}\ \frac{ft}{s^2}$ ◀

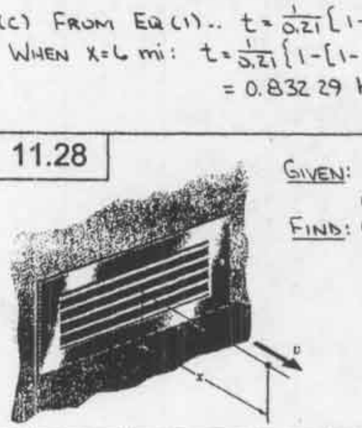
(c) FROM EQ.(1).. $t = \frac{1}{0.21}\left[1 - (1-0.04x)^{0.7}\right]$

WHEN $x = 6$ mi: $t = \frac{1}{0.21}\left\{1 - [1 - 0.04(6)]^{0.7}\right\}$

$= 0.832\,29$ h

OR $t = 49.9$ MIN ◀

11.28



GIVEN: $v = \frac{0.18v_0}{x}$ $v \sim \frac{m}{s}$, $x \sim$ m;
$v_0 = 3.6$ m/s

FIND: (a) a WHEN $x = 2$ m
(b) TIME FOR AIR TO FLOW FROM $x = 1$ m TO $x = 3$ m

(a) HAVE $a = v\frac{dv}{dx}$

$= \frac{0.18v_0}{x}\frac{d}{dx}\left(\frac{0.18v_0}{x}\right)$

$= -\frac{0.0324\,v_0^2}{x^3}$

WHEN $x = 2$ m: $a = -\frac{0.0324(3.6)^2}{(2)^3}$

OR $a = -0.0525\ \frac{m}{s^2}$ ◀

(b) HAVE $\frac{dx}{dt} = v = \frac{0.18v_0}{x}$

FROM $x = 1$ m TO $x = 3$ m: $\int_1^3 x\,dx = \int_{t_1}^{t_3} 0.18v_0\,dt$

OR $\left[\frac{1}{2}x^2\right]_1^3 = 0.18v_0(t_3 - t_1)$

OR $(t_3 - t_1) = \frac{\frac{1}{2}(9-1)}{0.18(3.6)}$

OR $t_3 - t_1 = 6.17$ s ◀

11.29

GIVEN: $a = -32.2/[1+(y/20.9\times10^6)]^2$

$a \sim \mathrm{ft/s^2}$, $y \sim \mathrm{ft}$

FIND: y_{MAX} WHEN

(a) $v_0 = 1800\ \mathrm{ft/s}$

(b) $v_0 = 3000\ \mathrm{ft/s}$

(c) $v_0 = 36{,}700\ \mathrm{ft/s}$

HAVE $v\dfrac{dv}{dy} = a = -\dfrac{32.2}{\left(1+\dfrac{y}{20.9\times10^6}\right)^2}$

WHEN $y=0$, $v=v_0$

$y = y_{MAX}$, $v=0$

THEN.. $$\int_{v_0}^{0} v\,dv = \int_0^{y_{MAX}} \frac{-32.2}{\left(1+\frac{y}{20.9\times10^6}\right)^2}\,dy$$

OR $$-\frac{1}{2}v_0^2 = -32.2\left[-20.9\times10^6\,\frac{1}{1+\frac{y}{20.9\times10^6}}\right]_0^{y_{MAX}}$$

OR $$v_0^2 = 1345.96\times10^6\left(1-\frac{1}{1+\frac{y_{MAX}}{20.9\times10^6}}\right)$$

OR $$y_{MAX} = \frac{v_0^2}{64.4-\frac{v_0^2}{20.9\times10^6}}$$

(a) $v_0 = 1800\ \frac{\mathrm{ft}}{\mathrm{s}}$:

$$y_{MAX} = \frac{(1800)^2}{64.4-\frac{(1800)^2}{20.9\times10^6}}$$

OR $y_{MAX} = 50.4\times10^3\ \mathrm{ft}$ ◀

(b) $v_0 = 3000\ \frac{\mathrm{ft}}{\mathrm{s}}$:

$$y_{MAX} = \frac{(3000)^2}{64.4-\frac{(3000)^2}{20.9\times10^6}}$$

OR $y_{MAX} = 140.7\times10^3\ \mathrm{ft}$ ◀

(c) $v_0 = 36{,}700\ \frac{\mathrm{ft}}{\mathrm{s}}$:

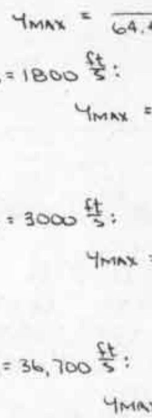
$$y_{MAX} = \frac{(36{,}700)^2}{64.4-\frac{(36{,}700)^2}{20.9\times10^6}}$$

OR $y_{MAX} = -3.03\times10^{10}\ \mathrm{ft}$

THE VELOCITY $36{,}700\ \frac{\mathrm{ft}}{\mathrm{s}}$ IS APPROXIMATELY THE ESCAPE VELOCITY v_e FROM THE EARTH. FOR v_e

$y_{MAX} \to \infty$ ◀

11.30



GIVEN: $a = -\dfrac{gR^2}{r^2}$, $R = 3960$ mi;

WHEN $r=\infty$, $v=0$

FIND: v_e

HAVE $v\dfrac{dv}{dr} = a = -\dfrac{gR^2}{r^2}$

WHEN $r=R$, $v=v_e$

$r=\infty$, $v=0$

THEN $$\int_{v_e}^{0} v\,dv = \int_R^{\infty} -\frac{gR^2}{r^2}\,dr$$ (CONTINUED)

11.30 CONTINUED

OR $-\frac{1}{2}v_e^2 = gR^2\left[\frac{1}{r}\right]_R^{\infty}$

OR $v_e = \sqrt{2gR}$

$= \left(2\times32.2\ \frac{\mathrm{ft}}{\mathrm{s}^2}\times3960\ \mathrm{mi}\times\frac{5280\ \mathrm{ft}}{1\ \mathrm{mi}}\right)^{1/2}$

OR $v_e = 36.7\times10^3\ \frac{\mathrm{ft}}{\mathrm{s}}$ ◀

11.31

GIVEN: $v = v_0[1-\sin(\frac{\pi t}{T})]$; AT $t=0$, $x=0$, $v=v_0$

FIND: (a) x AND a AT $t=3T$

(b) v_{AVE} DURING $t=0$ TO $t=T$

(a) HAVE $\dfrac{dx}{dt} = v = v_0[1-\sin(\frac{\pi t}{T})]$

AT $t=0$, $x=0$: $\int_0^x dx = \int_0^t v_0[1-\sin(\frac{\pi t}{T})]dt$

OR $x = v_0\left[t+\frac{T}{\pi}\cos(\frac{\pi t}{T})\right]_0^t$

$= v_0\left[t+\frac{T}{\pi}\cos(\frac{\pi t}{T})-\frac{T}{\pi}\right]$ (1)

AT $t=3T$: $x_{3T} = v_0\left[3T+\frac{T}{\pi}\cos(\frac{\pi\times3T}{T})-\frac{T}{\pi}\right]$

$= v_0(3T-\frac{2T}{\pi})$

OR $x_{3T} = 2.36\,v_0T$ ◀

ALSO.. $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\{v_0[1-\sin(\frac{\pi t}{T})]\}$

$= -v_0\frac{\pi}{T}\cos\frac{\pi t}{T}$

AT $t=3T$: $a_{3T} = -v_0\frac{\pi}{T}\cos\frac{\pi\times3T}{T}$

OR $a_{3T} = \dfrac{\pi v_0}{T}$ ◀

(b) USING EQ. (1)..

AT $t=0$: $x_0 = v_0[0+\frac{T}{\pi}\cos(0)-\frac{T}{\pi}] = 0$

AT $t=T$: $x_T = v_0[T+\frac{T}{\pi}\cos(\frac{\pi T}{T})-\frac{T}{\pi}]$

$= v_0(T-\frac{2T}{\pi})$

$= 0.363\,v_0T$

NOW.. $v_{AVE} = \dfrac{x_T-x_0}{\Delta t}$

$= \dfrac{0.363\,v_0T-0}{T-0}$

OR $v_{AVE} = 0.363\,v_0$ ◀

11.32

GIVEN: $v = v'\sin(\omega_N t+\phi)$; AT $t=0$, $x=x_0$, $v=v_0$; $x_{MAX} = 2x_0$

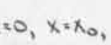
SHOW: (a) $v' = (v_0^2+x_0^2\omega_N^2)/2x_0\omega_N$

(b) v_{MAX} OCCURS WHEN

$x = x_0[3-(v_0/x_0\omega_N)^2]/2$

(a) AT $t=0$, $v=v_0$: $v_0 = v'\sin(0+\phi) = v'\sin\phi$

THEN $\cos\phi = \sqrt{v'^2-v_0^2}/v'$



NOW $\dfrac{dx}{dt} = v = v'\sin(\omega_N t+\phi)$

AT $t=0$, $x=x_0$: $\int_{x_0}^{x} dx = \int_0^t v'\sin(\omega_N t+\phi)dt$

OR $x-x_0 = v'\left[-\frac{1}{\omega_N}\cos(\omega_N t+\phi)\right]_0^t$

OR $x = x_0+\frac{v'}{\omega_N}[\cos\phi-\cos(\omega_N t+\phi)]$

NOW OBSERVE THAT x_{MAX} OCCURS WHEN $\cos(\omega_N t+\phi) = -1$. THEN--

$x_{MAX} = 2x_0 = x_0+\frac{v'}{\omega_N}[\cos\phi-(-1)]$

SUBSTITUTING FOR $\cos\phi$-- $x_0 = \frac{v'}{\omega_N}\left(\frac{\sqrt{v'^2-v_0^2}}{v'}+1\right)$

OR $x_0\omega_N - v' = \sqrt{v'^2-v_0^2}$

SQUARING BOTH SIDES OF THIS EQUATION--

$x_0^2\omega_N^2 - 2x_0\omega_N v' + v'^2 = v'^2 - v_0^2$

OR $v' = \dfrac{v_0^2+x_0^2\omega_N^2}{2x_0\omega_N}$ Q.E.D.

(CONTINUED)

11.32 CONTINUED

(b) FIRST OBSERVE THAT v_{MAX} OCCURS WHEN $\omega_N t + \phi = \frac{\pi}{2}$. THE CORRESPONDING VALUE OF x IS

$$x_{v_{MAX}} = x_0 + \frac{v'}{\omega_N}\left[\cos\phi - \cos\left(\frac{\pi}{2}\right)\right]$$
$$= x_0 + \frac{v'}{\omega_N}\cos\phi$$

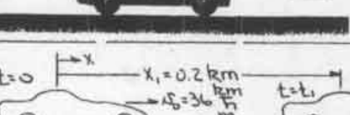
SUBSTITUTING FIRST FOR $\cos\phi$ AND THEN FOR v'..

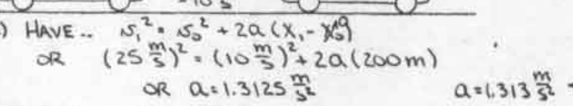
$$x_{v_{MAX}} = x_0 + \frac{v'}{\omega_N}\,\frac{\sqrt{v'^2 - v_0^2}}{v'}$$
$$= x_0 + \frac{1}{\omega_N}\left[\left(\frac{v_0^2 + x_0^2\omega_N^2}{2x_0\omega_N}\right)^2 - v_0^2\right]^{1/2}$$
$$= x_0 + \frac{1}{2x_0\omega_N^2}\left(v_0^4 + 2v_0^2x_0^2\omega_N^2 + x_0^4\omega_N^4 - 4x_0^2\omega_N^2 v_0^2\right)^{1/2}$$
$$= x_0 + \frac{1}{2x_0\omega_N^2}\left[\left(x_0^2\omega_N^2 - v_0^2\right)^2\right]^{1/2}$$
$$= x_0 + \frac{x_0^2\omega_N^2 - v_0^2}{2x_0\omega_N^2}$$
$$= \frac{x_0}{2}\left[3 - \left(\frac{v_0}{x_0\omega_N}\right)^2\right] \quad \text{Q.E.D.}$$

11.33

GIVEN: $v_0 = 36\ \mathrm{km/h}$, $v_1 = 90\ \mathrm{km/h}$; UNIFORM ACCELERATION; $\Delta x = 0.2\ \mathrm{km}$

FIND: (a) a
(b) t_1





(a) HAVE.. $v_1^2 = v_0^2 + 2a(x_1 - \cancel{x_0}^{0})$

OR $(25\ \tfrac{m}{s})^2 = (10\ \tfrac{m}{s})^2 + 2a(200\ m)$

OR $a = 1.3125\ \tfrac{m}{s^2}$ $\quad a = 1.313\ \tfrac{m}{s^2}$ ◀

(b) HAVE $v_1 = v_0 + at_1$

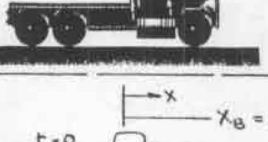
OR $25\ \tfrac{m}{s} = 10\ \tfrac{m}{s} + (1.3125\ \tfrac{m}{s^2})t_1$

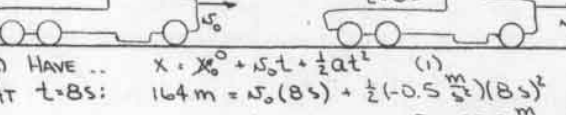
OR $t_1 = 11.43\ s$ ◀

11.34

GIVEN: $a = -0.5\ \mathrm{m/s^2}$ – CONSTANT; $\Delta x = 164\ m$, $\Delta t = 8\ s$

FIND: (a) v_0
(b) v AT 8 s
(c) Δx AT 0.6 s





(a) HAVE.. $x = \cancel{x_0}^{0} + v_0 t + \frac{1}{2}at^2 \quad (1)$

AT $t = 8\ s$: $164\ m = v_0(8\ s) + \frac{1}{2}(-0.5\ \tfrac{m}{s^2})(8\ s)^2$

OR $v_0 = 22.5\ \tfrac{m}{s}$ ◀

(b) HAVE.. $v = v_0 + at$

AT $t = 8\ s$: $v_B = 22.5\ \tfrac{m}{s} + (-0.5\ \tfrac{m}{s^2})(8\ s)$

OR $v_B = 18.5\ \tfrac{m}{s}$ ◀

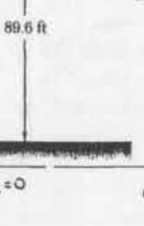
(c) USING EQ. (1).. $x_{0.6} = (22.5\ \tfrac{m}{s})(0.6\ s) + \frac{1}{2}(-0.5\ \tfrac{m}{s^2})(0.6\ s)^2$

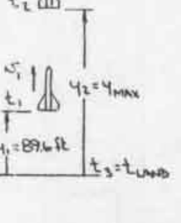
OR $x_{0.6} = 13.41\ m$ ◀

11.35

GIVEN: $a = -32.2\ \mathrm{ft/s^2}$; $t_{LAND} - t_1 = 16\ s$

FIND: (a) v_1
(b) y_{MAX}





(a) HAVE.. $y = y_1 + v_1 t + \frac{1}{2}at^2$

AT t_{LAND}, $y = 0$

THEN.. $0 = 89.6\ ft + v_1(16\ s) + \frac{1}{2}(-32.2\ \tfrac{ft}{s^2})(16\ s)^2$

OR $v_1 = 252\ \tfrac{ft}{s}$ ◀

(b) HAVE.. $v^2 = v_1^2 + 2a(y - y_1)$

AT $y = y_{MAX}$, $v = 0$

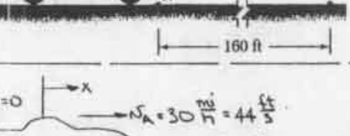
THEN.. $0 = (252\ \tfrac{ft}{s})^2 + 2(-32.2\ \tfrac{ft}{s^2})(y_{MAX} - 89.6)\ ft$

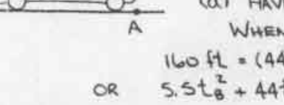
OR $y_{MAX} = 1076\ ft$ ◀

11.36

GIVEN: $a = 11\ \tfrac{ft}{s^2}$ ~ CONSTANT; $v_A = 30\ \mathrm{mi/h}$

FIND: (a) t_B
(b) v_B





(a) HAVE.. $x = \cancel{x_A}^{0} + v_A t + \frac{1}{2}at^2$

WHEN $x = x_B$..

$160\ ft = (44\ \tfrac{ft}{s})t_B + \frac{1}{2}(11\ \tfrac{ft}{s^2})t_B^2$

OR $5.5t_B^2 + 44t_B - 160 = 0 \quad (t_B \sim s)$

SOLVING FOR THE POSITIVE ROOT.. $t_B = 2.7150\ s$

OR $t_B = 2.72\ s$ ◀

(b) HAVE.. $v = v_A + at$

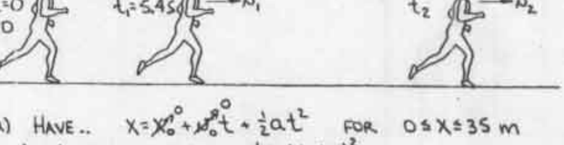
AT $t = t_B$.. $v_B = 44\ \tfrac{ft}{s} + (11\ \tfrac{ft}{s^2})(2.7150\ s)$
$= 73.865\ \tfrac{ft}{s}$

OR $v_B = 50.4\ \tfrac{mi}{h}$ ◀

11.37

GIVEN: $0 \le x \le 35\ m$, a = CONSTANT;
$35\ m < x \le 100\ m$, v = CONSTANT;
AT $t = 0$, $v = 0$; WHEN $x = 35\ m$, $t = 5.4\ s$

FIND: (a) a
(b) v WHEN $x = 100\ m$
(c) t WHEN $x = 100\ m$



(a) HAVE.. $x = \cancel{x_0}^{0} + \cancel{v_0}^{0}t + \frac{1}{2}at^2$ FOR $0 \le x \le 35\ m$

AT $t = 5.4\ s$: $35\ m = \frac{1}{2}a(5.4\ s)^2$

OR $a = 2.4005\ \tfrac{m}{s^2}$ $\quad a = 2.40\ \tfrac{m}{s^2}$ ◀

(b) FIRST NOTE THAT $v = v_{MAX}$ FOR $35\ m \le x \le 100\ m$

NOW.. $v^2 = \cancel{v_0^2}^{0} + 2a(x - \cancel{x_0}^{0})$ FOR $0 \le x \le 35\ m$

WHEN $x = 35\ m$: $v_{MAX}^2 = 2(2.4005\ \tfrac{m}{s^2})(35\ m)$

(CONTINUED)

11.37 CONTINUED

OR $v_{MAX} = 12.9628\ \frac{m}{s}$ $\qquad v_{MAX} = 12.96\ \frac{m}{s}$ ◀

(c) HAVE.. $x = x_1 + v_0(t - t_1)$ FOR $35\text{ m} < x \le 100\text{ m}$

WHEN $x = 100\text{ m}$: $100\text{ m} = 35\text{ m} + (12.9628\ \frac{m}{s})(t_2 - 5.4)\text{ s}$

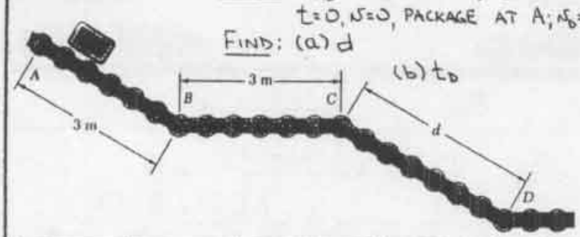
OR $t_2 = 10.41\text{ s}$ ◀

11.38

GIVEN: $a_{AB} = a_{CD} = 4.8\ \frac{m}{s^2}$; v_{BC} = CONSTANT; $t = 0$, $v = 0$, PACKAGE AT A; $v_D = 7.2\ \frac{m}{s}$

FIND: (a) d

(b) t_D



(a) FOR A → B AND C → D HAVE.. $v^2 = v_0^2 + 2a(x - x_0)$

THEN.. AT B.. $v_{BC}^2 = \cancel{v_A^2}^{\,0} + 2(4.8\ \frac{m}{s^2})(3 - 0)\text{m}$

$= 28.8\ \frac{m^2}{s^2}$ $\qquad (v_{BC} = 5.3666\ \frac{m}{s})$

AND AT D.. $v_D^2 = v_{BC}^2 + 2a_{CD}(x_D - x_C)$ $\qquad d = x_D - x_C$

OR $(7.2\ \frac{m}{s})^2 = (28.8\ \frac{m^2}{s^2}) + 2(4.8\ \frac{m}{s^2})d$

OR $d = 2.40\text{ m}$ ◀

(b) FOR A → B AND C → D HAVE.. $v = v_0 + at$

THEN.. A → B.. $5.3666\ \frac{m}{s} = 0 + (4.8\ \frac{m}{s^2})t_{AB}$

OR $t_{AB} = 1.11804\text{ s}$

AND C → D.. $7.2\ \frac{m}{s} = 5.3666\ \frac{m}{s} + (4.8\ \frac{m}{s^2})t_{CD}$

OR $t_{CD} = 0.38196\text{ s}$

NOW.. FOR B → C HAVE $x_C = x_B + v_{BC}t_{BC}$

OR $3\text{ m} = (5.3666\ \frac{m}{s})t_{BC}$

OR $t_{BC} = 0.55901\text{ s}$

FINALLY, $t_D = t_{AB} + t_{BC} + t_{CD} = (1.11804 + 0.55901 + 0.38196)\text{s}$

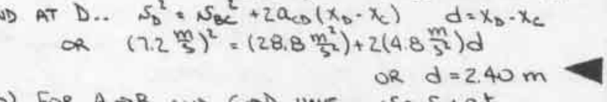
OR $t_D = 2.06\text{ s}$ ◀

11.39

GIVEN: AT $t = 0$, $x_M = x_P = 0$; AT $t = 42\text{ s}$, $x_M = x_P$; v_M = CONSTANT; FOR $0 \le t \le 18\text{ s}$, $v_P = 0$; FOR $18\text{ s} < t \le 26\text{ s}$, a_P = CONSTANT; AT $t = 26\text{ s}$, $v_P = 90\ \frac{km}{h}$; FOR $26\text{ s} < t \le 42\text{ s}$, $v_P = 90\ \frac{km}{h}$

FIND: (a) x_P AT $t = 42\text{ s}$

(b) v_M



$t = 18\text{ s}$ $\qquad t = 26\text{ s}$ $\qquad t = 42\text{ s}$

$(v_P)_{18} = 0$ $\qquad (v_P)_{26} = 90\ \frac{km}{h} = 25\ \frac{m}{s}$ $\qquad (v_P)_{42} = 90\ \frac{km}{h} = 25\ \frac{m}{s}$

(a) PATROL CAR: FOR $18\text{ s} < t \le 26\text{ s}$: $v_P = \cancel{(v_P)_{18}}^{\,0} + a_P(t - 18)$

AT $t = 26\text{ s}$: $25\ \frac{m}{s} = a_P(26 - 18)\text{s}$

OR $a_P = 3.125\ \frac{m}{s^2}$

ALSO, $x_P = \cancel{(x_P)_{18}}^{\,0} + \cancel{(v_P)_{18}}^{\,0}(t - 18) + \frac{1}{2}a_P(t - 18)^2$

AT $t = 26\text{ s}$: $(x_P)_{26} = \frac{1}{2}(3.125\ \frac{m}{s^2})(26 - 18)^2 = 100\text{ m}$

FOR $26\text{ s} < t \le 42\text{ s}$: $x_P = (x_P)_{26} + (v_P)_{26}(t - 26)$

AT $t = 42\text{ s}$: $(x_P)_{42} = 100\text{ m} + (25\ \frac{m}{s})(42 - 26)\text{s}$

$= 500\text{ m}$ $\qquad (x_P)_{42} = 0.5\text{ km}$ ◀

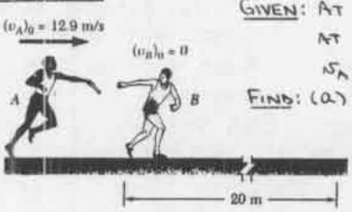
(b) FOR THE MOTORIST'S CAR.. $x_M = \cancel{(x_M)_0}^{\,0} + v_M t$

AT $t = 42\text{ s}$, $x_M = x_P$: $500\text{ m} = v_M(42\text{ s})$

OR $v_M = 11.9048\ \frac{m}{s}$

OR $v_M = 42.9\ \frac{km}{h}$ ◀

11.40



GIVEN: AT $t = 0$, $x_A = x_B = 0$; AT $t = 1.82\text{ s}$, $x_A = x_B = 20\text{ m}$, $v_A = v_B$

FIND: (a) a_A AND a_B KNOWING THAT BOTH ARE UNIFORM

(b) t_B WHEN RUNNER B STARTS TO RUN

(a) FOR RUNNER A: $x_A = \cancel{(x_A)_0}^{\,0} + (v_A)_0 t + \frac{1}{2}a_A t^2$

AT $t = 1.82\text{ s}$: $20\text{ m} = (12.9\ \frac{m}{s})(1.82\text{ s}) + \frac{1}{2}a_A(1.82\text{ s})^2$

OR $a_A = -2.10\ \frac{m}{s^2}$ ◀

ALSO.. $v_A = (v_A)_0 + a_A t$

AT $t = 1.82\text{ s}$: $(v_A)_{1.82} = (12.9\ \frac{m}{s}) + (-2.10\ \frac{m}{s^2})(1.82\text{ s})$

$= 9.078\ \frac{m}{s}$

FOR RUNNER B: $v_B^2 = \cancel{(v_B)_0^2}^{\,0} + 2a_B[x_B - \cancel{(x_B)_0}^{\,0}]$

WHEN $x_B = 20\text{ m}$, $v_B = v_A$: $(9.078\ \frac{m}{s})^2 = 2a_B(20\text{ m})$

OR $a_B = 2.0603\ \frac{m}{s^2}$

$a_B = 2.06\ \frac{m}{s^2}$ ◀

(b) FOR RUNNER B: $v_B = \cancel{(v_B)_0}^{\,0} + a_B(t - t_B)$

WHERE t_B IS THE TIME AT WHICH HE BEGINS TO RUN.

AT $t = 1.82\text{ s}$: $9.078\ \frac{m}{s} = (2.0603\ \frac{m}{s^2})(1.82 - t_B)\text{s}$

OR $t_B = -2.59\text{ s}$

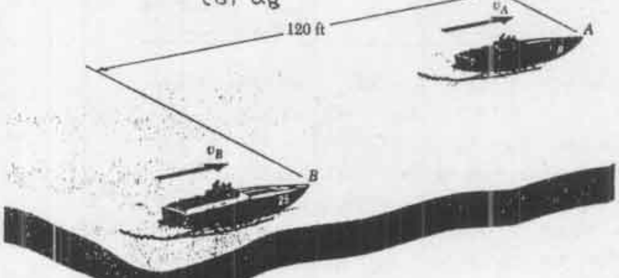
∴ RUNNER B SHOULD START TO RUN 2.59 s BEFORE A REACHES THE EXCHANGE ZONE. ◀

11.41

GIVEN: AT $t = 0$, $x_{A/B} = 120\text{ ft}$, $v_A = v_B = 105\ \frac{mi}{h}$, a_A, a_B = CONSTANTS; AT $t = 8\text{ s}$, $x_A = x_B$, $v_A = 135\ \frac{mi}{h}$

FIND: (a) a_A

(b) a_B



(a) HAVE.. $v_A = (v_A)_0 + a_A t$ $\qquad (v_A)_0 = 105\ \frac{mi}{h} = 154\ \frac{ft}{s}$

AT $t = 8\text{ s}$: $v_A = 135\ \frac{mi}{h} = 198\ \frac{ft}{s}$

THEN $198\ \frac{ft}{s} = 154\ \frac{ft}{s} + a_A(8\text{ s})$

OR $a_A = 5.50\ \frac{ft}{s^2}$ ◀

(b) HAVE.. $x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2}a_A t^2$ $\qquad (x_A)_0 = 120\text{ ft}$

AND $x_B = \cancel{(x_B)_0}^{\,0} + (v_B)_0 t + \frac{1}{2}a_B t^2$ $\qquad (v_B)_0 = 154\ \frac{ft}{s}$

AT $t = 8\text{ s}$: $x_A = x_B$

$\therefore\ 120\text{ ft} + (154\ \frac{ft}{s})(8\text{ s}) + \frac{1}{2}(5.50\ \frac{ft}{s^2})(8\text{ s})^2 = (154\ \frac{ft}{s})(8\text{ s}) + \frac{1}{2}a_B(8\text{ s})^2$

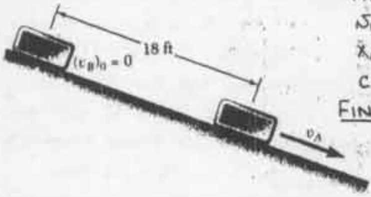
OR $a_B = 9.25\ \frac{ft}{s^2}$ ◀

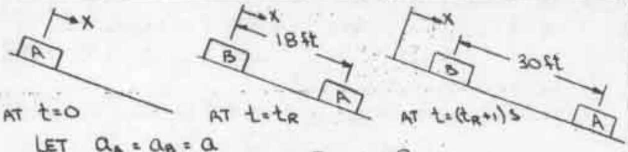
11.42

GIVEN: AT $t=0$, $x_A=0$, $v_A=0$; AT $t=t_R$, $x_A=18$ ft, $x_B=0$, $v_B=0$; AT $t=(t_R+1)$ s, $x_{A/B}=30$ ft; $a_A=a_B=$ CONSTANT

FIND: (a) t_R

(b) a_A AND a_B





LET $a_A=a_B=a$

(a) FOR $t\geq 0$: $x_A = \cancelto{0}{(x_A)_0} + \cancelto{0}{(v_A)_0}t + \frac{1}{2}at^2$

$t\geq t_R$: $x_B = \cancelto{0}{(x_B)_0} + \cancelto{0}{(v_B)_0}t + \frac{1}{2}a(t-t_R)^2$

AT $t=t_R$, $x_A=18$ ft: $\quad 18 = \frac{1}{2}at_R^2 \quad (1)$

$t=(t_R+1)$ s, $x_A-x_B=30$ ft:

$$30 = \tfrac{1}{2}a(t_R+1)^2 - \tfrac{1}{2}a[(t_R+1)-t_R]^2$$
$$= \tfrac{1}{2}a(t_R^2+2t_R) \quad (2)$$

EQ. (1) $\Rightarrow \frac{1}{2}a = \frac{18}{t_R^2}$ SO THAT $30 = \frac{18}{t_R^2}(t_R^2+2t_R)$

OR $t_R = 3$ s

(b) SUBSTITUTING INTO EQ. (1) .. $18 = \frac{1}{2}a(3)^2$

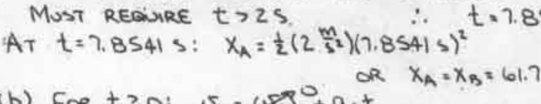
OR $a = a_A = a_B = 4\ \frac{\text{ft}}{\text{s}^2}$

11.43

GIVEN: FOR $t\geq 0$, $a_A = 2$ m/s^2; FOR $t\geq 2$ s, $a_B = 3.6$ m/s^2; CARS START FROM REST

FIND: (a) t AND x WHEN $x_A=x_B$

(b) v_A AND v_B WHEN $x_A=x_B$



(a) FOR $t\geq 0$: $x_A = \cancelto{0}{(x_A)_0} + \cancelto{0}{(v_A)_0}t + \frac{1}{2}a_At^2$

$t\geq 2$ s: $x_B = \cancelto{0}{(x_B)_0} + \cancelto{0}{(v_B)_0}t + \frac{1}{2}a_B(t-2)^2$

WHEN $x_A=x_B$ .. $\frac{1}{2}(2\ \frac{\text{m}}{\text{s}^2})t^2 = \frac{1}{2}(3.6\ \frac{\text{m}}{\text{s}^2})(t-2)^2$

EXPANDING AND SIMPLIFYING ... $t^2-9t+9=0$

SOLVING .. $t = 1.1459$ s AND $t = 7.8541$ s

MUST REQUIRE $t>2$ s $\quad\therefore\ t = 7.85$ s

AT $t=7.8541$ s: $x_A = \frac{1}{2}(2\ \frac{\text{m}}{\text{s}^2})(7.8541\ \text{s})^2$

OR $x_A=x_B=61.7$ m

(b) FOR $t\geq 0$: $v_A = \cancelto{0}{(v_A)_0} + a_At$

AT $t=7.8541$ s: $v_A = (2\ \frac{\text{m}}{\text{s}^2})(7.8541\ \text{s})$

OR $v_A = 15.71\ \frac{\text{m}}{\text{s}}$

FOR $t\geq 2$ s: $v_B = (v_B)_0 + a_B(t-2)$

AT $t=7.8541$ s: .. $v_B = (3.6\ \frac{\text{m}}{\text{s}^2})(7.8541-2)$ s

OR $v_B = 21.1\ \frac{\text{m}}{\text{s}}$

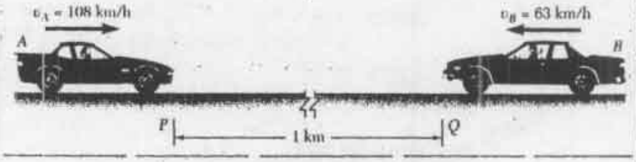
11.44

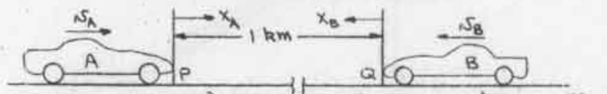
GIVEN: AT $t=0$, CAR A IS AT P, CAR B IS AT Q; AT $t=40$ s, CAR A IS AT Q; AT $t=42$ s, CAR B IS AT P

FIND: (a) a_A AND a_B (a_A AND a_B ARE CONSTANT)

(b) t WHEN CARS MEET

(c) v_B WHEN CARS MEET





(a) HAVE.. $x_A = \cancelto{0}{(x_A)_0} + (v_A)_0t + \frac{1}{2}a_At^2 \quad (v_A)_0 = 108\ \frac{\text{km}}{\text{h}} = 30\ \frac{\text{m}}{\text{s}}$

AT $t=40$ s: $\quad 1000\ \text{m} = (30\ \frac{\text{m}}{\text{s}})(40\ \text{s}) + \frac{1}{2}a_A(40\ \text{s})^2$

OR $a_A = -0.250\ \frac{\text{m}}{\text{s}^2}$

ALSO.. $x_B = \cancelto{0}{(x_B)_0} + (v_B)_0t + \frac{1}{2}a_Bt^2 \quad (v_B)_0 = 63\ \frac{\text{km}}{\text{h}} = 17.5\ \frac{\text{m}}{\text{s}}$

AT $t=42$ s: $\quad 1000\ \text{m} = (17.5\ \frac{\text{m}}{\text{s}})(42\ \text{s}) + \frac{1}{2}a_B(42\ \text{s})^2$

OR $a_B = 0.30045\ \frac{\text{m}}{\text{s}^2} \quad a_B = 0.300\ \frac{\text{m}}{\text{s}^2}$

(b) WHEN THE CARS PASS EACH OTHER

$x_A + x_B = 1000$ m

THEN.. $(30\ \frac{\text{m}}{\text{s}})t_{AB} + \frac{1}{2}(-0.250\ \frac{\text{m}}{\text{s}^2})t_{AB}^2 + (17.5\ \frac{\text{m}}{\text{s}})t_{AB}$

$+ \frac{1}{2}(0.30045\ \frac{\text{m}}{\text{s}^2})t_{AB}^2 = 1000$ m

OR $\quad 0.05045t_{AB}^2 + 95t_{AB} - 2000 = 0$

SOLVING.. $t = 20.822$ s AND $t = -1904$ s

$t>0 \Rightarrow t_{AB} = 20.8$ s

(c) HAVE.. $v_B = (v_B)_0 + a_Bt$

AT $t=t_{AB}$: $\quad v_B = 17.5\ \frac{\text{m}}{\text{s}} + (0.30045\ \frac{\text{m}}{\text{s}^2})(20.822\ \text{s})$

$= 23.756\ \frac{\text{m}}{\text{s}}$

OR $v_B = 85.5\ \frac{\text{km}}{\text{h}}$

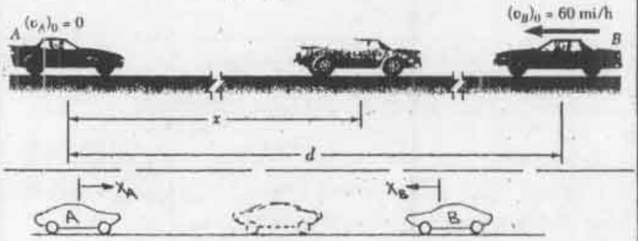
11.45

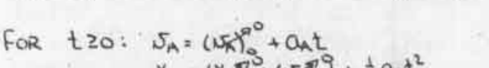
GIVEN: AT $t=0$, $v_A=0$, $v_B = 60\ \frac{\text{mi}}{\text{h}}$; FOR $t\geq 0$, a_A CONSTANT; FOR $t\geq 5$ s, $a_B = -a_A/6$; WHEN CARS MEET, $x = 294$ ft, $v_A=v_B$

FIND: (a) a_A

(b) t WHEN $x=294$ ft

(c) d





FOR $t\geq 0$: $v_A = \cancelto{0}{(v_A)_0} + a_At$

$x_A = \cancelto{0}{(x_A)_0} + \cancelto{0}{(v_A)_0}t + \frac{1}{2}a_At^2$

$0\leq t<5$ s: $\quad x_B = \cancelto{0}{(x_B)_0} + (v_B)_0t \quad (v_B)_0 = 60\ \frac{\text{mi}}{\text{h}} = 88\ \frac{\text{ft}}{\text{s}}$

AT $t=5$ s: $x_B = (88\ \frac{\text{ft}}{\text{s}})(5\ \text{s}) = 440$ ft

FOR $t\geq 5$ s: $v_B = (v_B)_0 + a_B(t-5) \quad a_B = -\frac{1}{6}a_A$

$x_B = (x_B)_5 + (v_B)_0(t-5) + \frac{1}{2}a_B(t-5)^2$

ASSUME $t>5$ s WHEN THE CARS PASS EACH OTHER.

AT THAT TIME (t_{AB}),

$v_A=v_B$: $\quad a_At_{AB} = (88\ \frac{\text{ft}}{\text{s}}) - \frac{a_A}{6}(t_{AB}-5)$

$x_A = 294$ ft: $\quad 294\ \text{ft} = \frac{1}{2}a_At_{AB}^2 \qquad$ (CONTINUED)

11.45 CONTINUED

THEN $\dfrac{a_A(\frac{7}{6}t_{AB}-\frac{5}{6})}{\frac{1}{2}a_A t_{AB}^2} = \dfrac{88}{294}$

OR $44t_{AB}^2 - 343t_{AB} + 245 = 0$

SOLVING... $t_{AB} = 0.795$ s AND $t_{AB} = 7.00$ s

(a) WITH $t_{AB} > 5$ s, $294\text{ ft} = \frac{1}{2}a_A(7.00\text{ s})^2$

OR $a_A = 12.00\ \frac{\text{ft}}{\text{s}^2}$ ◀

(b) FROM ABOVE $t_{AB} = 7.00$ s ◀

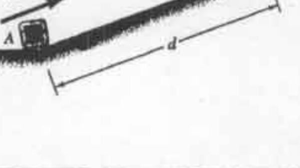
NOTE: AN ACCEPTABLE SOLUTION CANNOT BE FOUND IF IT IS ASSUMED THAT $t_{AB} \le 5$ s.

(c) HAVE.. $d = x + (x_B)_{t_{AB}}$

$= 294\text{ ft} + [440\text{ ft} + (88\ \frac{\text{ft}}{\text{s}})(7.00-5)\text{s} + \frac{1}{2}(-\frac{1}{6}\times 12.00\ \frac{\text{ft}}{\text{s}^2})(7.00-5)^2\text{s}^2]$

OR $d = 906$ ft ◀

11.46

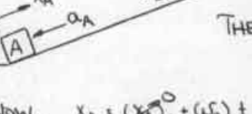


GIVEN: $(v_A)_0$ AND $(v_B)_0$; AT $t=1$ s, BLOCKS PASS EACH OTHER, AT $t=3.4$ s, $x_B = d$; $(x_A)_{MAX} = 21$ ft; a_A, a_B ARE CONSTANT AND ↙

FIND: (a) a_A AND a_B

(b) d

(c) v_A AT $t = 1$ s



(a) HAVE.. $v_A^2 = (v_A)_0^2 + 2a_A[x_A - \cancelto{0}{(x_A)_0}]$

WHEN $x_A = (x_A)_{MAX}$, $v_A = 0$

THEN.. $0 = (27\ \frac{\text{ft}}{\text{s}})^2 + 2a_A(21\text{ ft})$

OR $a_A = -17.3571\ \frac{\text{ft}}{\text{s}^2}$

OR $a_A = 17.36\ \frac{\text{ft}}{\text{s}^2}$ ↙ ◀

NOW.. $x_A = \cancelto{0}{(x_A)_0} + (v_A)_0 t + \frac{1}{2}a_A t^2$

AND $x_B = \cancelto{0}{(x_B)_0} + \cancelto{0}{(v_B)_0}t + \frac{1}{2}a_B t^2$

AT $t = 1$ s, THE BLOCKS PAST EACH OTHER

$\therefore\ (x_A)_1 + (x_B)_1 = d$

AT $t = 3.4$ s, $x_B = d$

THUS.. $(x_A)_1 + (x_B)_1 = (x_B)_{3.4}$

OR $[(27\ \frac{\text{ft}}{\text{s}})(1\text{ s}) + \frac{1}{2}(-17.3571\ \frac{\text{ft}}{\text{s}^2})(1\text{ s})^2] + [\frac{1}{2}a_B(1\text{ s})^2] = \frac{1}{2}a_B(3.4\text{ s})^2$

OR $a_B = 3.4700\ \frac{\text{ft}}{\text{s}^2}$ $\quad a_B = 3.47\ \frac{\text{ft}}{\text{s}^2}$ ↙ ◀

(b) AT $t = 3.4$ s, $x_B = d$: $d = \frac{1}{2}(3.4700\ \frac{\text{ft}}{\text{s}^2})(3.4\text{ s})^2$

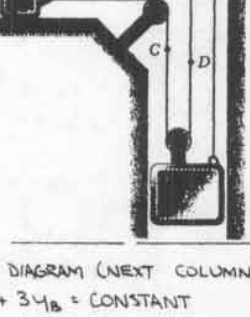
OR $d = 20.1$ ft ◀

(c) HAVE.. $v_A = (v_A)_0 + a_A t$

AT $t = 1$ s: $v_A = 27\ \frac{\text{ft}}{\text{s}} + (-17.3571\ \frac{\text{ft}}{\text{s}^2})(1\text{ s})$

OR $v_A = 9.64\ \frac{\text{ft}}{\text{s}}$ ◀

11.47 and 11.48



GIVEN: BLOCKS A AND B AND THE PULLEY/CABLE SYSTEM SHOWN

FROM THE DIAGRAM (NEXT COLUMN) HAVE..

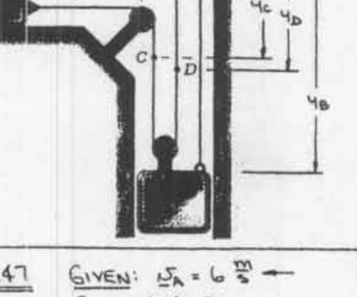
$x_A + 3y_B = \text{CONSTANT}$

THEN $v_A + 3v_B = 0 \quad (1)$

AND $a_A + 3a_B = 0 \quad (2)$

(CONTINUED)

11.47 and 11.48 CONTINUED



11.47 GIVEN: $v_A = 6\ \frac{\text{m}}{\text{s}}$ ⟵

FIND: (a) v_B

(b) v_D

(c) $v_{C/D}$

(a) SUBSTITUTING INTO EQ. (1).. $6\ \frac{\text{m}}{\text{s}} + 3v_B = 0$

OR $v_B = 2\ \frac{\text{m}}{\text{s}}$ ↑ ◀

(b) FROM THE DIAGRAM.. $y_B + y_D = \text{CONSTANT}$

THEN $v_B + v_D = 0$

$\therefore\ v_D = 2\ \frac{\text{m}}{\text{s}}$ ↓ ◀

(c) FROM THE DIAGRAM.. $x_A + y_C = \text{CONSTANT}$

THEN $v_A + v_C = 0 \quad \therefore\ v_C = -6\ \frac{\text{m}}{\text{s}}$

NOW.. $v_{C/D} = v_C - v_D$

$= (-6\ \frac{\text{m}}{\text{s}}) - (2\ \frac{\text{m}}{\text{s}}) = -8\ \frac{\text{m}}{\text{s}}$

$\therefore\ v_{C/D} = 8\ \frac{\text{m}}{\text{s}}$ ↑ ◀

11.48 GIVEN: AT $t = 0$, $v_B = 0$; a_B = CONSTANT ↓; WHEN $|\Delta x_A| = 0.4$ m, $|v_A| = 4\ \frac{\text{m}}{\text{s}}$

FIND: (a) a_A AND a_B

(b) v_B AND $[y_B - (y_B)_0]$ AT $t = 2$ s

(a) EQ. (2): $a_A + 3a_B = 0$ AND a_B IS CONSTANT AND POSITIVE ⇒ a_A IS CONSTANT AND NEGATIVE

ALSO, EQ. (1) AND $(v_B)_0 = 0$ ⇒ $(v_A)_0 = 0$

THEN $v_A^2 = \cancelto{0}{(v_A)_0^2} + 2a_A[x_A - (x_A)_0]$

WHEN $|\Delta x_A| = 0.4$ m: $(4\ \frac{\text{m}}{\text{s}})^2 = 2a_A(0.4\text{ m})$

OR $a_A = 20\ \frac{\text{m}}{\text{s}^2}$ ⟶ ◀

THEN.. SUBSTITUTING INTO EQ (2).. $-20\ \frac{\text{m}}{\text{s}^2} + 3a_B = 0$

OR $a_B = \frac{20}{3}\ \frac{\text{m}}{\text{s}^2}$ $\quad a_B = 6.67\ \frac{\text{m}}{\text{s}^2}$ ↓ ◀

(b) HAVE.. $v_B = \cancelto{0}{(v_B)_0} + a_B t$

AT $t = 2$ s: $v_B = (\frac{20}{3}\ \frac{\text{m}}{\text{s}^2})(2\text{ s})$

OR $v_B = 13.33\ \frac{\text{m}}{\text{s}}$ ↓ ◀

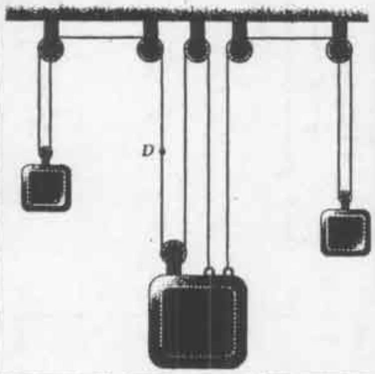
ALSO.. $y_B = (y_B)_0 + \cancelto{0}{(v_B)_0}t + \frac{1}{2}a_B t^2$

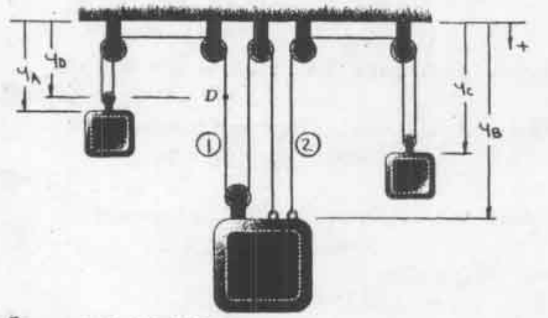
AT $t = 2$ s: $y_B - (y_B)_0 = \frac{1}{2}(\frac{20}{3}\ \frac{\text{m}}{\text{s}^2})(2\text{ s})^2$

OR $y_B - (y_B)_0 = 13.33$ m ↓ ◀

11.49 and 11.50

GIVEN: BLOCKS A, B, AND C AND THE PULLEY / CABLE SYSTEM SHOWN





FROM THE DIAGRAM..

CABLE 1: $2y_A + 3y_B = \text{CONSTANT}$

THEN.. $2v_A + 3v_B = 0$ (1)

AND $2a_A + 3a_B = 0$ (2)

CABLE 2: $y_B + 2y_C = \text{CONSTANT}$

THEN.. $v_B + 2v_C = 0$ (3)

AND $a_B + 2a_C = 0$ (4)

11.49 GIVEN: $v_B = 24\,\frac{\text{in.}}{\text{s}}\downarrow$

FIND: (a) v_A
(b) v_C
(c) v_D
(d) $v_{D/B}$

(a) SUBSTITUTING INTO EQ. (1).. $2v_A + 3(24\,\frac{\text{in.}}{\text{s}}) = 0$

OR $v_A = 36\,\frac{\text{in.}}{\text{s}}\uparrow$ ◀

(b) SUBSTITUTING INTO EQ. (3).. $(24\,\frac{\text{in.}}{\text{s}}) + 2v_C = 0$

OR $v_C = 12\,\frac{\text{in.}}{\text{s}}\uparrow$ ◀

(c) FROM THE DIAGRAM.. $2y_A + y_D = \text{CONSTANT}$

THEN.. $2v_A + v_D = 0$

SUBSTITUTING FOR v_A.. $2(-36\,\frac{\text{in.}}{\text{s}}) + v_D = 0$

OR $v_D = 72\,\frac{\text{in.}}{\text{s}}\downarrow$ ◀

(d) HAVE.. $v_{D/B} = v_D - v_B$
$= 72\,\frac{\text{in.}}{\text{s}} - 24\,\frac{\text{in.}}{\text{s}}$

OR $v_{D/B} = 48\,\frac{\text{in.}}{\text{s}}\downarrow$ ◀

11.50 GIVEN: $(v_C)_0 = 0$; $a_C = \text{CONSTANT}\downarrow$; AT $t = 12\text{ s}$, $v_A = 18\text{ in./s}$

FIND: (a) a_A, a_B, AND a_C
(b) v_B AND $[y_B - (y_B)_0]$ AT $t = 8\text{ s}$

(a) EQS. (3) AND (1) AND $(v_C)_0 = 0 \Rightarrow (v_A)_0 = (v_B)_0 = 0$

ALSO, EQS. (4) AND (2) AND a_C IS CONSTANT AND POSITIVE $\Rightarrow$ a_B IS CONSTANT AND NEGATIVE
a_A IS CONSTANT AND POSITIVE

THEN.. $v_A = \cancel{(v_A)_0}^{\,0} + a_A t$

AT $t = 12\text{ s}$: $18\,\frac{\text{in.}}{\text{s}} = a_A(12\text{ s})$ OR $a_A = 1.5\,\frac{\text{in.}}{\text{s}^2}\downarrow$ ◀

(CONTINUED)

11.50 CONTINUED

SUBSTITUTING INTO EQ. (2).. $2(1.5\,\frac{\text{in.}}{\text{s}^2}) + 3a_B = 0$

OR $a_B = 1.0\,\frac{\text{in.}}{\text{s}^2}\uparrow$ ◀

SUBSTITUTING INTO EQ. (4).. $(-1.0\,\frac{\text{in.}}{\text{s}^2}) + 2a_C = 0$

OR $a_C = 0.5\,\frac{\text{in.}}{\text{s}^2}\downarrow$ ◀

(b) HAVE.. $v_B = \cancel{(v_B)_0}^{\,0} + a_B t$

AT $t = 8\text{ s}$: $v_B = (-1.0\,\frac{\text{in.}}{\text{s}^2})(8\text{ s})$

OR $v_B = 8.0\,\frac{\text{in.}}{\text{s}}\uparrow$ ◀

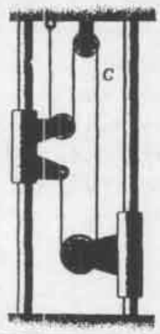
ALSO.. $y_B = (y_B)_0 + \cancel{(v_B)_0}^{\,0}t + \frac{1}{2}a_B t^2$

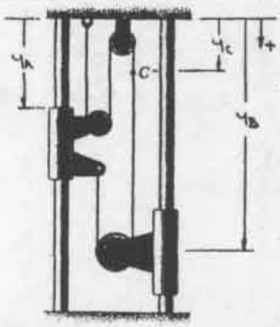
AT $t = 8\text{ s}$: $y_B - (y_B)_0 = \frac{1}{2}(-1.0\,\frac{\text{in.}}{\text{s}^2})(8\text{ s})^2$

OR $y_B - (y_B)_0 = 32.0\text{ in.}\uparrow$ ◀

11.51 and 11.52

GIVEN: COLLARS A AND B AND THE PULLEY / CABLE SYSTEM SHOWN





FROM THE DIAGRAM..

$2y_A + y_B + (y_B - y_A) = \text{CONSTANT}$

THEN.. $v_A + 2v_B = 0$ (1)

AND $a_A + 2a_B = 0$ (2)

11.51 GIVEN: $(v_A)_0 = 0$; $a_A = \text{CONSTANT}\uparrow$; AT $t = 8\text{ s}$, $|v_{B/A}| = 24\text{ in./s}$

FIND: (a) a_A AND a_B
(b) v_B AND $y_B - (y_B)_0$ AT $t = 6\text{ s}$

(a) EQ. (1) AND $(v_A)_0 = 0 \Rightarrow (v_B)_0$

ALSO, EQ. (2) AND a_A IS CONSTANT AND NEGATIVE
$\Rightarrow$ a_B IS CONSTANT AND POSITIVE

THEN.. $v_A = \cancel{(v_A)_0}^{\,0} + a_A t$ $\quad v_B = \cancel{(v_B)_0}^{\,0} + a_B t$

NOW.. $v_{B/A} = v_B - v_A = (a_B - a_A)t$

FROM EQ. (2).. $a_B = -\frac{1}{2}a_A$ SO THAT $v_{B/A} = -\frac{3}{2}a_A t$

AT $t = 8\text{ s}$: $24\,\frac{\text{in.}}{\text{s}} = -\frac{3}{2}a_A(8\text{ s})$

OR $a_A = 2\,\frac{\text{in.}}{\text{s}^2}\uparrow$ ◀

AND THEN.. $a_B = -\frac{1}{2}(-2\,\frac{\text{in.}}{\text{s}^2})$ OR $a_B = 1\,\frac{\text{in.}}{\text{s}^2}\downarrow$ ◀

(b) AT $t = 6\text{ s}$: $v_B = (1\,\frac{\text{in.}}{\text{s}^2})(6\text{ s})$

OR $v_B = 6\,\frac{\text{in.}}{\text{s}}\downarrow$ ◀

NOW.. $y_B = (y_B)_0 + \cancel{(v_B)_0}^{\,0}t + \frac{1}{2}a_B t^2$

AT $t = 6\text{ s}$: $y_B - (y_B)_0 = \frac{1}{2}(1\,\frac{\text{in.}}{\text{s}^2})(6\text{ s})^2$

OR $y_B - (y_B)_0 = 18\text{ in.}\downarrow$ ◀

(CONTINUED)

11.52 CONTINUED

11.52 GIVEN: $v_B = 12 \frac{in.}{s}$ ↓

FIND: (a) v_A

(b) v_C

(c) $v_{C/B}$

(a) SUBSTITUTING INTO EQ. (1).. $v_A + 2(12 \frac{in.}{s}) = 0$

OR $v_A = 24 \frac{in.}{s}$ ↑ ◀

(b) FROM THE DIAGRAM.. $2y_A + y_C = \text{CONSTANT}$

THEN.. $2v_A + v_C = 0$

SUBSTITUTING.. $2(-24 \frac{in.}{s}) + v_C = 0$

OR $v_C = 48 \frac{in.}{s}$ ↓ ◀

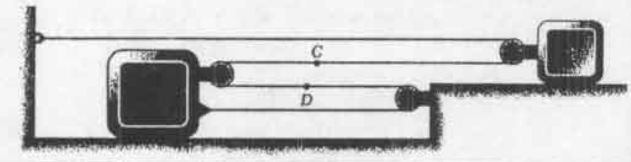
(c) HAVE.. $v_{C/B} = v_C - v_B$

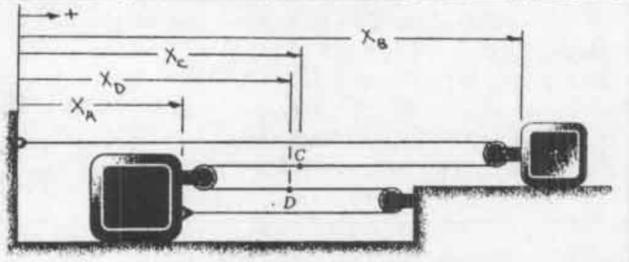
$= (48 \frac{in.}{s}) - (12 \frac{in.}{s})$

OR $v_{C/B} = 36 \frac{in.}{s}$ ↓ ◀

11.53 and 11.54

GIVEN: BLOCKS A AND B AND THE PULLEY/CABLE SYSTEM SHOWN





FROM THE DIAGRAM.. $x_B + (x_B - x_A) - 2x_A = \text{CONSTANT}$

THEN.. $2v_B - 3v_A = 0$ (1)

AND $2a_B - 3a_A = 0$ (2)

11.53 GIVEN: $v_B = 300 \text{ mm/s}$ →

FIND: (a) v_A

(b) v_C

(c) v_D

(d) $v_{C/A}$

(a) SUBSTITUTING INTO EQ. (1).. $2(300 \frac{mm}{s}) - 3v_A = 0$

OR $v_A = 200 \frac{mm}{s}$ → ◀

(b) FROM THE DIAGRAM.. $x_B + (x_B - x_C) = \text{CONSTANT}$

THEN.. $2v_B - v_C = 0$

SUBSTITUTING.. $2(300 \frac{mm}{s}) - v_C = 0$

OR $v_C = 600 \frac{mm}{s}$ → ◀

(c) FROM THE DIAGRAM.. $(x_C - x_A) + (x_D - x_A) = \text{CONSTANT}$*

THEN.. $v_C - 2v_A + v_D = 0$

SUBSTITUTING... $600 \frac{mm}{s} - 2(200 \frac{mm}{s}) + v_D = 0$

OR $v_D = 200 \frac{mm}{s}$ ← ◀

(d) HAVE.. $v_{C/A} = v_C - v_A$

$= 600 \frac{mm}{s} - 200 \frac{mm}{s}$

OR $v_{C/A} = 400 \frac{mm}{s}$ → ◀

*ALSO HAVE.. $-x_D - x_A = \text{CONSTANT}$

THEN.. $v_D + v_A = 0$ (3)

(CONTINUED)

11.54 CONTINUED

11.54 GIVEN: $(v_B)_0 = 150 \frac{mm}{s}$; $a_B = \text{CONSTANT}$; WHEN $x_A - (x_A)_0 = 240 \text{ mm}$ →, $v_A = 60 \frac{mm}{s}$

FIND: (a) a_A AND a_B

(b) a_D

(c) v_B AND $x_B - (x_B)_0$ AT $t = 4$ s

(a) FIRST OBSERVE THAT IF BLOCK A MOVES TO THE RIGHT, v_A → AND EQ. (1) ⇒ v_B →. THEN, USING EQ. (1) AT $t = 0$.. $2(150 \frac{mm}{s}) - 3(v_A)_0 = 0$

OR $(v_A)_0 = 100 \frac{mm}{s}$

ALSO, EQ. (2) AND $a_B = \text{CONSTANT}$ ⇒ $a_A = \text{CONSTANT}$

THEN.. $v_A^2 = (v_A)_0^2 + 2a_A[x_A - (x_A)_0]$

WHEN $x_A - (x_A)_0 = 240 \text{ mm}$: $(60 \frac{mm}{s})^2 = (100 \frac{mm}{s})^2 + 2a_A(240 \text{ mm})$

OR $a_A = -\frac{40}{3} \frac{mm}{s^2}$

OR $a_A = 13.33 \frac{mm}{s^2}$ ← ◀

THEN, SUBSTITUTING INTO EQ. (2)..

$2a_B - 3(-\frac{40}{3} \frac{mm}{s^2}) = 0$

OR $a_B = -20 \frac{mm}{s^2}$ $\quad$ $a_B = 20.0 \frac{mm}{s^2}$ ← ◀

(b) FROM THE SOLUTION TO PROBLEM 11.53..

$v_D + v_A = 0$

THEN.. $a_D + a_A = 0$

SUBSTITUTING.. $a_D + (-\frac{40}{3} \frac{mm}{s^2}) = 0$

OR $a_D = 13.33 \frac{mm}{s^2}$ → ◀

(c) HAVE.. $v_B = (v_B)_0 + a_B t$

AT $t = 4$ s: $v_B = 150 \frac{mm}{s} + (-20.0 \frac{mm}{s^2})(4 \text{ s})$

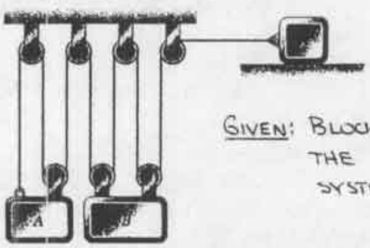
OR $v_B = 70.0 \frac{mm}{s}$ → ◀

ALSO.. $y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$

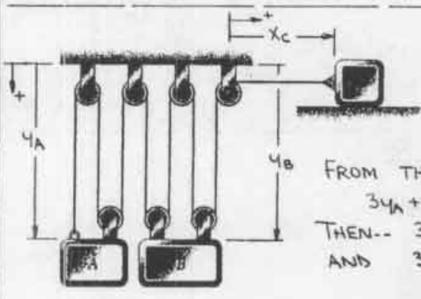
AT $t = 4$ s: $y_B - (y_B)_0 = (150 \frac{mm}{s})(4 \text{ s}) + \frac{1}{2}(-20.0 \frac{mm}{s^2})(4 \text{ s})^2$

OR $y_B - (y_B)_0 = 440 \text{ mm}$ → ◀

11.55 and 11.56



GIVEN: BLOCKS A, B, AND C AND THE PULLEY/CABLE SYSTEM SHOWN



FROM THE DIAGRAM..

$3y_A + 4y_B + x_C = \text{CONSTANT}$

THEN.. $3v_A + 4v_B + v_C = 0$ (1)

AND $3a_A + 4a_B + a_C = 0$ (2)

(CONTINUED)

11.55 and 11.56 CONTINUED

11.55 GIVEN: $v_B = 20\ \frac{mm}{s}\downarrow$; $(v_A)_0 = 30\ \frac{mm}{s}\uparrow$;
a_A = CONSTANT; AT $t = 3\ s$,
$x_C - (x_C)_0 = 57\ mm \rightarrow$

FIND: (a) $(v_C)_0$
(b) a_A AND a_C
(c) $y_A - (y_A)_0$ AT $t = 5\ s$

(a) SUBSTITUTING INTO EQ. (1) AT $t = 0$..
$$3\left(-30\ \frac{mm}{s}\right) + 4\left(20\ \frac{mm}{s}\right) + (v_C)_0 = 0$$
OR $(v_C)_0 = 10\ \frac{mm}{s} \rightarrow$ ◀

(b) HAVE.. $x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2}a_C t^2$

AT $t = 3\ s$: $57\ mm = \left(10\ \frac{mm}{s}\right)(3\ s) + \frac{1}{2}a_C(3\ s)^2$
OR $a_C = 6\ \frac{mm}{s^2} \rightarrow$ ◀

NOW.. v_B = CONSTANT $\Rightarrow a_B = 0$

THEN.. SUBSTITUTING INTO EQ. (2)..
$$3a_A + 4(0) + \left(6\ \frac{mm}{s^2}\right) = 0$$
OR $a_A = 2\ \frac{mm}{s^2}\uparrow$ ◀

(c) HAVE.. $y_A = (y_A)_0 + (v_A)_0 t + \frac{1}{2}a_A t^2$

AT $t = 5\ s$: $y_A - (y_A)_0 = \left(-30\ \frac{mm}{s}\right)(5\ s) + \frac{1}{2}\left(-2\ \frac{mm}{s^2}\right)(5\ s)^2$
OR $y_A - (y_A)_0 = 175\ mm\uparrow$ ◀

11.56 GIVEN: $(v_B)_0 = 0$, a_A = CONSTANT,
$(a_C) = 75\ \frac{mm}{s^2} \rightarrow$; AT $t = 2\ s$,
$v_B = 480\ \frac{mm}{s}\downarrow$, $v_C = 280\ \frac{mm}{s} \rightarrow$

FIND: (a) a_A AND a_B
(b) $(v_A)_0$ AND $(v_C)_0$
(c) $x_C - (x_C)_0$ AT $t = 3\ s$

(a) EQ. (2) AND a_A = CONSTANT AND
a_C = CONSTANT $\Rightarrow a_B$ = CONSTANT

THEN.. $v_B = (v_B)_0^{\,\nearrow 0} + a_B t$

AT $t = 2\ s$: $480\ \frac{mm}{s} = a_B(2\ s)$
OR $a_B = 240\ \frac{mm}{s^2}\downarrow$ ◀

SUBSTITUTING INTO EQ. (2)..
$$3a_A + 4\left(240\ \frac{mm}{s^2}\right) + \left(75\ \frac{mm}{s^2}\right) = 0$$
OR $a_A = 345\ \frac{mm}{s^2}\uparrow$ ◀

(b) HAVE.. $v_C = (v_C)_0 + a_C t$

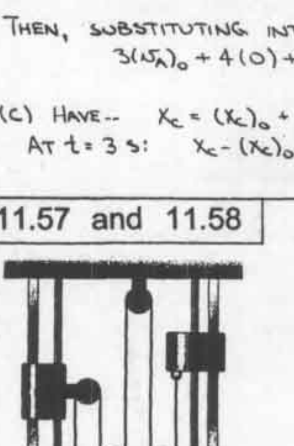
AT $t = 2\ s$: $280\ \frac{mm}{s} = (v_C)_0 + \left(75\ \frac{mm}{s^2}\right)(2\ s)$
OR $(v_C)_0 = 130\ \frac{mm}{s} \rightarrow$ ◀

THEN, SUBSTITUTING INTO EQ. (1) AT $t = 0$..
$$3(v_A)_0 + 4(0) + \left(130\ \frac{mm}{s}\right) = 0$$
OR $(v_A)_0 = 43.3\ \frac{mm}{s}\uparrow$ ◀

(c) HAVE.. $x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2}a_C t^2$

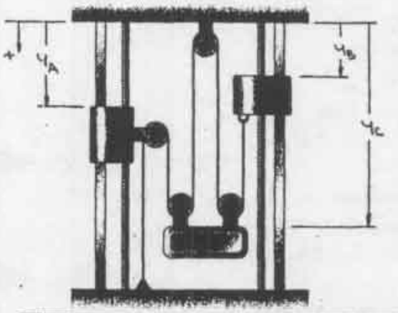
AT $t = 3\ s$: $x_C - (x_C)_0 = \left(130\ \frac{mm}{s}\right)(3\ s) + \frac{1}{2}\left(75\ \frac{mm}{s^2}\right)(3\ s)^2$
OR $x_C - (x_C)_0 = 728\ mm \rightarrow$ ◀

11.57 and 11.58



GIVEN: COLLARS A AND B, BLOCK C, AND THE PULLEY/CABLE SYSTEM SHOWN

11.57 and 11.58 CONTINUED



FROM THE DIAGRAM..
$$-y_A + (y_C - y_A) + 2y_C + (y_C - y_B) = \text{CONSTANT}$$
THEN..
$$-2v_A - v_B + 4v_C = 0 \quad (1)$$
AND
$$-2a_A - a_B + 4a_C = 0 \quad (2)$$

11.57 GIVEN: $(v_A)_0 = 0$, $(a_A) = 7\ \frac{in.}{s^2}\downarrow$; $(v_B)_0 = 8\ \frac{in.}{s}\uparrow$,
a_B = CONSTANT$\uparrow$; AT $t = 2\ s$, $y_B - (y_B)_0 = 20\ in.\uparrow$

FIND: (a) a_B AND a_C
(b) t WHEN $v_C = 0$
(c) $y_C - (y_C)_0$ WHEN $v_C = 0$

(a) HAVE.. $y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2}a_B t^2$

AT $t = 2\ s$: $-20\ in. = \left(-8\ \frac{in.}{s}\right)(2\ s) + \frac{1}{2}a_B(2\ s)^2$
OR $a_B = 2\ \frac{in.}{s^2}\uparrow$ ◀

THEN.. SUBSTITUTING INTO EQ. (2)..
$$-2\left(7\ \frac{in.}{s^2}\right) - \left(-2\ \frac{in.}{s^2}\right) + 4a_C = 0$$
OR $a_C = 3\ \frac{in.}{s^2}\downarrow$ ◀

(b) SUBSTITUTING INTO EQ (1) AT $t = 0$..
$$-2(0) - \left(-8\ \frac{in.}{s}\right) + 4(v_C)_0 = 0 \quad \text{OR } (v_C)_0 = -2\ \frac{in.}{s}$$

NOW.. $v_C = (v_C)_0 + a_C t$

WHEN $v_C = 0$: $0 = \left(-2\ \frac{in.}{s}\right) + \left(3\ \frac{in.}{s^2}\right)t$
OR $t = \frac{2}{3}\ s$ $\quad t = 0.667\ s$ ◀

(c) HAVE.. $y_C = (y_C)_0 + (v_C)_0 t + \frac{1}{2}a_C t^2$

AT $t = \frac{2}{3}\ s$: $y_C - (y_C)_0 = \left(-2\ \frac{in.}{s}\right)\left(\frac{2}{3}\ s\right) + \frac{1}{2}\left(3\ \frac{in.}{s^2}\right)\left(\frac{2}{3}\ s\right)^2$
OR $y_C - (y_C)_0 = 0.667\ in.\uparrow$ ◀

11.58 GIVEN: $(v_A)_0 = 0$, $(v_B)_0 = 0$; $a_A = 3t^2\ \frac{in.}{s^2}\uparrow$,
a_B = CONSTANT$\downarrow$; WHEN $y_B - (y_B)_0 = 32\ in.\downarrow$,
$v_B = 8\ \frac{in.}{s}$

FIND: (a) a_C
(b) DISTANCE TRAVELED BY C AT $t = 3\ s$

(a) HAVE.. $v_B^2 = (v_B)_0^{2\,\nearrow 0} + 2a_B[y_B - (y_B)_0]$

WHEN $y_B - (y_B)_0 = 32\ in.$: $\left(8\ \frac{in.}{s}\right)^2 = 2a_B(32\ in.)$
OR $a_B = 1\ \frac{in.}{s^2}$

THEN, SUBSTITUTING INTO EQ. (2)..
$$-2\left(-3t^2\ \frac{in.}{s^2}\right) - \left(1\ \frac{in.}{s^2}\right) + 4a_C = 0$$
OR $a_C = \frac{1}{4}(1 - 6t^2)\ \frac{in.}{s^2}$ ◀

(b) SUBSTITUTING INTO EQ. (1) AT $t = 0$..
$$-2(0) - (0) + 4(v_C)_0 = 0 \quad \text{OR } (v_C)_0 = 0$$

NOW.. $\frac{dv_C}{dt} = a_C = \frac{1}{4}(1 - 6t^2)$

AT $t = 0$, $v_C = 0$: $\int_0^{v_C} dv_C = \int_0^t \frac{1}{4}(1 - 6t^2)dt$
OR $v_C = \frac{1}{4}(t - 2t^3)$

THUS, $v_C = 0$ AT $\frac{1}{4}t(1 - 2t^2) = 0$ OR $t = 0$, $t = \frac{1}{\sqrt{2}}\ s$

THEREFORE, BLOCK C INITIALLY MOVES DOWNWARD ($v_C > 0$) AND THEN MOVES UPWARD ($v_C < 0$).

NOW.. $\frac{dy_C}{dt} = v_C = \frac{1}{4}(t - 2t^3)$

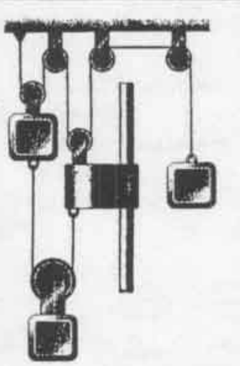
AT $t = 0$, $y_C = (y_C)_0$: $\int_{(y_C)_0}^{y_C} dy_C = \int_0^t \frac{1}{4}(t - 2t^3)dt$
OR $y_C - (y_C)_0 = \frac{1}{8}(t^2 - t^4)$

AT $t = \frac{1}{\sqrt{2}}\ s$: $y_C - (y_C)_0 = \frac{1}{8}\left[\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^4\right] = \frac{1}{32}\ in.$

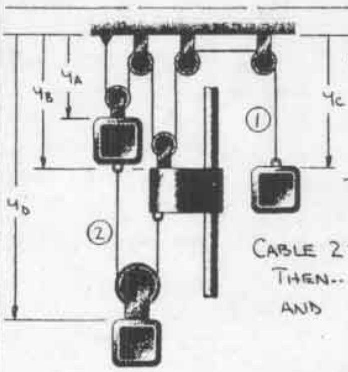
AT $t = 3\ s$: $y_C - (y_C)_0 = \frac{1}{8}\left[(3)^2 - (3)^4\right] = -9\ in.$

$\therefore$ TOTAL DISTANCE TRAVELED $= \left(\frac{1}{32}\right) + \left|-9 - \frac{1}{32}\right| = 9\frac{1}{16}\ in.$
$= 9.06\ in.$ ◀

11.59 and * 11.60



GIVEN: BLOCKS A, C, AND D, COLLAR B, AND THE PULLEY/CABLE SYSTEM SHOWN



FROM THE DIAGRAM..

CABLE 1: $2y_A + 2y_B + y_C$ = CONSTANT

THEN.. $2v_A + 2v_B + v_C = 0$ (1)

AND $2a_A + 2a_B + a_C = 0$ (2)

CABLE 2: $(y_D - y_A) + (y_D - y_B)$ = CONSTANT

THEN.. $-v_A - v_B + 2v_D = 0$ (3)

AND $-a_A - a_B + 2a_D = 0$ (4)

11.59 GIVEN: AT $t=0$, $v=0$; ALL ACCELERATIONS CONSTANT; $a_{C/B} = 60\ \frac{mm}{s^2}\uparrow$, $a_{D/A} = 110\ \frac{mm}{s^2}\downarrow$

FIND: (a) v_C AT $t = 3\ s$

(b) $y_D - (y_D)_0$ AT $t = 5\ s$

(a) HAVE.. $a_{C/B} = a_C - a_B = -60$ OR $a_B = a_C + 60$

AND $a_{D/A} = a_D - a_A = 110$ OR $a_A = a_D - 110$

SUBSTITUTING INTO EQS (2) AND (4)..

EQ.(2): $2(a_D - 110) + 2(a_C + 60) + a_C = 0$

OR $3a_C + 2a_D = 100$ (5)

EQ.(4): $-(a_D - 110) - (a_C + 60) + 2a_D = 0$

OR $-a_C + a_D = -50$ (6)

SOLVING EQS. (5) AND (6) FOR a_C AND a_D---

$a_C = 40\ \frac{mm}{s^2}$ $a_D = -10\ \frac{mm}{s^2}$

NOW.. $v_C = \cancelto{0}{(v_C)_0} + a_C t$

AT $t = 3\ s$: $v_C = (40\ \frac{mm}{s^2})(3s)$

OR $v_C = 120\ \frac{mm}{s}\downarrow$

(b) HAVE.. $y_D = (y_D)_0 + \cancelto{0}{(v_D)_0}t + \frac{1}{2}a_D t^2$

AT $t = 5\ s$: $y_D - (y_D)_0 = \frac{1}{2}(-10\ \frac{mm}{s^2})(5\ s)^2$

OR $y_D - (y_D)_0 = 125\ mm\uparrow$

11.60 GIVEN: AT $t=0$, $v=0$, $(y_A)_0 = (y_B)_0 = (y_C)_0$; ALL ACCELERATIONS CONSTANT; AT $t = 2\ s$, $y_{C/A} = 280\ mm\uparrow$; WHEN $v_{B/A} = 80\ \frac{mm}{s}\downarrow$, $y_A - (y_A)_0 = 160\ mm\downarrow$, $y_B - (y_B)_0 = 320\ mm\downarrow$; $a_B > 10\ \frac{mm}{s^2}$

FIND: (a) a_A AND a_B

(b) $y_D - (y_D)_0$ WHEN $v_C = 600\ \frac{mm}{s}\uparrow$

(a) HAVE.. $y_A = (y_A)_0 + \cancelto{0}{(v_A)_0}t + \frac{1}{2}a_A t^2$

AND $y_C = (y_C)_0 + \cancelto{0}{(v_C)_0}t + \frac{1}{2}a_C t^2$

(CONTINUED)

11.60 CONTINUED

THEN.. $y_{C/A} = y_C - y_A = \frac{1}{2}(a_C - a_A)t^2$

AT $t = 2\ s$, $y_{C/A} = -280\ mm$: $-280\ mm = \frac{1}{2}(a_C - a_A)(2\ s)^2$

OR $a_C = a_A - 140$ (5)

SUBSTITUTING INTO EQ.(2).. $2a_A + 2a_B + (a_A - 140) = 0$

OR $a_A = \frac{1}{3}(140 - 2a_B)$ (6)

NOW.. $v_B = \cancelto{0}{(v_B)_0} + a_B t$ $v_A = \cancelto{0}{(v_A)_0} + a_A t$

$\therefore v_{B/A} = v_B - v_A = (a_B - a_A)t$

ALSO.. $y_B = (y_B)_0 + \cancelto{0}{(v_B)_0}t + \frac{1}{2}a_B t^2$

WHEN $v_{B/A} = 80\ \frac{mm}{s}\downarrow$: $80 = (a_B - a_A)t$ (7)

$\Delta y_A = 160\ mm\downarrow$: $160 = \frac{1}{2}a_A t^2$

$\Delta y_B = 320\ mm\downarrow$: $320 = \frac{1}{2}a_B t^2$

THEN $160 = \frac{1}{2}(a_B - a_A)t^2$

USING EQ.(7).. $320 = (80)t$ OR $t = 4\ s$

THEN $160 = \frac{1}{2}a_A(4)^2$ OR $a_A = 20\ \frac{mm}{s^2}\downarrow$

AND $320 = \frac{1}{2}a_B(4)^2$ OR $a_B = 40\ \frac{mm}{s^2}\downarrow$

NOTE THAT EQ. (6) IS NOT USED; THUS, THE PROBLEM IS OVER DETERMINED.

ALTERNATIVE SOLUTION:

HAVE.. $v_A^2 = \cancelto{0}{(v_A)_0^2} + 2a_A[y_A - (y_A)_0]$ $v_B^2 = \cancelto{0}{(v_B)_0^2} + 2a_B[y_B - (y_B)_0]$

THEN.. $v_{B/A} = v_B - v_A = \sqrt{2a_B[y_B - (y_B)_0]} - \sqrt{2a_A[y_A - (y_A)_0]}$

WHEN $v_{B/A} = 80\ \frac{mm}{s}\downarrow$: $80\ \frac{mm}{s} = \sqrt{2}\left[\sqrt{a_B(320\ mm)} - \sqrt{a_A(160\ mm)}\right]$

OR $20 = \sqrt{2}(\sqrt{20a_B} - \sqrt{10a_A})$ (8)

SOLVING EQS (6) AND (8) YIELDS a_A AND a_B.

(b) SUBSTITUTING INTO EQ. (5)..

$a_C = 20 - 140 = -120\ \frac{mm}{s^2}$

AND INTO EQ. (4).. $-(20\ \frac{mm}{s^2}) - (40\ \frac{mm}{s^2}) + 2a_D = 0$

OR $a_D = 30\ \frac{mm}{s^2}$

NOW.. $v_C = \cancelto{0}{(v_C)_0} + a_C t$

WHEN $v_C = -600\ \frac{mm}{s}$: $-600\ \frac{mm}{s} = (-120\ \frac{mm}{s^2})t$

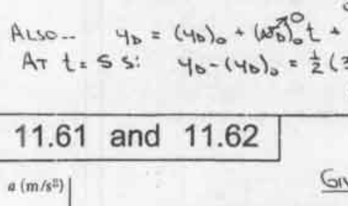
OR $t = 5\ s$

ALSO.. $y_D = (y_D)_0 + \cancelto{0}{(v_D)_0}t + \frac{1}{2}a_D t^2$

AT $t = 5\ s$: $y_D - (y_D)_0 = \frac{1}{2}(30\ \frac{mm}{s^2})(5\ s)^2$

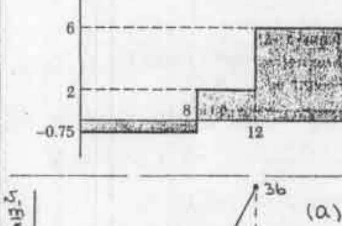
OR $y_D - (y_D)_0 = 375\ mm\downarrow$

11.61 and 11.62



GIVEN: a-t CURVE FOR THE STRAIGHT LINE MOTION OF A PARTICLE; AT $t=0$, $x=0$, $v=-2\ \frac{m}{s}$

CONSTRUCT: (a) v-t AND a-t CURVES FOR $0 < t < 18\ s$

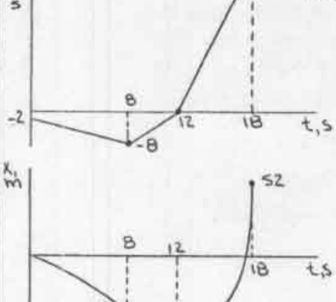


(a) $v_2 = v_1$ + (AREA UNDER a-t CURVE FROM t_1 TO t_2)

AT $t = 8\ s$: $v_8 = -2 - 8(0.75) = -8.0\ \frac{m}{s}$

$t = 12\ s$: $v_{12} = -8.0 + 4(2) = 0$

$t = 18\ s$: $v_{18} = 0 + 6(6) = 36\ \frac{m}{s}$



$x_2 = x_1$ + (AREA UNDER v-t CURVE FROM t_1 TO t_2)

AT $t = 8\ s$: $x_8 = 0 - 8(\frac{2+8}{2}) = -40\ m$

(CONTINUED)

11.61 and 11.62 CONTINUED

AT $t = 12s$: $x_{12} = -40 - \frac{1}{2}(4)(8) = -56m$

$t = 18s$: $x_{18} = -56 + \frac{1}{2}(6)(36) = 52m$

11.61 FIND: (b) x, v, AND TOTAL DISTANCE TRAVELED AT $t = 18s$

(b) READING FROM THE CURVES.. $x_{18} = 52m$ ◄

$v_{18} = 36\frac{m}{s}$ ◄

FROM $t=0$ TO $t=12s$: DISTANCE TRAVELED = 56m

$t = 12s$ TO $t = 18s$: DISTANCE TRAVELED = $52 - (-56)$ = 108 m

∴ TOTAL DISTANCE TRAVELED = $(56+108)$m = 164 m ◄

11.62 FIND: (b) v_{MIN}

(c) x_{MIN}

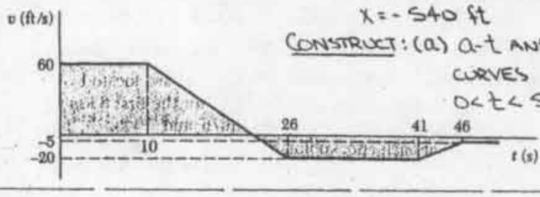
(b) READING FROM THE v-t CURVE.. $v_{MIN} = -8\frac{m}{s}$ ◄

(c) READING FROM THE x-t CURVE.. $x_{MIN} = -56m$ ◄

11.63 and 11.64

GIVEN: v-t CURVE FOR THE STRAIGHT LINE MOTION OF A PARTICLE; AT $t=0$, $x = -540$ ft

CONSTRUCT: (a) a-t AND x-t CURVES FOR $0 < t < 50s$



(a) a_t = SLOPE OF v-t CURVE AT TIME t

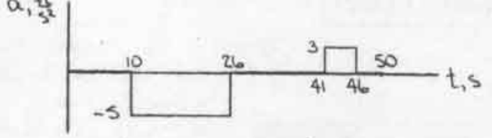
FROM $t=0$ TO $t=10s$: v = CONSTANT ⇒ $a=0$

$t=10s$ TO $t=26s$: $a = \frac{-20-60}{26-10} = -5\frac{ft}{s^2}$

$t = 26s$ TO $t=41s$: v = CONSTANT ⇒ $a=0$

$t=41s$ TO $t=46s$: $a = \frac{-5-(-20)}{46-41} = 3\frac{ft}{s^2}$

$t > 46s$: v = CONSTANT ⇒ $a = 0$



$x_2 = x_1 +$ (AREA UNDER v-t CURVE FROM t_1 TO t_2)

AT $t=10s$: $x_{10} = -540 + 10(60) = 60$ ft

NEXT FIND TIME AT WHICH $v=0$. USING SIMILAR TRIANGLES.. $\frac{t_{v=0} - 10}{60} = \frac{26-10}{80}$ OR $t_{v=0} = 22s$

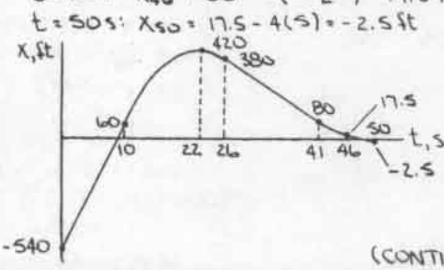
AT $t=22s$: $x_{22} = 60 + \frac{1}{2}(12)(60) = 420$ ft

$t = 26s$: $x_{26} = 420 - \frac{1}{2}(4)(20) = 380$ ft

$t=41s$: $x_{41} = 380 - 15(20) = 80$ ft

$t=46s$: $x_{46} = 80 - 5\left(\frac{20+5}{2}\right) = 17.5$ ft

$t = 50s$: $x_{50} = 17.5 - 4(5) = -2.5$ ft



(CONTINUED)

11.63 and 11.64 CONTINUED

11.63 FIND: (b) TOTAL DISTANCE TRAVELED AT $t = 50s$

(c) t WHEN $x=0$

(b) FROM $t=0$ TO $t=22s$: DISTANCE TRAVELED = $420 - (-540)$ = 960 ft

$t = 22s$ TO $t=50s$: DISTANCE TRAVELED = $|-2.5 - 420|$ = 422.5 ft

∴ TOTAL DISTANCE TRAVELED = $(960 + 422.5)$ ft = 1382.5 ft = 1383 ft ◄

(C) USING SIMILAR TRIANGLES..

BETWEEN 0 AND 10s: $\frac{(t_{x=0})_1 - 0}{540} = \frac{10}{600}$

OR $(t_{x=0})_1 = 9s$ ◄

BETWEEN 46s AND 50s: $\frac{(t_{x=0})_2 - 46}{17.5} = \frac{4}{20}$

OR $(t_{x=0})_2 = 49.5s$ ◄

11.64 FIND: (b) x_{MAX}

(c) t WHEN $x = 100$ ft

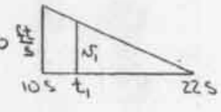
(b) READING FROM THE x-t CURVE.. $x_{MAX} = 420$ ft ◄

(C) BETWEEN 10s AND 22s..

100 ft = 420 ft − (AREA UNDER v-t CURVE FROM t_1 TO 22s) ft

OR $100 = 420 - \frac{1}{2}(22-t_1)(v_1)$

OR $(22-t_1)(v_1) = 640$



USING SIMILAR TRIANGLES..

$\frac{v_1}{22-t_1} = \frac{60}{12}$ OR $v_1 = 5(22-t_1)$

THEN.. $(22-t_1)[5(22-t_1)] = 640$

OR $t_1 = 10.69s$ AND $t_1 = 33.3s$

HAVE $10s < t_1 < 22s$ ⇒ $t_1 = 10.69s$ ◄

BETWEEN 26s AND 41s

USING SIMILAR TRIANGLES..

$\frac{41 - t_2}{20} = \frac{15}{300}$

OR $t_2 = 40s$ ◄

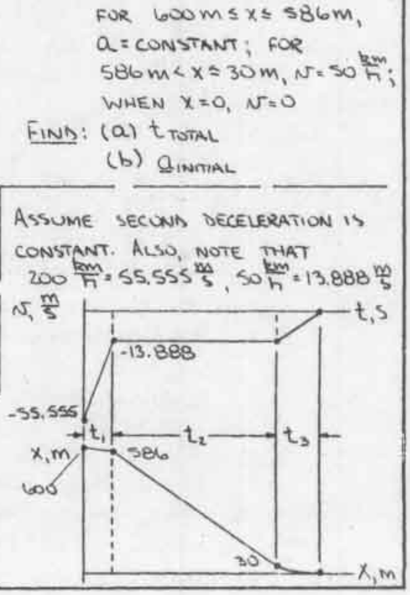
11.65

GIVEN: AT $t=0$, $v = 200\frac{km}{h}$, $x = 600m$; FOR $600m \le x \le 586m$, a = CONSTANT; FOR $586m < x \le 30m$, $v = 50\frac{km}{h}$; WHEN $x=0$, $v=0$

FIND: (a) t_{TOTAL}

(b) $a_{INITIAL}$

ASSUME SECOND DECELERATION IS CONSTANT. ALSO, NOTE THAT $200\frac{km}{h} = 55.555\frac{m}{s}$, $50\frac{km}{h} = 13.888\frac{m}{s}$



(CONTINUED)

11.65 CONTINUED

(a) NOW.. Δx = AREA UNDER v-t CURVE FOR GIVEN TIME INTERVAL

THEN.. $(586-600)\text{m} = -t_1\left(\frac{55.555+13.888}{2}\right)\frac{\text{m}}{\text{s}}$

OR $t_1 = 0.4032\ \text{s}$

$(30-586)\text{m} = -t_2(13.888\ \frac{\text{m}}{\text{s}})$

OR $t_2 = 40.0346\ \text{s}$

$(0-30)\text{m} = -\frac{1}{2}(t_3)(13.888\ \frac{\text{m}}{\text{s}})$

OR $t_3 = 4.3203\ \text{s}$

$\therefore\ t_{TOTAL} = (0.4032 + 40.0346 + 4.3203)\text{s}$

OR $t_{TOTAL} = 44.8\ \text{s}$ ◀

(b) HAVE.. $a_{INITIAL} = \frac{\Delta v_{INITIAL}}{t_1}$

$= \frac{[-13.888-(55.555)]\frac{\text{m}}{\text{s}}}{0.4032\ \text{s}}$

$= 103.3\ \frac{\text{m}}{\text{s}^2}$

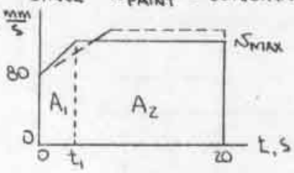
OR $a_{INITIAL} = 103.3\ \frac{\text{m}}{\text{s}^2}$ ↑ ◀

11.66

GIVEN: AT $t = 20\ \text{s}$, $x = 4\ \text{m}$; $v_0 = 80\ \frac{\text{mm}}{\text{s}}$, $a_{MAX} = 60\ \frac{\text{mm}}{\text{s}^2}$; $t_{PAINT} = 15\ \text{s}$, v_{PAINT} = CONSTANT

FIND: $(v_{MAX})_{MIN}$

FIRST NOTE THAT $(80\ \frac{\text{mm}}{\text{s}})(20\ \text{s}) < 4000\ \text{mm}$, SO THAT THE SPEED OF THE PALLET MUST BE INCREASED. SINCE v_{PAINT} = CONSTANT, IT FOLLOWS



THAT $v_{PAINT} = v_{MAX}$ AND THEN $t_1 \leq 5\ \text{s}$. FROM THE v-t CURVE, $A_1 + A_2 = 4000\ \text{mm}$

AND IT IS SEEN THAT $(v_{MAX})_{MIN}$ OCCURS WHEN $a_1\ (= \frac{v_{MAX}-80}{t_1})$ IS MAXIMUM. THUS

$\frac{(v_{MAX}-80)\frac{\text{mm}}{\text{s}}}{t_1\ (\text{s})} = 60\ \frac{\text{mm}}{\text{s}}$

OR $t_1 = (v_{MAX}-80)/60$

AND $t_1\left(\frac{80+v_{MAX}}{2}\right) + (20-t_1)(v_{MAX}) = 4000$

SUBSTITUTING FOR t_1..

$\frac{(v_{MAX}-80)}{60}\left(\frac{80+v_{MAX}}{2}\right) + \left(20 - \frac{v_{MAX}-80}{60}\right)v_{MAX} = 4000$

SIMPLIFYING.. $v_{MAX}^2 - 2560v_{MAX} + 486\,400 = 0$

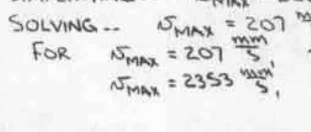
SOLVING.. $v_{MAX} = 207\ \frac{\text{mm}}{\text{s}}$ AND $v_{MAX} = 2353\ \frac{\text{mm}}{\text{s}}$

FOR $v_{MAX} = 207\ \frac{\text{mm}}{\text{s}}$, $t_1 < 5\ \text{s}$

$v_{MAX} = 2353\ \frac{\text{mm}}{\text{s}}$, $t_1 > 5\ \text{s}$

$\therefore\ (v_{MAX})_{MIN} = 207\ \frac{\text{mm}}{\text{s}}$ ◀

11.67



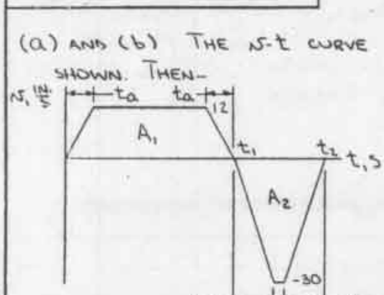
GIVEN: $(v_{MAX})_{RIGHT} = 12\ \frac{\text{in.}}{\text{s}}$, $(v_{MAX})_{LEFT} = 30\ \frac{\text{in.}}{\text{s}}$; $a_{RIGHT} = \pm 6\ \frac{\text{in.}}{\text{s}^2}$, $a_{LEFT} = \pm 20\ \frac{\text{in.}}{\text{s}^2}$

FIND: (a) t_{CYCLE}

CONSTRUCT (b) v-t AND x-t CURVES

11.67 CONTINUED

(a) AND (b) THE v-t CURVE IS FIRST DRAWN AS SHOWN. THEN–



$t_a = \frac{v_{RIGHT}}{a_{RIGHT}} = \frac{12\ \frac{\text{in.}}{\text{s}}}{6\ \frac{\text{in.}}{\text{s}^2}} = 2\ \text{s}$

$t_d = \frac{v_{LEFT}}{a_{LEFT}} = \frac{30\ \frac{\text{in.}}{\text{s}}}{20\ \frac{\text{in.}}{\text{s}^2}} = 1.5\ \text{s}$

NOW.. $A_1 = 60\ \text{in.}$

OR $[(t_1-2)\text{s}](12\ \frac{\text{in.}}{\text{s}}) = 60\ \text{in.}$

OR $t_1 = 7\ \text{s}$

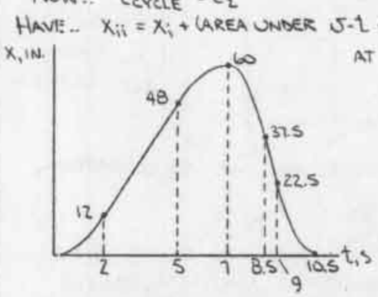
AND $A_2 = 60\ \text{in.}$

OR $\{[(t_2-7)-1.5]\text{s}\}(30\ \frac{\text{in.}}{\text{s}}) = 60\ \text{in.}$

OR $t_2 = 10.5\ \text{s}$

NOW.. $t_{CYCLE} = t_2$ $\therefore\ t_{CYCLE} = 10.5\ \text{s}$ ◀

HAVE.. $x_{ii} = x_i$ + (AREA UNDER v-t CURVE FROM t_i TO t_{ii})



AT $t = 2\ \text{s}$: $x_2 = \frac{1}{2}(2)(12) = 12\ \text{in.}$

$t = 5\ \text{s}$: $x_5 = 12 + (5-2)(12) = 48\ \text{in.}$

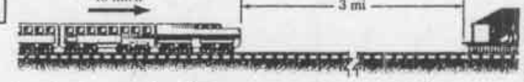
$t = 7\ \text{s}$: $x_7 = 60\ \text{in.}$

$t = 8.5\ \text{s}$: $x_{8.5} = 60 - \frac{1}{2}(1.5)(30) = 37.5\ \text{in.}$

$t = 9\ \text{s}$: $x_9 = 37.5 - (0.5)(30) = 22.5\ \text{in.}$

$t = 10.5\ \text{s}$: $x_{10.5} = 0$

11.68



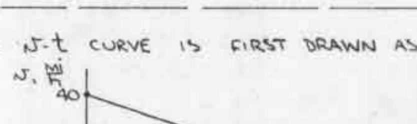
GIVEN: AT $t = 0$, $v = 40\ \frac{\text{mi}}{\text{h}}$, $x = 0$; WHEN $x = 2.5\ \text{mi}$, $v = 20\ \frac{\text{mi}}{\text{h}}$; AT $t = 7.5\ \text{MIN}$, $x = 3\ \text{mi}$; CONSTANT DECELERATIONS

FIND: (a) t WHEN $x = 2.5\ \text{mi}$

(b) v WHEN $x = 3\ \text{mi}$

(c) a_{FINAL}

THE v-t CURVE IS FIRST DRAWN AS SHOWN.



(a) HAVE.. $A_1 = 2.5\ \text{mi}$

OR $(t_1\ \text{MIN})\left(\frac{40+20}{2}\right)\frac{\text{mi}}{\text{h}} \times \frac{1\ \text{h}}{60\ \text{MIN}} = 2.5\ \text{mi}$

OR $t_1 = 5\ \text{MIN}$ ◀

(b) HAVE.. $A_2 = 0.5\ \text{mi}$

OR $(7.5-5)\text{MIN} \times \left(\frac{20+v_2}{2}\right)\frac{\text{mi}}{\text{h}} \times \frac{1\ \text{h}}{60\ \text{MIN}} = 0.5\ \text{mi}$

OR $v_2 = 4\ \frac{\text{mi}}{\text{h}}$ ◀

(c) HAVE $a_{FINAL} = a_{12}$

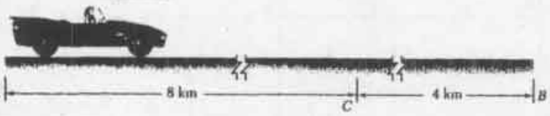
$= \frac{(4-20)\frac{\text{mi}}{\text{h}}}{(7.5-5)\text{MIN}} \times \frac{5280\ \text{ft}}{\text{mi}} \times \frac{1\ \text{MIN}}{60\ \text{s}} \times \frac{1\ \text{h}}{3600\ \text{s}}$

OR $a_{FINAL} = -0.1564\ \frac{\text{ft}}{\text{s}^2}$ ◀

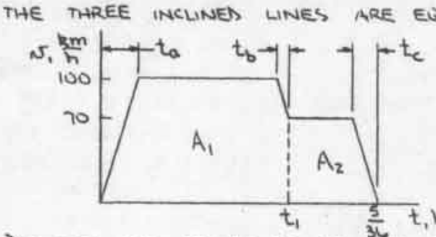
11.69

GIVEN: $(v_{MAX})_{AC} = 100\ \frac{km}{h}$, $(v_{MAX})_{CB} = 70\ \frac{km}{h}$; $v_A = v_B = 0$; $t_{AB} = 8$ MIN, 20 S; $|a|$ = CONSTANT; $v = v_{MAX}$ AS MUCH AS POSSIBLE

FIND: a



THE v-t CURVE IS FIRST DRAWN AS SHOWN, WHERE THE MAGNITUDES OF THE SLOPES (ACCELERATIONS) OF THE THREE INCLINED LINES ARE EQUAL.



NOTE: 8 MIN 20 S $= \frac{5}{36}$ h

AT t_1, $x = 8$ km

$\frac{5}{36}$ h, $x = 12$ km

DENOTING THE MAGNITUDE OF THE ACCELERATIONS BY a, HAVE..

$a = \frac{100}{t_a}$ $\quad a = \frac{30}{t_b}$ $\quad a = \frac{70}{t_c}$

WHERE a IS IN km/h^2 AND THE TIMES ARE IN h.

NOW.. $A_1 = 8$ km: $(t_1)(100) - \frac{1}{2}(t_a)(100) - \frac{1}{2}(t_b)(30) = 8$

SUBSTITUTING... $100t_1 - \frac{1}{2}(\frac{100}{a})(100) - \frac{1}{2}(\frac{30}{a})(30) = 8$

OR $t_1 = 0.08 + \frac{54.5}{a}$

ALSO.. $A_2 = 4$ km: $(\frac{5}{36} - t_1)(70) - \frac{1}{2}(t_c)(70) = 4$

SUBSTITUTING.. $(\frac{5}{36} - t_1)(70) - \frac{1}{2}(\frac{70}{a})(70) = 4$

OR $t_1 = \frac{103}{1260} - \frac{35}{a}$

THEN $0.08 + \frac{54.5}{a} = \frac{103}{1260} - \frac{35}{a}$

OR $a = 51\,259\ \frac{km}{h^2} \times \frac{1000\ m}{km} \times \left(\frac{1\ h}{3600\ s}\right)^2$

OR $a = 3.96\ \frac{m}{s^2}$

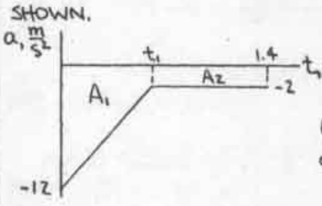
11.70

GIVEN: $v_0 = 6\ \frac{m}{s}$; FOR $0 \le t \le t_1$, $a \propto t$; FOR $t_1 \le t \le 1.4$ s, $a = -2\ \frac{m}{s^2}$; AT $t = 0$, $a = -12\ \frac{m}{s^2}$; AT $t = t_1$, $a = -2\ \frac{m}{s^2}$, $v = 1.8\ \frac{m}{s}$

FIND: (a) t_1

(b) v AND x AT $t = 1.4$ s

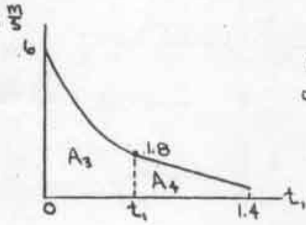
THE a-t AND v-t CURVES ARE FIRST DRAWN AS SHOWN.



(a) HAVE.. $v_{t_1} = v_0 + A_1$

OR $1.8\ \frac{m}{s} = 6\ \frac{m}{s} - (t_1\ s)\left(\frac{12+2}{2}\right)\frac{m}{s^2}$

OR $t_1 = 0.6$ s



(b) HAVE.. $v_{1.4} = v_{t_1} + A_2$

OR $v_{1.4} = 1.8\ \frac{m}{s} - (1.4 - 0.6)s \times 2\ \frac{m}{s^2}$

OR $v_{1.4} = 0.20\ \frac{m}{s}$

(CONTINUED)

11.70 CONTINUED

NOW.. $x_{1.4} = A_3 + A_4$, WHERE A_3 IS MOST EASILY DETERMINED USING INTEGRATING. THUS..

FOR $0 \le t \le t_1$: $a = \frac{-2-(-12)}{0.6}t - 12 = \frac{50}{3}t - 12$

NOW.. $\frac{dv}{dt} = a = \frac{50}{3}t - 12$

AT $t = 0$, $v = 6\ \frac{m}{s}$: $\int_6^v dv = \int_0^t (\frac{50}{3}t - 12)dt$

OR $v = 6 + \frac{25}{3}t^2 - 12t$

HAVE.. $\frac{dx}{dt} = v = 6 - 12t + \frac{25}{3}t^2$

THEN.. $A_3 = \int_0^{x_{t_1}} dx = \int_0^{0.6}(6 - 12t + \frac{25}{3}t^2)dt$

$= [6t - 6t^2 + \frac{25}{9}t^3]_0^{0.6} = 2.04$ m

ALSO.. $A_4 = (1.4 - 0.6)\left(\frac{1.8 + 0.2}{2}\right) = 0.8$ m

THEN.. $x_{1.4} = (2.04 + 0.8)$ m OR $x_{1.4} = 2.84$ m

11.71

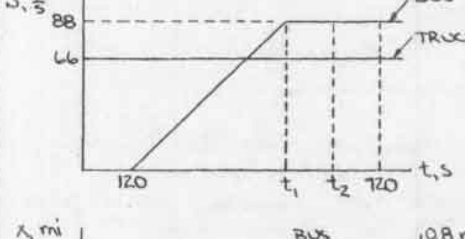
GIVEN: $v_T = 45\ \frac{mi}{h}$; AT $t = 0$, $x_T = 0$, $x_B = 0$; FOR $0 \le t \le 2$ MIN, $v_B = 0$; FOR $t > 2$ MIN, a_B = CONST UNTIL $v_B = 60\ \frac{mi}{h}$, THEN $a_B = 0$; AT $t = 12$ MIN, $x_B - x_T = 0.8$ mi

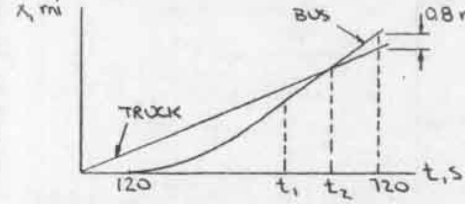
FIND: (a) t AND x WHEN $x_B = x_T$

(b) a_B

FIRST NOTE.. $45\ \frac{mi}{h} = 66\ \frac{ft}{s}$ $\quad 60\ \frac{mi}{h} = 88\ \frac{ft}{s}$

(a) ASSUMING THAT THE BUS REACHES $60\ \frac{mi}{h}$ (AT TIME t_1) BEFORE IT PASSES THE TRUCK (AT TIME t_2), THE v-t AND x-t CURVES CAN THEN BE DRAWN AS SHOWN.





AT $t = 720$ s (12 MIN): $x_B - x_T = 0.8$ mi

OR $[\frac{1}{2}(t_1 - 120)s \times (88\ \frac{ft}{s}) + (720 - t_1)s \times (88\ \frac{ft}{s})] - [720\ s(66\ \frac{ft}{s})]$

$= 0.8\ mi \times \frac{5280\ ft}{mi}$

OR $t_1 = 144$ s

AT $t = t_2$: $x_B = x_T$

OR $\frac{1}{2}(144 - 120)s \times (88\ \frac{ft}{s}) + (t_2 - 144)s \times (88\ \frac{ft}{s}) = (t_2\ s)(66\ \frac{ft}{s})$

OR $t_2 = 528$ s

THEN $t_2 > t_1 \Rightarrow$ ASSUMPTION CORRECT

$\therefore t_2 = 528$ s OR $t_2 = 8$ MIN 48 S

AT $t = t_2$: $x_B = x_T = (528\ s)(66\ \frac{ft}{s}) = 34\,848$ ft

OR $x_2 = 6.60$ mi

(b) HAVE.. $a_B = \frac{(v_B)_{t_1} - 0}{t_1 - 120}$

$= \frac{88\ ft/s}{(144 - 120)s}$

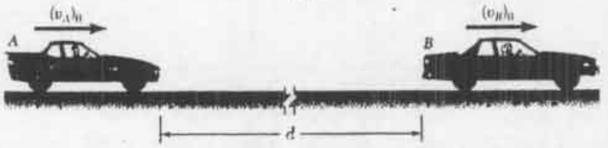
OR $a_B = 3.67\ \frac{ft}{s^2}$

11.72

GIVEN: AT $t=0$, $d=200$ ft ; $(v_A)_0 = 20\ \frac{mi}{h}$, $(v_B)_0 = 15\ \frac{mi}{h}$; AT $t = 45$ s, $x_A = x_B$; FOR $t > 0$, a_A = CONSTANT

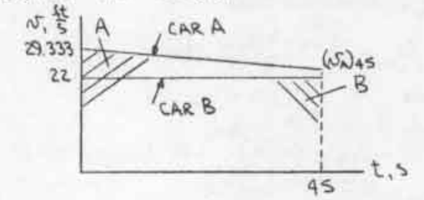
FIND: (a) a_A

(b) $v_{A/B}$ AT $t = 45$ s



(a) FIRST NOTE.. $20\ \frac{mi}{h} = 29.333\ \frac{ft}{s}$ $\quad 15\ \frac{mi}{h} = 22\ \frac{ft}{s}$

THE v-t CURVES FOR THE TWO CARS ARE THEN DRAWN AS SHOWN.



AT $t = 45$ s, $x_A = x_B$: $(AREA)_A = (AREA)_B + 200$ ft

OR $(45\ s)\left(\frac{29.333 + v_A}{2}\right)\frac{ft}{s} = (45\ s)(22\ \frac{ft}{s}) + 200$ ft

OR $(v_A)_{45} = 23.555\ \frac{ft}{s}$

THEN $a_A = \frac{(v_A)_{45} - (v_A)_0}{t_{45}} = \frac{(23.555 - 29.333)\frac{ft}{s}}{45\ s}$

OR $a_A = -0.1284\ \frac{ft}{s^2}$ ◄

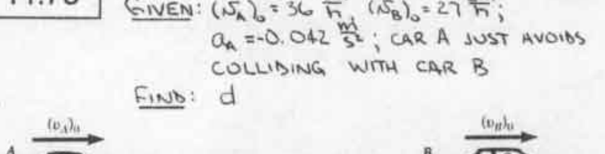
(b) HAVE -- $v_{A/B} = v_A - v_B = (23.555 - 22)\frac{ft}{s}$
$= 1.555\ \frac{ft}{s}$

OR $v_{A/B} = 1.060\ \frac{mi}{h}$ ◄

11.73

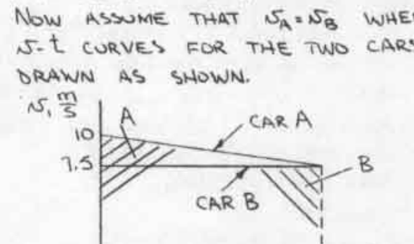
GIVEN: $(v_A)_0 = 36\ \frac{km}{h}$, $(v_B)_0 = 27\ \frac{km}{h}$; $a_A = -0.042\ \frac{m}{s^2}$; CAR A JUST AVOIDS COLLIDING WITH CAR B

FIND: d



FIRST NOTE.. $36\ \frac{km}{h} = 10\ \frac{m}{s}$ $\quad 27\ \frac{km}{h} = 7.5\ \frac{m}{s}$

NOW ASSUME THAT $v_A = v_B$ WHEN $x_A = x_B$; THE v-t CURVES FOR THE TWO CARS ARE THEN DRAWN AS SHOWN.



NOW.. $a_A = \frac{(v_A)_{t_1} - (v_A)_0}{t_1}$

OR $-0.042\ \frac{m}{s^2} = \frac{(7.5 - 10)\ m/s}{t_1}$

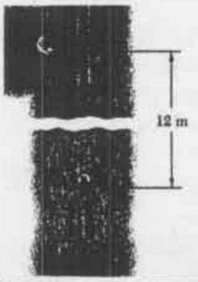
OR $t_1 = 59.524$ s

AT $t = t_1$, $x_A = x_B$: $(AREA)_A = (AREA)_B + d$

OR $(59.524\ s)\left(\frac{10 + 7.5}{2}\right)\frac{m}{s} = (59.524\ s)(7.5\ \frac{m}{s}) + d$

OR $d = 74.4$ m ◄

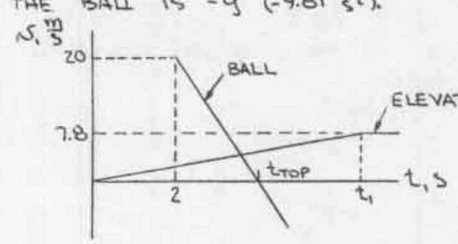
11.74



GIVEN: AT $t=0$, $v_E = 0$; FOR $0 \le v_E < 7.8\ \frac{m}{s}$, $a_E = 1.2\ \frac{m}{s^2}$↑; FOR $v_E = 7.8\ \frac{m}{s}$, $a_E = 0$; AT $t = 2$ s, $v_B = 20\ \frac{m}{s}$↑

FIND: t WHEN THE BALL HITS THE ELEVATOR

THE v-t CURVES OF THE BALL AND THE ELEVATOR ARE FIRST DRAWN AS SHOWN. NOTE THAT THE INITIAL SLOPE OF THE CURVE FOR THE ELEVATOR IS $1.2\ \frac{m}{s^2}$, WHILE THE SLOPE OF THE CURVE FOR THE BALL IS $-g$ $(-9.81\ \frac{m}{s^2})$.



THE TIME t_1 IS THE TIME WHEN v_E REACHES $7.8\ \frac{m}{s}$. THUS.. $v_E = \cancel{(v_E)_0}^{\,0} + a_E t$

OR $7.8\ \frac{m}{s} = (1.2\ \frac{m}{s^2})t_1$ OR $t_1 = 6.5$ s

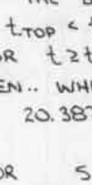
THE TIME t_{TOP} IS THE TIME AT WHICH THE BALL REACHES THE TOP OF ITS TRAJECTORY.

THUS.. $v_B = (v_B)_0 - g(t-2)$

OR $0 = 20\ \frac{m}{s} - (9.81\ \frac{m}{s^2})(t_{TOP} - 2)$s

OR $t_{TOP} = 4.0387$ s

USING THE COORDINATE SYSTEM SHOWN, HAVE..



$0 \le t \le t_1$: $y_E = -12\ m + (\frac{1}{2}a_E t^2)$ m

AT $t = t_{TOP}$: $y_B = \frac{1}{2}(4.0387 - 2)s \times (20\ \frac{m}{s})$
$= 20.387$ m

AND $y_E = -12\ m + \frac{1}{2}(1.2\ \frac{m}{s^2})(4.0387\ s)^2$
$= -2.213$ m

AT $t = [2 + 2(4.0387 - 2)]s = 6.0774$ s, $y_B = 0$

AND AT $t = t_1$, $y_E = -12\ m + \frac{1}{2}(6.5\ s)(7.8\ \frac{m}{s}) = 13.35$ m

∴ THE BALL HITS THE ELEVATOR ($y_B = y_E$) WHEN $t_{TOP} < t < t_1$.

FOR $t \ge t_{TOP}$: $y_B = 20.387\ m - [\frac{1}{2}g(t - t_{TOP})^2]$ m

THEN.. WHEN $y_B = y_E$ --

$20.387\ m - \frac{1}{2}(9.81\ \frac{m}{s^2})(t - 4.0387)^2$
$= -12\ m + \frac{1}{2}(1.2\ \frac{m}{s^2})(t\ s)^2$

OR $5.505t^2 - 39.6196t + 47.619 = 0$

SOLVING.. $t = 1.525$ s AND $t = 5.67$ s

NOW.. $t_{TOP} < t < t_1$ ⟹ $\quad t = 5.67$ s ◄

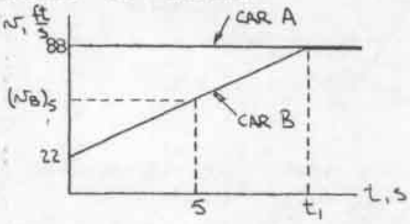
11.75

GIVEN: $(v_A)_0 = 60\ \frac{mi}{h}$, $(v_B)_0 = 15\ \frac{mi}{h}$; AT $t = 0$, $(x_A)_0 = -380$ ft, $(x_B)_0 = 0$; AT $t = 5$ s, $x_B = 200$ ft; FOR $15\ \frac{mi}{h} \le v_B < 60\ \frac{mi}{h}$, a_B = CONSTANT; FOR $v_B = 60\ \frac{mi}{h}$, $a_B = 0$

FIND: $x_{B/A}$ WHEN $v_B = 60\ \frac{mi}{h}$

FIRST NOTE.. $60\ \frac{mi}{h} = 88\ \frac{ft}{s}$ $\quad 15\ \frac{mi}{h} = 22\ \frac{ft}{s}$

THE v-t CURVES OF THE TWO CARS ARE THEN DRAWN AS SHOWN.



USING THE COORDINATE SYSTEM SHOWN, HAVE..

AT $t = 5$ s, $x_B = 200$ ft:

$$(5\text{ s})\left[\frac{22 + (v_B)_5}{2}\right]\frac{ft}{s} = 200\text{ ft}$$

OR $(v_B)_5 = 58\ \frac{ft}{s}$

THEN, USING SIMILAR TRIANGLES, HAVE..

$$\frac{(88-22)\text{ ft/s}}{t_1} = \frac{(58-22)\text{ ft/s}}{5\text{ s}} \quad (= a_B)$$

OR $t_1 = 9.1667$ s

FINALLY, AT $t = t_1$

$$x_{B/A} = x_B - x_A = \left[(9.1667\text{ s})\left(\frac{22+88}{2}\right)\frac{ft}{s}\right] - \left[-380\text{ ft} + (9.1667\text{ s})\left(88\ \frac{ft}{s}\right)\right]$$

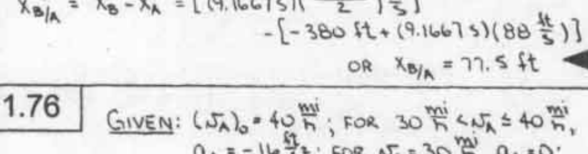
OR $x_{B/A} = 77.5$ ft ◀

11.76

GIVEN: $(v_A)_0 = 40\ \frac{mi}{h}$; FOR $30\ \frac{mi}{h} < v_A \le 40\ \frac{mi}{h}$, $a_A = -16\ \frac{ft}{s^2}$; FOR $v_A = 30\ \frac{mi}{h}$, $a_A = 0$; $(x_{A/B})_0 = 60$ ft; $(v_B)_0 = 45\ \frac{mi}{h}$; WHEN $x_B = 0$, $a_B = -20\ \frac{ft}{s^2}$; FOR $v_B = 28\ \frac{mi}{h}$, $a_B = 0$

FIND: (a) $(x_{A/B})_{MIN}$

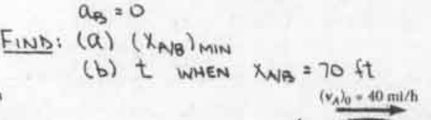
(b) t WHEN $x_{A/B} = 70$ ft



FIRST NOTE.. $40\ \frac{mi}{h} = 58.667\ \frac{ft}{s}$ $\quad 30\ \frac{mi}{h} = 44\ \frac{ft}{s}$

$45\ \frac{mi}{h} = 66\ \frac{ft}{s}$ $\quad 28\ \frac{mi}{h} = 41.067\ \frac{ft}{s}$

AT $t = 0$...



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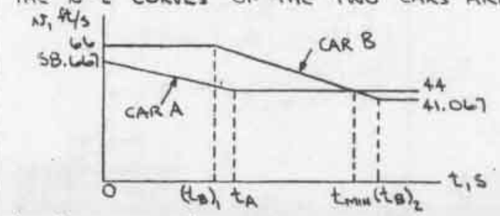
11.76 CONTINUED

THE v-t CURVES OF THE TWO CARS ARE AS SHOWN.



AT $t = 0$: CAR A ENTERS THE SPEED ZONE

$t = (t_B)_1$: CAR B ENTERS THE SPEED ZONE

$t = t_A$: CAR A REACHES ITS FINAL SPEED

$t = t_{MIN}$: $v_A = v_B$

$t = (t_B)_2$: CAR B REACHES ITS FINAL SPEED

(a) HAVE.. $a_A = \dfrac{(v_A)_{FINAL} - (v_A)_0}{t_A}$

OR $-16\ \frac{ft}{s^2} = \dfrac{(44 - 58.667)\ \frac{ft}{s}}{t_A}$

OR $t_A = 0.91669$ s

ALSO.. $60\text{ ft} = (t_B)_1 (v_B)_0$

OR $60\text{ ft} = (t_B)_1 (66\ \frac{ft}{s})$ OR $(t_B)_1 = 0.90909$ s

AND $a_B = \dfrac{(v_B)_{FINAL} - (v_B)_0}{(t_B)_2 - (t_B)_1}$

OR $-20\ \frac{ft}{s^2} = \dfrac{(41.067 - 66)\ \frac{ft}{s}}{[(t_B)_2 - 0.90909]\text{ s}}$

OR $(t_B)_2 = 2.15574$ s

CAR B WILL CONTINUE TO OVERTAKE CAR A WHILE $v_B > v_A$. THEREFORE, $(x_{A/B})_{MIN}$ WILL OCCUR WHEN $v_A = v_B$, WHICH OCCURS FOR

$(t_B)_1 < t_{MIN} < (t_B)_2$

FOR THIS TIME INTERVAL..

$v_A = 44\ \frac{ft}{s}$ $\quad v_B = (v_B)_0 + a_B[t - (t_B)_1]$

THEN.. AT $t = t_{MIN}$:

$$44\ \frac{ft}{s} = 66\ \frac{ft}{s} + \left(-20\ \frac{ft}{s^2}\right)(t_{MIN} - 0.90909)\text{ s}$$

OR $t_{MIN} = 2.00909$ s

FINALLY..

$$(x_{A/B})_{MIN} = (x_A)_{t_{MIN}} - (x_B)_{t_{MIN}}$$

$$= \left\{t_A\left[\frac{(v_A)_0 + (v_A)_{FINAL}}{2}\right] + (t_{MIN} - t_A)(v_A)_{FINAL}\right\}$$

$$- \left\{(x_B)_0 + (t_B)_1 (v_B)_0 + [t_{MIN} - (t_B)_1]\left[\frac{(v_B)_0 + (v_A)_{FINAL}}{2}\right]\right\}$$

$$= \left[(0.91669\text{ s})\left(\frac{58.667 + 44}{2}\right)\frac{ft}{s} + (2.00909 - 0.91669)\text{ s} \times \left(44\ \frac{ft}{s}\right)\right]$$

$$- \left[-60\text{ ft} + (0.90909\text{ s})\left(66\ \frac{ft}{s}\right) + (2.00909 - 0.90909)\text{ s} \times \left(\frac{66+44}{2}\right)\frac{ft}{s}\right]$$

$$= (47.057 + 48.066)\text{ ft} - (-60 + 60.000 + 60.500)\text{ ft}$$

$= 34.623$ ft OR $(x_{A/B})_{MIN} = 34.6$ ft ◀

(b) SINCE $(x_{A/B}) \le 60$ ft FOR $t \le t_{MIN}$, IT FOLLOWS THAT $x_{A/B} = 70$ ft FOR $t > (t_B)_2$ [NOTE.. $(t_B)_2 \simeq t_{MIN}$]. THEN, FOR $t > (t_B)_2$ --

$$x_{A/B} = (x_{A/B})_{MIN} + [(t - t_{MIN})(v_A)_{FINAL}] - \left\{[(t_B)_2 - (t_{MIN})]\left[\frac{(v_A)_{FINAL} + (v_B)_{FINAL}}{2}\right] + [t - (t_B)_2](v_B)_{FINAL}\right\}$$

OR

$$70\text{ ft} = 34.623\text{ ft} + \left[(t - 2.00909)\text{ s} \times \left(44\ \frac{ft}{s}\right)\right] - \left[(2.15574 - 2.00909)\text{ s} \times \left(\frac{44 + 41.067}{2}\right)\frac{ft}{s} + (t - 2.15574)\text{ s} \times \left(41.067\ \frac{ft}{s}\right)\right]$$

OR $t = 14.14$ s ◀

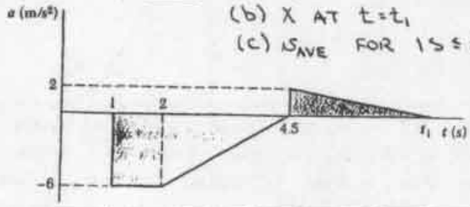
11.77

GIVEN: At $t=0$, $x=0$, $v=54\ \frac{km}{h}$; for $t>t_1$, $v=54\ \frac{km}{h}$

FIND: (a) t_1

(b) x at $t=t_1$

(c) v_{AVE} for $1\,s \le t \le t_1$



First note.. $54\ \frac{km}{h} = 15\ \frac{m}{s}$

(a) Have.. $v_b = v_a +$ (area under a-t curve from t_a to t_b)

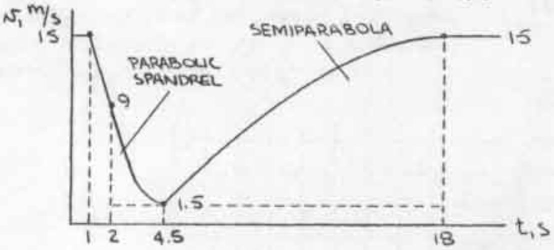
Then.. at $t=2\,s$: $v = 15-(1)(6) = 9\ \frac{m}{s}$

$t=4.5\,s$: $v = 9-\frac{1}{2}(2.5)(6) = 1.5\ \frac{m}{s}$

$t=t_1$: $15 = 1.5+\frac{1}{2}(t_1-4.5)(2)$

or $t_1 = 18\ s$ ◀

(b) Using the above values of the velocities, the v-t curve is drawn as shown.



Now.. x at $t=18\,s$..

$x_{18} = \cancel{x_0}^{\,0} + \Sigma$(area under the v-t curve from $t=0$ to $t=18\,s$)

$= (1\,s)(15\ \frac{m}{s}) + (1\,s)(\frac{15+9}{2})\frac{m}{s}$

$+[(2.5\,s)(1.5\ \frac{m}{s}) + \frac{1}{3}(2.5\,s)(7.5\ \frac{m}{s})]$

$+[(13.5\,s)(1.5\ \frac{m}{s}) + \frac{2}{3}(13.5\,s)(13.5\ \frac{m}{s})]$

$= [15+12+(3.75+6.25)+(20.25+121.50)]\ m$

$= 178.75\ m$ or $x_{18} = 178.8\ m$ ◀

(c) First note.. $x_1 = 15\ m$ $x_{18} = 178.75\ m$

Now.. $v_{AVE} = \frac{\Delta x}{\Delta t} = \frac{(178.75-15)\ m}{(18-1)\ s} = 9.6324\ \frac{m}{s}$

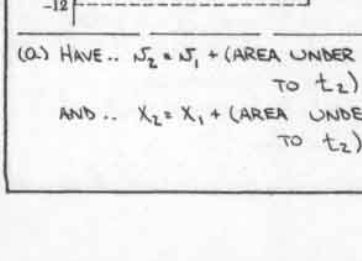
or $v_{AVE} = 34.7\ \frac{km}{h}$ ◀

11.78

GIVEN: At $t=0$, $x=0$, $v=8\ \frac{m}{s}$

CONSTRUCT: (a) v-t and x-t curves for $0<t<4\,s$

FIND: (b) x at $t=3\,s$



(a) Have.. $v_2 = v_1 +$ (area under a-t curve from t_1 to t_2)

and.. $x_2 = x_1 +$ (area under v-t curve from t_1 to t_2)

(CONTINUED)

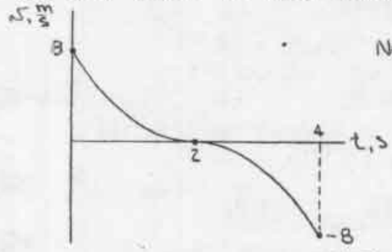
11.78 CONTINUED

Then, using the formula for the area of a parabolic spandrel, have..

At $t=2\,s$: $v = 8-\frac{1}{3}(2)(12) = 0$

$t=4\,s$: $v = 0-\frac{1}{3}(2)(12) = -8\ \frac{m}{s}$

The v-t curve is then drawn as shown.

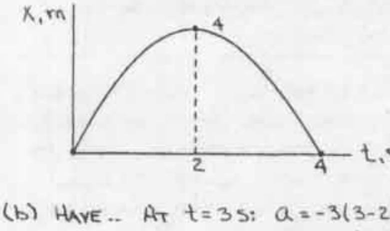


Note: The area under each portion of the curve is a spandrel of order $n=3$.

Now.. at $t=2\,s$: $x = 0+\frac{(2)(8)}{3+1} = 4\ m$

$t=4\,s$: $x = 4-\frac{(2)(8)}{3+1} = 0$

The x-t curve is then drawn as shown.



(b) Have.. At $t=3\,s$: $a = -3(3-2)^2 = -3\ \frac{m}{s^2}$

$v = 0-\frac{1}{3}(1)(3) = -1\ \frac{m}{s}$

$x = 4-\frac{(1)(1)}{3+1}$

or $x_3 = 3.75\ m$ ◀

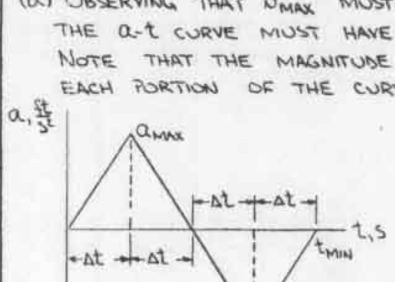
11.79

GIVEN: At $t=0$, $x=0$, $v=0$; $x_{MAX} = 1.2\ ft$; when $x=x_{MAX}$, $v=0$; $|(\frac{da}{dt})_{MAX}| = 4.8\ \frac{ft}{s^3}$

FIND: (a) t_{MIN} for $x_{MAX} = 1.2\ ft$

(b) v_{MAX} and v_{AVE} for $0 \le t \le t_{MIN}$

(a) Observing that v_{MAX} must occur at $t=\frac{1}{2}t_{MIN}$, the a-t curve must have the shape shown. Note that the magnitude of the slope of each portion of the curve is $4.8\ ft/s^2/s$.

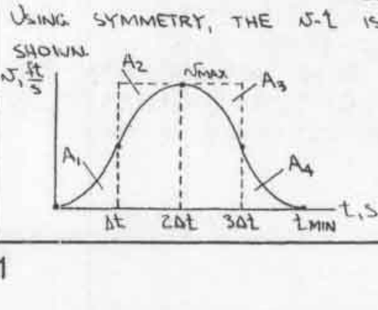


Have.. at $t=\Delta t$: $v = 0+\frac{1}{2}(\Delta t)(a_{MAX}) = \frac{1}{2}a_{MAX}\Delta t$

$t=2\Delta t$: $v_{MAX} = \frac{1}{2}a_{MAX}\Delta t + \frac{1}{2}(\Delta t)(a_{MAX})$

$= a_{MAX}\Delta t$

Using symmetry, the v-t is then drawn as shown.



(CONTINUED)

11.79 CONTINUED

NOTING THAT $A_1 = A_2 = A_3 = A_4$ AND THAT THE AREA UNDER THE v-t CURVE IS EQUAL TO X_{MAX}, HAVE..

$$(2\Delta t)(v_{MAX}) = X_{MAX}$$

$v_{MAX} = a_{MAX}\,\Delta t \Rightarrow 2a_{MAX}\,\Delta t^2 = X_{MAX}$

NOW.. $\frac{a_{MAX}}{\Delta t} = 4.8\ \mathrm{ft/s^2/s}$ SO THAT

$$2\left(4.8\,\Delta t\ \tfrac{\mathrm{ft}}{\mathrm{s^3}}\right)\Delta t^2 = 1.2\ \mathrm{ft}$$

OR $\Delta t = 0.5\ \mathrm{s}$

THEN $t_{MIN} = 4\Delta t$ OR $t_{MIN} = 2.00\ \mathrm{s}$ ◀

(b) HAVE.. $v_{MAX} = a_{MAX}\,\Delta t = (4.8\ \tfrac{\mathrm{ft}}{\mathrm{s^3}} \times \Delta t)\Delta t$
$= 4.8\ \mathrm{ft/s^2/s} \times (0.5\ \mathrm{s})^2$

OR $v_{MAX} = 1.2\ \frac{\mathrm{ft}}{\mathrm{s}}$ ◀

ALSO.. $v_{AVE} = \frac{\Delta x}{\Delta t_{TOTAL}} = \frac{1.2\ \mathrm{ft}}{2.00\ \mathrm{s}}$

OR $v_{AVE} = 0.6\ \frac{\mathrm{ft}}{\mathrm{s}}$ ◀

11.80

GIVEN: $X_{MAX} = 1.6\ \mathrm{mi}$; $|a_{MAX}| = 4\ \frac{\mathrm{ft}}{\mathrm{s^2}}$, $|(\frac{da}{dt})_{MAX}| = 0.8\ \mathrm{ft/s^2/s}$; $v_{MAX} = 20\ \frac{\mathrm{mi}}{\mathrm{h}}$

FIND: (a) t_{MIN} FOR $X_{MAX} = 1.6\ \mathrm{mi}$

(b) v_{AVE}

FIRST NOTE.. $20\ \frac{\mathrm{mi}}{\mathrm{h}} = 29.333\ \frac{\mathrm{ft}}{\mathrm{s}}$ $\quad 1.6\ \mathrm{mi} = 8448\ \mathrm{ft}$

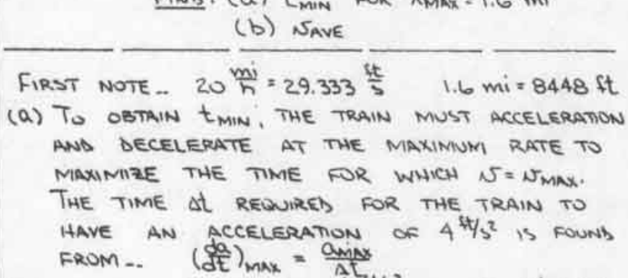
(a) TO OBTAIN t_{MIN}, THE TRAIN MUST ACCELERATION AND DECELERATE AT THE MAXIMUM RATE TO MAXIMIZE THE TIME FOR WHICH $v = v_{MAX}$. THE TIME Δt REQUIRED FOR THE TRAIN TO HAVE AN ACCELERATION OF $4\ \mathrm{ft/s^2}$ IS FOUND FROM.. $\left(\frac{da}{dt}\right)_{MAX} = \frac{a_{MAX}}{\Delta t}$

OR $\Delta t = \frac{4\ \mathrm{ft/s^2}}{0.8\ \mathrm{ft/s^2/s}}$ OR $\Delta t = 5\ \mathrm{s}$

NOW.. AFTER 5 s THE SPEED OF THE TRAIN IS.. $v_5 = \frac{1}{2}(\Delta t)(a_{MAX})$ (SINCE $\frac{da}{dt}$ = CONSTANT)

OR $v_5 = \frac{1}{2}(5\ \mathrm{s})(4\ \frac{\mathrm{ft}}{\mathrm{s^2}}) = 10\ \frac{\mathrm{ft}}{\mathrm{s}}$

THEN, SINCE $v_5 < v_{MAX}$, THE TRAIN WILL CONTINUE TO ACCELERATE AT $4\ \mathrm{ft/s^2}$ UNTIL $v = v_{MAX}$. THE a-t CURVE MUST THEN HAVE THE SHAPE SHOWN. NOTE THAT THE MAGNITUDE OF THE SLOPE OF EACH INCLINED PORTION OF THE CURVE IS $0.8\ \mathrm{ft/s^2/s}$.



NOW.. AT $t = (10 + \Delta t_1)\ \mathrm{s}$, $v = v_{MAX}$:

$$\therefore\ 2\left[\tfrac{1}{2}(5\ \mathrm{s})(4\ \tfrac{\mathrm{ft}}{\mathrm{s^2}})\right] + (\Delta t_1)(4\ \tfrac{\mathrm{ft}}{\mathrm{s^2}}) = 29.333\ \tfrac{\mathrm{ft}}{\mathrm{s}}$$

OR $\Delta t_1 = 2.3333\ \mathrm{s}$

THEN.. AT $t = 5\ \mathrm{s}$: $v = 0 + \frac{1}{2}(5)(4) = 10\ \frac{\mathrm{ft}}{\mathrm{s}}$

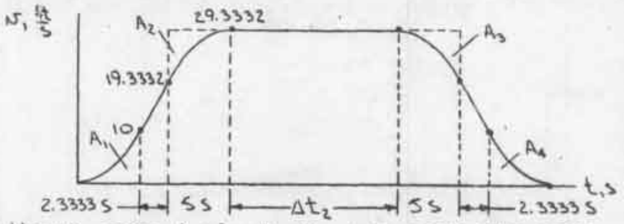
$t = 7.3333\ \mathrm{s}$: $v = 10 + (2.3333)(4) = 19.3332\ \frac{\mathrm{ft}}{\mathrm{s}}$

$t = 12.3333\ \mathrm{s}$: $v = 19.3332 + \frac{1}{2}(5)(4) = 29.3332\ \frac{\mathrm{ft}}{\mathrm{s}}$

USING SYMMETRY, THE v-t CURVE IS THEN DRAWN AS SHOWN.

(CONTINUED)

11.80 CONTINUED



NOTING THAT $A_1 = A_2 = A_3 = A_4$ AND THAT THE AREA UNDER THE v-t CURVE IS EQUAL TO X_{MAX}, HAVE..

$$2\left[(2.3333\ \mathrm{s})\left(\tfrac{10 + 19.3332}{2}\right)\tfrac{\mathrm{ft}}{\mathrm{s}}\right] + (10 + \Delta t_2)\mathrm{s} \times (29.3332\ \tfrac{\mathrm{ft}}{\mathrm{s}}) = 8448\ \mathrm{ft}$$

OR $\Delta t_2 = 275.67\ \mathrm{s}$

THEN.. $t_{MIN} = 4(5\ \mathrm{s}) + 2(2.3333\ \mathrm{s}) + 275.67\ \mathrm{s}$
$= 300.34\ \mathrm{s}$ OR $t_{MIN} = 5.01\ \mathrm{min}$ ◀

(b) HAVE.. $v_{AVE} = \frac{\Delta x}{\Delta t} = \frac{1.6\ \mathrm{mi}}{300.34\ \mathrm{s}} \times \frac{3600\ \mathrm{s}}{1\ \mathrm{h}}$

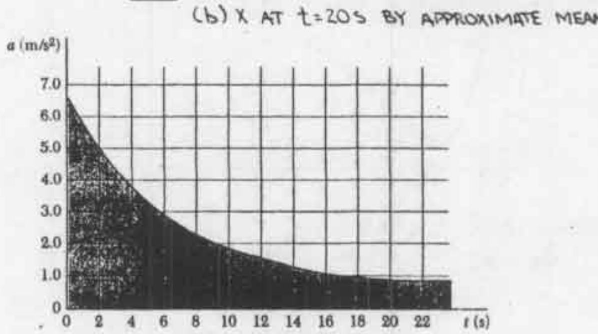
OR $v_{AVE} = 19.18\ \frac{\mathrm{mi}}{\mathrm{h}}$ ◀

11.81

GIVEN: a-t CURVE; AT $t = 0$, $x = 0$, $v = 0$

FIND: (a) v AT $t = 8\ \mathrm{s}$ BY APPROXIMATE MEANS

(b) x AT $t = 20\ \mathrm{s}$ BY APPROXIMATE MEANS



SOLUTION PROCEDURE

1. THE a-t CURVE IS FIRST APPROXIMATED WITH A SERIES OF RECTANGLES, EACH OF WIDTH $\Delta t = 2\ \mathrm{s}$. THE AREA $(\Delta t)(a_{AVE})$ OF EACH RECTANGLE IS APPROXIMATELY EQUAL TO THE CHANGE IN VELOCITY Δv FOR THE SPECIFIED INTERVAL OF TIME. THUS,
$$\Delta v \simeq a_{AVE}\,\Delta t$$
WHERE THE VALUES OF a_{AVE} AND Δv ARE GIVEN IN COLUMNS 1 AND 2, RESPECTIVELY, OF THE FOLLOWING TABLE.
2. NOTING THAT $v_0 = 0$ AND THAT
$$v_2 = v_1 + \Delta v_{12}$$
WHERE Δv_{12} IS THE CHANGE IN VELOCITY BETWEEN TIMES t_1 AND t_2, THE VELOCITY AT THE END OF EACH 2 s INTERVAL CAN BE COMPUTED; SEE COLUMN 3 OF THE TABLE AND THE v-t CURVE.
3. THE v-t CURVE IS NEXT APPROXIMATED WITH A SERIES OF RECTANGLES, EACH OF WIDTH $\Delta t = 2\ \mathrm{s}$. THE AREA $(\Delta t)(v_{AVE})$ OF EACH RECTANGLE IS APPROXIMATELY EQUAL TO THE CHANGE IN POSITION Δx FOR THE SPECIFIED INTERVAL OF TIME. THUS,
$$\Delta x \simeq v_{AVE}\,\Delta t$$

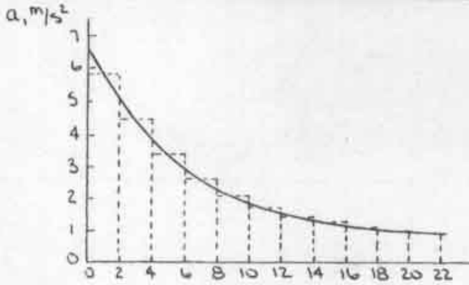
(CONTINUED)

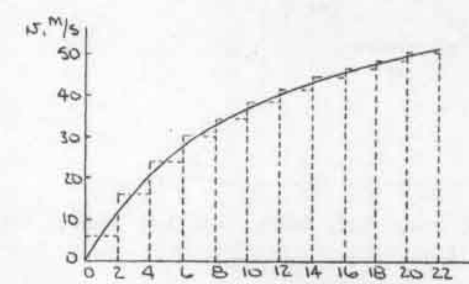
11.81 CONTINUED

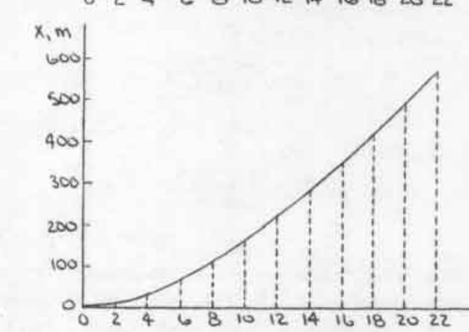
WHERE v_{AVE} AND Δx ARE GIVEN IN COLUMNS 4 AND 5, RESPECTIVELY, OF THE TABLE.

4. WITH $x_0 = 0$ AND NOTING THAT
$$x_2 = x_1 + \Delta x_{12}$$
WHERE Δx_{12} IS THE CHANGE IN POSITION BETWEEN TIMES t_1 AND t_2, THE POSITION AT THE END OF EACH 2 S INTERVAL CAN BE COMPUTED; SEE COLUMN 6 OF THE TABLE AND THE X-t CURVE.

		1	2	3	4	5	6
t, s	a, m/s²	a_{AVE}, m/s²	Δv, m/s	v, m/s	v_{AVE}, m/s	Δx, m	x, m
0	6.63			0			0
		5.86	11.72		5.86	11.72	
2	5.08			11.72			11.72
		4.47	8.94		16.19	32.38	
4	3.86			20.66			44.10
		3.38	6.76		24.04	48.08	
6	2.90			27.42			92.18
		2.58	5.16		30.00	60.00	
8	2.25			32.58			152.18
		2.06	4.12		34.64	69.28	
10	1.87			36.70			221.46
		1.71	3.42		38.41	76.82	
12	1.54			40.12			298.28
		1.42	2.84		41.54	83.08	
14	1.29			42.96			381.36
		1.23	2.46		44.19	88.38	
16	1.16			45.42			469.74
		1.10	2.20		46.52	93.04	
18	1.03			47.62			562.78
		1.00	2.00		48.62	97.24	
20	0.97			49.62			660.02
		0.94	1.88		50.56	101.12	
22	0.90			51.50			761.14



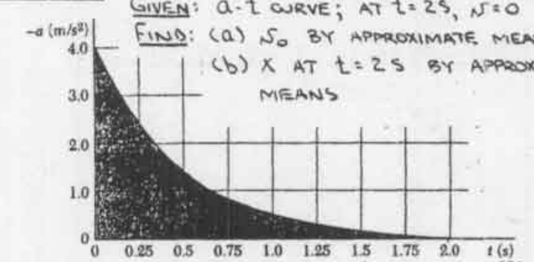




(a) AT $t = 8$ S, $v = 32.58 \frac{m}{s}$ OR $v = 117.3 \frac{km}{h}$ ◀

(b) AT $t = 20$ S, $x = 660$ m ◀

11.82



GIVEN: a-t CURVE; AT $t = 2$ S, $v = 0$

FIND: (a) v_0 BY APPROXIMATE MEANS

(b) X AT $t = 2$ S BY APPROXIMATE MEANS

SOLUTION PROCEDURE

1. THE a-t CURVE IS FIRST APPROXIMATED WITH A SERIES OF RECTANGLES, EACH OF WIDTH $\Delta t = 0.25$ S. THE AREA $(\Delta t)(a_{AVE})$ OF EACH RECTANGLE IS APPROXIMATELY EQUAL TO THE CHANGE IN VELOCITY Δv FOR THE SPECIFIED INTERVAL OF TIME. THUS,
$$\Delta v \simeq a_{AVE}\,\Delta t$$
WHERE THE VALUES OF a_{AVE} AND Δv ARE GIVEN IN COLUMNS 1 AND 2, RESPECTIVELY, OF THE FOLLOWING TABLE.
2. NOW.. $v(2) = v_0 + \int_0^2 a\,dt = 0$
AND APPROXIMATING THE AREA $\int_0^2 a\,dt$ UNDER THE a-t CURVE BY $\sum a_{AVE}\,\Delta t = \sum \Delta v$, THE INITIAL VELOCITY IS THEN EQUAL TO
$$v_0 = -\sum \Delta v$$
FINALLY, USING
$$v_2 = v_1 + \Delta v_{12}$$
WHERE Δv_{12} IS THE CHANGE IN VELOCITY BETWEEN TIMES t_1 AND t_2, THE VELOCITY AT THE END OF EACH 0.25 INTERVAL CAN BE COMPUTED; SEE COLUMN 3 OF THE TABLE AND THE v-t CURVE.
3. THE v-t CURVE IS THEN APPROXIMATED WITH A SERIES OF RECTANGLES, EACH OF WIDTH 0.25 S. THE AREA $(\Delta t)(v_{AVE})$ OF EACH RECTANGLE IS APPROXIMATELY EQUAL TO THE CHANGE IN POSITION Δx FOR THE SPECIFIED INTERVAL OF TIME. THUS.. $\Delta x \simeq v_{AVE}\,\Delta t$
WHERE v_{AVE} AND Δx ARE GIVEN IN COLUMNS 4 AND 5, RESPECTIVELY, OF THE TABLE.
4. WITH $x_0 = 0$ AND NOTING THAT
$$x_2 = x_1 + \Delta x_{12}$$
WHERE Δx_{12} IS THE CHANGE IN POSITION BETWEEN TIMES t_1 AND t_2, THE POSITION AT THE END OF EACH 0.25 S INTERVAL CAN BE COMPUTED; SEE COLUMN 6 OF THE TABLE AND THE X-t CURVE.

		1	2	3	4	5	6
t, s	a, m/s²	a_{AVE}, m/s²	Δv, m/s	v, m/s	v_{AVE}, m/s	Δx, m	x, m
0	-4.00			1.914			0
		-3.215	-0.804		1.512	0.378	
0.25	-2.43			1.110			0.378
		-1.915	-0.479		0.871	0.218	
0.50	-1.40			0.631			0.596
		-1.125	-0.281		0.491	0.123	
0.75	-0.85			0.350			0.719
		-0.675	-0.169		0.266	0.067	
1.00	-0.50			0.181			0.786
		-0.390	-0.098		0.132	0.033	
1.25	-0.28			0.083			0.819
		-0.205	-0.051		0.058	0.015	
1.50	-0.13			0.032			0.834
		-0.095	-0.024		0.020	0.005	
1.75	-0.06			0.008			0.839
		-0.030	-0.008		0.004	0.001	
2.00	0			0			0.840

$\sum \Delta v = -1.914$ m/s

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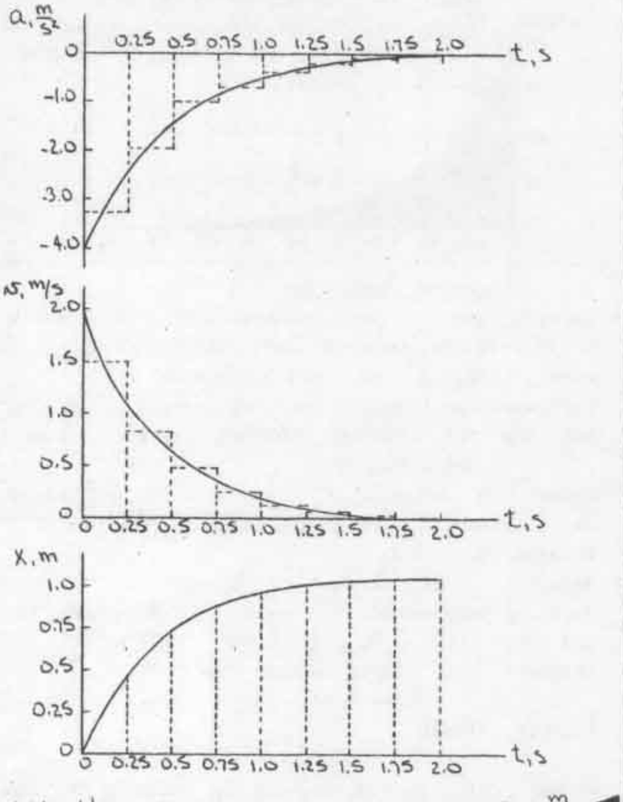
11.82 CONTINUED



(a) HAD FOUND $\qquad v_0 = 1.914 \frac{m}{s}$

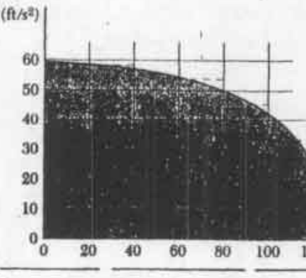
(b) AT $t = 2$ s $\qquad x = 0.840$ m

11.83

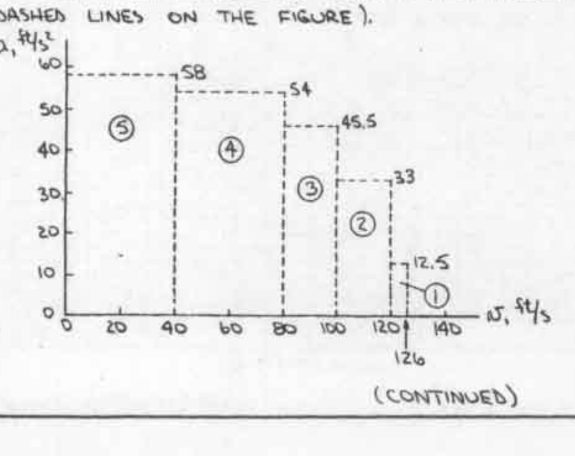
GIVEN: a-v CURVE; $v_0 = 126$ ft/s

FIND: (a) t WHEN $v=0$ BY APPROXIMATE MEANS

(b) x WHEN $v=0$ BY APPROXIMATE MEANS



THE GIVEN CURVE IS APPROXIMATED BY A SERIES OF UNIFORMLY ACCELERATED MOTIONS (THE HORIZONTAL DASHED LINES ON THE FIGURE).



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11.83 CONTINUED

FOR UNIFORMLY ACCELERATED MOTION..

$$v_2^2 = v_1^2 + 2a(x_2 - x_1) \qquad v_2 = v_1 + a(t_2 - t_1)$$

OR

$$\Delta x = \frac{v_2^2 - v_1^2}{2a} \qquad \Delta t = \frac{v_2 - v_1}{a}$$

FOR THE FIVE REGIONS SHOWN ABOVE, HAVE..

REGION	v_1, ft/s	v_2, ft/s	a, ft/s²	Δx, ft	Δt, s
1	126	120	-12.5	59.0	0.480
2	120	100	-33	66.7	0.606
3	100	80	-45.5	39.6	0.440
4	80	40	-54	44.4	0.741
5	40	0	-58	13.8	0.690
Σ				223.5	2.957

(a) FROM THE TABLE, WHEN $v=0$ $\qquad t = 2.96$ s

(b) FROM THE TABLE AND ASSUMING $x_0 = 0$, WHEN $v=0$ $\qquad x = 224$ ft

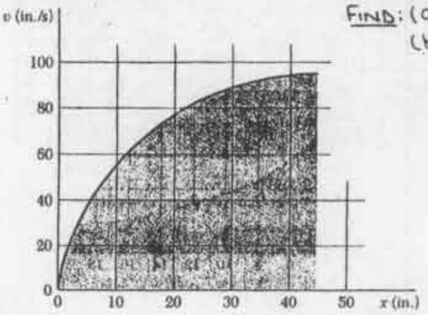
11.84

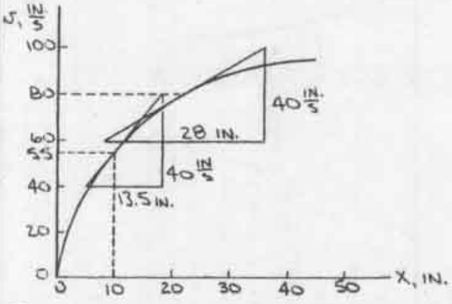
GIVEN: v-x CURVE

FIND: (a) a WHEN $x = 10$ in.

(b) a WHEN $v = 80 \frac{in.}{s}$

USE APPROXIMATE MEANS





FIRST NOTE THAT THE SLOPE OF THE ABOVE CURVE IS $\frac{dv}{dx}$. NOW...

$$a = v \frac{dv}{dx}$$

(a) WHEN $x = 10$ IN., $v = 55 \frac{in.}{s}$

THEN..
$$a = 55 \frac{in.}{s}\left(\frac{40 \text{ in./s}}{13.5 \text{ in.}}\right)$$

OR $\qquad a = 163.0 \frac{in.}{s^2}$

(b) WHEN $v = 80 \frac{in.}{s}$, HAVE

$$a = 80 \frac{in.}{s}\left(\frac{40 \text{ in./s}}{28 \text{ in.}}\right)$$

OR $\qquad a = 114.3 \frac{in.}{s^2}$

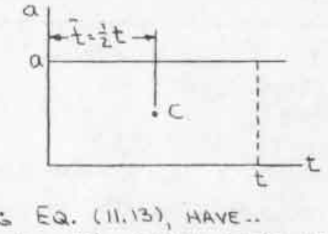
NOTE: TO USE THE METHOD OF MEASURING THE SUBNORMAL OUTLINED AT THE END OF SECTION 11.8, IT IS NECESSARY THAT THE SAME SCALE BE USED FOR THE X AND v AXES (e.g., 1 IN. = 50 IN., 1 IN. = 50 IN./s). IN THE ABOVE SOLUTION, Δv AND Δx WERE MEASURED DIRECTLY, SO DIFFERENT SCALES COULD BE USED.

11.85

GIVEN: MOMENT-AREA METHOD OF SECTION 11.8

DERIVE: $x = x_0 + v_0 t + \frac{1}{2}at^2$ FOR A PARTICLE IN UNIFORMLY ACCELERATED RECTILINEAR MOTION

THE a-t CURVE FOR UNIFORMLY ACCELERATED MOTION IS AS SHOWN.



USING EQ. (11.13), HAVE..

$x = x_0 + v_0 t + (\text{AREA UNDER a-t CURVE})(t - \bar{t})$

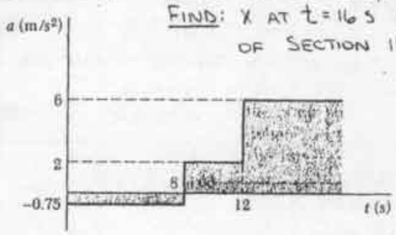
$= x_0 + v_0 t + (t \times a)(t - \frac{1}{2}t)$

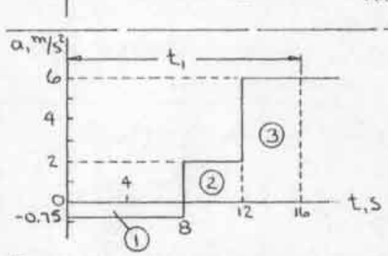
$= x_0 + v_0 t + \frac{1}{2}at^2$ Q.E.D.

11.86

GIVEN: a-t CURVE; $v_0 = -2\ \frac{m}{s}$

FIND: x AT t = 16 s USING THE METHOD OF SECTION 11.86





THE AREA UNDER THE CURVE IS FIRST DIVIDED INTO THREE REGIONS AS SHOWN.

FROM THE DISCUSSION FOLLOWING EQ. (11.13) AND ASSUMING $x_0 = 0$, HAVE..

$x = \cancel{x_0}^{0} + v_0 t_1 + \Sigma A(t_1 - \bar{t})$

WHERE A IS THE AREA OF A REGION AND $\bar{t}$ IS THE DISTANCE TO ITS CENTROID. THEN, FOR $t_1 = 16$ s...

$x = (-2\ \frac{m}{s})(16\ s) + \{[(8\ s)(-0.75\ \frac{m}{s^2})](16-4)s + [(4\ s)(2\ \frac{m}{s^2})](16-10)s + [(4\ s)(6\ \frac{m}{s^2})](16-14)s\}$

$= [-32 + (-72 + 48 + 48)]$ m

OR $x = -8.00$ m

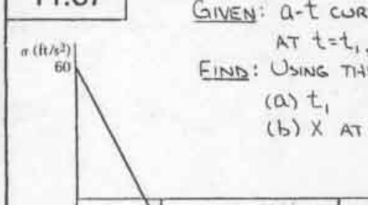
11.87

GIVEN: a-t CURVE; AT t = 0, $v = 7.5$ ft/s; AT $t = t_1$, $v = 0$

FIND: USING THE METHOD OF SECTION 11.8

(a) t_1

(b) x AT $t = t_1$



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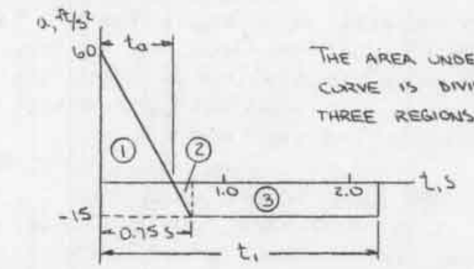
11.87 CONTINUED



THE AREA UNDER THE CURVE IS DIVIDED INTO THREE REGIONS AS SHOWN

(a) FIRST NOTE.. $\frac{t_a}{60} = \frac{0.75}{75}$ OR $t_a = 0.60$ s

NOW.. $v = v_0 + \int_0^t a\,dt$

WHERE THE INTEGRAL IS EQUAL TO THE AREA UNDER THE a-t CURVE. THEN, WITH $v_0 = 7.5\ \frac{ft}{s}$, $v_{t_1} = 0$

HAVE.. $0 = 7.5\ \frac{ft}{s} + [\frac{1}{2}(0.6\ s)(60\ \frac{ft}{s^2}) - \frac{1}{2}(0.15\ s)(15\ \frac{ft}{s^2}) - (t_1 - 0.75)s \times (15\ \frac{ft}{s^2})]$

OR $t_1 = 2.375$ s $\quad t_1 = 2.38$ s

(b) FROM THE DISCUSSION FOLLOWING EQ. (11.13) AND ASSUMING $x_0 = 0$, HAVE

$x = \cancel{x_0}^{0} + v_0 t_1 + \Sigma A(t_1 - \bar{t})$

WHERE A IS THE AREA OF A REGION AND $\bar{t}$ IS THE DISTANCE TO ITS CENTROID. THEN FOR $t_1 = 2.375$ s..

$x = (7.5\ \frac{ft}{s})(2.375\ s) + \{[\frac{1}{2}(0.6\ s)(60\ \frac{ft}{s^2})](2.375 - 0.2)s$
$- [\frac{1}{2}(0.15\ s)(15\ \frac{ft}{s^2})](2.375 - 0.70)s$
$- [(1.625\ s)(15\ \frac{ft}{s^2})][2.375 - (0.75 + \frac{1}{2} \times 1.625)]s\}$

$= [17.8125 + (39.1500 - 1.8844 - 19.8047)]$ ft

OR $x = 35.3$ ft

11.88

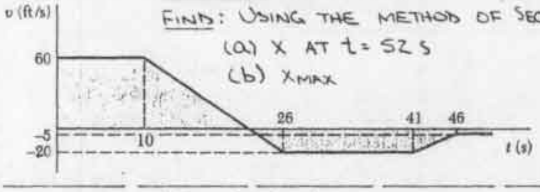
GIVEN: v-t CURVE FOR THE STRAIGHT LINE MOTION OF A PARTICLE; AT t = 0, $x = -540$ ft

CONSTRUCT: a-t CURVE

FIND: USING THE METHOD OF SECTION 11.8

(a) x AT t = 52 s

(b) x_{MAX}



HAVE.. $a = \frac{dv}{dt}$ WHERE $\frac{dv}{dt}$ IS THE SLOPE OF THE v-t CURVE. THEN..

FROM t = 0 TO t = 10 s: v = CONSTANT ⇒ a = 0

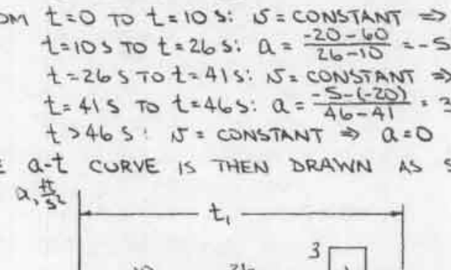
t = 10 s TO t = 26 s: $a = \frac{-20-60}{26-10} = -5$ ft/s²

t = 26 s TO t = 41 s: v = CONSTANT ⇒ a = 0

t = 41 s TO t = 46 s: $a = \frac{-5-(-20)}{46-41} = 3$ ft/s²

t > 46 s: v = CONSTANT ⇒ a = 0

THE a-t CURVE IS THEN DRAWN AS SHOWN.



(a) FROM THE DISCUSSION FOLLOWING EQ. (11.13), HAVE.. $x = x_0 + v_0 t_1 + \Sigma A(t_1 - \bar{t})$

(CONTINUED)

11.88 CONTINUED

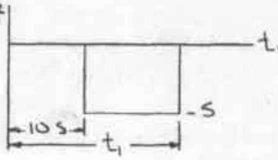
WHERE A IS THE AREA OF A REGION AND $\bar{t}$ IS THE DISTANCE TO ITS CENTROID. THEN, FOR $t_1 = 52$ s...

$$x = -540 \text{ ft} + (60 \tfrac{ft}{s})(52 s) + \{-[(16 s)(5 \tfrac{ft}{s^2})](52-18)s + [(5 s)(3 \tfrac{ft}{s^2})](52-43.5)s\}$$

$$= [-540 + (3120) + (-2720 + 127.5)] \text{ ft}$$

OR $x = -12.50$ ft ◀

(b) NOTING THAT x_{MAX} OCCURS WHEN $v = 0$ $(\frac{dx}{dt} = 0)$, IT IS SEEN FROM THE v-t CURVE THAT x_{MAX} OCCURS FOR $10 s < t < 26 s$. ALTHOUGH "SIMILAR TRIANGLES" COULD BE USED TO DETERMINE THE TIME AT WHICH $x = x_{MAX}$ (SEE THE SOLUTION TO PROBLEM 11.63), THE FOLLOWING METHOD WILL BE USED.



FOR $10 s < t_1 < 26 s$, HAVE

$$x = -540 + 60t_1 - [(t_1 - 10)(5)][\tfrac{1}{2}(t_1 - 10)] \quad (\text{ft})$$

$$= -540 + 60t_1 - \tfrac{5}{2}(t_1 - 10)^2$$

WHEN $x = x_{MAX}$: $\frac{dx}{dt} = 60 - 5(t_1 - 10) = 0$

OR $(t_1)_{x_{MAX}} = 22$ s

THEN.. $x_{MAX} = -540 + 60(22) - \frac{5}{2}(22-10)^2$

OR $x_{MAX} = 420$ ft ◀

11.89

GIVEN: $x = 4t^4 - 6t$, $y = 6t^3 - 2t^2$ $x, y \sim$ mm, $t \sim$ s

FIND: v AND a AT
(a) $t = 1$ s
(b) $t = 2$ s
(c) $t = 4$ s

HAVE.. $x = 4t^4 - 6t$ $\qquad y = 6t^3 - 2t^2$

THEN $v_x = \frac{dx}{dt} = 16t^3 - 6$ $\qquad v_y = \frac{dy}{dt} = 18t^2 - 4t$

AND $a_x = \frac{dv_x}{dt} = 48t^2$ $\qquad a_y = \frac{dv_y}{dt} = 36t - 4$

(a) AT $t = 1$ s: $v_x = 16(1)^3 - 6 = 10 \frac{mm}{s}$ $\qquad v_y = 18(1)^2 - 4(1) = 14 \frac{mm}{s}$

OR $v = 17.20 \frac{mm}{s} \measuredangle 54.5°$ ◀

$a_x = 48(1)^2 = 48 \frac{mm}{s^2}$ $\qquad a_y = 36(1) - 4 = 32 \frac{mm}{s^2}$

OR $a = 57.7 \frac{mm}{s^2} \measuredangle 33.7°$ ◀

(b) AT $t = 2$ s: $v_x = 16(2)^3 - 6 = 122 \frac{mm}{s}$ $\qquad v_y = 18(2)^2 - 4(2) = 64 \frac{mm}{s}$

OR $v = 137.8 \frac{mm}{s} \measuredangle 27.7°$ ◀

$a_x = 48(2)^2 = 192 \frac{mm}{s^2}$ $\qquad a_y = 36(2) - 4 = 68 \frac{mm}{s^2}$

OR $a = 204 \frac{mm}{s^2} \measuredangle 19.50°$ ◀

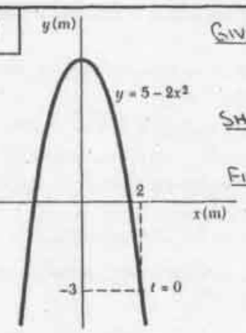
(c) AT $t = 4$ s: $v_x = 16(4)^3 - 6 = 1018 \frac{mm}{s}$ $\qquad v_y = 18(4)^2 - 4(4) = 272 \frac{mm}{s}$

OR $v = 1054 \frac{mm}{s} \measuredangle 14.96°$ ◀

$a_x = 48(4)^2 = 768 \frac{mm}{s^2}$ $\qquad a_y = 36(4) - 4 = 140 \frac{mm}{s^2}$

OR $a = 781 \frac{mm}{s^2} \measuredangle 10.33°$ ◀

11.90



GIVEN: $x = 2\cos \pi t$, $y = 1 - 4\cos 2\pi t$ $x, y \sim$ m, $t \sim$ s

SHOW: PATH IS THE PARABOLA $y = 5 - 2x^2$

FIND: v AND a AT
(a) $t = 0$
(b) $t = 1.5$ s

HAVE.. $x = 2\cos \pi t$ $\qquad y = 1 - 4\cos 2\pi t$

THEN.. $y = 1 - 4(2\cos^2 \pi t - 1)$

$= 5 - 8(\frac{x}{2})^2$

OR $y = 5 - 2x^2$ Q.E.D. ◀

NOW.. $v_x = \frac{dx}{dt} = -2\pi \sin \pi t$ $\qquad v_y = \frac{dy}{dt} = 8\pi \sin 2\pi t$

AND $a_x = \frac{dv_x}{dt} = -2\pi^2 \cos \pi t$ $\qquad a_y = \frac{dv_y}{dt} = 16\pi^2 \cos 2\pi t$

(a) AT $t = 0$: $v_x = 0$ $\qquad v_y = 0$ $\qquad \therefore v = 0$ ◀

$a_x = -2\pi^2 \frac{m}{s^2}$ $\qquad a_y = 16\pi^2 \frac{m}{s^2}$

OR $a = 159.1 \frac{m}{s^2} \measuredangle 82.9°$ ◀

(b) AT $t = 1.5$ s: $v_x = -2\pi \sin(1.5\pi)$ $\qquad v_y = 8\pi \sin(2\pi \cdot 1.5)$

$= 2\pi \frac{m}{s}$ $\qquad = 0$

OR $v = 6.28 \frac{m}{s} \longrightarrow$ ◀

$a_x = -2\pi^2 \cos(1.5\pi)$ $\qquad a_y = 16\pi^2 \cos(2\pi \cdot 1.5)$

$= 0$ $\qquad = -16\pi^2$

OR $a = 157.9 \frac{m}{s^2} \downarrow$ ◀

11.91

GIVEN: $x = \frac{1}{12}(t-2)^3 + t^2$, $y = \frac{t^3}{12} - \frac{1}{2}(t-1)^2$
$x, y \sim$ ft, $t \sim$ s

FIND: (a) v_{MIN}
(b) t, x, y, AND DIRECTION OF v WHEN $v = v_{MIN}$

(a) HAVE.. $x = \frac{1}{12}(t-2)^3 + t^2$ $\qquad y = \frac{t^3}{12} - \frac{1}{2}(t-1)^2$

THEN.. $v_x = \frac{dx}{dt} = \frac{1}{4}(t-2)^2 + 2t$ $\qquad v_y = \frac{dy}{dt} = \frac{1}{4}t^2 - (t-1)$

$= \frac{1}{4}t^2 + t + 1$ $\qquad = \frac{1}{4}t^2 - t + 1$

$= \frac{1}{4}(t+2)^2$ $\qquad = \frac{1}{4}(t-2)^2$

NOW.. $v^2 = v_x^2 + v_y^2 = \frac{1}{16}[(t+2)^4 + (t-2)^4]$

NOTING THAT v IS MINIMUM WHEN v^2 IS MINIMUM, HAVE.. $\frac{dv^2}{dt} = \frac{1}{4}[(t+2)^3 + (t-2)^3] = 0$

EXPANDING.. $(t^3 + 6t^2 + 12t + 8) + (t^3 - 6t^2 + 12t - 8) = 0$

OR $2(t^3 + 12t) = 0$

THE ONLY REAL ROOT OF THIS EQUATION IS $t = 0$.

$\therefore v_{MIN}^2 = \frac{1}{16}[(0+2)^4 + (0-2)^4] = 2$

OR $v_{MIN} = 1.414 \frac{ft}{s}$ ◀

(b) WHEN $v = v_{MIN}$ $\qquad t = 0$ ◀

$x = \frac{1}{12}(0-2)^3 + (0)^2$ $\qquad$ OR $x = -0.667$ ft ◀

$y = \frac{1}{12}(0)^3 - \frac{1}{2}(0-1)^2$ $\qquad$ OR $y = -0.500$ ft ◀

AND $v_x = \frac{1}{4}(0+2)^2 = 1 \frac{ft}{s}$ $\qquad v_y = \frac{1}{4}(0-2)^2 = 1 \frac{ft}{s}$

THEN $\tan\theta = \frac{v_y}{v_x} = 1$

OR $\theta_{v_{MIN}} = 45° \measuredangle$ ◀

11.92

<u>GIVEN:</u> $x = 4t - 2\sin t$, $y = 4 - 2\cos t$

$x, y \sim$ IN., $t \sim$ S

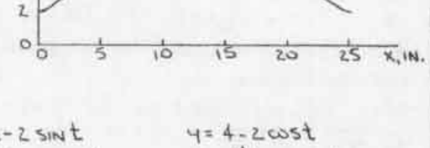
<u>SKETCH:</u> PATH OF THE PARTICLE

<u>FIND:</u> (a) v_{MIN} AND v_{MAX}

(b) t, x, y, AND DIRECTION OF $\underline{v}$ WHEN $v = v_{MIN}$ AND $v = v_{MAX}$

HAVE.. $x = 4t - 2\sin t$ $\quad y = 4 - 2\cos t$

t, s	x, in.	y, in.
0	0	2.0
$\frac{\pi}{2}$	4.28	4.0
π	12.57	6.0
$\frac{3\pi}{2}$	20.8	4.0
2π	25.1	2.0



(a) HAVE.. $x = 4t - 2\sin t$ $\quad y = 4 - 2\cos t$

THEN.. $v_x = \frac{dx}{dt} = 4 - 2\cos t$ $\quad v_y = \frac{dy}{dt} = 2\sin t$

NOW.. $v^2 = v_x^2 + v_y^2 = (4 - 2\cos t)^2 + (2\sin t)^2$

$= 20 - 16\cos t$

BY OBSERVATION.. FOR v_{MIN}, $\cos t = 1$ SO THAT

$v_{MIN}^2 = 4$ $\quad$ OR $v_{MIN} = 2\ \frac{\text{IN.}}{\text{S}}$

FOR v_{MAX}, $\cos t = -1$ SO THAT

$v_{MAX}^2 = 36$ $\quad$ OR $v_{MAX} = 6\ \frac{\text{IN.}}{\text{S}}$

(b) WHEN $v = v_{MIN}$: $\quad \cos t = 1$

OR $t = 2N\pi$ S

WHERE $N = 0, 1, 2, \ldots$

THEN.. $x = 4(2N\pi) - 2\sin(2N\pi)$ $\quad$ OR $x = 8N\pi$ IN.

$y = 4 - 2(1)$ $\quad$ OR $y = 2$ IN.

ALSO.. $v_x = 4 - 2(1) = 2\ \frac{\text{IN.}}{\text{S}}$ $\quad v_y = 2\sin(2N\pi) = 0$

$\therefore\ \theta_{v_{MIN}} = 0 \longrightarrow$

WHEN $v = v_{MAX}$: $\quad \cos t = -1$

OR $t = (2N+1)\pi$ S

WHERE $N = 0, 1, 2, \ldots$

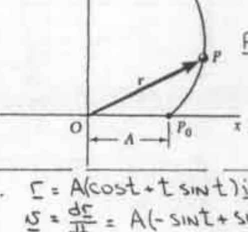
THEN $x = 4(2N+1)\pi - 2\sin(2N+1)\pi$ $\quad$ OR $x = 4(2N+1)\pi$ IN.

$y = 4 - 2(-1)$ $\quad$ OR $y = 6$ IN.

ALSO.. $v_x = 4 - 2(-1) = 6\ \frac{\text{IN.}}{\text{S}}$ $\quad v_y = 2\sin(2N+1)\pi = 0$

$\therefore\ \theta_{v_{MAX}} = 0 \longrightarrow$

11.93



<u>GIVEN:</u> $\underline{r} = A(\cos t + t\sin t)\underline{i} + A(\sin t - t\cos t)\underline{j}$

$t \sim$ S

<u>FIND:</u> (a) t SO THAT $\underline{r}$ AND $\underline{a}$ ARE PERPENDICULAR

(b) t SO THAT $\underline{r}$ AND $\underline{a}$ ARE PARALLEL

HAVE.. $\underline{r} = A(\cos t + t\sin t)\underline{i} + A(\sin t - t\cos t)\underline{j}$

THEN $\underline{v} = \frac{d\underline{r}}{dt} = A(-\sin t + \sin t + t\cos t)\underline{i} + A(\cos t - \cos t + t\sin t)\underline{j}$

$= A(t\cos t)\underline{i} + A(t\sin t)\underline{j}$

AND $\underline{a} = \frac{d\underline{v}}{dt} = A(\cos t - t\sin t)\underline{i} + A(\sin t + t\cos t)\underline{j}$

(a) WHEN $\underline{r}$ AND $\underline{a}$ ARE PERPENDICULAR, $\underline{r} \cdot \underline{a} = 0$

$\therefore\ A[(\cos t + t\sin t)\underline{i} + (\sin t - t\cos t)\underline{j}] \cdot A[(\cos t - t\sin t)\underline{i} + (\sin t + t\cos t)\underline{j}] = 0$

OR $(\cos t + t\sin t)(\cos t - t\sin t) + (\sin t - t\cos t)(\sin t + t\cos t) = 0$

(CONTINUED)

11.93 CONTINUED

OR $(\cos^2 t - t^2\sin^2 t) + (\sin^2 t - t^2\cos^2 t) = 0$

OR $1 - t^2 = 0$ $\quad$ OR $t = 1$ S

(b) WHEN $\underline{r}$ AND $\underline{a}$ ARE PARALLEL, $\underline{r} \times \underline{a} = 0$

$\therefore\ A[(\cos t + t\sin t)\underline{i} + (\sin t - t\cos t)\underline{j}] \times A[(\cos t - t\sin t)\underline{i} + (\sin t + t\cos t)\underline{j}] = 0$

OR $[(\cos t + t\sin t)(\sin t + t\cos t) - (\sin t - t\cos t)(\cos t - t\sin t)]\underline{k} = 0$

EXPANDING.. $(\sin t\cos t + t + t^2\sin t\cos t) - (\sin t\cos t - t + t^2\sin t\cos t) = 0$

OR $2t = 0$ $\quad$ OR $t = 0$

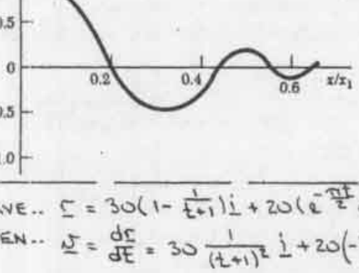
11.94

<u>GIVEN:</u> $\underline{r} = x_1\left(1 - \frac{1}{t+1}\right)\underline{i} + \left(y_1 e^{-\frac{\pi t}{2}}\cos 2\pi t\right)\underline{j}$

$t \sim$ S; $x_1 = 30$ mm, $y_1 = 20$ mm

<u>FIND:</u> (a) $\underline{r}$, $\underline{v}$, AND $\underline{a}$ AT $t = 0$

(b) $\underline{r}$, $\underline{v}$, AND $\underline{a}$ AT $t = 1.5$ s



HAVE.. $\underline{r} = 30\left(1 - \frac{1}{t+1}\right)\underline{i} + 20\left(e^{-\frac{\pi t}{2}}\cos 2\pi t\right)\underline{j}$

THEN.. $\underline{v} = \frac{d\underline{r}}{dt} = 30\frac{1}{(t+1)^2}\underline{i} + 20\left(-\frac{\pi}{2}e^{-\frac{\pi t}{2}}\cos 2\pi t - 2\pi e^{-\frac{\pi t}{2}}\sin 2\pi t\right)\underline{j}$

$= 30\frac{1}{(t+1)^2}\underline{i} - 20\pi\left[e^{-\frac{\pi t}{2}}\left(\frac{1}{2}\cos 2\pi t + 2\sin 2\pi t\right)\right]\underline{j}$

AND $\underline{a} = \frac{d\underline{v}}{dt} = -30\frac{2}{(t+1)^3}\underline{i} - 20\pi\left[-\frac{\pi}{2}e^{-\frac{\pi t}{2}}\left(\frac{1}{2}\cos 2\pi t + 2\sin 2\pi t\right) + e^{-\frac{\pi t}{2}}(-\pi\sin 2\pi t + 4\cos 2\pi t)\right]\underline{j}$

$= -\frac{60}{(t+1)^3}\underline{i} + 10\pi^2 e^{-\frac{\pi t}{2}}(4\sin 2\pi t - 7.5\cos 2\pi t)\underline{j}$

(a) AT $t = 0$: $\quad \underline{r} = 30\left(1 - \frac{1}{1}\right)\underline{i} + 20(1)\underline{j}$

OR $\underline{r} = 20$ mm $\uparrow$

$\underline{v} = 30\left(\frac{1}{1}\right)\underline{i} - 20\pi\left[(1)\left(\frac{1}{2} + 0\right)\right]\underline{j}$

OR $\underline{v} = 43.4\ \frac{\text{mm}}{\text{s}}\ \measuredangle\ 46.3°$

$\underline{a} = -\frac{60}{(1)}\underline{i} + 10\pi^2(1)(0 - 7.5)\underline{j}$

OR $\underline{a} = 743\ \frac{\text{mm}}{\text{s}^2}\ \measuredangle\ 85.4°$

(b) AT $t = 1.5$ s: $\quad \underline{r} = 30\left(1 - \frac{1}{2.5}\right)\underline{i} + 20e^{-0.75\pi}(\cos 3\pi)\underline{j}$

$= (18\text{ mm})\underline{i} + (-1.8956\text{ mm})\underline{j}$

OR $\underline{r} = 18.10$ mm $\measuredangle\ 6.01°$

$\underline{v} = \frac{30}{(2.5)^2}\underline{i} - 20\pi e^{-0.75\pi}\left(\frac{1}{2}\cos 3\pi + 0\right)\underline{j}$

$= \left(4.80\ \frac{\text{mm}}{\text{s}}\right)\underline{i} + \left(2.9778\ \frac{\text{mm}}{\text{s}}\right)\underline{j}$

OR $\underline{v} = 5.65\ \frac{\text{mm}}{\text{s}}\ \measuredangle\ 31.8°$

$\underline{a} = -\frac{60}{(2.5)^3}\underline{i} + 10\pi^2 e^{-0.75\pi}(0 - 7.5\cos 3\pi)\underline{j}$

$= \left(-3.84\ \frac{\text{mm}}{\text{s}^2}\right)\underline{i} + \left(70.1582\ \frac{\text{mm}}{\text{s}^2}\right)\underline{j}$

OR $\underline{a} = 70.3\ \frac{\text{mm}}{\text{s}^2}\ \measuredangle\ 86.9°$

11.95

GIVEN: $\underline{r} = (Rt\cos\omega_N t)\underline{i} + ct\underline{j} + (Rt\sin\omega_N t)\underline{k}$

FIND: v AND a

HAVE.. $\underline{r} = (Rt\cos\omega_N t)\underline{i} + ct\underline{j} + (Rt\sin\omega_N t)\underline{k}$

THEN.. $\underline{v} = \frac{d\underline{r}}{dt} = R(\cos\omega_N t - \omega_N t\sin\omega_N t)\underline{i} + c\underline{j}$
$+ R(\sin\omega_N t + \omega_N t\cos\omega_N t)\underline{k}$

AND $\underline{a} = \frac{d\underline{v}}{dt} = R(-\omega_N\sin\omega_N t - \omega_N\sin\omega_N t - \omega_N^2 t\cos\omega_N t)\underline{i}$
$+ R(\omega_N\cos\omega_N t + \omega_N\cos\omega_N t - \omega_N^2 t\sin\omega_N t)\underline{k}$
$= R(-2\omega_N\sin\omega_N t - \omega_N^2 t\cos\omega_N t)\underline{i}$
$+ R(2\omega_N\cos\omega_N t - \omega_N^2 t\sin\omega_N t)\underline{k}$

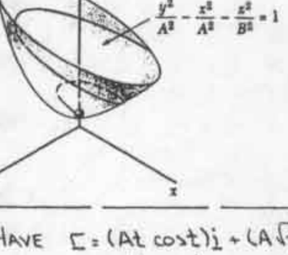
NOW.. $v^2 = v_x^2 + v_y^2 + v_z^2$
$= [R(\cos\omega_N t - \omega_N t\sin\omega_N t)]^2 + (c)^2$
$+ [R(\sin\omega_N t + \omega_N t\cos\omega_N t)]^2$
$= R^2[(\cos^2\omega_N t - 2\omega_N t\sin\omega_N t\cos\omega_N t$
$+ \omega_N^2 t^2\sin^2\omega_N t) + (\sin^2\omega_N t$
$+ 2\omega_N t\sin\omega_N t\cos\omega_N t + \omega_N^2 t^2\cos^2\omega_N t)]$
$+ c^2$
$= R^2(1 + \omega_N^2 t^2) + c^2$

OR $v = \sqrt{R^2(1+\omega_N^2 t^2) + c^2}$ ◀

ALSO.. $a^2 = a_x^2 + a_y^2 + a_z^2$
$= [R(-2\omega_N\sin\omega_N t - \omega_N^2 t\cos\omega_N t)]^2 + (0)^2$
$+ [R(2\omega_N\cos\omega_N t - \omega_N^2 t\sin\omega_N t)]^2$
$= R^2[(4\omega_N^2\sin^2\omega_N t + 4\omega_N^3 t\sin\omega_N t\cos\omega_N t$
$+ \omega_N^4 t^2\cos^2\omega_N t) + (4\omega_N^2\cos^2\omega_N t$
$- 4\omega_N^3 t\sin\omega_N t\cos\omega_N t + \omega_N^4 t^2\sin^2\omega_N t)]$
$= R^2(4\omega_N^2 + \omega_N^4 t^2)$

OR $a = R\omega_N\sqrt{4 + \omega_N^2 t^2}$ ◀

* 11.96



GIVEN: $\underline{r} = (At\cos t)\underline{i} + (A\sqrt{t^2+1})\underline{j} + (Bt\sin t)\underline{k}$
$r \sim$ ft, $t \sim$ s; $A = 3$, $B = 1$

SHOW: $(\frac{y}{A})^2 - (\frac{x}{A})^2 - (\frac{z}{B})^2 = 1$

FIND: (a) v AND a AT $t = 0$
(b) t_{min} ($t \neq 0$) SO THAT $\underline{r}$ AND $\underline{v}$ ARE PERPENDICULAR

HAVE $\underline{r} = (At\cos t)\underline{i} + (A\sqrt{t^2+1})\underline{j} + (Bt\sin t)\underline{k}$

OR $x = At\cos t$ $\quad y = A\sqrt{t^2+1}$ $\quad z = Bt\sin t$

THEN $\cos t = \frac{x}{At}$ $\quad \sin t = \frac{z}{Bt}$ $\quad t^2 = (\frac{y}{A})^2 - 1$

NOW.. $\cos^2 t + \sin^2 t = 1 \Rightarrow (\frac{x}{At})^2 + (\frac{z}{Bt})^2 = 1$

OR $t^2 = (\frac{x}{A})^2 + (\frac{z}{B})^2$

THEN.. $(\frac{y}{A})^2 - 1 = (\frac{x}{A})^2 + (\frac{z}{B})^2$

OR $(\frac{y}{A})^2 - (\frac{x}{A})^2 - (\frac{z}{B})^2 = 1$ Q.E.D. ◀

(a) WITH $A = 3$ AND $B = 1$, HAVE..

$\underline{v} = \frac{d\underline{r}}{dt} = 3(\cos t - t\sin t)\underline{i} + 3\frac{t}{\sqrt{t^2+1}}\underline{j} + (\sin t + t\cos t)\underline{k}$

AND $\underline{a} = \frac{d\underline{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\underline{i} + 3\frac{\sqrt{t^2+1} - t(\frac{t}{\sqrt{t^2+1}})}{(t^2+1)}\underline{j}$
$+ (\cos t + \cos t - t\sin t)\underline{k}$
$= -3(2\sin t + t\cos t)\underline{i} + 3\frac{1}{(t^2+1)^{3/2}}\underline{j}$
$+ (2\cos t - t\sin t)\underline{k}$

AT $t = 0$: $\underline{v} = 3(1-0)\underline{i} + (0)\underline{j} + (0)\underline{k}$

OR $v = 3\,\frac{\text{ft}}{\text{s}}$ ◀

(CONTINUED)

11.96 CONTINUED

AND $\underline{a} = -3(0)\underline{i} + 3(1)\underline{j} + (2-0)\underline{k}$

THEN $a^2 = (0)^2 + (3)^2 + (2)^2 = 13$ OR $a = 3.61\,\frac{\text{ft}}{\text{s}^2}$ ◀

(b) IF $\underline{r}$ AND $\underline{v}$ ARE PERPENDICULAR, $\underline{r}\cdot\underline{v} = 0$

$\therefore [(3t\cos t)\underline{i} + (3\sqrt{t^2+1})\underline{j} + (t\sin t)\underline{k}]$
$\cdot [3(\cos t - t\sin t)\underline{i} + (3\frac{t}{\sqrt{t^2+1}})\underline{j} + (\sin t + t\cos t)\underline{k}] = 0$

OR $(3t\cos t)[3(\cos t - t\sin t)] + (3\sqrt{t^2+1})(3\frac{t}{\sqrt{t^2+1}})$
$+ (t\sin t)(\sin t + t\cos t) = 0$

EXPANDING.. $(9t\cos^2 t - 9t^2\sin t\cos t) + (9t)$
$+ (t\sin^2 t + t^2\sin t\cos t) = 0$

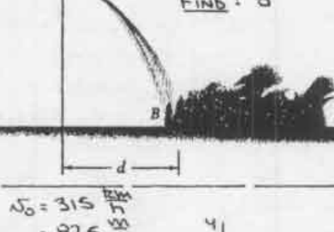
OR (WITH $t \neq 0$) $\quad 10 + 8\cos^2 t - 8t\sin t\cos t = 0$

OR $\quad 7 + 2\cos 2t - 2t\sin 2t = 0$

USING "TRIAL AND ERROR" OR NUMERICAL METHODS, THE SMALLEST ROOT IS $t = 3.82$ s ◀

NOTE: THE NEXT ROOT IS $t = 4.38$ s.

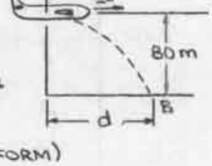
11.97



GIVEN: $v_0 = 315\,\frac{\text{km}}{\text{h}}$; $h = 80$ m

FIND: d

FIRST NOTE.. $v_0 = 315\,\frac{\text{km}}{\text{h}}$
$= 87.5\,\frac{\text{m}}{\text{s}}$



VERTICAL MOTION

(UNIFORMLY ACCEL. MOTION)

$y = \cancel{y_0}^{0} + \cancel{(v_y)_0}^{0}t - \frac{1}{2}gt^2$

AT B.. $-80\text{ m} = -\frac{1}{2}(9.81\,\frac{\text{m}}{\text{s}^2})t^2$

OR $t_B = 4.038\,55$ s

HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t$

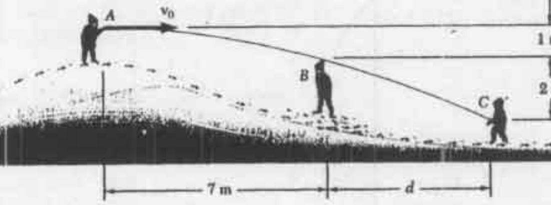
AT B.. $d = (87.5\,\frac{\text{m}}{\text{s}})(4.038\,55\text{ s})$

OR $d = 353$ m ◀

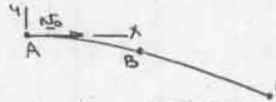
11.98

GIVEN: v_0 IS HORIZONTAL; PATH OF SNOWBALL

FIND: (a) v_0
(b) d



(a) VERTICAL MOTION



(UNIFORMLY ACCEL. MOTION)

$y = \cancel{y_0}^{0} + \cancel{(v_y)_0}^{0}t - \frac{1}{2}gt^2$

AT B.. $-1\text{ m} = -\frac{1}{2}(9.81\,\frac{\text{m}}{\text{s}^2})t^2$ OR $t_B = 0.451\,524$ s

HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t$

AT B.. $7\text{ m} = v_0(0.451\,524\text{ s})$

OR $v_0 = 15.5031\,\frac{\text{m}}{\text{s}}$ $\quad v_0 = 15.50\,\frac{\text{m}}{\text{s}}$ ◀

(b) VERTICAL MOTION: AT C.. $-3\text{ m} = -\frac{1}{2}(9.81\,\frac{\text{m}}{\text{s}^2})t^2$

OR $t_C = 0.782\,062$ s

(CONTINUED)

11.98 CONTINUED

HORIZONTAL MOTION

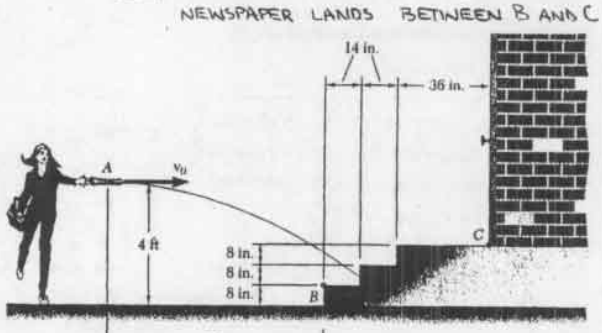
AT C.. $(7+d)\,m = (15.5031\,\frac{m}{s})(0.782\,062\,s)$

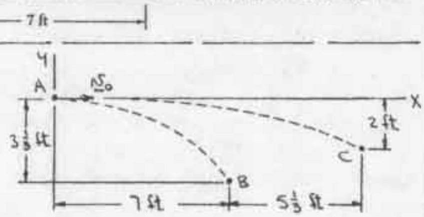
OR $d = 5.12\,m$ ◀

11.99

GIVEN: v_0 IS HORIZONTAL

FIND: RANGE OF VALUES OF v_0 IF NEWSPAPER LANDS BETWEEN B AND C





VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = \cancel{y_0}^{0} + \cancel{(v_y)_0}^{0}t - \frac{1}{2}gt^2$$

HORIZONTAL MOTION (UNIFORM)

$$x = \cancel{x_0}^{0} + (v_x)_0 t = v_0 t$$

AT B: y: $-3\frac{1}{3}\,ft = -\frac{1}{2}(32.2\,\frac{ft}{s^2})t^2$ OR $t_B = 0.455\,016\,s$

THEN.. x: $7\,ft = (v_0)_B(0.455\,016\,s)$

OR $(v_0)_B = 15.38\,\frac{ft}{s}$

AT C: y: $-2\,ft = -\frac{1}{2}(32.2\,\frac{ft}{s^2})t^2$ OR $t_C = 0.352\,454\,s$

THEN.. x: $12\frac{1}{3}\,ft = (v_0)_C(0.352\,454\,s)$

OR $(v_0)_C = 35.0\,\frac{ft}{s}$

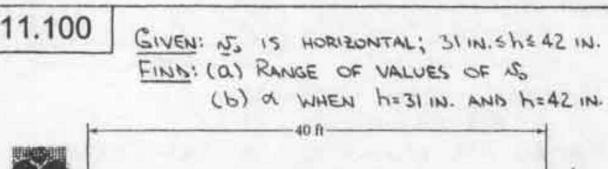
$\therefore\ 15.38\,\frac{ft}{s} \le v_0 \le 35.0\,\frac{ft}{s}$ ◀

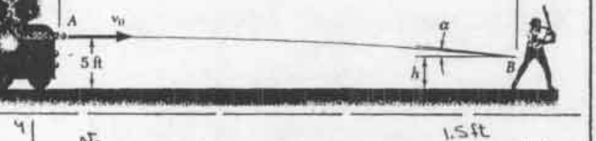
11.100

GIVEN: v_0 IS HORIZONTAL; $31\,IN. \le h \le 42\,IN.$

FIND: (a) RANGE OF VALUES OF v_0

(b) α WHEN $h = 31$ IN. AND $h = 42$ IN.





(a) VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = \cancel{y_0}^{0} + \cancel{(v_y)_0}^{0}t - \frac{1}{2}gt^2$$

HORIZONTAL MOTION (UNIFORM)

$$x = \cancel{x_0}^{0} + (v_x)_0 t = v_0 t$$

WHEN $h = 31$ IN., $y = -2\frac{5}{12}\,ft$: $-2\frac{5}{12}\,ft = -\frac{1}{2}(32.2\,\frac{ft}{s^2})t^2$

OR $t_{31} = 0.387\,432\,s$

(CONTINUED)

11.100 CONTINUED

THEN.. $40\,ft = (v_0)_{31}(0.387\,432\,s)$

OR $(v_0)_{31} = 103.244\,\frac{ft}{s} = 70.4\,\frac{mi}{h}$

WHEN $h = 42$ IN., $y = -1.5\,ft$: $-1.5\,ft = -\frac{1}{2}(32.2\,\frac{ft}{s^2})t^2$

OR $t_{42} = 0.305\,234\,s$

THEN.. $40\,ft = (v_0)_{42}(0.305\,234\,s)$

OR $(v_0)_{42} = 131.047\,\frac{ft}{s} = 89.4\,\frac{mi}{h}$

$\therefore\ 70.4\,\frac{mi}{h} \le v_0 \le 89.4\,\frac{mi}{h}$ ◀

(b) FOR THE VERTICAL MOTION

$$v_y = \cancel{(v_y)_0}^{0} - gt$$

NOW $\tan\alpha = \frac{|(v_y)_B|}{(v_x)_B} = \frac{gt}{v_0}$

WHEN $h = 31$ IN.: $\tan\alpha = \frac{(32.2\,\frac{ft}{s^2})(0.387\,432\,s)}{103.244\,ft/s} = 0.120833$

OR $\alpha_{31} = 6.89°$ ◀

WHEN $h = 42$ IN.: $\tan\alpha = \frac{(32.2\,ft/s^2)(0.305\,234\,s)}{131.047\,ft/s} = 0.075\,000$

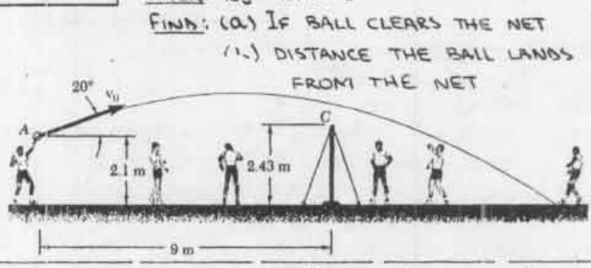
OR $\alpha_{42} = 4.29°$ ◀

11.101

GIVEN: $v_0 = 13.40\,\frac{m}{s}$

FIND: (a) IF BALL CLEARS THE NET

(b) DISTANCE THE BALL LANDS FROM THE NET



FIRST NOTE.. $(v_x)_0 = (13.40\,\frac{m}{s})\cos 20° = 12.5919\,\frac{m}{s}$

$(v_y)_0 = (13.40\,\frac{m}{s})\sin 20° = 4.5831\,\frac{m}{s}$

(a) HORIZONTAL MOTION (UNIFORM)

$$x = \cancel{x_0}^{0} + (v_x)_0 t$$

AT C.. $9\,m = (12.5919\,\frac{m}{s})t$ OR $t_C = 0.71475\,s$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

AT C.. $y_C = 2.1\,m + (4.5831\,\frac{m}{s})(0.71475\,s) - \frac{1}{2}(9.81\,\frac{m}{s^2})(0.71475\,s)^2$

$= 2.87\,m$

$\therefore\ y_C > 2.43\,m$ (HEIGHT OF NET) ⇒ BALL CLEARS NET ◀

(b) AT B, $y = 0$: $0 = 2.1\,m + (4.5831\,\frac{m}{s})t - \frac{1}{2}(9.81\,\frac{m}{s^2})t^2$

SOLVING.. $t_B = 1.271\,175\,s$ (THE OTHER ROOT IS NEGATIVE)

THEN.. $d = (v_x)_0 t_B = (12.5919\,\frac{m}{s})(1.271\,175\,s)$

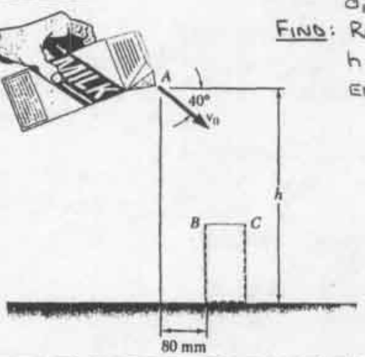
$= 16.01\,m$

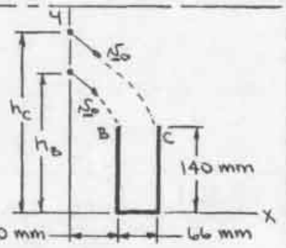
$\therefore$ THE BALL LANDS $b = (16.01 - 9.00)\,m = 7.01\,m$ FROM THE NET ◀

11.102

GIVEN: $v_0 = 1.2\ \frac{m}{s}$; $h_G = 140$ mm, $d_{BC} = 66$ mm

FIND: RANGE OF VALUES OF h SO THAT MILK ENTERS THE GLASS





FIRST NOTE..

$(v_x)_0 = (1.2\ \frac{m}{s})\cos 40° = 0.919\,25\ \frac{m}{s}$

$(v_y)_0 = -(1.2\ \frac{m}{s})\sin 40° = -0.771\,35\ \frac{m}{s}$

HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$

MILK ENTERS GLASS AT B

x: $0.08\text{ m} = (0.919\,25\ \frac{m}{s})t$ OR $t_B = 0.087\,028$ s

y: $0.140\text{ m} = h_B + (-0.771\,35\ \frac{m}{s})(0.087\,028\text{ s}) - \frac{1}{2}(9.81\ \frac{m}{s^2})(0.087\,028\text{ s})^2$

OR $h_B = 0.244$ m

MILK ENTERS GLASS AT C

x: $0.146\text{ m} = (0.919\,25\ \frac{m}{s})t$ OR $t_C = 0.158\,825$ s

y: $0.140\text{ m} = h_C + (-0.771\,35\ \frac{m}{s})(0.158\,825\text{ s}) - \frac{1}{2}(9.81\ \frac{m}{s^2})(0.158\,825\text{ s})^2$

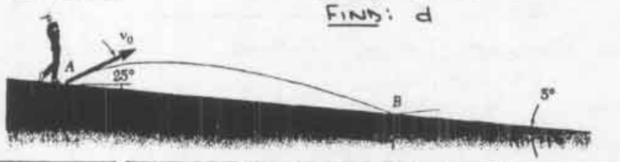
OR $h_C = 0.386$ m

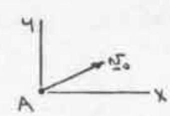
$\therefore\ 0.244\text{ m} \le h \le 0.386\text{ m}$

11.103

GIVEN: $v_0 = 160\ \frac{ft}{s}$

FIND: d





FIRST NOTE.. $(v_x)_0 = (160\ \frac{ft}{s})\cos 25°$

$(v_y)_0 = (160\ \frac{ft}{s})\sin 25°$

AND AT B.. $x_B = d\cos 5°$ $y_B = -d\sin 5°$

NOW.. HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t$

AT B.. $d\cos 5° = (160\cos 25°)t$ OR $t_B = \frac{\cos 5°}{160\cos 25°}d$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2}gt^2$ $(g = 32.2\text{ ft/s}^2)$

AT B.. $-d\sin 5° = (160\sin 25°)t_B - \frac{1}{2}gt_B^2$

SUBSTITUTING FOR t_B.. $-d\sin 5° = (160\sin 25°)\left(\frac{\cos 5°}{160\cos 25°}\right)d - \frac{1}{2}g\left(\frac{\cos 5°}{160\cos 25°}\right)^2 d^2$

OR $d = \frac{2}{32.2\cos 5°}(160\cos 25°)^2(\tan 5° + \tan 25°)$

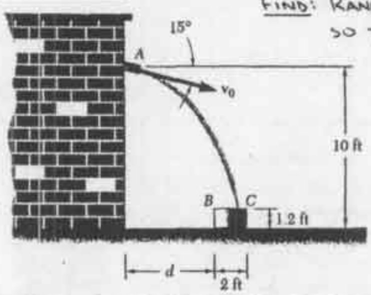
$= 726.06$ ft

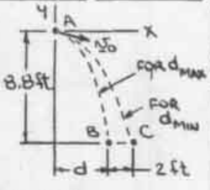
OR $d = 242$ yd

11.104

GIVEN: $v_0 = 2.5$ ft/s

FIND: RANGE OF VALUES OF d SO THAT WATER ENTERS THE TROUGH





FIRST NOTE.. $(v_x)_0 = (2.5\ \frac{ft}{s})\cos 15° = 2.4148\ \frac{ft}{s}$

$(v_y)_0 = -(2.5\ \frac{ft}{s})\sin 15° = -0.647\,05\ \frac{ft}{s}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2}gt^2$

AT THE TOP OF THE TROUGH..

$-8.8\text{ ft} = (-0.647\,05\ \frac{ft}{s})t - \frac{1}{2}(32.2\ \frac{ft}{s^2})t^2$

OR $t_{BC} = 0.719\,491$ s (THE OTHER ROOT IS NEGATIVE)

HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t$

IN TIME t_{BC}.. $x_{BC} = (2.4148\ \frac{ft}{s})(0.719\,491\text{ s}) = 1.737$ ft

THUS, THE TROUGH MUST BE PLACED SO THAT

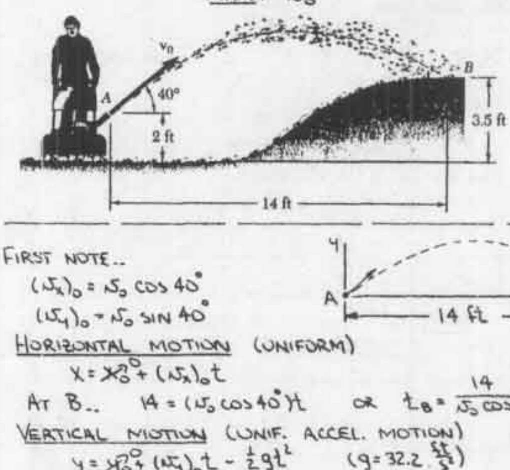
$x_B \le 1.737$ ft $x_C \ge 1.737$ ft

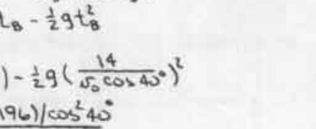
SINCE THE TROUGH IS 2 ft WIDE, IT THEN FOLLOWS THAT $0 \le d \le 1.737$ ft

11.105

GIVEN: SNOW DISCHARGED AS SHOWN

FIND: v_0





FIRST NOTE..

$(v_x)_0 = v_0\cos 40°$

$(v_y)_0 = v_0\sin 40°$

HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t$

AT B.. $14 = (v_0\cos 40°)t$ OR $t_B = \frac{14}{v_0\cos 40°}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2}gt^2$ $(g = 32.2\ \frac{ft}{s^2})$

AT B.. $1.5 = (v_0\sin 40°)t_B - \frac{1}{2}gt_B^2$

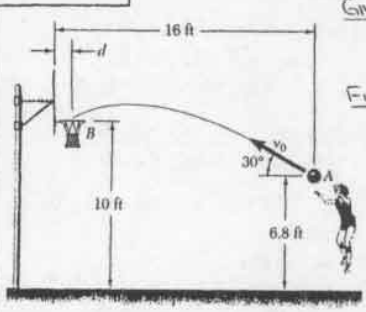
SUBSTITUTING FOR t_B..

$1.5 = (v_0\sin 40°)\left(\frac{14}{v_0\cos 40°}\right) - \frac{1}{2}g\left(\frac{14}{v_0\cos 40°}\right)^2$

OR $v_0^2 = \frac{\frac{1}{2}(32.2)(196)/\cos^2 40°}{-1.5 + 14\tan 40°}$

OR $v_0 = 22.9\ \frac{ft}{s}$

11.106



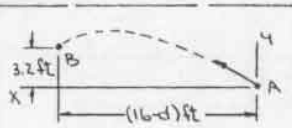
GIVEN: TRAJECTORY OF A BASKETBALL AS SHOWN

FIND: (a) v_0 WHEN d = 9 IN.
(b) v_0 WHEN d = 17 IN.

FIRST NOTE..

$(v_x)_0 = v_0 \cos 30^\circ$

$(v_y)_0 = v_0 \sin 30^\circ$



HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t$

AT B.. $(16-d) = (v_0 \cos 30^\circ)t$ OR $t_B = \frac{16-d}{v_0 \cos 30^\circ}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2} g t^2 \qquad (g = 32.2\ \frac{ft}{s^2})$

AT B.. $3.2 = (v_0 \sin 30^\circ) t_B - \frac{1}{2} g t_B^2$

SUBSTITUTING FOR t_B..

$$3.2 = (v_0 \sin 30^\circ)\left(\frac{16-d}{v_0 \cos 30^\circ}\right) - \frac{1}{2} g \left(\frac{16-d}{v_0 \cos 30^\circ}\right)^2$$

OR $$v_0^2 = \frac{2g(16-d)^2}{3\left[\frac{1}{\sqrt{3}}(16-d) - 3.2\right]} \qquad d \sim ft$$

(a) d = 9 IN.: $$v_0^2 = \frac{2(32.2)(16 - \frac{9}{12})^2}{3\left[\frac{1}{\sqrt{3}}(16 - \frac{9}{12}) - 3.2\right]}$$

OR $v_0 = 29.8\ \frac{ft}{s}$ ◀

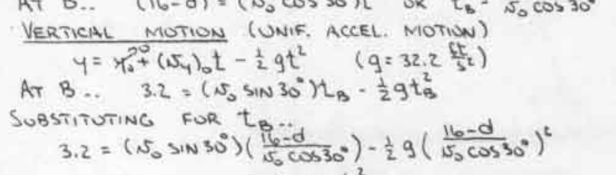
(b) d = 17 IN.: $$v_0^2 = \frac{2(32.2)(16 - \frac{17}{12})^2}{3\left[\frac{1}{\sqrt{3}}(16 - \frac{17}{12}) - 3.2\right]}$$

OR $v_0 = 29.6\ \frac{ft}{s}$ ◀

11.107

GIVEN: TRAJECTORY OF A BALL AS SHOWN

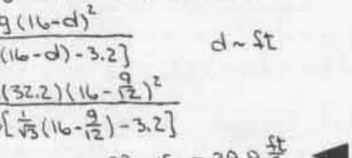
FIND: RANGE OF VALUES OF v_0 SO THAT BALL GOES THROUGH THE TIRE



FIRST NOTE..

$(v_x)_0 = v_0 \cos 3^\circ$

$(v_y)_0 = v_0 \sin 3^\circ$



HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t$

WHEN x = 6 m: $6 = (v_0 \cos 3^\circ)t$ OR $t_6 = \frac{6}{v_0 \cos 3^\circ}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2} g t^2 \qquad (g = 9.81\ \frac{m}{s^2})$

WHEN THE BALL REACHES THE TIRE, $t = t_6$.

$\therefore\ y_{B,C} = (v_0 \sin 3^\circ)\left(\frac{6}{v_0 \cos 3^\circ}\right) - \frac{1}{2} g \left(\frac{6}{v_0 \cos 3^\circ}\right)^2$

OR $$v_0^2 = \frac{18(9.81)}{\cos^2 3^\circ (6 \tan 3^\circ - y_{B,C})}$$

(CONTINUED)

11.107 CONTINUED

OR $$v_0^2 = \frac{177.065}{0.314447 - y_{B,C}}$$

AT B, y = −0.53 m: $$v_0^2 = \frac{177.065}{0.314447 - (-0.53)}$$

OR $(v_0)_B = 14.48\ \frac{m}{s}$

AT C, y = −1.25 m: $$v_0^2 = \frac{177.065}{0.314447 - (-1.25)}$$

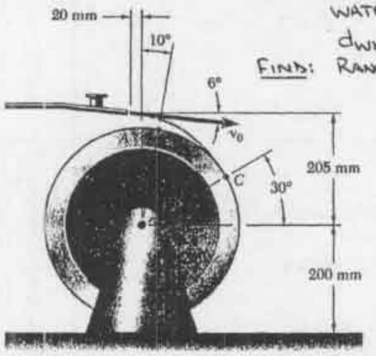
OR $(v_0)_C = 10.64\ \frac{m}{s}$

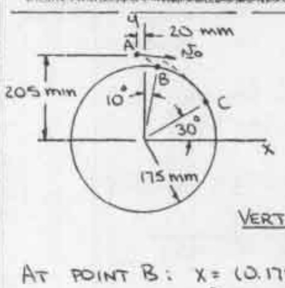
$\therefore\ 10.64\ \frac{m}{s} \le v_0 \le 14.48\ \frac{m}{s}$ ◀

11.108

GIVEN: TRAJECTORY OF COOLING WATER AS SHOWN; $d_{WHEEL} = 350$ mm

FIND: RANGE OF VALUES OF v_0 SO THAT THE WATER LANDS BETWEEN POINTS B AND C





FIRST NOTE..

$(v_x)_0 = v_0 \cos 6^\circ$

$(v_y)_0 = -v_0 \sin 6^\circ$

HORIZONTAL MOTION (UNIFORM)

$x = x_0 + (v_x)_0 t$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2 \quad (g = 9.81\ \frac{m}{s^2})$

AT POINT B: $x = (0.175\ m)\sin 10^\circ$, $y = (0.175\ m)\cos 10^\circ$

x: $0.175 \sin 10^\circ = -0.020 + (v_0 \cos 6^\circ)t$

OR $t_B = \frac{0.050388}{v_0 \cos 6^\circ}$

y: $0.175 \cos 10^\circ = 0.205 + (-v_0 \sin 6^\circ)t_B - \frac{1}{2} g t_B^2$

SUBSTITUTING FOR t_B..

$$-0.032659 = (-v_0 \sin 6^\circ)\left(\frac{0.050388}{v_0 \cos 6^\circ}\right) - \frac{1}{2}(9.81)\left(\frac{0.050388}{v_0 \cos 6^\circ}\right)^2$$

OR $$v_0^2 = \frac{\frac{1}{2}(9.81)(0.050388)^2}{\cos^2 6^\circ (0.032659 - 0.050388 \tan 6^\circ)}$$

OR $(v_0)_B = 0.678\ \frac{m}{s}$

AT POINT C: $x = (0.175\ m)\cos 30^\circ$, $y = (0.175\ m)\sin 30^\circ$

x: $0.175 \cos 30^\circ = -0.020 + (v_0 \cos 6^\circ)t$

OR $t_C = \frac{0.171554}{v_0 \cos 6^\circ}$

y: $0.175 \sin 30^\circ = 0.205 + (-v_0 \sin 6^\circ)t_C - \frac{1}{2} g t_C^2$

SUBSTITUTING FOR t_C..

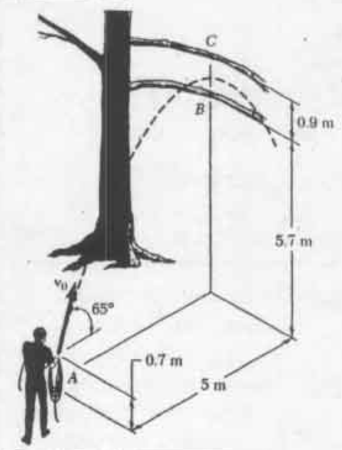
$$-0.117500 = (-v_0 \sin 6^\circ)\left(\frac{0.171554}{v_0 \cos 6^\circ}\right) - \frac{1}{2}(9.81)\left(\frac{0.171554}{v_0 \cos 6^\circ}\right)^2$$

OR $$v_0^2 = \frac{\frac{1}{2}(9.81)(0.171554)^2}{\cos^2 6^\circ (0.117500 - 0.171554 \tan 6^\circ)}$$

OR $(v_0)_C = 1.211\ \frac{m}{s}$

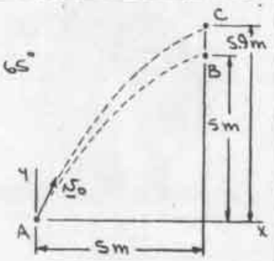
$\therefore\ 0.678\ \frac{m}{s} \le v_0 \le 1.211\ \frac{m}{s}$ ◀

11.109



GIVEN: TRAJECTORY OF A ROPE AS SHOWN.
FIND: RANGE OF VALUES OF v_0 SO THAT THE ROPE GOES OVER ONLY THE LOWEST LIMB.

FIRST NOTE..
$(v_x)_0 = v_0 \cos 65°$ $\quad (v_y)_0 = v_0 \sin 65°$



HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t$

AT EITHER B OR C, $x = 5$ m

$5 = (v_0 \cos 65°) t_{BC}$

OR $t_{BC} = \dfrac{5}{(v_0 \cos 65°)}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2} g t^2 \qquad (g = 9.81 \frac{m}{s^2})$

AT THE TREE LIMBS, $t = t_{BC}$

$y_{B,C} = (v_0 \sin 65°)\left(\dfrac{5}{v_0 \cos 65°}\right) - \frac{1}{2} g \left(\dfrac{5}{v_0 \cos 65°}\right)^2$

OR $\quad v_0^2 = \dfrac{\frac{1}{2}(9.81)(25)}{\cos^2 65° (5 \tan 65° - y_{B,C})}$

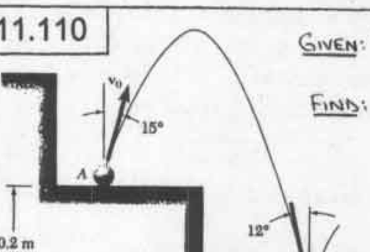
$= \dfrac{686.566}{5 \tan 65° - y_{B,C}}$

AT POINT B: $\quad v_0^2 = \dfrac{686.566}{5 \tan 65° - 5}$ OR $(v_0)_B = 10.95 \frac{m}{s}$

AT POINT C: $\quad v_0^2 = \dfrac{686.566}{5 \tan 65° - 5.9}$ OR $(v_0)_C = 11.93 \frac{m}{s}$

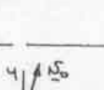
$\therefore 10.95 \frac{m}{s} \le v_0 \le 11.93 \frac{m}{s}$ ◀

11.110



GIVEN: TRAJECTORY OF A BALL AS SHOWN
FIND: v_0

FIRST NOTE.. $(v_x)_0 = v_0 \sin 15°$
$(v_y)_0 = v_0 \cos 15°$



HORIZONTAL MOTION (UNIFORM)

$v_x = (v_x)_0 = v_0 \sin 15°$

(CONTINUED)

11.110 CONTINUED

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$v_y = (v_y)_0 - gt \qquad y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2} g t^2$

$= v_0 \cos 15° - gt \qquad = (v_0 \cos 15°) t - \frac{1}{2} g t^2$

AT POINT B, $v_y < 0$

THEN.. $\quad \tan 12° = \dfrac{(v_x)_B}{|(v_y)_B|} = \dfrac{v_0 \sin 15°}{g t_B - v_0 \cos 15°}$

OR $\quad t_B = \dfrac{v_0}{g}\left(\dfrac{\sin 15°}{\tan 12°} + \cos 15°\right) \qquad g = 9.81 \frac{m}{s^2}$

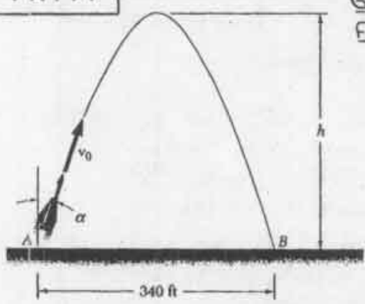
$= 0.222\,59\, v_0$

NOTING THAT $y_B = -0.2$ m, HAVE..

$-0.2 = (v_0 \cos 15°)(0.222\,59 v_0)$
$\qquad - \frac{1}{2}(9.81)(0.222\,59 v_0)^2$

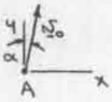
OR $v_0 = 2.67 \frac{m}{s}$ ◀

11.111



GIVEN: $v_0 = 280 \frac{ft}{s}$
FIND: (a) α
(b) h
(c) t_B

FIRST NOTE.. $(v_x)_0 = v_0 \sin\alpha = (280 \frac{ft}{s}) \sin\alpha$
$(v_y)_0 = v_0 \cos\alpha = (280 \frac{ft}{s}) \cos\alpha$



(a) HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t = (280 \sin\alpha) t$

AT POINT B: $\quad 340 = (280 \sin\alpha) t$

OR $\quad t_B = \dfrac{17}{14 \sin\alpha}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2} g t^2 = (280 \cos\alpha) t - \frac{1}{2} g t^2 \qquad (g = 32.2 \frac{ft}{s^2})$

AT POINT B, $t = t_B$, $y = 0$:

$0 = (280 \cos\alpha)\left(\dfrac{17}{14 \sin\alpha}\right) - \frac{1}{2} g \left(\dfrac{17}{14 \sin\alpha}\right)^2$

OR $\quad 280 \sin\alpha \cos\alpha - \frac{1}{2} g \left(\frac{17}{14}\right) = 0$

OR $\quad \sin 2\alpha = \frac{1}{2}\left(\frac{17}{14}\right)\left(\frac{32.2}{140}\right)$

OR $\quad \alpha = 4.013\,59°$ $\qquad \alpha = 4.01°$ ◀

(b) HAVE.. $v_y = (v_y)_0 - gt = 280 \cos\alpha - gt$

WHEN $y = y_{MAX} = h$, $v_y = 0$: $\quad 0 = 280 \cos\alpha - gt$

OR $\quad t_h = \dfrac{280 \cos 4.013\,59°}{32.2} = 8.674\,33$ s

THEN.. $\quad h = (280 \cos\alpha) t_h - \frac{1}{2} g t_h^2$

$= (280 \cos 4.013\,59°)(8.674\,33) - \frac{1}{2}(32.2)(8.674\,33)^2$

OR $h = 1211$ ft ◀

(c) HAD FOUND.. $t_B = \dfrac{17}{14 \sin\alpha}$

$= \dfrac{17}{14 \sin 4.013\,59°}$

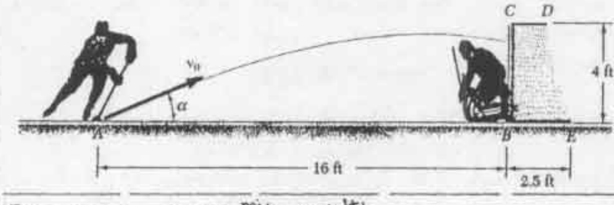
OR $t_B = 17.35$ s ◀

11.112

GIVEN: $v_0 = 105\ \frac{mi}{h}$

FIND: (a) α_{MAX} ($<45°$) FOR WHICH THE PUCK ENTERS THE NET

(b) t WHEN $\alpha = \alpha_{MAX}$



FIRST NOTE.. $v_0 = 105\ \text{mi/h} = 154\ \text{ft/s}$

AND $(v_x)_0 = v_0 \cos\alpha = (154\ \frac{ft}{s})\cos\alpha$

$(v_y)_0 = v_0 \sin\alpha = (154\ \frac{ft}{s})\sin\alpha$

(a) HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t = (154\cos\alpha)t$

AT THE FRONT OF THE NET, $x = 16$ ft

THEN... $16 = (154\cos\alpha)t$

OR $t_{ENTER} = \frac{8}{77\cos\alpha}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2}gt^2 = (154\sin\alpha)t - \frac{1}{2}gt^2 \quad (g = 32.2\ \frac{ft}{s^2})$

AT THE FRONT OF THE NET..

$y_{FRONT} = (154\sin\alpha)t_{ENTER} - \frac{1}{2}g t_{ENTER}^2$

$= (154\sin\alpha)\left(\frac{8}{77\cos\alpha}\right) - \frac{1}{2}g\left(\frac{8}{77\cos\alpha}\right)^2$

$= 16\tan\alpha - \frac{32g}{5929\cos^2\alpha}$

NOW.. $\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$

THEN $y_{FRONT} = 16\tan\alpha - \frac{32g}{5929}(1 + \tan^2\alpha)$

OR $\tan^2\alpha - \frac{5929}{2g}\tan\alpha + \left(1 + \frac{5929}{32g}y_{FRONT}\right) = 0$

THEN $\tan\alpha = \dfrac{\frac{5929}{2g} \pm \left[\left(-\frac{5929}{2g}\right)^2 - 4\left(1 + \frac{5929}{32g}y_{FRONT}\right)\right]^{1/2}}{2}$

OR $\tan\alpha = \frac{5929}{4\times 32.2} \pm \left[\left(-\frac{5929}{4\times 32.2}\right)^2 - \left(1 + \frac{5929}{32\times 32.2}y_{FRONT}\right)\right]^{1/2}$

OR $\tan\alpha = 46.0326 \pm \left[(46.0326)^2 - (1 + 5.7541\,y_{FRONT})\right]^{1/2}$

NOW.. $0 \le y_{FRONT} \le 4$ ft SO THAT THE POSITIVE ROOT WILL YIELD VALUES OF $\alpha > 45°$ FOR ALL VALUES OF y_{FRONT}. WHEN THE NEGATIVE ROOT IS SELECTED, α INCREASES AS y_{FRONT} IS INCREASED. THEREFORE, FOR α_{MAX} SET

$y_{FRONT} = y_C = 4$ ft

THEN.. $\tan\alpha = 46.0326 - \left[(46.0326)^2 - (1 + 5.7541\times 4)\right]^{1/2}$

OR $\alpha_{MAX} = 14.6604°$ $\quad \alpha_{MAX} = 14.66°$ ◀

(b) HAD FOUND $t_{ENTER} = \frac{8}{77\cos\alpha}$

$= \frac{8}{77\cos 14.6604°}$

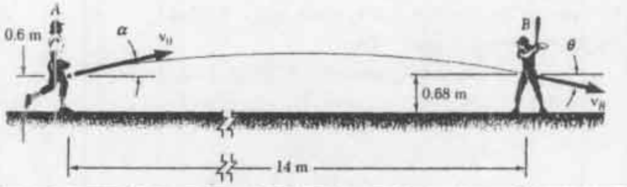
OR $t_{ENTER} = 0.1074$ s ◀

11.113

GIVEN: $v_0 = 72\ \frac{km}{h}$

FIND: (a) α

(b) θ



FIRST NOTE.. $v_0 = 72\ \frac{km}{h} = 20\ \frac{m}{s}$

AND $(v_x)_0 = v_0\cos\alpha = (20\ \frac{m}{s})\cos\alpha$

$(v_y)_0 = v_0\sin\alpha = (20\ \frac{m}{s})\sin\alpha$

(a) HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t = (20\cos\alpha)t$

AT POINT B: $14 = (20\cos\alpha)t$ OR $t_B = \frac{7}{10\cos\alpha}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2}gt^2 = (20\sin\alpha)t - \frac{1}{2}gt^2 \quad (g = 9.81\ \frac{m}{s^2})$

AT POINT B: $0.08 = (20\sin\alpha)t_B - \frac{1}{2}g t_B^2$

SUBSTITUTING FOR t_B..

$0.08 = (20\sin\alpha)\left(\frac{7}{10\cos\alpha}\right) - \frac{1}{2}g\left(\frac{7}{10\cos\alpha}\right)^2$

OR $8 = 1400\tan\alpha - \frac{1}{2}g\frac{49}{\cos^2\alpha}$

NOW.. $\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$

THEN.. $8 = 1400\tan\alpha - 24.5g(1 + \tan^2\alpha)$

OR $240.345\tan^2\alpha - 1400\tan\alpha + 248.345 = 0$

SOLVING.. $\alpha = 10.3786°$ AND $\alpha = 79.949°$

REJECTING THE SECOND ROOT BECAUSE IT IS NOT PHYSICALLY REASONABLE, HAVE

$\alpha = 10.38°$ ◀

(b) HAVE $v_x = (v_x)_0 = 20\cos\alpha$

AND $v_y = (v_y)_0 - gt = 20\sin\alpha - gt$

AT POINT B: $(v_y)_B = 20\sin\alpha - g t_B$

$= 20\sin\alpha - \frac{7g}{10\cos\alpha}$

NOTING THAT AT POINT B, $v_y < 0$, HAVE

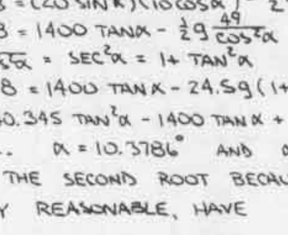
$\tan\theta = \frac{|(v_y)_B|}{v_x}$

$= \dfrac{\frac{7g}{10\cos\alpha} - 20\sin\alpha}{20\cos\alpha}$

$= \dfrac{\frac{7}{200}\frac{9.81}{\cos 10.3786°} - \sin 10.3786°}{\cos 10.3786°}$

OR $\theta = 9.74°$ ◀

* 11.114



GIVEN: CLIMBER JUMPS FROM A TO B

FIND: $(v_0)_{MIN}$ AND α

FIRST NOTE.. $(v_x)_0 = v_0\cos\alpha$

$(v_y)_0 = v_0\sin\alpha$

HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t = (v_0\cos\alpha)t$

AT POINT B: $1.8 = (v_0\cos\alpha)t$

OR $t_B = \frac{1.8}{v_0\cos\alpha}$

(CONTINUED)

11.114 CONTINUED

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2}gt^2 = (v_0 \sin\alpha)t - \frac{1}{2}gt^2 \qquad (g = 9.81\ \frac{m}{s^2})$

AT POINT B: $-1.4 = (v_0 \sin\alpha)t_B - \frac{1}{2}gt_B^2$

SUBSTITUTING FOR t_B..

$$-1.4 = (v_0 \sin\alpha)\left(\frac{1.8}{v_0 \cos\alpha}\right) - \frac{1}{2}g\left(\frac{1.8}{v_0 \cos\alpha}\right)^2$$

OR $$v_0^2 = \frac{1.62g}{\cos^2\alpha(1.8 \tan\alpha + 1.4)}$$

$$= \frac{1.62g}{0.9 \sin 2\alpha + 1.4\cos^2\alpha}$$

NOW MINIMIZE v_0^2 WITH RESPECT TO α. HAVE..

$$\frac{dv_0^2}{d\alpha} = 1.62g \frac{-(1.8\cos 2\alpha - 2.8\cos\alpha \sin\alpha)}{(0.9\sin 2\alpha + 1.4\cos^2\alpha)^2} = 0$$

OR $1.8 \cos 2\alpha - 1.4 \sin 2\alpha = 0$

OR $\tan 2\alpha = \frac{18}{14}$

OR $\alpha = 26.0625°$ AND $\alpha = 206.06°$

REJECTING THE SECOND VALUE BECAUSE IT IS NOT PHYSICALLY POSSIBLE, HAVE..

$\alpha = 26.1°$ ◀

FINALLY, $$v_0^2 = \frac{1.62 \times 9.81}{\cos^2 26.0625°(1.8 \tan 26.0625° + 1.4)}$$

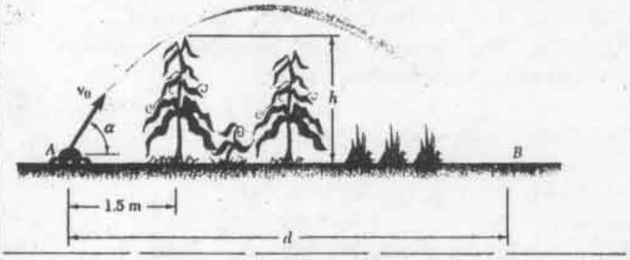
OR $(v_0)_{MIN} = 2.94\ \frac{m}{s}$ ◀

11.115

GIVEN: $v_0 = 8\ \frac{m}{s}$

FIND: (a) d_{MAX} AND α WHEN $h = 0$

(b) d_{MAX} AND α WHEN $h = 1.8$ m



FIRST NOTE.. $(v_x)_0 = v_0 \cos\alpha = (8\ \frac{m}{s})\cos\alpha$

$(v_y)_0 = v_0 \sin\alpha = (8\ \frac{m}{s})\sin\alpha$

HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t = (8\cos\alpha)t$

AT POINT B, $x = d$: $d = (8\cos\alpha)t$ OR $t_B = \frac{d}{8\cos\alpha}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2}gt^2 = (8\sin\alpha)t - \frac{1}{2}gt^2 \qquad (g = 9.81\ \frac{m}{s^2})$

AT POINT B: $0 = (8\sin\alpha)t_B - \frac{1}{2}gt_B^2$

SIMPLIFYING AND SUBSTITUTING FOR t_B..

$$0 = 8\sin\alpha - \frac{1}{2}g\left(\frac{d}{8\cos\alpha}\right)$$

OR $d = \frac{64}{g}\sin 2\alpha \qquad (1)$

(a) WHEN $h = 0$, THE WATER CAN FOLLOW ANY PHYSICALLY POSSIBLE TRAJECTORY. IT THEN FOLLOWS FROM EQ. (1) THAT d IS MAXIMUM WHEN $2\alpha = 90°$

OR $\alpha = 45°$ ◀

THEN $d = \frac{64}{9.81}\sin(2 \times 45°)$

OR $d_{MAX} = 6.52$ m ◀

(b) BASED ON EQ. (1) AND THE RESULTS OF PART a, IT CAN BE CONCLUDED THAT d INCREASES IN VALUE AS α INCREASES IN VALUE FROM

(CONTINUED)

11.115 CONTINUED

0 TO 45° AND THEN d DECREASES AS α IS FURTHER INCREASED. THUS, d_{MAX} OCCURS FOR THE VALUE OF α CLOSEST TO 45° AND FOR WHICH THE WATER JUST PASSES OVER THE FIRST ROW OF CORN PLANTS. AT THIS ROW $x_{CORN} = 1.5$ m

SO THAT $t_{CORN} = \frac{1.5}{8\cos\alpha}$

ALSO, WITH $y_{CORN} = h$, HAVE

$$h = (8\sin\alpha)t_{CORN} - \frac{1}{2}gt_{CORN}^2$$

SUBSTITUTING FOR t_{CORN} AND NOTING $h = 1.8$ m,

$$1.8 = (8\sin\alpha)\left(\frac{1.5}{8\cos\alpha}\right) - \frac{1}{2}g\left(\frac{1.5}{8\cos\alpha}\right)^2$$

OR $1.8 = 1.5\tan\alpha - \frac{2.25g}{128\cos^2\alpha}$

NOW.. $\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$

THEN $1.8 = 1.5\tan\alpha - \frac{2.25(9.81)}{128}(1 + \tan^2\alpha)$

OR $0.172441\tan^2\alpha - 1.5\tan\alpha + 1.972441 = 0$

SOLVING.. $\alpha = 58.229°$ AND $\alpha = 81.965°$

FROM THE ABOVE DISCUSSION, IT FOLLOWS THAT $d = d_{MAX}$ WHEN

$\alpha = 58.2°$ ◀

FINALLY, USING EQ (1)

$$d = \frac{64}{9.81}\sin(2 \times 58.229°)$$

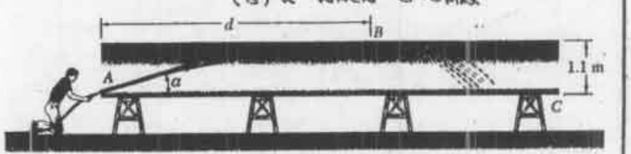
OR $d_{MAX} = 5.84$ m ◀

11.116

GIVEN: $v_0 = 11.5\ \frac{m}{s}$

FIND: (a) d_{MAX}

(b) α WHEN $d = d_{MAX}$



FIRST NOTE.. $(v_x)_0 = v_0 \cos\alpha = (11.5\ \frac{m}{s})\cos\alpha$

$(v_y)_0 = v_0 \sin\alpha = (11.5\ \frac{m}{s})\sin\alpha$

BY OBSERVATION, d_{MAX} OCCURS WHEN $y_{MAX} = 1.1$ m.

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$v_y = (v_y)_0 - gt = (11.5\sin\alpha) - gt$

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2}gt^2 = (11.5\sin\alpha)t - \frac{1}{2}gt^2$

WHEN $y = y_{MAX}$ AT B, $(v_y)_B = 0$

THEN $(v_y)_B = 0 = (11.5\sin\alpha) - gt$

OR $t_B = \frac{11.5\sin\alpha}{g} \qquad (g = 9.81\ \frac{m}{s^2})$

AND $y_B = (11.5\sin\alpha)t_B - \frac{1}{2}gt_B^2$

SUBSTITUTING FOR t_B AND NOTING $y_B = 1.1$ m..

$$1.1 = (11.5\sin\alpha)\left(\frac{11.5\sin\alpha}{g}\right) - \frac{1}{2}g\left(\frac{11.5\sin\alpha}{g}\right)^2$$

$$= \frac{1}{2g}(11.5)^2\sin^2\alpha$$

OR $\sin^2\alpha = \frac{2.2 \times 9.81}{11.5^2} \qquad \alpha = 23.8265°$

(a) HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t = (11.5\cos\alpha)t$

AT POINT B, $x = d_{MAX}$ AND $t = t_B$

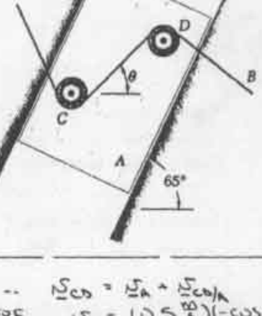
WHERE $t_B = \frac{11.5}{9.81}\sin 23.8265° = 0.47356$ s

THEN.. $d_{MAX} = (11.5)(\cos 23.8265°)(0.47356)$

OR $d_{MAX} = 4.98$ m ◀

(b) FROM ABOVE $\alpha = 23.8°$ ◀

11.117



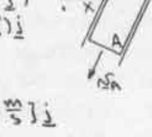
GIVEN: $\mathbf{v}_A = 0.5\ \frac{m}{s} \measuredangle 65^\circ$
$\mathbf{v}_{CD/A} = 2\ \frac{m}{s} \measuredangle \theta$

FIND: (a) $\mathbf{v}_{CD}$ WHEN $\theta = 45^\circ$
(b) $\mathbf{v}_{CD}$ WHEN $\theta = 60^\circ$

HAVE.. $\mathbf{v}_{CD} = \mathbf{v}_A + \mathbf{v}_{CD/A}$

WHERE $\mathbf{v}_A = (0.5\ \frac{m}{s})(-\cos 65^\circ \mathbf{i} - \sin 65^\circ \mathbf{j})$

$= -(0.21131\ \frac{m}{s})\mathbf{i} - (0.45315\ \frac{m}{s})\mathbf{j}$



AND $\mathbf{v}_{CD/A} = (2\ \frac{m}{s})(\cos\theta\, \mathbf{i} + \sin\theta\, \mathbf{j})$

THEN.. $\mathbf{v}_{CD} = [(-0.21131 + 2\cos\theta)\ \frac{m}{s}]\mathbf{i} + [(-0.45315 + 2\sin\theta)\ \frac{m}{s}]\mathbf{j}$

(a) HAVE.. $\mathbf{v}_{CD} = (-0.21131 + 2\cos 45^\circ)\mathbf{i} + (-0.45315 + 2\sin 45^\circ)\mathbf{j}$

$= (1.20290\ \frac{m}{s})\mathbf{i} + (0.96106\ \frac{m}{s})\mathbf{j}$

OR $\mathbf{v}_{CD} = 1.540\ \frac{m}{s} \measuredangle 38.6^\circ$ ◀

(b) HAVE.. $\mathbf{v}_{CD} = (-0.21131 + 2\cos 60^\circ)\mathbf{i} + (-0.45315 + 2\sin 60^\circ)\mathbf{j}$

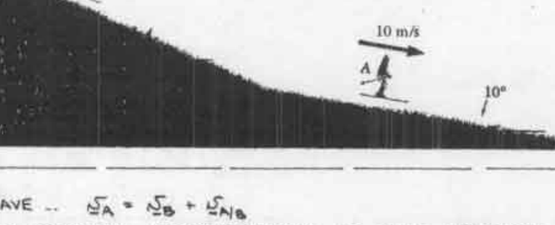
$= (0.78869\ \frac{m}{s})\mathbf{i} + (1.27890\ \frac{m}{s})\mathbf{j}$

OR $\mathbf{v}_{CD} = 1.503\ \frac{m}{s} \measuredangle 58.3^\circ$ ◀

11.118

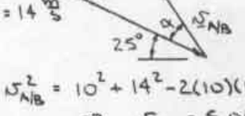
GIVEN: VELOCITIES OF SKIERS A AND B AS SHOWN

FIND: $\mathbf{v}_{A/B}$



HAVE.. $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



THEN.. $v_{A/B}^2 = 10^2 + 14^2 - 2(10)(14)\cos 15^\circ$

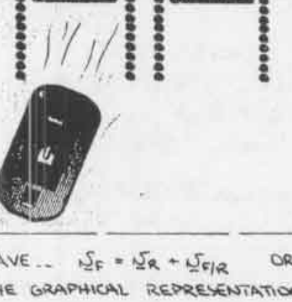
OR $v_{A/B} = 5.05379\ \frac{m}{s}$

AND $\frac{10}{\sin\alpha} = \frac{5.05379}{\sin 15^\circ}$

OR $\alpha = 30.8^\circ$

$\therefore\ \mathbf{v}_{A/B} = 5.05\ \frac{m}{s} \measuredangle 55.8^\circ$ ◀

11.119



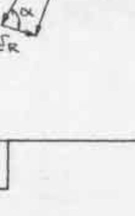
GIVEN: $v_F = 9.8$ KNOTS $\measuredangle 70^\circ$
$\mathbf{v}_{F/R} = 10$ KNOTS $\measuredangle 30^\circ$

FIND: $\mathbf{v}_R$

NOTE: "F" DENOTES THE FERRY AND "R" DENOTES THE RIVER.

HAVE.. $\mathbf{v}_F = \mathbf{v}_R + \mathbf{v}_{F/R}$ OR $\mathbf{v}_F = \mathbf{v}_{F/R} + \mathbf{v}_R$

THE GRAPHICAL REPRESENTATION OF THE SECOND EQUATION IS THEN AS SHOWN.



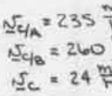
HAVE.. $v_R^2 = 9.8^2 + 10^2 - 2(9.8)(10)\cos 10^\circ$

OR $v_R = 1.737147$ KNOTS

AND $\frac{9.8}{\sin\alpha} = \frac{1.737147}{\sin 10^\circ}$

OR $\alpha = 78.41^\circ$

NOTING THAT

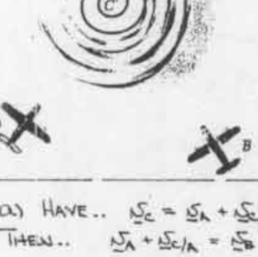


$\therefore\ \mathbf{v}_R = 1.737$ KNOTS $\measuredangle 18.41^\circ$ ◀

11.120

GIVEN: $\mathbf{v}_{C/A} = 235\ \frac{mi}{h} \measuredangle 75^\circ$
$\mathbf{v}_{C/B} = 260\ \frac{mi}{h} \measuredangle 40^\circ$
$\mathbf{v}_C = 24\ \frac{mi}{h} \uparrow$

FIND: (a) $\mathbf{v}_{B/A}$
(b) $\mathbf{v}_A$
(c) $\Delta\mathbf{r}_{C/B}$ FOR $\Delta t = 15$ MIN



(a) HAVE.. $\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$ AND $\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

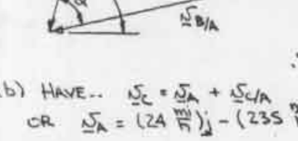
THEN.. $\mathbf{v}_A + \mathbf{v}_{C/A} = \mathbf{v}_B + \mathbf{v}_{C/B}$

OR $\mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{C/A} - \mathbf{v}_{C/B}$

NOW.. $\mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{B/A}$ SO THAT

$\mathbf{v}_{B/A} = \mathbf{v}_{C/A} - \mathbf{v}_{C/B}$ OR $\mathbf{v}_{C/A} = \mathbf{v}_{C/B} + \mathbf{v}_{B/A}$

THE GRAPHICAL REPRESENTATION OF THE LAST EQUATION IS THEN AS SHOWN.



HAVE.. $v_{B/A}^2 = 235^2 + 260^2 - 2(235)(260)\cos 65^\circ$

OR $v_{B/A} = 266.798\ \frac{mi}{h}$

AND $\frac{260}{\sin\alpha} = \frac{266.798}{\sin 65^\circ}$

OR $\alpha = 62.03^\circ$

$\therefore\ \mathbf{v}_{B/A} = 267\ \frac{mi}{h} \measuredangle 12.97^\circ$ ◀

(b) HAVE.. $\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$

OR $\mathbf{v}_A = (24\ \frac{mi}{h})\mathbf{j} - (235\ \frac{mi}{h})(-\cos 75^\circ \mathbf{i} - \sin 75^\circ \mathbf{j})$

(CONTINUED)

11.120 CONTINUED

$\mathbf{v}_A = (60.822\ \frac{mi}{h})\mathbf{i} + (250.99\ \frac{mi}{h})\mathbf{j}$

OR $v_A = 258\ \frac{mi}{h} \angle 76.4°$ ◀

(c) NOTING THAT THE VELOCITIES OF B AND C ARE CONSTANT, HAVE..

$\mathbf{r}_B = (\mathbf{r}_B)_0 + \mathbf{v}_B t \qquad \mathbf{r}_C = (\mathbf{r}_C)_0 + \mathbf{v}_C t$

NOW.. $\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = [(\mathbf{r}_C)_0 - (\mathbf{r}_B)_0] + (\mathbf{v}_C - \mathbf{v}_B)t$

$= [(\mathbf{r}_C)_0 - (\mathbf{r}_B)_0] + \mathbf{v}_{C/B} t$

THEN.. $\Delta \mathbf{r}_{C/B} = (\mathbf{r}_{C/B})_{t_2} - (\mathbf{r}_{C/B})_{t_1} = \mathbf{v}_{C/B}(t_2 - t_1)$

$= \mathbf{v}_{C/B}\,\Delta t$

FOR $\Delta t = 15$ MIN: $\Delta r_{C/B} = (260\ \frac{mi}{h})(\frac{1}{4}\ h) = 65$ mi

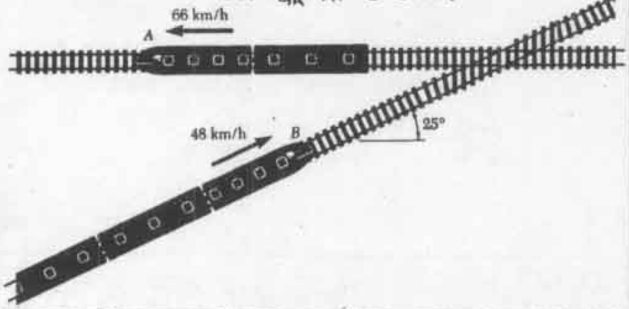
$\therefore \Delta \mathbf{r}_{C/B} = 65$ mi $\measuredangle 40°$ ◀

11.121

GIVEN: CONSTANT VELOCITIES OF TRAINS A AND B; AT $t = 0$, A IS AT THE CROSSING; AT $t = 10$ MIN, B IS AT THE CROSSING

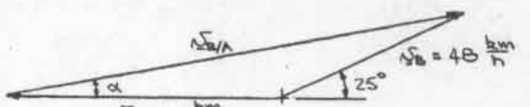
FIND: (a) $\mathbf{v}_{B/A}$

(b) $r_{B/A}$ AT $t = 3$ MIN



(a) HAVE.. $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



THEN.. $v_{B/A}^2 = 66^2 + 48^2 - 2(66)(48)\cos 155°$

OR $v_{B/A} = 111.366$ km/h

AND $\dfrac{48}{\sin\alpha} = \dfrac{111.366}{\sin 155°}$

OR $\alpha = 10.50°$

$\therefore \mathbf{v}_{B/A} = 111.4\ \frac{km}{h} \measuredangle 10.50°$ ◀

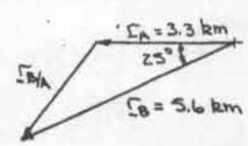
(b) FIRST NOTE THAT

AT $t = 3$ MIN, A IS $(66\ \frac{km}{h})(\frac{3}{60}) = 3.3$ km WEST OF THE CROSSING.

AT $t = 3$ MIN, B IS $(48\ \frac{km}{h})(\frac{7}{60}) = 5.6$ km SOUTHWEST OF THE CROSSING.

NOW.. $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$

THEN AT $t = 3$ MIN HAVE..



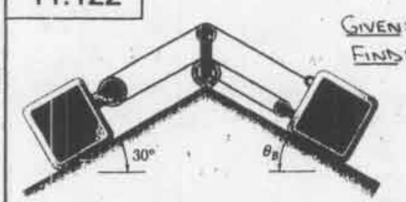
$r_{B/A}^2 = 3.3^2 + 5.6^2 - 2(3.3)(5.6)\cos 25°$

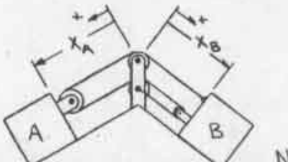
OR $r_{B/A} = 2.96$ km ◀

11.122

GIVEN: $v_{B/A} = 5.6\ \frac{m}{s} \measuredangle 70°$

FIND: $\mathbf{v}_A$ AND $\mathbf{v}_B$





FROM THE DIAGRAM..

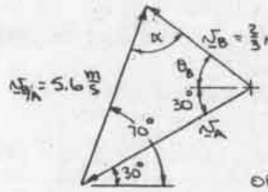
$2x_A + 3x_B =$ CONSTANT

THEN.. $2v_A + 3v_B = 0$

OR $|v_B| = \frac{2}{3} v_A$

NOW.. $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

AND NOTING THAT $\mathbf{v}_A$ AND $\mathbf{v}_B$ MUST BE PARALLEL TO SURFACES A AND B, RESPECTIVELY, THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN. NOTE: ASSUMING THAT $\mathbf{v}_A$ IS DIRECTED UP THE INCLINE LEADS TO A VELOCITY DIAGRAM THAT DOES NOT "CLOSE."



FIRST NOTE..

$\alpha = 180° - (40° + 30° + \theta_B)$

$= 110° - \theta_B$

THEN

$\dfrac{v_A}{\sin(110° - \theta_B)} = \dfrac{\frac{2}{3}v_A}{\sin 40°} = \dfrac{5.6}{\sin(30° + \theta_B)}$

OR $v_A \sin 40° = \frac{2}{3} v_A \sin(110° - \theta_B)$

OR $\sin(110° - \theta_B) = 0.964\,18$

OR $\theta_B = 35.3817°$ AND $\theta_B = 4.6183°$

FOR $\theta_B = 35.3817°$:

$v_B = \frac{2}{3} v_A = \dfrac{5.6 \sin 40°}{\sin(30° + 35.3817°)}$

OR $v_A = 5.94\ \frac{m}{s}$ $\quad v_B = 3.96\ \frac{m}{s}$

$\therefore \mathbf{v}_A = 5.94\ \frac{m}{s} \measuredangle 30°$ ◀

$\mathbf{v}_B = 3.96\ \frac{m}{s} \measuredangle 35.4°$ ◀

FOR $\theta_B = 4.6183°$:

$v_B = \frac{2}{3} v_A = \dfrac{5.6 \sin 40°}{\sin(30° + 4.6183°)}$

OR $v_A = 9.50\ \frac{m}{s}$ $\quad v_B = 6.34\ \frac{m}{s}$

$\therefore \mathbf{v}_A = 9.50\ \frac{m}{s} \measuredangle 30°$ ◀

$\mathbf{v}_B = 6.34\ \frac{m}{s} \measuredangle 4.62°$ ◀

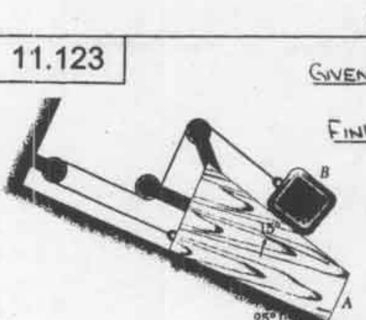
11.123

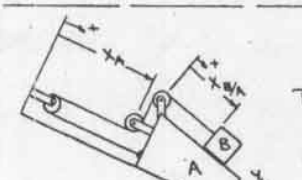
GIVEN: $v_A = 8\ \frac{in.}{s} \measuredangle 25°$

$a_A = 6\ \frac{in.}{s^2} \measuredangle 25°$

FIND: (a) $\mathbf{v}_B$

(b) $\mathbf{a}_B$





FROM THE DIAGRAM..

$2x_A + x_{B/A} =$ CONSTANT

THEN.. $2v_A + v_{B/A} = 0$

OR $|v_{B/A}| = 16\ \frac{in.}{s}$

AND $2a_A + a_{B/A} = 0$

OR $|a_{B/A}| = 12\ \frac{in.}{s^2}$

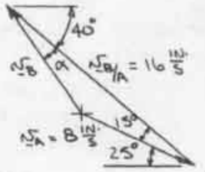
(CONTINUED)

11.123 CONTINUED

NOTE THAT $\underline{v}_{B/A}$ AND $\underline{a}_{B/A}$ MUST BE PARALLEL TO THE TOP SURFACE OF BLOCK A.

(a) HAVE.. $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN. NOTE THAT BECAUSE A IS MOVING DOWNWARD, B MUST BE MOVING UPWARD RELATIVE TO A.



HAVE..

$v_B^2 = 8^2 + 16^2 - 2(8)(16)\cos 15^\circ$

OR $v_B = 8.5278\ \frac{in.}{s}$

AND $\frac{8}{\sin\alpha} = \frac{8.5278}{\sin 15^\circ}$

OR $\alpha = 14.05^\circ$

$\therefore\ \underline{v}_B = 8.53\ \frac{in.}{s}\ \measuredangle 54.1^\circ$ ◀

(b) THE SAME TECHNIQUE THAT WAS USED TO DETERMINE $\underline{v}_B$ CAN BE USED TO DETERMINE $\underline{a}_B$. AN ALTERNATIVE METHOD IS AS FOLLOWS.

HAVE.. $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

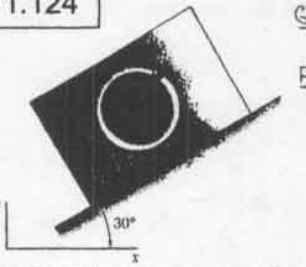
$= (6\,\underline{i}) + 12(-\cos 15^\circ \underline{i} + \sin 15^\circ \underline{j})$ *

$= -(5.5911\ \frac{in.}{s^2})\underline{i} + (3.1058\ \frac{in.}{s^2})\underline{j}$

OR $\underline{a}_B = 6.40\ \frac{in.}{s^2}\ \measuredangle 54.1^\circ$ ◀

* NOTE THE ORIENTATION OF THE COORDINATE AXES ON THE SKETCH OF THE SYSTEM.

11.124



GIVEN: $v_{P/A} = 200\ \frac{mm}{s}$ ↻

$v_A = 120\ \frac{mm}{s}\ \measuredangle 30^\circ$

FIND: (a) $\underline{v}_P$ WHEN $\theta = 30^\circ$

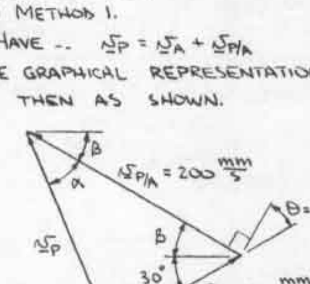
(b) $\underline{v}_P$ WHEN $\theta = 135^\circ$

NOTE: RATHER THAN APPLY THE SAME METHOD OF SOLUTION TWICE, TWO EQUALLY APPLICABLE TECHNIQUES WILL BE USED.

(a) METHOD 1.

HAVE.. $\underline{v}_P = \underline{v}_A + \underline{v}_{P/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



FIRST NOTE..

$\beta = 90^\circ - (30^\circ + 30^\circ) = 30^\circ$

THEN..

$v_P^2 = 120^2 + 200^2 - 2(120)(200)\cos 60^\circ$

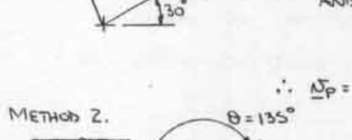
OR $v_P = 174.356\ \frac{mm}{s}$

AND $\frac{120}{\sin\alpha} = \frac{174.356}{\sin 60^\circ}$

OR $\alpha = 36.6^\circ$

$\therefore\ \underline{v}_P = 174.4\ \frac{mm}{s}\ \measuredangle 66.6^\circ$ ◀

(b) METHOD 2.



(CONTINUED)

11.124 CONTINUED

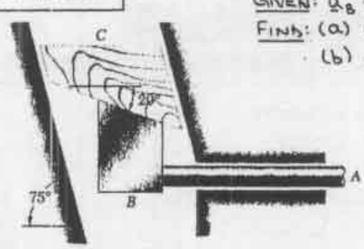
HAVE.. $\underline{v}_P = \underline{v}_A + \underline{v}_{P/A}$

$= 120(\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j}) + 200(-\cos 75^\circ \underline{i} - \sin 75^\circ \underline{j})$

$= (52.159\ \frac{mm}{s})\underline{i} - (133.185\ \frac{mm}{s})\underline{j}$

OR $\underline{v}_P = 143.0\ \frac{mm}{s}\ \measuredangle 68.6^\circ$ ◀

11.125



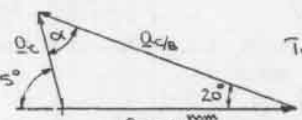
GIVEN: $a_B = 2\ \frac{mm}{s^2}$ →; $(v_B)_0 = (v_C)_0 = 0$

FIND: (a) $\underline{a}_C$

(b) $\underline{v}_C$ AT $t = 10\,s$

(a) HAVE.. $\underline{a}_C = \underline{a}_B + \underline{a}_{C/B}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



FIRST NOTE.. $\alpha = 180^\circ - (20^\circ + 105^\circ)$

$= 55^\circ$

THEN.. $\frac{a_C}{\sin 20^\circ} = \frac{2}{\sin 55^\circ}$

$a_C = 0.83506\ \frac{mm}{s^2}$

$\therefore\ \underline{a}_C = 0.835\ \frac{mm}{s^2}\ \measuredangle 75^\circ$ ◀

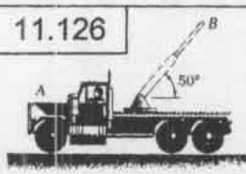
(b) FOR UNIFORMLY ACCELERATED MOTION..

$v_C = (v_C)_0^{\ 0} + a_C t$

AT $t = 10\,s$: $v_C = (0.83506\ \frac{mm}{s^2})(10\,s) = 8.3506\ \frac{mm}{s}$

OR $\underline{v}_C = 8.35\ \frac{mm}{s}\ \measuredangle 75^\circ$ ◀

11.126



GIVEN: $a_A = 1.2\ \frac{m}{s^2}$ →; $(v_B)_0 = 0$

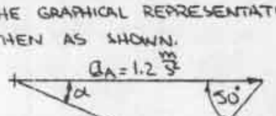
$a_{B/A} = 0.5\ \frac{m}{s^2}\ \measuredangle 50^\circ$

FIND: (a) $\underline{a}_B$

(b) $\underline{v}_B$ AT $t = 2\,s$

(a) HAVE.. $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



HAVE.. $a_B^2 = 1.2^2 + 0.5^2 - 2(1.2)(0.5)\cos 50^\circ$

OR $a_B = 0.95846\ \frac{m}{s^2}$

AND $\frac{0.5}{\sin\alpha} = \frac{0.95846}{\sin 50^\circ}$

OR $\alpha = 23.6^\circ$

$\therefore\ \underline{a}_B = 0.958\ \frac{m}{s^2}\ \measuredangle 23.6^\circ$ ◀

(b) FOR UNIFORMLY ACCELERATED MOTION...

$v_B = (v_B)_0^{\ 0} + a_B t$

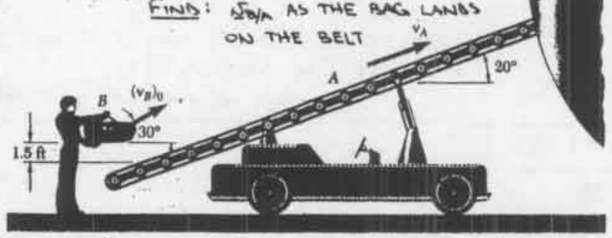
AT $t = 2\,s$: $v_B = (0.95846\ \frac{m}{s^2})(2\,s) = 1.91692\ \frac{m}{s}$

OR $\underline{v}_B = 1.917\ \frac{m}{s}\ \measuredangle 23.6^\circ$ ◀

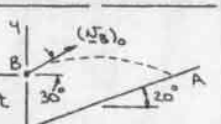
11.127

GIVEN: $v_A = 4\ \frac{ft}{s}$; $(v_B)_o = 2.5\ \frac{ft}{s}$

FIND: $v_{B/A}$ AS THE BAG LANDS ON THE BELT



FIRST DETERMINE THE VELOCITY OF THE BAG AS IT LANDS ON THE BELT. NOW..



$[(v_B)_x]_o = (v_B)_o \cos 30° = (2.5\ \frac{ft}{s})\cos 30°$

$[(v_B)_y]_o = (v_B)_o \sin 30° = (2.5\ \frac{ft}{s})\sin 30°$

HORIZONTAL MOTION (UNIFORM)

$x = x_o^{\,0} + [(v_B)_x]_o t \qquad (v_B)_x = [(v_B)_x]_o$

$= (2.5\cos 30°)t \qquad = 2.5\cos 30°$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = y_o + [(v_B)_y]_o t - \frac{1}{2}gt^2 \qquad (v_B)_y = [(v_B)_y]_o - gt$

$= 1.5 + (2.5\sin 30°)t - \frac{1}{2}gt^2 \qquad = 2.5\sin 30° - gt$

THE EQUATION OF THE LINE COLLINEAR WITH THE TOP SURFACE OF THE BELT IS

$y = x \tan 20°$

THUS, WHEN THE BAG REACHES THE BELT..

$1.5 + (2.5\sin 30°)t - \frac{1}{2}gt^2 = [(2.5\cos 30°)t]\tan 20°$

OR $\frac{1}{2}(32.2)t^2 + 2.5(\cos 30° \tan 20° - \sin 30°)t - 1.5 = 0$

OR $16.1t^2 - 0.46198t - 1.5 = 0$

SOLVING -- $t = 0.31992$ s AND $t = -0.29122$ s (REJECT)

THE VELOCITY v_B OF THE BAG AS IT LANDS ON THE BELT IS THEN..

$v_B = (2.5\cos 30°)\mathbf{i} + [2.5\sin 30° - 32.2(0.31992)]\mathbf{j}$

$= (2.1651\ \frac{ft}{s})\mathbf{i} - (9.0514\ \frac{ft}{s})\mathbf{j}$

FINALLY -- $v_B = v_A + v_{B/A}$

OR $v_{B/A} = (2.1651\mathbf{i} - 9.0514\mathbf{j}) - 4(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$

$= -(1.59367\ \frac{ft}{s})\mathbf{i} - (10.4195\ \frac{ft}{s})\mathbf{j}$

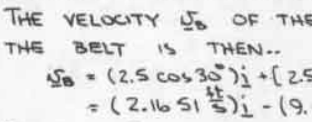
OR $v_{B/A} = 10.54\ \frac{ft}{s}\ \measuredangle 81.3°$

11.128

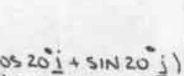
GIVEN: $(v_C)_o = 6\ \frac{ft}{s}\ \measuredangle 50°$

FIND: (a) v_B IF $v_{C/B}$ IS VERTICAL

(b) v_B IF $v_{C/B} = (v_{C/B})_{MIN}$



FIRST DETERMINE THE VELOCITY OF THE COAL AS IT LANDS ON THE BELT. NOW..



$[(v_C)_x]_o = -(v_C)_o \cos 50° = -(6\ \frac{ft}{s})\cos 50°$

$[(v_C)_y]_o = -(v_C)_o \sin 50° = -(6\ \frac{ft}{s})\sin 50°$

HORIZONTAL MOTION (UNIFORM)

$(v_C)_x = [(v_C)_x]_o = -6\cos 50°$

$= -3.8567\ \frac{ft}{s}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$(v_C)_y^2 = [(v_C)_y]_o^2 - 2g(y - y_o^{\,0}) \qquad (g = 32.2\ \frac{ft}{s})$

AT THE BELT: $(v_C)_y^2 = (-6\sin 50°)^2 - 2(32.2)(-5)$

OR $(v_C)_y = -18.5237\ \frac{ft}{s}$

(CONTINUED)

11.128 CONTINUED

THEN $v_C = -(3.8567\ \frac{ft}{s})\mathbf{i} - (18.5237\ \frac{ft}{s})\mathbf{j}$

$= 18.9209\ \frac{ft}{s}\ \measuredangle 78.239°$

(a) HAVE.. $v_C = v_B + v_{C/B}$

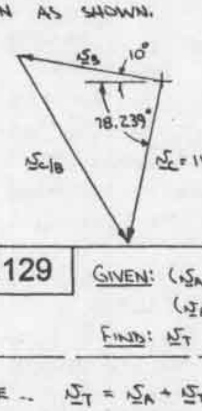
IF $v_{C/B}$ IS TO VERTICAL, THEN $(v_{C/B})_x = 0$ WHICH IMPLIES $(v_C)_x = (v_B)_x$

$\therefore\ -v_B \cos 10° = -3.8567$

OR $v_B = 3.92\ \frac{ft}{s}\ \measuredangle 10°$

(b) HAVE.. $v_C = v_B + v_{C/B}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



FOR $v_{C/B}$ TO BE MINIMUM, $v_{C/B}$ MUST BE PERPENDICULAR TO v_B.

$\therefore\ v_B = 18.9209\cos 88.239°$

OR $v_B = 0.581\ \frac{ft}{s}\ \measuredangle 10°$

11.129

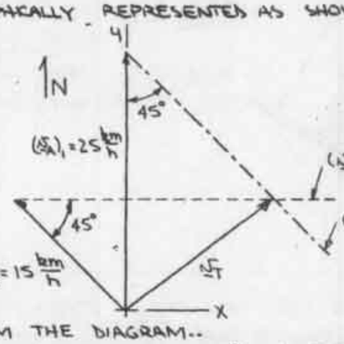
GIVEN: $(v_A)_1 = 25\ \frac{km}{h}\uparrow$, $(v_{T/A})_1\ \measuredangle 45°$

$(v_A)_2 = 15\ \frac{km}{h}\ \measuredangle 45°$, $(v_{T/A})_2 \rightarrow$

FIND: v_T, WHERE v_T IS CONSTANT

HAVE -- $v_T = v_A + v_{T/A}$

USING THIS EQUATION, THE TWO CASES ARE THEN GRAPHICALLY REPRESENTED AS SHOWN.



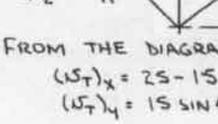
FROM THE DIAGRAM..

$(v_T)_x = 25 - 15\sin 45° = 14.3934\ \frac{km}{h}$

$(v_T)_y = 15\sin 45° = 10.6066\ \frac{km}{h}$

$\therefore\ v_T = 17.88\ \frac{km}{h}\ \measuredangle 36.4°$

11.130



GIVEN: $(v_B)_1 = 5\ \frac{km}{h}\uparrow$, $(v_{W/B})_1\ \measuredangle 50°$

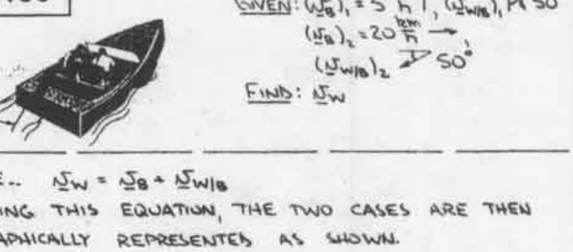
$(v_B)_2 = 20\ \frac{km}{h} \rightarrow$

$(v_{W/B})_2\ \measuredangle 50°$

FIND: v_W

HAVE.. $v_W = v_B + v_{W/B}$

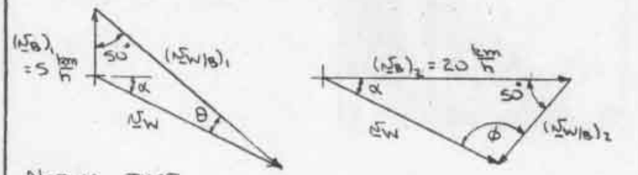
USING THIS EQUATION, THE TWO CASES ARE THEN GRAPHICALLY REPRESENTED AS SHOWN.



(CONTINUED)

11.130 CONTINUED

WITH $\mathbf{v}_W$ NOW DEFINED, THE ABOVE DIAGRAM IS REDRAWN FOR THE TWO CASES FOR CLARITY.



NOTING THAT

$$\theta = 180° - (50° + 90° + \alpha) = 40° - \alpha \qquad \phi = 180° - (50° + \alpha) = 130° - \alpha$$

HAVE $\quad \dfrac{v_W}{\sin 50°} = \dfrac{5}{\sin(40° - \alpha)} \qquad \dfrac{v_W}{\sin 50°} = \dfrac{20}{\sin(130° - \alpha)}$

THEREFORE $\quad \dfrac{5}{\sin(40° - \alpha)} = \dfrac{20}{\sin(130° - \alpha)}$

OR $\quad \sin 130° \cos\alpha - \cos 130° \sin\alpha = 4(\sin 40° \cos\alpha - \cos 40° \sin\alpha)$

OR $\quad \tan\alpha = \dfrac{\sin 130° - 4\sin 40°}{\cos 130° - 4\cos 40°}$

OR $\quad \alpha = 25.964°$

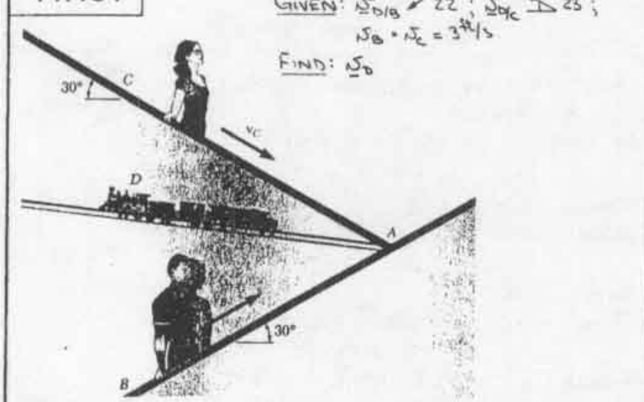
THEN $\quad v_W = \dfrac{5 \sin 50°}{\sin(40° - 25.964°)} = 15.79\ \frac{km}{h}$

$\therefore\ \mathbf{v}_W = 15.79\ \frac{km}{h} \measuredangle 26.0°$ ◀

11.131

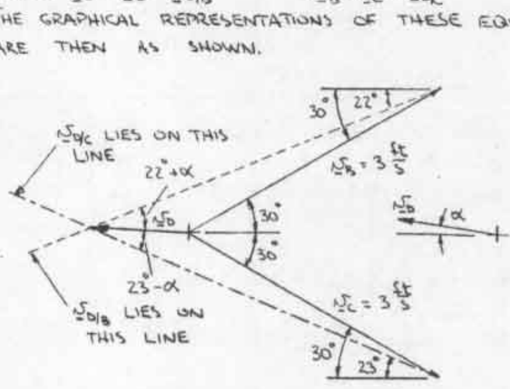
GIVEN: $\mathbf{v}_{D/B} \measuredangle 22°$; $\mathbf{v}_{D/C} \measuredangle 23°$; $v_B = v_C = 3\ ft/s$

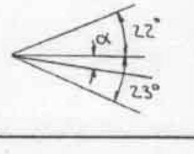
FIND: $\mathbf{v}_D$



HAVE.. $\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} \qquad \mathbf{v}_D = \mathbf{v}_C + \mathbf{v}_{D/C}$

THE GRAPHICAL REPRESENTATIONS OF THESE EQUATIONS ARE THEN AS SHOWN.





(CONTINUED)

11.131 CONTINUED

THEN.. $\quad \dfrac{v_D}{\sin 8°} = \dfrac{3}{\sin(22° + \alpha)} \qquad \dfrac{v_D}{\sin 7°} = \dfrac{3}{\sin(23° - \alpha)}$

EQUATING THE EXPRESSIONS FOR $\frac{v_D}{3}$ ..

$$\frac{\sin 8°}{\sin(22° + \alpha)} = \frac{\sin 7°}{\sin(23° - \alpha)}$$

OR $\quad \sin 8°(\sin 23° \cos\alpha - \cos 23° \sin\alpha) = \sin 7°(\sin 22° \cos\alpha + \cos 22° \sin\alpha)$

OR $\quad \tan\alpha = \dfrac{\sin 8° \sin 23° - \sin 7° \sin 22°}{\sin 8° \cos 23° + \sin 7° \cos 22°}$

OR $\quad \alpha = 2.0728°$

THEN $\quad v_D = \dfrac{3 \sin 8°}{\sin(22° + 2.0728°)} = 1.024\ \frac{ft}{s}$

$\therefore\ \mathbf{v}_D = 1.024\ \frac{ft}{s} \measuredangle 2.07°$ ◀

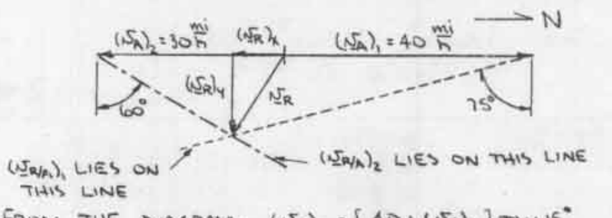
11.132

GIVEN: $(\mathbf{v}_A)_1 = 40\ \frac{mi}{h}$ N, $(\mathbf{v}_{R/A})_1 \measuredangle 75°$
$(\mathbf{v}_A)_2 = 30\ \frac{mi}{h}$ S, $(\mathbf{v}_{R/A})_2$ 60° WITH THE VERTICAL

FIND: $\mathbf{v}_R$

HAVE.. $\mathbf{v}_R = (\mathbf{v}_A)_1 + (\mathbf{v}_{R/A})_1 \qquad \mathbf{v}_R = (\mathbf{v}_A)_2 + (\mathbf{v}_{R/A})_2$

THE GRAPHICAL REPRESENTATIONS OF THESE EQUATIONS ARE THEN AS SHOWN. NOTE THAT THE LINE OF ACTION OF $(\mathbf{v}_{R/A})_2$ MUST BE DIRECTED AS SHOWN SO THAT THE SECOND VELOCITY DIAGRAM "CLOSES."



FROM THE DIAGRAM.. $(v_R)_y = [40 + (v_R)_x] \tan 15°$
AND $(v_R)_y = [30 - (v_R)_x] \tan 30°$

EQUATING THE EXPRESSIONS FOR $(v_R)_y$ --

$$[40 + (v_R)_x] \tan 15° = [30 - (v_R)_x] \tan 30°$$

OR $\quad (v_R)_x = 7.8109\ \frac{mi}{h}$

THEN.. $\quad (v_R)_y = (40 + 7.8109) \tan 15° = 12.8109\ \frac{mi}{h}$

$\therefore\ \mathbf{v}_R = 15.00\ \frac{mi}{h} \measuredangle 58.6°$ ◀

11.133

GIVEN: $v = 18\ \frac{m}{s}$, $\rho = 14$ m

FIND: a_n

HAVE.. $\quad a_n = \dfrac{v^2}{\rho} = \dfrac{(18\ \frac{m}{s})^2}{14\ m}$

OR $\quad a_n = 23.1\ \frac{m}{s^2}$ ◀

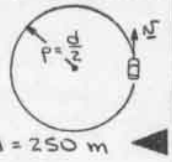
11.134

GIVEN: CIRCULAR TRACK OF DIAMETER d

FIND: (a) d WHEN $v = 72\ \frac{km}{h}$, $a_n = 3.2\ \frac{m}{s^2}$

(b) v WHEN $d = 180$ m, $a_n = 0.6g$

(a) FIRST NOTE.. $v = 72\ \frac{km}{h} = 20\ \frac{m}{s}$



NOW.. $a_n = \frac{v^2}{\rho}$

OR $\frac{d}{2} = \frac{(20\ \frac{m}{s})^2}{3.2\ \frac{m}{s^2}}$

OR $d = 250$ m ◄

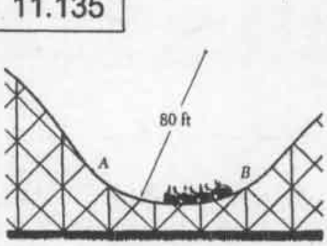
(b) HAVE $a_n = \frac{v^2}{\rho}$

THEN.. $v^2 = (0.6 \times 9.81\ \frac{m}{s^2})(\frac{1}{2} \times 180\ m)$

OR $v = 23.016\ \frac{m}{s}$

OR $v = 82.9\ \frac{km}{h}$ ◄

11.135



GIVEN: $(a_n)_{AB} \le 3g$

FIND: $(v_{MAX})_{AB}$

HAVE.. $a_n = \frac{v^2}{\rho}$

THEN.. $(v_{MAX})^2_{AB} = (3 \times 32.2\ \frac{ft}{s^2})(80\ ft)$

OR $(v_{MAX})_{AB} = 87.909\ \frac{ft}{s}$

OR $(v_{MAX})_{AB} = 59.9\ \frac{mi}{h}$ ◄

11.136

GIVEN: $[(a_c)_n]_A = 26\ \frac{in.}{s^2}$

$[(a_c)_n]_B = 267\ \frac{in.}{s^2}$

B ROLLS ON A

FIND: d_B

FIRST NOTE THAT "ROLLING WITHOUT SLIPPING" IMPLIES $(v_c)_A = (v_c)_B = v_c$

NOW.. $[(a_c)_n]_A = \frac{v_c^2}{\rho_A}$ AND $[(a_c)_n]_B = \frac{v_c^2}{\rho_B}$

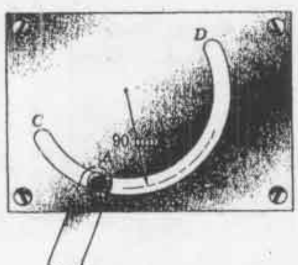
WHERE $\rho_B = \frac{d_B}{2}$

THEN.. $\rho_A[(a_c)_n]_A = [(a_c)_n]_B(\frac{d_B}{2})$

SUBSTITUTING.. $(2.6\ in.)(26\ \frac{in.}{s^2}) = (267\ \frac{in.}{s^2})(\frac{d_B}{2})$

OR $d_B = 0.506$ in. ◄

11.137



GIVEN: $(v_A)_0 = 0$;

$(a_A)_t = 20\ \frac{mm}{s^2}$

FIND: (a) a_A AT $t = 0$

(b) a_A AT $t = 2$ s

(a) AT $t = 0$, $v_A = 0$ WHICH IMPLIES $(a_A)_n = 0$

$\therefore\ a_A = (a_A)_t$

OR $a_A = 20\ \frac{mm}{s^2}$ ◄

(b) HAVE UNIFORMLY ACCELERATED MOTION...

$\therefore\ v_A = \cancel{(v_A)_0}^{\,0} + (a_A)_t t$

AT $t = 2$ s: $v_A = (20\ mm/s^2)(2\ s) = 40\ \frac{mm}{s}$

NOW.. $(a_A)_n = \frac{v_A^2}{\rho_A} = \frac{(40\ mm/s)^2}{90\ mm} = 17.7778\ \frac{mm}{s^2}$

FINALLY, $a_A^2 = (a_A)_t^2 + (a_A)_n^2$

$= (20)^2 + (17.7778)^2$

OR $a_A = 26.8\ \frac{mm}{s^2}$ ◄

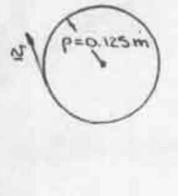
11.138

GIVEN: $d = 250$ mm; $v_0 = 45\ \frac{m}{s}$; a_t = CONSTANT;

AT $t = 9$ s, $v = 0$

FIND: t WHEN $a = 40\ \frac{m}{s^2}$

HAVE UNIFORMLY DECELERATED MOTION..



$\therefore\ v = v_0 + a_t t$

AT $t = 9$ s: $0 = 45\ \frac{m}{s} + a_t(9\ s)$

OR $a_t = -5\ \frac{m}{s^2}$

NOW.. $a^2 = a_t^2 + a_n^2$

WHEN $a = 40\ \frac{m}{s^2}$: $40^2 = (-5)^2 + a_n^2$

OR $a_n = 39.686\ \frac{m}{s^2}$

HAVE.. $a_n = \frac{v^2}{\rho}$

THEN $v^2 = (39.686\ \frac{m}{s^2})(0.125\ m)$

OR $v = 2.227\ \frac{m}{s}$

FINALLY.. $2.227\ \frac{m}{s} = 45\ \frac{m}{s} + (-5\ \frac{m}{s^2})t$

OR $t = 8.55$ s ◄

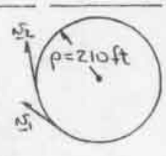
11.139

GIVEN: $d = 420$ ft; a_t = CONSTANT; $v_1 = 14\ \frac{ft}{s}$,

$v_2 = 24\ \frac{ft}{s}$, $\Delta s_{12} = 95$ ft

FIND: a AT $t = 2$ s

HAVE UNIFORMLY ACCELERATED MOTION..



$\therefore\ v_2^2 = v_1^2 + 2a_t \Delta s_{12}$

SUBSTITUTING.. $(24\ \frac{ft}{s})^2 = (14\ \frac{ft}{s})^2 + 2a_t(95\ ft)$

OR $a_t = 2\ \frac{ft}{s^2}$

ALSO.. $v = v_1 + a_t t$

AT $t = 2$ s: $v = 14\ \frac{ft}{s} + (2\ \frac{ft}{s^2})(2\ s) = 18\ \frac{ft}{s}$

NOW.. $a_n = \frac{v^2}{\rho}$

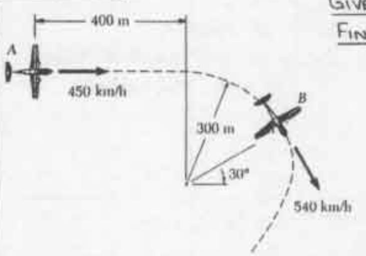
AT $t = 2$ s: $a_n = \frac{(18\ ft/s)^2}{210\ ft} = 1.54286\ \frac{ft}{s^2}$

FINALLY.. $a^2 = a_t^2 + a_n^2$

AT $t = 2$ s: $a^2 = 2^2 + 1.54286^2$

OR $a = 2.53\ \frac{ft}{s^2}$ ◄

11.140



GIVEN: $a_A = 8\ \frac{m}{s^2}$, $(a_B)_t = -3\ \frac{m}{s^2}$

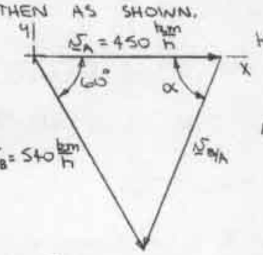
FIND: (a) $v_{B/A}$

(b) $a_{B/A}$

FIRST NOTE.. $v_A = 450\ \frac{km}{h}$ $\quad v_B = 540\ \frac{km}{h} = 150\ \frac{m}{s}$

(a) HAVE.. $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



HAVE..

$$v_{B/A}^2 = 450^2 + 540^2 - 2(450)(540)\cos 60°$$

OR $v_{B/A} = 501.10\ \frac{km}{h}$

AND $\frac{540}{\sin\alpha} = \frac{501.10}{\sin 60°}$

OR $\alpha = 68.9°$

$\therefore\ \underline{v}_{B/A} = 501\ \frac{km}{h} \measuredangle 68.9°$

(b) FIRST NOTE.. $\underline{a}_A = 8\ \frac{m}{s^2} \rightarrow$ $\quad (\underline{a}_B)_t = 3\ \frac{m}{s^2} \measuredangle 60°$

NOW.. $(a_B)_n = \frac{v_B^2}{\rho_B} = \frac{(150\ m/s)^2}{300\ m}$ OR $(\underline{a}_B)_n = 75\ \frac{m}{s^2} \measuredangle 30°$

THEN.. $\underline{a}_B = (\underline{a}_B)_t + (\underline{a}_B)_n$

$$= 3(-\cos 60°\,\underline{i} + \sin 60°\,\underline{j}) + 75(-\cos 30°\,\underline{i} - \sin 30°\,\underline{j})$$

$$= -(66.452\ \tfrac{m}{s^2})\underline{i} - (34.902\ \tfrac{m}{s^2})\underline{j}$$

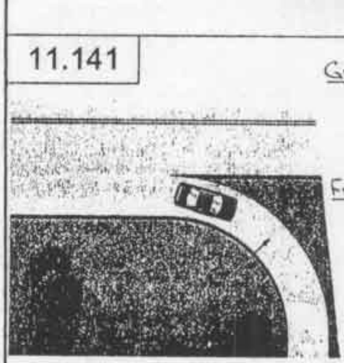
FINALLY.. $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

OR $\underline{a}_{B/A} = (-66.452\underline{i} - 34.902\underline{j}) - (8\underline{i})$

$$= -(74.452\ \tfrac{m}{s^2})\underline{i} - (34.902\ \tfrac{m}{s^2})\underline{j}$$

OR $\underline{a}_{B/A} = 82.2\ \frac{m}{s^2} \measuredangle 25.1°$

11.141



GIVEN: $a_{STRAIGHT} = a_t =$ CONSTANT; AT $t=0$, CAR ENTERS EXIT RAMP; FOR $t > 10$ s, $v = 20\ \frac{mi}{h}$, $a = \frac{1}{4} a_{ST}$.

FIND: a_{MAX}

FIRST NOTE.. $v_{10} = 20\ \frac{mi}{h} = \frac{88}{3}\ \frac{ft}{s}$

WHILE THE CAR IS ON THE STRAIGHT PORTION OF THE HIGHWAY

$$a = a_{STRAIGHT} = a_t$$

AND FOR THE CIRCULAR EXIT RAMP

$$a = \sqrt{a_t^2 + a_n^2}$$

WHERE $\quad a_n = \frac{v^2}{\rho}$

BY OBSERVATION, a_{MAX} OCCURS WHEN v IS MAXIMUM, WHICH IS AT $t=0$ WHEN THE CAR FIRST ENTERS THE RAMP.

FOR UNIFORMLY DECELERATED MOTION

$$v = v_0 + a_t t$$

(CONTINUED)

11.141 CONTINUED

AND AT t 10 s: $v =$ CONSTANT $\Rightarrow a = a_n = \frac{v_{10}^2}{\rho}$

$$a = \tfrac{1}{4} a_{ST.}$$

THEN $a_{ST} = a_t \Rightarrow \frac{1}{4} a_t = \frac{v_{10}^2}{\rho} = \frac{(\frac{88}{3}\ \frac{ft}{s})^2}{560\ ft}$

OR $a_t = -6.1460\ \frac{ft}{s^2}$

(THE CAR IS DECELERATING; HENCE, THE MINUS SIGN).

THEN AT $t = 10$ s: $\frac{88}{3}\ \frac{ft}{s} = v_0 + (-6.1460\ \frac{ft}{s^2})(10\ s)$

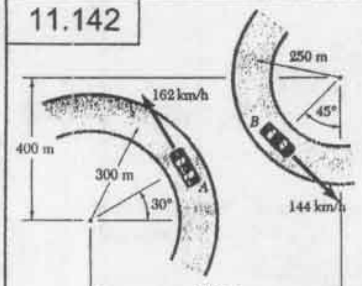
OR $v_0 = 90.793\ \frac{ft}{s}$

THEN AT $t = 0$: $a_{MAX} = \sqrt{a_t^2 + \left(\frac{v_0^2}{\rho}\right)^2}$

$$= \left\{(-6.1460\ \tfrac{ft}{s^2})^2 + \left[\frac{(90.793\ \frac{ft}{s})^2}{560\ ft}\right]^2\right\}^{1/2}$$

OR $a_{MAX} = 15.95\ \frac{ft}{s^2}$

11.142



GIVEN: $(a_A)_t = -7\ \frac{m}{s^2}$

$(a_B)_t = 2\ \frac{m}{s^2}$

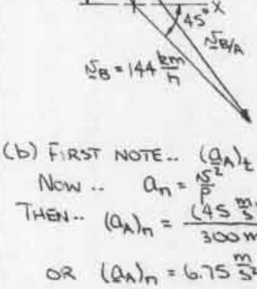
FIND: (a) $v_{B/A}$

(b) $a_{B/A}$

FIRST NOTE.. $v_A = 162\ \frac{km}{h} = 45\ \frac{m}{s}$ $\quad v_B = 144\ \frac{km}{h} = 40\ \frac{m}{s}$

(a) HAVE.. $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

THE GRAPHICAL REPRESENTATION OF THIS EQUATION IS THEN AS SHOWN.



HAVE..

$$v_{B/A}^2 = 162^2 + 144^2 - 2(162)(144)\cos 165°$$

OR $v_{B/A} = 303.39\ \frac{km}{h}$

AND $\frac{144}{\sin\alpha} = \frac{303.39}{\sin 165°}$

OR $\alpha = 7.056°$

$\therefore\ \underline{v}_{B/A} = 303\ \frac{km}{h} \measuredangle 52.9°$

(b) FIRST NOTE.. $(\underline{a}_A)_t = 7\ \frac{m}{s^2} \measuredangle 60°$ $\quad (\underline{a}_B)_t = 2\ \frac{m}{s^2} \measuredangle 45°$

NOW.. $a_n = \frac{v^2}{\rho}$

THEN.. $(a_A)_n = \frac{(45\ \frac{m}{s})^2}{300\ m}$ $\quad (a_B)_n = \frac{(40\ \frac{m}{s})^2}{250\ m}$

OR $(\underline{a}_A)_n = 6.75\ \frac{m}{s^2} \measuredangle 30°$ $\quad (\underline{a}_B)_n = 6.40\ \frac{m}{s^2} \measuredangle 45°$

NOTING THAT $\quad \underline{a} = \underline{a}_t + \underline{a}_n$

HAVE.. $\underline{a}_A = 7(\cos 60°\,\underline{i} - \sin 60°\,\underline{j}) + 6.75(-\cos 30°\,\underline{i} - \sin 30°\,\underline{j})$

$$= -(2.3457\ \tfrac{m}{s^2})\underline{i} - (9.4372\ \tfrac{m}{s^2})\underline{j}$$

AND $\quad \underline{a}_B = 2(\cos 45°\,\underline{i} - \sin 45°\,\underline{j}) + 6.40(\cos 45°\,\underline{i} + \sin 45°\,\underline{j})$

$$= (5.9397\ \tfrac{m}{s^2})\underline{i} + (3.1113\ \tfrac{m}{s^2})\underline{j}$$

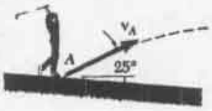
FINALLY.. $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

OR $\underline{a}_{B/A} = (5.9397\underline{i} + 3.1113\underline{j}) - (-2.3457\underline{i} - 9.4372\underline{j})$

$$= (8.2854\ \tfrac{m}{s^2})\underline{i} + (12.5485\ \tfrac{m}{s^2})\underline{j}$$

OR $\underline{a}_{B/A} = 15.04\ \frac{m}{s^2} \measuredangle 56.6°$

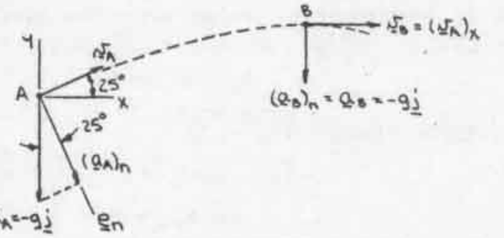
11.143



GIVEN: $v_A = 50\ \frac{m}{s}$

FIND: (a) ρ AT POINT A

(b) ρ AT THE HIGHEST POINT OF THE TRAJECTORY



(a) HAVE.. $(a_A)_n = \frac{v_A^2}{\rho_A}$

OR $\rho_A = \frac{(50\ \frac{m}{s})^2}{(9.81\ \frac{m}{s^2})\cos 25°}$

OR $\rho_A = 281$ m ◀

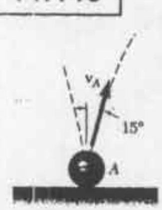
(b) HAVE.. $(a_B)_n = \frac{v_B^2}{\rho_B}$

WHERE POINT B IS THE HIGHEST POINT OF THE TRAJECTORY, SO THAT $v_B = (v_A)_x = v_A \cos 25°$

THEN.. $\rho_B = \frac{[(50\ \frac{m}{s})\cos 25°]^2}{9.81\ \frac{m}{s^2}}$

OR $\rho_B = 209$ m ◀

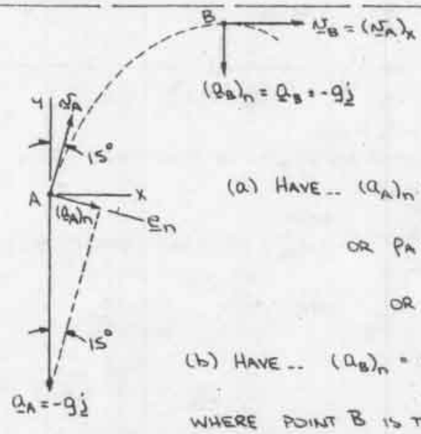
11.145



GIVEN: $v_A = 7.5\ \frac{ft}{s}$

FIND: (a) ρ AT POINT A

(b) ρ AT THE HIGHEST POINT OF THE TRAJECTORY



(a) HAVE.. $(a_A)_n = \frac{v_A^2}{\rho_A}$

OR $\rho_A = \frac{(7.5\ \frac{ft}{s})^2}{(32.2\ \frac{ft}{s^2})\sin 15°}$

OR $\rho_A = 6.75$ ft ◀

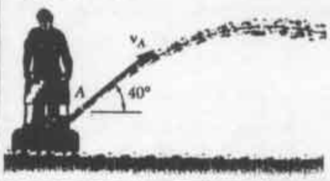
(b) HAVE.. $(a_B)_n = \frac{v_B^2}{\rho_B}$

WHERE POINT B IS THE HIGHEST POINT OF THE TRAJECTORY, SO THAT $v_B = (v_A)_x = v_A \sin 15°$

THEN.. $\rho_B = \frac{[(7.5\ \frac{ft}{s})\sin 15°]^2}{32.2\ ft/s^2}$

OR $\rho_B = 0.1170$ ft ◀

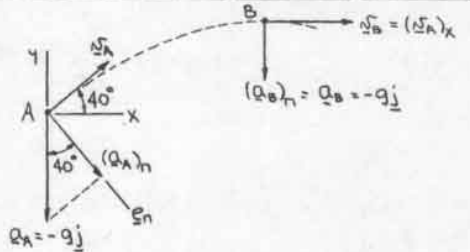
11.144



GIVEN: $\rho_A = 8.5$ m

FIND: (a) v_A

(b) ρ AT THE HIGHEST POINT OF THE TRAJECTORY



(a) HAVE.. $(a_A)_n = \frac{v_A^2}{\rho_A}$

OR $v_A^2 = (9.81 \cos 40°)(8.5\ m) = 63.8766\ m^2/s^2$

OR $v_A = 7.99\ \frac{m}{s} \measuredangle 40°$ ◀

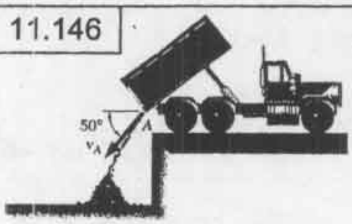
(b) HAVE.. $(a_B)_n = \frac{v_B^2}{\rho_B}$

WHERE POINT B IS THE HIGHEST POINT OF THE TRAJECTORY, SO THAT $v_B = (v_A)_x = v_A \cos 40°$

THEN.. $\rho_B = \frac{(63.8766\ \frac{m^2}{s^2})\cos^2 40°}{9.81\ \frac{m}{s^2}}$

OR $\rho_B = 3.82$ m ◀

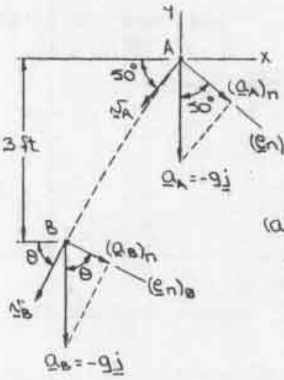
11.146



GIVEN: $v_A = 6\ \frac{ft}{s}$

FIND: (a) ρ AT POINT A

(b) ρ AT THE POINT ON THE TRAJECTORY 3 ft BELOW A



(a) HAVE.. $(a_A)_n = \frac{v_A^2}{\rho_A}$

OR $\rho_A = \frac{(6\ ft/s)^2}{(32.2\ \frac{ft}{s^2})\cos 50°}$

OR $\rho_A = 1.739$ ft ◀

(b) HORIZONTAL MOTION (UNIFORM)

$(v_B)_x = (v_A)_x = (6\ \frac{ft}{s})\cos 50° = 3.8567\ \frac{ft}{s}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

HAVE.. $v_y^2 = (v_A)_y^2 - 2g(y - \overset{0}{y_A})$ $\quad (g = 32.2\ \frac{ft}{s^2})$

WHERE $(v_A)_y = (6\ \frac{ft}{s})\sin 50° = 4.5963\ \frac{ft}{s}$

AT POINT B, $y = -3$ ft: $(v_B)_y^2 = (4.5963\ \frac{ft}{s})^2 - 2(32.2\ \frac{ft}{s^2})(-3\ ft)$

OR $(v_B)_y = 14.6399\ \frac{ft}{s}$

THEN.. $v_B = \sqrt{(v_B)_x^2 + (v_B)_y^2} = \sqrt{(3.8567)^2 + (14.6399)^2} = 15.1394\ \frac{ft}{s}$

AND $\tan\theta = \frac{v_y}{v_x} = \frac{14.6399}{3.8567}$ OR $\theta = 75.24°$

(CONTINUED)

11.146 CONTINUED

Now.. $(a_B)_n = \frac{v_B^2}{\rho_B}$

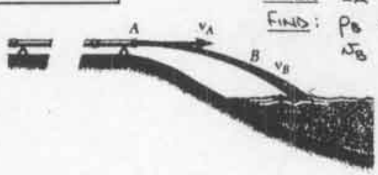
or $\rho_B = \frac{(15.1394\ \frac{ft}{s})^2}{(32.2\ \frac{ft}{s^2})\cos 75.24°}$

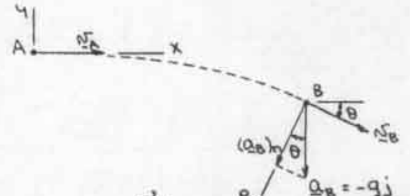
or $\rho_B = 27.9$ ft

11.147

GIVEN: $v_A = v_A \rightarrow$

FIND: ρ_B IN TERMS OF v_A AND v_B





HAVE.. $(a_B)_n = \frac{v_B^2}{\rho_B}$

WHERE $(a_B)_n = a_B \cos\theta = g\cos\theta$

NOTING THAT THE HORIZONTAL MOTION IS UNIFORM,

HAVE.. $(v_B)_x = v_A$

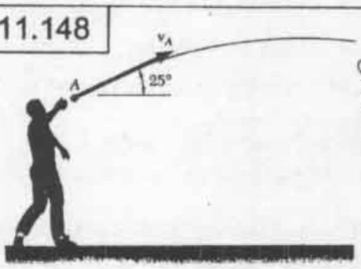
WHERE $(v_B)_x = v_B \cos\theta$

$\therefore \cos\theta = \frac{v_A}{v_B}$

THEN -- $\rho_B = \frac{v_B^2}{g\left(\frac{v_A}{v_B}\right)}$

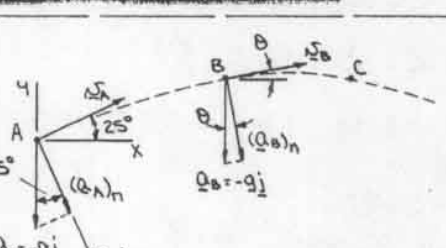
or $\rho_B = \frac{v_B^3}{g v_A}$

11.148



GIVEN: $v_A = 20\ \frac{m}{s}$

FIND: v AT THOSE POINTS WHERE $\rho = \frac{3}{4}\rho_A$



ASSUME THAT POINTS B AND C ARE THE POINTS OF INTEREST, WHERE $y_B = y_C$ AND $v_B = v_C$. NOW..

$(a_A)_n = \frac{v_A^2}{\rho_A}$

or $\rho_A = \frac{v_A^2}{g\cos 25°}$

THEN $\rho_B = \frac{3}{4}\rho_A = \frac{3}{4}\frac{v_A^2}{g\cos 25°}$

(CONTINUED)

11.148 CONTINUED

HAVE $(a_B)_n = \frac{v_B^2}{\rho_B}$ WHERE $(a_B)_n = g\cos\theta$

SO THAT $\frac{3}{4}\frac{v_A^2}{g\cos 25°} = \frac{v_B^2}{g\cos\theta}$

or $v_B^2 = \frac{3}{4}\frac{\cos\theta}{\cos 25°}v_A^2 \qquad (1)$

NOTING THAT THE HORIZONTAL MOTION IS UNIFORM,

HAVE.. $(v_A)_x = (v_B)_x$

WHERE $(v_A)_x = v_A\cos 25° \qquad (v_B)_x = v_B\cos\theta$

THEN $v_A\cos 25° = v_B\cos\theta$

or $\cos\theta = \frac{v_A}{v_B}\cos 25°$

SUBSTITUTING FOR $\cos\theta$ IN EQ. (1), HAVE..

$v_B^2 = \frac{3}{4}\left(\frac{v_A}{v_B}\cos 25°\right)\frac{v_A^2}{\cos 25°}$

or $v_B^3 = \frac{3}{4}v_A^3 = \frac{3}{4}\left(20\ \frac{m}{s}\right)^3$

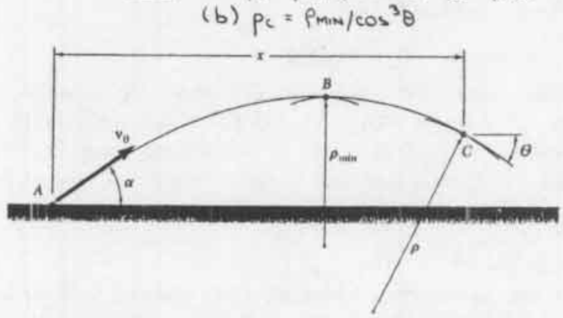
or $v_B = v_C = 18.17\ \frac{m}{s}$

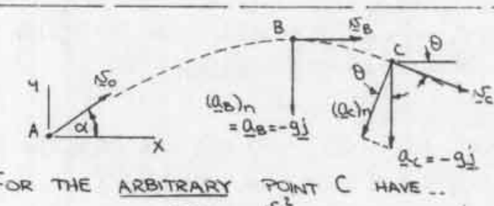
11.149

GIVEN: THE INITIAL VELOCITY v_0 AND THE TRAJECTORY OF THE PROJECTILE AS SHOWN

SHOW: (a) $\rho_B = \rho_{MIN}$, WHERE $y_B = y_{MAX}$

(b) $\rho_C = \rho_{MIN}/\cos^3\theta$





FOR THE ARBITRARY POINT C HAVE..

$(a_C)_n = \frac{v_C^2}{\rho_C}$

or $\rho_C = \frac{v_C^2}{g\cos\theta}$

NOTING THAT THE HORIZONTAL MOTION IS UNIFORM,

HAVE.. $(v_A)_x = (v_C)_x$

WHERE $(v_A)_x = v_0\cos\alpha \qquad (v_C)_x = v_C\cos\theta$

THEN $v_0\cos\alpha = v_C\cos\theta$

or $v_C = \frac{\cos\alpha}{\cos\theta}v_0$

SO THAT $\rho_C = \frac{1}{g\cos\theta}\left(\frac{\cos\alpha}{\cos\theta}v_0\right)^2 = \frac{v_0^2\cos^2\alpha}{g\cos^3\theta}$

(a) IN THE EXPRESSION FOR ρ_C, v_0, α, AND g ARE CONSTANTS, SO THAT ρ_C IS MINIMUM WHERE $\cos\theta$ IS MAXIMUM. BY OBSERVATION, THIS OCCURS AT POINT B WHERE $\theta = 0$.

$\therefore \rho_{MIN} = \rho_B = \frac{v_0^2\cos^2\alpha}{g}$ Q.E.D.

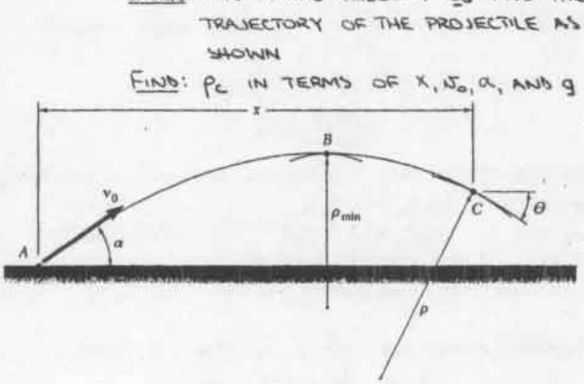
(b) $\rho_C = \frac{1}{\cos^3\theta}\left(\frac{v_0^2\cos^2\alpha}{g}\right)$

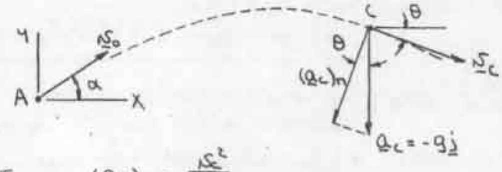
or $\rho_C = \frac{\rho_{MIN}}{\cos^3\theta}$ Q.E.D.

11.150

GIVEN: THE INITIAL VELOCITY v_0 AND THE TRAJECTORY OF THE PROJECTILE AS SHOWN

FIND: ρ_C IN TERMS OF x, v_0, α, AND g





HAVE.. $(a_C)_n = \frac{v_C^2}{\rho_C}$

OR $\rho_C = \frac{v_C^2}{g\cos\theta}$

NOTING THAT THE HORIZONTAL MOTION IS UNIFORM,
HAVE $(v_A)_x = (v_C)_x$ $\quad x = x_0^{\,0} + (v_0)_x t = (v_0\cos\alpha)t$
WHERE $(v_A)_x = v_0\cos\alpha$ $\quad (v_C)_x = v_C\cos\theta$
THEN $v_0\cos\alpha = v_C\cos\theta$ AND $(v_C)_x = v_0\cos\alpha$ (1)
OR $\cos\theta = \frac{v_0}{v_C}\cos\alpha$

SO THAT $\rho_C = \frac{v_C^3}{g v_0\cos\alpha}$

FOR THE UNIFORMLY ACCELERATED VERTICAL MOTION HAVE
$(v_C)_y = (v_0)_y - gt = v_0\sin\alpha - gt$
FROM ABOVE.. $x = (v_0\cos\alpha)t$ OR $t = \frac{x}{v_0\cos\alpha}$
THEN.. $(v_C)_y = v_0\sin\alpha - g\frac{x}{v_0\cos\alpha}$ (2)

NOW.. $v_C^2 = (v_C)_x^2 + (v_C)_y^2$
SUBSTITUTING FOR $(v_C)_x$ [EQ.(1)] AND $(v_C)_y$ [EQ.(2)]

$$v_C^2 = (v_0\cos\alpha)^2 + \left(v_0\sin\alpha - g\frac{x}{v_0\cos\alpha}\right)^2 = v_0^2\left(1 - \frac{2gx\tan\alpha}{v_0^2} + \frac{g^2x^2}{v_0^4\cos^2\alpha}\right)$$

OR $v_C^3 = v_0^3\left(1 - \frac{2gx\tan\alpha}{v_0^2} + \frac{g^2x^2}{v_0^4\cos^2\alpha}\right)^{3/2}$

FINALLY, SUBSTITUTING INTO THE EXPRESSION FOR ρ_C, OBTAIN--

$$\rho = \frac{v_0^2}{g\cos\alpha}\left(1 - \frac{2gx\tan\alpha}{v_0^2} + \frac{g^2x^2}{v_0^4\cos^2\alpha}\right)^{3/2}$$

* 11.151

GIVEN: $\mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$

FIND: ρ AT $t=0$

HAVE.. $\mathbf{v} = \frac{d\mathbf{r}}{dt} = R(\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$

AND.. $\mathbf{a} = \frac{d\mathbf{v}}{dt} = R(-\omega_n\sin\omega_n t - \omega_n\sin\omega_n t - \omega_n^2 t\cos\omega_n t)\mathbf{i} + R(\omega_n\cos\omega_n t + \omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t)\mathbf{k}$

(CONTINUED)

11.151 CONTINUED

OR $\mathbf{a} = \omega_n R[-(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{i} + (2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{k}]$

NOW.. $v^2 = R^2(\cos\omega_n t - \omega_n t\sin\omega_n t)^2 + c^2 + R^2(\sin\omega_n t + \omega_n t\cos\omega_n t)^2$
$= R^2(1+\omega_n^2t^2) + c^2$

THEN $v = [R^2(1+\omega_n^2t^2)+c^2]^{1/2}$

AND $\frac{dv}{dt} = \frac{R^2\omega_n^2 t}{[R^2(1+\omega_n^2t^2)+c^2]^{1/2}}$

NOW.. $a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$

AT $t=0$: $\frac{dv}{dt} = 0$
$\mathbf{a} = \omega_n R(2\mathbf{k})$ OR $a = 2\omega_n R$
$v^2 = R^2 + c^2$

THEN, WITH $\frac{dv}{dt}=0$, HAVE.. $a = \frac{v^2}{\rho}$
OR $2\omega_n R = \frac{R^2+c^2}{\rho}$

OR $\rho = \frac{R^2+c^2}{2\omega_n R}$

* 11.152

GIVEN: $\mathbf{r} = (At\cos t)\mathbf{i} + (A\sqrt{t^2+1})\mathbf{j} + (Bt\sin t)\mathbf{k}$, $r \sim$ ft, $t \sim$ s;
$A=3$, $B=1$

FIND: ρ AT $t=0$

WITH $A=3$, $B=1$ HAVE..
$\mathbf{r} = (3t\cos t)\mathbf{i} + (3\sqrt{t^2+1})\mathbf{j} + (t\sin t)\mathbf{k}$

NOW.. $\mathbf{v} = \frac{d\mathbf{r}}{dt} = 3(\cos t - t\sin t)\mathbf{i} + \left(\frac{3t}{\sqrt{t^2+1}}\right)\mathbf{j} + (\sin t + t\cos t)\mathbf{k}$

AND.. $\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\mathbf{i} + 3\left[\frac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{t^2+1}\right]\mathbf{j} + (\cos t + \cos t - t\sin t)\mathbf{k}$
$= -3(2\sin t + t\cos t)\mathbf{i} + 3\frac{1}{(t^2+1)^{3/2}}\mathbf{j} + (2\cos t - t\sin t)\mathbf{k}$

THEN.. $v^2 = 9(\cos t - t\sin t)^2 + 9\frac{t^2}{t^2+1} + (\sin t + t\cos t)^2$

EXPANDING AND SIMPLIFYING YIELDS--
$v^2 = t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3+t)\sin 2t$

THEN $v = [t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3+t)\sin 2t]^{1/2}$

AND

$$\frac{dv}{dt} = \frac{4t^3 + 38t + 8(-2\cos t\sin t + 4t^3\sin^2 t + 2t^4\sin t\cos t) - 8[(3t^2+1)\sin 2t + 2(t^3+t)\cos 2t]}{2[t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3+t)\sin 2t]^{1/2}}$$

NOW.. $a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$

AT $t=0$: $\mathbf{a} = 3\mathbf{j} + 2\mathbf{k}$ OR $a = \sqrt{13}\ \frac{\text{ft}}{\text{s}^2}$
$\frac{dv}{dt} = 0$
$v^2 = 9\left(\frac{\text{ft}}{\text{s}}\right)^2$

THEN, WITH $\frac{dv}{dt}=0$, HAVE.. $a = \frac{v^2}{\rho}$

OR $\rho = \frac{9\ \frac{\text{ft}^2}{\text{s}^2}}{\sqrt{13}\ \frac{\text{ft}}{\text{s}^2}}$

OR $\rho = 2.50$ ft

11.153

GIVEN: $a_n = g\frac{R^2}{r^2}$; $g_{VENUS} = 8.53\ \frac{m}{s^2}$
$R_{VENUS} = 6161$ km

FIND: $v_{CIRC.}$ WHEN $h = 160$ km

HAVE.. $a_n = g\frac{R^2}{r^2}$ AND $a_n = \frac{v^2}{r}$

THEN $g\frac{R^2}{r^2} = \frac{v^2}{r}$

OR $v_{CIRC.} = R\sqrt{\frac{g}{r}}$ WHERE $r = R + h$

FOR THE GIVEN DATA..

$$v_{CIRC.} = 6161\ \text{km}\sqrt{\frac{8.53\ \text{m/s}^2}{(6161+160)\times 10^3\ \text{m}}} \times \frac{3600\ \text{s}}{1\ \text{h}}$$

OR $v_{CIRC.} = 25.8\times 10^3\ \frac{km}{h}$ ◀

11.154

GIVEN: $a_n = g\frac{R^2}{r^2}$; $g_{MARS} = 3.83\ \frac{m}{s^2}$
$R_{MARS} = 3332$ km

FIND: $v_{CIRC.}$ WHEN $h = 160$ km

HAVE.. $a_n = g\frac{R^2}{r^2}$ AND $a_n = \frac{v^2}{r}$

THEN $g\frac{R^2}{r^2} = \frac{v^2}{r}$

OR $v_{CIRC.} = R\sqrt{\frac{g}{r}}$ WHERE $r = R + h$

FOR THE GIVEN DATA..

$$v_{CIRC.} = 3332\ \text{km}\sqrt{\frac{3.83\ \text{m/s}^2}{(3332+160)\times 10^3\ \text{m}}} \times \frac{3600\ \text{s}}{1\ \text{h}}$$

OR $v_{CIRC.} = 12.56\times 10^3\ \frac{km}{h}$ ◀

11.155

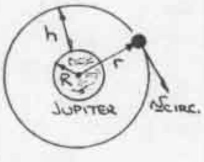
GIVEN: $a_n = g\frac{R^2}{r^2}$, $g_{JUPITER} = 26.0\ \frac{m}{s^2}$,
$R_{JUPITER} = 69\,893$ km

FIND: $v_{CIRC.}$ WHEN $h = 160$ km

HAVE.. $a_n = g\frac{R^2}{r^2}$ AND $a_n = \frac{v^2}{r}$

THEN $g\frac{R^2}{r^2} = \frac{v^2}{r}$

OR $v_{CIRC.} = R\sqrt{\frac{g}{r}}$ WHERE $r = R + h$



FOR THE GIVEN DATA..

$$v_{CIRC.} = 69\,893\sqrt{\frac{26.0\ \text{m/s}^2}{(69\,893+160)\times 10^3\ \text{m}}} \times \frac{3600\ \text{s}}{1\ \text{h}}$$

OR $v_{CIRC.} = 153.3\times 10^3\ \frac{km}{h}$ ◀

11.156

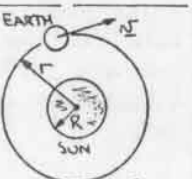
GIVEN: $a_n = g\frac{R^2}{r^2}$; $d_{SUN} = 864{,}000$ mi,
$g_{SUN} = 900\ \text{ft/s}^2$;
$[(v_{MEAN})_{ORBIT}]_{EARTH} = 66{,}600\ \frac{mi}{h}$

FIND: r_{EARTH}

HAVE.. $a_n = g\frac{R^2}{r^2}$ AND $a_n = \frac{v^2}{r}$

THEN $g\frac{R^2}{r^2} = \frac{v^2}{r}$

OR $r = g\left(\frac{R}{v}\right)^2$ WHERE $R = \frac{1}{2}d$



FOR THE GIVEN DATA..

$$r_{EARTH} = \left(900\ \frac{ft}{s^2}\right)\left(\frac{\frac{1}{2}\times 864{,}000\ \text{mi}}{66{,}600\ \text{mi/h}}\right)^2 \times \frac{1\ \text{mi}}{5280\ \text{ft}} \times \left(\frac{3600\ \text{s}}{1\ \text{h}}\right)^2$$

OR $r_{EARTH} = 92.9\times 10^6$ mi ◀

11.157

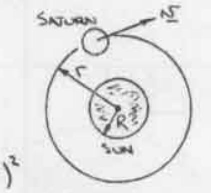
GIVEN: $a_n = g\frac{R^2}{r^2}$; $d_{SUN} = 864{,}000$ mi,
$g_{SUN} = 900\ \text{ft/s}^2$;
$[(v_{MEAN})_{ORBIT}]_{SATURN} = 21{,}580\ \frac{mi}{h}$

FIND: r_{SATURN}

HAVE.. $a_n = g\frac{R^2}{r^2}$ AND $a_n = \frac{v^2}{r}$

THEN $g\frac{R^2}{r^2} = \frac{v^2}{r}$

OR $r = g\left(\frac{R}{v}\right)^2$ WHERE $R = \frac{1}{2}d$



FOR THE GIVEN DATA..

$$r_{SATURN} = \left(900\ \frac{ft}{s^2}\right)\left(\frac{\frac{1}{2}\times 864{,}000\ \text{mi}}{21{,}580\ \text{mi/h}}\right)^2 \times \frac{1\ \text{mi}}{5280\ \text{ft}} \times \left(\frac{3600\ \text{s}}{1\ \text{h}}\right)^2$$

OR $r_{SATURN} = 885\times 10^6$ mi ◀

11.158

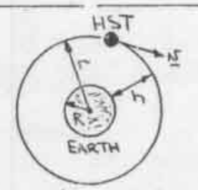
GIVEN: $a_n = g\frac{R^2}{r^2}$; $R_{EARTH} = 6370$ km;
$h = 590$ km

FIND: t_{ORBIT}

HAVE.. $a_n = g\frac{R^2}{r^2}$ AND $a_n = \frac{v^2}{r}$

THEN $g\frac{R^2}{r^2} = \frac{v^2}{r}$

OR $v = R\sqrt{\frac{g}{r}}$ WHERE $r = R + h$



THE CIRCUMFERENCE s OF THE CIRCULAR ORBIT IS EQUAL TO

$$s = 2\pi r$$

ASSUMING THAT THE SPEED OF THE TELESCOPE IS CONSTANT, HAVE

$$s = v\,t_{ORBIT}$$

SUBSTITUTING FOR s AND v..

$$2\pi r = R\sqrt{\frac{g}{r}}\,t_{ORBIT}$$

OR
$$t_{ORBIT} = \frac{2\pi}{R}\,\frac{r^{3/2}}{\sqrt{g}}$$

$$= \frac{2\pi}{6370\ \text{km}}\,\frac{[(6370+590)\ \text{km}]^{3/2}}{[9.81\times 10^{-3}\ \text{km/s}^2]^{1/2}} \times \frac{1\ \text{h}}{3600\ \text{s}}$$

OR $t_{ORBIT} = 1.606$ h ◀

11.159

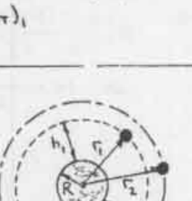
GIVEN: $a_n = g\frac{R^2}{r^2}$; $R_{MARS} = 2071$ mi; $h_1 = 180$ mi;
$(t_{ORBIT})_2 = 1.1\,(t_{ORBIT})_1$

FIND: h_2

HAVE.. $a_n = g\frac{R^2}{r^2}$ AND $a_n = \frac{v^2}{r}$

THEN $g\frac{R^2}{r^2} = \frac{v^2}{r}$

OR $v = R\sqrt{\frac{g}{r}}$ WHERE $r = R + h$



THE CIRCUMFERENCE s OF A CIRCULAR ORBIT IS EQUAL TO

$$s = 2\pi r$$

ASSUMING THAT THE SPEED OF THE SATELLITE IN EACH ORBIT IS CONSTANT, HAVE

$$s = v\,t_{ORBIT}$$

SUBSTITUTING FOR s AND v..

$$2\pi r = R\sqrt{\frac{g}{r}}\,t_{ORBIT}$$

OR
$$t_{ORBIT} = \frac{2\pi}{R}\,\frac{r^{3/2}}{\sqrt{g}} = \frac{2\pi}{R}\,\frac{(R+h)^{3/2}}{\sqrt{g}}$$

(CONTINUED)

11.159 CONTINUED

NOW.. $(t_{ORBIT})_2 = 1.1(t_{ORBIT})_1$

OR $\frac{2\pi}{R}\frac{(R+h_2)^{3/2}}{\sqrt{g}} = 1.1\frac{2\pi}{R}\frac{(R+h_1)^{3/2}}{\sqrt{g}}$

OR $h_2 = (1.1)^{2/3}(R+h_1) - R$

$= (1.1)^{2/3}(2071+180)\text{ mi} - (2071\text{ mi})$

OR $h_2 = 328\text{ mi}$ ◀

11.160

GIVEN: $a_n = g\frac{R^2}{r^2}$; $h_A = 120$ mi, $h_B = 200$ mi; $R_{EARTH} = 3960$ mi; AT $t=0$, A AND B ALIGNED AS SHOWN

FIND: t WHEN A AND B ARE NEXT RADIALLY ALIGNED

HAVE.. $a_n = g\frac{R^2}{r^2}$ AND $a_n = \frac{v^2}{r}$

THEN $g\frac{R^2}{r^2} = \frac{v^2}{r}$ OR $v = R\sqrt{\frac{g}{r}}$ WHERE $r = R+h$

THE CIRCUMFERENCE s OF A CIRCULAR ORBIT IS EQUAL TO $s = 2\pi r$

ASSUMING THAT THE SPEEDS OF THE SATELLITES ARE CONSTANT, HAVE

$$s = v t_{ORBIT}$$

SUBSTITUTING FOR s AND v..

$$2\pi r = R\sqrt{\frac{g}{r}}\,t_{ORBIT}$$

OR $t_{ORBIT} = \frac{2\pi}{R}\frac{r^{3/2}}{\sqrt{g}} = \frac{2\pi}{R}\frac{(R+h)^{3/2}}{\sqrt{g}}$

NOW $h_B > h_A \Rightarrow (t_{ORBIT})_B > (t_{ORBIT})_A$

NEXT LET TIME t_{TOTAL} BE THE TIME AT WHICH THE SATELLITES ARE NEXT RADIALLY ALIGNED. THEN, IF IN TIME t_{TOTAL} SATELLITE B COMPLETES N ORBITS, SATELLITE A MUST COMPLETE (N+1) ORBITS. THUS,

$$t_{TOTAL} = N(t_{ORBIT})_B = (N+1)(t_{ORBIT})_A$$

OR $N\left[\frac{2\pi}{R}\frac{(R+h_B)^{3/2}}{\sqrt{g}}\right] = (N+1)\left[\frac{2\pi}{R}\frac{(R+h_A)^{3/2}}{\sqrt{g}}\right]$

OR $N = \frac{(R+h_A)^{3/2}}{(R+h_B)^{3/2}-(R+h_A)^{3/2}} = \frac{1}{\left(\frac{R+h_B}{R+h_A}\right)^{3/2}-1}$

$= \frac{1}{\left(\frac{3960+200}{3960+120}\right)^{3/2}-1} = 33.835$ ORBITS

THEN.. $t_{TOTAL} = N(t_{ORBIT})_B = N\frac{2\pi}{R}\frac{(R+h_B)^{3/2}}{\sqrt{g}}$

$= 33.835\,\frac{2\pi}{3960\text{ mi}}\,\frac{[(3960+200)\text{mi}]^{3/2}}{\left(32.2\frac{\text{ft}}{\text{s}^2}\times\frac{1\text{ mi}}{5280\text{ ft}}\right)^{1/2}}\times\frac{1\text{ h}}{3600\text{ s}}$

OR $t_{TOTAL} = 51.2\text{ h}$ ◀

ALTERNATIVE SOLUTION

FROM ABOVE HAVE $(t_{ORBIT})_B > (t_{ORBIT})_A$

THUS, WHEN THE SATELLITES ARE NEXT RADIALLY ALIGNED, THE ANGLES θ_A AND θ_B SWEPT OUT

(CONTINUED)

11.160 CONTINUED

BY RADIAL LINES DRAWN TO THE SATELLITES MUST DIFFER BY 2π. THAT IS,

$$\theta_A = \theta_B + 2\pi$$

FOR A CIRCULAR ORBIT.. $s = r\theta$

FROM ABOVE.. $s = vt$ AND $v = R\sqrt{\frac{g}{r}}$

THEN $\theta = \frac{s}{r} = \frac{vt}{r} = \frac{1}{r}\left(R\sqrt{\frac{g}{r}}\right)t = \frac{R\sqrt{g}}{r^{3/2}}t = \frac{R\sqrt{g}}{(R+h)^{3/2}}t$

AT TIME t_{TOTAL}: $\frac{R\sqrt{g}}{(R+h_A)^{3/2}}t_{TOTAL} = \frac{R\sqrt{g}}{(R+h_B)^{3/2}}t_{TOTAL} + 2\pi$

OR $t_{TOTAL} = \frac{2\pi}{R\sqrt{g}\left[\frac{1}{(R+h_A)^{3/2}} - \frac{1}{(R+h_B)^{3/2}}\right]}$

$= \frac{2\pi}{(3960\text{ mi})\left(32.2\,\text{ft/s}^2\times\frac{1\text{ mi}}{5280\text{ ft}}\right)^{1/2}}\times\frac{1}{\frac{1}{[(3960+120)\text{mi}]^{3/2}} - \frac{1}{[(3960+200)\text{mi}]^{3/2}}}\times\frac{1\text{ h}}{3600\text{ s}}$

OR $t_{TOTAL} = 51.2\text{ h}$ ◀

11.161

GIVEN: $r = 3(2-e^{-t})$, $\theta = 4(t+2e^{-t})$ $r\sim$m, $t\sim$s, $\theta\sim$RAD

FIND: (a) $\underline{v}$ AND $\underline{a}$ AT $t=0$

(b) $\underline{v}$ AND $\underline{a}$ AS $t\to\infty$; THE FINAL PATH OF THE PARTICLE

HAVE.. $r = 3(2-e^{-t})$ $\qquad\theta = 4(t+2e^{-t})$

THEN $\dot{r} = 3e^{-t}$ $\qquad\dot{\theta} = 4(1-2e^{-t})$

AND $\ddot{r} = -3e^{-t}$ $\qquad\ddot{\theta} = 8e^{-t}$

NOW.. $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = 3e^{-t}\underline{e}_r + 12(2-e^{-t})(1-2e^{-t})\underline{e}_\theta$

AND $\underline{a} = (\ddot{r}-r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta}+2\dot{r}\dot{\theta})\underline{e}_\theta$

$= [-3e^{-t} - 48(2-e^{-t})(1-2e^{-t})^2]\underline{e}_r + [24(2-e^{-t})e^{-t} + 24e^{-t}(1-2e^{-t})]\underline{e}_\theta$

(a) AT $t=0$: $\underline{v} = 3\underline{e}_r + 12(2-1)(1-2)\underline{e}_\theta$

OR $\underline{v} = (3\frac{\text{m}}{\text{s}})\underline{e}_r - (12\frac{\text{m}}{\text{s}})\underline{e}_\theta$ ◀

$\underline{a} = [-3-48(2-1)(1-2)^2]\underline{e}_r + [24(2-1)+24(1-2)]\underline{e}_\theta$

OR $\underline{a} = -(51\frac{\text{m}}{\text{s}^2})\underline{e}_r$ ◀

(b) AS $t\to\infty$: $\underline{v} = (0)\underline{e}_r + 12(2-0)(1-0)\underline{e}_\theta$

OR $\underline{v} = (24\frac{\text{m}}{\text{s}})\underline{e}_\theta$ ◀

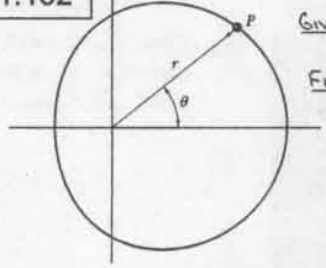
$\underline{a} = [0-48(2-0)(1-0)^2]\underline{e}_r + (0+0)\underline{e}_\theta$

OR $\underline{a} = -(96\frac{\text{m}}{\text{s}^2})\underline{e}_r$ ◀

AS $t\to\infty$, $r\to 6$ m.. A CONSTANT. THUS, THE FINAL PATH IS A CIRCLE OF RADIUS 6 m. ◀

NOTE THAT THE SPEED OF THE PARTICLE IS CONSTANT ($24\frac{\text{m}}{\text{s}}$); THUS, THE TRANSVERSE (TANGENTIAL) COMPONENT OF THE ACCELERATION IS ZERO.

11.162



GIVEN: $r = b(2+\cos\pi t)$, $\theta = \pi t$ $t \sim s$, $\theta \sim RAD$

FIND: (a) $\underline{v}$ AND $\underline{a}$ AT $t = 2\,s$

(b) θ FOR WHICH $v = v_{MAX}$

HAVE.. $r = b(2+\cos\pi t)$ $\qquad \theta = \pi t$

THEN $\dot{r} = -\pi b \sin\pi t$ $\qquad \dot\theta = \pi$

AND $\ddot{r} = -\pi^2 b\cos\pi t$ $\qquad \ddot\theta = 0$

NOW.. $\underline{v} = \dot r \underline{e}_r + r\dot\theta \underline{e}_\theta = -(\pi b \sin\pi t)\underline{e}_r + \pi b(2+\cos\pi t)\underline{e}_\theta$

AND $\underline{a} = (\ddot r - r\dot\theta^2)\underline{e}_r + (r\ddot\theta + 2\dot r\dot\theta)\underline{e}_\theta$

$= [-\pi^2 b\cos\pi t - \pi^2 b(2+\cos\pi t)]\underline{e}_r + (0 - 2\pi^2 b\sin\pi t)\underline{e}_\theta$

$= -2\pi^2 b[(1+\cos\pi t)\underline{e}_r + (\sin\pi t)\underline{e}_\theta]$

(a) AT $t = 2\,s$: $\underline{v} = -(0)\underline{e}_r + \pi b(2+1)\underline{e}_\theta$

OR $\underline{v} = 3\pi b \underline{e}_\theta$ ◀

$\underline{a} = -2\pi^2 b[(1+1)\underline{e}_r + (0)\underline{e}_\theta]$

OR $\underline{a} = -4\pi^2 b \underline{e}_r$ ◀

(b) HAVE.. $v = \pi b\sqrt{(-\sin\pi t)^2 + (2+\cos\pi t)^2}$

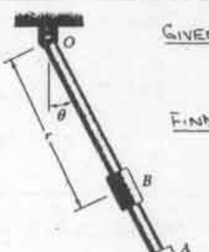
$= \pi b\sqrt{5 + 4\cos\pi t}$ $\qquad \theta = \pi t$

$= \pi b\sqrt{5+4\cos\theta}$

BY OBSERVATION, $v = v_{MAX}$ WHEN $\cos\theta = 1$

OR $\theta = 2N\pi$, $N = 0, 1, 2, \ldots$ ◀

11.163



GIVEN: $\theta = \pi(4t^2 - 8t)$, $r = 10 + 6\sin\pi t$ $\theta \sim RAD$, $t \sim s$, $r \sim IN.$

FIND: (a) $\underline{v}_B$ AT $t = 1\,s$

(b) $\underline{a}_B$ AT $t = 1\,s$

(c) $\underline{a}_{B/OA}$

HAVE.. $r = 10 + 6\sin\pi t$ $\qquad \theta = \pi(4t^2 - 8t)$

THEN $\dot r = 6\pi\cos\pi t$ $\qquad \dot\theta = 8\pi(t-1)$

AND $\ddot r = -6\pi^2\sin\pi t$ $\qquad \ddot\theta = 8\pi$

AT $t = 1\,s$: $r = 10$ IN. $\qquad \theta = -4\pi$ RAD

$\dot r = -6\pi$ IN./s $\qquad \dot\theta = 0$

$\ddot r = 0$ $\qquad \ddot\theta = 8\pi$ RAD/s²

(a) HAVE.. $\underline{v} = \dot r\underline{e}_r + r\dot\theta\underline{e}_\theta$

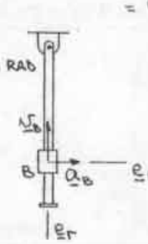
SO THAT $\underline{v}_B = -(6\pi \frac{IN.}{s})\underline{e}_r$ ◀

(b) HAVE.. $\underline{a}_B = (\ddot r - r\dot\theta^2)\underline{e}_r + (r\ddot\theta + 2\dot r\dot\theta)\underline{e}_\theta$

$= (10)(8\pi)\underline{e}_\theta$

OR $\underline{a}_B = (80\pi \frac{IN.}{s^2})\underline{e}_\theta$ ◀

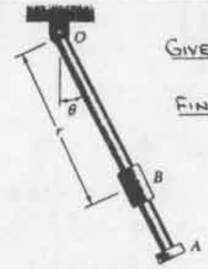
$\theta = -4\pi$ RAD



(c) HAVE.. $a_{B/OA} = \ddot r$

SO THAT $a_{B/OA} = 0$ ◀

11.164



GIVEN: $r = \frac{25}{t+4}$, $\theta = \frac{2}{\pi}\sin\pi t$ $\theta \sim RAD$, $t \sim s$, $r \sim IN.$

FIND: (a) $\underline{v}_B$ AT $t = 1\,s$

(b) $\underline{a}_B$ AT $t = 1\,s$

(c) $\underline{a}_{B/OA}$ AT $t = 1\,s$

HAVE.. $r = \frac{25}{t+4}$ $\qquad \theta = \frac{2}{\pi}\sin\pi t$

THEN $\dot r = -\frac{25}{(t+4)^2}$ $\qquad \dot\theta = 2\cos\pi t$

AND $\ddot r = \frac{50}{(t+4)^3}$ $\qquad \ddot\theta = -2\pi\sin\pi t$

AT $t = 1\,s$: $r = 5$ IN. $\qquad \theta = 0$

$\dot r = -1$ IN./s $\qquad \dot\theta = -2$ RAD/s

$\ddot r = 0.4$ IN./s² $\qquad \ddot\theta = 0$

(a) HAVE.. $\underline{v}_B = \dot r\underline{e}_r + r\dot\theta\underline{e}_\theta = (-1)\underline{e}_r + (5)(-2)\underline{e}_\theta$

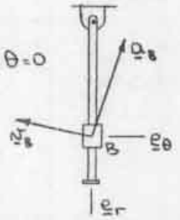
OR $\underline{v}_B = -(1\frac{IN.}{s})\underline{e}_r - (10\frac{IN.}{s})\underline{e}_\theta$ ◀

(b) HAVE.. $\underline{a}_B = (\ddot r - r\dot\theta^2)\underline{e}_r + (r\ddot\theta + 2\dot r\dot\theta)\underline{e}_\theta$

$= [0.4 - (5)(2)^2]\underline{e}_r + [0 + 2(-1)(-2)]\underline{e}_\theta$

OR $\underline{a}_B = -(19.6\frac{IN.}{s^2})\underline{e}_r + (4\frac{IN.}{s^2})\underline{e}_\theta$ ◀

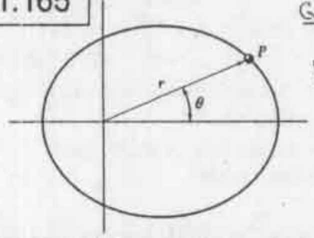
$\theta = 0$



(c) HAVE $a_{B/OA} = \ddot r$

SO THAT $\underline{a}_{B/OA} = (0.4\frac{IN.}{s^2})\underline{e}_r$ ◀

11.165



GIVEN: $r = \frac{2}{2-\cos\pi t}$, $\theta = \pi t$ $r \sim m$, $t \sim s$, $\theta \sim RAD$

FIND: (a) $\underline{v}$ AND $\underline{a}$ AT $t = 0$

(b) $\underline{v}$ AND $\underline{a}$ AT $t = 0.5\,s$

HAVE.. $r = \frac{2}{2-\cos\pi t}$ $\qquad \theta = \pi t$

THEN $\dot r = \frac{-2\pi\sin\pi t}{(2-\cos\pi t)^2}$ $\qquad \dot\theta = \pi$

AND $\ddot r = -2\pi\frac{\pi\cos\pi t(2-\cos\pi t) - \sin\pi t(2\pi\sin\pi t)}{(2-\cos\pi t)^3}$ $\qquad \ddot\theta = 0$

$= -2\pi^2\frac{2\cos\pi t - 1 - \sin^2\pi t}{(2-\cos\pi t)^3}$

(a) AT $t = 0$: $r = 2$ m $\qquad \theta = 0$

$\dot r = 0$ $\qquad \dot\theta = \pi$ $\frac{RAD}{s}$

$\ddot r = -2\pi^2\frac{m}{s^2}$ $\qquad \ddot\theta = 0$

NOW.. $\underline{v} = \dot r\underline{e}_r + r\dot\theta\underline{e}_\theta = (2)(\pi)\underline{e}_\theta$

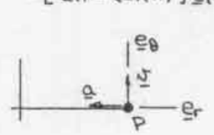
OR $\underline{v} = (2\pi\frac{m}{s})\underline{e}_\theta$ ◀

AND.. $\underline{a} = (\ddot r - r\dot\theta^2)\underline{e}_r + (r\ddot\theta + 2\dot r\dot\theta)\underline{e}_\theta$

$= [-2\pi^2 - (2)(\pi)^2]\underline{e}_r$

OR $\underline{a} = -(4\pi^2\frac{m}{s^2})\underline{e}_r$ ◀

$\theta = 0$



(CONTINUED)

11.165 CONTINUED

(b) AT $t = 0.5\,s$: $r = 1\,m$ $\qquad \theta = \frac{\pi}{2}$ RAD

$\dot{r} = \frac{-2\pi}{(2)^2} = -\frac{\pi}{2}\,\frac{m}{s}$ $\qquad \dot{\theta} = \pi\ \frac{RAD}{s}$

$\ddot{r} = -2\pi^2\,\frac{-1-1}{(2)^3} = \frac{\pi^2}{2}\,\frac{m}{s^2}$ $\qquad \ddot{\theta} = 0$

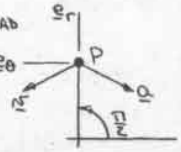
NOW.. $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta = (-\frac{\pi}{2})\underline{e}_r + (1)(\pi)\underline{e}_\theta$

OR $\underline{v} = -(\frac{\pi}{2}\,\frac{m}{s})\underline{e}_r + (\pi\,\frac{m}{s})\underline{e}_\theta$ ◀

AND.. $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta$

$= [\frac{\pi^2}{2} - (1)(\pi)^2]\underline{e}_r + [2(-\frac{\pi}{2})(\pi)]\underline{e}_\theta$

OR $\underline{a} = -(\frac{\pi^2}{2}\,\frac{m}{s^2})\underline{e}_r - (\pi^2\,\frac{m}{s^2})\underline{e}_\theta$ ◀



11.166

GIVEN: $r = 2a\cos\theta$, $\theta = \frac{1}{2}bt^2$

FIND: (a) v AND a

(b) ρ; PATH OF THE PARTICLE

(a) HAVE.. $r = 2a\cos\theta$ $\qquad \theta = \frac{1}{2}bt^2$

THEN $\dot{r} = -2a\dot{\theta}\sin\theta$ $\qquad \dot{\theta} = bt$

AND $\ddot{r} = -2a(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)$ $\qquad \ddot{\theta} = b$

SUBSTITUTING FOR $\dot{\theta}$ AND $\ddot{\theta}$

$\dot{r} = -2abt\sin\theta$

$\ddot{r} = -2ab(\sin\theta + bt^2\cos\theta)$

NOW.. $v_r = \dot{r}$ $\qquad v_\theta = r\dot{\theta}$

$= -2abt\sin\theta$ $\qquad = 2abt\cos\theta$

THEN.. $v = \sqrt{v_r^2 + v_\theta^2} = 2abt[(-\sin\theta)^2 + (\cos\theta)^2]^{1/2}$

OR $v = 2abt$ ◀

ALSO.. $a_r = \ddot{r} - r\dot{\theta}^2 = -2ab(\sin\theta + bt^2\cos\theta) - 2ab^2t^2\cos\theta$

$= -2ab(\sin\theta + 2bt^2\cos\theta)$

AND $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2ab\cos\theta - 4ab^2t^2\sin\theta$

$= 2ab(\cos\theta - 2bt^2\sin\theta)$

THEN.. $a = \sqrt{a_r^2 + a_\theta^2}$

$= 2ab[(\sin\theta + 2bt^2\cos\theta)^2 + (\cos\theta - 2bt^2\sin\theta)^2]^{1/2}$

OR $a = 2ab\sqrt{1 + 4b^2t^4}$ ◀

(b) NOW.. $a^2 = a_t^2 + a_n^2 = (\frac{dv}{dt})^2 + (\frac{v^2}{\rho})^2$

THEN.. $\frac{dv}{dt} = \frac{d}{dt}(2abt) = 2ab$

SO THAT $(2ab\sqrt{1 + 4b^2t^4})^2 = (2ab)^2 + a_n^2$

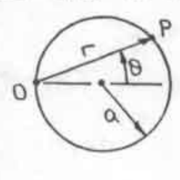
OR $4a^2b^2(1 + 4b^2t^4) = 4a^2b^2 + a_n^2$

OR $a_n = 4ab^2t^2$

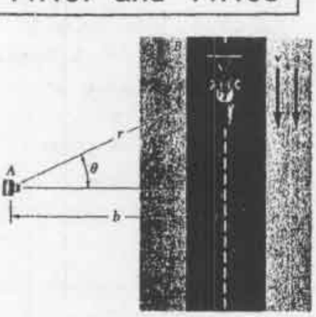
FINALLY.. $a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{(2abt)^2}{4ab^2t^2}$

OR $\rho = a$ ◀

SINCE THE RADIUS OF CURVATURE IS A CONSTANT, THE PATH IS A CIRCLE OF RADIUS a. ◀



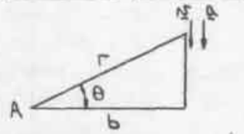
11.167 and 11.168



GIVEN: THE RECTILINEAR MOTION OF A RACE CAR AS SHOWN

HAVE.. $r = \frac{b}{\cos\theta}$

THEN $\dot{r} = \frac{b\dot{\theta}\sin\theta}{\cos^2\theta}$



11.167 FIND: v IN TERMS OF b, θ, AND $\dot{\theta}$

HAVE.. $v^2 = v_r^2 + v_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$= (\frac{b\dot{\theta}\sin\theta}{\cos^2\theta})^2 + (\frac{b\dot{\theta}}{\cos\theta})^2$

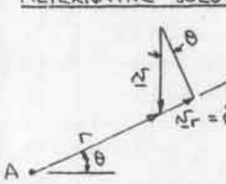
$= \frac{b^2\dot{\theta}^2}{\cos^2\theta}(\frac{\sin^2\theta}{\cos^2\theta} + 1) = \frac{b^2\dot{\theta}^2}{\cos^4\theta}$

OR $v = \pm\frac{b\dot{\theta}}{\cos^2\theta}$

FOR THE POSITION OF THE CAR SHOWN, θ IS DECREASING; THUS, THE NEGATIVE ROOT IS CHOSEN.

$\therefore v = -\frac{b\dot{\theta}}{\cos^2\theta}$ ◀

ALTERNATIVE SOLUTION



FROM THE DIAGRAM..

$\dot{r} = -v\sin\theta$

OR $\frac{b\dot{\theta}\sin\theta}{\cos^2\theta} = -v\sin\theta$

OR $v = -\frac{b\dot{\theta}}{\cos^2\theta}$ ◀

11.168 FIND: a IN TERMS OF b, θ, $\dot{\theta}$, AND $\ddot{\theta}$

FOR RECTILINEAR MOTION $\quad a = \frac{dv}{dt}$

FROM THE SOLUTION TO PROBLEM 11.167

$v = -\frac{b\dot{\theta}}{\cos^2\theta}$

THEN $a = \frac{d}{dt}(-\frac{b\dot{\theta}}{\cos^2\theta}) = -b\,\frac{\ddot{\theta}\cos^2\theta - \dot{\theta}(-2\dot{\theta}\cos\theta\sin\theta)}{\cos^4\theta}$

OR $a = -\frac{b}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)$ ◀

ALTERNATIVE SOLUTION

FROM ABOVE.. $r = \frac{b}{\cos\theta}$ $\qquad \dot{r} = \frac{b\dot{\theta}\sin\theta}{\cos^2\theta}$

THEN.. $\ddot{r} = b\,\frac{(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)(\cos^2\theta) - (\dot{\theta}\sin\theta)(-2\dot{\theta}\cos\theta\sin\theta)}{\cos^4\theta}$

$= b[\frac{\ddot{\theta}\sin\theta}{\cos^2\theta} + \frac{\dot{\theta}^2(1 + \sin^2\theta)}{\cos^3\theta}]$

NOW.. $a^2 = a_r^2 + a_\theta^2$

WHERE $a_r = \ddot{r} - r\dot{\theta}^2 = b[\frac{\ddot{\theta}\sin\theta}{\cos^2\theta} + \frac{\dot{\theta}^2(1 + \sin^2\theta)}{\cos^3\theta}] - \frac{b\dot{\theta}^2}{\cos\theta}$

$= \frac{b}{\cos^2\theta}(\ddot{\theta}\sin\theta + \frac{2\dot{\theta}^2\sin^2\theta}{\cos\theta})$ (CONTINUED)

11.168 CONTINUED

$$a_r = \frac{b\sin\theta}{\cos^2\theta}\left(\ddot{\theta} + 2\dot{\theta}^2\tan\theta\right)$$

AND $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \dfrac{b\ddot{\theta}}{\cos\theta} + 2\dfrac{b\dot{\theta}^2\sin\theta}{\cos^2\theta}$

$$= \frac{b\cos\theta}{\cos^2\theta}\left(\ddot{\theta} + 2\dot{\theta}^2\tan\theta\right)$$

THEN $a = \pm\dfrac{b}{\cos^2\theta}\left(\ddot{\theta} + 2\dot{\theta}^2\tan\theta\right)\left[(\sin\theta)^2 + (\cos\theta)^2\right]^{1/2}$

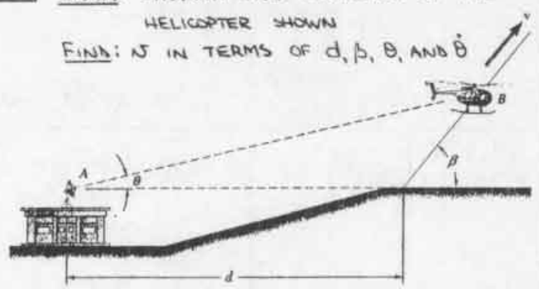
FOR THE POSITION OF THE CAR SHOWN, $\ddot{\theta}$ IS NEGATIVE; FOR a TO BE POSITIVE, THE NEGATIVE ROOT IS CHOSEN.

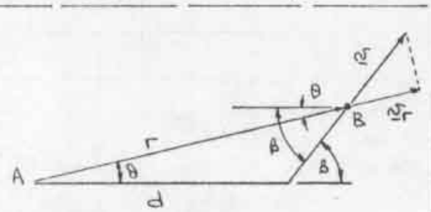
$$\therefore\ a = -\frac{b}{\cos^2\theta}\left(\ddot{\theta} + 2\dot{\theta}^2\tan\theta\right) ◀$$

11.169

GIVEN: STRAIGHT LINE TRAJECTORY OF THE HELICOPTER SHOWN

FIND: v IN TERMS OF d, β, θ, AND $\dot{\theta}$





FROM THE DIAGRAM..

$$\frac{r}{\sin(180^\circ-\beta)} = \frac{d}{\sin(\beta-\theta)}$$

OR $d\sin\beta = r(\sin\beta\cos\theta - \cos\beta\sin\theta)$

OR $r = d\dfrac{\tan\beta}{\tan\beta\cos\theta - \sin\theta}$

THEN $\dot{r} = d\tan\beta\dfrac{-(-\tan\beta\sin\theta - \cos\theta)}{(\tan\beta\cos\theta - \sin\theta)^2}\dot{\theta}$

$$= d\dot{\theta}\tan\beta\frac{\tan\beta\sin\theta + \cos\theta}{(\tan\beta\cos\theta - \sin\theta)^2}$$

FROM THE DIAGRAM

$v_r = v\cos(\beta-\theta)$ WHERE $v_r = \dot{r}$

THEN

$$d\dot{\theta}\tan\beta\frac{\tan\beta\sin\theta + \cos\theta}{(\tan\beta\cos\theta - \sin\theta)^2} = v(\cos\beta\cos\theta + \sin\beta\sin\theta)$$

$$= v\cos\beta(\tan\beta\sin\theta + \cos\theta)$$

OR $v = \dfrac{d\dot{\theta}\tan\beta\sec\beta}{(\tan\beta\cos\theta - \sin\theta)^2}$ ◀

ALTERNATIVE SOLUTION

HAVE.. $v^2 = v_r^2 + v_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$

USING THE EXPRESSIONS FOR r AND $\dot{r}$ FROM ABOVE..

$$v^2 = \left[d\dot{\theta}\tan\beta\frac{\tan\beta\sin\theta + \cos\theta}{(\tan\beta\cos\theta - \sin\theta)^2}\right]^2 + \left(d\dot{\theta}\frac{\tan\beta}{\tan\beta\cos\theta - \sin\theta}\right)^2$$

(CONTINUED)

11.169 CONTINUED

OR $v = \pm\dfrac{d\dot{\theta}\tan\beta}{(\tan\beta\cos\theta - \sin\theta)}\left[\dfrac{(\tan\beta\sin\theta + \cos\theta)^2}{(\tan\beta\cos\theta - \sin\theta)^2} + 1\right]^{1/2}$

$$= \pm\frac{d\dot{\theta}\tan\beta}{(\tan\beta\cos\theta - \sin\theta)}\left[\frac{\tan^2\beta + 1}{(\tan\beta\cos\theta - \sin\theta)^2}\right]^{1/2}$$

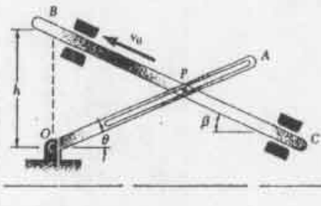
NOTE THAT AS θ INCREASES, THE HELICOPTER MOVES IN THE INDICATED DIRECTION. THUS, THE POSITIVE ROOT IS CHOSEN.

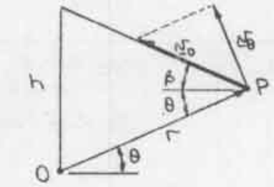
$$\therefore\ v = \frac{d\dot{\theta}\tan\beta\sec\beta}{(\tan\beta\cos\theta - \sin\theta)^2} ◀$$

* 11.170

GIVEN: v_0 = CONSTANT

FIND: $\dot{\theta}$ IN TERMS OF v_0, h, β, AND θ





FROM THE DIAGRAM..

$$\frac{r}{\sin(90^\circ-\beta)} = \frac{h}{\sin(\beta+\theta)}$$

OR $r(\sin\beta\cos\theta + \cos\beta\sin\theta) = h\cos\beta$

OR $r = \dfrac{h}{\tan\beta\cos\theta + \sin\theta}$

ALSO.. $v_\theta = v_0\sin(\beta+\theta)$ WHERE $v_\theta = r\dot{\theta}$

THEN

$$\frac{h\dot{\theta}}{\tan\beta\cos\theta + \sin\theta} = v_0(\sin\beta\cos\theta + \cos\beta\sin\theta)$$

OR $\dot{\theta} = \dfrac{v_0\cos\beta}{h}(\tan\beta\cos\theta + \sin\theta)^2$ ◀

ALTERNATIVE SOLUTION

FROM ABOVE.. $r = \dfrac{h}{\tan\beta\cos\theta + \sin\theta}$

THEN $\dot{r} = h\dfrac{\tan\beta\sin\theta - \cos\theta}{(\tan\beta\cos\theta + \sin\theta)^2}\dot{\theta}$

NOW.. $v_0^2 = v_r^2 + v_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$

OR $v_0^2 = \left[h\dot{\theta}\dfrac{\tan\beta\sin\theta - \cos\theta}{(\tan\beta\cos\theta + \sin\theta)^2}\right]^2 + \left(\dfrac{h\dot{\theta}}{\tan\beta\cos\beta + \sin\theta}\right)^2$

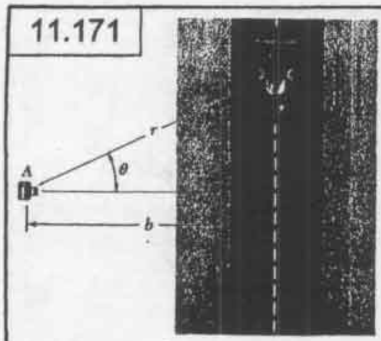
OR $v_0 = \pm\dfrac{h\dot{\theta}}{\tan\beta\cos\theta + \sin\theta}\left[\dfrac{(\tan\beta\sin\theta - \cos\theta)^2}{(\tan\beta\cos\theta + \sin\theta)^2} + 1\right]^{1/2}$

$$= \pm\frac{h\dot{\theta}}{\tan\beta\cos\theta + \sin\theta}\left[\frac{\tan^2\beta + 1}{(\tan\beta\cos\theta + \sin\theta)^2}\right]^{1/2}$$

NOTE THAT AS θ INCREASES, MEMBER BC MOVES IN THE INDICATED DIRECTION. THUS, THE POSITIVE ROOT IS CHOSEN.

$$\therefore\ \dot{\theta} = \frac{v_0\cos\beta}{h}(\tan\beta\cos\theta + \sin\theta)^2 ◀$$

11.171

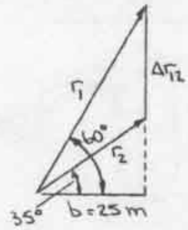


GIVEN: $\theta_1 = 60^\circ$, $\theta_2 = 35^\circ$, $\Delta t_{12} = 0.5\ s$; $b = 25\ m$

FIND: v_{AVE}

FROM THE DIAGRAM..

$$\Delta r_{12} = 25 \tan 60^\circ - 25 \tan 35^\circ = 25.796\ m$$



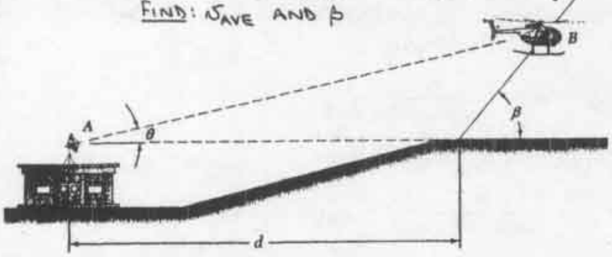
NOW.. $v_{AVE} = \dfrac{\Delta r_{12}}{\Delta t_{12}} = \dfrac{25.796\ m}{0.5\ s} = 51.592\ \frac{m}{s}$

OR $v_{AVE} = 185.7\ \frac{km}{h}$ ◀

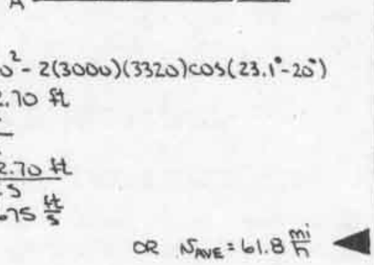
11.172

GIVEN: AT $t = 0$, $r_{AB} = 3000\ ft$, $\theta = 20^\circ$
AT $t = 4\ s$, $r_{AB} = 3320\ ft$, $\theta = 23.1^\circ$

FIND: v_{AVE} AND β



HAVE.. $r_0 = 3000\ ft$ $\quad \theta_0 = 20^\circ$
$r_4 = 3320\ ft$ $\quad \theta_4 = 23.1^\circ$



FROM THE DIAGRAM..

$$\Delta r^2 = 3000^2 + 3320^2 - 2(3000)(3320)\cos(23.1^\circ - 20^\circ)$$

OR $\Delta r = 362.70\ ft$

NOW.. $v_{AVE} = \dfrac{\Delta r}{\Delta t} = \dfrac{362.70\ ft}{4\ s} = 90.675\ \frac{ft}{s}$

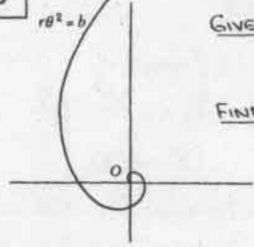
OR $v_{AVE} = 61.8\ \frac{mi}{h}$ ◀

ALSO.. $\Delta r \cos\beta = r_4 \cos\theta_4 - r_0 \cos\theta_0$

OR $\cos\beta = \dfrac{3320 \cos 23.1^\circ - 3000 \cos 20^\circ}{362.70}$

OR $\beta = 49.7^\circ$ ◀

11.173



GIVEN: A PARTICLE MOVES ALONG THE SPIRAL SHOWN

FIND: v IN TERMS OF b, θ, AND $\dot{\theta}$

HAVE.. $r = \dfrac{b}{\theta^2}$

THEN $\dot{r} = -\dfrac{2b}{\theta^3}\dot{\theta}$

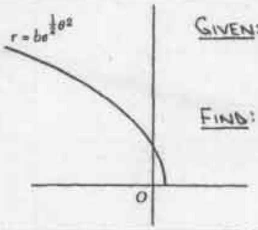
NOW.. $v^2 = v_r^2 + v_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$$= \left(-\frac{2b}{\theta^3}\dot{\theta}\right)^2 + \left(\frac{b}{\theta^2}\dot{\theta}\right)^2$$

$$= \left(\frac{b\dot{\theta}}{\theta^2}\right)^2\left(\frac{4}{\theta^2} + 1\right)$$

OR $v = \dfrac{b\dot{\theta}}{\theta^3}\sqrt{4 + \theta^2}$ ◀

11.174



GIVEN: A PARTICLE MOVES ALONG THE SPIRAL SHOWN

FIND: v IN TERMS OF b, θ, AND $\dot{\theta}$

HAVE.. $r = be^{\frac{1}{2}\theta^2}$

THEN $\dot{r} = b\theta\dot{\theta}e^{\frac{1}{2}\theta^2}$

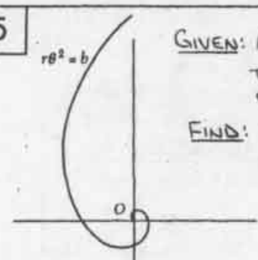
NOW $v^2 = v_r^2 + v_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$

$$= \left(b\theta\dot{\theta}e^{\frac{1}{2}\theta^2}\right)^2 + \left(b\dot{\theta}e^{\frac{1}{2}\theta^2}\right)^2$$

$$= \left(b\dot{\theta}e^{\frac{1}{2}\theta^2}\right)^2(\theta^2 + 1)$$

OR $v = b\dot{\theta}e^{\frac{1}{2}\theta^2}\sqrt{1 + \theta^2}$ ◀

11.175



GIVEN: A PARTICLE MOVES ALONG THE SPIRAL SHOWN; $\dot{\theta} = \omega$ = CONSTANT

FIND: a IN TERMS OF b, θ, AND ω

HAVE.. $r = \dfrac{b}{\theta^2}$

THEN $\dot{r} = -\dfrac{2b\dot{\theta}}{\theta^3} = -\dfrac{2b\omega}{\theta^3}$

AND $\ddot{r} = -2b\omega\dfrac{-3\dot{\theta}}{\theta^4} = \dfrac{6b\omega^2}{\theta^4}$

NOW.. $a^2 = a_r^2 + a_\theta^2 = (\ddot{r} - r\dot{\theta}^2)^2 + (r\overset{0}{\cancel{\ddot{\theta}}} + 2\dot{r}\dot{\theta})^2$

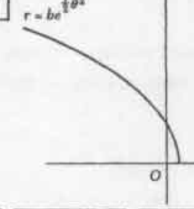
$$= \left(\frac{6b\omega^2}{\theta^4} - \frac{b}{\theta^2}\omega^2\right)^2 + \left[2\left(-\frac{2b\omega}{\theta^3}\right)\omega\right]^2$$

$$= \left(\frac{b\omega^2}{\theta^4}\right)^2\left[(6 - \theta^2)^2 + (-4\theta)^2\right]$$

$$= \left(\frac{b\omega^2}{\theta^4}\right)^2(36 + 4\theta^2 + \theta^4)$$

OR $a = \dfrac{b\omega^2}{\theta^4}\sqrt{36 + 4\theta^2 + \theta^4}$ ◀

11.176



GIVEN: A PARTICLE MOVES ALONG THE SPIRAL SHOWN; $\dot\theta = \omega =$ CONSTANT

FIND: a IN TERMS OF b, θ, AND ω

HAVE.. $r = be^{\frac{1}{2}\theta^2}$

THEN $\dot r = b\theta\dot\theta e^{\frac{1}{2}\theta^2} = b\omega\theta e^{\frac{1}{2}\theta^2}$

AND $\ddot r = b\omega(\dot\theta e^{\frac{1}{2}\theta^2} + \theta^2\dot\theta e^{\frac{1}{2}\theta^2}) = b\omega^2 e^{\frac{1}{2}\theta^2}(1+\theta^2)$

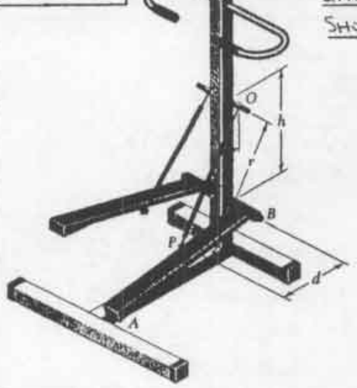
NOW.. $a^2 = a_r^2 + a_\theta^2 = (\ddot r - r\dot\theta^2)^2 + (r\overset{0}{\ddot\theta} + 2\dot r\dot\theta)^2$

$= [b\omega^2 e^{\frac{1}{2}\theta^2}(1+\theta^2) - b\omega^2 e^{\frac{1}{2}\theta^2}]^2 + (2b\omega^2\theta e^{\frac{1}{2}\theta^2})^2$

$= (b\omega^2 e^{\frac{1}{2}\theta^2})^2(\theta^4 + 4\theta^2)$

OR $a = b\theta\omega^2 e^{\frac{1}{2}\theta^2}\sqrt{4+\theta^2}$

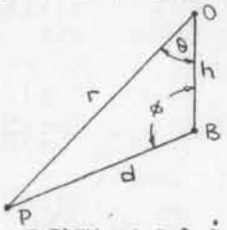
11.177



GIVEN: $\dot{\underline{\Phi}} = \dot\phi\mathbf{j}$, $\dot\phi$ = CONSTANT

SHOW: $\dot r = h\dot\phi\sin\theta$

FROM THE DIAGRAM..



$r^2 = d^2 + h^2 - 2dh\cos\phi$

THEN.. $2r\dot r = 2dh\dot\phi\sin\phi$

NOW.. $\dfrac{r}{\sin\phi} = \dfrac{d}{\sin\theta}$

OR $r = \dfrac{d\sin\phi}{\sin\theta}$

SUBSTITUTING FOR r IN THE EXPRESSION FOR $\dot r$

$\left(\dfrac{d\sin\phi}{\sin\theta}\right)\dot r = dh\dot\phi\sin\phi$

OR $\dot r = h\dot\phi\sin\theta$ Q.E.D.

ALTERNATIVE SOLUTION

FIRST NOTE.. $\alpha = 180° - (\phi + \theta)$

NOW.. $\mathbf{v} = \mathbf{v}_r + \mathbf{v}_\theta = \dot r\mathbf{e}_r + r\dot\theta\mathbf{e}_\theta$

WITH B AS THE ORIGIN..

$v_P = d\dot\phi$ (d = CONSTANT $\Rightarrow \dot d = 0$)

WITH O AS THE ORIGIN..

$(v_P)_r = \dot r$

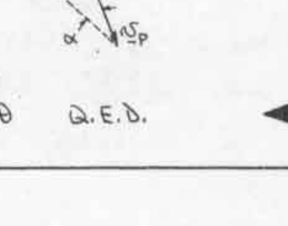
WHERE $(v_P)_r = v_P\sin\alpha$

THEN $\dot r = d\dot\phi\sin\alpha$

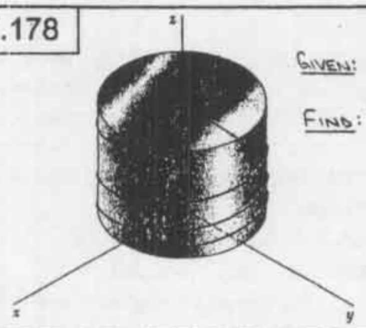
NOW.. $\dfrac{h}{\sin\alpha} = \dfrac{d}{\sin\theta}$

OR $d\sin\alpha = h\sin\theta$

THEN.. $\dot r = h\dot\phi\sin\theta$ Q.E.D.



11.178



GIVEN: $R = A$, $\theta = 2\pi t$, $z = \frac{1}{4}At^2$

FIND: v AND a

HAVE.. $R = A$ $\quad \theta = 2\pi t$ $\quad z = \frac{1}{4}At^2$

THEN $\dot R = 0$ $\quad \dot\theta = 2\pi$ $\quad \dot z = \frac{1}{2}At$

AND $\ddot R = 0$ $\quad \ddot\theta = 0$ $\quad \ddot z = \frac{1}{2}A$

NOW.. $v^2 = v_R^2 + v_\theta^2 + v_z^2 = (\overset{0}{\dot R})^2 + (R\dot\theta)^2 + (\dot z)^2$

$= 0 + (A \times 2\pi)^2 + (\frac{1}{2}At)^2$

$= A^2(4\pi^2 + \frac{1}{4}t^2)$

OR $v = \frac{1}{2}A\sqrt{16\pi^2 + t^2}$

AND.. $a^2 = a_R^2 + a_\theta^2 + a_z^2 = (\overset{0}{\ddot R} - R\dot\theta^2)^2 + (R\overset{0}{\ddot\theta} + 2\overset{0}{\dot R}\dot\theta)^2 + (\ddot z)^2$

$= [-A(2\pi)^2]^2 + 0 + (\frac{1}{2}A)^2$

$= A^2(16\pi^4 + \frac{1}{4})$

OR $a = \frac{1}{2}A\sqrt{64\pi^4 + 1}$

11.179

GIVEN: $R = \dfrac{A}{t+1}$, $\theta = Bt$, $z = \dfrac{Ct}{t+1}$

FIND: (a) v AND a AT $t = 0$

(b) v AND a AS $t \to \infty$

HAVE.. $R = \dfrac{A}{t+1}$ $\quad \theta = Bt$ $\quad z = \dfrac{Ct}{t+1}$

THEN $\dot R = -\dfrac{A}{(t+1)^2}$ $\quad \dot\theta = B$ $\quad \dot z = C\dfrac{(t+1) - t}{(t+1)^2} = \dfrac{C}{(t+1)^2}$

AND $\ddot R = \dfrac{2A}{(t+1)^3}$ $\quad \ddot\theta = 0$ $\quad \ddot z = -\dfrac{2C}{(t+1)^3}$

NOW.. $v^2 = (v_R)^2 + (v_\theta)^2 + (v_z)^2 = (\dot R)^2 + (R\dot\theta)^2 + (\dot z)^2$

AND $a^2 = (a_R)^2 + (a_\theta)^2 + (a_z)^2 = (\ddot R - R\dot\theta^2)^2 + (R\overset{0}{\ddot\theta} + 2\dot R\dot\theta)^2 + (\ddot z)^2$

(a) AT $t = 0$: $R = A$

$\dot R = -A$ $\quad \dot\theta = B$ $\quad \dot z = C$

$\ddot R = 2A$ $\quad \ddot z = -2C$

THEN.. $v^2 = (-A)^2 + (AB)^2 + (C)^2$

OR $v = \sqrt{(1+B^2)A^2 + C^2}$

AND $a^2 = (2A - AB^2)^2 + [2(-A)(B)]^2 + (-2C)^2$

$= 4A^2[(1 - \frac{1}{2}B^2)^2 + B^2 + \frac{C^2}{A^2}]$

$= 4[(1 + \frac{1}{4}B^4)A^2 + C^2]$

OR $a = 2\sqrt{(1+\frac{1}{4}B^4)A^2 + C^2}$

(b) AS $t \to \infty$: $R = 0$

$\dot R = 0$ $\quad \dot\theta = B$ $\quad \dot z = 0$

$\ddot R = 0$ $\quad \ddot z = 0$

$\therefore\ v = 0$

AND $a = 0$

* 11.180

GIVEN: $\underline{r} = (Rt\cos\omega_N t)\underline{i} + ct\,\underline{j} + (Rt\sin\omega_N t)\underline{k}$

FIND: THE ANGLE THAT THE OSCULATING PLANE FORMS WITH THE y AXIS

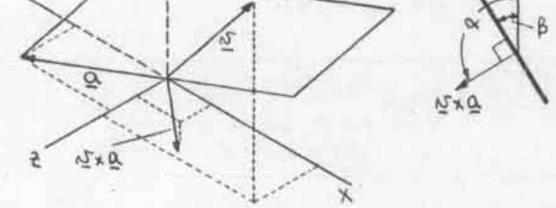
FIRST NOTE THAT THE VECTORS $\underline{v}$ AND $\underline{a}$ LIE IN THE OSCULATING PLANE.

NOW.. $\underline{r} = (Rt\cos\omega_N t)\underline{i} + ct\,\underline{j} + (Rt\sin\omega_N t)\underline{k}$

THEN $\underline{v} = \frac{d\underline{r}}{dt} = R(\cos\omega_N t - \omega_N t\sin\omega_N t)\underline{i} + c\underline{j}$
$+ R(\sin\omega_N t + \omega_N t\cos\omega_N t)\underline{k}$

AND $\underline{a} = \frac{d\underline{v}}{dt} = R(-\omega_N\sin\omega_N t - \omega_N\sin\omega_N t - \omega_N^2 t\cos\omega_N t)\underline{i}$
$+ R(\omega_N\cos\omega_N t + \omega_N\cos\omega_N t - \omega_N^2 t\sin\omega_N t)\underline{k}$
$= \omega_N R[-(2\sin\omega_N t + \omega_N t\cos\omega_N t)\underline{i}$
$+ (2\cos\omega_N t - \omega_N t\sin\omega_N t)\underline{k}]$

IT THEN FOLLOWS THAT THE VECTOR $(\underline{v}\times\underline{a})$ IS PERPENDICULAR TO THE OSCULATING PLANE.



$$(\underline{v}\times\underline{a}) = \omega_N R\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ R(\cos\omega_N t - \omega_N t\sin\omega_N t) & c & R(\sin\omega_N t + \omega_N t\cos\omega_N t) \\ -(2\sin\omega_N t + \omega_N t\cos\omega_N t) & 0 & (2\cos\omega_N t - \omega_N t\sin\omega_N t) \end{vmatrix}$$

$$= \omega_N R\{c(2\cos\omega_N t - \omega_N t\sin\omega_N t)\underline{i}$$
$$+ R[-(\sin\omega_N t + \omega_N t\cos\omega_N t)(2\sin\omega_N t + \omega_N t\cos\omega_N t)$$
$$-(\cos\omega_N t - \omega_N t\sin\omega_N t)(2\cos\omega_N t - \omega_N t\sin\omega_N t)]\underline{j}$$
$$+ c(2\sin\omega_N t + \omega_N t\cos\omega_N t)\underline{k}\}$$

$$= \omega_N R[c(2\cos\omega_N t - \omega_N t\sin\omega_N t)\underline{i} - R(2+\omega_N^2 t^2)\underline{j}$$
$$+ c(2\sin\omega_N t + \omega_N t\cos\omega_N t)\underline{k}]$$

THE ANGLE α FORMED BY THE VECTOR $(\underline{v}\times\underline{a})$ AND THE y AXIS IS FOUND FROM..

$$\cos\alpha = \frac{(\underline{v}\times\underline{a})\cdot\underline{j}}{|(\underline{v}\times\underline{a})||\underline{j}|}$$

WHERE $|\underline{j}| = 1$

$(\underline{v}\times\underline{a})\cdot\underline{j} = -\omega_N R^2(2+\omega_N^2 t^2)$

$|(\underline{v}\times\underline{a})| = \omega_N R[c^2(2\cos\omega_N t - \omega_N t\sin\omega_N t)^2 + R^2(2+\omega_N^2t^2)^2 + c^2(2\sin\omega_N t + \omega_N t\cos\omega_N t)^2]^{1/2}$

$= \omega_N R[c^2(4+\omega_N^2 t^2) + R^2(2+\omega_N^2 t^2)^2]^{1/2}$

THEN
$$\cos\alpha = \frac{-\omega_N R^2(2+\omega_N^2 t^2)}{\omega_N R[c^2(4+\omega_N^2t^2) + R^2(2+\omega_N^2t^2)^2]^{1/2}}$$
$$= \frac{-R(2+\omega_N^2t^2)}{[c^2(4+\omega_N^2t^2) + R^2(2+\omega_N^2t^2)^2]^{1/2}}$$

(CONTINUED)

11.180 CONTINUED

THE ANGLE β THAT THE OSCULATING PLANE FORMS WITH THE y AXIS (SEE THE ABOVE DIAGRAM) IS EQUAL TO

$$\beta = \alpha - 90^\circ$$

THEN $\cos\alpha = \cos(\beta + 90^\circ) = -\sin\beta$

$$\therefore\ -\sin\beta = \frac{-R(2+\omega_N^2t^2)}{[c^2(4+\omega_N^2t^2) + R^2(2+\omega_N^2t^2)^2]^{1/2}}$$

Triangle with angle β, opposite side $R(2+\omega_N^2t^2)$, adjacent side $c\sqrt{4+\omega_N^2t^2}$.

THEN $\tan\beta = \dfrac{R(2+\omega_N^2t^2)}{c\sqrt{4+\omega_N^2t^2}}$

OR $\beta = \tan^{-1}\left[\dfrac{R(2+\omega_N^2t^2)}{c\sqrt{4+\omega_N^2t^2}}\right]$ ◀

* 11.181

GIVEN: $\underline{r} = (At\cos t)\underline{i} + (A\sqrt{t^2+1})\underline{j} + (Bt\sin t)\underline{k}$
$r \sim ft,\ t \sim s;\ A = 3,\ B = 1$

FIND: (a) DIRECTION OF $\underline{e}_b$ AT $t=0$
(b) DIRECTION OF $\underline{e}_b$ AT $t = \frac{\pi}{2}$ s

FIRST NOTE THAT $\underline{e}_b$ IS GIVEN BY
$$\underline{e}_b = \frac{\underline{v}\times\underline{a}}{|\underline{v}\times\underline{a}|}$$

NOW.. $\underline{r} = (3t\cos t)\underline{i} + (3\sqrt{t^2+1})\underline{j} + (t\sin t)\underline{k}$

THEN $\underline{v} = \frac{d\underline{r}}{dt} = 3(\cos t - t\sin t)\underline{i} + \frac{3t}{\sqrt{t^2+1}}\underline{j}$
$+ (\sin t + t\cos t)\underline{k}$

AND $\underline{a} = \frac{d\underline{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\underline{i} + 3\dfrac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{t^2+1}\underline{j}$
$+ (\cos t + \cos t - t\sin t)\underline{k}$
$= -3(2\sin t + t\cos t)\underline{i} + \frac{3}{(t^2+1)^{3/2}}\underline{j}$
$+ (2\cos t - t\sin t)\underline{k}$

(a) AT $t=0$: $\underline{v} = (3\ \tfrac{ft}{s})\underline{j}$ $\quad \underline{a} = (3\ \tfrac{ft}{s^2})\underline{j} + (2\ \tfrac{ft}{s^2})\underline{k}$

THEN $\underline{v}\times\underline{a} = 3\underline{j}\times(3\underline{j} + 2\underline{k})$
$= 3(-2\underline{j} + 3\underline{k})$

AND $|\underline{v}\times\underline{a}| = 3\sqrt{(-2)^2 + (3)^2} = 3\sqrt{13}$

THEN $\underline{e}_b = \dfrac{3(-2\underline{j} + 3\underline{k})}{3\sqrt{13}} = \dfrac{1}{\sqrt{13}}(-2\underline{j} + 3\underline{k})$

$\therefore\ \cos\theta_x = 0 \qquad \cos\theta_y = -\frac{2}{\sqrt{13}} \qquad \cos\theta_z = \frac{3}{\sqrt{13}}$

OR $\theta_x = 90^\circ \qquad \theta_y = 123.7^\circ \qquad \theta_z = 33.7^\circ$ ◀

(b) AT $t = \frac{\pi}{2}$ s: $\underline{v} = -(\frac{3\pi}{2}\ \tfrac{ft}{s})\underline{i} + (\frac{3\pi}{\sqrt{\pi^2+4}}\ \tfrac{ft}{s})\underline{j} + (1\ \tfrac{ft}{s})\underline{k}$

$\underline{a} = -(6\ \tfrac{ft}{s^2})\underline{i} + \left[\frac{24}{(\pi^2+4)^{3/2}}\ \tfrac{ft}{s^2}\right]\underline{j} - (\frac{\pi}{2}\ \tfrac{ft}{s^2})\underline{k}$

$$\text{THEN.. } \underline{v}\times\underline{a} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -\frac{3\pi}{2} & \frac{3\pi}{(\pi^2+4)^{1/2}} & 1 \\ -6 & \frac{24}{(\pi^2+4)^{3/2}} & -\frac{\pi}{2} \end{vmatrix}$$

$$= -\left[\frac{3\pi^2}{2(\pi^2+4)^{1/2}} + \frac{24}{(\pi^2+4)^{3/2}}\right]\underline{i} - (6 + \tfrac{3\pi^2}{4})\underline{j}$$
$$+ \left[-\frac{36\pi}{(\pi^2+4)^{3/2}} + \frac{18\pi}{(\pi^2+4)^{1/2}}\right]\underline{k}$$

$= -4.43984\underline{i} - 13.40220 + 12.99459\underline{k}$

AND $|\underline{v}\times\underline{a}| = [(-4.43984)^2 + (-13.40220)^2 + (12.99459)^2]^{1/2}$
$= 19.18829$

THEN.. $\underline{e}_b = \frac{1}{19.18829}(-4.43984\underline{i} - 13.40220\underline{j} + 12.99459\underline{k})$

$\therefore\ \cos\theta_x = -\frac{4.43984}{19.18829} \quad \cos\theta_y = -\frac{13.40220}{19.18829} \quad \cos\theta_z = \frac{12.99459}{19.18829}$

OR $\theta_x = 103.4^\circ \quad \theta_y = 134.3^\circ \quad \theta_z = 47.4^\circ$ ◀

11.182

GIVEN: $x = 2t^3 - 15t^2 + 24t + 4 \quad x \sim m,\ t \sim s$

FIND: (a) t WHEN $v = 0$

(b) x AND TOTAL DISTANCE TRAVELED WHEN $a = 0$

HAVE... $x = 2t^3 - 15t^2 + 24t + 4$

THEN $v = \frac{dx}{dt} = 6t^2 - 30t + 24$

AND $a = \frac{dv}{dt} = 12t - 30$

(a) WHEN $v = 0$: $6t^2 - 30t + 24 = 0$

OR $(t-1)(t-4) = 0$

OR $t = 1\ s$ AND $t = 4\ s$ ◀

(b) WHEN $a = 0$: $12t - 30 = 0$ OR $t = 2.5\ s$

AT $t = 2.5\ s$: $x_{2.5} = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4$

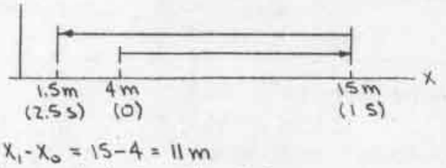
OR $x_{2.5} = 1.5\ m$ ◀

FIRST OBSERVE THAT $0 \le t < 1\ s \quad v > 0$

$1\ s < t \le 2.5\ s \quad v < 0$

NOW... AT $t = 0$: $x_0 = 4\ m$

$t = 1\ s$: $x_1 = 2(1)^3 - 15(1)^2 + 24(1) + 4 = 15\ m$



THEN... $x_1 - x_0 = 15 - 4 = 11\ m$

$|x_{2.5} - x_1| = |1.5 - 15| = 13.5\ m$

$\therefore$ TOTAL DISTANCE TRAVELED $= (11 + 13.5)\ m = 24.5\ m$ ◀

11.183

GIVEN: $a = -60x^{-1.5} \quad a \sim \frac{m}{s^2},\ x \sim m$; AT $t = 0$, $v = 0$, $x = 4\ m$

FIND: (a) v WHEN $x = 2\ m$

(b) v WHEN $x = 1\ m$

(c) v WHEN $x = 0.1\ m$

HAVE... $v\frac{dv}{dx} = a = -60x^{-1.5}$

WHEN $x = 4\ m$, $v = 0$: $\int_0^v v\,dv = \int_4^x (-60x^{-1.5})\,dx$

OR $\frac{1}{2}v^2 = 120[x^{-0.5}]_4^x$

OR $v^2 = 240\left(\frac{1}{\sqrt{x}} - \frac{1}{2}\right)$

(a) WHEN $x = 2\ m$: $v^2 = 240\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)$

OR $v = -7.05\ \frac{m}{s}$ ◀

(b) WHEN $x = 1\ m$: $v^2 = 240\left(1 - \frac{1}{2}\right)$

OR $v = -10.95\ \frac{m}{s}$ ◀

(c) WHEN $x = 0.1\ m$: $v^2 = 240\left(\frac{1}{\sqrt{0.1}} - \frac{1}{2}\right)$

OR $v = -25.3\ \frac{m}{s}$ ◀

11.184

GIVEN: $v = v_0 - kx \quad v \sim \frac{ft}{s},\ x \sim ft$; AT $t = 0$, $x = 0$, $v_0 = 900\ \frac{ft}{s}$; WHEN $x = 4$ IN., $v = 0$

FIND: (a) a AT $t = 0$

(b) t WHEN $x = 3.9$ IN.

FIRST NOTE... WHEN $x = \frac{4}{12}\ ft$, $v = 0$: $0 = (900\ \frac{ft}{s}) - k(\frac{4}{12}\ ft)$

OR $k = 2700\ \frac{1}{s}$

(a) HAVE... $v = v_0 - kx$

THEN $a = \frac{dv}{dt} = \frac{d}{dt}(v_0 - kx) = -kv$

(CONTINUED)

11.184 CONTINUED

OR $a = -k(v_0 - kx)$

AT $t = 0$: $a = -2700\ \frac{1}{s}(900\ \frac{ft}{s} - 0)$

OR $a_0 = -2.43 \times 10^6\ \frac{ft}{s^2}$ ◀

(b) HAVE... $\frac{dx}{dt} = v = v_0 - kx$

AT $t = 0$, $x = 0$: $\int_0^x \frac{dx}{v_0 - kx} = \int_0^t dt$

OR $-\frac{1}{k}[\ln(v_0 - kx)]_0^x = t$

OR $t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - kx}\right) = \frac{1}{k}\ln\left(\frac{1}{1 - \frac{k}{v_0}x}\right)$

WHEN $x = 3.9$ IN.: $t = \frac{1}{2700\ \frac{1}{s}}\ln\left[\frac{1}{1 - \frac{2700\ 1/s}{900\ ft/s}\left(\frac{3.9}{12}\ ft\right)}\right]$

OR $t = 1.366 \times 10^{-3}\ s$ ◀

11.185

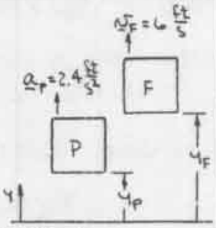
GIVEN: $v_F = 6\ \frac{ft}{s}\uparrow$; AT $t = 0$, $y_F = y_P = 0$; FOR $t \le 4\ s$, $v_P = 0$; $a_P = 2.4\ ft/s^2\uparrow$

FIND: (a) t AND y WHEN $y_F = y_P$

(b) v_P WHEN $y_F = y_P$

(a) FOR $t \ge 0$: $y_F = (y_F)_0^{\,0} + v_F t$

$t \ge 4\ s$: $y_P = (y_P)_0^{\,0} + (v_P)_0^{\,0}(t-4) + \frac{1}{2}a_P(t-4)^2$



WHEN $y_F = y_P$...

$(6\ \frac{ft}{s})t = \frac{1}{2}(2.4\ \frac{ft}{s^2})(t-4)^2$

EXPANDING AND SIMPLIFYING...

$t^2 - 13t + 16 = 0$

SOLVING... $t = 1.3765\ s$ AND $t = 11.6235\ s$

MOST REQUIRE $t > 4\ s$ $\quad \therefore t = 11.62\ s$ ◀

AT $t = 11.6235\ s$: $y_F = (6\ \frac{ft}{s})(11.6235\ s)$

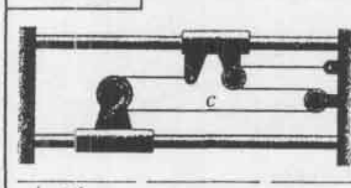
OR $y_F = y_P = 69.7\ ft$ ◀

(b) FOR $t \ge 4\ s$: $v_P = (v_P)_0^{\,0} + a_P(t-4)$

AT $t = 11.6235\ s$: $v_P = (2.4\ \frac{ft}{s^2})(11.6235 - 4)\ s$

OR $v_P = 18.30\ \frac{ft}{s}$ ◀

11.186

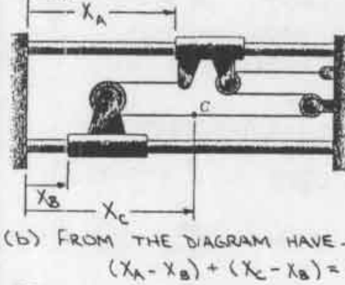


GIVEN: $v_B = 150\ \frac{mm}{s} \leftarrow$

FIND: (a) v_A

(b) v_C

(c) $v_{C/B}$



(a) FROM THE DIAGRAM HAVE...

$(x_A - x_B) + (-x_B) + 2(-x_A) = \text{CONSTANT}$

THEN... $v_A + 2v_B = 0$

SUBSTITUTING...

$v_A + 2(-150\ \frac{mm}{s}) = 0$

OR $v_A = 300\ \frac{mm}{s} \rightarrow$ ◀

(b) FROM THE DIAGRAM HAVE...

$(x_A - x_B) + (x_C - x_B) = \text{CONSTANT}$

THEN... $v_A - 2v_B + v_C = 0$

SUBSTITUTING... $300\ \frac{mm}{s} - 2(-150\ \frac{mm}{s}) + v_C = 0$

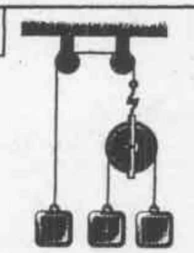
OR $v_C = 600\ \frac{mm}{s} \leftarrow$ ◀

(c) HAVE... $v_{C/B} = v_C - v_B$

$= -600\ \frac{mm}{s} - (-150\ \frac{mm}{s})$

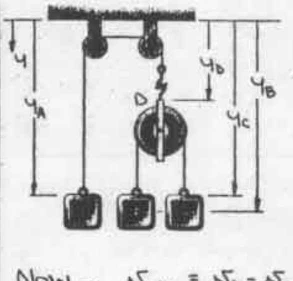
OR $v_{C/B} = 450\ \frac{mm}{s} \leftarrow$ ◀

11.187



GIVEN: v_A, v_B, v_C CONSTANTS;
$v_{A/C} = 300\ \frac{mm}{s}\uparrow$,
$v_{B/A} = 200\ \frac{mm}{s}\downarrow$

FIND: v_A, v_B, AND v_C



FROM THE DIAGRAM...

CABLE 1: $y_A + y_D$ = CONSTANT

THEN.. $v_A + v_D = 0$ (1)

CABLE 2: $(y_B - y_D) + (y_C - y_D)$ = CONSTANT

THEN.. $v_B + v_C - 2v_D = 0$ (2)

COMBINING EQS. (1) AND (2) TO ELIMINATE v_D--

$2v_A + v_B + v_C = 0$ (3)

NOW -- $v_{A/C} = v_A - v_C = -300\ \frac{mm}{s}$ (4)

AND $v_{B/A} = v_B - v_A = 200\ \frac{mm}{s}$ (5)

THEN $(3) + (4) - (5) \Rightarrow$

$(2v_A + v_B + v_C) + (v_A - v_C) - (v_B - v_A) = (-300) - (200)$

OR $v_A = 125\ \frac{mm}{s}\uparrow$ ◀

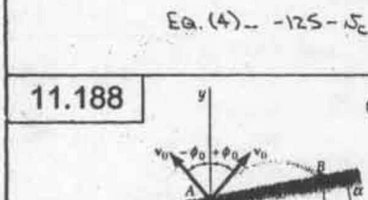
AND USING EQ(5).. $v_B - (-125) = 200$

OR $v_B = 75\ \frac{mm}{s}\downarrow$ ◀

EQ. (4).. $-125 - v_C = -300$

OR $v_C = 175\ \frac{mm}{s}\downarrow$ ◀

11.188



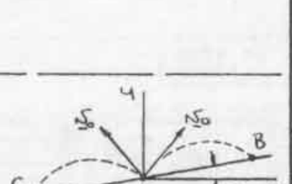
GIVEN: $v_0 = 30\ \frac{ft}{s}$, $\phi_0 = 40°$, $\alpha = 10°$

FIND: d_B AND d_C

FIRST NOTE..

$(v_0)_x = v_0 \sin\phi = (30\ \frac{ft}{s})\sin\phi$

$(v_0)_y = v_0 \cos\phi = (30\ \frac{ft}{s})\cos\phi$



ALSO, ALONG INCLINE CAB..

$y = x \tan 10°$

HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_0)_x t = (30 \sin\phi)t$ OR $t = \frac{x}{30 \sin\phi}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_0)_y t - \frac{1}{2}gt^2 = (30\cos\phi)t - \frac{1}{2}gt^2$

SUBSTITUTING FOR t --

$y = (30\cos\phi)\left(\frac{x}{30\sin\phi}\right) - \frac{1}{2}g\left(\frac{x}{30\sin\phi}\right)^2$

$= \frac{x}{\tan\phi} - \frac{g}{1800}\frac{x^2}{\sin^2\phi}$ $(g = 32.2\ \frac{ft}{s^2})$

AT B: $\phi = 40°$, $x = d_B$: $d_B \tan 10° = \frac{d_B}{\tan 40°} - \frac{32.2}{1800}\frac{d_B^2}{\sin^2 40°}$

OR $d_B = 23.5$ ft ◀

AT C: $\phi = -40°$, $x = -d_C$: $-d_C \tan 10° = \frac{-d_C}{\tan(-40°)} - \frac{32.2}{1800}\frac{(-d_C)^2}{\sin^2(-40°)}$

OR $d_C = 31.6$ ft ◀

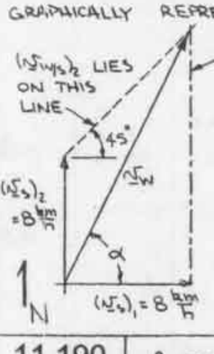
11.189

GIVEN: $(v_S)_1 = 8\ \frac{km}{h} \rightarrow$, $(v_{W/S})_1 \downarrow$;
$(v_S)_2 = 8\ \frac{km}{h}\uparrow$, $(v_{W/S})_2 \measuredangle 45°$

FIND: v_W, WHERE v_W IS CONSTANT

HAVE -- $v_W = v_S + v_{W/S}$

USING THIS EQUATION, THE TWO CASES ARE THEN GRAPHICALLY REPRESENTED AS SHOWN.



FROM THE DIAGRAM..

$v_W^2 = (8)^2 + (8+8)^2$

OR $v_W = 17.89\ \frac{km}{h}$

AND $\tan\alpha = \frac{16}{8}$

OR $\alpha = 63.4°$

$\therefore v_W = 17.89\ \frac{km}{h} \measuredangle 63.4°$ ◀

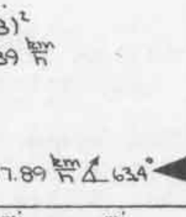
11.190

GIVEN: $\rho = 1500$ ft ; $v_1 = 45\ \frac{mi}{h}$, $v_2 = 30\ \frac{mi}{h}$,
$\Delta s_{12} = 750$ ft ; a_t = CONSTANT

FIND: a WHEN $\Delta s = 500$ ft

FIRST NOTE.. $v_1 = 45\ \frac{mi}{h} = 66\ \frac{ft}{s}$

$v_2 = 30\ \frac{mi}{h} = 44\ \frac{ft}{s}$



HAVE UNIFORMLY DECELERATED MOTION..

$\therefore v^2 = v_1^2 + 2a_t(s - s_1)$

WHEN $v = v_2$: $(44\ \frac{ft}{s})^2 = (66\ \frac{ft}{s})^2 + 2a_t(750\ ft)$

OR $a_t = -1.61333\ ft/s^2$

THEN WHEN $\Delta s = 500$ ft :

$v^2 = (66\ \frac{ft}{s})^2 + 2(-1.61333\ \frac{ft}{s^2})(500\ ft)$

$= 2742.67\ \frac{ft^2}{s^2}$

NOW.. $a_n = \frac{v^2}{\rho} = \frac{2742.67\ ft^2/s^2}{1500\ ft} = 1.82845\ \frac{ft}{s^2}$

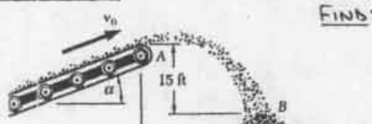
FINALLY.. $a^2 = a_t^2 + a_n^2 = (-1.61333\ \frac{ft}{s^2})^2 + (1.82845\ \frac{ft}{s^2})^2$

OR $a = 2.44\ \frac{ft}{s^2}$ ◀

11.191

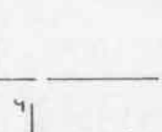
GIVEN: $v_0 = 24\ \frac{ft}{s}$

FIND: α



FIRST NOTE.. $(v_x)_0 = v_0 \cos\alpha = (24\ \frac{ft}{s})\cos\alpha$

$(v_y)_0 = v_0 \sin\alpha = (24\ \frac{ft}{s})\sin\alpha$



HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_x)_0 t = (24\cos\alpha)t$

AT POINT B: $25 = (24\cos\alpha)t$

OR $t_B = \frac{25}{24\cos\alpha}$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_y)_0 t - \frac{1}{2}gt^2 = (24\sin\alpha)t - \frac{1}{2}gt^2$ $(g = 32.2\ \frac{ft}{s^2})$

AT POINT B: $-15 = (24\sin\alpha)t_B - \frac{1}{2}gt_B^2$

SUBSTITUTING FOR t_B ..

(CONTINUED)

11.191 CONTINUED

$$-15 = (24\sin\alpha)\left(\frac{25}{24\cos\alpha}\right) - \frac{1}{2}g\left(\frac{25}{24\cos\alpha}\right)^2$$

OR $-3 = 5\tan\alpha - \dfrac{125g}{1152\cos^2\alpha}$

NOW.. $\dfrac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$

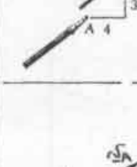
THEN.. $-3 = 5\tan\alpha - \dfrac{125\times 32.2}{1152}(1+\tan^2\alpha)$

OR $3.4939\tan^2\alpha - 5\tan\alpha + 0.49392 = 0$

SOLVING -- $\tan\alpha = 0.106746$ AND $\tan\alpha = 1.32432$

THEN -- $\alpha = 6.09^\circ$ AND $\alpha = 52.9^\circ$ ◀

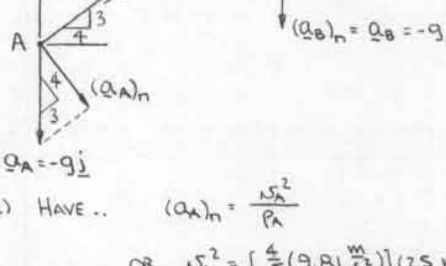
11.192



GIVEN: $\rho_A = 25\text{ m}$

FIND: (a) $\underline{v}_A$

(b) ρ_B, WHERE $y_B = y_{MAX}$



(a) HAVE.. $(a_A)_n = \dfrac{v_A^2}{\rho_A}$

OR $v_A^2 = \left[\frac{4}{5}(9.81\ \frac{\text{m}}{\text{s}^2})\right](25\text{ m})$

OR $v_A = 14.0071\ \frac{\text{m}}{\text{s}}$

$\therefore\ \underline{v}_A = 14.01\ \frac{\text{m}}{\text{s}} \measuredangle 36.9^\circ$ ◀

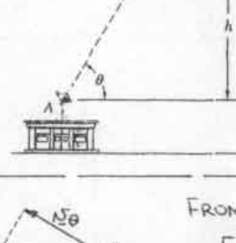
(b) HAVE.. $(a_B)_N = \dfrac{v_B^2}{\rho_B}$

WHERE $v_B = (v_A)_x = \frac{4}{5}v_A$

THEN.. $\rho_B = \dfrac{(\frac{4}{5}\times 14.0071\ \frac{\text{m}}{\text{s}})^2}{9.81\ \frac{\text{m}}{\text{s}^2}}$

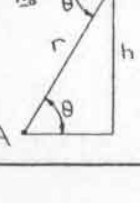
OR $\rho_B = 12.80\text{ m}$ ◀

11.193



GIVEN: $\underline{v}_0 = v_0 \leftarrow$, $v_0 =$ CONSTANT

FIND: $\dot\theta$ AND $\ddot\theta$ IN TERMS OF v_0, h, AND θ



FROM THE DIAGRAM

$r = \dfrac{h}{\sin\theta}$ $\quad v_\theta = v_0\sin\theta$

NOW.. $v_\theta = r\dot\theta$

SUBSTITUTING FOR v_θ AND r..

$v_0\sin\theta = \left(\dfrac{h}{\sin\theta}\right)\dot\theta$

(CONTINUED)

11.193 CONTINUED

OR $\dot\theta = \dfrac{v_0}{h}\sin^2\theta$ ◀

HAVE $\dot\theta = \dfrac{v_0}{h}\sin^2\theta$

THEN.. $\ddot\theta = \dfrac{v_0}{h}(2\dot\theta\sin\theta\cos\theta)$

SUBSTITUTING FOR $\dot\theta$..

$$\ddot\theta = \frac{v_0}{h}(2\sin\theta\cos\theta)\left(\frac{v_0}{h}\sin^2\theta\right)$$

OR $\ddot\theta = 2\dfrac{v_0^2}{h^2}\sin^3\theta\cos\theta$ ◀

ALTERNATIVE SOLUTIONS

HAVE.. $r = \dfrac{h}{\sin\theta}$

THEN $\dot r = -\dfrac{h\cos\theta}{\sin^2\theta}\dot\theta$

NOW.. $v^2 = v_r^2 + v_\theta^2 = (\dot r)^2 + (r\dot\theta)^2$

OR
$$v_0^2 = \left(-\frac{h\cos\theta}{\sin^2\theta}\dot\theta\right)^2 + \left(\frac{h}{\sin\theta}\dot\theta\right)^2$$
$$= \left(\frac{h\dot\theta}{\sin\theta}\right)^2\left(\frac{\cos^2\theta}{\sin^2\theta} + 1\right)$$
$$= \left(\frac{h\dot\theta}{\sin^2\theta}\right)^2$$

OR $\dot\theta = \pm\dfrac{v_0}{h}\sin^2\theta$

NOTE THAT AS θ INCREASES, THE AIRPLANE MOVES IN THE INDICATED DIRECTION. THUS, THE POSITIVE ROOT IS CHOSEN.

$\therefore\ \dot\theta = \dfrac{v_0}{h}\sin^2\theta$ ◀

HAVE.. $\underline{a} = \underline{a}_r + \underline{a}_\theta$

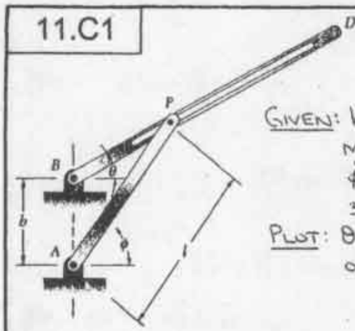
NOW $\underline{v}_0 = \text{CONSTANT} \Rightarrow a = 0$

$\therefore\ a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0$

OR
$$\ddot\theta = -2\,\frac{\left(-\frac{h\cos\theta}{\sin^2\theta}\times\frac{v_0}{h}\sin^2\theta\right)\left(\frac{v_0}{h}\sin^2\theta\right)}{\frac{h}{\sin\theta}}$$

OR $\ddot\theta = 2\dfrac{v_0^2}{h^2}\sin^3\theta\cos\theta$ ◀

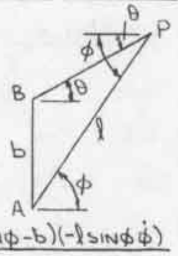
11.C1



GIVEN: WHITWORTH QUICK-RETURN MECHANISM SHOWN; $\dot{\phi} = 1\ \frac{RAD}{S}$; $l = 4$ IN.; $b = 2.5$ IN., 3.0 IN., 3.5 IN.

PLOT: θ VS. ϕ AND $\dot{\theta}$ VS. ϕ FOR ONE REVOLUTION OF ROD AP

ANALYSIS



HAVE.. $\dfrac{b}{\sin(\phi-\theta)} = \dfrac{l}{\sin(90°+\theta)}$

OR $b\cos\theta = l(\sin\phi\cos\theta - \cos\phi\sin\theta)$

OR $b = l(\sin\phi - \cos\phi\tan\theta)$

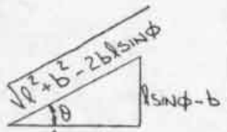
OR $\tan\theta = \dfrac{l\sin\phi - b}{l\cos\phi}$ (1)

THEN $\sec^2\theta\,\dot{\theta} = \dfrac{(l\cos\phi\,\dot{\phi})(l\cos\phi) - (l\sin\phi - b)(-l\sin\phi\,\dot{\phi})}{(l\cos\phi)^2}$

OR $\dot{\theta} = \cos^2\theta\,\dfrac{l^2 - bl\sin\phi}{(l\cos\phi)^2}\,\dot{\phi}$

USING EQ. (1)..

$\cos\theta = \dfrac{l\cos\phi}{\sqrt{l^2+b^2-2bl\sin\phi}}$



THEN.. $\dot{\theta} = \left[\dfrac{(l\cos\phi)^2}{l^2+b^2-2bl\sin\phi}\right] \times \dfrac{l^2 - bl\sin\phi}{(l\cos\phi)^2}\,\dot{\phi}$

$= l\,\dfrac{l - b\sin\phi}{l^2+b^2-2bl\sin\phi}\,\dot{\phi}$ (2)

NOTE: FOR $0 \le \phi < \tan^{-1}\left(\dfrac{b}{\sqrt{l^2-b^2}}\right)$

EQ. (1) => $-90 \le \theta < 0$

THUS, FOR THESE VALUES OF ϕ MUST USE

$\theta = \tan^{-1}\left(\dfrac{l\sin\phi - b}{l\cos\phi}\right) + 360°$

WHEN PLOTTING THE GRAPH.

SIMILARLY,

FOR $90° < \phi < 270°$, EQ. (1) => $-90° < \theta < 90°$

$\therefore\ \theta = \tan^{-1}\left(\dfrac{l\sin\phi - b}{l\cos\phi}\right) + 180°$

FOR $270° < \phi \le 360°$, EQ. (1) => $-90° < \theta \le 0$

$\therefore\ \theta = \tan^{-1}\left(\dfrac{l\sin\phi - b}{l\cos\phi}\right) + 360°$

OUTLINE OF PROGRAM

INPUT VALUE OF b

CONSTRUCT BORDER FOR GRAPH OF θ VS. ϕ; LABEL AXES

FOR VALUES OF ϕ FROM 0 TO 360° IN INCREMENTS OF 1°

COMPUTE θ:

FOR $0 \le \phi < \tan^{-1}\left(\dfrac{b}{\sqrt{16-b^2}}\right)$,

$\theta = \tan^{-1}\left(\dfrac{4\sin\phi - b}{4\cos\phi}\right) + 360°$

FOR $\tan^{-1}\left(\dfrac{b}{\sqrt{16-b^2}}\right) \le \phi < 90°$

(CONTINUED)

11.C1 continued

$\theta = \tan^{-1}\left(\dfrac{4\sin\phi - b}{4\cos\phi}\right)$

FOR $\phi = 90°$, $\theta = 90°$

FOR $90° < \phi < 270°$, $\theta = \tan^{-1}\left(\dfrac{4\sin\phi - b}{4\cos\phi}\right) + 180°$

FOR $\phi = 270°$, $\theta = 270°$

FOR $270° < \phi \le 360°$, $\theta = \tan^{-1}\left(\dfrac{4\sin\phi - b}{4\cos\phi}\right) + 360°$

PLOT (ϕ, θ)

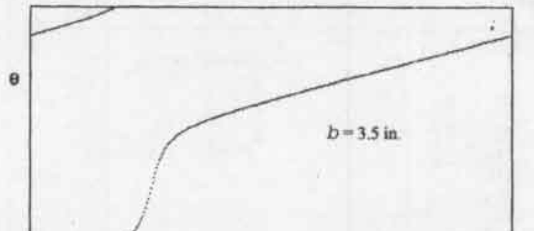
CONSTRUCT BORDER FOR GRAPH OF $\dot{\theta}$ VS. ϕ; LABEL AXES

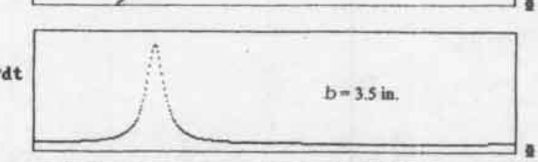
FOR VALUES OF ϕ FROM 0 TO 360° IN INCREMENTS OF 1°

COMPUTE $\dot{\theta}$: $\dot{\theta} = 4\,\dfrac{4 - b\sin\phi}{16 + b^2 - 8b\sin\phi}$

PLOT $(\phi, \dot{\theta})$

PROGRAM OUTPUT

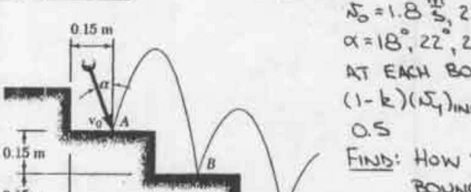




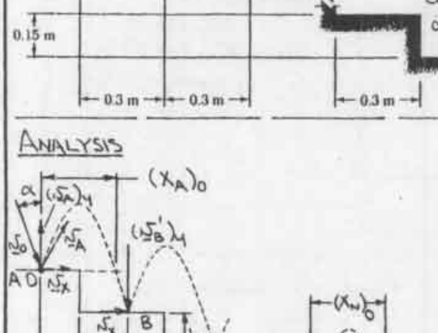
11.C2

GIVEN: EIGHT STEPS AS SHOWN; $v_0 = 1.8\ \frac{m}{s}, 2.4\ \frac{m}{s}, 3.0\ \frac{m}{s}$; $\alpha = 18°, 22°, 26°$; v_x = CONSTANT; AT EACH BOUNCE, $(v_y)_{FINAL} = (1-k)(v_y)_{INITIAL}$, $k = 0.4, 0.5$

FIND: HOW THE BALL BOUNCES DOWN THE STEPS FOR EACH COMBINATION OF v_0, α, AND k



ANALYSIS



(CONTINUED)

FIRST NOTE.. $v_x = v_0 \sin\alpha \quad (v_A)_y = (1-k)v_0\cos\alpha$

WITH THE ORIGIN OF A RECTANGULAR COORDINATE SYSTEM AT POINT O..

HORIZONTAL MOTION (UNIFORM)

$$x = x_0^{\,0} + v_x t \quad \text{OR} \quad t = x/v_x$$

VERTICAL MOTION (UNIF. ACCEL. MOTION)

$$y = y_0^{\,0} + (v_A)_y t - \tfrac{1}{2}gt^2 \qquad v_y = (v_A)_y - gt$$

SUBSTITUTING FOR t..

$$y = \frac{(v_A)_y}{v_x}x - \frac{1}{2}g\frac{x^2}{v_x^2} \qquad v_y = (v_A)_y - g\frac{x}{v_x}$$

CONSIDER THE MOTION OF THE BALL AFTER IT LANDS ON A GIVEN STEP

1. DETERMINE IF THE BALL BOUNCES TWICE ON STEP A:

ON STEP A, $y=0$: $\quad 0 = \frac{(v_A)_y}{v_x}(x_A)_0 - \frac{1}{2}g\frac{(x_A)_0^2}{v_x^2}$

$$\text{OR} \quad (x_A)_0 = \frac{2}{g}v_x(v_A)_y$$

$\therefore$ IF $(x_A)_0 < l$, THE BALL BOUNCES TWICE ON STEP A.

IN GENERAL, THE BALL BOUNCES TWICE ON STEP N $(N = A, B, C, \ldots, H)$ IF

$$(x_N)_0 < l + (N-1)b - \sum_{j=A}^{N-1} x_j$$

WHERE $(x_N)_0 = \frac{2}{g}v_x(v_N)_y$

AND x_N AND $(v_N)_y$ ARE GIVEN BELOW.

2. DETERMINE IF THE LANDS ON STEP B:

ON STEP B, $y=-h$: $\quad -h = \frac{(v_A)_y}{v_x}x_A - \frac{1}{2}g\frac{x_A^2}{v_x^2}$

SOLVING FOR x_A AND TAKING THE POSITIVE ROOT $(x_A > 0)$, HAVE..

$$x_A = \frac{\frac{(v_A)_y}{v_x} + \left\{\left[-(v_A)_y/v_x\right]^2 - 4\left(g/2v_x^2\right)(-h)\right\}^{1/2}}{2\left(\frac{g}{2v_x^2}\right)}$$

$$= \frac{v_x}{g}\left\{(v_A)_y + \sqrt{\left[(v_A)_y\right]^2 + 2gh}\right\}$$

$\therefore$ IF $x_A \le l + b$, THE BALL BOUNCES ON STEP B.

IN GENERAL, AFTER THE BALL BOUNCES ON STEP N, IT NEXT BOUNCES ON STEP i IF

$$\sum_{j=A}^{N} x_N \le l + (i-1)b$$

WHERE $x_N = \frac{v_x}{g}\left\{(v_N)_y + \sqrt{\left[(v_N)_y\right]^2 + 2g\left[(i-N)h\right]}\right\}$

FINALLY, IF THE BALL BOUNCES ON STEP B, HAVE USING THE EXPRESSION DERIVED ABOVE FOR v_y..

$$(v_B')_y = (v_A)_y - g\frac{x_A}{v_x}$$

NOTING THAT $(v_B')_y < 0$ AND THAT THE MAGNITUDE OF THE VERTICAL COMPONENT $(v_B)_y$ OF THE VELOCITY AFTER THE BOUNCE IS

$$(v_B)_y = (1-k)\left[g\frac{x_A}{v_x} - (v_A)_y\right]$$

HAVE IN GENERAL..

$$(v_N)_y = (1-k)\left[g\frac{x_{N-1}}{v_x} - (v_{N-1})_y\right]$$

(CONTINUED)

OUTLINE OF PROGRAM

FOR INITIAL ANGLES α: $\alpha = 18°, 22°, 26°$

FOR VALUES OF k: $k = 0.4, 0.5$

FOR INITIAL VELOCITIES v_0: $v_0 = 1.8\ \frac{m}{s}, 2.4\ \frac{m}{s}, 3.0\ \frac{m}{s}$

FOR EACH COMBINATION OF α, k, AND v_0

COMPUTE v_x AND $(v_A)_y$:

$$v_x = v_0 \sin\alpha \qquad (v_A)_y = (1-k)v_0\cos\alpha$$

SET INITIAL CONDITIONS: $N=1$, $i=2$, $x_{TOTAL} = 0$ WHERE 1, 2, 3, ..., 8 CORRESPOND TO STEPS A, B, C, ..., H AND x_{TOTAL} IS THE SUM OF THE HORIZONTAL DISTANCES BETWEEN SUCCESSIVE POINTS OF IMPACT.

DETERMINE IF THE BALL BOUNCES TWICE ON STEP N:

IF $\frac{2}{g}v_x(v_N)_y \le 0.15 + (N-1)(0.3) - x_{TOTAL}$

PRINT: "BALL FIRST BOUNCES TWICE ON STEP N."

CONSIDER THE NEXT COMBINATION OF α, k, AND v_0

DETERMINE THE NEXT STEP ON WHICH THE BALL BOUNCES

UPDATE x_{TOTAL}: $x_{TOTAL} = x_{TOTAL} + x_N$

WHERE $x_N = \frac{v_x}{g}\left\{(v_N)_y + \sqrt{\left[(v_N)_y\right]^2 + 0.3g(i-N)}\right\}$

DETERMINE IF THE BALL BOUNCES ON CONSECUTIVE STEPS

IF $x_{TOTAL} > 0.15 + (i-1)(0.3)$ AND $i \le 8$ PRINT: "BALL MISSES STEP i."

RESET x_{TOTAL}: $x_{TOTAL} = x_{TOTAL} - x_N$

UPDATE i: $i = i+1$

IF $i < 8$, COMPUTE NEW x_N AND x_{TOTAL} AND REPEAT CHECK

IF $i \ge 8$, CONSIDER THE NEXT COMBINATION OF α, k, AND v_0

DETERMINE HOW THE BALL BOUNCES DOWN THE REMAINING STEPS

IF $N \ge 8$ PRINT: "BALL CONTINUES TO BOUNCE DOWN THE STEPS."

IF $N < 8$, UPDATE VALUES FOR THE NEXT STEP:

v_y: $(v_i)_y = (1-k)\left[g\frac{x_N}{v_x} - (v_N)_y\right]$

N: $N = i$

i: $i = i+1$

PROGRAM OUTPUT

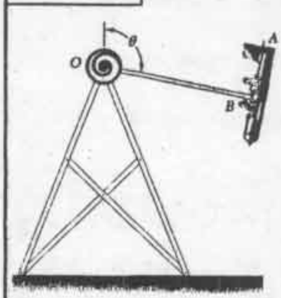
α	k	v_0	
18°	40%	1.8 m/s	Ball first bounces twice on step A
		2.4 m/s	Ball first bounces twice on step C
		3.0 m/s	Ball misses step D Ball continues to bounce down the steps
	50%	1.8 m/s	Ball first bounces twice on step A
		2.4 m/s	Ball first bounces twice on step B
		3.0 m/s	Ball first bounces twice on step H
22°	40%	1.8 m/s	Ball first bounces twice on step A
		2.4 m/s	Ball continues to bounce down the steps

(CONTINUED)

11.C2 continued

		3.0 m/s	Ball misses step B Ball misses step E Ball misses step G
	50%	1.8 m/s	Ball first bounces twice on step A
		2.4 m/s	Ball first bounces twice on step C
		3.0 m/s	Ball misses step C Ball misses step H
26°	40%	1.8 m/s	Ball first bounces twice on step B
		2.4 m/s	Ball misses step D Ball misses step G
		3.0 m/s	Ball misses step B Ball misses step D Ball misses step F Ball misses step H
	50%	1.8 m/s	Ball first bounces twice on step A
		2.4 m/s	Ball continues to bounce down the steps
		3.0 m/s	Ball misses step B Ball misses step E Ball misses step G

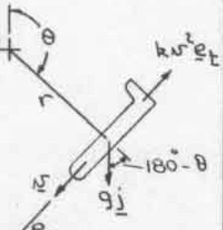
11.C3



GIVEN: $L_{OB} = 10$ m; $a_{DRAG} = -kv^2$, $k = 0$, 2×10^{-4} m^{-1}, 4×10^{-2} m^{-1}; $\theta_0 = 70°, 100°, 130°$

FIND: v_{MAX} AND THE FIRST TWO VALUES OF θ FOR WHICH $v = 0$ FOR EACH COMBINATION OF θ_0 AND k

ANALYSIS



IN THE TANGENTIAL DIRECTION, THE TANGENTIAL COMPONENT OF THE ACCELERATION OF THE AIRPLANE IS

$$a_t = g\sin(180° - \theta) - kv^2$$
$$= g\sin\theta - kv^2$$

RECALLING THAT $a_t = \frac{dv}{dt}$

HAVE $\frac{dv}{dt} = g\sin\theta - kv^2$

NOW, SINCE r = CONSTANT, HAVE $v = r\dot{\theta}$

THEREFORE, THE DIFFERENTIAL EQUATIONS

$$\frac{dv}{dt} = g\sin\theta - kv^2$$
$$\frac{d\theta}{dt} = \frac{1}{r}v$$

DEFINE THE MOTION OF THE AIRPLANE.

OUTLINE OF PROGRAM

INPUT VALUE OF k

INPUT VALUE OF θ_0

CASE 1: DETERMINE THE VALUE OF THE VELOCITY AT THE SPECIFIED ANGLE θ_f

INPUT θ_f

USE, FOR EXAMPLE, THE MODIFIED EULER METHOD (SECOND-ORDER RUNGE-KUTTA METHOD -- SEE CHAPRA AND CANALE, NUMERICAL METHODS FOR ENGINEERS, 2d

(CONTINUED)

11.C3 continued

ED., MCGRAW-HILL, 1988.) WITH A STEP SIZE $\Delta t = 0.008$ s TO NUMERICALLY INTEGRATE THE EQUATIONS

$$\frac{dv}{dt} = \begin{cases} g\sin\theta - kv^2 & \theta \le 180° \\ -g\sin\theta - kv^2 & \theta > 180° \end{cases}$$
$$\frac{d\theta}{dt} = \frac{1}{r}v$$

UNTIL $\theta_1' \le \theta_f \le \theta_2$, WHERE θ_1' AND θ_2 ARE THE VALUES OF θ AT THE MIDPOINT AND END, RESPECTIVELY, OF THE FINAL TIME INTERVAL.

USE LINEAR INTERPOLATION TO DETERMINE THE FINAL VELOCITY v_f:

$$v_f = v_1' + \frac{\theta_f - \theta_1'}{\theta_2 - \theta_1'}(v_2 - v_1')$$

PRINT THE VALUES OF k, θ_0, θ_f, AND v_f

CASE 2: DETERMINE THE VALUE OF θ FOR WHICH THE VELOCITY IS FIRST ZERO

USE THE MODIFIED EULER METHOD WITH A STEP SIZE $\Delta t = 0.008$ s TO NUMERICALLY INTEGRATE THE EQUATIONS

$$\frac{dv}{dt} = \begin{cases} g\sin\theta - kv^2 & \theta_0, \theta_1 < 180° \text{ OR } \theta_0, \theta_1 > 180° \\ -g\sin\theta - kv^2 & \theta_0 < 180°, \theta_1 > 180° \text{ OR } \theta_0 > 180°, \theta_1 < 180° \end{cases}$$

$$\frac{d\theta}{dt} = \frac{1}{r}v$$

WHERE θ_1 IS THE VALUE OF θ AT THE BEGINNING OF A TIME INTERVAL, UNTIL $v_2 < 0$, WHERE v_2 IS THE VELOCITY AT THE END OF A TIME INTERVAL.

USE LINEAR INTERPOLATION TO DETERMINE THE FINAL ANGLE θ_f:

$$\theta_f = \theta_1' + \frac{0 - v_1'}{v_2 - v_1'}(\theta_2 - \theta_1')$$

PRINT THE VALUES OF k, θ_0, AND θ_f

SUMMARY OF PROGRAM OUTPUT

Maximum velocity attained for a release angle θ_0

	v_{max}, m/s			
θ_0	$k = 0$	$k = 2\times10^{-4}$ m^{-1}	$k = 4\times10^{-2}$ m^{-1}	$k = 0$, theory
70°	16.23	16.19	11.67	16.23
100°	12.73	12.71	9.78	12.73
130°	8.37	8.36	6.97	8.37

First [$(\theta_0)_1$] and second [$(\theta_0)_2$] rest positions for a release angle (θ_0)

	$k = 0$		$k = 2\times10^{-4}$ m^{-1}		$k = 4\times10^{-2}$ m^{-1}	
θ_0	$(\theta_0)_1$	$(\theta_0)_2$	$(\theta_0)_1$	$(\theta_0)_2$	$(\theta_0)_1$	$(\theta_0)_2$
70°	290.0°	70.0°	289.2°	71.6°	229.4°	146.7°
100°	260.0°	100.0°	259.7°	100.6°	223.7°	149.3°
130°	230.0°	130.0°	229.9°	130.2°	213.6°	154.6°

11.C4

GIVEN: CAR TRAVELING ON AN EXIT RAMP; $v_0 = 60\ \frac{mi}{h}$, $v_{FINAL} = 0$; $|a_{MAX}| \le 10\ ft/s^2$; RAMP IS EITHER STRAIGHT OR CURVED ($\rho = 800$ ft); $\frac{dv}{dt}$ IS EITHER CONSTANT OR VARIES LINEARLY DURING TIME INTERVALS OF 1 S

FIND: t_{STOP} AND DISTANCE TRAVELED ON THE RAMP FOR EACH COMBINATION OF RAMP TYPE AND $\frac{dv}{dt}$

ANALYSIS

CASE 1: STRAIGHT RAMP, $\frac{dv}{dt}$ = CONSTANT

FOR THIS UNIFORMLY DECELERATED RECTILINEAR MOTION HAVE..

$$\frac{dv}{dt} = a = -10\ \frac{ft}{s^2}$$

THEN $v = v_0 + (-10)t$

AND $v^2 = v_0^2 + 2(-10)(x - \overset{0}{x_0})$

NOTING THAT a IS CONSTANT AND $v_{FINAL} = 0$, HAVE

$$t_{STOP} = \frac{v_0}{10} \quad (s)$$

$$x_{TOTAL} = \frac{v_0^2}{20} \quad (ft)$$

WHERE t_{STOP} AND x_{TOTAL} ARE THE TIME FOR THE CAR TO COME TO REST AND THE TOTAL DISTANCE TRAVELED BY THE CAR ON THE RAMP, RESPECTIVELY. ALSO,

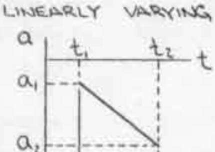
$$v_0 = 60\ mi/h = 88\ ft/s$$

CASE 2: STRAIGHT RAMP, $\frac{dv}{dt}$ LINEARLY VARYING

HAVE $a = \frac{dv}{dt}$

AND ASSUMING THAT FOR ANY TIME INTERVAL

$a_1 = 0 \qquad a_2 = -10\ ft/s^2$



HAVE $\frac{dv}{dt} = a = -\frac{10}{\Delta t}(t - t_1) \quad (\frac{ft}{s^2})$

AT $t = t_1, v = v_1$: $\int_{v_1}^{v} dv = \int_{t_1}^{t} -\frac{10}{\Delta t}(t - t_1)dt$

OR $v = v_1 - \frac{5}{\Delta t}(t - t_1)^2 \qquad (1)$

NOW.. $\frac{dx}{dt} = v$

AT $t = t_1, x = x_1$: $\int_{x_1}^{x} dx = \int_{t_1}^{t} [v_1 - \frac{5}{\Delta t}(t - t_1)^2]dt$

OR $x = x_1 + v_1(t - t_1) - \frac{5}{3\Delta t}(t - t_1)^3 \qquad (2)$

FOR $\Delta t = 1$ S AND WHEN $t = t_2$, HAVE..

$(1) \Rightarrow \quad v_2 = v_1 - 5 \quad (\frac{ft}{s})$

$(2) \Rightarrow \quad x_2 = x_1 + v_1 - \frac{5}{3} \quad (ft)$

FOR THE FINAL TIME INTERVAL ($\Delta t_{FINAL} < 1$ S), $v = 0$ AT $t = t_{FINAL}$. THEN, ASSUMING $t_1 = 0$ (FOR CONVENIENCE) HAVE..

$(1) \Rightarrow \quad 0 = v_1 - \frac{5}{\Delta t}(t_{FINAL}) \qquad \Delta t = 1$ S

OR $t_{FINAL} = \sqrt{\frac{v_1}{5}} \quad (s)$

AND $(2) \Rightarrow x_{FINAL} = x_1 + v_1 t_{FINAL} - \frac{5}{3}t_{FINAL}^3 \quad (ft)$

WHERE x_{FINAL} IS THE TOTAL DISTANCE, t_{FINAL} IS THE TIME DURATION OF THE FINAL TIME

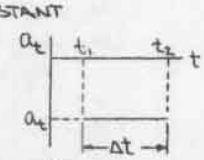
(CONTINUED)

11.C4 continued

INTERVAL, AND v_1 AND x_1 ARE THE VELOCITY AND DISTANCE, RESPECTIVELY, AT THE BEGINNING OF THE FINAL TIME INTERVAL.

CASE 3: CURVED RAMP, $\frac{dv}{dt}$ = CONSTANT

HAVE.. $a_t = \frac{dv}{dt}$ = CONSTANT



NOW.. $a^2 = a_t^2 + a_n^2$

$\quad = a_t^2 + (\frac{v^2}{\rho})^2$

WHERE $\rho = 800$ ft AND $|a_{MAX}| = 10\ ft/s^2$

FOR EACH TIME INTERVAL, a_t IS CONSTANT AND a_n IS MAXIMUM AT TIME t_1 SINCE THE VELOCITY DECREASES FROM t_1 TO t_2.

$\therefore \quad a_{MAX}^2 = a_t^2 + (\frac{v_1^2}{\rho})^2$

OR $\quad a_t = -\sqrt{a_{MAX}^2 - \frac{v_1^4}{\rho^2}} \quad (\frac{ft}{s^2})$

FOR EACH TIME INTERVAL.

NOW.. a_t = CONSTANT (UNIF. ACCEL. MOTION)

THEN.. $v = v_1 + a_t(t - t_1) \qquad (3)$

AND $\quad x = x_1 + v_1(t - t_1) + \frac{1}{2}a_t(t - t_1)^2 \qquad (4)$

FOR $\Delta t = 1$ S AND WHEN $t = t_2$, HAVE..

$$a_t = -\sqrt{a_{MAX}^2 - \frac{v_1^4}{\rho^2}} \quad (\frac{ft}{s^2})$$

$(3) \Rightarrow \quad v_2 = v_1 + a_t \quad (\frac{ft}{s})$

$(4) \Rightarrow \quad x_2 = x_1 + v_1 + \frac{1}{2}a_t \quad (ft)$

FOR THE FINAL TIME INTERVAL, $v = 0$ AT $t = t_{FINAL}$.

THEN, ASSUMING $t_1 = 0$ HAVE..

$$a_t = -\sqrt{a_{MAX}^2 - \frac{v_1^4}{\rho^2}} \quad (\frac{ft}{s^2})$$

$(3) \Rightarrow \quad 0 = v_1 + a_t(t_{FINAL})$

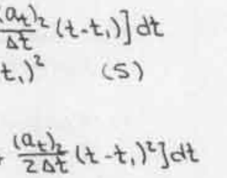
OR $\quad t_{FINAL} = \frac{v_1}{a_t} \quad (s)$

$(4) \Rightarrow x_{FINAL} = x_1 + v_1 t_{FINAL} + \frac{1}{2}a_t t_{FINAL}^2 \quad (ft)$

WHERE v_1 AND x_1 ARE THE VELOCITY AND DISTANCE, RESPECTIVELY, AT THE BEGINNING OF THE FINAL TIME INTERVAL.

CASE 4: CURVED RAMP, $\frac{dv}{dt}$ LINEARLY VARYING

ASSUMING FOR ANY TIME INTERVAL $\quad (a_t)_1 = 0$



HAVE.. $\quad a_t = \frac{(a_t)_2}{\Delta t}(t - t_1)$

NOW.. $\frac{dv}{dt} = a_t = \frac{(a_t)_2}{\Delta t}(t - t_1)$

AT $t = t_1, v = v_1$: $\int_{v_1}^{v} dv = \int_{t_1}^{t} [\frac{(a_t)_2}{\Delta t}(t - t_1)]dt$

OR $\quad v = v_1 + \frac{(a_t)_2}{2\Delta t}(t - t_1)^2 \qquad (5)$

ALSO, $\frac{dx}{dt} = v$

AT $t = t_1, x = x_1$: $\int_{x_1}^{x} dx = \int_{t_1}^{t} [v_1 + \frac{(a_t)_2}{2\Delta t}(t - t_1)^2]dt$

OR $\quad x = x_1 + v_1(t - t_1) + \frac{(a_t)_2}{6\Delta t}(t - t_1)^3 \qquad (6)$

NOW.. $\quad a^2 = a_t^2 + a_n^2$

$\quad = a_t^2 + (\frac{v^2}{\rho})^2$.

(CONTINUED)

11.C4 continued

WHERE $\rho = 800$ ft AND $|a_{max}| = 10$ ft/s².
NOW, FOR ANY TIME INTERVAL,
$(a_n)_{max}$ OCCURS AT $t = t_1$ (WHEN THE VELOCITY IS MAXIMUM)
$(a_t)_{max}$ OCCURS AT $t = t_2$
$(a_n)_{max_2} < a_{max}$ AT ALL TIMES (NOTE.. $\frac{v_2^2}{\rho} < 10$ ft/s²)

∴ ASSUME $a = a_{max}$ AT $t = t_2$. THEN..
$$a_{max}^2 = (a_t)_2^2 + \left(\frac{v_2^2}{\rho}\right)^2 \qquad (7)$$
FOR $\Delta t = 1$ s AND WHEN $t = t_2$, HAVE
$$(5) \Rightarrow v_2 = v_1 + \tfrac{1}{2}(a_t)_2$$
$$\text{OR} \quad (a_t)_2 = 2(v_2 - v_1) \qquad (8)$$
$$(6) \Rightarrow x_2 = x_1 + v_1 + \tfrac{1}{6}(a_t)_2 \qquad (\text{ft})$$
COMBINING EQS. (7) AND (8) TO ELIMINATE $(a_t)_2$..
$$a_{max}^2 = [2(v_2 - v_1)]^2 + \frac{v_2^4}{\rho^2}$$
OR $\frac{v_2^4}{\rho^2} + 4v_2^2 - 8v_1v_2 + (4v_1^2 - a_{max}^2) = 0$
-- A QUARTIC EQUATION WHICH DEFINES v_2.
FOR THE FINAL TIME INTERVAL, $v = 0$ AT $t = t_{FINAL}$. THEN, ASSUMING $t_1 = 0$ HAVE..
EQ. (8): $(a_t)_2 = 2(v_2 - v_1)$ WHERE $v_2 < 0$
$$(5) \Rightarrow 0 = v_1 + \tfrac{1}{2}(a_t)_2 t_{FINAL}^2$$
$$\text{OR} \quad t_{FINAL} = \sqrt{-\frac{2v_1}{(a_t)_2}} \qquad (5)$$
$$(6) \Rightarrow x_{FINAL} = x_1 + v_1 t_{FINAL} + \tfrac{1}{6}(a_t)_2 t_{FINAL}^3 \qquad (\text{ft})$$
WHERE v_1 AND x_1 ARE THE VELOCITY AND DISTANCE, RESPECTIVELY, AT THE BEGINNING OF THE FINAL TIME INTERVAL.

OUTLINE OF PROGRAM

INPUT INITIAL VELOCITY v_1
CONSIDER EACH CASE:
- CASE 1: STRAIGHT RAMP, $\frac{dv}{dt}$ = CONSTANT
  - COMPUTE TIME t_{STOP}: $t_{STOP} = \frac{v_1}{10}$
  - COMPUTE DISTANCE x_{TOTAL}: $x_{TOTAL} = \frac{v_1^2}{20}$
  - PRINT THE VALUES OF t_{STOP} AND x_{TOTAL}
- CASE 2: STRAIGHT RAMP, $\frac{dv}{dt}$ LINEARLY VARYING
  - FOR EACH SUCCESSIVE TIME INTERVAL
    - COMPUTE v_2: $v_2 = v_1 - 5$
    - WHILE $v_2 > 0$
      - UPDATE DISTANCE x_1: $x_1 = x_1 + v_1 - \frac{5}{3}$
      - UPDATE TIME AND SPEED: $t = t + 1$; $v_1 = v_2$
  - FOR THE FINAL TIME INTERVAL
    - COMPUTE t_{FINAL}: $t_{FINAL} = \sqrt{\frac{1}{5}v_1}$
    - COMPUTE TIME t_{STOP}: $t_{STOP} = t + t_{FINAL}$
    - COMPUTE DISTANCE x_{TOTAL}: $x_{TOTAL} = x_1 + v_1 t_{FINAL} - \frac{5}{3}t_{FINAL}^3$
  - PRINT THE VALUES OF t_{STOP} AND x_{TOTAL}
- CASE 3: CURVED RAMP, $\frac{dv}{dt}$ = CONSTANT
  - FOR EACH SUCCESSIVE TIME INTERVAL

(CONTINUED)

11.C4 continued

- - COMPUTE a_t: $a_t = -\left(100 - \frac{v_1^4}{64\times10^4}\right)^{1/2}$
    - COMPUTE v_2: $v_2 = v_1 + a_t$
    - WHILE $v_2 > 0$
      - UPDATE DISTANCE x_1: $x_1 = x_1 + v_1 + \frac{1}{2}a_t$
      - UPDATE TIME AND SPEED: $t = t + 1$; $v_1 = v_2$
  - FOR THE FINAL TIME INTERVAL
    - COMPUTE a_t: $a_t = -\left(100 - \frac{v_1^4}{64\times10^4}\right)^{1/2}$
    - COMPUTE t_{FINAL}: $t_{FINAL} = -\frac{v_1}{a_t}$
    - COMPUTE TIME t_{STOP}: $t_{STOP} = t + t_{FINAL}$
    - COMPUTE DISTANCE x_{TOTAL}: $x_{TOTAL} = x_1 + v_1 t_{FINAL} + \frac{1}{2}a_t t_{FINAL}^2$
  - PRINT THE VALUES OF t_{STOP} AND x_{TOTAL}
- CASE 4: CURVED RAMP, $\frac{dv}{dt}$ LINEARLY VARYING
  - FOR EACH SUCCESSIVE TIME INTERVAL
    - SOLVE THE EQUATION
      $$\frac{v_2^4}{64\times10^4} + 4v_2^2 - 8v_1v_2 + (4v_1^2 - 100) = 0$$
      FOR v_2 USING NEWTON'S METHOD (SEE, FOR EXAMPLE, CHAPRA AND CANALE, <u>NUMERICAL METHODS FOR ENGINEERS</u>, 2d ED., MCGRAW-HILL, 1988.)
    - WHILE $v_2 > 0$
      - COMPUTE $(a_t)_2$: $(a_t)_2 = 2(v_2 - v_1)$
      - UPDATE DISTANCE x_1: $x_1 = x_1 + v_1 + \frac{1}{6}(a_t)_2$
      - UPDATE TIME AND SPEED: $t = t + 1$; $v_1 = v_2$; $v_2 = 0$
  - FOR THE FINAL TIME INTERVAL
    - COMPUTE $(a_t)_2$: $(a_t)_2 = 2(v_2 - v_1)$
    - COMPUTE t_{FINAL}: $t_{FINAL} = \left[-2\frac{v_1}{(a_t)_2}\right]^{1/2}$
    - COMPUTE TIME t_{STOP}: $t_{STOP} = t + t_{FINAL}$
    - COMPUTE DISTANCE x_{TOTAL}: $x_{TOTAL} = x_1 + v_1 t_{FINAL} + \frac{1}{6}(a_t)_2 t_{FINAL}^3$
  - PRINT THE VALUES OF t_{STOP} AND x_{TOTAL}

PROGRAM OUTPUT

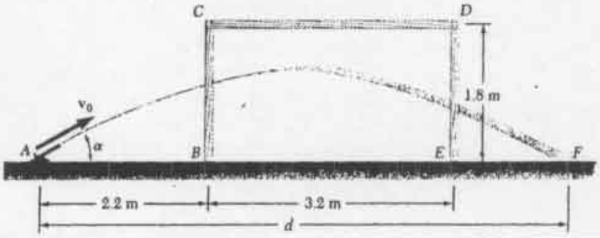
```
For a straight highway and a constant rate of change
  of the speed,
     time to stop = 8.80 s
     distance traveled = 387.2 ft

For a straight highway and a uniformly varying rate of
  change of the speed,
     time to stop = 17.77 s
     distance traveled = 789.2 ft

For a curved highway and a constant rate of change of
  the speed,
     time to stop = 11.29 s
     distance traveled = 581.4 ft

For a curved highway and a uniformly varying rate of
  change of the speed,
     time to stop = 20.71 s
     distance traveled = 1015.3 ft
```

11.C5

GIVEN: $v_0 = 10\,\frac{ft}{s}$; $\alpha = 20°$ TO $80°$ IN $5°$ INCREMENTS

FIND: (a) d FOR EACH VALUE OF α

(b) d_{MAX} AND α WHEN $d = d_{MAX}$

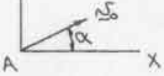


ANALYSIS

HORIZONTAL MOTION (UNIFORM)

$x = \cancel{x_0}^{0} + (v_0 \cos\alpha)t$

OR $t = \dfrac{x}{v_0 \cos\alpha}$



VERTICAL MOTION (UNIF. ACCEL. MOTION)

$y = \cancel{y_0}^{0} + (v_0 \sin\alpha)t - \frac{1}{2}gt^2 \qquad v_y = v_0 \sin\alpha - gt$

SUBSTITUTING FOR t..

$$y = (\tan\alpha)x - \frac{1}{2}g\frac{x^2}{v_0^2 \cos^2\alpha}$$

AT POINT F, $x = d$ AND $y = 0$:

$$0 = (\tan\alpha)d - \frac{1}{2}g\frac{d^2}{v_0^2\cos^2\alpha}$$

OR $d = \dfrac{v_0^2}{g}\sin 2\alpha$

AT THE MAXIMUM THEORETICAL HEIGHT y_{MAX} OF THE WATER, $v_y = 0$. THEN..

$$0 = v_0 \sin\alpha - g t_{y_{MAX}} \quad \text{OR} \quad t_{y_{MAX}} = \frac{v_0}{g}\sin\alpha$$

THEN $y_{MAX} = (v_0 \sin\alpha)\left(\frac{v_0}{g}\sin\alpha\right) - \frac{1}{2}g\left(\frac{v_0}{g}\sin\alpha\right)^2$

$= \frac{1}{2}\frac{v_0^2}{g}\sin^2\alpha$

AND $x_{y_{MAX}} = (v_0\cos\alpha)\left(\frac{v_0}{g}\sin\alpha\right)$

$= \dfrac{v_0^2}{2g}\sin 2\alpha$

IF THE WATER HITS THE ARBOR, $y = 1.8$ m AT THE POINT OF IMPACT. THE CORRESPONDING VALUE OF X IS THEN..

$$1.8 = (\tan\alpha)x - \frac{1}{2}g\frac{x^2}{v_0^2\cos^2\alpha}$$

OR $$x_{ARBOR} = \frac{\tan\alpha \pm \sqrt{(-\tan\alpha)^2 - \frac{3.6g}{v_0^2\cos^2\alpha}}}{\frac{g}{v_0^2\cos^2\alpha}}$$

WHERE THE (+) AND (−) SIGNS CORRESPOND TO THE WATER HITTING THE ARBOR FROM ABOVE AND FROM BELOW, RESPECTIVELY.

OUTLINE OF PROGRAM

INPUT MINIMUM AND MAXIMUM VALUES OF α

INPUT SIZE OF INCREMENT OF α

FOR EACH VALUE OF α

COMPUTE y AT $x = 2.2$ m:

$$y_{2.2} = 2.2\tan\alpha - \frac{0.02429}{\cos^2\alpha}$$

COMPUTE y AT $x = 5.4$ m:

$$y_{5.4} = 5.4\tan\alpha - \frac{0.14589}{\cos^2\alpha}$$

(CONTINUED)

11.C5 continued

(1) IF $y_{2.2} > 1.8$ m AND $y_{5.4} > 1.8$ m

COMPUTE d: $d = \frac{100}{g}\sin 2\alpha$

PRINT THE VALUES OF α AND d

NEXT VALUE OF α

(2) IF $y_{2.2} > 1.8$ m AND $y_{5.4} \le$ m

COMPUTE $(x_{ARBOR})_{ABOVE}$:

$$(x_{ARBOR})_{ABOVE} = \frac{100\cos\alpha}{g}\left(\sin\alpha + \sqrt{\sin^2\alpha - 0.0369}\right)$$

PRINT THE VALUES OF α AND $(x_{ARBOR})_{ABOVE}$

NEXT VALUE OF α

COMPUTE y_{MAX}: $y_{MAX} = \frac{50}{g}\sin^2\alpha$

COMPUTE $x_{y_{MAX}}$: $x_{y_{MAX}} = \frac{50}{g}\sin 2\alpha$

(3) IF $y_{MAX} < 1.8$ m

COMPUTE d: $d = \frac{100}{g}\sin 2\alpha$

PRINT THE VALUES OF α AND d

NEXT VALUE OF α

(4) IF $2.2 \text{ m} \le x_{y_{MAX}} \le 5.4$ m

COMPUTE $(x_{ARBOR})_{BELOW}$:

$$(x_{ARBOR})_{BELOW} = \frac{100\cos\alpha}{g}\left(\sin\alpha - \sqrt{\sin^2\alpha - 0.0369}\right)$$

PRINT THE VALUES OF α AND $(x_{ARBOR})_{BELOW}$

NEXT VALUE OF α

(5) IF $y_{2.2} = 1.8$ m

PRINT "THE WATER HITS THE ARBOR AT CORNER C."

NEXT VALUE OF α

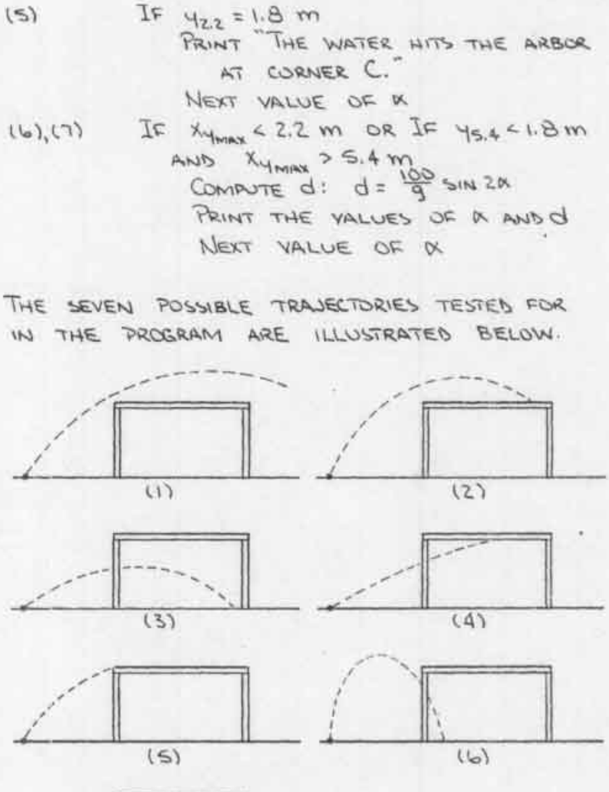
(6),(7) IF $x_{y_{MAX}} < 2.2$ m OR IF $y_{5.4} < 1.8$ m AND $x_{y_{MAX}} > 5.4$ m

COMPUTE d: $d = \frac{100}{g}\sin 2\alpha$

PRINT THE VALUES OF α AND d

NEXT VALUE OF α

THE SEVEN POSSIBLE TRAJECTORIES TESTED FOR IN THE PROGRAM ARE ILLUSTRATED BELOW.



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11.C5 continued

PROGRAM OUTPUT

(a)

```
For α = 20.00°, the water hits the ground at d = 6.552 m
For α = 25.00°, the water hits the ground at d = 7.809 m
For α = 30.00°, the water hits the ground at d = 8.828 m
For α = 35.00°, the water hits the ground at d = 9.579 m
For α = 40.00°, the water hits the top of the arbor from
        below at x = 3.106 m
For α = 45.00°, the water hits the top of the arbor from
        below at x = 2.335 m
For α = 50.00°, the water hits the ground at d = ‡10.039 m
For α = 55.00°, the water hits the ground at d = 9.579 m
For α = 60.00°, the water hits the ground at d = 8.828 m
For α = 65.00°, the water hits the ground at d = 7.809 m
For α = 70.00°, the water hits the ground at d = 6.552 m
For α = 75.00°, the water hits the top of the arbor from
        above at x = 4.557 m
For α = 80.00°, the water hits the top of the arbor from
        above at x = 3.133 m
```

(b)

```
For α = 46.20°, the water hits the top of the arbor from
        below at x = 2.202 m
For α = 46.21°, the water hits the top of the arbor from
        below at x = 2.201 m
For α = 46.22°, the water hits the top of the arbor from
        below at x = 2.200 m
For α = 46.23°, the water hits the ground at d = ‡10.184 m
For α = 46.24°, the water hits the ground at d = ‡10.184 m
For α = 46.25°, the water hits the ground at d = ‡10.184 m
For α = 46.26°, the water hits the ground at d = ‡10.184 m
For α = 46.27°, the water hits the ground at d = ‡10.184 m
For α = 46.28°, the water hits the ground at d = ‡10.184 m
For α = 46.29°, the water hits the ground at d = ‡10.183 m
For α = 46.30°, the water hits the ground at d = ‡10.183 m
```

12.1

GIVEN: $g = 9.7807(1+0.0053\sin^2\phi)\ \mathrm{m/s^2}$; $m = 2\ \mathrm{kg}$

FIND: (a) m AND W AT $\phi = 0$

(b) m AND W AT $\phi = 45^\circ$

(c) m AND W AT $\phi = 60^\circ$

FIRST NOTE THAT AT ALL LATITUDES

$$m = 2.000\ \mathrm{kg}$$

NOW.. $g = 9.7807(1+0.0053\sin^2\phi)\ \mathrm{m/s^2}$

AND $W = mg$

THEN..

(a) $\phi = 0$: $W = 2\ \mathrm{kg} \times 9.7807(1+0.0053\sin^2 0)\ \frac{\mathrm{m}}{\mathrm{s}^2}$

OR $W = 19.56\ \mathrm{N}$

(b) $\phi = 45^\circ$: $W = 2\ \mathrm{kg} \times 9.7807(1+0.0053\sin^2 45^\circ)\ \frac{\mathrm{m}}{\mathrm{s}^2}$

OR $W = 19.61\ \mathrm{N}$

(c) $\phi = 60^\circ$: $W = 2\ \mathrm{kg} \times 9.7807(1+0.0053\sin^2 60^\circ)\ \frac{\mathrm{m}}{\mathrm{s}^2}$

OR $W = 19.64\ \mathrm{N}$

12.2

GIVEN: $g = 12.3\ \mathrm{ft/s^2}$; $m = 50\ \mathrm{lb}$

FIND: (a) m (lb)

(b) m ($\mathrm{lb \cdot s^2/ft}$)

(c) W (lb)

(a) GIVEN.. $m = 50\ \mathrm{lb}$

(b) HAVE.. $m = 50\ \mathrm{lb} \times \dfrac{1\ \mathrm{lb \cdot s^2/ft}}{32.2\ \mathrm{lb}}$

$= 1.55280\ \frac{\mathrm{lb \cdot s^2}}{\mathrm{ft}}$

OR $m = 1.553\ \frac{\mathrm{lb \cdot s^2}}{\mathrm{ft}}$

(c) HAVE.. $W = mg$

$= 1.55280\ \frac{\mathrm{lb \cdot s^2}}{\mathrm{ft}} \times 12.3\ \frac{\mathrm{ft}}{\mathrm{s}^2}$

OR $W = 19.10\ \mathrm{lb}$

12.3

GIVEN: $m = 200\ \mathrm{kg}$; $v = 23.4 \times 10^3\ \mathrm{km/h}$

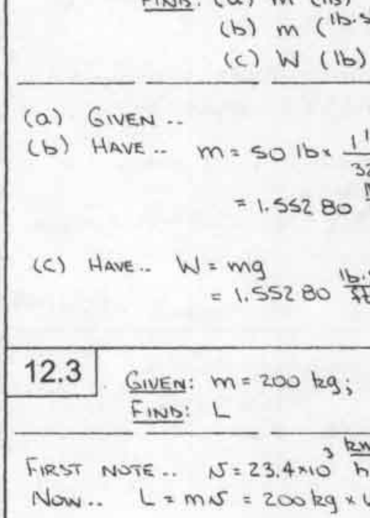
FIND: L

FIRST NOTE.. $v = 23.4 \times 10^3\ \frac{\mathrm{km}}{\mathrm{h}} = 6500\ \frac{\mathrm{m}}{\mathrm{s}}$

NOW.. $L = mv = 200\ \mathrm{kg} \times 6500\ \frac{\mathrm{m}}{\mathrm{s}}$

OR $L = 1.30 \times 10^6\ \frac{\mathrm{kg \cdot m}}{\mathrm{s}}$

12.4



GIVEN: LEVER ARMS OF SCALE B ARE OF EQUAL LENGTH; WHEN $a_E = 4\ \frac{\mathrm{ft}}{\mathrm{s}^2}\uparrow$, $R_A = 14.1\ \mathrm{lb}$

FIND: (a) W

(b) R_A AND m_B WHEN $a_E = 4\ \frac{\mathrm{ft}}{\mathrm{s}^2}\uparrow$

(a) WHEN THE ELEVATOR IS MOVING DOWNWARDS HAVE..

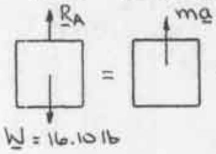
$+\uparrow \Sigma F_y = ma$: $W - 14.1\ \mathrm{lb} = \dfrac{W}{32.2\ \frac{\mathrm{ft}}{\mathrm{s}^2}} \times 4\ \frac{\mathrm{ft}}{\mathrm{s}^2}$

OR $W = 16.10\ \mathrm{lb}$

(CONTINUED)

12.4 CONTINUED

(b) WHEN THE ELEVATOR IS MOVING UPWARD HAVE..



$+\uparrow \Sigma F_y = ma$: $R_A - 16.10\ \mathrm{lb} = \dfrac{16.10\ \mathrm{lb}}{32.2\ \frac{\mathrm{ft}}{\mathrm{s}^2}} \times 4\ \frac{\mathrm{ft}}{\mathrm{s}^2}$

OR $R_A = 18.10\ \mathrm{lb}$

NOW OBSERVE THAT BECAUSE THE LEVER ARMS OF SCALE B ARE EQUAL,

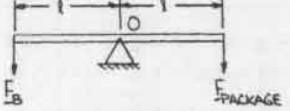
$m_B = m$ $\quad$ ($m = m_{PACKAGE}$)

REGARDLESS OF THE ACCELERATION OF THE ELEVATOR.* THEN..

$m_B = m = \dfrac{W}{g} = \dfrac{16.10\ \mathrm{lb}}{32.2\ \mathrm{ft/s^2}}$

OR $m_B = 0.500\ \frac{\mathrm{lb \cdot s^2}}{\mathrm{ft}}$

* PROOF

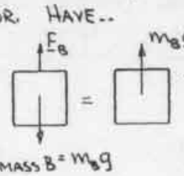


$+\circlearrowleft \Sigma M_O = 0$: $lF_B - lF_{PACKAGE} = 0$

OR $F_B = F_{PACKAGE}$

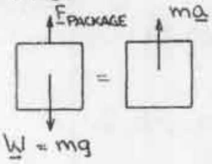
NEXT CONSIDER THE MASS m_B AND THE PACKAGE FOR AN ARBITRARY ACCELERATION $\underline{a}$ OF THE ELEVATOR. HAVE..

MASS B:



$+\uparrow \Sigma F_y = ma$: $F_B - m_B g = m_B a$ $\quad$ (1)

PACKAGE:



$+\uparrow \Sigma F_y = ma$: $F_{PACKAGE} - mg = ma$ $\quad$ (2)

SUBTRACTING EQ. (2) FROM EQ. (1) AND RECALLING $F_B = F_{PACKAGE}$ --

$$-m_B g - (-mg) = m_B a - ma$$

OR $\quad -(m_B - m)g = (m_B - m)a$

SINCE, IN GENERAL, $a \neq g$, IT FOLLOWS THAT

$m_B = m$ $\quad$ Q.E.D.

12.5

GIVEN: A PUCK WITH AN INITIAL VELOCITY v_0; AT $t = 9\ \mathrm{s}$, $v = 0$, $x = 30\ \mathrm{m}$

FIND: (a) v_0

(b) μ_k BETWEEN THE PUCK AND THE ICE

(a) ASSUME UNIFORMLY DECELERATED MOTION.

THEN $\quad v = v_0 + at$

AT $t = 9\mathrm{s}$: $0 = v_0 + a(9)$ $\quad$ OR $\quad a = -\frac{v_0}{9}$

ALSO.. $v^2 = v_0^2 + 2a(x - x_0^{\,0})$

AT $t = 9\mathrm{s}$: $0 = v_0^2 + 2a(30)$

(CONTINUED)

12.5 CONTINUED

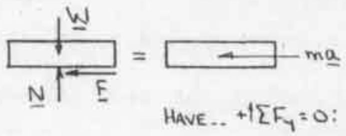
SUBSTITUTING FOR a.. $0 = v_0^2 + 2\left(-\frac{v_0}{9}\right)(30) = 0$

OR $v_0 = 6.6667 \ \frac{m}{s}$

OR $v_0 = 6.67 \ \frac{m}{s}$ ◀

AND $a = -\frac{6.6667}{9} = -0.74074 \ \frac{m}{s^2}$

(b)



HAVE.. $+\uparrow \Sigma F_y = 0$: $N - W = 0$

OR $N = W = mg$

SLIDING: $F = \mu_k N$

$= \mu_k mg$

$\xrightarrow{+} \Sigma F_x = ma$: $-F = ma$

OR $-\mu_k mg = ma$

OR $\mu_k = -\frac{a}{g} = -\frac{-0.74074 \ m/s^2}{9.81 \ m/s^2}$

OR $\mu_k = 0.0755$ ◀

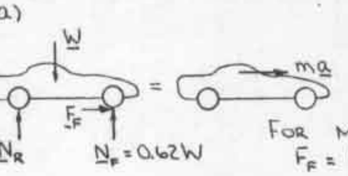
12.6

GIVEN: AN AUTOMOBILE INITIALLY AT REST; $\mu_s = 0.80$ BETWEEN THE TIRES AND THE PAVEMENT

FIND: (a) v_{MAX} WHEN $x = 400$ m FOR FRONT-WHEEL DRIVE, $W_{FRONT}/W = 0.62$

(b) v_{MAX} WHEN $x = 400$ m FOR REAR-WHEEL DRIVE, $W_{REAR}/W = 0.43$

(a)



FOR MAXIMUM ACCELERATION..

$F_F = F_{MAX} = \mu_s N_F = 0.8(0.62W)$

$= 0.496 \ W = 0.496 \ mg$

NOW.. $\xrightarrow{+} \Sigma F_x = ma$: $F_F = ma$

OR $0.496 \ mg = ma$

THEN $a = 0.496\left(9.81 \ \frac{m}{s^2}\right) = 4.86576 \ \frac{m}{s^2}$

SINCE a IS CONSTANT, HAVE --

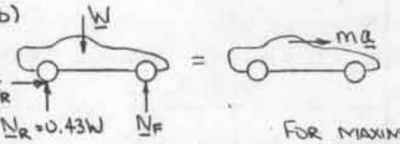
$v^2 = \cancel{v_0^2}^{\,0} + 2a(x - \cancel{x_0}^{\,0})$

WHEN $x = 400$ m: $v_{MAX}^2 = 2\left(4.86576 \ \frac{m}{s^2}\right)(400 \ m)$

OR $v_{MAX} = 62.391 \ \frac{m}{s}$

OR $v_{MAX} = 225 \ \frac{km}{h}$ ◀

(b)



FOR MAXIMUM ACCELERATION..

$F_R = F_{MAX} = \mu_s N_R = 0.8(0.43W)$

$= 0.344 \ W = 0.344 \ mg$

NOW.. $\xrightarrow{+} \Sigma F_x = ma$: $F_R = ma$

OR $0.344 \ mg = ma$

THEN $a = 0.344\left(9.81 \ \frac{m}{s^2}\right) = 3.37464 \ \frac{m}{s^2}$

SINCE a IS CONSTANT, HAVE --

$v^2 = \cancel{v_0^2}^{\,0} + 2a(x - \cancel{x_0}^{\,0})$

WHEN $x = 400$ m: $v_{MAX}^2 = 2\left(3.37464 \ \frac{m}{s^2}\right)(400 \ m)$

OR $v_{MAX} = 51.959 \ \frac{m}{s}$

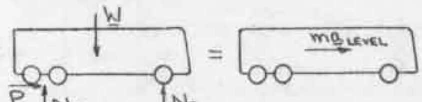
OR $v_{MAX} = 187.1 \ \frac{km}{h}$ ◀

12.7

GIVEN: $(a)_{LEVEL} = 3 \ ft/s^2$; $\theta_{UPGRADE} = 7°$; $(v_0)_{UPGRADE} = 60 \ mi/h$; P = CONSTANT

FIND: $x_{UPGRADE}$ WHEN $v = 50 \ mi/h$

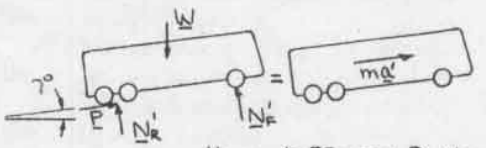
FIRST CONSIDER WHEN THE BUS IS ON THE LEVEL SECTION OF THE HIGHWAY.



$a_{LEVEL} = 3 \ \frac{ft}{s^2}$

HAVE.. $\xrightarrow{+} \Sigma F_x = ma$: $P = \frac{W}{g} a_{LEVEL}$

NOW CONSIDER WHEN THE BUS IS ON THE UPGRADE.



HAVE.. $\xrightarrow{+} \Sigma F_x = ma$: $P - W \sin 7° = \frac{W}{g} a'$

SUBSTITUTING FOR P.. $\frac{W}{g} a_{LEVEL} - W \sin 7° = \frac{W}{g} a'$

OR $a' = a_{LEVEL} - g \sin 7° = (3 - 32.2 \sin 7°) \ \frac{ft}{s^2}$

$= -0.92419 \ \frac{ft}{s^2}$

FOR THE UNIFORMLY DECELERATED MOTION..

$v^2 = (v_0)^2_{UPGRADE} + 2a'(x_{UPGRADE} - \cancel{x_0}^{\,0})$

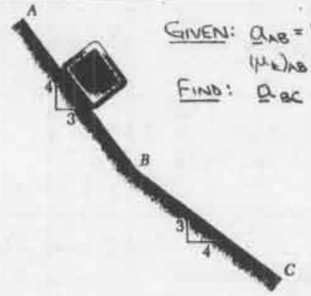
NOTING THAT $60 \ \frac{mi}{h} = 88 \ \frac{ft}{s}$, THEN WHEN $v = 50 \ \frac{mi}{h}$ $\left(= \frac{5}{6} v_0\right)$, HAVE..

$\left(\frac{5}{6} \times 88 \ \frac{ft}{s}\right)^2 = \left(88 \ \frac{ft}{s}\right)^2 + 2\left(-0.92419 \ \frac{ft}{s^2}\right) x_{UPGRADE}$

OR $x_{UPGRADE} = 1280.16$ ft

OR $x_{UPGRADE} = 0.242$ mi ◀

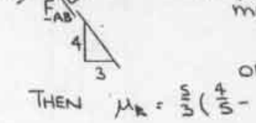
12.8



GIVEN: $a_{AB} = 18 \ ft/s^2$ ↘; $(\mu_k)_{AB} = (\mu_k)_{BC} = \mu_k$

FIND: a_{BC}

FIRST CONSIDER THE MOTION OF THE PACKAGE ON SECTION AB.



$+\nearrow \Sigma F_y = 0$: $N_{AB} - \frac{3}{5} W = 0$

OR $N_{AB} = \frac{3}{5} W$

SLIDING: $F_{AB} = \mu_k N_{AB}$

$= \frac{3}{5} \mu_k W$

$+\searrow \Sigma F_x = ma$: $\frac{4}{5} W - F_{AB} = ma_{AB}$

OR $\frac{4}{5} W - \frac{3}{5} \mu_k W = \frac{W}{g} a_{AB}$

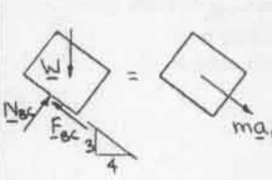
THEN $\mu_k = \frac{5}{3}\left(\frac{4}{5} - \frac{18 \ ft/s^2}{32.2 \ ft/s^2}\right)$

$= 0.40166$

NOW CONSIDER SECTION BC.

(CONTINUED)

12.8 CONTINUED



$+\nearrow \Sigma F_y = 0: \quad N_{BC} - \frac{4}{5}W = 0$

OR $N_{BC} = \frac{4}{5}W$

SLIDING: $F_{BC} = \mu_k N_{BC} = \frac{4}{5}\mu_k W$

$\searrow \Sigma F_x = ma: \quad \frac{3}{5}W - F_{BC} = ma_{BC}$

OR $\frac{3}{5}W - \frac{4}{5}\mu_k W = \frac{W}{g}a_{BC}$

OR $a_{BC} = (32.2\ \frac{ft}{s^2})(\frac{3}{5} - \frac{4}{5}\times 0.40166)$

OR $a_{BC} = 8.97\ \frac{ft}{s^2} \measuredangle 36.9°$

12.9

GIVEN: AN AUTOMOBILE'S BRAKING DISTANCE, X_{BR}, FROM 90 km/h ON LEVEL PAVEMENT IS 50 m

FIND: (a) X_{BR} FROM 90 km/h FOR A 5° INCLINE - UP

(b) X_{BR} FROM 90 km/h FOR A 3% INCLINE - DOWN

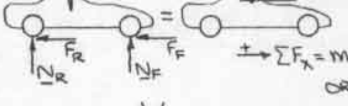
FIRST CONSIDER BRAKING ON LEVEL PAVEMENT. ASSUMING UNIFORMLY DECELERATED MOTION, HAVE..

$v^2 = (v_0)^2 + 2a_{LEVEL}(x - \cancel{x_0}^{0})$

NOTING THAT $90\ km/h = 25\ m/s$ HAVE..

$0 = (25\ \frac{m}{s})^2 + 2a_{LEVEL}(50\ m)$

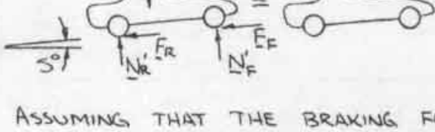
OR $a_{LEVEL} = -6.25\ \frac{m}{s^2}$



$\xrightarrow{+} \Sigma F_x = ma: -(F_F + F_R) = -m|a_{LEVEL}|$

OR $F_F + F_R = (6.25\ m)$ N

(a)



ASSUMING THAT THE BRAKING FORCE $(F_F + F_R)$ IS INDEPENDENT OF THE GRADE, HAVE..

$\xrightarrow{+} \Sigma F_x = ma: \quad -(F_F + F_R) - W\sin 5° = ma_{UP}$

OR $-6.25\ m - mg\sin 5° = ma_{UP}$

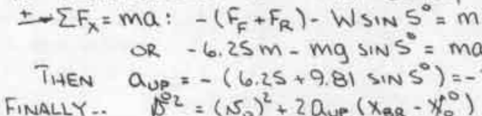
THEN $a_{UP} = -(6.25 + 9.81\sin 5°) = -7.1050\ \frac{m}{s^2}$

FINALLY.. $\cancel{v}^{0\,2} = (v_0)^2 + 2a_{UP}(x_{BR} - \cancel{x_0}^{0})$

SUBSTITUTING.. $0 = (25\ \frac{m}{s})^2 + 2(-7.1050\ \frac{m}{s^2})x_{BR}$

OR $x_{BR} = 44.0$ m

(b)



HAVE.. $\searrow \Sigma F_x = ma: \quad W\sin\theta - (F_F + F_R) = ma_{DOWN}$

NOW.. $\tan\theta = 0.03 \Rightarrow \theta$ SMALL $\Rightarrow \sin\theta \approx \tan\theta$

THEN.. $mg\tan\theta - 6.25\ m = ma_{DOWN}$

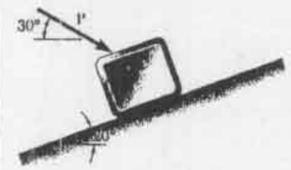
OR $a_{DOWN} = 9.81(0.03) - 6.25 = -5.9557\ \frac{m}{s^2}$

FINALLY.. $\cancel{v}^{0\,2} = (v_0)^2 + 2a_{DOWN}(x_{BR} - \cancel{x_0}^{0})$

SUBSTITUTING.. $0 = (25\ \frac{m}{s})^2 + 2(-5.9557\ \frac{m}{s^2})x_{BR}$

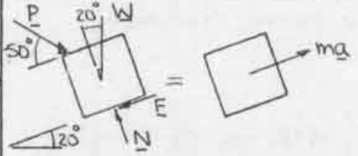
OR $x_{BR} = 52.5$ m

12.10



GIVEN: $m = 20$ kg; $v_0 = 0$; AT $t = 10$ s, $\Delta x = 5$ m; $\mu_s = 0.4$, $\mu_k = 0.3$

FIND: P



FIRST OBSERVE THAT THE PACKAGE IS UNIFORMLY ACCELERATED SINCE ALL OF THE FORCES ARE CONSTANT. THEN..

$x = x_0 + \cancel{v_0}^{0} t + \frac{1}{2}at^2$

AT $t = 10$ s: $5\ m = \frac{1}{2}a(10\ s)^2$

OR $a = 0.10\ \frac{m}{s^2}$

NOW.. $+\nwarrow \Sigma F_y = 0: \quad N - W\cos 20° - P\sin 50° = 0$

OR $N = mg\cos 20° + P\sin 50°$

SLIDING: $F = \mu_k N = \mu_k(mg\cos 20° + P\sin 50°)$

$\xrightarrow{+} \Sigma F_x = ma: \quad P\cos 50° - W\sin 20° - F = ma$

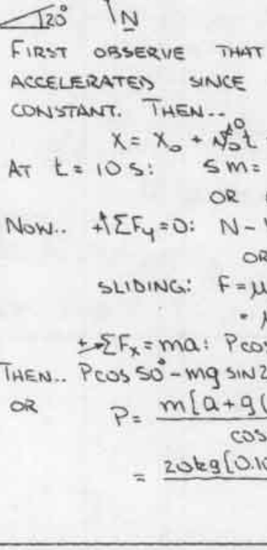
THEN.. $P\cos 50° - mg\sin 20° - \mu_k(mg\cos 20° + P\sin 50°) = ma$

OR $P = \dfrac{m[a + g(\sin 20° + \mu_k\cos 20°)]}{\cos 50° - \mu_k\sin 50°}$

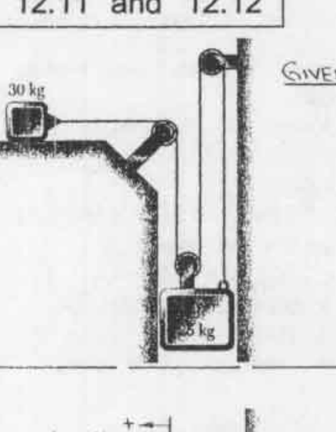
$= \dfrac{20\,kg\,[0.10\ \frac{m}{s^2} + 9.81\ \frac{m}{s^2}(\sin 20° + 0.3\cos 20°)]}{\cos 50° - 0.3\sin 50°}$

OR $P = 301$ N

12.11 and 12.12



GIVEN: BLOCKS A AND B AND THE PULLEY/CABLE SYSTEM, WHICH IS OF NEGLIGIBLE MASS, SHOWN; $(v_A)_0 = (v_B)_0 = 0$



FROM THE DIAGRAM..

$x_A + 3y_B =$ CONSTANT

THEN.. $v_A + 3v_B = 0$

AND $a_A + 3a_B = 0$

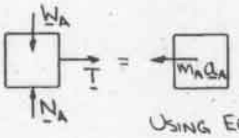
OR $a_A = -3a_B$ (1)

(CONTINUED)

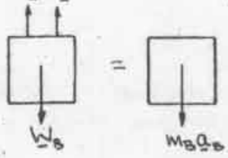
12.11 and 12.12 CONTINUED

12.11 GIVEN: $\mu_A = 0$
FIND: (a) a_A AND a_B
(b) T

(a)

A:


$\xrightarrow{+}\Sigma F_x = m_A a_A$: $-T = m_A a_A$
USING EQ. (1).. $T = 3 m_A a_B$

B:


$+\downarrow \Sigma F_y = m_B a_B$: $W_B - 3T = m_B a_B$
SUBSTITUTING FOR T..
$m_B g - 3(3 m_A a_B) = m_B a_B$

OR $a_B = \dfrac{g}{1 + 9\frac{m_A}{m_B}} = \dfrac{9.81 \frac{m}{s^2}}{1 + 9\frac{30\ kg}{25\ kg}}$

$= 0.83136 \frac{m}{s^2}$

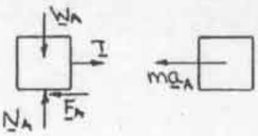
THEN -- $a_A = 2.49 \frac{m}{s^2} \rightarrow$

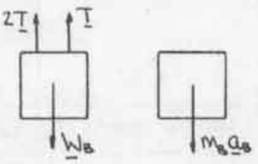
AND $a_B = 0.831 \frac{m}{s^2} \downarrow$

(b) HAVE.. $T = 3 \times 30\ kg \times 0.83136 \frac{m}{s^2}$

OR $T = 74.8\ N$

12.12 GIVEN: $(\mu_s)_A = 0.25$, $(\mu_k)_A = 0.20$
FIND: (a) a_A AND a_B
(b) T

A:


B:


FIRST DETERMINE IF THE BLOCKS WILL MOVE.
WITH $a_A = a_B = 0$, HAVE..

B: $+\downarrow \Sigma F_y = 0$: $W_B - 3T = 0$ OR $T = \frac{1}{3} m_B g$

A: $\xrightarrow{+}\Sigma F_x = 0$: $F_A - T = 0$

THEN $F_A = \frac{1}{3} \times 25\ kg \times 9.81 \frac{m}{s^2} = 81.75\ N$

$+\downarrow \Sigma F_y = 0$: $W_A - N_A = 0$ OR $N_A = m_A g$

ALSO.. $(F_A)_{MAX} = (\mu_s)_A N_A = (\mu_s)_A m_A g$
$= 0.25 \times 30\ kg \times 9.81 \frac{m}{s^2}$
$= 73.575\ N$

$\therefore$ $F_A > (F_A)_{MAX}$ WHICH IMPLIES THAT THE BLOCKS WILL MOVE.

(a) A: $+\downarrow \Sigma F_y = 0$: $W_A - N_A = 0$ OR $N_A = m_A g$

SLIDING: $F_A = (\mu_k)_A N_A = 0.20 m_A g$

$\xrightarrow{+}\Sigma F_x = m_A a_A$: $F_A - T = m_A a_A$

USING EQ. (1) .. $T = 0.20 m_A g + 3 m_A a_B$

B: $+\downarrow \Sigma F_y = m_B a_B$: $W_B - 3T = m_B a_B$

OR $m_B g - 3(0.20 m_A g + 3 m_A a_B) = m_B a_B$

OR $a_B = \dfrac{g\left(1 - 0.6\frac{m_A}{m_B}\right)}{1 + 9\frac{m_A}{m_B}} = \dfrac{(9.81 \frac{m}{s^2})\left(1 - 0.6\frac{30\ kg}{25\ kg}\right)}{1 + 9\frac{30\ kg}{25\ kg}}$

$= 0.23278\ m/s^2$

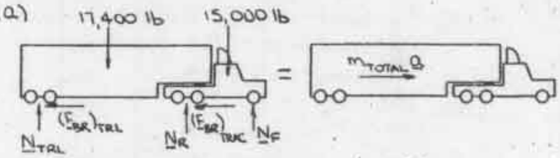
THEN.. $a_A = 0.698 \frac{m}{s^2} \rightarrow$

AND $a_B = 0.233 \frac{m}{s^2} \downarrow$

(b) HAVE.. $T = (30\ kg)(0.20 \times 9.81 + 3 \times 0.23278) \frac{m}{s^2}$ OR $T = 79.8\ N$

12.13

GIVEN: AT $t = 0$, $v = 60 \frac{mi}{h}$, BRAKES ARE APPLIED; $(F_{BR})_{TRAC} = 3600\ lb$, $(F_{BR})_{TRL} = 13{,}700\ lb$
FIND: (a) Δx WHEN $v = 0$
(b) P_{HITCH}

(a)


$\xrightarrow{+}\Sigma F_x = ma$: $-(F_{BR})_{TRAC} - (F_{BR})_{TRL} = \dfrac{W_{TOTAL}}{g} a$

OR $a = -\dfrac{32.2\ ft/s^2}{(15{,}000 + 17{,}400)\ lb}(3600 + 13{,}700)\ lb$

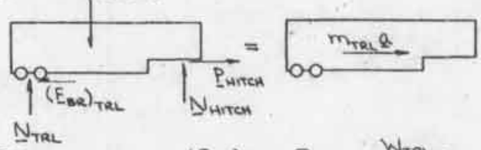
$= -17.1932\ ft/s^2$

FOR UNIFORMLY DECELERATED MOTION..

$v^2 = v_0^2 + 2a(x - x_0)$ $\qquad v_0 = 60 \frac{mi}{h} = 88 \frac{ft}{s}$

WHEN $v = 0$: $0 = (88 \frac{ft}{s})^2 + 2(-17.1932 \frac{ft}{s^2})(\Delta x)$

OR $\Delta x = 225\ ft$

(b)


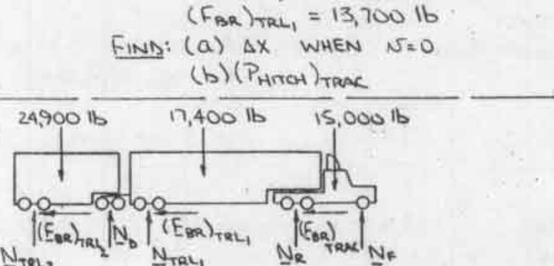
$\xrightarrow{+}\Sigma F_x = m_{TRL} a$: $-(F_{BR})_{TRL} + P_{HITCH} = \dfrac{W_{TRL}}{g} a$

THEN.. $P_{HITCH} = 13{,}700\ lb + \dfrac{17{,}400\ lb}{32.2\ ft/s^2} \times (-17.1932 \frac{ft}{s^2})$

OR $P_{HITCH} = 4410\ lb\ (T)$

12.14

GIVEN: TRACTOR-TRAILER OF PROBLEM 12.13 WITH A SECOND TRAILER..
$(W)_{TRL_2} = 24{,}900\ lb$, $(F_{BR})_{TRL_2} = 12{,}900\ lb$;
AT $t = 0$, $v = 60 \frac{mi}{h}$, BRAKES ARE APPLIED; $(F_{BR})_{TRAC} = 3600\ lb$,
$(F_{BR})_{TRL_1} = 13{,}700\ lb$
FIND: (a) Δx WHEN $v = 0$
(b) $(P_{HITCH})_{TRAC}$

(a)


$\xrightarrow{+}\Sigma F_x = ma$: $-(F_{BR})_{TRAC} - (F_{BR})_{TRL_1} - (F_{BR})_{TRL_2} = \dfrac{W_{TOTAL}}{g} a$

OR $a = -\dfrac{32.2\ ft/s^2}{(15{,}000 + 17{,}400 + 24{,}900)\ lb}(3600 + 13{,}700 + 12{,}900)\ lb$

$= -16.9710 \frac{ft}{s^2}$ (CONTINUED)

12.14 CONTINUED

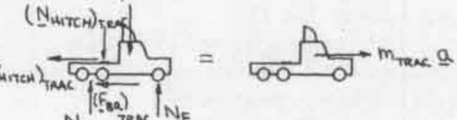
FOR UNIFORMLY DECELERATED MOTION..

$v^2 = v_0^2 + 2a(x - x_0)$ $\quad v_0 = 60\ \frac{mi}{h} = 88\ \frac{ft}{s}$

WHEN $v = 0$: $\quad 0 = (88\ \frac{ft}{s})^2 + 2(-16.9710\ \frac{ft}{s^2})(\Delta x)$

OR $\Delta x = 228$ ft ◀

(b) 15,000 lb



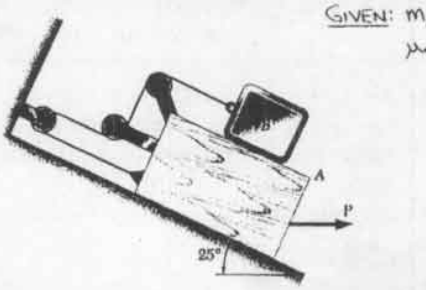
$\xrightarrow{+} \Sigma F_x = m_{TRAC}\, a: \ -(F_B)_{TRAC} - (P_{HITCH})_{TRAC} = \frac{W_{TRAC}}{g} a$

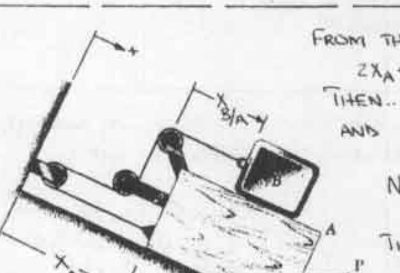
THEN .. $(P_{HITCH})_{TRAC} = -3600\text{ lb} - \frac{15{,}000\text{ lb}}{32.2\ ft/s^2}\left(-16.9710\ \frac{ft}{s^2}\right)$

OR $(P_{HITCH})_{TRAC} = 4310$ lb (T) ◀

12.15 and 12.16

GIVEN: $m_A = 40$ kg, $m_B = 8$ kg; $\mu_s = 0.20$, $\mu_k = 0.15$





FROM THE DIAGRAM..

$2x_A + x_{B/A} = \text{CONSTANT}$

THEN.. $2v_A + v_{B/A} = 0$

AND $2a_A + a_{B/A} = 0$

NOW..

$a_B = a_A + a_{B/A}$

THEN

$a_B = a_A + (-2a_A)$

OR $a_B = -a_A \quad (1)$

12.15 GIVEN: $P = 0$, $\theta = 25°$

FIND: (a) a_B

(b) T

FIRST DETERMINE IF THE BLOCKS WILL MOVE FOR THE GIVEN VALUE OF θ. THUS, SEEK THE VALUE OF θ FOR WHICH THE BLOCKS ARE IN IMPENDING MOTION, WITH THE IMPENDING MOTION OF A DOWN THE INCLINE.

B:

$+\nearrow \Sigma F_y = 0: \ N_{AB} - W_B \cos\theta = 0$

OR $N_{AB} = m_B g \cos\theta$

NOW.. $F_{AB} = \mu_s N_{AB}$

$= 0.2\, m_B g \cos\theta$

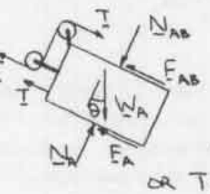
$\stackrel{+}{\searrow} \Sigma F_x = 0: \ -T + F_{AB} + W_B \sin\theta = 0$

OR $T = m_B g(0.2\cos\theta + \sin\theta)$

(CONTINUED)

12.15 and 12.16 CONTINUED

A:



$+\nearrow \Sigma F_y = 0: \ N_A - N_{AB} - W_A \cos\theta = 0$

OR $N_A = (m_A + m_B) g \cos\theta$

NOW.. $F_A = \mu_s N_A$

$= 0.2(m_A + m_B) g \cos\theta$

$\stackrel{+}{\searrow} \Sigma F_x = 0: \ -T - F_A - F_{AB} + W_A \sin\theta = 0$

OR $T = m_A g \sin\theta - 0.2(m_A + m_B) g\cos\theta$

$- 0.2\, m_B g \cos\theta$

$= g[m_A \sin\theta - 0.2(m_A + 2m_B)\cos\theta]$

EQUATING THE TWO EXPRESSIONS FOR T...

$m_B g(0.2\cos\theta + \sin\theta) = g[m_A \sin\theta - 0.2(m_A + 2m_B)\cos\theta]$

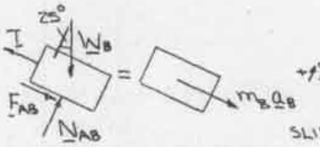
OR $8(0.2 + \tan\theta) = [40\tan\theta - 0.2(40 + 2 \times 8)]$

OR $\tan\theta = 0.4$

OR $\theta = 21.8°$ FOR IMPENDING MOTION. SINCE $\theta < 25°$, THE BLOCKS WILL MOVE. NOW CONSIDER THE MOTION OF THE BLOCKS.

(a)

B:



$+\nearrow \Sigma F_y = 0: \ N_{AB} - W_B \cos 25° = 0$

OR $N_{AB} = m_B g \cos 25°$

SLIDING: $F_{AB} = \mu_k N_{AB}$

$= 0.15\, m_B g \cos 25°$

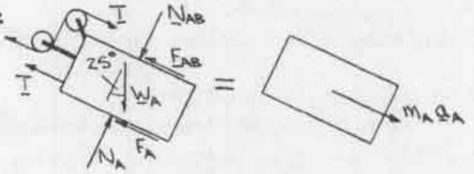
$\stackrel{+}{\searrow} \Sigma F_x = m_B a_B: \ -T + F_{AB} + W_B \sin 25° = m_B a_B$

OR $T = m_B[g(0.15\cos 25° + \sin 25°) - a_B]$

$= 8[9.81(0.15\cos 25° + \sin 25°) - a_B]$

$= 8(5.47952 - a_B) \quad (N)$

A:



$+\nearrow \Sigma F_y = 0: \ N_A - N_{AB} - W_A \cos 25° = 0$

OR $N_A = (m_A + m_B) g \cos 25°$

SLIDING: $F_A = \mu_k N_A = 0.15(m_A + m_B) g \cos 25°$

$\stackrel{+}{\searrow} \Sigma F_x = m_A a_A: \ -T - F_A - F_{AB} + W_A \sin 25° = m_A a_A$

SUBSTITUTING AND USING EQ. (1)..

$T = m_A g \sin 25° - 0.15(m_A + m_B) g \cos 25° - 0.15\, m_B g \cos 25°$

$- m_A(-a_B)$

$= g[m_A \sin 25° - 0.15(m_A + 2m_B)\cos 25°] + m_A a_B$

$= 9.81[40\sin 25° - 0.15(40 + 2 \times 8)\cos 25°] + 40\, a_B$

$= 91.15202 + 40\, a_B \quad (N)$

EQUATING THE TWO EXPRESSIONS FOR T..

$8(5.47952 - a_B) = 91.15202 + 40\, a_B$

OR $a_B = -0.98575\ \frac{m}{s^2}$

$\therefore\ a_B = 0.986\ \frac{m}{s^2}\ \nwarrow 25°$ ◀

(b) HAVE.. $T = 8[5.47952 - (-0.98575)]$

OR $T = 51.7$ N ◀

(CONTINUED)

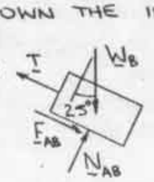
12.16 CONTINUED

12.16 GIVEN: $P = 40\text{ N} \rightarrow$, $\theta = 25°$

FIND: (a) $\underline{a}_B$

(b) T

FIRST DETERMINE IF THE BLOCKS WILL MOVE FOR THE GIVEN VALUE OF $\underline{P}$. THUS, SEEK THE VALUE OF $\underline{P}$ FOR WHICH THE BLOCKS ARE IN IMPENDING MOTION, WITH THE IMPENDING MOTION OF A DOWN THE INCLINE.

B: 

$+\nearrow \Sigma F_y = 0$: $N_{AB} - W_B \cos 25° = 0$

OR $N_{AB} = m_B g \cos 25°$

NOW.. $F_{AB} = \mu_s N_{AB}$

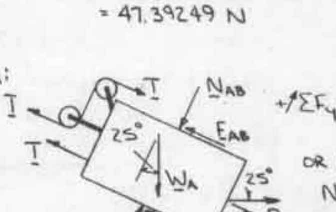
$= 0.2\, m_B g \cos 25°$

$\searrow^{+} \Sigma F_x = 0$: $-T + F_{AB} + W_B \sin 25° = 0$

OR $T = 0.2\, m_B g \cos 25° + m_B g \sin 25°$

$= (8\text{ kg})(9.81\ \frac{m}{s^2})(0.2 \cos 25° + \sin 25°)$

$= 47.39249\text{ N}$

A: 

$+\nearrow \Sigma F_y = 0$: $N_A - N_{AB} - W_A \cos 25° + P \sin 25° = 0$

OR $N_A = (m_A + m_B) g \cos 25° - P \sin 25°$

NOW.. $F_A = \mu_s N_A$

OR $F_A = 0.2[(m_A + m_B) g \cos 25° - P \sin 25°]$

$\searrow^{+} \Sigma F_x = 0$: $-T - F_A - F_{AB} + W_A \sin 25° + P \cos 25° = 0$

OR $-T - 0.2[(m_A + m_B) g \cos 25° - P \sin 25°] - 0.2\, m_B g \cos 25° + m_A g \sin 25° + P \cos 25° = 0$

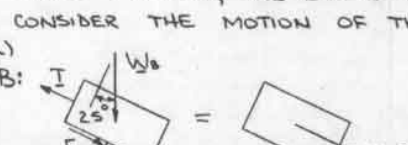
OR $P(0.2 \sin 25° + \cos 25°) = T + 0.2[(m_A + 2m_B) g \cos 25°] - m_A g \sin 25°$

THEN $P(0.2 \sin 25° + \cos 25°) = 47.39249\text{ N} + 9.81\frac{m}{s^2}\{0.2[(40 + 2\times 8)\cos 25° - 40 \sin 25°]\text{kg}\}$

OR $P = -19.04\text{ N}$ FOR IMPENDING MOTION.

SINCE $P < 40\text{ N}$, THE BLOCKS WILL MOVE. NOW CONSIDER THE MOTION OF THE BLOCKS.

(a)

B: 

$+\nearrow \Sigma F_y = 0$: $N_{AB} - W_B \cos 25° = 0$

OR $N_{AB} = m_B g \cos 25°$

SLIDING: $F_{AB} = \mu_k N_{AB}$

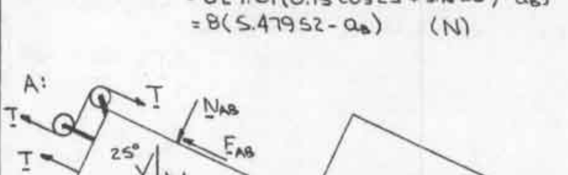
$= 0.15\, m_B g \cos 25°$

$\searrow^{+} \Sigma F_x = m_B a_B$: $-T + F_{AB} + W_B \sin 25° = m_B a_B$

OR $T = m_B[g(0.15 \cos 25° + \sin 25°) - a_B]$

$= 8[9.81(0.15 \cos 25° + \sin 25°) - a_B]$

$= 8(5.47952 - a_B)$ (N)

A: 

(CONTINUED)

12.16 CONTINUED

$+\nearrow \Sigma F_y = 0$: $N_A - N_{AB} - W_A \cos 25° + P \sin 25° = 0$

OR $N_A = (m_A + m_B) g \cos 25° - P \sin 25°$

SLIDING: $F_A = \mu_k N_A$

$= 0.15[(m_A + m_B) g \cos 25° - P \sin 25°]$

$\searrow^{+} \Sigma F_x = m_A a_A$: $-T - F_A - F_{AB} + W_A \sin 25° + P \cos 25° = m_A a_A$

SUBSTITUTING AND USING EQ. (1)..

$T = m_A g \sin 25° - 0.15[(m_A + m_B) g \cos 25° - P \sin 25°] - 0.15\, m_B g \cos 25° + P \cos 25° - m_A(-a_B)$

$= g[m_A \sin 25° - 0.15(m_A + 2m_B)\cos 25°] + P(0.15 \sin 25° + \cos 25°) + m_A a_B$

$= 9.81[40 \sin 25° - 0.15(40 + 2\times 8)\cos 25°] + 40(0.15 \sin 25° + \cos 25°) + 40 a_B$

$= 129.94004 + 40 a_B$ (N)

EQUATING THE TWO EXPRESSIONS FOR T..

$8(5.47952 - a_B) = 129.94004 + 40 a_B$

OR $a_B = -1.79383\ \frac{m}{s^2}$

$\therefore \underline{a}_B = 1.794\ \frac{m}{s^2} \measuredangle 25°$

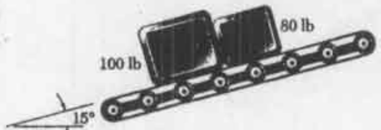
(b) HAVE.. $T = 8[5.47952 - (-1.79383)]$

OR $T = 58.2\text{ N}$

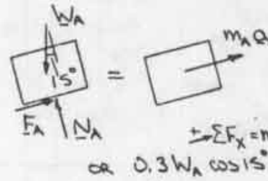
12.17

GIVEN: AT $t = 0$, $v_A = v_B = 0$, BELT BEGINS TO MOVE $\nearrow$ SO THAT SLIPPING OF BOTH BOXES OCCURS; $(\mu_k)_A = 0.30$, $(\mu_k)_B = 0.32$

FIND: $\underline{a}_A$ AND $\underline{a}_B$



ASSUME THAT $\underline{a}_B > \underline{a}_A$ SO THAT THE NORMAL FORCE N_{AB} BETWEEN THE BOXES IS ZERO.

A: 

$+\nwarrow \Sigma F_y = 0$: $N_A - W_A \cos 15° = 0$

OR $N_A = W_A \cos 15°$

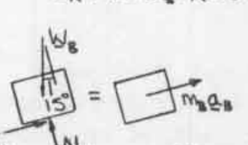
SLIPPING: $F_A = (\mu_k)_A N_A$

$= 0.3\, W_A \cos 15°$

$\nearrow^{+} \Sigma F_x = m_A a_A$: $F_A - W_A \sin 15° = m_A a_A$

OR $0.3\, W_A \cos 15° - W_A \sin 15° = \frac{W_A}{g} a_A$

OR $a_A = (32.2\ \frac{ft}{s^2})(0.3 \cos 15° - \sin 15°) = 0.997\ \frac{ft}{s^2}$

B: 

$+\nwarrow \Sigma F_y = 0$: $N_B - W_B \cos 15° = 0$

OR $N_B = W_B \cos 15°$

SLIPPING: $F_B = (\mu_k)_B N_B$

$= 0.32\, W_B \cos 15°$

$\nearrow^{+} \Sigma F_x = m_B a_B$: $F_B - W_B \sin 15° = m_B a_B$

OR $0.32\, W_B \cos 15° - W_B \sin 15° = \frac{W_B}{g} a_B$

OR $a_B = (32.2\ \frac{ft}{s^2})(0.32 \cos 15° - \sin 15°) = 1.619\ \frac{ft}{s^2}$

$a_B > a_A \Rightarrow$ ASSUMPTION IS CORRECT

$\therefore \underline{a}_A = 0.997\ \frac{ft}{s^2} \measuredangle 15°$

$\underline{a}_B = 1.619\ \frac{ft}{s^2} \measuredangle 15°$

NOTE: IF IT IS ASSUMED THAT THE BOXES REMAIN IN CONTACT ($N_{AB} \neq 0$), THEN

(CONTINUED)

12.17 CONTINUED

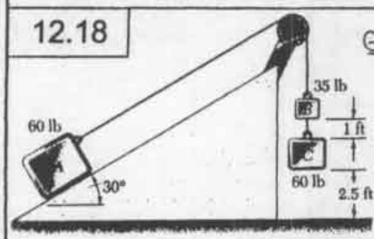
$a_A = a_B$ AND FIND $(\Sigma F_x = ma)$

A: $0.3W_A \cos 15° - W_A \sin 15° - N_{AB} = \frac{W_A}{g} a$

B: $0.32 W_B \cos 15° - W_B \sin 15° + N_{AB} = \frac{W_B}{g} a$

SOLVING YIELDS $a = 1.273\ ft/s^2$ AND $N_{AB} = -0.859$ lb, WHICH CONTRADICTS THE ASSUMPTION.

12.18



GIVEN: $\mu_s = 0.35, \mu_k = 0.30$; $t = 0$, $v = 0$

FIND: (a) $(v_A)_{max}$
(b) Δx_A WHEN $v_A = 0$

FIRST DETERMINE THE COMBINED MINIMUM WEIGHT OF BLOCKS B AND C FOR IMPENDING MOTION OF PACKAGE A UP THE INCLINE.

$+\nwarrow \Sigma F_y = 0$: $N_A - W_A \cos 30° = 0$

OR $N_A = W_A \cos 30°$

NOW.. $F_A = \mu_s N_A$

$= 0.35 W_A \cos 30°$

$+\nearrow \Sigma F_x = 0$: $T - F_A - W_A \sin 30° = 0$

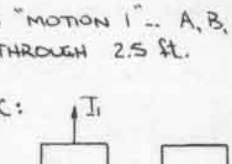
OR $T = W_A (0.35 \cos 30° + \sin 30°)$

$W_A = 60$ lb $\Rightarrow$ $T_{MIN} = 48.2$ lb

THEREFORE, SINCE T_{MIN} IS LESS THAN T_{B+C} (95 lb), PACKAGE A WILL MOVE UP THE INCLINE WHEN BLOCKS B AND C ARE RELEASED.

(a) "MOTION 1".. A, B, AND C MOVE TOGETHER THROUGH 2.5 ft.

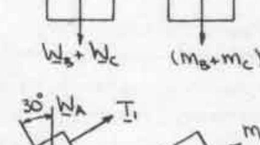
B+C:



$+\downarrow \Sigma F_y = ma_1$: $W_B + W_C - T_1 = \frac{W_B + W_C}{g} a_1$

OR $T_1 = 95\left(1 - \frac{a_1}{g}\right)$ (1)

A:



$+\nwarrow \Sigma F_y = 0$: $N_A - W_A \cos 30° = 0$

OR $N_A = W_A \cos 30°$

SLIDING: $F_A = \mu_k N_A$

$= 0.3 W_A \cos 30°$

$+\nearrow \Sigma F_x = m_A a_1$: $T_1 - F_A - W_A \sin 30° = \frac{W_A}{g} a_1$

OR $T_1 = 60\left(0.3 \cos 30° + \sin 30° + \frac{a_1}{g}\right)$

$= 60\left(0.759\,808 + \frac{a_1}{g}\right)$ (2)

EQUATING THE TWO EXPRESSIONS FOR T_1--

$$95\left(1 - \frac{a_1}{32.2}\right) = 60\left(0.759\,808 + \frac{a_1}{32.2}\right)$$

OR $a_1 = 10.2648\ \frac{ft}{s^2}$

"MOTION 2".. C IS AT REST, A AND B MOVE TOGETHER THROUGH 1 ft. FOR THIS CASE, EQS. (1) AND (2) BECOME..

$T_2 = 35\left(1 - \frac{a_2}{g}\right)$ (1')

(CONTINUED)

12.18 continued

$T_2 = 60\left(0.759\,808 + \frac{a_2}{g}\right)$ (2')

THEN $35\left(1 - \frac{a_2}{32.2}\right) = 60\left(0.759\,808 + \frac{a_2}{32.2}\right)$

OR $a_2 = -3.5889\ \frac{ft}{s^2}$

$\therefore$ SINCE $a_2 < 0$, A BEGINS TO DECELERATE AFTER BLOCK C REACHES THE GROUND; THUS, $(v_A)_{MAX}$ OCCURS AT THE END OF "MOTION 1." FOR THE UNIFORMLY ACCELERATION OF "MOTION 1," HAVE--

$v_A^2 = (v_A)_0^2 + 2a_1 (x - x_0)$

WHEN $\Delta x = 2.5$ ft: $(v_A)_{MAX}^2 = 2\left(10.2648\ \frac{ft}{s^2}\right)(2.5\ ft)$

OR $(v_A)_{MAX} = 7.1641\ \frac{ft}{s}$ $\quad (v_A)_{MAX} = 7.16\ \frac{ft}{s} \measuredangle 30°$ ◀

(b) FIRST NOTE THAT AT THE END OF "MOTION 2," THE SPEED OF PACKAGE A IS..

$(v_A)_2^2 = (v_A)_{MAX}^2 + 2a_2 \Delta x_2$

$= \left(7.1641\ \frac{ft}{s}\right)^2 + 2\left(-3.5889\ \frac{ft}{s^2}\right)(1\ ft)$

OR $(v_A)_2 = 6.6443\ \frac{ft}{s}$

"MOTION 3".. B AND C ARE AT REST, A CONTINUES UP THE INCLINE AND FINALLY COMES TO REST. FOR THIS CASE, $T = 0$ SO THAT EQ (2) BECOMES

$60\left(0.759\,808 + \frac{a_3}{g}\right) = 0$ (2")

THEN.. $a_3 = -0.759808\,(32.2) = -24.466\ \frac{ft}{s^2}$

THEN.. $v_A^2 = (v_A)_2^2 + 2a_3 (x - x_0)_3$

WHEN $v_A = 0$: $0 = \left(6.6443\ \frac{ft}{s}\right)^2 + 2\left(-24.466\ \frac{ft}{s^2}\right)\Delta x_3$

OR $\Delta x_3 = 0.9022$ ft

THE TOTAL DISTANCE Δx_A TRAVELED BY A UP THE INCLINE BEFORE COMING TO REST IS THEN..

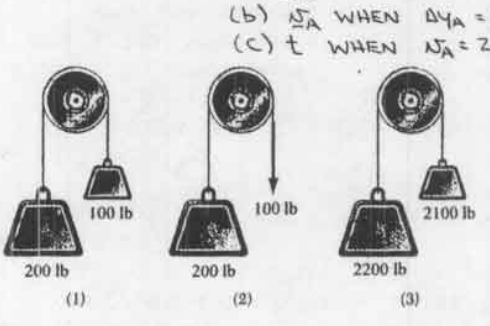
$\Delta x_A = \Delta x_1 + \Delta x_2 + \Delta x_3 = (2.5 + 1 + 0.9022)$ ft

OR $\Delta x_A = 4.40$ ft ◀

12.19

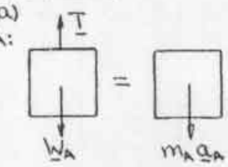
GIVEN: THE THREE SYSTEMS SHOWN; $v_0 = 0$

FIND (FOR EACH SYSTEM):
(a) a_A
(b) v_A WHEN $\Delta y_A = 10$ ft
(c) t WHEN $v_A = 20\ \frac{ft}{s}$



SYSTEM 1

(a)
A:



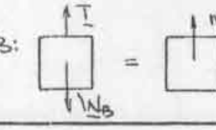
A: $+\downarrow \Sigma F_y = m_A a_A$: $W_A - T = \frac{W_A}{g} a_A$

OR $T = 200\left(1 - \frac{a_A}{g}\right)$

B: $+\uparrow \Sigma F_y = m_B a_B$: $T - W_B = \frac{W_B}{g} a_B$

OR $T = 100\left(1 + \frac{a_B}{g}\right)$

B:



(CONTINUED)

12.19 continued

EQUATING THE TWO EXPRESSIONS FOR T AND NOTING THAT $|a_A| = |a_B|$ ..

$$200\left(1-\frac{a_A}{g}\right) = 100\left(1+\frac{a_A}{g}\right)$$

OR $a_A = \frac{1}{3}g = \frac{1}{3}(32.2\ \frac{ft}{s^2}) = 10.7333\ \frac{ft}{s^2}$

$\therefore\ a_A = 10.73\ \frac{ft}{s^2}\downarrow$ ◀

(b) HAVE.. $v_A^2 = (v_A)_0^{2} + 2a_A(y-y_0)$ (with $(v_A)_0 = 0$)

WHEN $\Delta y_A = 10$ ft: $v_A^2 = 2(10.7333\ \frac{ft}{s^2})(10\ ft)$

OR $v_A = 14.65\ \frac{ft}{s}\downarrow$ ◀

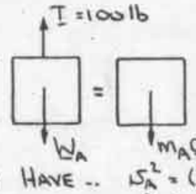
(c) HAVE.. $v_A = (v_A)_0 + a_A t$

WHEN $v_A = 20\ \frac{ft}{s}$: $20\ ft/s = (10.7333\ \frac{ft}{s^2})t$

OR $t = 1.863$ s ◀

SYSTEM 2

(a)



$+\downarrow\Sigma F_y = m_A a_A$: $W_A - T = \frac{W_A}{g}a_A$

OR.. $a_A = (32.2)\left(1-\frac{100}{200}\right)$

OR $a_A = 16.1\ \frac{ft}{s^2}\downarrow$ ◀

(b) HAVE.. $v_A^2 = (v_A)_0^2 + 2a_A(y-y_0)$

WHEN $\Delta y_A = 10$ ft: $v_A^2 = 2(16.1\ \frac{ft}{s^2})(10\ ft)$

OR $v_A = 17.94\ \frac{ft}{s}\downarrow$ ◀

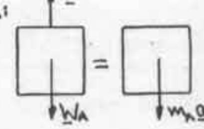
(c) HAVE.. $v_A = (v_A)_0 + a_A t$

WHEN $v_A = 20\ \frac{ft}{s}$: $20\ ft/s = (16.1\ \frac{ft}{s^2})t$

OR $t = 1.242$ s ◀

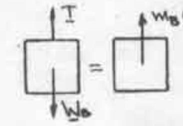
SYSTEM 3

(a)



A: $+\downarrow\Sigma F_y = m_A a_A$: $W_A - T = \frac{W_A}{g}a_A$ OR $T = 2200\left(1-\frac{a_A}{g}\right)$

B: $+\uparrow\Sigma F_y = m_B a_B$: $T - W_B = \frac{W_B}{g}a_B$ OR $T = 2100\left(1+\frac{a_B}{g}\right)$



EQUATING THE TWO EXPRESSIONS FOR T AND NOTING THAT $|a_A| = |a_B|$ ..

$$2200\left(1-\frac{a_A}{g}\right) = 2100\left(1+\frac{a_A}{g}\right)$$

OR $a_A = \frac{1}{43}g = \frac{1}{43}(32.2\ ft/s^2) = 0.74884\ \frac{ft}{s^2}$

$\therefore\ a_A = 0.749\ \frac{ft}{s^2}\downarrow$ ◀

(b) HAVE.. $v_A^2 = (v_A)_0^2 + 2a_A(y-y_0)$

WHEN $\Delta y = 10$ ft: $v_A^2 = 2(0.74884\ \frac{ft}{s^2})(10\ ft)$

OR $v_A = 3.87\ \frac{ft}{s}\downarrow$ ◀

(c) HAVE.. $v = (v_A)_0 + a_A t$

WHEN $v_A = 20\ \frac{ft}{s}$: $20\ \frac{ft}{s} = (0.74884\ \frac{ft}{s^2})t$

OR $t = 26.7$ s ◀

12.20

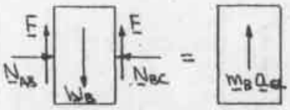
GIVEN: a_B = CONSTANT; $m_B = 3$ kg; MOTION OF B IS IMPENDING; $\mu_s = 0.30$, $\mu_k = 0.25$

FIND: (a) a_{EL} WHEN $a_{EL}\uparrow$ AND $N_{AB} = N_{BC} = 2W_B$

(b) N_{AB} AND N_{BC} WHEN $a_{EL} = 2.0\ \frac{m}{s^2}\downarrow$

FIRST OBSERVE THAT BECAUSE B IS NOT MOVING RELATIVE TO A AND TO C THAT $a_B = a_{EL}$.

(a)



HAVE.. $F = \mu_s N = 0.30(2W_B) = 0.6W_B = 0.6m_B g$

FOR a_{EL} TO BE ↑, THE NET VERTICAL FORCE MUST BE ↑, WHICH REQUIRES THAT THE FRICTIONAL FORCES BE ACTING AS SHOWN. IT THEN FOLLOWS THAT THE IMPENDING MOTION OF B RELATIVE TO A AND C IS DOWNWARD. THEN..

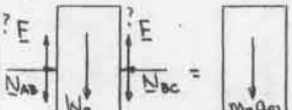
$+\uparrow\Sigma F_y = m_B a_{EL}$: $2F - W_B = m_B a_{EL}$

OR $2(0.6\,m_B g) - m_B g = m_B a_{EL}$

OR $a_{EL} = 0.2\times 9.81\ \frac{m}{s^2}$

OR $a_{EL} = 1.962\ \frac{m}{s^2}\uparrow$ ◀

(b)



HAVE.. $F = \mu_s N = 0.30N$

NOW OBSERVE THAT BECAUSE THE DIRECTION OF THE IMPENDING MOTION IS UNKNOWN, THE DIRECTIONS OF THE FRICTIONAL FORCES IS ALSO UNKNOWN (ALTHOUGH F_{NET} MUST BE DOWNWARD).

$+\uparrow\Sigma F_y = m_B a_{EL}$: $\pm 2F - W_B = -m_B|a_{EL}|$

OR $\pm 2F = m_B(g - |a_{EL}|) = 3\ kg\times(9.81-2)\ \frac{m}{s^2}$

SINCE THE MAGNITUDE OF F MUST BE POSITIVE, IT THEN FOLLOWS THAT F ↑ AND THAT THE IMPENDING MOTION OF B RELATIVE TO A AND C IS DOWNWARD. FINALLY..

$2(0.30\,N) = 3\ kg\times(9.81-2)\ \frac{m}{s^2}$

OR $N_{AB} = N_{BC} = 39.1$ N ◀

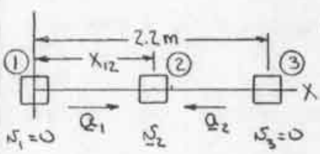
12.21

GIVEN: At $t=0$, $v=0$; for $0<t\le 1.3\text{ s}$, $a_{BELT} = 2\ \frac{m}{s^2}\rightarrow$; for $t>1.3\text{ s}$, $a_{BELT} = a_2 \leftarrow$; when $\Delta x_{BELT} = 2.2\text{ m}$, $v_{BELT}=0$; $\mu_s = 0.35$, $\mu_k = 0.25$

FIND: (a) a_2

(b) $x_{PACKAGE/BELT}$ when $v_{BELT} = 0$

(a) For the uniformly accelerated motion of a point on the belt have..



$1\rightarrow 2$: $x_{12} = \overset{0}{x_1} + \overset{0}{v_1}t + \frac{1}{2}a_1 t_{12}^2$

$= \frac{1}{2}(2\ \frac{m}{s^2})(1.3\text{ s})^2$

$= 1.69\text{ m}$

and $v_2 = \overset{0}{v_1} + a_1 t_{12}$

$= (2\ \frac{m}{s^2})(1.3\text{ s})$

$= 2.6\ \frac{m}{s}$

$2\rightarrow 3$: $\overset{0}{v_3^2} = v_2^2 + 2a_2(x_3 - x_2)$

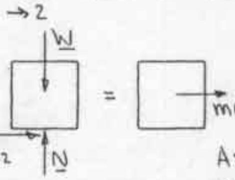
$0 = (2.6\ \frac{m}{s})^2 + 2a_2(2.2 - 1.69)\text{ m}$

or $a_2 = -6.62745\ \frac{m}{s^2}$

$\therefore\ a_2 = 6.63\ \frac{m}{s^2}\leftarrow$

(b) Now consider the package for each portion of the motion.

$1\rightarrow 2$



$+\uparrow\Sigma F_y = 0$: $N - W = 0$

or $N = W$

Now.. $F_{MAX} = \mu_s N$

$= 0.35\text{ W}$

Assume that the package does not slip nor is in impending motion relative to the belt.

Then $F_{12} < F_{MAX}$ $\quad (a_{PACK})_1 = a_1$

and .. $\xrightarrow{+}\Sigma F_x = ma_1$: $F_{12} = ma_1 \quad m = \frac{W}{g}$

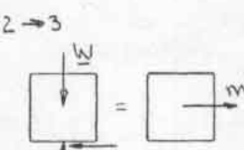
$= W\left(\frac{2\ m/s^2}{9.81\ m/s^2}\right)$

$= 0.204\text{ W}$

$\therefore\ F_{12}\ (0.204\text{ W}) < F_{MAX}\ (0.35\text{ W}) \Rightarrow$ assumption is correct (no slipping) so that

$(x_{PACKAGE/BELT})_{12} = 0$.

$2\rightarrow 3$



$+\uparrow\Sigma F_y = 0$: $N - W = 0$

or $N = W$

Now.. $F_{MAX} = \mu_s N$

$= 0.35\text{ W}$

Repeating the above assumtion implies

$F < F_{MAX}$ $\quad (a_{PACK})_2 = a_2$

Then $\xrightarrow{+}\Sigma F_x = ma_2$: $-F_{23} = ma_2 \quad m = \frac{W}{g}$

$= W\left(\frac{-6.62745\ m/s^2}{9.81\ m/s^2}\right)$

or $F_{23} = 0.676\text{ W}$

$\therefore\ F_{23}\ (0.676\text{ W}) > F_{MAX}\ (0.35\text{ W}) \Rightarrow$ assumption is incorrect, so that the package slips on the belt as the belt comes to rest.

(CONTINUED)

12.21 continued

Then.. slipping: $F'_{23} = \mu_k N$

$= 0.25\ mg$

$\xrightarrow{+}\Sigma F_x = m(a_{PACK})_2$: $-F'_{23} = m(a_{PACK})_2$

or $-0.25\ mg = m(a_{PACK})_2$

or $(a_{PACK})_2 = -0.25(9.81\ m/s^2) = -2.4525\ m/s^2$

Now.. $(a_{PACK})_2 = a_2 + (a_{PACK/BELT})_2$

or $(a_{PACK/BELT})_2 = -2.4525\ \frac{m}{s^2} - (-6.62745\ \frac{m}{s^2})$

$= 4.17495\ \frac{m}{s^2}$

For the belt.. $\overset{0}{v_3} = v_2 + a_2 t_{23}$

or $0 = 2.6\ \frac{m}{s} + (-6.62745\ \frac{m}{s^2})t_{23}$

or $t_{23} = 0.392\ 31\text{ s}$

Then.. $(x_{PACK/BELT})_{23} = \overset{0}{x_0} + \overset{0}{v_0}t_{23} + \frac{1}{2}(a_{PACK/BELT})_2 t_{23}^2$

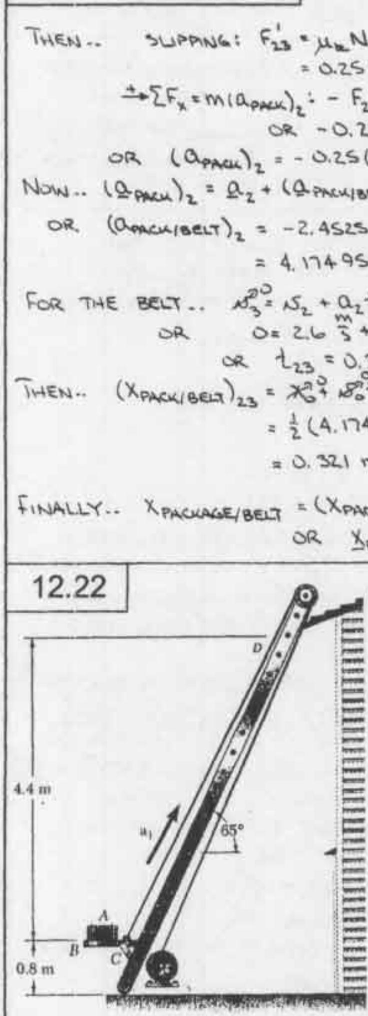
$= \frac{1}{2}(4.17495\ \frac{m}{s^2})(0.392\ 31\text{ s})^2$

$= 0.321\text{ m}$

Finally.. $x_{PACKAGE/BELT} = (x_{PACK/BELT})_{12} + (x_{PACK/BELT})_{23}$

or $x_{PACKAGE/BELT} = 0.321\text{ m}\rightarrow$

12.22

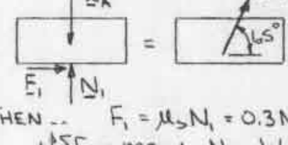


GIVEN: $(v_{BC})_0 = 0$, $(v_{BC})_D = 0$; BC moves constant accelerations a_1 and a_2; $\mu_s = 0.30$

FIND: $(a_1)_{MAX}$ and $(a_2)_{MAX}$ if sliding of shingles A is not to occur

---

a_1:



Note that the direction of a_1 fixes the direction of F_1 and that for $(a_1)_{MAX}$, $F_1 = (F_1)_{MAX}$

Then .. $F_1 = \mu_s N_1 = 0.3 N_1$

$+\uparrow\Sigma F_y = m a_y$: $N_1 - W_A = m_A a_1 \sin 65°$

or $N_1 = m_A(g + a_1 \sin 65°)$

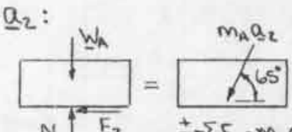
$\xrightarrow{+}\Sigma F_x = m_A a_x$: $F_1 = m_A a_1 \cos 65°$

Then.. $0.3[m_A(g + a_1\sin 65°)] = m_A a_1 \cos 65°$

or $a_1 = \frac{0.3(9.81\ m/s^2)}{\cos 65° - 0.3\sin 65°}$

or $(a_1)_{MAX} = 19.53\ \frac{m}{s^2}\ \angle 65°$

a_2:



Require .. $F_2 = \mu_s N_2 = 0.3 N_2$

$+\uparrow\Sigma F_y = m_A a_y$: $N_2 - W_A = -m_A a_2 \sin 65°$

or $N_2 = m_A(g - a_2 \sin 65°)$

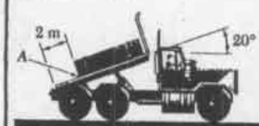
$\xrightarrow{+}\Sigma F_x = m_A a_x$: $-F_2 = -m_A a_2 \cos 65°$

Then .. $0.3[m_A(g - a_2 \sin 65°)] = m_A a_2 \cos 65°$

or $a_2 = \frac{0.3(9.81\ m/s^2)}{\cos 65° + 0.3\sin 65°}$

or $(a_2)_{MAX} = 4.24\ \frac{m}{s^2}\ \angle 65°$

12.23



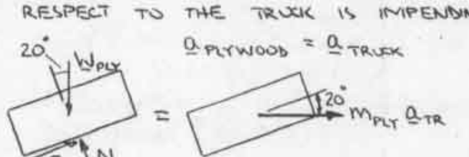
GIVEN: $t=0,\ v=0$; $\mu_s = 0.40$, $\mu_k = 0.30$

FIND: (a) $(a_{TRUCK})_{MIN}$ SO THAT PLYWOOD SLIDES

(b) a_{TRUCK} SO THAT $\Delta x_{PLYWOOD/TRUCK} = 2\text{ m}$ AT $t = 0.9\text{ s}$

(a) SEEK THE VALUE OF a_{TRUCK} SO THAT RELATIVE MOTION OF THE PLYWOOD WITH RESPECT TO THE TRUCK IS IMPENDING. NOTE..

$$\underline{a}_{PLYWOOD} = \underline{a}_{TRUCK}$$



HAVE.. $F = \mu_s N$

$= 0.4\,N \qquad (1)$

$+\!\nearrow \Sigma F_y = m_{PLY} a_y$: $\quad N - W_{PLY}\cos 20° = -m_{PLY} a_{TR} \sin 20°$

OR $\quad N = m_{PLY}(g\cos 20° - a_{TR}\sin 20°)$

$\xrightarrow{+} \Sigma F_x = m_{PLY} a_x$: $\quad F - W_{PLY}\sin 20° = m_{PLY} a_{TR}\cos 20°$

OR $\quad F = m_{PLY}(g \sin 20° + a_{TR}\cos 20°)$

SUBSTITUTING INTO EQ. (1)..

$$m_{PLY}(g\sin 20° + a_{TR}\cos 20°) = 0.4\,m_{PLY}(g\cos 20° - a_{TR}\sin 20°)$$

OR $\quad a_{TR} = \dfrac{g(0.4\cos 20° - \sin 20°)}{\cos 20° + 0.4\sin 20°} = (9.81\tfrac{m}{s^2})\dfrac{0.4 - \tan 20°}{1 + 0.4\tan 20°}$

OR $\quad (\underline{a}_{TRUCK})_{MIN} = 0.309\,\tfrac{m}{s^2} \rightarrow$ ◀

(b) FIRST NOTE THAT BECAUSE ALL OF THE FORCES ARE CONSTANT, THE ACCELERATIONS ARE ALSO CONSTANT. THEN--

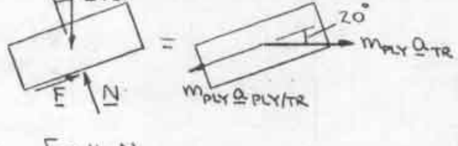
$$x_{PLY/TR} = (x_{PLY/TR})_0 + (v_{PLY/TR})_0 t + \tfrac{1}{2} a_{PLY/TR}\, t^2$$

AT $t = 0.9$ s: $\quad 2\text{ m} = \tfrac{1}{2} a_{PLY/TR}(0.9\text{ s})^2$

OR $\quad \underline{a}_{PLY/TR} = 4.93827\,\tfrac{m}{s^2} \measuredangle 20°$

NOW-- $\quad \underline{a}_{PLY} = \underline{a}_{TR} + \underline{a}_{PLY/TR}$

THEN



HAVE.. $F = \mu_k N$

$= 0.3\,N \qquad (1)$

$\xrightarrow{+} \Sigma F_x = m_{PLY} a_x$: $\quad F - W_{PLY}\sin 20° = m_{PLY}(a_{TR}\cos 20° - a_{PLY/TR})$

OR $\quad F = m_{PLY}(g\sin 20° + a_{TR}\cos 20° - a_{PLY/TR})$

$+\!\nearrow \Sigma F_y = m_{PLY} a_y$: $\quad N - W_{PLY}\cos 20° = -m_{PLY} a_{TR}\sin 20°$

OR $\quad N = m_{PLY}(g\cos 20° - a_{TR}\sin 20°)$

SUBSTITUTING INTO EQ. (1) --

$$m_{PLY}(g\sin 20° + a_{TR}\cos 20° - a_{PLY/TR}) = 0.3[m_{PLY}(g\cos 20° - a_{TR}\sin 20°)]$$

OR $\quad a_{TR} = \dfrac{g(0.3\cos 20° - \sin 20°) + a_{PLY/TR}}{\cos 20° + 0.3\sin 20°}$

$$= \frac{(9.81\tfrac{m}{s^2})(0.3\cos 20° - \sin 20°) + 4.93827\tfrac{m}{s^2}}{\cos 20° + 0.3\sin 20°}$$

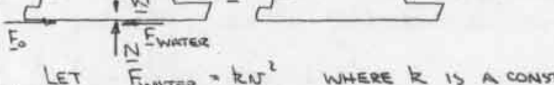
OR $\quad \underline{a}_{TRUCK} = 4.17\,\tfrac{m}{s^2} \rightarrow$ ◀

12.24

GIVEN: SHIP OF WEIGHT W HAVING A PROPULSIVE FORCE F_0; AT $t=0$, $v = v_0$ ($= v_{MAX}$), FORWARD, ENGINES ARE REVERSED; $F_{WATER} \propto v^2$

FIND: x WHEN $v = 0$

FIRST CONSIDER WHEN THE SHIP IS MOVING FORWARD.

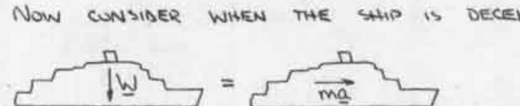


LET $\quad F_{WATER} = kv^2 \quad$ WHERE k IS A CONSTANT

$\xrightarrow{+} \Sigma F_x = 0$: $\quad F_0 - kv_0^2 = 0$

OR $\quad k = \dfrac{F_0}{v_0^2}$

NOW CONSIDER WHEN THE SHIP IS DECELERATING.



$\xrightarrow{+} \Sigma F_x = ma$: $\quad -F_0 - F_{WATER} = \dfrac{W}{g}a$

OR $\quad a = -\dfrac{g}{W}\left(F_0 + \dfrac{F_0}{v_0^2}v^2\right) = -\dfrac{g}{v_0^2}\dfrac{F_0}{W}(v^2 + v_0^2)$

NOW.. $\quad v\dfrac{dv}{dx} = a = -\dfrac{g}{v_0^2}\dfrac{F_0}{W}(v^2 + v_0^2)$

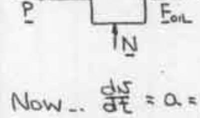
AT $t=0,\ x=0,\ v=v_0$:

$$\int_0^x dx = -\frac{v_0^2}{g}\frac{W}{F_0}\int_{v_0}^{0}\frac{v\,dv}{v^2+v_0^2}$$

OR $\quad x = -\dfrac{v_0^2}{g}\dfrac{W}{F_0}\left[\tfrac{1}{2}\ln(v^2+v_0^2)\right]_{v_0}^{0} = -\dfrac{1}{2}\dfrac{v_0^2}{g}\dfrac{W}{F_0}\ln\dfrac{v_0^2}{v_0^2+v_0^2}$

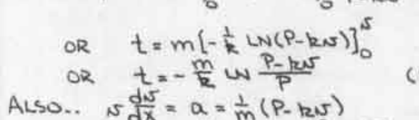
OR $\quad x = \dfrac{1}{2}\dfrac{W}{g}\dfrac{v_0^2}{F_0}\ln 2$ ◀

12.25



GIVEN: CONSTANT FORCE $\underline{P}$; PISTON AND ROD OF MASS m; $F_{OIL} = kv$; AT $t=0,\ x=0,\ v=0$

SHOW: $f(x, v, t) = 0$ IS LINEAR IN x, v, AND t



$\xrightarrow{+} \Sigma F_x = ma$: $\quad P - F_{OIL} = ma$

OR $\quad a = \dfrac{1}{m}(P - kv)$

NOW.. $\quad \dfrac{dv}{dt} = a = \dfrac{1}{m}(P - kv)$

AT $t=0,\ v=0$: $\quad \displaystyle\int_0^t dt = m\int_0^v \frac{dv}{P - kv}$

OR $\quad t = m\left[-\tfrac{1}{k}\ln(P-kv)\right]_0^v$

OR $\quad t = -\dfrac{m}{k}\ln\dfrac{P-kv}{P} \qquad (1)$

ALSO.. $\quad v\dfrac{dv}{dx} = a = \dfrac{1}{m}(P - kv)$

AT $x=0,\ v=0$: $\quad \displaystyle\int_0^x dx = m\int_0^v \frac{v\,dv}{P-kv}$

OR $\quad x = m\left\{\displaystyle\int_0^v\left[-\frac{1}{k} + \frac{P}{k(P-kv)}\right]dv\right\}$

$= m\left[-\dfrac{v}{k} - \dfrac{P}{k^2}\ln(P-kv)\right]_0^v$

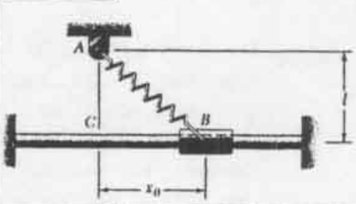
$= -m\left(\dfrac{v}{k} + \dfrac{P}{k^2}\ln\dfrac{P-kv}{P}\right)$

USING EQ. (1).. $\quad x = -\dfrac{mv}{k} + \dfrac{P}{k}t$

OR $\quad xk + mv - Pt = 0$ ◀

WHICH IS LINEAR IN x, v, AND t.

12.26

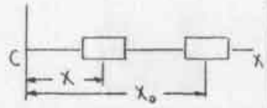


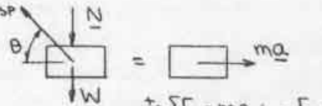
GIVEN: SPRING CONSTANT k, $m_{col} = m$; $(x_{SP})_{UNS.} = l$; AT $t=0$, $x = x_0$, $v = 0$

FIND: v AT C

FIRST NOTE..

$F_{SP} = k[L_{AB} - (x_{SP})_{UNS.}]$
$= k(\sqrt{x^2+l^2} - l)$





$\xrightarrow{+} \Sigma F_x = ma: \ -F_{SP}\cos\theta = ma$

WHERE $\cos\theta = \frac{x}{\sqrt{x^2+l^2}}$

THEN.. $a = -\frac{k}{m}(\sqrt{x^2+l^2} - l)\left(\frac{x}{\sqrt{x^2+l^2}}\right) = -\frac{k}{m}\left(x - \frac{lx}{\sqrt{x^2+l^2}}\right)$

NOW.. $v\frac{dv}{dx} = a = -\frac{k}{m}\left(x - \frac{lx}{\sqrt{x^2+l^2}}\right)$

AT $x = x_0$, $v = 0$: $\int_0^{(v_{col})_C} v\,dv = -\frac{k}{m}\int_{x_0}^{0}\left(x - \frac{lx}{\sqrt{x^2+l^2}}\right)dx$

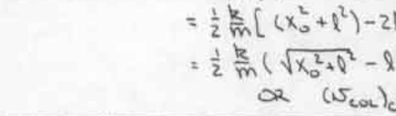
OR $\frac{1}{2}(v_{col})_C^2 = -\frac{k}{m}\left[\frac{1}{2}x^2 - l\sqrt{x^2+l^2}\right]_{x_0}^{0}$
$= -\frac{k}{m}\left\{(-l^2) - \left[\frac{1}{2}x_0^2 - l\sqrt{x_0^2+l^2}\right]\right\}$
$= \frac{1}{2}\frac{k}{m}\left[(x_0^2+l^2) - 2l\sqrt{x_0^2+l^2} + l^2\right]$
$= \frac{1}{2}\frac{k}{m}(\sqrt{x_0^2+l^2} - l)^2$

OR $(v_{col})_C = \sqrt{\frac{k}{m}}(\sqrt{x_0^2+l^2} - l)$ ◀

12.27

GIVEN: AUTOMOBILE WEIGHING 2700 lb, FRONT-WHEEL DRIVE, $W_{FR} = 0.62W$; $\mu_s = 0.70$, $D = 0.012v^2$ $D \sim$ lb, $v \sim$ ft/s; AT $t=0$, $x=0$, $v=0$

FIND: v_{MAX} WHEN $x = 0.25$ mi



$\underline{N}_R = 0.38W$ $\quad \underline{N}_F = 0.62W$

$F = F_{MAX}$ FOR $v = v_{MAX}$
$\therefore F = \mu_s N_F = 0.70(0.62W)$
$= 0.434W$

$\xrightarrow{+} \Sigma F_x = ma: \ F - D = \frac{W}{g}a$
OR $a = \frac{g}{W}(0.434W - 0.012v^2)$
$= 0.002\frac{g}{W}(217W - 6v^2)$

NOW.. $v\frac{dv}{dx} = a = 0.002\frac{g}{W}(217W - 6v^2)$

AT $x=0$, $v=0$: $0.002\frac{g}{W}\int_0^x dx = \int_0^v \frac{v\,dv}{217W - 6v^2}$

OR $0.002\frac{g}{W}x = -\frac{1}{12}[\ln(217W - 6v^2)] = -\frac{1}{12}\ln\left(\frac{217W - 6v^2}{217W}\right)$

OR $\frac{217W - 6v^2}{217W} = e^{-0.024\frac{g}{W}x}$

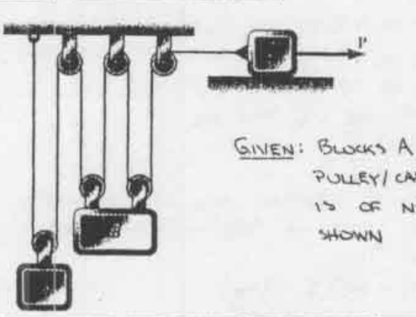
OR $v = \left[\frac{217}{6}W\left(1 - e^{-0.024\frac{g}{W}x}\right)\right]^{1/2}$

WHEN $x = 0.25$ mi $= 1320$ ft:

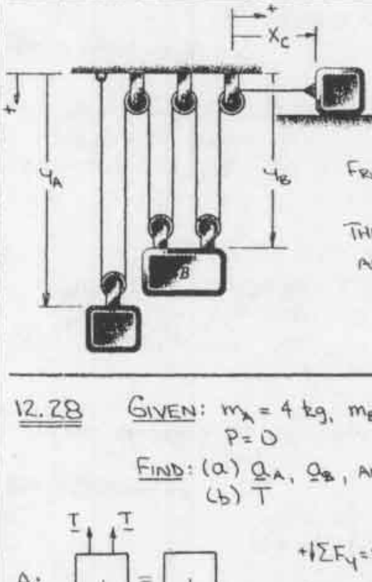
$v_{MAX} = \left[\frac{217}{6}(2700)\left(1 - e^{-0.024\frac{32.2}{2700}\times 1320}\right)\right]^{1/2} = 175.285\ \frac{ft}{s}$

OR $v_{MAX} = 119.5\ \frac{mi}{h}$ ◀

12.28 and 12.29



GIVEN: BLOCKS A, B, AND C AND THE PULLEY/CABLE SYSTEM, WHICH IS OF NEGLIGIBLE MASS, SHOWN

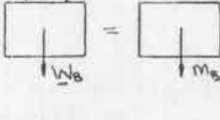


FROM THE DIAGRAM..
$2y_A + 4y_B + x_C = \text{CONSTANT}$
THEN.. $2v_A + 4v_B + v_C = 0$
AND $2a_A + 4a_B + a_C = 0$ (1)

12.28 GIVEN: $m_A = 4$ kg, $m_B = 10$ kg, $m_C = 2$ kg; $P = 0$

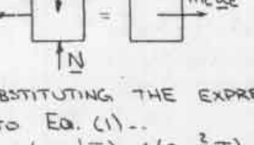
FIND: (a) $\underline{a}_A$, $\underline{a}_B$, AND $\underline{a}_C$
(b) T

A:



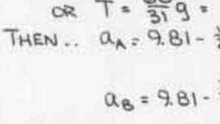
$+\downarrow \Sigma F_y = m_A a_A: \ W_A - 2T = m_A a_A$
OR $a_A = \frac{1}{m_A}(m_A g - 2T)$
$= g - \frac{2}{4}T$
$= g - \frac{1}{2}T$

B:



$+\downarrow \Sigma F_y = m_B a_B: \ W_B - 4T = m_B a_B$
OR $a_B = \frac{1}{m_B}(m_B g - 4T)$
$= g - \frac{4}{10}T$
$= g - \frac{2}{5}T$

C:



$\xrightarrow{+} \Sigma F_x = m_C a_C: \ -T = m_C a_C$
OR $a_C = -\frac{1}{2}T$

SUBSTITUTING THE EXPRESSIONS FOR a_A, a_B, AND a_C INTO EQ. (1)..

$2(g - \frac{1}{2}T) + 4(g - \frac{2}{5}T) + (-\frac{1}{2}T) = 0$

OR $T = \frac{60}{31}g = \frac{60}{31}(9.81) = 18.9871$ N

(a) THEN.. $a_A = 9.81 - \frac{1}{2}(18.9871)$

OR $\underline{a}_A = 0.316\ \frac{m}{s^2}\downarrow$ ◀

$a_B = 9.81 - \frac{2}{5}(18.9871)$

OR $\underline{a}_B = 2.22\ \frac{m}{s^2}\downarrow$ ◀

$a_C = -\frac{1}{2}(18.9871)$

OR $\underline{a}_C = 9.49\ \frac{m}{s^2}\leftarrow$ ◀

(b) HAVE.. $T = 18.99$ N ◀

(CONTINUED)

12.29 continued

12.29 GIVEN: $m_A = 8\text{ kg}$, $m_B = 16\text{ kg}$, $m_C = 10\text{ kg}$; $\mu_s = 0.30$, $\mu_k = 0.20$; AT $t=0$, $v=0$; AT $t = 0.8\text{ s}$, $\Delta y_B = 2\text{ m}\downarrow$

FIND: (a) $\underline{a}_A$, $\underline{a}_B$, AND $\underline{a}_C$
(b) T
(c) $\underline{P}$

(a) FIRST NOTE THAT BECAUSE ALL OF THE FORCES ARE CONSTANT, ALL OF THE ACCELERATIONS ARE CONSTANT. THEN..

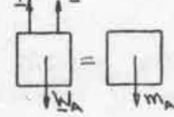
$$y_B = (y_B)_0 + (v_B)_0^{\,0} t + \tfrac{1}{2} a_B t^2$$

AT $t = 0.8\text{ s}$: $2\text{ m} = \frac{1}{2} a_B (0.8\text{ s})^2$

OR $a_B = 6.25 \frac{m}{s^2}$

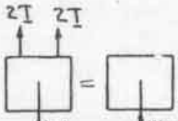
$\therefore \quad \underline{a}_B = 6.25 \frac{m}{s^2}\downarrow$ ◄

A:



$+\downarrow \Sigma F_y = m_A a_A$: $W_A - 2T = m_A a_A$
OR $m_A g - 2T = m_A a_A$
OR $8g - 2T = 8a_A$ (2)

B:



$+\downarrow \Sigma F_y = m_B a_B$: $W_B - 4T = m_B a_B$
OR $m_B g - 4T = m_B a_B$
OR $16g - 4T = 16a_B$ (3)

COMPARING EQS (2) AND (3), IT FOLLOWS THAT
$a_A = a_B$

$\therefore \quad \underline{a}_A = 6.25 \frac{m}{s^2}\downarrow$ ◄

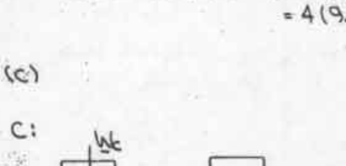
SUBSTITUTING INTO EQ. (1)..
$2(6.25) + 4(6.25) + a_C = 0$
$a_C = -37.5 \frac{m}{s^2}$

$\therefore \quad \underline{a}_C = 37.5 \frac{m}{s^2} \leftarrow$ ◄

(b) FROM EQ (2).. $T = 4(g - a_A)$
$= 4(9.81 - 6.25)$
OR $T = 14.24\text{ N}$ ◄

(c)

C:

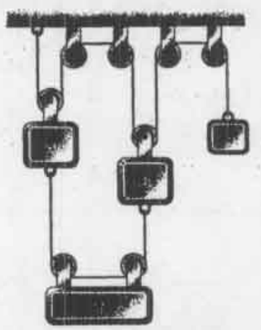


$+\downarrow \Sigma F_y = 0$: $W_C - N = 0$
OR $N = m_C g$

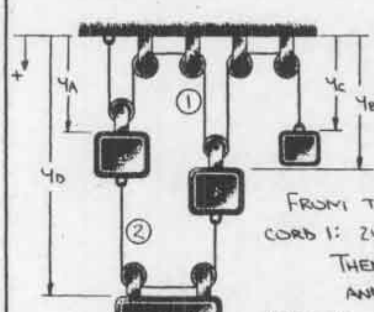
SLIDING: $F = \mu_k N$
$= 0.2 m_C g$

$\xrightarrow{+} \Sigma F_x = m_C a_C$: $P + F - T = m_C a_C$
OR $P = T + m_C(a_C - 0.2g)$
$= 14.24\text{ N} + (10\text{ kg})(-37.5 - 0.2 \times 9.81)\frac{m}{s^2}$
OR $\underline{P} = 380\text{ N} \leftarrow$ ◄

12.30 and 12.31



GIVEN: BLOCKS A, B, C, AND D AND THE PULLEY/CABLE SYSTEM, WHICH IS OF NEGLIGIBLE WEIGHT, SHOWN; $W_A = W_B = 20\text{ lb}$, $W_C = 14\text{ lb}$, $W_D = 16\text{ lb}$



NOTE: AS SHOWN, THE SYSTEM IS IN EQUILIBRIUM.

FROM THE DIAGRAM..

CORD 1: $2y_A + 2y_B + y_C = \text{CONSTANT}$
THEN.. $2v_A + 2v_B + v_C = 0$
AND $2a_A + 2a_B + a_C = 0$ (1)

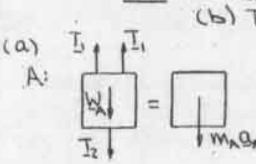
CORD 2: $(y_D - y_A) + (y_D - y_B) = \text{CONSTANT}$
THEN.. $2v_D - v_A - v_B = 0$
AND $2a_D - a_A - a_B = 0$ (2)

12.30 GIVEN: $(F_D)_{EXT} = 24\text{ lb}\downarrow$

FIND: (a) $\underline{a}_A$, $\underline{a}_B$, $\underline{a}_C$, AND $\underline{a}_D$
(b) T_1 ($= T_{ABC}$)

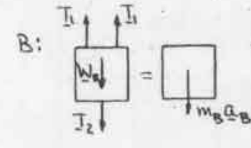
(a)

A:



$+\downarrow \Sigma F_y = m_A a_A$: $W_A - 2T_1 + T_2 = \frac{W_A}{g} a_A$
OR $20 - 2T_1 + T_2 = \frac{20}{g} a_A$ (3)

B:

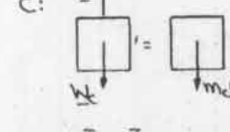


$+\downarrow \Sigma F_y = m_B a_B$: $W_B - 2T_1 + T_2 = \frac{W_B}{g} a_B$
OR $20 - 2T_1 + T_2 = \frac{20}{g} a_B$ (4)

NOTE: EQS. (3) AND (4) $\Rightarrow$
$\underline{a}_A = \underline{a}_B$

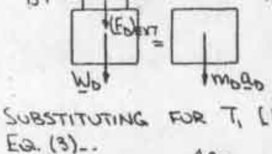
THEN.. EQ(1) $\Rightarrow a_C = -4a_A$ EQ(2) $\Rightarrow a_D = a_A$

C:



$+\downarrow \Sigma F_y = m_C a_C$: $W_C - T_1 = \frac{W_C}{g} a_C$
OR $T_1 = 14\left(1 - \frac{a_C}{g}\right)$
$= 14\left(1 + \frac{4a_A}{g}\right)$ (5)

D:



$+\downarrow \Sigma F_y = m_D a_D$: $W_D - 2T_2 + (F_D)_{EXT} = \frac{W_D}{g} a_D$
OR $T_2 = \frac{1}{2}\left[16\left(1 - \frac{a_D}{g}\right) + 24\right]$
$= 20 - 8\frac{a_A}{g}$

SUBSTITUTING FOR T_1 [EQ. (5)] AND T_2 [EQ. (6)] IN EQ. (3)..

$$20 - 2 \times 14\left(1 + \frac{4a_A}{g}\right) + \left(20 - 8\frac{a_A}{g}\right) = \frac{20}{g} a_A$$

OR $a_A = \frac{3}{35} g = \frac{3}{35} \times 32.2 \frac{ft}{s^2} = 2.76 \frac{ft}{s^2}$

(CONTINUED)

12.30 and 12.31 continued

$\therefore\ \underline{a}_A = \underline{a}_B = \underline{a}_D = 2.76\ \frac{ft}{s^2}\downarrow$ ◀

AND $a_C = -4(2.76\ \frac{ft}{s^2})$ OR $\underline{a}_C = 11.04\ \frac{ft}{s^2}\uparrow$ ◀

(b) SUBSTITUTING INTO EQ. (5)..

$T_1 = 14\left(1+\frac{4\times 2.76}{32.2}\right)$

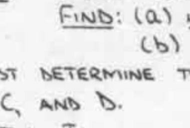
OR $T_1 = 18.80$ lb ◀

12.31 GIVEN: $(\underline{F}_B)_{EXT} = 10$ lb$\downarrow$; AT $t=0$, $v=0$

FIND: (a) $v_{D/A}$ AT $t=3$ s

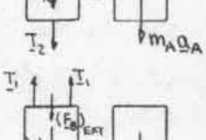
(b) $v_{C/D}$ AT $t=3$ s

FIRST DETERMINE THE ACCELERATIONS OF BLOCKS A, C, AND D.

A: 

$+\downarrow\Sigma F_y = m_A a_A$: $W_A - 2T_1 + T_2 = \frac{W_A}{g}a_A$

OR $20 - 2T_1 + T_2 = \frac{20}{g}a_A$ (3)

B: 

$+\downarrow\Sigma F_y = m_B a_B$: $W_B - 2T_1 + T_2 + (F_B)_{EXT} = \frac{W_B}{g}a_B$

OR $20 - 2T_1 + T_2 + 10 = \frac{20}{g}a_B$ (4)

FORMING (3)−(4) $\Rightarrow -10 = \frac{20}{g}(a_A - a_B)$

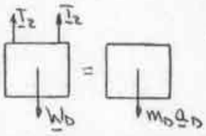
OR $a_B = a_A + \frac{1}{2}g$

THEN.. EQ. (1): $2a_A + 2(a_A + \frac{1}{2}g) + a_C = 0$

OR $a_C = -4a_A - g$

EQ. (2): $2a_D - a_A - (a_A + \frac{1}{2}g) = 0$

OR $a_D = a_A + \frac{1}{4}g$

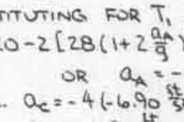
C: 

$+\downarrow\Sigma F_y = m_C a_C$: $W_C - T_1 = \frac{W_C}{g}a_C$

OR $T_1 = 14(1 - \frac{a_C}{g})$

$= 14[1 - \frac{1}{g}(-4a_A - g)]$

$= 28(1 + 2\frac{a_A}{g})$ (5)

D: 

$+\downarrow\Sigma F_y = m_D a_D$: $W_D - 2T_2 = \frac{W_D}{g}a_D$

OR $T_2 = \frac{1}{2}\times 16(1 - \frac{a_D}{g})$

$= 8[1 - \frac{1}{g}(a_A + \frac{1}{4}g)]$

$= 8(\frac{3}{4} - \frac{a_A}{g})$ (6)

SUBSTITUTING FOR T_1 [EQ. (5)] AND T_2 [EQ. (6)] IN EQ. (3)..

$20 - 2[28(1 + 2\frac{a_A}{g})] + 8(\frac{3}{4} - \frac{a_A}{g}) = \frac{20}{g}a_A$

OR $a_A = -\frac{3}{14}g = -\frac{3}{14}(32.2\ \frac{ft}{s^2}) = -6.90\ \frac{ft}{s^2}$

THEN.. $a_C = -4(-6.90\ \frac{ft}{s^2}) - 32.2\ \frac{ft}{s^2} = -4.60\ \frac{ft}{s^2}$

$a_D = -6.90\ \frac{ft}{s^2} + \frac{1}{4}(32.2\ \frac{ft}{s^2}) = 1.15\ \frac{ft}{s^2}$

NOTE: HAVE UNIFORMLY ACCELERATED MOTION, SO THAT

$v = \cancel{v_0}^{\,0} + at$

(a) HAVE.. $v_{D/A} = v_D - v_A$

OR $v_{D/A} = a_D t - a_A t$

$= [1.15 - (-6.90)]\ \frac{ft}{s^2}\times 3$ s

OR $v_{D/A} = 24.2\ \frac{ft}{s}\downarrow$ ◀

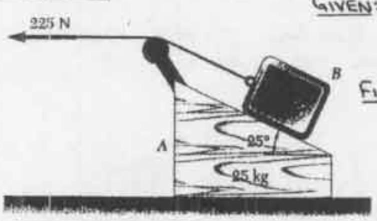
(b) HAVE.. $v_{C/D} = v_C - v_D$

OR $v_{C/D} = a_C t - a_D t$

$= (-4.60 - 1.15)\ \frac{ft}{s^2}\times 3$ s

OR $v_{C/D} = 17.25\ \frac{ft}{s}\uparrow$ ◀

12.32

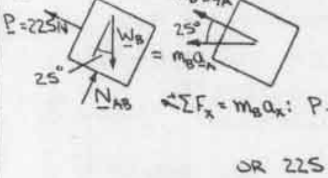


GIVEN: BLOCKS A AND B AND THE 225 N FORCE SHOWN

FIND: (a) $\underline{a}_A$

(b) $\underline{a}_{B/A}$

(a) FIRST NOTE.. $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$ WHERE $\underline{a}_{B/A}$ IS DIRECTED ALONG THE INCLINED SURFACE OF A.

B: 

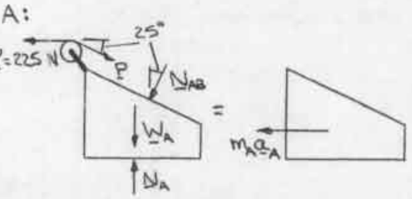
$\overset{+}{\nwarrow}\Sigma F_x = m_B a_x$: $P - W_B\sin 25° = m_B a_A\cos 25° + m_B a_{B/A}$

OR $225 - 15g\sin 25° = 15(a_A\cos 25° + a_{B/A})$

OR $15 - g\sin 25° = a_A\cos 25° + a_{B/A}$ (1)

$+\nearrow\Sigma F_y = m_B a_y$: $N_{AB} - W_B\cos 25° = -m_B a_A\sin 25°$

OR $N_{AB} = 15(g\cos 25° - a_A\sin 25°)$

A: 

$\overset{+}{\leftarrow}\Sigma F_x = m_A a_A$: $P - P\cos 25° + N_{AB}\sin 25° = m_A a_A$

OR $N_{AB} = [25a_A - 225(1 - \cos 25°)]/\sin 25°$

EQUATING THE TWO EXPRESSIONS FOR N_{AB}..

$15(g\cos 25° - a_A\sin 25°) = \frac{25a_A - 225(1-\cos 25°)}{\sin 25°}$

OR $a_A = \frac{3(9.81)\cos 25°\sin 25° + 45(1-\cos 25°)}{5 + 3\sin^2 25°}$

$= 2.7979\ \frac{m}{s^2}$

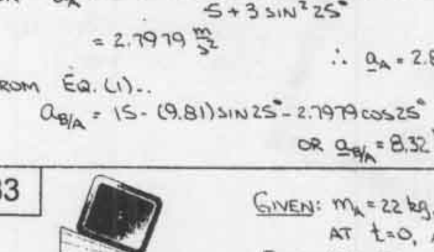
$\therefore\ \underline{a}_A = 2.80\ \frac{m}{s^2}\leftarrow$ ◀

(b) FROM EQ. (1)..

$a_{B/A} = 15 - (9.81)\sin 25° - 2.7979\cos 25°$

OR $\underline{a}_{B/A} = 8.32\ \frac{m}{s^2}\ \measuredangle 25°$ ◀

12.33

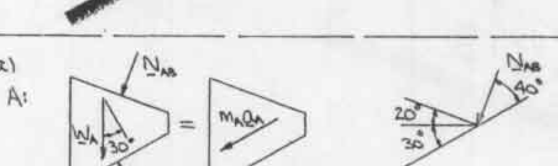


GIVEN: $m_A = 22$ kg, $m_B = 10$ kg; AT $t=0$, $v=0$

FIND: (a) $\underline{a}_B$

(b) $v_{B/A}$ AT $t = 0.5$ s

(a)

A: 

$\overset{+}{\swarrow}\Sigma F_x = m_A a_A$: $W_A\sin 30° + N_{AB}\cos 40° = m_A a_A$

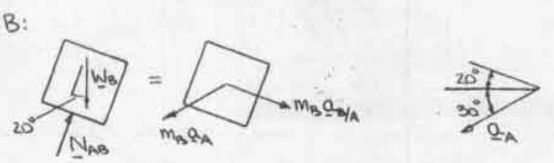
OR $N_{AB} = \frac{22(a_A - \frac{1}{2}g)}{\cos 40°}$

(CONTINUED)

12.33 continued

NOW NOTE: $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$ WHERE $\underline{a}_{B/A}$ IS DIRECTED ALONG THE TOP SURFACE OF A.

B:



$+\nearrow \Sigma F_{y'} = m_B a_{y'}$: $N_{AB} - W_B \cos 20° = -m_B a_A \sin 50°$

OR $N_{AB} = 10(g\cos 20° - a_A \sin 50°)$

EQUATING THE TWO EXPRESSIONS FOR N_{AB}..

$$\frac{22(a_A - \frac{1}{2}g)}{\cos 40°} = 10(g\cos 20° - a_A \sin 50°)$$

OR $a_A = \dfrac{(9.81)(1.1 + \cos 20° \cos 40°)}{2.2 + \cos 40° \sin 50°} = 6.4061\ \frac{m}{s^2}$

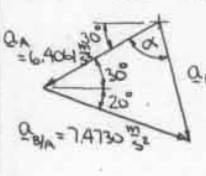
$\stackrel{+}{\searrow} \Sigma F_{x'} = m_B a_{x'}$: $W_B \sin 20° = m_B a_{B/A} - m_B a_A \cos 50°$

OR $a_{B/A} = g\sin 20° + a_A \cos 50°$

$= (9.81 \sin 20° + 6.4061 \cos 50°)\ \frac{m}{s^2}$

$= 7.4730\ \frac{m}{s^2}$

FINALLY.. $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$



HAVE.. $a_B^2 = 6.4061^2 + 7.4730^2 - 2(6.4061)(7.4730)\cos 50°$

OR $a_B = 5.9447\ \frac{m}{s^2}$

AND $\dfrac{7.4730}{\sin \alpha} = \dfrac{5.9447}{\sin 50°}$

OR $\alpha = 74.4°$

$\therefore\ \underline{a}_B = 5.94\ \frac{m}{s^2} \measuredangle 75.6°$ ◀

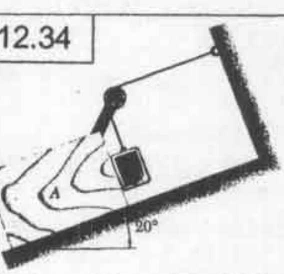
(b) NOTE: HAVE UNIFORMLY ACCELERATED MOTION, SO THAT $v = \cancel{v_0} + at$

NOW.. $v_{B/A} = v_B - v_A = \underline{a}_B t - \underline{a}_A t = \underline{a}_{B/A} t$

AT $t = 0.5$ s: $v_{B/A} = 7.4730\ \frac{m}{s^2} \times 0.5$ s

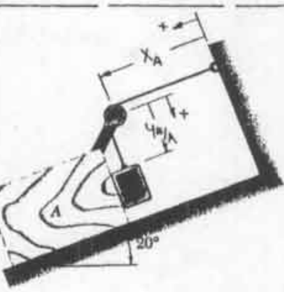
OR $\underline{v}_{B/A} = 3.74\ \frac{m}{s} \measuredangle 20°$ ◀

12.34



GIVEN: $W_A = 50$ lb, $W_B = 30$ lb

FIND: a_A AND T



FROM THE DIAGRAM..

$x_A + y_{B/A} = $ CONSTANT

THEN.. $v_A + v_{B/A} = 0$

AND $a_A + a_{B/A} = 0$

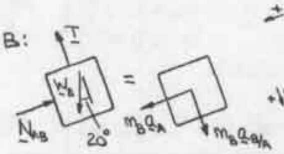
OR $a_{B/A} = -a_A$ (1)

FIRST NOTE: $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$ WHERE $\underline{a}_{B/A}$ IS DIRECTED ALONG THE SIDE OF A

(CONTINUED)

12.34 continued

B:



$\stackrel{+}{\nearrow} \Sigma F_x = m_B a_x$: $W_B \sin 20° - N_{AB} = m_B a_A$

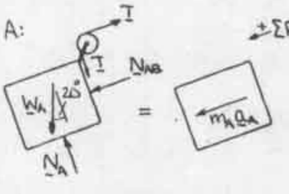
OR $N_{AB} = W_B(\sin 20° - \frac{a_A}{g})$

$+\nwarrow \Sigma F_y = m_B a_y$: $W_B \cos 20° - T = m_B a_{B/A}$

USING EQ. (1)..

$T = W_B(\cos 20° + \frac{a_A}{g})$

A:



$\stackrel{+}{\nearrow} \Sigma F_x = m_A a_A$: $W_A \sin 20° + N_{AB} - T = \frac{W_A}{g} a_A$ (2)

NOW SUBSTITUTE THE EXPRESSIONS FOR N_{AB} AND T INTO EQ. (2)..

$50 \sin 20° + 30(\sin 20° - \frac{a_A}{g}) - 30(\cos 20° + \frac{a_A}{g}) = 50\frac{a_A}{g}$

OR $a_A = \frac{1}{11}(32.2\ \frac{ft}{s^2})(8\sin 20° - 3\cos 20°)$

$= -0.24272\ \frac{ft}{s^2}$

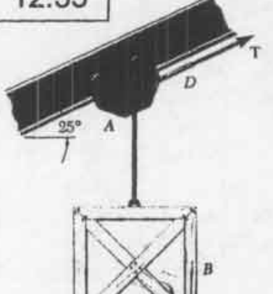
$\therefore\ \underline{a}_A = 0.243\ \frac{ft}{s^2} \measuredangle 20°$ ◀

USING THE ABOVE EXPRESSION FOR T..

$T = (30\text{ lb})(\cos 20° + \dfrac{-0.24272\ ft/s^2}{32.2\ ft/s^2})$

OR $T = 28.0$ lb

12.35



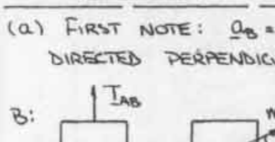
GIVEN: $W_A = 40$ lb, $W_B = 500$ lb; $\underline{a}_A = 1.2\ \frac{ft}{s^2} \nearrow$

FIND: (a) $\underline{a}_{B/A}$

(b) T_{CD}

(a) FIRST NOTE: $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$ WHERE $\underline{a}_{B/A}$ IS DIRECTED PERPENDICULAR TO CABLE AB

B:



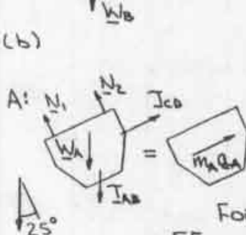
$\stackrel{+}{\rightarrow} \Sigma F_x = m_B a_x$: $0 = -m_B a_{B/A} + m_B a_A \cos 25°$

OR $a_{B/A} = (1.2\ ft/s^2)\cos 25°$

OR $\underline{a}_{B/A} = 1.088\ \frac{ft}{s^2} \leftarrow$ ◀

(b)

A:



FOR CRATE B..

$+\uparrow \Sigma F_y = m_B a_y$: $T_{AB} - W_B = \frac{W_B}{g} a_A \sin 25°$

OR $T_{AB} = (500\text{ lb})\left[1 + \dfrac{(1.2\ ft/s^2)\sin 25°}{32.2\ ft/s^2}\right]$

$= 507.87$ lb

FOR TROLLEY A..

$\stackrel{+}{\nearrow} \Sigma F_{x'} = m_A a_A$: $T_{CD} - T_{AB}\sin 25° - W_A \sin 25° = \frac{W_A}{g} a_A$

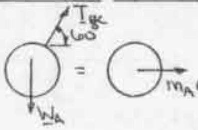
OR $T_{CD} = (507.87\text{ lb})\sin 25° + (40\text{ lb})(\sin 25° + \frac{1.2\ ft/s^2}{32.2\ ft/s^2})$

OR $T_{CD} = 233$ lb ◀

12.36

GIVEN: $m_A = 7.1$ kg; $v_A =$ CONSTANT; $\rho = 0.93$ m, $\theta = 60°$

FIND: (a) T_{BC}
(b) v_A



FIRST NOTE.. $a_A = a_n = \frac{v_A^2}{\rho}$

(a) $+\uparrow \Sigma F_y = 0$: $T_{BC} \sin 60° - W_A = 0$

OR $T_{BC} = \frac{7.1 \text{ kg} \times 9.81 \text{ m/s}^2}{\sin 60°}$

$= 80.426$ N

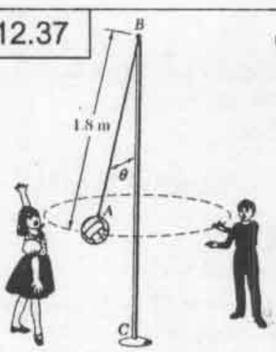
$\therefore T_{BC} = 80.4$ N ◀

(b) $\xrightarrow{+} \Sigma F_x = m_A a_A$: $T_{BC} \cos 60° = m_A \frac{v_A^2}{\rho}$

OR $v_A^2 = \frac{(80.426 \text{ N}) \cos 60° \times 0.93 \text{ m}}{7.1 \text{ kg}}$

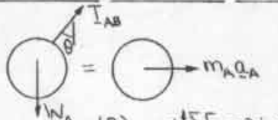
OR $v_A = 2.30 \frac{\text{m}}{\text{s}}$ ◀

12.37



GIVEN: $m_A = 0.450$ kg; $v_A = 4 \frac{\text{m}}{\text{s}}$

FIND: (a) θ
(b) T_{AB}



FIRST NOTE.. $a_A = a_n = \frac{v_A^2}{\rho}$
WHERE $\rho = l_{AB} \sin\theta$

(a) $+\uparrow \Sigma F_y = 0$: $T_{AB} \cos\theta - W_A = 0$

OR $T_{AB} = \frac{m_A g}{\cos\theta}$

$\xrightarrow{+} \Sigma F_x = m_A a_A$: $T_{AB} \sin\theta = m_A \frac{v_A^2}{\rho}$

SUBSTITUTING FOR T_{AB} AND ρ--

$\frac{m_A g}{\cos\theta} \sin\theta = m_A \frac{v_A^2}{l_{AB} \sin\theta}$ $\qquad \sin^2\theta = 1 - \cos^2\theta$

$1 - \cos^2\theta = \frac{(4 \text{ m/s})^2}{1.8 \text{ m} \times 9.81 \frac{\text{m}}{\text{s}^2}} \cos\theta$

OR $\cos^2\theta + 0.906105 \cos\theta - 1 = 0$

SOLVING-- $\cos\theta = 0.64479$

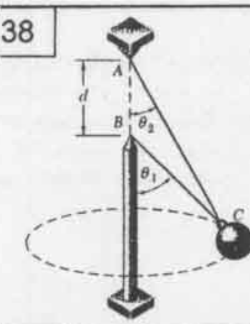
OR $\theta = 49.9°$ ◀

(b) FROM ABOVE.. $T_{AB} = \frac{m_A g}{\cos\theta}$

$= \frac{0.450 \text{ kg} \times 9.81 \frac{\text{m}}{\text{s}^2}}{0.64479}$

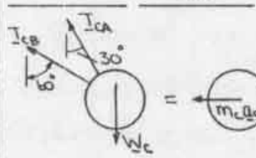
OR $T_{AB} = 6.85$ N ◀

12.38



GIVEN: $L_{ACB} = 80$ IN.;
$v_C =$ CONSTANT;
$\theta_1 = 60°$, $\theta_2 = 30°$;
$T_{CA} = T_{CB} = T$

FIND: v_C



FIRST NOTE.. $a_C = a_n = \frac{v_C^2}{\rho}$
WHERE $\rho = L_{AC} \sin 30° = L_{BC} \sin 60°$

NOW.. $L_{AC} + L_{BC} = L_{ABC}$

OR $\rho \left(\frac{1}{\sin 30°} + \frac{1}{\sin 60°} \right) = 80$ IN.

OR $\rho = 25.359$ IN.

$+\uparrow \Sigma F_y = 0$: $\qquad T_{CA} \cos 30° + T_{CB} \cos 60° - W_C = 0$

OR $T = \frac{m_C g}{\cos 30° + \cos 60°} = 0.73205\, m_C g$

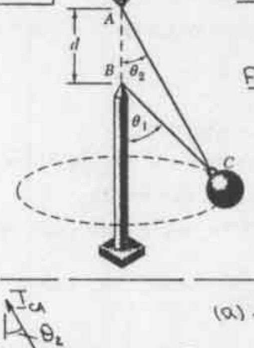
$\xleftarrow{+} \Sigma F_x = m_C a_C$: $\quad T_{CA} \sin 30° + T_{CB} \sin 60° = m_C \frac{v_C^2}{\rho}$

OR $0.73205\, m_C g (\sin 30° + \sin 60°) = m_C \frac{v_C^2}{\rho}$

OR $v_C^2 = 0.73205 (32.2 \frac{\text{ft}}{\text{s}^2}) (\frac{25.359}{12} \text{ ft})$
$\times (\sin 30° + \sin 60°)$

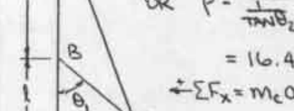
OR $v_C = 8.25 \frac{\text{ft}}{\text{s}}$ ◀

12.39



GIVEN: $W_C = 12$ lb; $v_C =$ CONSTANT;
$T_{CA} = T_{CB} = 7.6$ lb;
$\theta_1 = 50°$, $d = 30$ IN.

FIND: (a) θ_2
(b) v_C



(a) $+\uparrow \Sigma F_y = 0$: $T_{CA} \cos\theta_2 + T_{CB} \cos 50°$
$- W_C = 0$

OR $(7.6 \text{ lb})(\cos\theta_2 + \cos 50°) = 12$ lb

OR $\theta_2 = 20.584°$

$\therefore \theta_2 = 20.6°$ ◀

(b) FIRST NOTE.. $a_C = a_n = \frac{v_C^2}{\rho}$

WHERE $\rho = l \tan\theta_1$ AND $\rho = (d + l) \tan\theta_2$

THEN $\rho = (d + \frac{\rho}{\tan\theta_1}) \tan\theta_2$

OR $\rho = \frac{d}{\frac{1}{\tan\theta_2} - \frac{1}{\tan\theta_1}} = \frac{30 \text{ IN.}}{\frac{1}{\tan 20.584°} - \frac{1}{\tan 50°}}$

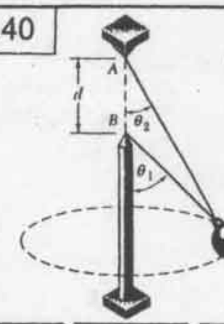
$= 16.4508$ IN.

$\xleftarrow{+} \Sigma F_x = m_C a_C$: $T_{CA} \sin\theta_2 + T_{CB} \sin\theta_1 = \frac{W_C}{g} \frac{v_C^2}{\rho}$

OR $v_C^2 = \frac{7.6 \text{ lb}}{12 \text{ lb}} \times (32.2 \frac{\text{ft}}{\text{s}^2}) (\frac{16.4508}{12} \text{ ft})$
$\times (\sin 20.584° + \sin 50°)$

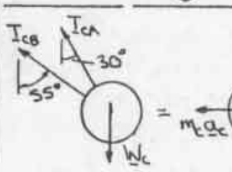
OR $v_C = 5.59 \frac{\text{ft}}{\text{s}}$ ◀

12.40



GIVEN: $m_C = 7$ kg; $\theta_1 = 55°$, $\theta_2 = 30°$, $d = 1.4$ m; v_C = CONSTANT

FIND: RANGE OF VALUES OF v_C SO THAT WIRES AC AND BC BOTH REMAIN TAUT



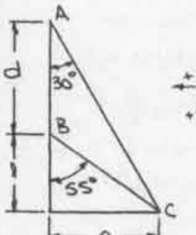
FIRST NOTE.. $a_C = a_n = \frac{v_C^2}{\rho}$

WHERE

$\rho = l \tan 55°$ AND $\rho = (d+l)\tan 30°$

THEN $\rho = \left(d + \frac{\rho}{\tan 55°}\right)\tan 30°$

OR $\rho = \dfrac{1.4 \text{ m}}{\frac{1}{\tan 30°} - \frac{1}{\tan 55°}} = 1.356\,80$ m



$\xrightarrow{+} \Sigma F_x = m_C a_C$: $T_{CA}\sin 30° + T_{CB}\sin 55° = m_C \frac{v_C^2}{\rho}$ (1)

$+\uparrow \Sigma F_y = 0$: $T_{CA}\cos 30° + T_{CB}\cos 55° - W_C = 0$

OR $T_{CA}\cos 30° + T_{CB}\cos 55° = m_C g$ (2)

CASE 1: $T_{CA} \to 0$: EQ. (2) $\Rightarrow T_{CB} = \frac{m_C g}{\cos 55°}$

SUBSTITUTING INTO EQ. (1).. $\frac{m_C g}{\cos 55°}\sin 55° = m_C \frac{v_C^2}{\rho}$

OR $(v_C^2)_{T_{CA}=0} = (1.356\,80 \text{ m})(9.81 \text{ m})\tan 55°$

OR $(v_C)_{T_{CA}=0} = 4.36 \frac{\text{m}}{\text{s}}$

NOW FORM $(\cos 30°)(1) - (\sin 30°)(2)$..

$T_{CB}\sin 55°\cos 30° - T_{CB}\cos 55°\sin 30° = m_C \frac{v_C^2}{\rho}\cos 30° - m_C g \sin 30°$

OR $T_{CB}\sin 25° = m_C \frac{v_C^2}{\rho}\cos 30° - m_C g \sin 30°$

$\therefore (v_C)_{MAX}$ OCCURS WHEN $T_{CB} = (T_{CB})_{MAX}$, WHICH OCCURS WHEN $T_{CA} = 0$.

$\therefore (v_C)_{MAX} = 4.36 \frac{\text{m}}{\text{s}}$ AND WIRE AC WILL BE TAUT IF $v_C < 4.36 \frac{\text{m}}{\text{s}}$.

CASE 2: $T_{CB} \to 0$: EQ. (2) $\Rightarrow T_{CA} = \frac{m_C g}{\cos 30°}$

SUBSTITUTING INTO EQ. (1).. $\frac{m_C g}{\cos 30°}\sin 30° = m_C \frac{v_C^2}{\rho}$

OR $(v_C^2)_{T_{CB}=0} = (1.356\,80 \text{ m})(9.81 \frac{\text{m}}{\text{s}^2})\tan 30°$

OR $(v_C)_{T_{CB}=0} = 2.77 \frac{\text{m}}{\text{s}}$

NOW FORM $(\cos 55°)(1) - (\sin 55°)(2)$..

$T_{CA}\sin 30°\cos 55° - T_{CA}\cos 30°\sin 55° = m_C \frac{v_C^2}{\rho}\cos 55° - m_C g \sin 55°$

OR $-T_{CA}\sin 25° = m_C \frac{v_C^2}{\rho}\cos 55° - m_C g \sin 55°$

$\therefore (v_C)_{MIN}$ OCCURS WHEN $T_{CA} = (T_{CA})_{MAX}$, WHICH OCCURS WHEN $T_{CB} = 0$.

$\therefore (v_C)_{MIN} = 2.77 \frac{\text{m}}{\text{s}}$ AND WIRE BC WILL BE TAUT IF $v_C > 2.77 \frac{\text{m}}{\text{s}}$.

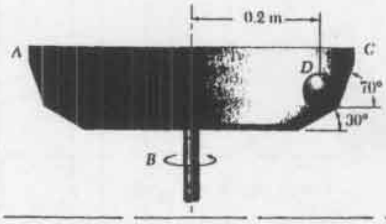
$\therefore$ BOTH WIRES ARE TAUT WHEN

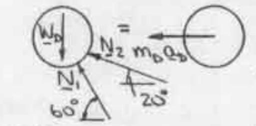
$$2.77 \frac{\text{m}}{\text{s}} < v_C < 4.36 \frac{\text{m}}{\text{s}}$$

12.41

GIVEN: $m_D = 0.1$ kg; v_D = CONSTANT

FIND: RANGE OF VALUES OF v_D SO THAT NEITHER OF THE NORMAL FORCES EXCEEDS 1.1 N





FIRST NOTE.. $a_D = a_n = \frac{v_D^2}{\rho}$ WHERE $\rho = 0.2$ m

$\xrightarrow{+} \Sigma F_x = m_D a_D$: $N_1\cos 60° + N_2 \cos 20° = m_D \frac{v_D^2}{\rho}$ (1)

$+\uparrow \Sigma F_y = 0$: $N_1 \sin 60° + N_2 \sin 20° - W_D = 0$

OR $N_1 \sin 60° + N_2 \sin 20° = m_D g$ (2)

CASE 1: N_1 IS MAXIMUM

LET $N_1 = 1.1$ N

EQ. (2).. $(1.1 \text{ N})\sin 60° + N_2 \sin 20° = (0.1 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})$

OR $N_2 = 0.082\,954$ N

$\therefore (N_2)_{(N_1)_{MAX}} < 1.1$ N.. O.K.

EQ. (1)..

$(v_D^2)_{(N_1)_{MAX}} = \frac{0.2 \text{ m}}{0.1 \text{ kg}}(1.1\cos 60° + 0.082\,954\cos 20°)$ N

OR $(v_D)_{(N_1)_{MAX}} = 1.121 \frac{\text{m}}{\text{s}}$

NOW FORM $(\sin 20°)(1) - (\cos 20°)(2)$...

$N_1\cos 60°\sin 20° - N_1 \sin 60°\cos 20° = m_D \frac{v_D^2}{\rho}\sin 20° - m_D g \cos 20°$

OR $-N_1 \sin 40° = m_D \frac{v_D^2}{\rho}\sin 20° - m_D g\cos 20°$

$\therefore (v_D)_{MIN}$ OCCURS WHEN $N_1 = (N_1)_{MAX}$

$\therefore (v_D)_{MIN} = 1.121 \frac{\text{m}}{\text{s}}$

CASE 2: N_2 IS MAXIMUM

LET $N_2 = 1.1$ N

EQ. (2).. $N_1 \sin 60° + (1.1 \text{ N})\sin 20° = (0.1 \text{ kg})(9.81 \frac{\text{m}}{\text{s}^2})$

OR $N_1 = 0.698\,34$ N

$\therefore (N_1)_{(N_2)_{MAX}} < 1.1$ N.. O.K.

EQ. (1)..

$(v_D^2)_{(N_2)_{MAX}} = \frac{0.2 \text{ m}}{0.1 \text{ kg}}(0.698\,34\cos 60° + 1.1\cos 20°)$ N

OR $(v_D)_{(N_2)_{MAX}} = 1.663 \frac{\text{m}}{\text{s}}$

NOW FORM $(\sin 60°)(1) - (\cos 60°)(2)$..

$N_2 \cos 20° \sin 60° - N_2 \sin 20°\cos 60° = m_D \frac{v_D^2}{\rho}\sin 60° - m_D g \cos 60°$

OR $N_2 \cos 40° = m_D \frac{v_D^2}{\rho}\sin 60° - m_D g\cos 60°$

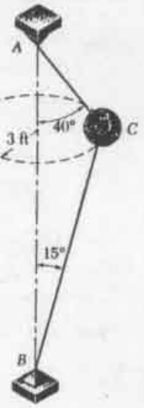
$\therefore (v_D)_{MAX}$ OCCURS WHEN $N_2 = (N_2)_{MAX}$

$\therefore (v_D)_{MAX} = 1.663 \frac{\text{m}}{\text{s}}$

$\therefore$ FOR $N_1, N_2 \leq 1.1$ N

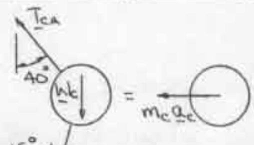
$$1.121 \frac{\text{m}}{\text{s}} \leq v_D \leq 1.663 \frac{\text{m}}{\text{s}}$$

* 12.42



GIVEN: $W_C = 12$ lb; v_C = CONSTANT; $0 < T_{CA}, T_{CB} \le 26$ lb

FIND: RANGE OF VALUES OF v_C



FIRST NOTE.. $a_C = a_n = \frac{v_C^2}{\rho}$ WHERE $\rho = 3$ ft

$\overset{+}{\leftarrow}\Sigma F_x = m_C a_C$: $T_{CA}\sin 40° + T_{CB}\sin 15° = \frac{W_C}{g}\frac{v_C^2}{\rho}$ (1)

$+\uparrow\Sigma F_y = 0$: $T_{CA}\cos 40° - T_{CB}\cos 15° - W_C = 0$ (2)

NOTE THAT EQ. (2) IMPLIES THAT

(a) WHEN $T_{CB} = (T_{CB})_{MAX}$, $T_{CA} = (T_{CA})_{MAX}$

(b) WHEN $T_{CB} = (T_{CB})_{MIN}$, $T_{CA} = (T_{CA})_{MIN}$

CASE 1: T_{CA} IS MAXIMUM

LET $T_{CA} = 26$ lb

EQ. (2).. $(26\text{ lb})\cos 40° - T_{CB}\cos 15° - (12\text{ lb}) = 0$

OR $T_{CB} = 8.1964$ lb

$\therefore (T_{CB})_{(T_{CA})_{max}} < 26$ lb .. OK $[(T_{CB})_{MAX} = 8.1964\text{ lb}]$

EQ. (1)..

$$(v_C^2)_{(T_{CA})_{max}} = \frac{(32.2\frac{ft}{s^2})(3\text{ ft})}{12\text{ lb}}(26\sin 40° + 8.1964\sin 15°)\text{ lb}$$

OR $(v_C)_{(T_{CA})_{max}} = 12.31\ \frac{ft}{s}$

NOW FORM $(\cos 15°)(1) + (\sin 15°)(2)$...

$T_{CA}\sin 40°\cos 15° + T_{CA}\cos 40°\sin 15° = \frac{W_C}{g}\frac{v_C^2}{\rho}\cos 15° + W_C\sin 15°$

OR $T_{CA}\sin 55° = \frac{W_C}{g}\frac{v_C^2}{\rho}\cos 15° + W_C\sin 15°$ (3)

$\therefore (v_C)_{MAX}$ OCCURS WHEN $T_{CA} = (T_{CA})_{MAX}$

$\therefore (v_C)_{MAX} = 12.31\ \frac{ft}{s}$

CASE 2: T_{CA} IS MINIMUM

BECAUSE $(T_{CA})_{MIN}$ OCCURS WHEN $T_{CB} = (T_{CB})_{MIN}$, LET $T_{CB} = 0$ (NOTE THAT WIRE BC WILL NOT BE TAUT).

EQ. (2).. $T_{CA}\cos 40° - (12\text{ lb}) = 0$

OR $T_{CA} = 15.6649$ lb < 26 lb.. OK

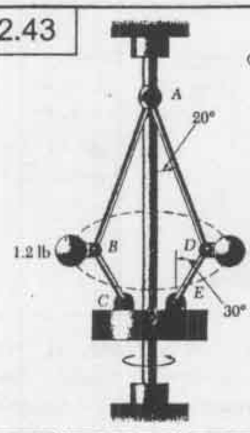
NOTE: EQ (3) IMPLIES THAT WHEN $T_{CA} = (T_{CA})_{MIN}$, $v_C = (v_C)_{MIN}$. THEN..

$$\text{EQ. (1)}..\ (v_C^2)_{MIN} = \frac{(32.2\frac{ft}{s^2})(3\text{ ft})}{12\text{ lb}}(15.6649\text{ lb})\sin 40°$$

OR $(v_C)_{MIN} = 9.00\ \frac{ft}{s}$

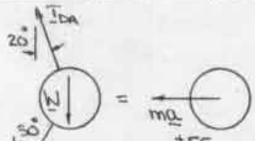
$\therefore 0 < T_{CA}, T_{CB} \le 26$ lb WHEN $9.00\frac{ft}{s} < v_C \le 12.31\frac{ft}{s}$ ◀

* 12.43



GIVEN: $v_{FLYBALL} = v$ = CONSTANT; $\rho = 6$ in.; W_{AB}, W_{BC}, W_{AD}, AND W_{DE} NEGLIGIBLE; $0 \le T_{AB}, T_{BC}, T_{AD}, T_{DE} \le 17$ lb

FIND: RANGE OF VALUES OF v



FIRST NOTE .. $a = a_n = \frac{v^2}{\rho}$ WHERE $\rho = 0.5$ ft

$\overset{+}{\leftarrow}\Sigma F_x = ma$: $T_{DA}\sin 20° + T_{DE}\sin 30° = \frac{W}{g}\frac{v^2}{\rho}$ (1)

$+\uparrow\Sigma F_y = 0$: $T_{DA}\cos 20° - T_{DE}\cos 30° - W = 0$ (2)

NOTE THAT EQ. (2) IMPLIES THAT

(a) WHEN $T_{DE} = (T_{DE})_{MAX}$, $T_{DA} = (T_{DA})_{MAX}$

(b) WHEN $T_{DE} = (T_{DE})_{MIN}$, $T_{DA} = (T_{DA})_{MIN}$

CASE 1: T_{DA} IS MAXIMUM

LET $T_{DA} = 17$ lb

EQ (2).. $(17\text{ lb})\cos 20° - T_{DE}\cos 30° - (1.2\text{ lb}) = 0$

OR $T_{DE} = 17.06$ lb .. UNACCEPTABLE (> 17 lb)

NOW LET $T_{DE} = 17$ lb

EQ (2).. $T_{DA}\cos 20° - (17\text{ lb})\cos 30° - (1.2\text{ lb}) = 0$

OR $T_{DA} = 16.9443$ lb .. OK (< 17 lb)

$\therefore (T_{DA})_{MAX} = 16.9443$ lb $(T_{DE})_{MAX} = 17$ lb

EQ. (1)..

$$(v^2)_{(T_{DA})_{max}} = \frac{(32.2\frac{ft}{s^2})(0.5\text{ ft})}{1.2\text{ lb}}(16.9443\sin 20° + 17\sin 30°)\text{ lb}$$

OR $v_{(T_{DA})_{max}} = 13.85\ \frac{ft}{s}$

NOW FORM $(\cos 30°)(1) + (\sin 30°)(2)$..

$T_{DA}\sin 20°\cos 30° + T_{DA}\cos 20°\sin 30° = \frac{W}{g}\frac{v^2}{\rho}\cos 30° + W\sin 30°$

OR $T_{DA}\sin 50° = \frac{W}{g}\frac{v^2}{\rho}\cos 30° + W\sin 30°$ (3)

$\therefore v_{MAX}$ OCCURS WHEN $T_{DA} = (T_{DA})_{MAX}$

$\therefore v_{MAX} = 13.85\ \frac{ft}{s}$

CASE 2: T_{DA} IS MINIMUM

BECAUSE $(T_{DA})_{MIN}$ OCCURS WHEN $T_{DE} = (T_{DE})_{MIN}$, LET $T_{DE} = 0$.

EQ. (2).. $T_{DA}\cos 20° - (1.2\text{ lb}) = 0$

OR $T_{DA} = 1.27701$ lb < 17 lb.. OK

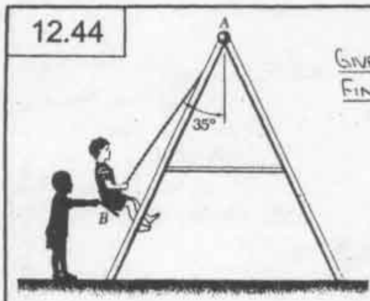
NOTE: EQ. (3) IMPLIES THAT WHEN $T_{DA} = (T_{DA})_{MIN}$, $v = v_{MIN}$. THEN..

$$\text{EQ. (1)}\ (v^2)_{MIN} = \frac{(32.2\frac{ft}{s^2})(0.5\text{ ft})}{1.2\text{ lb}}(1.27701\text{ lb})\sin 20°$$

OR $v_{MIN} = 2.42\ \frac{ft}{s}$

$\therefore 0 \le T_{AB}, T_{BC}, T_{AD}, T_{DE} \le 17$ lb WHEN $2.42\frac{ft}{s} \le v \le 13.85\frac{ft}{s}$ ◀

12.44



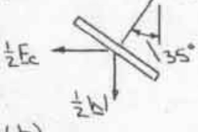
GIVEN: $m = 22$ kg

FIND: (a) T_{BA} WHEN $F_c = F_c \leftarrow$

(b) T_{BA} AT $t = 0$ WHEN $F_c = 0$

NOTE: THE FACTORS OF "$\frac{1}{2}$" ARE INCLUDED IN THE FOLLOWING FREE-BODY DIAGRAMS BECAUSE THERE ARE TWO ROPES AND ONLY ONE IS CONSIDERED.

(a)



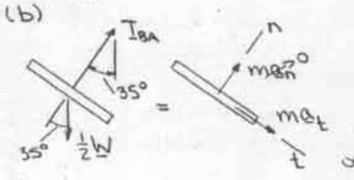
FOR THE SWING AT REST...

$\Sigma F_y = 0$: $T_{BA}\cos 35° - \frac{1}{2}W = 0$

OR $T_{BA} = \dfrac{22\text{ kg} \times 9.81\text{ m/s}^2}{2\cos 35°}$

OR $T_{BA} = 131.7$ N

(b)



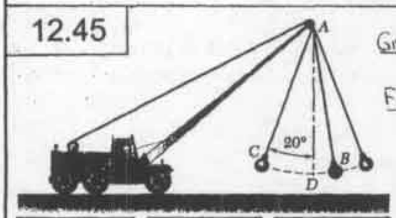
AT $t = 0$, $v = 0$ SO THAT $a_n = \frac{v^2}{\rho} = 0$

$+\nearrow\Sigma F_n = 0$: $T_{BA} - \frac{1}{2}W\cos 35° = 0$

OR $T_{BA} = \frac{1}{2}(22\text{ kg})(9.81\,\frac{\text{m}}{\text{s}^2})\cos 35°$

OR $T_{BA} = 88.4$ N

12.45

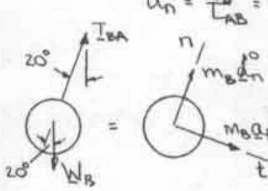


GIVEN: $m_B = 60$ kg, $L_{AB} = 15$ m; $(v_B)_D = 4.2$ m/s

FIND: (a) T_{BA} AT C

(b) T_{BA} AT D

(a) AT C, THE TOP OF THE SWING, $v_B = 0$; THUS $a_n = \dfrac{v_B^2}{L_{AB}} = 0$

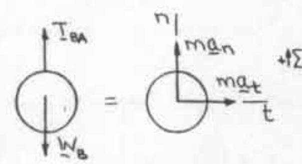


$+\nearrow\Sigma F_n = 0$: $T_{BA} - W_B\cos 20° = 0$

OR $T_{BA} = (60\text{ kg})(9.81\,\frac{\text{m}}{\text{s}^2}) \times \cos 20°$

OR $T_{BA} = 553$ N

(b)

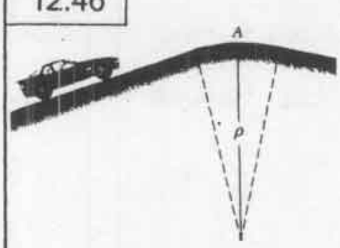


$+\uparrow\Sigma F_n = ma_n$: $T_{BA} - W_B = m_B\dfrac{(v_B)_D^2}{L_{AB}}$

OR $T_{BA} = (60\text{ kg})\left[9.81\,\frac{\text{m}}{\text{s}^2} + \dfrac{(4.2\,\frac{\text{m}}{\text{s}})^2}{15\text{ m}}\right]$

OR $T_{BA} = 659$ N

12.46

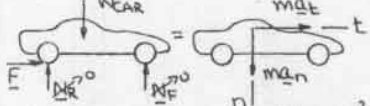


GIVEN: ROAD WITH RADIUS OF CURVATURE ρ AS SHOWN

FIND: (a) ρ FOR $W_{CAR} = 2400$ lb, $v_A = 100\,\frac{\text{mi}}{\text{h}}$, $N_{ROAD} = 0$

(b) $\underline{N}$ FOR $W = 160$ lb, $v_A = 50\,\frac{\text{mi}}{\text{h}}$

(a) NOTE: $100\,\frac{\text{mi}}{\text{h}} = 146.667\,\frac{\text{ft}}{\text{s}}$

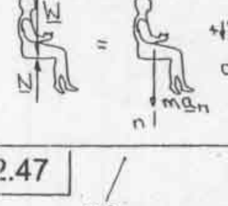


$+\downarrow\Sigma F_n = ma_n$: $W_{CAR} = \dfrac{W_{CAR}}{g}\dfrac{v_A^2}{\rho}$

OR $\rho = \dfrac{(146.667\,\text{ft/s})^2}{32.2\,\text{ft/s}^2} = 668.05$ ft

OR $\rho = 668$ ft

(b) NOTE: v IS CONSTANT $\Rightarrow a_t = 0$; $50\,\frac{\text{mi}}{\text{h}} = 73.333\,\frac{\text{ft}}{\text{s}}$

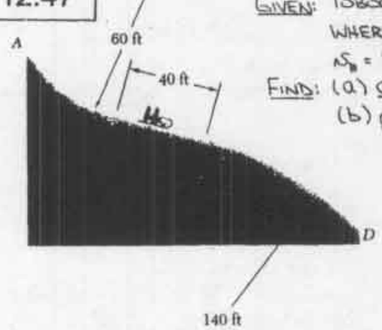


$+\downarrow\Sigma F_n = ma_n$: $W - N = \dfrac{W}{g}\dfrac{v_A^2}{\rho}$

OR $N = (160\text{ lb})\left[1 - \dfrac{(73.333\,\frac{\text{ft}}{\text{s}})^2}{(32.2\,\frac{\text{ft}}{\text{s}^2})(668.05\text{ ft})}\right]$

OR $\underline{N} = 120.0\text{ lb}\uparrow$

12.47

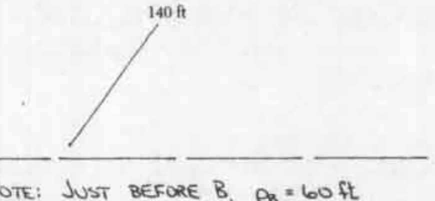


GIVEN: TOBOGGAN RUN SHOWN, WHERE $L_{BC} \measuredangle 20°$; $\mu_k = 0.10$; $v_B = 25$ ft/s

FIND: (a) a_t JUST BEFORE B

(b) a_t JUST AFTER C

(a) NOTE: JUST BEFORE B, $\rho_B = 60$ ft



$+\nearrow\Sigma F_n = ma_n$: $N - W\cos 20° = \dfrac{W}{g}\dfrac{v_B^2}{\rho_B}$

OR $N = W\left(\cos 20° + \dfrac{v_B^2}{g\rho_B}\right)$

SLIDING: $F = \mu_k N$
$= \mu_k W\left(\cos 20° + \dfrac{v_B^2}{g\rho_B}\right)$

$+\searrow\Sigma F_t = ma_t$: $W\sin 20° - F = \dfrac{W}{g}a_t$

OR $a_t = g(\sin 20° - \mu_k\cos 20°) - \mu_k\dfrac{v_B^2}{\rho_B}$

(CONTINUED)

12.47 continued

THEN.. $a_t = (32.2\ \frac{ft}{s^2})(\sin 20^\circ - 0.1\cos 20^\circ) - 0.1\ \frac{(25\ \frac{ft}{s})^2}{60\ ft}$

OR $\underline{a}_t = 6.95\ \frac{ft}{s^2}\ \measuredangle 20^\circ$

(b) IT IS FIRST NECESSARY TO DETERMINE v_C.

FOR SECTION BC..

$+\nearrow\Sigma F_y = 0$: $N_{BC} - W\cos 20^\circ = 0$

OR $N_{BC} = W\cos 20^\circ$

SLIDING: $F_{BC} = \mu_k N_{BC} = \mu_k W\cos 20^\circ$

$+\searrow\Sigma F_x = ma_{BC}$: $W\sin 20^\circ - F_{BC} = \frac{W}{g}a_{BC}$

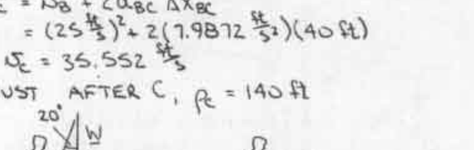
OR $a_{BC} = g(\sin 20^\circ - \mu_k\cos 20^\circ)$
$= (32.2\ ft/s^2)(\sin 20^\circ - 0.1\cos 20^\circ)$
$= 7.9872\ ft/s^2$

FOR THIS UNIFORMLY ACCELERATED MOTION HAVE..

$v_C^2 = v_B^2 + 2a_{BC}\Delta x_{BC}$
$= (25\ \frac{ft}{s})^2 + 2(7.9872\ \frac{ft}{s^2})(40\ ft)$

OR $v_C = 35.552\ \frac{ft}{s}$

NOW.. JUST AFTER C, $\rho_C = 140\ ft$



$+\nearrow\Sigma F_n = ma_n$: $W\cos 20^\circ - N = \frac{W}{g}\frac{v_C^2}{\rho_C}$

OR $N = W(\cos 20^\circ - \frac{v_C^2}{g\rho_C})$

SLIDING: $F = \mu_k N$
$= \mu_k W(\cos 20^\circ - \frac{v_C^2}{g\rho_C})$

$+\searrow\Sigma F_t = ma_t$: $W\sin 20^\circ - F = \frac{W}{g}a_t$

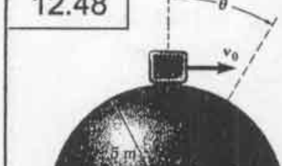
OR $a_t = g(\sin 20^\circ - \mu_k\cos 20^\circ) + \mu_k\frac{v_C^2}{\rho_C}$

NOTE: $g(\sin 20^\circ - \mu_k\cos 20^\circ) = a_{BC}$

THEN.. $a_t = 7.9872\ \frac{ft}{s^2} + 0.1\ \frac{(35.552\ ft/s)^2}{140\ ft}$

OR $\underline{a}_t = 8.89\ \frac{ft}{s^2}\ \measuredangle 20^\circ$

12.48

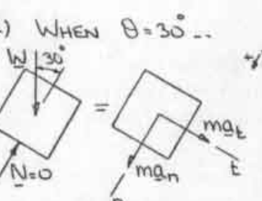


GIVEN: $m = 0.5\ kg$; AT $t = 0$, $v = v_0$; WHEN $\theta = 30^\circ$, $\underline{N} \to 0$

FIND: (a) v_0

(b) FORCE EXERTED ON THE SURFACE BY THE BLOCK WHEN $v = v_0$

(a) WHEN $\theta = 30^\circ$..



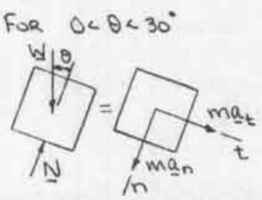
$+\swarrow\Sigma F_n = ma_n$: $W\cos 30^\circ = m\frac{v_{30}^2}{\rho}$

OR $v_{30}^2 = \rho g\cos 30^\circ$

(CONTINUED)

12.48 continued

FOR $0 < \theta < 30^\circ$



$+\searrow\Sigma F_t = ma_t$: $W\sin\theta = ma_t$

OR $a_t = g\sin\theta$

NOW.. $a_t = v\frac{dv}{ds}$ AND $ds = \rho d\theta$

THEN $\frac{1}{\rho}v\frac{dv}{d\theta} = a_t = g\sin\theta$

AT $\theta = 0$, $v = v_0$: $\int_{v_0}^{v_{30}} v\,dv = \rho g\int_0^{30^\circ}\sin\theta\,d\theta$

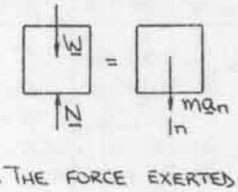
OR $\frac{1}{2}[v^2]_{v_0}^{v_{30}} = \rho g[-\cos\theta]_0^{30^\circ}$

OR $v_{30}^2 - v_0^2 = 2\rho g(1 - \cos 30^\circ)$

THEN $v_0^2 = \rho g\cos 30^\circ - 2\rho g(1 - \cos 30^\circ)$
$= \rho g(3\cos 30^\circ - 2)$
$= (1.5\ m)(9.81\ \frac{m}{s^2})(3\cos 30^\circ - 2)$
$= 8.80069\ m^2/s^2$

OR $v_0 = 2.97\ \frac{m}{s}$

(b) WHEN $\theta = 0$..

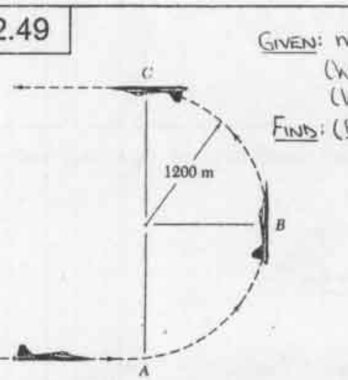


$+\downarrow\Sigma F_n = ma_n$: $W - N = m\frac{v_0^2}{\rho}$

OR $N = m(g - \frac{v_0^2}{\rho})$
$= (0.5\ kg)(9.81\ \frac{m}{s^2} - \frac{8.80069\ \frac{m^2}{s^2}}{1.5\ m})$
$= 1.971\ N$

∴ THE FORCE EXERTED ON THE SURFACE BY THE BLOCK IS $1.971\ N\downarrow$

12.49

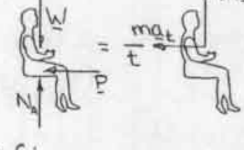


GIVEN: $m = 54\ kg$; a_t = CONSTANT
$(W_{APP})_A = 1680\ N$,
$(W_{APP})_C = 350\ N$

FIND: $(\underline{F}_{PILOT})_B$

FIRST NOTE THAT THE PILOT'S APPARENT WEIGHT IS EQUAL TO THE VERTICAL FORCE THAT SHE EXERTS ON THE SEAT OF THE JET TRAINER.

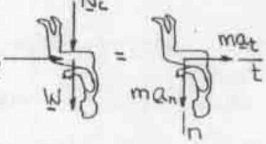
AT A:



$+\uparrow\Sigma F_n = ma_n$: $N_A - W = m\frac{v_A^2}{\rho}$

OR $v_A^2 = (1200\ m)(\frac{1680\ N}{54\ kg} - 9.81\ \frac{m}{s^2})$
$= 25\ 561.3\ \frac{m^2}{s^2}$

AT C:



$+\downarrow\Sigma F_n = ma_n$: $N_C + W = m\frac{v_C^2}{\rho}$

OR $v_C^2 = (1200\ m)(\frac{350\ N}{54\ kg} + 9.81\ \frac{m}{s^2})$
$= 19\ 549.8\ \frac{m^2}{s^2}$

(CONTINUED)

12.49 continued

SINCE a_t = CONSTANT, HAVE FROM A TO C..

$v_C^2 = v_A^2 + 2a_t \Delta s_{AC}$

OR $19\,549.8 \frac{m^2}{s^2} = 25\,561.3 \frac{m^2}{s^2} + 2a_t(\pi \times 1200\text{ m})$

OR $a_t = -0.79730 \frac{m}{s^2}$

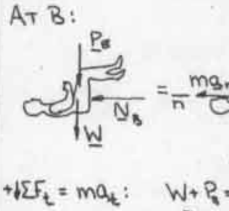
THEN FROM A TO B--

$v_B^2 = v_A^2 + 2a_t \Delta s_{AB}$

$= 25\,561.3 \frac{m^2}{s^2} + 2(-0.79730 \frac{m}{s^2})(\frac{\pi}{2} \times 1200\text{ m})$

$= 22\,555 \frac{m^2}{s^2}$

AT B:



$\overset{+}{\leftarrow}\Sigma F_n = ma_n$: $N_B = m\frac{v_B^2}{\rho}$

OR $N_B = 54\text{ kg}\,\frac{22\,555 \frac{m^2}{s^2}}{1200\text{ m}}$

OR $\underline{N}_B = 1014.98\text{ N} \leftarrow$

$+\downarrow\Sigma F_t = ma_t$: $W + P_B = m|a_t|$

OR $P_B = (54\text{ kg})(0.79730 - 9.81)\frac{m}{s^2}$

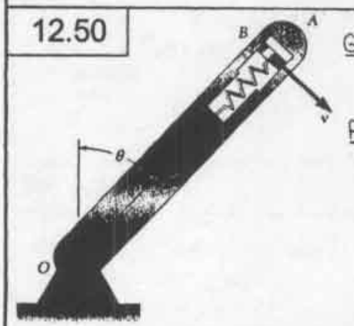
OR $\underline{P}_B = 486.69\text{ N}\uparrow$

FINALLY-- $(F_{PILOT})_B = \sqrt{N_B^2 + P_B^2} = \sqrt{(1014.98)^2 + (486.69)^2}$

$= 1126\text{ N}$

OR $(\underline{F}_{PILOT})_B = 1126\text{ N} \measuredangle 25.6°$ ◀

12.50

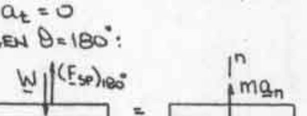


GIVEN: $W_B = 0.5$ lb; $\dot{\theta}$ = CONSTANT; WHEN $\theta = 180°$, $N_{FACE} = 0.8$ lb

FIND: RANGE OF VALUES OF θ SO THAT $N_{FACE} = 0$

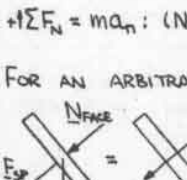
FIRST NOTE THAT $\dot{\theta}$ = CONSTANT $\Rightarrow v_B$ = CONSTANT $\Rightarrow a_t = 0$

WHEN $\theta = 180°$:



$+\uparrow\Sigma F_N = ma_n$: $(N_{FACE})_{180°} + (F_{SP})_{180°} - W = m\frac{v_B^2}{\rho_{MAX}}$

FOR AN ARBITRARY VALUE OF θ:



$\Sigma F_n = ma_n$: $N_{FACE} + F_{SP} + W\cos\theta = m\frac{v_B^2}{\rho}$

NOW-- AS BLOCK B LOSES CONTACT WITH THE CAVITY AT A, $N_{FACE} \to 0$, $F_{SP} \approx (F_{SP})_{180°}$, $\rho \approx \rho_{MAX}$

THEN-- $(F_{SP})_{180°} + W\cos\theta = (N_{FACE})_{180°} + (F_{SP})_{180°} - W$ $(= m\frac{v_B^2}{\rho_{MAX}})$

OR $\cos\theta = \frac{(N_{FACE})_{180°}}{W} - 1 = \frac{0.8\text{ lb}}{0.5\text{ lb}} - 1 = 0.6$

OR $\theta = \pm 53.1°$

$\therefore$ BLOCK B IS NOT IN CONTACT WITH THE FACE OF THE CAVITY AT END A WHEN

$-53.1 \le \theta \le 53.1°$ ◀

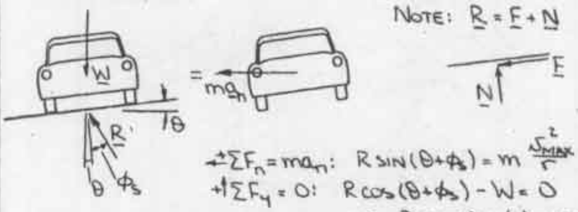
12.51

GIVEN: CAR TRAVELING AT A CONSTANT SPEED v ON A ROAD BANKED AT AN ANGLE θ

FIND: RANGE OF VALUES OF v SO THAT THE CAR DOES NOT SKID; $v = f(r, \theta, \phi_s)$

CASE 1: $v = v_{MAX}$

NOTE: $\underline{R} = \underline{F} + \underline{N}$



$\overset{+}{\leftarrow}\Sigma F_n = ma_n$: $R\sin(\theta + \phi_s) = m\frac{v_{MAX}^2}{r}$ (1)

$+\uparrow\Sigma F_y = 0$: $R\cos(\theta + \phi_s) - W = 0$

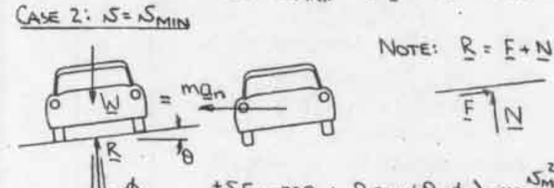
OR $R\cos(\theta + \phi_s) = mg$ (2)

FORMING $\frac{(1)}{(2)}$-- $\frac{R\sin(\theta + \phi_s)}{R\cos(\theta + \phi_s)} = \frac{m\frac{v_{MAX}^2}{r}}{mg}$

OR $v_{MAX} = \sqrt{gr\tan(\theta + \phi_s)}$

CASE 2: $v = v_{MIN}$

NOTE: $\underline{R} = \underline{F} + \underline{N}$



$\overset{+}{\leftarrow}\Sigma F_n = ma_n$: $R\sin(\theta - \phi_s) = m\frac{v_{MIN}^2}{r}$ (3)

$+\uparrow\Sigma F_y = 0$: $R\cos(\theta - \phi_s) - W = 0$

OR $R\cos(\theta - \phi_s) = mg$ (4)

FORMING $\frac{(3)}{(4)}$-- $\frac{R\sin(\theta - \phi_s)}{R\cos(\theta - \phi_s)} = \frac{m\frac{v_{MIN}^2}{r}}{mg}$

OR $v_{MIN} = \sqrt{gr\tan(\theta - \phi_s)}$

$\therefore$ FOR THE CAR NOT TO SKID..

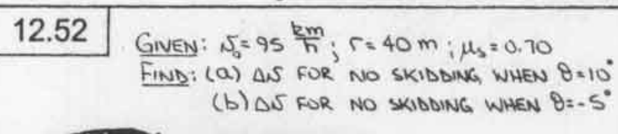
$\sqrt{gr\tan(\theta - \phi_s)} \le v \le \sqrt{gr\tan(\theta + \phi_s)}$ ◀

12.52

GIVEN: $v_0 = 95 \frac{km}{h}$; $r = 40$ m; $\mu_s = 0.70$

FIND: (a) Δv FOR NO SKIDDING WHEN $\theta = 10°$

(b) Δv FOR NO SKIDDING WHEN $\theta = -5°$

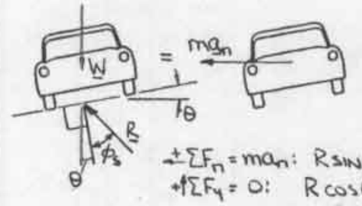


FIRST NOTE.. $\tan\phi_s = 0.70$ $(= \mu_s)$

OR $\phi_s = 34.992°$

ALSO, REQUIRING THAT THE SPEED OF THE CAR BE DECREASED TO AVOID SKIDDING, IMPLIES THAT IMPENDING SKIDDING IS "OUTWARD."

(a) $\theta = 10°$



$\overset{+}{\leftarrow}\Sigma F_n = ma_n$: $R\sin(\theta + \phi_s) = m\frac{v^2}{r}$ (1)

$+\uparrow\Sigma F_y = 0$: $R\cos(\theta + \phi_s) - W = 0$

OR $R\cos(\theta + \phi_s) = mg$ (2)

FORMING $\frac{(1)}{(2)}$.. $\frac{R\sin(\theta + \phi_s)}{R\cos(\theta + \phi_s)} = \frac{m\frac{v^2}{r}}{mg}$

(CONTINUED)

12.52 continued

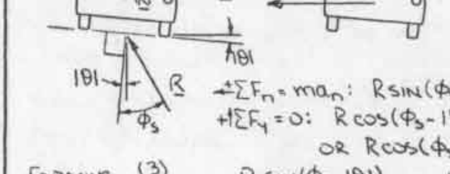
OR $v^2 = gr\tan(\theta+\phi_s) = (9.81\ \frac{m}{s^2})(40\ m)\tan(10^\circ+34.992^\circ)$

OR $v = 19.8063\ \frac{m}{s} = 71.302\ km/h$

THEN.. $\Delta v = v_s - v = (95 - 71.302)\ km/h$

OR $\Delta v = 23.7\ \frac{km}{h}$ ◀

(b) $\theta = -5^\circ$



$\xleftarrow{+}\Sigma F_n = ma_n$: $R\sin(\phi_s - |\theta|) = m\frac{v^2}{r}$ (3)

$+\uparrow\Sigma F_y = 0$: $R\cos(\phi_s - |\theta|) - W = 0$

OR $R\cos(\phi_s - |\theta|) = mg$ (4)

FORMING $\frac{(3)}{(4)}$.. $\frac{R\sin(\phi_s - |\theta|)}{R\cos(\phi_s - |\theta|)} = \frac{m\frac{v^2}{r}}{mg}$

OR $v^2 = gr\tan(\phi_s - |\theta|) = (9.81\ \frac{m}{s^2})(40\ m)\tan(34.992^\circ - 5^\circ)$

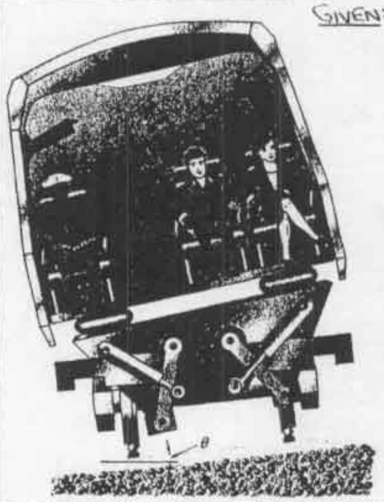
OR $v = 15.0492\ \frac{m}{s} = 54.177\ km/h$

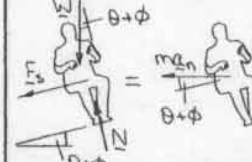
THEN.. $\Delta v = v_s - v = (95 - 54.177)\ km/h$

OR $\Delta v = 40.8\ \frac{km}{h}$ ◀

12.53 and 12.54

GIVEN: $\theta = 6^\circ$; $v_r = 60\ \frac{mi}{h}$ FOR A CURVE OF RADIUS ρ; $v = 100\ \frac{mi}{h}$





$\xleftarrow{+}\Sigma F_x = ma_x$: $F_s + W\sin(\theta+\phi) = \frac{W}{g}\frac{v^2}{\rho}\cos(\theta+\phi)$

WHEN $v = v_r$ (THE RATED SPEED), $F_s = 0$ (FOR $\phi = 0$)

$\therefore\ W\sin\theta = \frac{W}{g}\frac{v_r^2}{\rho}\cos\theta$

OR $\frac{1}{g\rho} = \frac{\tan\theta}{v_r^2}$ FOR THE GIVEN CURVE

SUBSTITUTING FOR $\frac{1}{g\rho}$ IN THE ABOVE EQUATION..

$F_s = W\left[\frac{v^2}{v_r^2}\tan\theta\cos(\theta+\phi) - \sin(\theta+\phi)\right]$ (1)

(CONTINUED)

12.53 and 12.54 continued

12.53 GIVEN: A PASSENGER OF WEIGHT W

FIND: (a) F_s WHEN $\phi = 0$

(b) ϕ FOR $F_s = 0$

(a) SUBSTITUTING THE KNOWN VALUES INTO EQ. (1)..

$F_s = W\left[\frac{(100\ mi/h)^2}{(60\ mi/h)^2}\tan 6^\circ\cos 6^\circ - \sin 6^\circ\right]$

$= W\left(\frac{25}{9} - 1\right)\sin 6^\circ$

OR $F_s = 0.1858\ W$ ◀

(b) SETTING $F_s = 0$ IN EQ. (1)..

$0 = W\left[\frac{(100\ mi/h)^2}{(60\ mi/h)^2}\tan 6^\circ\cos(6^\circ+\phi) - \sin(6^\circ+\phi)\right]$

OR $\tan(6^\circ+\phi) = \frac{25}{9}\tan 6^\circ$

OR $6^\circ + \phi = 16.28^\circ$

OR $\phi = 10.28^\circ$ ◀

12.54 GIVEN: $F_s = 0.1\ W$

FIND: ϕ

SUBSTITUTING THE KNOWN VALUES INTO EQ. (1)..

$0.1\ W = W\left[\frac{(100\ mi/h)^2}{(60\ mi/h)^2}\tan 6^\circ\cos(6^\circ+\phi) - \sin(6^\circ+\phi)\right]$

OR $[0.1 + \sin(6^\circ+\phi)]^2 = \left[\frac{25}{9}\tan 6^\circ\cos(6^\circ+\phi)\right]^2$

OR $0.01 + 0.2\sin(6^\circ+\phi) + \sin^2(6^\circ+\phi) = 0.085238[1 - \sin^2(6^\circ+\phi)]$

OR $1.085238\sin^2(6^\circ+\phi) + 0.2\sin(6^\circ+\phi) - 0.075238 = 0$

SOLVING FOR THE POSITIVE ROOT..

$\sin(6^\circ+\phi) = 0.186816$

OR $6^\circ + \phi = 10.77^\circ$

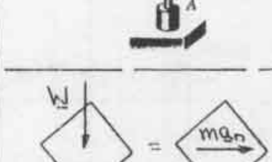
OR $\phi = 4.77^\circ$ ◀

12.55

GIVEN: $m_D = 0.3\ kg$; $\alpha = 40^\circ$

$\dot{\theta}_{ABC} = 5\ \frac{RAD}{s}$ (CONSTANT)

FIND: r IF r = CONSTANT



FIRST NOTE .. $v_D = r\dot{\theta}_{ABC}$

$+\uparrow\Sigma F_y = 0$: $N\sin 40^\circ - W = 0$

OR $N = \frac{mg}{\sin 40^\circ}$

$\xrightarrow{+}\Sigma F_n = ma_n$: $N\cos 40^\circ = m\frac{v_D^2}{r}$

OR $\frac{mg}{\sin 40^\circ}\cos 40^\circ = m\frac{(r\dot{\theta}_{ABC})^2}{r}$

OR $r = \frac{g}{\dot{\theta}_{ABC}^2}\frac{1}{\tan 40^\circ}$

$= \frac{9.81\ m/s^2}{(5\ RAD/s)^2}\frac{1}{\tan 40^\circ}$

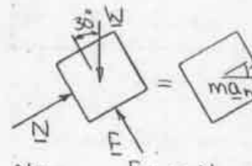
$= 0.468\ m$ OR $r = 468\ mm$ ◀

12.56

GIVEN: $m_D = 0.2\text{ kg}$; $\alpha = 30°$, $r = 0.6\text{ m}$, $\mu_s = 0.30$; $\dot{\theta}_{ABC}$ = CONSTANT

FIND: RANGE OF VALUES OF v SO THAT COLLAR D DOES NOT SLIDE ON THE ROD

CASE 1: $v = v_{MIN}$, IMPENDING MOTION DOWNWARD



$\xrightarrow{+}\Sigma F_x = ma_x$: $N - W\sin 30° = m\frac{v^2}{r}\cos 30°$

OR $N = m(g\sin 30° + \frac{v^2}{r}\cos 30°)$

$+\nwarrow\Sigma F_y = ma_y$: $F - W\cos 30° = -m\frac{v^2}{r}\sin 30°$

OR $F = m(g\cos 30° - \frac{v^2}{r}\sin 30°)$

NOW.. $F = \mu_s N$

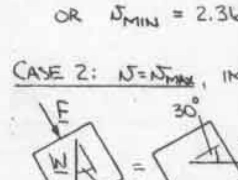
THEN.. $m(g\cos 30° - \frac{v^2}{r}\sin 30°) = \mu_s \times m(g\sin 30° + \frac{v^2}{r}\cos 30°)$

OR $v^2 = gr\,\dfrac{1 - \mu_s\tan 30°}{\mu_s + \tan 30°}$

$= (9.81\,\frac{m}{s^2})(0.6\text{ m})\,\dfrac{1 - 0.3\tan 30°}{0.3 + \tan 30°}$

OR $v_{MIN} = 2.36\,\frac{m}{s}$

CASE 2: $v = v_{MAX}$, IMPENDING MOTION UPWARD



$\xrightarrow{+}\Sigma F_x = ma_x$: $N - W\sin 30° = m\frac{v^2}{r}\cos 30°$

OR $N = m(g\sin 30° + \frac{v^2}{r}\cos 30°)$

$+\nwarrow\Sigma F_y = ma_y$: $F + W\cos 30° = m\frac{v^2}{r}\sin 30°$

OR $F = m(-g\cos 30° + \frac{v^2}{r}\sin 30°)$

NOW.. $F = \mu_s N$

THEN $m(-g\cos 30° + \frac{v^2}{r}\sin 30°) = \mu_s \times m(g\sin 30° + \frac{v^2}{r}\cos 30°)$

OR $v^2 = gr\,\dfrac{1 + \mu_s\tan 30°}{\tan 30° - \mu_s}$

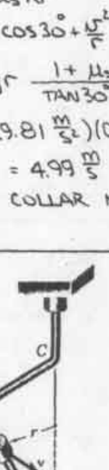
$= (9.81\,\frac{m}{s^2})(0.6\text{ m})\,\dfrac{1 + 0.3\tan 30°}{\tan 30° - 0.3}$

OR $v_{MAX} = 4.99\,\frac{m}{s}$

FOR THE COLLAR NOT TO SLIDE..

$2.36\,\frac{m}{s} \le v \le 4.99\,\frac{m}{s}$ ◀

12.57



GIVEN: $W_D = 0.6\text{ lb}$; $r = 8\text{ in.}$, $\dot{\theta} = 10\text{ RAD/s}$ (CONSTANT); COLLAR D DOES NOT SLIDE ON THE ROD

FIND: (a) $(\mu_s)_{MIN}$ WHEN $\alpha = 15°$

(b) $(\mu_s)_{MIN}$ WHEN $\alpha = 45°$

(CONTINUED)

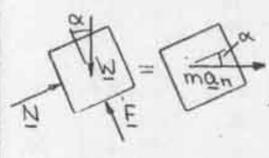
12.57 continued

FIRST NOTE THAT $v = r\dot{\theta}_{ABC} = (\frac{8}{12}\text{ft})(10\,\frac{\text{RAD}}{s}) = \frac{20}{3}\,\frac{ft}{s}$ AND THAT REQUIRING $\mu_s = (\mu_s)_{MIN}$ IMPLIES THAT SLIDING OF COLLAR D IS IMPENDING. ALSO,

$\mu_s = \tan\phi_s$

NOW CONSIDER THE TWO POSSIBLE CASES OF IMPENDING MOTION.

CASE 1: IMPENDING MOTION DOWNWARD



$\xrightarrow{+}\Sigma F_x = ma_x$: $N - W\sin\alpha = \frac{W}{g}\frac{v^2}{r}\cos\alpha$

OR $N = W(\sin\alpha + \frac{v^2}{gr}\cos\alpha)$

$+\nwarrow\Sigma F_y = ma_y$: $F - W\cos\alpha = -\frac{W}{g}\frac{v^2}{r}\sin\alpha$

OR $F = W(\cos\alpha - \frac{v^2}{gr}\sin\alpha)$

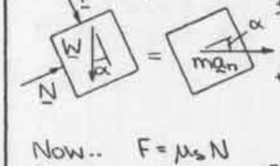
NOW.. $F = \mu_s N$

THEN.. $W(\cos\alpha - \frac{v^2}{gr}\sin\alpha) = \mu_s \times W(\sin\alpha + \frac{v^2}{gr}\cos\alpha)$

OR $\dfrac{v^2}{gr} = \dfrac{1 - \mu_s\tan\alpha}{\tan\alpha + \mu_s} = \dfrac{1 - \tan\phi_s\tan\alpha}{\tan\alpha + \tan\phi_s}$

$= \dfrac{1}{\tan(\alpha + \phi_s)}$

CASE 2: IMPENDING MOTION UPWARD



$\xrightarrow{+}\Sigma F_x = ma_x$: $N - W\sin\alpha = \frac{W}{g}\frac{v^2}{r}\cos\alpha$

OR $N = W(\sin\alpha + \frac{v^2}{gr}\cos\alpha)$

$+\nwarrow\Sigma F_y = ma_y$: $F + W\cos\alpha = \frac{W}{g}\frac{v^2}{r}\sin\alpha)$

OR $F = W(-\cos\alpha + \frac{v^2}{gr}\sin\alpha)$

NOW.. $F = \mu_s N$

THEN.. $W(-\cos\alpha + \frac{v^2}{gr}\sin\alpha) = \mu_s \times W(\sin\alpha + \frac{v^2}{gr}\cos\alpha)$

OR $\dfrac{v^2}{gr} = \dfrac{1 + \mu_s\tan\alpha}{\tan\alpha - \mu_s} = \dfrac{1 + \tan\alpha\tan\phi_s}{\tan\alpha - \tan\phi_s}$

$= \dfrac{1}{\tan(\alpha - \phi_s)}$

NOW.. $\dfrac{gr}{v^2} = \dfrac{(32.2\,ft/s^2)(\frac{8}{12}\,ft)}{(\frac{20}{3}\,ft/s)^2} = 0.483$

THEN $\tan(\alpha \pm \phi_s) = 0.483$

OR $\alpha \pm \phi_s = 25.781°$, $\phi_s \ge 0$

AND WHERE THE "+" CORRESPONDS TO IMPENDING MOTION DOWNWARD AND THE "−" TO IMPENDING MOTION UPWARD.

(a) $\alpha = 15°$: HAVE $15° \pm \phi_s = 25.781°$

$\phi_s \ge 0 \Rightarrow$ "+" SO THAT $\phi_s = 10.781°$

THEN $(\mu_s)_{MIN} = \tan 10.781°$

OR $(\mu_s)_{MIN} = 0.1904$, MOTION IMPENDING DOWNWARD ◀

(b) $\alpha = 45°$: HAVE $45° \pm \phi_s = 25.781°$

$\phi_s \ge 0 \Rightarrow$ "−" SO THAT $\phi_s = 19.219°$

THEN $(\mu_s)_{MIN} = \tan 19.219°$

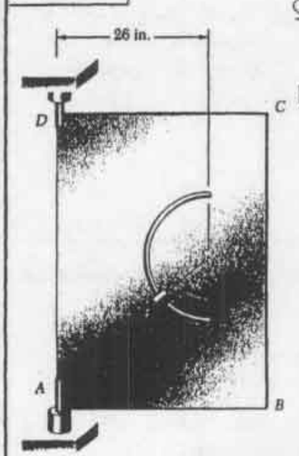
OR $(\mu_s)_{MIN} = 0.349$, MOTION IMPENDING UPWARD ◀

12.58

GIVEN: $r_{slot} = 10$ in, $\dot{\phi}_{ABCD} = 14 \frac{rad}{s}$; $W_E = 0.8$ lb; $\mu_s = 0.35$, $\mu_k = 0.25$

FIND: (a) $\underline{F}$ AND IF THE BLOCK SLIDES IN THE SLOT AT $t=0$ WHEN $\theta = 80°$

(b) $\underline{F}$ AND IF THE BLOCK SLIDES IN THE SLOT AT $t=0$ WHEN $\theta = 40°$



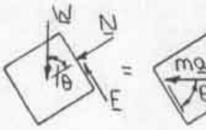
FIRST NOTE.. $\rho = \frac{1}{12}(26 - 10\sin\theta)$ ft

$v_E = \rho\dot{\phi}_{ABCD}$

THEN $a_n = \frac{v_E^2}{\rho} = \rho(\dot{\phi}_{ABCD})^2 = \left[\frac{1}{12}(26-10\sin\theta)\text{ ft}\right](14 \frac{rad}{s})^2$

$= \frac{98}{3}(13 - 5\sin\theta)$ ft/s²

ASSUME THAT THE BLOCK IS AT REST WITH RESPECT TO THE PLATE.



$+\nearrow \Sigma F_x = ma_x$: $N + W\cos\theta = m\frac{v_E^2}{\rho}\sin\theta$

OR $N = W(-\cos\theta + \frac{v_E^2}{g\rho}\sin\theta)$

$+\nwarrow \Sigma F_y = ma_y$: $-F + W\sin\theta = -m\frac{v_E^2}{\rho}\cos\theta$

OR $F = W(\sin\theta + \frac{v_E^2}{g\rho}\cos\theta)$

(a) HAVE $\theta = 80°$ .. THEN

$N = (0.8\text{ lb})[-\cos 80° + \frac{1}{32.2\text{ ft/s}^2} \times \frac{98}{3}(13 - 5\sin 80°)\frac{ft}{s^2} \times \sin 80°]$

$= 6.3159$ lb

$F = (0.8\text{ lb})[\sin 80° + \frac{1}{32.2\text{ ft/s}^2} \times \frac{98}{3}(13-5\sin 80°)\frac{ft}{s^2} \times \cos 80°]$

$= 1.92601$ lb

NOW -- $F_{MAX} = \mu_s N = 0.35(6.3159\text{ lb})$

$= 2.2106$ lb

$\therefore$ THE BLOCK DOES NOT SLIDE IN THE SLOT AND $\underline{F} = 1.926$ lb $\measuredangle 80°$ ◀

(b) HAVE $\theta = 40°$ .. THEN

$N = (0.8\text{ lb})[-\cos 40° + \frac{1}{32.2\text{ ft/s}^2} \times \frac{98}{3}(13-5\sin 40°)\frac{ft}{s^2} \times \sin 40°]$

$= 4.4924$ lb

$F = (0.8\text{ lb})[\sin 40° + \frac{1}{32.2\text{ ft/s}^2} \times \frac{98}{3}(13-5\sin 40°)\frac{ft}{s^2} \times \cos 40°]$

$= 6.5984$ lb

NOW .. $F_{MAX} = \mu_s N$ FROM WHICH IT FOLLOWS THAT $F > F_{MAX}$

$\therefore$ BLOCK E WILL SLIDE IN THE SLOT

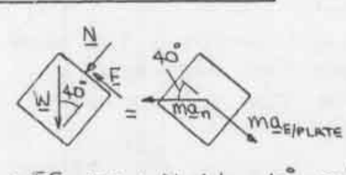
AND $\underline{a}_E = \underline{a}_n + \underline{a}_{E/PLATE}$

$= \underline{a}_n + (\underline{a}_{E/PLATE})_t + (\underline{a}_{E/PLATE})_n$

AT $t=0$, THE BLOCK IS AT REST RELATIVE TO THE PLATE. THUS, $(\underline{a}_{E/PLATE})_n = 0$ AT $t=0$, SO THAT $\underline{a}_{E/PLATE}$ MUST BE DIRECTED TANGENTIALLY TO THE SLOT.

(CONTINUED)

12.58 continued



$+\searrow \Sigma F_x = ma_x$: $N + W\cos 40° = m\frac{v_E^2}{\rho}\sin 40°$

OR $N = W(-\cos 40° + \frac{v_E^2}{g\rho}\sin 40°)$ (AS ABOVE)

$= 4.4924$ lb

SLIDING: $F = \mu_k N = 0.25(4.4924\text{ lb})$

$= 1.123$ lb

NOTING THAT $\underline{F}$ AND $\underline{a}_{E/PLANE}$ MUST BE DIRECTED AS SHOWN (IF THEIR DIRECTIONS ARE REVERSED, THEN ΣF_x IS ↘ WHILE ma_x IS ↖), HAVE

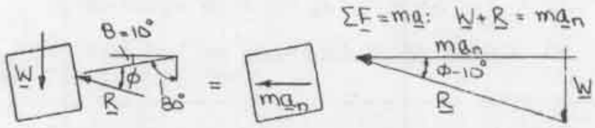
$\therefore$ THE BLOCK SLIDES DOWNWARD IN THE SLOT AND

$\underline{F} = 1.123$ lb $\measuredangle 40°$ ◀

ALTERNATIVE SOLUTIONS

(a) ASSUME THAT THE BLOCK IS AT REST WITH RESPECT TO THE PLATE.

$\Sigma \underline{F} = m\underline{a}$: $\underline{W} + \underline{R} = m\underline{a}_n$



THEN.. $\tan(\phi - 10°) = \frac{W}{ma_n} = \frac{W}{\frac{W}{g}\frac{v_E^2}{\rho}} = \frac{g}{\rho(\dot{\phi}_{ABCD})^2}$

$= \frac{32.2\text{ ft/s}^2}{\frac{98}{3}(13 - 5\sin 80°)\frac{ft}{s^2}}$ (FROM ABOVE)

OR $\phi - 10° = 6.9588°$

AND $\phi = 16.9588°$

NOW.. $\tan\phi_s = \mu_s$ $\mu_s = 0.35$

SO THAT $\phi_s = 19.29°$

$\therefore 0 < \phi < \phi_s \Rightarrow$ BLOCK DOES NOT SLIDE AND $\underline{R}$ IS DIRECTED AS SHOWN.

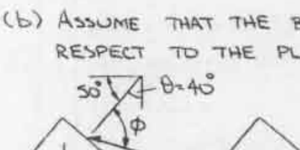
NOW -- $F = R\sin\phi$ AND $R = \frac{W}{\sin(\phi - 10°)}$

THEN.. $F = (0.8\text{ lb})\frac{\sin 16.9588°}{\sin 6.9588°} = 1.926$ lb

$\therefore$ THE BLOCK DOES NOT SLIDE IN THE SLOT AND

$\underline{F} = 1.926$ lb $\measuredangle 80°$ ◀

(b) ASSUME THAT THE BLOCK IS AT REST WITH RESPECT TO THE PLATE.



$\Sigma \underline{F} = m\underline{a}$: $\underline{W} + \underline{R} = m\underline{a}_n$

FROM PART a (ABOVE), IT THEN FOLLOWS THAT

$\tan(\phi - 50°) = \frac{g}{\rho(\dot{\phi}_{ABCD})^2} = \frac{32.2\text{ ft/s}^2}{\frac{98}{3}(13 - 5\sin 40°)\text{ ft/s}^2}$

OR $\phi - 50° = 5.752°$

AND $\phi = 55.752°$

NOW $\phi_s = 19.29°$ SO THAT $\phi > \phi_s$

$\therefore$ THE BLOCK WILL SLIDE IN THE SLOT

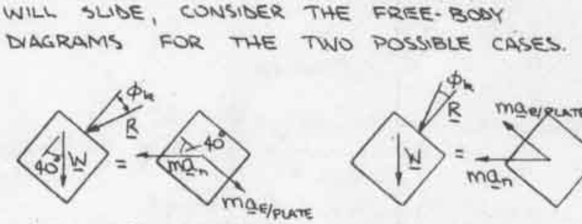
AND THEN $\phi = \phi_k$ WHERE $\tan\phi_k = \mu_k$ $\mu_k = 0.25$

OR $\phi_k = 14.0362°$

(CONTINUED)

12.58 continued

TO DETERMINE IN WHICH DIRECTION THE BLOCK WILL SLIDE, CONSIDER THE FREE-BODY DIAGRAMS FOR THE TWO POSSIBLE CASES.



SLIDING DOWNWARD SLIDING UPWARD

NOW.. $\Sigma \underline{F} = m\underline{a}$: $\underline{W} + \underline{R} = m\underline{a}_n + m\underline{a}_{B/PLATE}$

FROM THE DIAGRAMS IT CAN BE CONCLUDED THAT THIS EQUATION CAN BE SATISFIED ONLY IF THE BLOCK IS SLIDING DOWNWARD. THEN..

$+\!\!\nearrow \Sigma F_x = ma_x$: $W\cos 40° + R\cos\phi_k = m\frac{v_B^2}{\rho}\sin 40°$

NOW.. $F = R\sin\phi_k$

THEN.. $W\cos 40° + \frac{F}{\tan\phi_k} = \frac{W}{g}\frac{v_B^2}{\rho}\sin 40°$

OR $F = \mu_k W\left(-\cos 40° + \frac{v_B^2}{g\rho}\sin 40°\right)$

$= 1.123$ lb (SEE THE FIRST SOLUTION)

∴ THE BLOCK SLIDES DOWNWARD IN THE SLOT AND

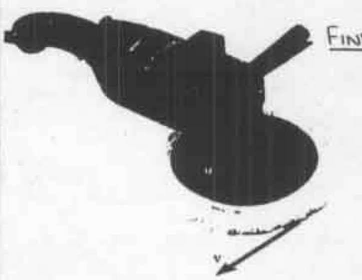
$\underline{F} = 1.123$ lb $\measuredangle 40°$ ◀

12.59

GIVEN: $d = 0.225$ m; $v_0 = 0$, $a_t = 4\ \frac{m}{s^2}$; $m = 1.6\times10^{-6}$ kg

FIND: (a) v AT $t = 3$ s

(b) F_{TUFT} AT $t = 3$ s



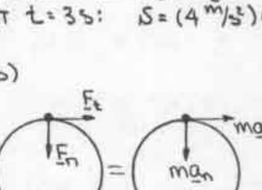
(a) a_t = CONSTANT ⇒ UNIFORMLY ACCELERATION MOTION

THEN.. $v = \cancel{v_0}^{\,0} + a_t t$

AT $t = 3$ s: $v = (4\ \text{m/s}^2)(3\ \text{s})$

OR $v = 12\ \frac{m}{s}$ ◀

(b)



$\Sigma F_t = ma_t$: $F_t = ma_t$

OR $F_t = (1.6\times10^{-6}\ \text{kg})(4\ \frac{m}{s^2})$

$= 6.4\times10^{-6}$ N

$\Sigma F_n = ma_n$: $F_n = m\frac{v^2}{\rho}$

AT $t = 3$ s: $F_n = (1.6\times10^{-6}\ \text{kg})\frac{(12\ \text{m/s})^2}{(\frac{0.225}{2}\ \text{m})}$

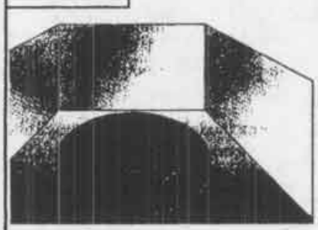
$= 2.048\times10^{-3}$ N

FINALLY.. $F_{TUFT} = \sqrt{F_t^2 + F_n^2}$

$= \sqrt{(6.4\times10^{-6}\ \text{N})^2 + (2.048\times10^{-3}\ \text{N})^2}$

OR $F_{TUFT} = 2.05\times10^{-3}$ N ◀

12.60



GIVEN: $v_0 = 0$, $(a_B)_t = 0.24\ \frac{m}{s^2}$

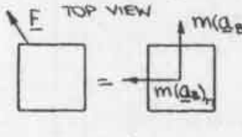
TRUNK B BEGINS TO SLIDE AT $t = 10$ s

FIND: μ_s

FIRST NOTE THAT $(a_B)_t$ = CONSTANT IMPLIES UNIFORMLY ACCELERATED MOTION.

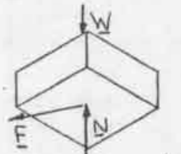
∴ $v_B = \cancel{v_0}^{\,0} + (a_B)_t t$

AT $t = 10$ s: $v_B = (0.24\ \frac{m}{s^2})(10\ \text{s}) = 2.4\ \frac{m}{s}$



IN THE PLANE OF THE TURNTABLE..

$\Sigma \underline{F} = m_B\underline{a}_B$: $\underline{F} = m_B(\underline{a}_B)_t + m_B(\underline{a}_B)_n$



THEN.. $F = m_B\sqrt{(a_B)_t^2 + (a_B)_n^2}$

$= m_B\sqrt{(a_B)_t^2 + \left(\frac{v_B^2}{\rho}\right)^2}$

$+\!\uparrow \Sigma F_y = 0$: $N - W = 0$

OR $N = m_B g$

AT $t = 10$ s: $F = \mu_s N = \mu_s m_B g$

THEN.. $\mu_s m_B g = m_B\sqrt{(a_B)_t^2 + \left(\frac{v_B^2}{\rho}\right)^2}$

OR $\mu_s = \frac{1}{9.81\ \text{m/s}^2}\left\{(0.24\ \frac{m}{s^2})^2 + \left[\frac{(2.4\ \text{m/s})^2}{2.5\ \text{m}}\right]^2\right\}^{1/2}$

OR $\mu_s = 0.236$ ◀

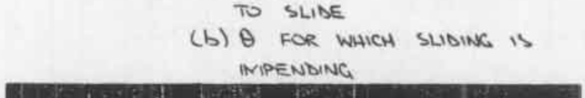
12.61

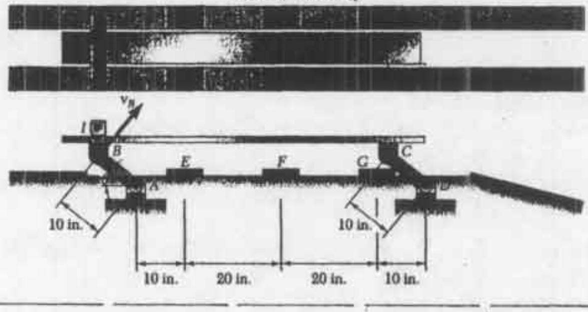
GIVEN: PARALLEL-LINK MECHANISM ABCD;

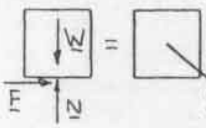
$v_B = 2.2$ ft/s

FIND: (a) $(\mu_s)_{MIN}$ IF COMPONENTS ARE NOT TO SLIDE

(b) θ FOR WHICH SLIDING IS IMPENDING







$\xrightarrow{+} \Sigma F_x = ma_x$: $F = \frac{W}{g}\frac{v_B^2}{\rho}\cos\theta$

$+\!\uparrow \Sigma F_y = ma_y$: $N - W = -\frac{W}{g}\frac{v_B^2}{\rho}\sin\theta$

OR $N = W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$

NOW.. $F_{MAX} = \mu_s N = \mu_s W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$

AND FOR THE COMPONENT NOT TO SLIDE

$F \le F_{MAX}$

OR $\frac{W}{g}\frac{v_B^2}{\rho}\cos\theta \le \mu_s W\left(1 - \frac{v_B^2}{g\rho}\sin\theta\right)$

(CONTINUED)

12.61 continued

OR $\mu_s \ge \dfrac{\cos\theta}{\frac{g\rho}{v_B^2} - \sin\theta}$

$\therefore$ MUST DETERMINE THE VALUES OF θ WHICH MAXIMIZE THE ABOVE EXPRESSION. THUS..

$$\frac{d}{d\theta}\left(\frac{\cos\theta}{\frac{g\rho}{v_B^2}-\sin\theta}\right) = \frac{-\sin\theta\left(\frac{g\rho}{v_B^2}-\sin\theta\right)-(\cos\theta)(-\cos\theta)}{\left(\frac{g\rho}{v_B^2}-\sin\theta\right)^2}=0$$

OR $\sin\theta = \dfrac{v_B^2}{g\rho}$ FOR $\mu_s = (\mu_s)_{MIN}$

NOW.. $\sin\theta = \dfrac{(2.2\ \text{ft/s})^2}{(32.2\ \frac{\text{ft}}{\text{s}^2})(\frac{10}{12}\ \text{ft})} = 0.180373$

OR $\theta = 10.3915^\circ$ AND $\theta = 169.609^\circ$

(a) FROM ABOVE,

$(\mu_s)_{MIN} = \dfrac{\cos\theta}{\frac{g\rho}{v_B^2}-\sin\theta}$ WHERE $\sin\theta = \dfrac{v_B^2}{g\rho}$

$\therefore (\mu_s)_{MIN} = \dfrac{\cos\theta}{\frac{1}{\sin\theta}-\sin\theta} = \dfrac{\cos\theta\sin\theta}{1-\sin^2\theta} = \tan\theta$

$= \tan 10.3915^\circ$

OR $(\mu_s)_{MIN} = 0.1834$ ◀

(b) HAVE IMPENDING MOTION

TO THE LEFT FOR $\theta = 10.39^\circ$ ◀

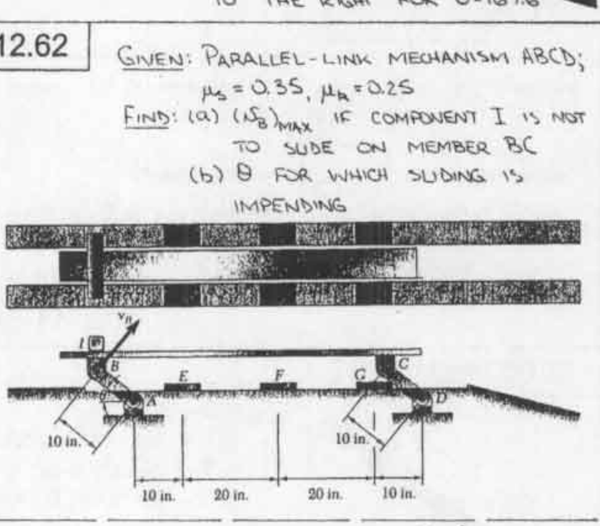
TO THE RIGHT FOR $\theta = 169.6^\circ$ ◀

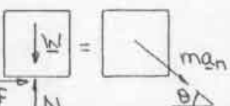
12.62

GIVEN: PARALLEL-LINK MECHANISM ABCD; $\mu_s = 0.35$, $\mu_k = 0.25$

FIND: (a) $(v_B)_{MAX}$ IF COMPONENT I IS NOT TO SLIDE ON MEMBER BC

(b) θ FOR WHICH SLIDING IS IMPENDING





$\xrightarrow{+}\Sigma F_x = ma_x$: $F = \dfrac{W}{g}\dfrac{v_B^2}{\rho}\cos\theta$

$+\uparrow\Sigma F_y = ma_y$: $N - W = -\dfrac{W}{g}\dfrac{v_B^2}{\rho}\sin\theta$

OR $N = W\left(1-\dfrac{v_B^2}{g\rho}\sin\theta\right)$

NOW.. $F_{MAX} = \mu_s N = \mu_s W\left(1-\dfrac{v_B^2}{g\rho}\sin\theta\right)$

AND FOR THE COMPONENT NOT TO SLIDE..

$F \le F_{MAX}$

OR $\dfrac{W}{g}\dfrac{v_B^2}{\rho}\cos\theta \le \mu_s W\left(1-\dfrac{v_B^2}{g\rho}\sin\theta\right)$

OR $v_B^2 \le \mu_s \dfrac{g\rho}{\cos\theta+\mu_s\sin\theta}$ (1)

TO ENSURE THAT THIS INEQUALITY IS SATISFIED, $(v_B^2)_{MAX}$ MUST BE LESS THAN OR EQUAL TO THE MINIMUM VALUE OF $\mu_s g\rho/(\cos\theta+\mu_s\sin\theta)$, WHICH OCCURS WHEN $(\cos\theta+\mu_s\sin\theta)$ IS MAXIMUM. THUS..

(CONTINUED)

12.62 continued

$\dfrac{d}{d\theta}(\cos\theta+\mu_s\sin\theta) = -\sin\theta+\mu_s\cos\theta = 0$

OR $\tan\theta = \mu_s$ $\quad \mu_s = 0.35$

OR $\theta = 19.2900^\circ$

(a) THE MAXIMUM ALLOWED VALUE OF v_B IS THEN..

$(v_B^2)_{MAX} = \mu_s\dfrac{g\rho}{\cos\theta+\mu_s\sin\theta}$ WHERE $\tan\theta = \mu_s$

$= g\rho\dfrac{\tan\theta}{\cos\theta+(\tan\theta)\sin\theta} = g\rho\sin\theta$

$= (32.2\ \frac{\text{ft}}{\text{s}^2})(\frac{10}{12}\ \text{ft})\sin 19.2900^\circ$

OR $(v_B)_{MAX} = 2.98\ \frac{\text{ft}}{\text{s}}$ ◀

(b) FIRST NOTE THAT FOR $90^\circ < \theta \le 180^\circ$, EQ. (1) BECOMES $\quad v_B^2 \le \mu_s\dfrac{g\rho}{\cos\alpha+\mu_s\sin\alpha}$

WHERE $\alpha = 180^\circ - \theta$. IT THEN FOLLOWS THAT THE SECOND VALUE OF θ FOR WHICH MOTION IS IMPENDING IS..

$\theta = 180^\circ - 19.2900^\circ = 160.7100^\circ$

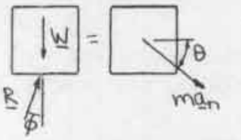
$\therefore$ HAVE IMPENDING MOTION

TO THE LEFT FOR $\theta = 19.29^\circ$ ◀

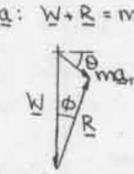
TO THE RIGHT FOR $\theta = 160.7^\circ$ ◀

ALTERNATIVE SOLUTION

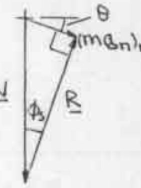
$\Sigma\underline{F} = m\underline{a}$: $\underline{W} + \underline{R} = m\underline{a}_n$



THEN



FOR IMPENDING MOTION, $\phi = \phi_s$. ALSO, AS SHOWN ABOVE, THE VALUES OF θ FOR WHICH MOTION IS IMPENDING MINIMIZE THE VALUE OF v_B, AND THUS THE VALUE OF a_n $(a_n = \frac{v_B^2}{\rho})$. FROM THE ABOVE DIAGRAM IT CAN BE CONCLUDED THAT a_n IS MINIMUM WHEN $m\underline{a}_n$ AND $\underline{R}$ ARE PERPENDICULAR. THEREFORE--



FROM THE DIAGRAM..

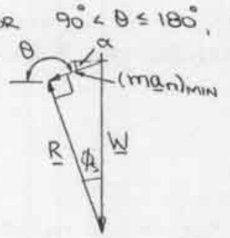
$\theta = \phi_s = \tan^{-1}\mu_s$ (AS ABOVE)

AND $ma_n = W\sin\phi_s$

OR $m\dfrac{v_B^2}{\rho} = mg\sin\theta$

OR $v_B^2 = g\rho\sin\theta$ (AS ABOVE)

FOR $90^\circ < \theta \le 180^\circ$, HAVE--



FROM THE DIAGRAM..

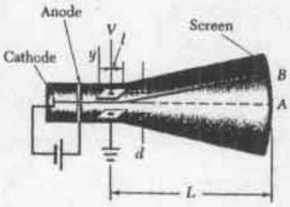
$\alpha = 180^\circ - \theta$ (AS ABOVE)

$\alpha = \phi_s$

AND $ma_n = W\sin\phi_s$

OR $v_B^2 = g\rho\sin\theta$ (AS ABOVE)

12.63 and 12.64



GIVEN: $v_x = v_0$ (= CONSTANT)
$(F_y)_{PLATE} = \frac{eV}{d}$

FIRST NOTE THAT THE HORIZONTAL COMPONENT OF THE VELOCITY OF AN ELECTRON IS A CONSTANT (v_0) REGARDLESS OF THE VALUE OF THE POTENTIAL V. THEN..

$x = x_0^{\,0} + v_0 t$

THE TIME t_{PLATE} FOR AN ELECTRON TO TRAVEL BETWEEN THE PLATES IS THEN..

$l = v_0(t_{PLATE})$

OR $t_{PLATE} = \frac{l}{v_0}$

AND THE TIME t_{SCREEN} TO TRAVEL FROM THE END OF THE PLATES TO THE SCREEN IS..

$(L - \frac{1}{2}l) = v_0(t_{SCREEN})$

OR $t_{SCREEN} = \frac{L - \frac{1}{2}l}{v_0}$

NEXT CONSIDER THE VERTICAL MOTION OF AN ELECTRON AS IT MOVES BETWEEN THE PLATES.

$+\uparrow \Sigma F_y = ma_y$: $(F_y)_{PLATE} = ma_y$

OR $a_y = \frac{eV}{md}$

THEN, FOR THE UNIFORMLY ACCELERATED MOTION IN THE y DIRECTION HAVE

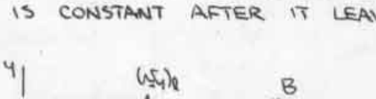
$v_y = (v_y)_0^{\,0} + a_y t = (\frac{eV}{md})t$ $\quad y = y_0^{\,0} + (v_y)_0^{\,0} t + \frac{1}{2}a_y t^2 = \frac{1}{2}(\frac{eV}{md})t^2$

AT THE END OF THE PLATES..

$(v_y)_l = (\frac{eV}{md})(\frac{l}{v_0}) = \frac{eVl}{mdv_0}$ $\quad y_l = \frac{1}{2}(\frac{eV}{md})(\frac{l}{v_0})^2 = \frac{eVl^2}{2mdv_0^2}$

12.63 FIND: δ IN TERMS OF V, v_0, e, m, d, l, L

FIRST NOTE THAT THE VELOCITY OF AN ELECTRON IS CONSTANT AFTER IT LEAVES THE PLATES.



THEN, FROM THE END OF THE PLATES TO THE SCREEN..

$y = y_l + (v_y)_l t = (\frac{eVl^2}{2mdv_0^2}) + (\frac{eVl}{mdv_0})t$

AT THE SCREEN: $\delta = \frac{eVl^2}{2mdv_0^2} + (\frac{eVl}{mdv_0})(\frac{L - \frac{1}{2}l}{v_0})$

OR $\delta = \frac{eVlL}{mdv_0^2}$ ◀

(CONTINUED)

12.64 continued

12.64 GIVEN: AT $x = l$, $(\frac{d}{2} - y)_{MIN} = 0.05d$
FIND: $(d/l)_{MIN}$ IN TERMS OF e, m, v_0, V

AT $x = l$, HAVE
$y = y_l = \frac{eVl^2}{2mdv_0^2}$

AND $\frac{d}{2} - y_l \geq 0.05d$

OR $0.45d \geq \frac{eVl^2}{2mdv_0^2}$

OR $\frac{d^2}{l^2} \geq \frac{eV}{0.9mv_0^2}$

THE MINIMUM VALUE OF $\frac{d}{l}$ IS THEN

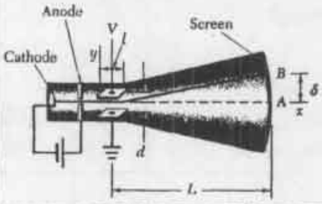
$(\frac{d}{l})_{MIN} = (\frac{eV}{0.9mv_0^2})^{1/2}$

OR $(\frac{d}{l})_{MIN} = \frac{1.054}{v_0}\sqrt{\frac{eV}{m}}$ ◀

12.65

GIVEN: $v_x = v_0$ (= CONSTANT);
$(F_y)_{PLATE} = \frac{eV}{d}$;
$L' = 0.6L$, $d' = 0.8d$,
δ, V, AND v_0 UNCHANGED

FIND: l'



FROM THE SOLUTION TO PROBLEM 12.63 HAVE

$\delta = \frac{eVlL}{mdv_0^2}$

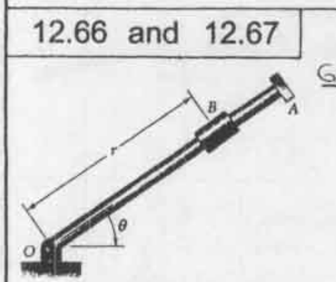
THEN, SINCE δ IS UNCHANGED, HAVE

$(\delta)_{MODIFIED} = (\delta)_{ORIGINAL}$: $\frac{eVl'L'}{md'v_0^2} = \frac{eVlL}{mdv_0^2}$

OR $\frac{l'(0.6L)}{0.8d} = \frac{lL}{d}$

OR $l' = 1.333\,l$ ◀

12.66 and 12.67



GIVEN: $m_B = 0.2$ kg;
$r = 250 + 150\sin \pi t$,
$\theta = \pi(4t^2 - 8t)$
$r \sim$ mm, $t \sim$ s, $\theta \sim$ RAD

HAVE.. $r = (0.25 + 0.15\sin \pi t)$ m $\quad \theta = \pi(4t^2 - 8t)$ RAD

THEN $\dot{r} = (0.15\pi\cos \pi t)\ \frac{m}{s}$ $\quad \dot{\theta} = \pi(8t - 8)\ \frac{RAD}{s}$

AND $\ddot{r} = -(0.15\pi^2\sin \pi t)\ \frac{m}{s^2}$ $\quad \ddot{\theta} = 8\pi\ \frac{RAD}{s^2}$

12.66 FIND: (a) F_r AND F_θ AT $t = 0$
(b) F_r AND F_θ AT $t = 0.5$ s

(a) AT $t = 0$: $r = 0.25$ m
$\dot{r} = 0.15\pi\ \frac{m}{s}$ $\quad \dot{\theta} = -8\pi\ \frac{RAD}{s}$
$\ddot{r} = 0$ $\quad \ddot{\theta} = 8\pi\ \frac{RAD}{s^2}$

(CONTINUED)

12.66 and 12.67 continued

Now.. $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (0.25\,m)(-8\pi\,\frac{RAD}{s})^2 = -16\pi^2\,\frac{m}{s^2}$

AND $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.25\,m)(8\pi\,\frac{RAD}{s^2}) + 2(0.15\pi\,\frac{m}{s})(-8\pi\,\frac{RAD}{s})$

$= \pi(2 - 2.4\pi)\,\frac{m}{s^2}$

FINALLY.. $F_r = m_B a_r = (0.2\,kg)(-16\pi^2\,\frac{m}{s^2})$

OR $F_r = -31.6\,N$ ◀

$F_\theta = m_B a_\theta = (0.2\,kg)[\pi(2 - 2.4\pi)\,\frac{m}{s^2}]$

OR $F_\theta = -3.48\,N$ ◀

(b) AT $t = 0.5\,s$: $r = 0.40\,m$

$\dot{r} = 0$ $\qquad \dot{\theta} = -4\pi\,\frac{RAD}{s}$

$\ddot{r} = -0.15\pi^2\,\frac{m}{s^2}$ $\qquad \ddot{\theta} = 8\pi\,\frac{RAD}{s^2}$

Now.. $a_r = \ddot{r} - r\dot{\theta}^2 = -(0.15\pi^2\,\frac{m}{s^2}) - (0.40\,m)(-4\pi\,\frac{RAD}{s})^2$

$= -6.55\pi^2\,\frac{m}{s^2}$

AND $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.40\,m)(8\pi\,\frac{RAD}{s^2}) + 0 = 3.20\pi\,\frac{m}{s^2}$

FINALLY.. $F_r = m_B a_r = (0.2\,kg)(-6.55\pi^2\,\frac{m}{s^2})$

OR $F_r = -12.93\,N$ ◀

$F_\theta = m_B a_\theta = (0.2\,kg)(3.20\pi\,\frac{m}{s^2})$

OR $F_\theta = 2.01\,N$ ◀

12.67 FIND: F_r AND F_θ AT $t = 1.5\,s$

AT $t = 1.5\,s$: $r = 0.10\,m$

$\dot{r} = 0$ $\qquad \dot{\theta} = 4\pi\,\frac{RAD}{s}$

$\ddot{r} = 0.15\pi^2\,\frac{m}{s^2}$ $\qquad \ddot{\theta} = 8\pi\,\frac{RAD}{s^2}$

Now.. $a_r = \ddot{r} - r\dot{\theta}^2 = (0.15\pi^2\,\frac{m}{s^2}) - (0.10\,m)(4\pi^2\,\frac{RAD}{s})^2$

$= -1.45\pi^2\,\frac{m}{s^2}$

AND $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (0.10\,m)(8\pi\,\frac{RAD}{s^2}) + 0 = 0.8\pi\,\frac{m}{s^2}$

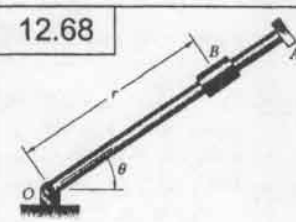
FINALLY.. $F_r = m_B a_r = (0.2\,kg)(-1.45\pi^2\,\frac{m}{s^2})$

OR $F_r = -2.86\,N$ ◀

$F_\theta = m_B a_\theta = (0.2\,kg)(0.8\pi\,\frac{m}{s^2})$

OR $F_\theta = 0.503\,N$ ◀

12.68



GIVEN: $W_B = 5\,lb$;
$r = \frac{10}{t+4}$, $\theta = \frac{2}{\pi}\sin\pi t$
$r \sim ft$, $t \sim s$, $\theta \sim RAD$

FIND: (a) F_r AND F_θ AT $t = 1\,s$
(b) F_r AND F_θ AT $t = 6\,s$

HAVE.. $r = \frac{10}{t+4}\,ft$ $\qquad \theta = (\frac{2}{\pi}\sin\pi t)\,RAD$

THEN.. $\dot{r} = -\frac{10}{(t+4)^2}\,\frac{ft}{s}$ $\qquad \dot{\theta} = (2\cos\pi t)\,\frac{RAD}{s}$

AND $\ddot{r} = \frac{20}{(t+4)^3}\,\frac{ft}{s^2}$ $\qquad \ddot{\theta} = -(2\pi\sin\pi t)\,\frac{RAD}{s^2}$

(a) AT $t = 1\,s$: $r = 2\,ft$

$\dot{r} = -0.4\,\frac{ft}{s}$ $\qquad \dot{\theta} = -2\,\frac{RAD}{s}$

$\ddot{r} = 0.16\,\frac{ft}{s^2}$ $\qquad \ddot{\theta} = 0$

Now.. $a_r = \ddot{r} - r\dot{\theta}^2 = (0.16\,\frac{ft}{s^2}) - (2\,ft)(-2\,\frac{RAD}{s})^2$

$= -7.84\,ft/s^2$

AND $a_\theta = \ddot{r} + 2\dot{r}\dot{\theta} = 0 + 2(-0.4\,\frac{ft}{s})(-2\,\frac{RAD}{s}) = 1.6\,\frac{ft}{s^2}$

FINALLY.. $F_r = m_B a_r = \frac{5\,lb}{32.2\,ft/s^2}(-7.84\,\frac{ft}{s^2})$

OR $F_r = -1.217\,lb$ ◀

$F_\theta = m_B a_\theta = \frac{5\,lb}{32.2\,ft/s^2}(1.6\,\frac{ft}{s^2})$

OR $F_\theta = 0.248\,lb$ ◀

(CONTINUED)

12.68 continued

(b) AT $t = 6\,s$: $r = 1\,ft$

$\dot{r} = -0.1\,\frac{ft}{s}$ $\qquad \dot{\theta} = 2\,\frac{RAD}{s}$

$\ddot{r} = 0.02\,\frac{ft}{s^2}$ $\qquad \ddot{\theta} = 0$

Now.. $a_r = \ddot{r} - r\dot{\theta}^2 = (0.02\,\frac{ft}{s^2}) - (1\,ft)(2\,\frac{RAD}{s})^2 = -3.98\,\frac{ft}{s^2}$

AND $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.1\,\frac{ft}{s})(2\,\frac{RAD}{s}) = -0.4\,\frac{ft}{s^2}$

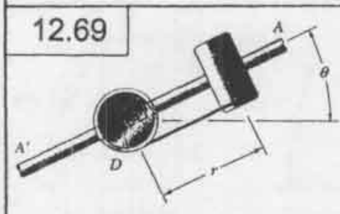
FINALLY.. $F_r = m_B a_r = \frac{5\,lb}{32.2\,ft/s^2}(-3.98\,\frac{ft}{s^2})$

OR $F_r = -0.618\,lb$ ◀

$F_\theta = m_B a_\theta = \frac{5\,lb}{32.2\,ft/s^2}(-0.4\,\frac{ft}{s^2})$

OR $F_\theta = -0.0621\,lb$ ◀

12.69



GIVEN: $\theta = ct$, c = CONSTANT,
$\dot{r} = -k$; AT $t = 0$, $r = r_0$

FIND: (a) T IN TERMS OF m, c, k, r_0, t
(b) Q, FORCE EXERTED ON B BY ARM AA'

KINEMATICS

HAVE.. $\frac{dr}{dt} = \dot{r} = -k$

AT $t = 0$, $r = r_0$: $\int_{r_0}^{r} dr = \int_0^t -k\,dt$

OR $r = r_0 - kt$

ALSO.. $\ddot{r} = 0$ $\qquad \dot{\theta} = ct$

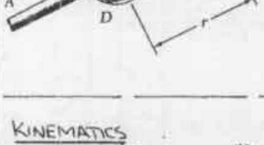
$\qquad \ddot{\theta} = c$

Now.. $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (r_0 - kt)(ct)^2 = -c^2(r_0 - kt)t^2$

AND $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (r_0 - kt)(c) + 2(-k)(ct)$

$= c(r_0 - 3kt)$

KINETICS



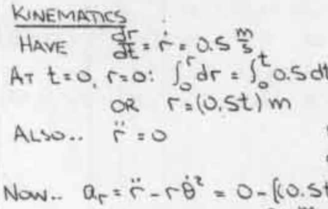
(a) $+\nwarrow \Sigma F_r = m a_r$: $-T = m[-c^2(r_0 - kt)t^2]$

OR $T = mc^2(r_0 - kt)t^2$ ◀

(b) $\nearrow \Sigma F_\theta = m a_\theta$: $Q = m[c(r_0 - 3kt)]$

OR $Q = mc(r_0 - 3kt)$ ◀

12.70



GIVEN: $m_B = 3\,kg$; $\dot{\theta} = 0.75t$
$\dot{\theta} \sim \frac{RAD}{s}$, $t \sim s$;
$\dot{r} = 0.5\,\frac{m}{s}$; AT $t = 0$, $r = 0$

FIND: t WHEN $T = Q$, WHERE Q IS THE FORCE ON B FROM AA'

KINEMATICS

HAVE $\frac{dr}{dt} = \dot{r} = 0.5\,\frac{m}{s}$

AT $t = 0$, $r = 0$: $\int_0^r dr = \int_0^t 0.5\,dt$

OR $r = (0.5t)\,m$

ALSO.. $\ddot{r} = 0$ $\qquad \dot{\theta} = (0.75t)\,\frac{RAD}{s}$

$\qquad \ddot{\theta} = 0.75\,\frac{RAD}{s^2}$

Now.. $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - [(0.5t)\,m][(0.75t)\,\frac{RAD}{s}]^2$

$= -(0.28125\,t^3)\,\frac{m}{s^2}$

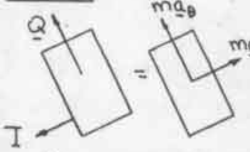
AND $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = [(0.5t)\,m](0.75\,\frac{RAD}{s^2}) + 2(0.5\,\frac{m}{s})[(0.75t)\,\frac{RAD}{s}]$

$= (1.125\,t)\,\frac{m}{s^2}$

(CONTINUED)

12.70 continued

KINETICS



$\xrightarrow{+}\Sigma F_r = m_B a_r$: $-T = (3\text{ kg})(-0.28125t^3)\frac{m}{s^2}$

OR $T = (0.84375t^3)$ N

$+\nearrow\Sigma F_\theta = m_B a_\theta$: $Q = (3\text{ kg})(1.125t)\frac{m}{s^2}$

OR $Q = (3.375t)$ N

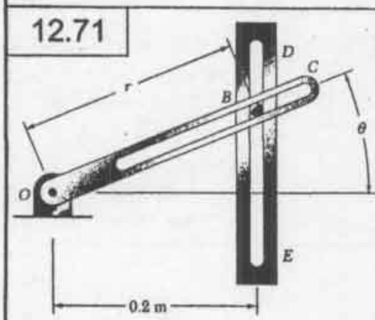
NOW REQUIRE THAT $T = Q$

OR $(0.84375t^3)\text{N} = (3.375t)\text{ N}$

OR $t^2 = 4.000$

OR $t = 2.00$ s ◀

12.71



GIVEN: $m_B = 0.1$ kg; $\dot\theta = \dot\theta_0 = 12\ \frac{\text{RAD}}{s}$

FIND: (a) F_r AND F_θ ON PIN B

(b) $\underline{P}$ AND $\underline{Q}$, WHERE $\underline{P}$ IS DUE TO OC AND $\underline{Q}$ IS DUE TO DE

KINEMATICS

FROM THE DRAWING OF THE SYSTEM HAVE..

$r = \frac{0.2}{\cos\theta}$ m

THEN $\dot r = (0.2\frac{\sin\theta}{\cos^2\theta}\dot\theta)\frac{m}{s}$ $\qquad \dot\theta = 12\ \frac{\text{RAD}}{s}$

$\ddot\theta = 0$

AND

$$\ddot r = 0.2\ \frac{\cos\theta(\cos^2\theta) - \sin\theta(-2\cos\theta\sin\theta)}{\cos^4\theta}\ \dot\theta^2$$

$$= (0.2\ \frac{1+\sin^2\theta}{\cos^3\theta}\ \dot\theta^2)\ \frac{m}{s^2}$$

SUBSTITUTING FOR $\dot\theta$..

$$\dot r = 0.2\ \frac{\sin\theta}{\cos^2\theta}(12) = (2.4\ \frac{\sin\theta}{\cos^2\theta})\ \frac{m}{s}$$

$$\ddot r = 0.2\ \frac{1+\sin^2\theta}{\cos^3\theta}(12)^2 = (28.8\ \frac{1+\sin^2\theta}{\cos^3\theta})\ \frac{m}{s^2}$$

NOW.. $a_r = \ddot r - r\dot\theta^2 = (28.8\ \frac{1+\sin^2\theta}{\cos^3\theta}) - (\frac{0.2}{\cos\theta})(12)^2$

$= (57.6\ \frac{\sin^2\theta}{\cos^3\theta})\ \frac{m}{s^2}$

AND $a_\theta = r\ddot\theta + 2\dot r\dot\theta = 0 + 2(2.4\ \frac{\sin\theta}{\cos^2\theta})(12)$

$= (57.6\ \frac{\sin\theta}{\cos^2\theta})\ \frac{m}{s^2}$

KINETICS

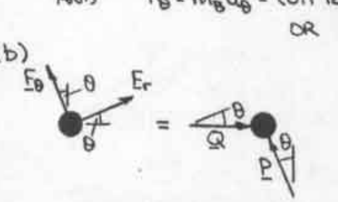
(a) HAVE.. $F_r = m_B a_r = (0.1\text{ kg})[(57.6\ \frac{\sin^2\theta}{\cos^3\theta})\frac{m}{s^2}]$

OR $F_r = (5.76\text{ N})\tan^2\theta\sec\theta$ ◀

AND $F_\theta = m_B a_\theta = (0.1\text{ kg})[(57.6\ \frac{\sin\theta}{\cos^2\theta})\frac{m}{s^2}]$

OR $F_\theta = (5.76\text{ N})\tan\theta\sec\theta$ ◀

(b)



NOW.. $+\uparrow\Sigma F_y$: $F_\theta\cos\theta + F_r\sin\theta = P\cos\theta$

OR $P = 5.76\tan\theta\sec\theta + (5.76\tan^2\theta\sec\theta)\tan\theta$

OR $\underline{P} = (5.76\text{ N})\tan\theta\sec^3\theta\ \forall\theta$ ◀

(CONTINUED)

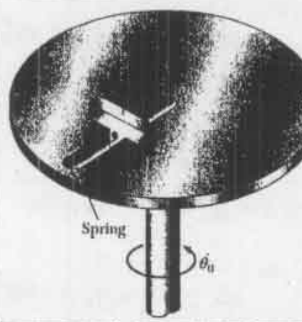
12.71 continued

$\xrightarrow{+}\Sigma F_r$: $F_r = Q\cos\theta$

OR $Q = (5.76\tan^2\theta\sec\theta)\frac{1}{\cos\theta}$

OR $\underline{Q} = (5.76\text{ N})\tan^2\theta\sec^2\theta \rightarrow$ ◀

12.72



GIVEN: $\dot\theta_0 = 15\ \frac{\text{RAD}}{s}$; $W_B = 0.5$ lb, $k = 4$ lb/ft; WHEN $r = 0$, $x_{sp} = 0$; $\ddot r = -40$ ft/s², $F_A = 2$ lb

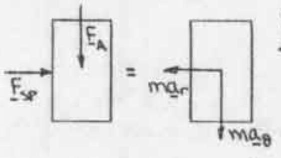
FIND: (a) r

(b) v_r

FIRST NOTE .. WHEN $r=0$, $x_{sp} = 0 \Rightarrow F_{sp} = kr$

AND $\dot\theta = \dot\theta_0 = 15\ \frac{\text{RAD}}{s}$

THEN $\ddot\theta = 0$



(a)

$\xrightarrow{+}\Sigma F_r = m_B a_r$: $-F_{sp} = \frac{W_B}{g}(\ddot r - r\dot\theta^2)$

OR $-kr = \frac{W_B}{g}(\ddot r - r\dot\theta_0^2)$

OR $r = \dfrac{\ddot r}{\dot\theta_0^2 - \frac{gk}{W_B}}$

THEN.. $r = \dfrac{-40\text{ ft/s}^2}{(15\frac{\text{RAD}}{s})^2 - \frac{(32.2\text{ ft/s}^2)(4\text{ lb/ft})}{0.5\text{ lb}}}$

OR $r = 1.227$ ft ◀

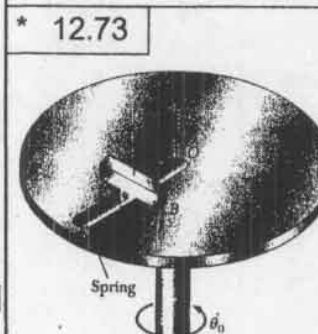
(b) $+\downarrow\Sigma F_\theta = m a_\theta$: $F_A = \frac{W_B}{g}(r\ddot\theta^{\,0} + 2\dot r\dot\theta)$

NOW.. $v_r = \dot r$

THEN $v_r = \dfrac{gF_A}{2W_B\dot\theta_0} = \dfrac{(32.2\frac{ft}{s^2})(2\text{ lb})}{2(0.5\text{ lb})(15\frac{\text{RAD}}{s})}$

OR $v_r = 4.29\ \frac{ft}{s}$ ◀

* 12.73



GIVEN: $\dot\theta_0 = 12\ \frac{\text{RAD}}{s}$; $W_B = 8.05$ oz; WHEN $r=0$, $x_{sp} = 0$; AT $t = 0$, $\dot r = 0$, $r = 15$ in.

FIND: (a) r AND F_A AT $t = 0.1$ s, $k = 2.25$ lb/ft

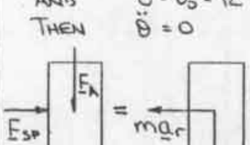
(b) r AND F_A AT $t = 0.1$ s, $k = 3.25$ lb/ft

WHERE F_A IS THE HORIZONTAL FORCE EXERTED ON THE SLIDER BY THE DISK

FIRST NOTE .. WHEN $r=0$, $x_{sp} = 0 \Rightarrow F_{sp} = kr$

AND $\dot\theta = \dot\theta_0 = 12\ \frac{\text{RAD}}{s}$ $\qquad r_0 = 15$ in. $= 1.25$ ft

THEN $\ddot\theta = 0$



$\xrightarrow{+}\Sigma F_r = m_B a_r$: $-F_{sp} = \frac{W_B}{g}(\ddot r - r\dot\theta_0^2)$

OR $\ddot r + (\frac{kg}{W_B} - \dot\theta_0^2)r = 0$ (1)

$+\downarrow\Sigma F_\theta = m_B a_\theta$: $F_A = \frac{W_B}{g}(r\ddot\theta^{\,0} + 2\dot r\dot\theta_0)$ (2)

(CONTINUED)

12.73 continued

(a) $k = 2.25$ lb/ft

SUBSTITUTING THE GIVEN VALUES INTO EQ (1)..

$$\ddot{r} + \left[\frac{2.25\,\text{lb/ft} \times 32.2\,\text{ft/s}^2}{\frac{8.05}{16}\,\text{lb}} - \left(12\,\frac{\text{RAD}}{\text{s}}\right)^2 \right] r = 0$$

OR $\ddot{r} = 0$

THEN.. $\frac{d\dot{r}}{dt} = \ddot{r} = 0$ AND AT $t=0$, $\dot{r}=0$:

$\therefore \int_0^{\dot{r}} d\dot{r} = \int_0^{0.1} (0)\,dt$

OR $\dot{r} = 0$

AND.. $\frac{dr}{dt} = \dot{r} = 0$ AND AT $t=0$, $r_0 = 1.25$ ft

$\therefore \int_{r_0}^{r} dr = \int_0^{0.1} (0)\,dt$

OR $r = r_0$

$\therefore r = 1.25$ ft ◀

NOTE: $\dot{r} = 0$ IMPLIES THAT THE SLIDER REMAINS AT ITS INITIAL RADIAL POSITION.

WITH $\dot{r}=0$, EQ. (2) IMPLIES

$F_H = 0$ ◀

(b) $k = 3.25$ lb/ft

SUBSTITUTING THE GIVEN VALUES INTO EQ (1)..

$$\ddot{r} + \left[\frac{3.25\,\text{lb/ft} \times 32.2\,\text{ft/s}^2}{\frac{8.05}{16}\,\text{lb}} - \left(12\,\frac{\text{RAD}}{\text{s}}\right)^2 \right] r = 0$$

OR $\ddot{r} + 64r = 0$

NOW.. $\ddot{r} = \frac{d}{dt}(\dot{r})$ $\quad \dot{r} = v_r \quad \frac{d}{dt} = \frac{dr}{dt}\frac{d}{dr} = v_r \frac{d}{dr}$

THEN $\ddot{r} = v_r \frac{dv_r}{dr}$

SO THAT $v_r \frac{dv_r}{dr} + 64r = 0$

AT $t=0$, $v_r = 0$, $r = r_0$: $\int_0^{v_r} v_r\,dv_r = -64\int_{r_0}^{r} r\,dr$

OR $v_r^2 = -64(r^2 - r_0^2)$

OR $v_r = 8\sqrt{r_0^2 - r^2}$

NOW.. $v_r = \frac{dr}{dt} = 8\sqrt{r_0^2 - r^2}$

AT $t=0$, $r = r_0$: $\int_{r_0}^{r} \frac{dr}{\sqrt{r_0^2 - r^2}} = \int_0^t 8\,dt = 8t$

LET $r = r_0 \sin\phi$, $\quad dr = r_0\cos\phi\,d\phi$

THEN $\int_{\frac{\pi}{2}}^{\sin^{-1}(\frac{r}{r_0})} \frac{r_0\cos\phi\,d\phi}{\sqrt{r_0^2 - r_0^2\sin^2\phi}} = 8t$

OR $\int_{\frac{\pi}{2}}^{\sin^{-1}(\frac{r}{r_0})} d\phi = 8t$

OR $\sin^{-1}\left(\frac{r}{r_0}\right) - \frac{\pi}{2} = 8t$

OR $r = r_0 \sin\left(8t + \frac{\pi}{2}\right) = r_0\cos 8t = (1.25\,\text{ft})\cos 8t$

THEN $\dot{r} = -\left(10\,\frac{\text{ft}}{\text{s}}\right)\sin 8t$

FINALLY.. AT $t = 0.1$ s:

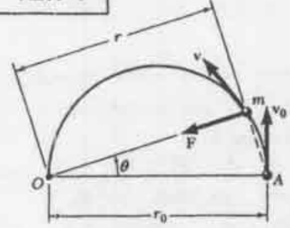
$r = (1.25\,\text{ft})\cos(8 \times 0.1)$

OR $r = 0.871$ ft ◀

EQ (2).. $F_A = \frac{(8.05/16)\,\text{lb}}{32.2\,\text{ft/s}^2} \times 2 \times \left[-\left(10\frac{\text{ft}}{\text{s}}\right)\sin(8\times 0.1)\right]\left(12\,\frac{\text{RAD}}{\text{s}}\right)$

OR $F_A = -2.69$ lb ◀

12.74



GIVEN: CENTRAL FORCE $\underline{F}$ AND SEMICIRCULAR PATH SHOWN; AT $t=0$, $\theta = 0$, $\underline{v} = \underline{v}_0$

SHOW: $v = v_0/\cos^2\theta$

HAVE – $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$

SO THAT $v^2 = \dot{r}^2 + r^2\dot{\theta}^2 \qquad (1)$

FROM THE DIAGRAM.. $r = r_0\cos\theta$

THEN $\dot{r} = -(r_0\sin\theta)\dot{\theta}$

SUBSTITUTING INTO EQ (1)..

$v^2 = (-r_0\dot{\theta}\sin\theta)^2 + (r_0\cos\theta)^2\dot{\theta}^2$

$= r_0^2(\sin^2\theta + \cos^2\theta)\dot{\theta}^2$

OR $v = r_0\dot{\theta} \qquad (2)$

AT $t=0$: $v_0 = r_0\dot{\theta}_0$

FROM EQ. (12.27): $r^2\dot{\theta} = r_0^2\dot{\theta}_0$

$= r_0 v_0$

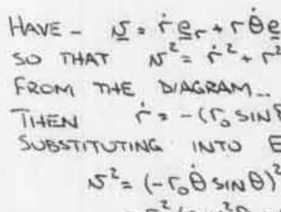
OR $\dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0}{(r_0\cos\theta)^2} = \frac{v_0}{r_0\cos^2\theta}$

SUBSTITUTING FOR $\dot{\theta}$ IN EQ. (2)..

$v = r_0\left(\frac{v_0}{r_0\cos^2\theta}\right)$

OR $v = \frac{v_0}{\cos^2\theta}$ Q.E.D. ◀

12.75



GIVEN: CENTRAL FORCE $\underline{F}$ AND SEMICIRCULAR PATH SHOWN; AT $t=0$, $\theta=0$, $\underline{v} = \underline{v}_0$, $\underline{r} = \underline{r}_0$

FIND: (a) F_t WHEN $\theta = 0$

(b) F_t WHEN $\theta = 45°$

FROM THE DIAGRAM.. $r = r_0\cos\theta$

THEN $\dot{r} = -(r_0\sin\theta)\dot{\theta}$

NOW.. $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$

SO THAT AT $t=0$.. $v_0 = r_0\dot{\theta}_0$

FROM EQ. (12.27): $r^2\dot{\theta} = r_0^2\dot{\theta}_0$

$= r_0 v_0$

OR $\dot{\theta} = \frac{r_0 v_0}{r^2} = \frac{r_0 v_0}{(r_0\cos\theta)^2} = \frac{v_0}{r_0}\frac{1}{\cos^2\theta}$

FROM PROBLEM 12.74: $v = \frac{v_0}{\cos^2\theta}$

NOW.. $a_t = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{v_0}{\cos^2\theta}\right) = v_0\frac{2\cos\theta\sin\theta}{\cos^4\theta}\dot{\theta}$

$= 2v_0\frac{\sin\theta}{\cos^3\theta}\cdot\left(\frac{v_0}{r_0}\frac{1}{\cos^2\theta}\right)$

$= 2\frac{v_0^2}{r_0}\frac{\sin\theta}{\cos^5\theta}$

FINALLY.. $F_t = ma_t = 2m\frac{v_0^2}{r_0}\frac{\sin\theta}{\cos^5\theta}$

(a) WHEN $\theta = 0$: $F_t = 0$ ◀

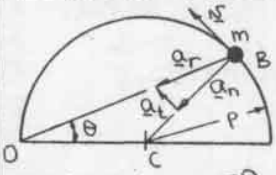
(b) WHEN $\theta = 45°$: $F_t = 2m\frac{v_0^2}{r_0}\frac{\sin 45°}{\cos^5 45°}$

OR $F_t = 8m\frac{v_0^2}{r_0}$ ◀

(CONTINUED)

12.75 continued

ALTERNATIVE SOLUTION



FIRST NOTE THAT TRIANGLE OBC IS AN ISOSCELES TRIANGLE.

$\therefore \angle OBC = \theta$

FOR CENTRAL FORCE MOTION $a_\theta = 0$

$\therefore \mathbf{a} = \mathbf{a}_r + \cancel{\mathbf{a}_\theta}^{\,0}$

NOW... $\mathbf{a} = \mathbf{a}_t + \mathbf{a}_n$ OR $\mathbf{a}_t + \mathbf{a}_n = \mathbf{a}_r$

FROM THE ABOVE DIAGRAM...

$$a_t = a_n \tan\theta$$

WHERE $a_n = \frac{v^2}{\rho}$ $\quad \rho = \frac{r_0}{2}$

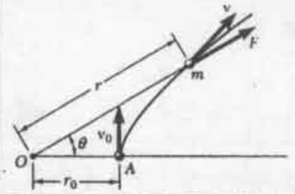
AND FROM PROBLEM 12.74

$$v = \frac{v_0}{\cos^2\theta}$$

THEN $a_t = \dfrac{(v_0/\cos^2\theta)^2}{\frac{r_0}{2}} \times \dfrac{\sin\theta}{\cos\theta} = 2\dfrac{v_0^2}{r_0}\dfrac{\sin\theta}{\cos^5\theta}$

FINALLY... $F_t = ma_t = 2m\dfrac{v_0^2}{r_0}\dfrac{\sin\theta}{\cos^5\theta}$ (AS ABOVE)

12.76 and 12.77



GIVEN: CENTRAL FORCE F AND PATH SHOWN; $r = r_0/\sqrt{\cos 2\theta}$; AT $t=0$, $\theta = 0$, $v = v_0$, $r = r_0$

HAVE $r = \dfrac{r_0}{\sqrt{\cos 2\theta}} = r_0(\cos 2\theta)^{-\frac{1}{2}}$

THEN $\dot r = r_0\left[-\frac{1}{2}(\cos 2\theta)^{-\frac{3}{2}}\right](-2\sin 2\theta)\dot\theta$

$= r_0\dfrac{\sin 2\theta}{\cos^{3/2} 2\theta}\dot\theta$

NOW... $\mathbf{v} = \dot r\,\mathbf{e}_r + r\dot\theta\,\mathbf{e}_\theta$

SO THAT AT $t=0$... $v_0 = r_0\dot\theta_0$

FROM EQ. (12.27): $r^2\dot\theta = r_0^2\dot\theta_0$

$= r_0 v_0$

OR $\dot\theta = \dfrac{r_0 v_0}{r^2} = \dfrac{r_0 v_0}{\left(\frac{r_0}{\sqrt{\cos 2\theta}}\right)^2} = \dfrac{v_0}{r_0}\cos 2\theta$

12.76 FIND: v_r AND v_θ AS FUNCTIONS OF θ

HAVE... $v_r = \dot r = r_0\dfrac{\sin 2\theta}{\cos^{3/2} 2\theta} \times \dfrac{v_0}{r_0}\cos 2\theta$

OR $v_r = v_0\dfrac{\sin 2\theta}{\sqrt{\cos 2\theta}}$ ◀

AND $v_\theta = r\dot\theta = \dfrac{r_0}{\sqrt{\cos 2\theta}} \times \dfrac{v_0}{r_0}\cos 2\theta$

OR $v_\theta = v_0\sqrt{\cos 2\theta}$ ◀

12.77 SHOW: (a) $v \propto r$ AND $F \propto r$

(b) $\rho \propto r^3$

(a) FROM THE SOLUTION TO PROBLEM 12.76 HAVE

$v_r = v_0\dfrac{\sin 2\theta}{\sqrt{\cos 2\theta}}$ $\quad v_\theta = v_0\sqrt{\cos 2\theta}$

(CONTINUED)

12.77 continued

NOW... $v^2 = v_r^2 + v_\theta^2$

$= \left(v_0\dfrac{\sin 2\theta}{\sqrt{\cos 2\theta}}\right)^2 + \left(v_0\sqrt{\cos 2\theta}\right)^2$

$= v_0^2\left(\dfrac{\sin^2 2\theta}{\cos 2\theta} + \cos 2\theta\right) = \dfrac{v_0^2}{\cos 2\theta}$

OR $v = \dfrac{v_0}{\sqrt{\cos 2\theta}}$

RECALLING THAT $r = \dfrac{r_0}{\sqrt{\cos 2\theta}}$

IT FOLLOWS THAT $v = \dfrac{v_0}{r_0}r$

OR $v \propto r$ Q.E.D. ◀

NOW... $\dot r = v_r = v_0\dfrac{\sin 2\theta}{\sqrt{\cos 2\theta}}$ AND $r = \dfrac{r_0}{\sqrt{\cos 2\theta}}$

COMBINING... $\dot r = \dfrac{v_0}{r_0}r\sin 2\theta$

THEN... $\ddot r = \dfrac{d}{dt}\left(\dfrac{v_0}{r_0}r\sin 2\theta\right) = \dfrac{v_0}{r_0}\left[\dot r\sin 2\theta + r(2\cos 2\theta)\dot\theta\right]$

NOTING THAT $r\dot\theta = v_\theta$ HAVE...

$\ddot r = \dfrac{v_0}{r_0}\left[\left(v_0\dfrac{\sin 2\theta}{\sqrt{\cos 2\theta}}\right)\sin 2\theta + 2\left(v_0\sqrt{\cos 2\theta}\right)\cos 2\theta\right]$

$= \dfrac{v_0^2}{r_0}\dfrac{1+\cos^2 2\theta}{\sqrt{\cos 2\theta}}$

$= \dfrac{v_0^2}{r_0^2}(1+\cos^2 2\theta)r$

NOW... $a_r = \ddot r - r\dot\theta^2$ $\quad \dot\theta = \dfrac{v_0}{r_0}\cos 2\theta$ (FROM ABOVE)

$= \dfrac{v_0^2}{r_0^2}(1+\cos^2 2\theta)r - r\left(\dfrac{v_0}{r_0}\cos 2\theta\right)^2$

$= \dfrac{v_0^2}{r_0^2}r$

FINALLY... $\bar F = \bar F_r + \bar F_\theta$ AND FOR CENTRAL FORCE MOTION, $F_\theta = 0$. THEN...

$F = F_r = ma_r = m\dfrac{v_0^2}{r_0^2}r$

OR $F \propto r$ Q.E.D. ◀

(b) FIRST NOTE... $v = \dfrac{v_0}{r_0}r$ (PART a)

AND $a_\theta = 0$ (CENTRAL FORCE MOTION)

NOW... $a_t = \dfrac{dv}{dt} = \dfrac{d}{dt}\left(\dfrac{v_0}{r_0}r\right) = \dfrac{v_0}{r_0}\dot r$

$= \dfrac{v_0}{r_0}\left(\dfrac{v_0}{r_0}r\sin 2\theta\right)$ (FROM PART a)

$= \dfrac{v_0^2}{r_0^2}r\sin 2\theta$

HAVE... $a^2 = a_t^2 + a_n^2 = a_r^2 + \cancel{a_\theta^2}^{\,0}$ $\quad a_r = \dfrac{v_0^2}{r_0^2}r$ (PART a)

SO THAT $a_n^2 = \left(\dfrac{v_0^2}{r_0^2}r\right)^2 - \left(\dfrac{v_0^2}{r_0^2}r\sin 2\theta\right)^2$

$= \dfrac{v_0^4}{r_0^4}r^2\cos^2 2\theta$ $\quad r = \dfrac{r_0}{\sqrt{\cos 2\theta}}$

$= \dfrac{v_0^4}{r^2}$

OR $a_n = \dfrac{v_0^2}{r}$

FINALLY... $a_n = \dfrac{v^2}{\rho}$ $\quad v = \dfrac{v_0}{r_0}r$ (FROM PART a)

OR $\dfrac{v_0^2}{r} = \dfrac{\left(\frac{v_0}{r_0}r\right)^2}{\rho}$

OR $\rho = \dfrac{1}{r_0^2}r^3$

OR $\rho \propto r^3$ Q.E.D. ◀

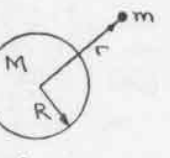
12.78

GIVEN: A PLANET OF RADIUS R AND OF DENSITY ρ; MOON HAVING ORBITAL RADIUS $r = 2R$

SHOW: $\tau = (24\pi/G\rho)^{1/2}$

HAVE.. $F = G\frac{Mm}{r^2}$ [EQ. (12.28)]

AND $F = F_n = ma_n = m\frac{v^2}{r}$



THEN $G\frac{Mm}{r^2} = m\frac{v^2}{r}$

OR $v^2 = \frac{GM}{r}$

FOR THE PLANET.. $M = \rho V = \rho\left(\frac{4}{3}\pi R^3\right)$

THEN $v^2 = \frac{G}{r}\left(\rho\frac{4}{3}\pi R^3\right) = \frac{4}{3}\pi G\rho\frac{R^3}{r}$

THE TIME τ FOR THE MOON TO COMPLETE ONE FULL REVOLUTION IS

$$\tau = \frac{2\pi r}{v} = 2\pi r\left(\frac{4}{3}\pi G\rho\frac{R^3}{r}\right)^{-\frac{1}{2}}$$

$$= \sqrt{\frac{3\pi}{G\rho}}\left(\frac{r}{R}\right)^{3/2}$$

FOR $r = 2R$ .. $\tau = \sqrt{\frac{3\pi}{G\rho}}\left(\frac{2R}{R}\right)^{3/2}$

OR $\tau = \sqrt{\frac{24\pi}{G\rho}}$ Q.E.D. ◀

12.79

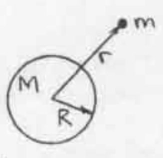
GIVEN: A PLANET OF RADIUS R HAVING AN ACCELERATION OF GRAVITY g AT ITS SURFACE; τ, THE ORBITAL PERIOD OF A MOON

SHOW: $r = f(R, g, \tau)$, WHERE r IS THE ORBITAL RADIUS OF THE MOON

FIND: g FOR JUPITER; $R = 71\,492$ km, $\tau_{EUROPA} = 3.551$ DAYS, $r_{EUROPA} = 670.9\times10^3$ km

HAVE.. $F = G\frac{Mm}{r^2}$ [EQ. (12.28)]

AND $F = F_n = ma_n = m\frac{v^2}{r}$



THEN $G\frac{Mm}{r^2} = m\frac{v^2}{r}$

OR $v^2 = \frac{GM}{r}$

NOW $GM = gR^2$ EQ. (12.30)

SO THAT $v^2 = \frac{gR^2}{r}$ OR $v = R\sqrt{\frac{g}{r}}$

FOR ONE ORBIT.. $\tau = \frac{2\pi r}{v} = \frac{2\pi r}{R\sqrt{\frac{g}{r}}}$

OR $r = \left(\frac{g\tau^2R^2}{4\pi^2}\right)^{1/3}$ Q.E.D. ◀

SOLVING FOR g.. $g = 4\pi^2\frac{r^3}{\tau^2R^2}$

AND NOTING THAT $\tau = 3.551$ DAYS $= 306\,806$ s,

THEN $g_{JUPITER} = 4\pi^2\frac{r_{EUR}^3}{\tau_{EUR}^2 R_{JUP}}$

$$= 4\pi^2\frac{(670.9\times10^6\text{ m})^3}{(306\,806\text{ s})^2(71.492\times10^6\text{ m})^2}$$

OR $g_{JUPITER} = 24.8\frac{m}{s^2}$ ◀

NOTE: $g_{JUPITER} \approx 2.53\,g_{EARTH}$

12.80

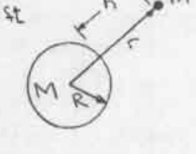
GIVEN: SATELLITE IN A GEOSYNCHRONOUS EARTH ORBIT; $\tau = 23.934$ h

FIND: (a) ALTITUDE h OF THE SATELLITE
(b) VELOCITY v OF THE SATELLITE

FIRST NOTE.. $\tau = 23.934$ h $= 86.1624\times10^3$ s

AND $R_{EARTH} = 3960$ mi $= 20.9088\times10^6$ ft

$R_{EARTH} = 6.37\times10^6$ m



(a) FROM THE SOLUTION TO PROBLEM 12.79 HAVE

$$r = \left(\frac{g\tau^2R^2}{4\pi^2}\right)^{1/3}$$

NOW.. $h = r - R$

THEN.. SI: $h = \left[\frac{9.81\frac{m}{s^2}\times(86.1624\times10^3\text{ s})^2\times(6.37\times10^6\text{ m})^2}{4\pi^2}\right]^{1/3} - 6.37\times10^6$ m

$= (42.145 - 6.37)\times10^6$ m

OR $h = 35.77\times10^3$ km ◀

U.S. UNITS: $h = \left[\frac{32.2\frac{ft}{s^2}\times(86.1624\times10^3\text{ s})^2\times(20.9088\times10^6\text{ ft})^2}{4\pi^2}\right]^{1/3} - 20.9088\times10^6$ ft

$= (138.3343 - 20.9088)\times10^6$ ft

OR $h = 22{,}240$ mi ◀

(b) HAVE.. $v = \frac{2\pi r}{\tau}$

THEN.. SI: $v = 2\pi\frac{42.145\times10^6\text{ m}}{86.1624\times10^3\text{ s}}$

OR $v = 3070\frac{m}{s}$ ◀

U.S. UNITS: $v = 2\pi\frac{138.3343\times10^6\text{ ft}}{86.1624\times10^3\text{ s}}$

OR $v = 10{,}090\frac{ft}{s}$ ◀

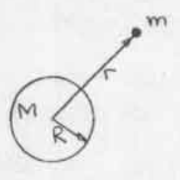
12.81

GIVEN: $r_{MOON} = 238{,}910$ mi, $\tau_{MOON} = 27.32$ DAYS

FIND: MASS M OF THE EARTH

HAVE.. $F = G\frac{Mm}{r^2}$ [EQ. (12.28)]

AND $F = F_n = ma_n = m\frac{v^2}{r}$



THEN $G\frac{Mm}{r^2} = m\frac{v^2}{r}$

OR $M = \frac{r}{G}v^2$

NOW.. $v = \frac{2\pi r}{\tau}$

SO THAT $M = \frac{r}{G}\left(\frac{2\pi r}{\tau}\right)^2 = \frac{1}{G}\left(\frac{2\pi}{\tau}\right)^2 r^3$

NOTING THAT $\tau = 27.32$ DAYS $= 2.3604\times10^6$ s

AND $r = 238{,}910$ mi $= 1.26144\times10^9$ ft

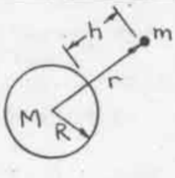
HAVE.. $M = \frac{1}{34.4\times10^{-9}\frac{ft^4}{lb\cdot s^4}}\left(\frac{2\pi}{2.3604\times10^6\text{ s}}\right)^2(1.26144\times10^9\text{ ft})^3$

OR $M = 413\times10^{21}\frac{lb\cdot s^2}{ft}$ ◀

12.82

GIVEN: ALTITUDE $h = 380$ km OF SPACECRAFT IN ORBIT ABOUT MARS; $\rho_{MARS} = 3.94\ \text{Mg/m}^3$
$R_{MARS} = 3397$ km

FIND: (a) TIME τ OF ONE ORBIT
(b) VELOCITY v OF THE SPACECRAFT



(a) FROM THE SOLUTION TO PROBLEM 12.78 HAVE

$$\tau = \sqrt{\frac{3\pi}{G\rho}}\left(\frac{r}{R}\right)^{3/2}$$

WHERE $r = R + h = (3397 + 380)$ km $= 3777$ km

THEN.. $$\tau = \left[\frac{3\pi}{(66.73\times10^{-12}\ \frac{m^3}{kg\cdot s^2})(3.94\times10^3\ \frac{kg}{m^3})}\right]^{1/2}\left(\frac{3777\ km}{3397\ km}\right)^{3/2}$$

$= 7019.5$ s

OR $\tau = 1$ h 57 MIN

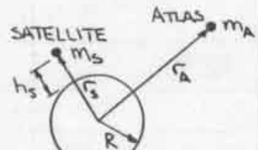
(b) HAVE.. $v = \dfrac{2\pi r}{\tau} = \dfrac{2\pi(3777\times10^3\ m)}{7019.5\ s}$

OR $v = 3380\ \frac{m}{s}$

12.83

GIVEN: ALTITUDE $h_s = 3400$ km OF SATELLITE IN ORBIT ABOUT SATURN; $v_s = 24.45\ \frac{km}{s}$; FOR MOON ATLAS, $r = 137.64\times10^3$ km, $\tau_{ATLAS} = 0.6019$ DAYS

FIND: (a) RADIUS R OF SATURN
(b) MASS M OF SATURN



HAVE.. $F = G\frac{Mm}{r^2}$ [EQ.(12.28)]
AND $F = F_n = ma_n = m\frac{v^2}{r}$
THEN $G\frac{Mm}{r^2} = m\frac{v^2}{r}$
OR $GM = rv^2$
EQ. (12.29): $g = \frac{GM}{R^2}$
AND THEN $gR^2 = rv^2$
OR $v = R\sqrt{\frac{g}{r}}$ (1) AND $R\sqrt{g} = v\sqrt{r} = v\sqrt{R+h}$ (2)

NOW.. $\tau = \frac{2\pi r}{v} = \frac{2\pi r}{R\sqrt{\frac{g}{r}}}$ [USING EQ.(1)]

OR $R\sqrt{g} = \frac{2\pi r^{3/2}}{\tau}$ (3)

(a) USING EQS. (2) AND (3)..

$$R_{SATURN}\sqrt{g_{SATURN}} = v_s\sqrt{R+h_s} = \frac{2\pi r_A^{3/2}}{\tau_A}$$

OR $R = \left(\frac{2\pi r_A^{3/2}}{v_s\tau_A}\right)^2 - h_s$

NOTING THAT $\tau_A = 0.6019$ DAYS $= 52.0042\times10^3$ s

HAVE.. $$R = \left[\frac{2\pi(137.64\times10^6\ m)^{3/2}}{(24.45\times10^3\ \frac{m}{s})(52.0042\times10^3\ s)}\right]^2 - 3400\times10^3\ m$$

$= 60.273\times10^6$ m

OR $R = 60.3\times10^3$ km

(b) FROM ABOVE.. $GM = rv^2$

THEN.. $M = \frac{v_s^2}{G}r_s = \frac{v_s^2}{G}(R+h_s)$

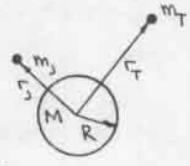
$$= \frac{(24.45\times10^3\ \frac{m}{s})^2}{66.73\times10^{-12}\ \frac{m^3}{kg\cdot s^2}}(60.273\times10^6 + 3400\times10^3)\ m$$

OR $M = 570\times10^{24}$ kg

12.84

GIVEN: FOR THE MOONS JULIET AND TITANIA OF URANUS, $\tau_J = 0.4931$ DAYS, $\tau_T = 8.706$ DAYS, $r_J = 40,000$ mi

FIND: (a) MASS M OF URANUS
(b) r_T



HAVE.. $F = G\frac{Mm}{r^2}$ [EQ.(12.28)]
AND $F = F_n = ma_n = m\frac{v^2}{r}$
THEN $G\frac{Mm}{r^2} = m\frac{v^2}{r}$
OR $M = \frac{r}{G}v^2$
NOW $v = \frac{2\pi r}{\tau}$
SO THAT $M = \frac{r}{G}\left(\frac{2\pi r}{\tau}\right)^2 = \frac{1}{G}\left(\frac{2\pi}{\tau}\right)^2 r^3$ (1)
NOW.. $\tau_J = 0.4931$ DAYS $= 42,604$ s
AND $r_J = 40,000$ mi $= 211.2\times10^6$ ft

(a) USING EQ. (1)..

$$M = \frac{1}{G}\left(\frac{2\pi}{\tau_J}\right)^2 r_J^3 = \frac{1}{34.4\times10^{-9}\ \frac{ft^4}{lb\cdot s^4}}\left(\frac{2\pi}{42,604\ s}\right)^2(211.2\times10^6\ ft)^3$$

OR $M = 5.96\times10^{24}\ \frac{lb\cdot s^2}{ft}$

(b) REWRITING EQ (1)..

$\frac{MG}{4\pi^2} = \frac{r^3}{\tau^2}$ AND THEN $\frac{r_T^3}{\tau_T^2} = \frac{r_J^3}{\tau_J^2}$

OR $r_T = \left(\frac{8.706\ \text{DAYS}}{0.4931\ \text{DAYS}}\right)^{2/3}\times(40,000\ mi)$

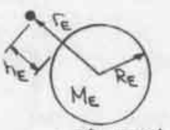
OR $r_T = 271.2\times10^3$ mi

12.85

GIVEN: SPACECRAFT OF WEIGHT $W = 1200$ lb; $h_E = 2800$ mi; $m_{MOON} = 0.01230\,m_{EARTH}$, $R_{MOON} = 1080$ mi

FIND: (a) GRAVITATIONAL FORCE F ON THE SPACECRAFT, EARTH ORBIT
(b) r_M, $\tau_E = \tau_M$
(c) g_{MOON}

FIRST NOTE THAT $R_E = 3960$ mi
THEN $r_E = R_E + h_E = (3960 + 2800)$ mi $= 6760$ mi



(a) HAVE.. $F = G\frac{Mm}{r^2}$ EQ.(12.28)
AND $GM = gR^2$ EQ.(12.29)
THEN.. $F = gR^2\frac{m}{r^2} = W\left(\frac{R}{r}\right)^2$

FOR THE EARTH ORBIT.. $F = (1200\ lb)\left(\frac{3960\ mi}{6760\ mi}\right)^2$

OR $F = 412$ lb

(b) FROM THE SOLUTION TO PROBLEM 12.81 HAVE
$M = \frac{1}{G}\left(\frac{2\pi}{\tau}\right)^2 r^3$
THEN $\tau = \frac{2\pi r^{3/2}}{\sqrt{GM}}$

NOW.. $\tau_E = \tau_M \Rightarrow \frac{2\pi r_E^{3/2}}{\sqrt{GM_E}} = \frac{2\pi r_M^{3/2}}{\sqrt{GM_M}}$ (1)

OR $r_M = \left(\frac{M_M}{M_E}\right)^{1/3} r_E = (0.01230)^{1/3}(6760\ mi)$

OR $r_M = 1560$ mi

(c) HAVE.. $GM = gR^2$ EQ. (12.29)
SUBSTITUTING INTO EQ. (1)

$$\frac{2\pi r_E^{3/2}}{R_E\sqrt{g_E}} = \frac{2\pi r_M^{3/2}}{R_M\sqrt{g_M}}$$

(CONTINUED)

12.85 continued

OR $g_M = \left(\frac{R_E}{R_M}\right)^2\left(\frac{r_m}{r_E}\right)^3 g_E = \left(\frac{R_E}{R_M}\right)^2\left(\frac{M_M}{M_E}\right)g_E$

USING THE RESULTS OF PART (b). THEN..

$g_M = \left(\frac{3960 \text{ mi}}{1080 \text{ mi}}\right)^2(0.01230)(32.2 \frac{ft}{s^2})$

OR $g_{MOON} = 5.32 \frac{ft}{s^2}$ ◀

NOTE: $g_{MOON} \approx \frac{1}{6} g_{EARTH}$

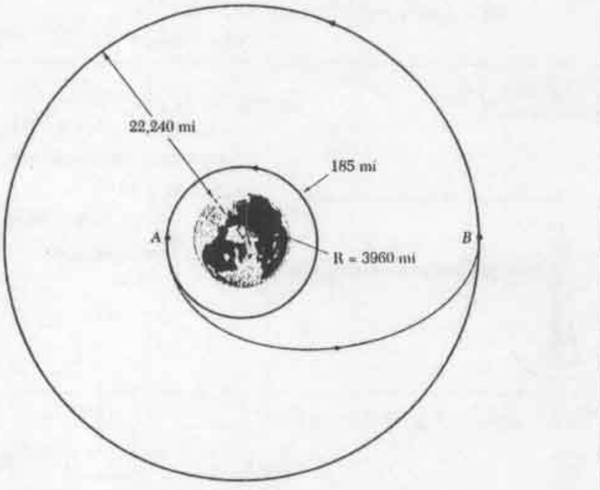
12.86

GIVEN: CIRCULAR ORBITS AND ELLIPTIC TRANSFER ORBIT AB SHOWN;

$\Delta v_B = 4810$ ft/s

FIND: (a) $(v_B)_{TR}$

(b) Δv_A



FIRST NOTE.. $R = 3960$ mi $= 20.9088 \times 10^6$ ft

$r_A = (3960 + 185)$ mi $= 4145$ mi $= 21.8856 \times 10^6$ ft

$r_B = (3960 + 22{,}240)$ mi $= 26{,}200$ mi $= 138.336 \times 10^6$ ft

FOR A CIRCULAR ORBIT.. $\Sigma F_n = ma_n$: $F = m\frac{v^2}{r}$

EQ. (12.28): $F = G\frac{Mm}{r^2}$

THEN $G\frac{Mm}{r^2} = m\frac{v^2}{r}$

OR $v^2 = \frac{GM}{r}$

EQ. (12.29): $GM = gR^2$

SO THAT $v^2 = \frac{gR^2}{r}$ FOR A CIRCULAR ORBIT

THEN.. $(v_A)^2_{CIRC.} = \frac{32.2 \text{ ft/s}^2 \times (20.9088 \times 10^6 \text{ ft})^2}{21.8856 \times 10^6 \text{ ft}}$

OR $(v_A)_{CIRC} = 25{,}362 \frac{ft}{s}$

AND $(v_B)^2_{CIRC} = \frac{32.2 \text{ ft/s}^2 \times (20.9088 \times 10^6 \text{ ft})^2}{138.336 \times 10^6 \text{ ft}}$

OR $(v_B)_{CIRC} = 10{,}088 \frac{ft}{s}$

(a) HAVE.. $(v_B)_{CIRC} = (v_B)_{TR} + \Delta v_B$

OR $(v_B)_{TR} = (10{,}088 - 4810)\frac{ft}{s} = 5278 \frac{ft}{s}$

OR $(v_B)_{TR} = 5280 \frac{ft}{s}$ ◀

(b) CONSERVATION OF ANGULAR MOMENTUM REQUIRES THAT $r_A m(v_A)_{TR} = r_B m(v_B)_{TR}$

OR $(v_A)_{TR} = \frac{26{,}200 \text{ mi}}{4145 \text{ mi}} \times 5278 \frac{ft}{s}$

$= 33{,}362 \frac{ft}{s}$

(CONTINUED)

12.86 continued

NOW.. $(v_A)_{TR} = (v_A)_{CIRC} + \Delta v_A$

OR $\Delta v_A = (33{,}362 - 25{,}362)\frac{ft}{s}$

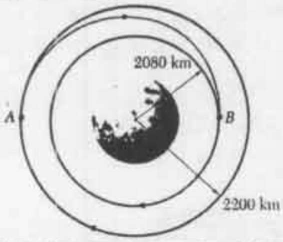
OR $\Delta v_A = 8000 \frac{ft}{s}$ ◀

12.87

GIVEN: CIRCULAR ORBITS ABOUT THE MOON AND ELLIPTIC TRANSFER ORBIT AB AS SHOWN; $\Delta v_A = -26.3 \frac{m}{s}$; $m_{MOON} = 73.49 \times 10^{21}$ kg

FIND: (a) $(v_B)_{TR}$

(b) Δv_B



FOR A CIRCULAR ORBIT.. $\Sigma F_n = ma_n$: $F = m\frac{v^2}{r}$

EQ. (12.28): $F = G\frac{Mm}{r^2}$

THEN.. $G\frac{Mm}{r^2} = m\frac{v^2}{r}$

OR $v^2 = \frac{GM}{r}$

THEN.. $(v_A)^2_{CIRC} = \frac{66.73 \times 10^{-12} \frac{m^3}{kg \cdot s^2} \times 73.49 \times 10^{21} \text{ kg}}{2200 \times 10^3 \text{ m}}$

OR $(v_A)_{CIRC} = 1493.0 \frac{m}{s}$

AND $(v_B)^2_{CIRC} = \frac{66.73 \times 10^{-12} \frac{m^3}{kg \cdot s^2} \times 73.49 \times 10^{21} \text{ kg}}{2080 \times 10^3 \text{ m}}$

OR $(v_B)_{CIRC} = 1535.5 \frac{m}{s}$

(a) HAVE.. $(v_A)_{TR} = (v_A)_{CIRC} + \Delta v_A = (1493.0 - 26.3)\frac{m}{s}$

$= 1466.7 \frac{m}{s}$

CONSERVATION OF ANGULAR MOMENTUM REQUIRES THAT $r_A m(v_A)_{TR} = r_B m(v_B)_{TR}$

OR $(v_B)_{TR} = \frac{2200 \text{ km}}{2080 \text{ km}} \times 1466.7 \frac{m}{s} = 1551.3 \frac{m}{s}$

OR $(v_B)_{TR} = 1551 \frac{m}{s}$ ◀

(b) NOW.. $(v_B)_{CIRC} = (v_B)_{TR} + \Delta v_B$

OR $\Delta v_B = (1535.5 - 1551.3)\frac{m}{s}$

OR $\Delta v_B = -15.8 \frac{m}{s}$ ◀

12.88

GIVEN: CIRCULAR ORBIT ABOUT VENUS AND ELLIPTIC TRANSFER ORBITS AB AND BC;

$r_C = 6420$ km;

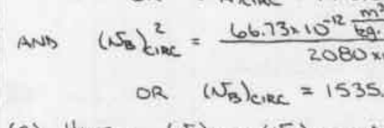
$v_A = 7420 \frac{m}{s}$, $h_A = 288$ km;

$\Delta v_B = 24.5 \frac{m}{s}$,

$\Delta v_C = -264 \frac{m}{s}$;

$m_{VENUS} = 4.869 \times 10^{24}$ kg

$R_{VENUS} = 6052$ km



Circular orbit

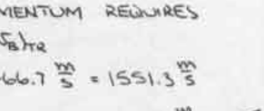
FIND: (a) $(v_B)_{TRAB}$

(b) h_B

FIRST NOTE.. $r_A = R + h_A$

$= (6052 + 288)$ km

$= 6340$ km



FOR A CIRCULAR ORBIT.. $\Sigma F_n = ma_n$: $F = m\frac{v^2}{r}$

EQ. (12.28): $F = G\frac{Mm}{r^2}$

THEN.. $G\frac{Mm}{r^2} = m\frac{v^2}{r}$ OR $v^2 = \frac{GM}{r}$

THEN $(v_C)^2_{CIRC} = \frac{66.73 \times 10^{-12} \frac{m^3}{kg \cdot s^2} \times 4.869 \times 10^{24} \text{ kg}}{6420 \times 10^3 \text{ m}}$

(CONTINUED)

12.88 continued

OR $(v_C)_{CIRC} = 7114.0 \frac{m}{s}$

NOW.. $(v_C)_{CIRC} = (v_C)_{TR_{BC}} + \Delta v_C$

OR $(v_C)_{TR_{BC}} = [7114.0 - (264)]\frac{m}{s} = 7378.0 \frac{m}{s}$

(a) CONSERVATION OF ANGULAR MOMENTUM REQUIRES THAT.. AB: $r_A m(v_A) = r_B m(v_B)_{TR_{AB}}$ (1)

BC: $r_B m(v_B)_{TR_{BC}} = r_C m(v_C)_{TR_{BC}}$ (2)

THEN $\frac{(2)}{(1)} \Rightarrow \frac{r_B (v_B)_{TR_{BC}}}{r_B (v_B)_{TR_{AB}}} = \frac{r_C (v_C)_{TR_{BC}}}{r_A (v_A)}$

NOW.. $(v_B)_{TR_{BC}} = (v_B)_{TR_{AB}} + \Delta v_B$

THEN.. $\frac{(v_B)_{TR_{AB}} + \Delta v_B}{(v_B)_{TR_{AB}}} = \frac{r_C (v_C)_{TR_{BC}}}{r_A (v_A)}$

OR $(v_B)_{TR_{AB}} = \frac{\Delta v_B}{\frac{r_C (v_C)_{TR_{AB}}}{r_A (v_A)} - 1} = \frac{24.5 \frac{m}{s}}{\frac{6420\ km}{6340\ km} \times \frac{7378.0 \frac{m}{s}}{7420 \frac{m}{s}} - 1}$

$= 3557.7 \frac{m}{s}$

OR $(v_B)_{TR_{AB}} = 3560 \frac{m}{s}$ ◀

(b) FROM EQ. (1)..

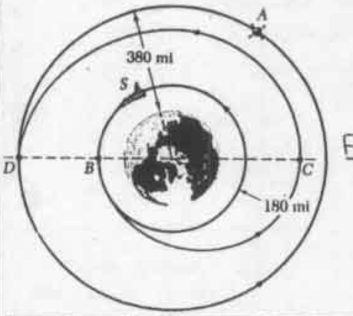
$r_B = \frac{v_A}{(v_B)_{TR_{AB}}} r_A = \frac{7420 \frac{m}{s}}{3557.7 \frac{m}{s}} \times 6340\ km = 13\,223\ km$

NOW.. $r_B = R + h_B$

OR $h_B = (13\,223 - 6052)\ km$

OR $h_B = 7170\ km$ ◀

12.89



GIVEN: CIRCULAR ORBITS A AND B ABOUT THE EARTH AND ELLIPTIC TRANSFER ORBITS BC AND CD; $\Delta v_B = 280 \frac{ft}{s}$, $\Delta v_C = 260 \frac{ft}{s}$; $r_C = 4289$ mi

FIND: Δv_D

FIRST NOTE.. $R = 3960\ mi = 20.9088 \times 10^6\ ft$

$r_A = (3960 + 380)\ mi = 4340\ mi = 22.9152 \times 10^6\ ft$

$r_B = (3960 + 180)\ mi = 4140\ mi = 21.8592 \times 10^6\ ft$

FOR A CIRCULAR ORBIT.. $\Sigma F_n = ma_n$: $F = m \frac{v^2}{r}$

EQ. (12.28): $F = G \frac{Mm}{r^2}$

THEN.. $G \frac{Mm}{r^2} = m \frac{v^2}{r}$

OR $v^2 = \frac{GM}{r} = \frac{gR^2}{r}$ USING EQ. (12.29)

THEN.. $(v_A)^2_{CIRC} = \frac{32.2\ ft/s^2 \times (20.9088 \times 10^6\ ft)^2}{22.9152 \times 10^6\ ft}$

OR $(v_A)_{CIRC} = 24{,}785\ ft/s$

AND $(v_B)^2_{CIRC} = \frac{32.2\ ft/s^2 \times (20.9088 \times 10^6\ ft)^2}{21.8592 \times 10^6\ ft}$

OR $(v_B)_{CIRC} = 25{,}377\ ft/s$

HAVE.. $(v_B)_{TR_{BC}} = (v_B)_{CIRC} + \Delta v_B = (25{,}377 + 280) \frac{ft}{s}$

$= 25{,}657\ ft/s$

(CONTINUED)

12.89 continued

CONSERVATION OF ANGULAR MOMENTUM REQUIRES THAT..

BC: $r_B m(v_B)_{TR_{BC}} = r_C m(v_C)_{TR_{BC}}$ (1)

CD: $r_C m(v_C)_{TR_{CD}} = r_A m(v_D)_{TR_{CD}}$ (2)

FROM EQ (1).. $(v_C)_{TR_{BC}} = \frac{r_B}{r_C}(v_B)_{TR_{BC}} = \frac{4140\ mi}{4289\ mi} \times 25{,}657 \frac{ft}{s}$

$= 24{,}766\ ft/s$

NOW.. $(v_C)_{TR_{CD}} = (v_C)_{TR_{BC}} + \Delta v_C = (24{,}766 + 260) \frac{ft}{s}$

$= 25{,}026\ ft/s$

FROM EQ. (2).. $(v_D)_{TR_{CD}} = \frac{r_C}{r_A}(v_C)_{TR_{CD}} = \frac{4289\ mi}{4340\ mi} \times 25{,}026 \frac{ft}{s}$

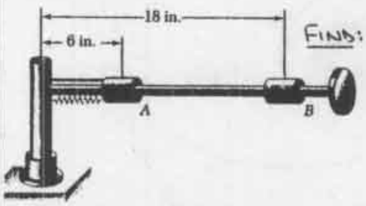
$= 24{,}732 \frac{ft}{s}$

FINALLY.. $(v_A)_{CIRC} = (v_D)_{TR_{CD}} + \Delta v_D$

OR $\Delta v_D = (24{,}785 - 24{,}732)\ ft/s$

OR $\Delta v_D = 53\ ft/s$ ◀

12.90



GIVEN: $r_0 = r_A$, $\dot{\theta}_0 = 16 \frac{RAD}{s}$, $(x_{SP})_0 = 0$; $k = 2\ lb/ft$; NEGLECT FRICTION AND m_{ROD}; $W = 3$ lb

FIND: (a) $(a_A)_r$ AND $(a_A)_\theta$

(b) $(a_{COLLAR/ROD})_A$

(c) $(v_B)_\theta$

FIRST NOTE.. $F_{SP} = k(r - r_A)$

F_{SP} ← [ ] = [ ] → ma_r, ↑ ma_θ

(a) $F_\theta = 0$ AND AT A, $F_r = -F_{SP} = 0$

$\therefore$ $(a_A)_r = 0$ ◀

$(a_A)_\theta = 0$ ◀

(b) $\pm \Sigma F_r = ma_r$: $-F_{SP} = m(\ddot{r} - r\dot{\theta}^2)$

NOTING THAT $a_{COLLAR/ROD} = \ddot{r}$, HAVE AT A..

$0 = m[a_{COLLAR/ROD} - (6\ in.)(16 \frac{RAD}{s})^2]$

OR $(a_{COLLAR/ROD})_A = 1536 \frac{in.}{s^2}$ ◀

(c) AFTER THE CORD IS CUT, THE ONLY HORIZONTAL FORCE ACTING ON THE COLLAR IS DUE TO THE SPRING. THUS, ANGULAR MOMENTUM ABOUT THE SHAFT IS CONSERVED.

$\therefore$ $r_A m(v_A)_\theta = r_B m(v_B)_\theta$ WHERE $(v_A)_\theta = r_A \dot{\theta}_0$

THEN.. $(v_B)_\theta = \frac{6\ in.}{18\ in.}[(6\ in.)(16 \frac{RAD}{s})]$

OR $(v_B)_\theta = 32.0 \frac{in.}{s}$ ◀

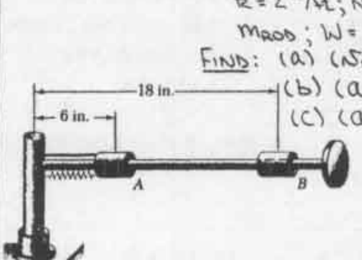
12.91

GIVEN: $r_0 = r_A$, $\dot{\theta}_0 = 12\ \frac{\text{RAD}}{\text{S}}$, $(x_{SP})_0 = 0$; $k = 2\ \text{lb/ft}$; NEGLECT FRICTION AND m_{ROD}; $W = 3\ \text{lb}$

FIND: (a) $(v_B)_\theta$

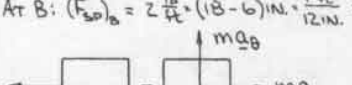
(b) $(a_B)_r$ AND $(a_B)_\theta$

(c) $(a_{COLLAR/ROD})_B$



FIRST NOTE.. $F_{SP} = k(r - r_A)$

AT B: $(F_{SP})_B = 2\ \frac{\text{lb}}{\text{ft}} \times (18-6)\,\text{IN.} \times \frac{1\ \text{ft}}{12\ \text{IN.}} = 2\ \text{lb}$



(a) AFTER THE CORD IS CUT, THE ONLY HORIZONTAL FORCE ACTING ON THE COLLAR IS DUE TO THE SPRING. THUS, ANGULAR MOMENTUM ABOUT THE SHAFT IS CONSERVED.

$\therefore\ r_A m (v_A)_\theta = r_B m (v_B)_\theta \qquad$ WHERE $(v_A)_\theta = r_A \dot{\theta}_0$

THEN.. $(v_B)_\theta = \frac{6\ \text{IN.}}{18\ \text{IN.}}\left[(6\ \text{IN.})(12\ \frac{\text{RAD}}{\text{S}})\right]$

OR $(v_B)_\theta = 24.0\ \frac{\text{IN.}}{\text{S}}$ ◀

(b) HAVE.. $F_\theta = 0 \qquad \therefore\ (a_B)_\theta = 0$ ◀

NOW.. $\xrightarrow{+} \Sigma F_r = m a_r$: $\ -(F_{SP})_B = \frac{W}{g}(a_B)_r$

OR $(a_B)_r = -\frac{2\ \text{lb}}{3\ \text{lb}} \times 32.2\ \frac{\text{ft}}{\text{s}^2} = -21.467\ \frac{\text{ft}}{\text{s}^2} = -257.6\ \frac{\text{IN.}}{\text{S}^2}$

OR $(a_B)_r = -258\ \frac{\text{IN.}}{\text{S}^2}$ ◀

(c) HAVE.. $a_r = \ddot{r} - r\dot{\theta}^2$

NOW.. $a_{COLLAR/ROD} = \ddot{r}$ AND $\dot{\theta}_B = \frac{(v_B)_\theta}{r_B}$

THEN.. AT B: $(a_{COLLAR/ROD})_B = -257.6\ \frac{\text{IN.}}{\text{S}^2} + 18\ \text{IN.} \times \left(\frac{24.0\ \frac{\text{IN.}}{\text{S}}}{18\ \text{IN.}}\right)^2$

OR $(a_{COLLAR/ROD})_B = -226\ \frac{\text{IN.}}{\text{S}^2}$ ◀

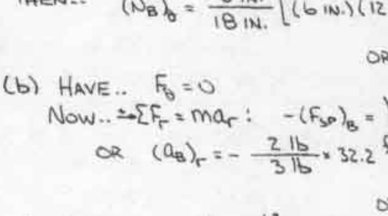
12.92

GIVEN: $m_A = 0.2\ \text{kg}$, $m_B = 0.4\ \text{kg}$, $m_{ROD} = 0$; $(v_A)_0 = 2.5\ \frac{\text{m}}{\text{s}}$; NEGLECT FRICTION; AT $t=0$, BALL B BEGINS TO MOVE FROM B TO C

FIND: (a) $(a_B)_r$ AND $(a_B)_\theta$ AT $t=0$

(b) $a_{B/ROD}$ AT $t=0$

(c) v_A WHEN BALL B IS AT C



(a) WHEN THE PIN HOLDING BALL B IS REMOVED, THERE ARE THEN NO HORIZONTAL FORCES ACTING ON THE BALL. THEREFORE,

AT $t=0$, $F_r = 0$ AND $F_\theta = 0$

(CONTINUED)

12.92 continued

SO THAT $\qquad [(a_B)_r]_0 = 0$ ◀

$[(a_B)_\theta]_0 = 0$ ◀

(b) HAVE.. $a_r = \ddot{r} - r\dot{\theta}^2$

NOW.. $a_{B/ROD} = \ddot{r}$ AND $\dot{\theta} = \frac{v_A}{r_A}$

THEN, AT $t=0$.. $(a_{B/ROD})_0 - (r_B)_0\left[\frac{(v_A)_0}{r_A}\right]^2 = 0$

OR $(a_{B/ROD})_0 = 0.2\ \text{m} \times \left(\frac{2.5\ \frac{\text{m}}{\text{s}}}{0.25\ \text{m}}\right)^2$

OR $(a_{B/ROD})_0 = 20.0\ \frac{\text{m}}{\text{s}^2}$ ◀

(c) NOW, $F_r = 0$ AND $F_\theta = 0$ WHILE B IS MOVING FROM ITS INITIAL TO ITS FINAL POSITION. THUS, ANGULAR MOMENTUM ABOUT THE SHAFT IS CONSERVED. THUS..

$r_A m_A (v_A)_0 + (r_B)_0 m_B (v_B)_0 = r_A m_A v_A' + r_B' m_B v_B'$

WHERE ( )' DENOTES THE STATE WHEN BALL B IS AT C. NOW..

$(v_B)_0 = (r_B)_0 \dot{\theta}_0 = (r_B)_0\left[\frac{(v_A)_0}{r_A}\right]$

AND $v_B' = r_B' \dot{\theta}' = r_B'\left(\frac{v_A'}{r_A}\right)$

THEN.. $r_A m_A (v_A)_0 + (r_B)_0 m_B\left[\frac{(r_B)_0}{r_A}(v_A)_0\right] = r_A m_A v_A' + r_B' m_B\left(\frac{r_B'}{r_A} v_A'\right)$

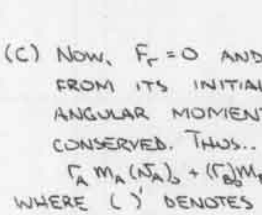
OR $\left\{1 + \frac{m_B}{m_A}\left[\frac{(r_B)_0}{r_A}\right]^2\right\}(v_A)_0 = \left[1 + \frac{m_B}{m_A}\left(\frac{r_B'}{r_A}\right)^2\right] v_A'$

SUBSTITUTING..

$\left[1 + \frac{0.4\ \text{kg}}{0.2\ \text{kg}}\left(\frac{0.2\ \text{m}}{0.25\ \text{m}}\right)^2\right](2.5\ \frac{\text{m}}{\text{s}}) = \left[1 + \frac{0.4\ \text{kg}}{0.2\ \text{kg}}\left(\frac{0.4\ \text{m}}{0.25\ \text{m}}\right)^2\right] v_A'$

OR $v_A' = 0.931\ \frac{\text{m}}{\text{s}}$ ◀

12.93

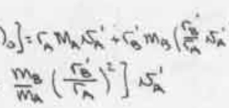


GIVEN: INITIAL STATE OF THE BALL DEFINED BY l_1, θ_1; FIND THE FINAL STATE DEFINED BY l_2, θ_2

FIND: (a) RELATION AMONG l_1, θ_1, l_2, AND θ_2

(b) θ_2 WHEN $l_1 = 0.8\ \text{m}$, $l_2 = 0.6\ \text{m}$, $\theta_1 = 35°$

(a) FOR STATE 1 OR 2..

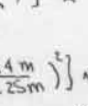


$+\!\uparrow \Sigma F_y = 0$: $\ T\cos\theta - W = 0$

OR $T = \frac{mg}{\cos\theta}$

$\xrightarrow{+} \Sigma F_n = m a_n$: $\ T\sin\theta = m\frac{v^2}{r}$

WHERE $r = l\sin\theta$



THEN $\left(\frac{mg}{\cos\theta}\right)\sin\theta = m\frac{v^2}{l\sin\theta}$

OR $v^2 = g l \sin\theta \tan\theta$

IT THEN FOLLOWS THAT

$$\frac{v_2^2}{v_1^2} = \frac{l_2 \sin\theta_2 \tan\theta_2}{l_1 \sin\theta_1 \tan\theta_1} \qquad (1)$$

NOW.. $\Sigma M_y = 0 \Rightarrow H_y = \text{CONSTANT}$

THUS.. $r_1 m v_1 = r_2 m v_2$

OR $\frac{v_2}{v_1} = \frac{l_1 \sin\theta_1}{l_2 \sin\theta_2} \qquad (2)$

COMBINING EQS. (1) AND (2).. $\left(\frac{l_1 \sin\theta_1}{l_2 \sin\theta_2}\right)^2 = \frac{l_2 \sin\theta_2 \tan\theta_2}{l_1 \sin\theta_1 \tan\theta_1}$

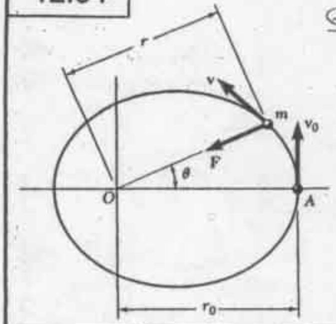
OR $l_1^3 \sin^3\theta_1 \tan\theta_1 = l_2^3 \sin^3\theta_2 \tan\theta_2$ ◀

(b) HAVE.. $(0.8\ \text{m})^3 \sin^3 35° \tan 35° = (0.6\ \text{m})^3 \sin^3\theta_2 \tan\theta_2$

OR $\sin^3\theta_2 \tan\theta_2 = 0.313197$

OR $\theta_2 = 43.6°$ ◀

12.94



GIVEN: PARTICLE OF MASS m MOVING UNDER THE CENTRAL FORCE $\underline{F}$ ALONG THE ELLIPSE $r = r_0/(2-\cos\theta)$; AT $t=0$, $\underline{v} = \underline{v}_0$

SHOW: $F \propto \frac{1}{r^2}$ USING EQ. (12.37)

HAVE $\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2}$ EQ. (12.37)

WHERE $u = \frac{1}{r}$ AND mh^2 = CONSTANT

$\therefore \quad F \propto u^2\left(\frac{d^2u}{d\theta^2} + u\right)$

NOW.. $u = \frac{1}{r} = \frac{1}{r_0}(2-\cos\theta)$

THEN.. $\frac{du}{d\theta} = \frac{d}{d\theta}\left[\frac{1}{r_0}(2-\cos\theta)\right] = \frac{1}{r_0}\sin\theta$

AND $\frac{d^2u}{d\theta^2} = \frac{1}{r_0}\cos\theta$

THEN.. $F \propto \left(\frac{1}{r}\right)^2\left[\left(\frac{1}{r_0}\cos\theta\right) + \frac{1}{r_0}(2-\cos\theta)\right] = \frac{2}{r_0}\frac{1}{r^2}$

$\therefore F \propto \frac{1}{r^2}$ Q.E.D. ◀

NOTE: $F > 0$ IMPLIES THAT $\underline{F}$ IS ATTRACTIVE.

12.95

GIVEN: PARTICLE OF MASS m MOVING UNDER A CENTRAL FORCE $\underline{F}$ ALONG THE PATH $r = r_0\sin\theta$

SHOW: $F \propto \frac{1}{r^5}$ USING EQ. (12.37)

HAVE $\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2}$ EQ. (12.37)

WHERE $u = \frac{1}{r}$ AND mh^2 = CONSTANT

$\therefore \quad F \propto u^2\left(\frac{d^2u}{d\theta^2} + u\right)$

NOW.. $u = \frac{1}{r} = \frac{1}{r_0\sin\theta}$

THEN.. $\frac{du}{d\theta} = \frac{d}{d\theta}\left(\frac{1}{r_0\sin\theta}\right) = -\frac{1}{r_0}\frac{\cos\theta}{\sin^2\theta}$

AND $\frac{d^2u}{d\theta^2} = -\frac{1}{r_0}\left[\frac{-\sin\theta(\sin^2\theta) - \cos\theta(2\sin\theta\cos\theta)}{\sin^4\theta}\right]$

$= \frac{1}{r_0}\frac{1+\cos^2\theta}{\sin^3\theta}$

THEN.. $F \propto \left(\frac{1}{r}\right)^2\left(\frac{1}{r_0}\frac{1+\cos^2\theta}{\sin^3\theta} + \frac{1}{r_0\sin\theta}\right)$

$= \frac{1}{r_0}\frac{1}{r^2}\left(\frac{1+\cos^2\theta}{\sin^3\theta} + \frac{\sin^2\theta}{\sin^3\theta}\right)$

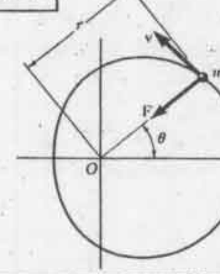
$= \frac{2}{r_0}\frac{1}{r^2}\frac{1}{\sin^3\theta} \qquad \sin^3\theta = \left(\frac{r}{r_0}\right)^3$

$= \frac{2r_0^2}{r^5}$

$\therefore F \propto \frac{1}{r^5}$ Q.E.D. ◀

NOTE: $F > 0$ IMPLIES THAT $\underline{F}$ IS ATTRACTIVE.

12.96



GIVEN: PARTICLE OF MASS m MOVING UNDER THE CENTRAL FORCE $\underline{F}$ ALONG THE CARDIOID $r = \frac{r_0}{2}(1+\cos\theta)$

SHOW: $F \propto \frac{1}{r^4}$ USING EQ. (12.37)

HAVE $\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2}$ EQ. (12.37)

WHERE $u = \frac{1}{r}$ AND mh^2 = CONSTANT

$\therefore \quad F \propto u^2\left(\frac{d^2u}{d\theta^2} + u\right)$

NOW.. $u = \frac{1}{r} = \frac{2}{r_0}\frac{1}{1+\cos\theta}$

THEN.. $\frac{du}{d\theta} = \frac{d}{d\theta}\left(\frac{2}{r_0}\frac{1}{1+\cos\theta}\right) = \frac{2}{r_0}\frac{\sin\theta}{(1+\cos\theta)^2}$

AND $\frac{d^2u}{d\theta^2} = \frac{2}{r_0}\frac{\cos\theta(1+\cos\theta)^2 - \sin\theta[2(1+\cos\theta)](-\sin\theta)}{(1+\cos\theta)^4}$

$= \frac{2}{r_0}\frac{1+\cos\theta+\sin^2\theta}{(1+\cos\theta)^3} = \frac{2}{r_0}\left[\frac{1}{(1+\cos\theta)^2} + \frac{1-\cos^2\theta}{(1+\cos\theta)^3}\right]$

$= \frac{2}{r_0}\frac{2-\cos\theta}{(1+\cos\theta)^2} = \frac{2}{r_0}\left(\frac{r_0}{2r}\right)^2\left[2 - \left(\frac{2r}{r_0} - 1\right)\right]$

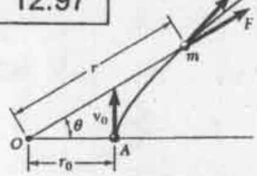
$= \frac{r_0}{2r^2}\left(3 - \frac{2r}{r_0}\right)$

THEN.. $F \propto \left(\frac{1}{r}\right)^2\left[\frac{r_0}{2r^2}\left(3 - \frac{2r}{r_0}\right) + \frac{1}{r}\right] = \frac{3}{2}\frac{r_0}{r^4}$

$\therefore F \propto \frac{1}{r^4}$ Q.E.D. ◀

NOTE: $F > 0$ IMPLIES THAT $\underline{F}$ IS ATTRACTIVE.

12.97



GIVEN: PARTICLE OF MASS m MOVING UNDER THE CENTRAL FORCE $\underline{F}$ ALONG THE PATH $r = r_0/\sqrt{\cos 2\theta}$; AT $t=0$, $\underline{v} = \underline{v}_0$

SHOW: $F \propto r$ USING EQ. (12.37)

HAVE $\frac{d^2u}{d\theta^2} + u = \frac{F}{mh^2u^2}$ EQ. (12.37)

WHERE $u = \frac{1}{r}$ AND mh^2 = CONSTANT

$\therefore \quad F \propto u^2\left(\frac{d^2u}{d\theta^2} + u\right)$

NOW.. $u = \frac{1}{r} = \frac{1}{r_0}\sqrt{\cos 2\theta}$

THEN.. $\frac{du}{d\theta} = \frac{d}{d\theta}\left(\frac{1}{r_0}\sqrt{\cos 2\theta}\right) = -\frac{1}{r_0}\frac{\sin 2\theta}{\sqrt{\cos 2\theta}}$

AND $\frac{d^2u}{d\theta^2} = -\frac{1}{r_0}\frac{2\cos 2\theta\sqrt{\cos 2\theta} - \sin 2\theta\left(-\sin 2\theta/\sqrt{\cos 2\theta}\right)}{\cos 2\theta}$

$= -\frac{1}{r_0}\frac{1+\cos^2 2\theta}{(\cos 2\theta)^{3/2}} = -\frac{1}{r_0}\left(\frac{r}{r_0}\right)^3\left[1 + \left(\frac{r_0}{r}\right)^4\right]$

$= -\frac{r^3}{r_0^4}\left[1 + \left(\frac{r_0}{r}\right)^4\right]$

THEN.. $F \propto \left(\frac{1}{r}\right)^2\left\{-\frac{r^3}{r_0^4}\left[1 + \left(\frac{r_0}{r}\right)^4\right] + \frac{1}{r}\right\} = -\frac{r}{r_0^4}$

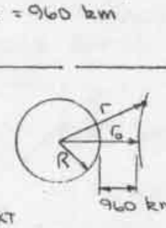
$\therefore F \propto r$ Q.E.D. ◀

NOTE: $F < 0$ IMPLIES THAT $\underline{F}$ IS REPULSIVE.

12.98

GIVEN: PARABOLIC TRAJECTORY OF GALILEO SPACECRAFT ABOUT THE EARTH; MINIMUM ALTITUDE = 960 km

FIND: v_{MAX}



FIRST NOTE.. $R = 6.37\times10^6$ m

SO THAT $r_0 = (6.37\times10^6 + 960\times10^3)$ m $= 7.33\times10^6$ m

NOW.. $v_{MAX} = v_0$

AND FROM PAGE 709 OF THE TEXT

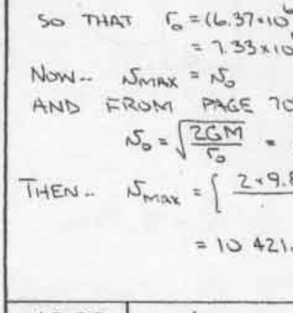
$$v_0 = \sqrt{\frac{2GM}{r_0}} = \sqrt{\frac{2gR^2}{r_0}} \quad \text{USING EQ. (12.30)}$$

THEN.. $v_{MAX} = \left[\dfrac{2\times9.81\,\text{m/s}^2\times(6.37\times10^6\,\text{m})^2}{7.33\times10^6\,\text{m}}\right]^{1/2}$

$= 10\,421.7\ \frac{\text{m}}{\text{s}}$

OR $v_{MAX} = 10.42\ \frac{\text{km}}{\text{s}}$ ◀

12.99



GIVEN: PARABOLIC APPROACH TRAJECTORY AND CIRCULAR ORBIT ABOUT VENUS; $m_{VENUS} = 4.87\times10^{24}$ kg, $R = 6052$ km

FIND: (a) $(v_A)_{PAR}$
(b) $|\Delta v_A|$

FIRST NOTE.. $r_A = (6052 + 280)$ km $= 6332$ km

(a) FROM PAGE 709 OF THE TEXT, THE VELOCITY AT THE POINT OF CLOSEST APPROACH ON A PARABOLIC TRAJECTORY IS GIVEN BY

$$v_0 = \sqrt{\frac{2GM}{r_0}}$$

THUS, $(v_A)_{PAR} = \left[\dfrac{2\times66.73\times10^{-12}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\times4.87\times10^{24}\,\text{kg}}{6332\times10^3\,\text{m}}\right]^{1/2}$

$= 10\,131.4\ \frac{\text{m}}{\text{s}}$

OR $(v_A)_{PAR} = 10.13\ \frac{\text{km}}{\text{s}}$ ◀

(b) HAVE.. $(v_A)_{CIRC} = (v_A)_{PAR} + \Delta v_A$

NOW.. $(v_A)_{CIRC} = \sqrt{\dfrac{GM}{r_0}}$ EQ. (12.44)

$= \frac{1}{\sqrt{2}}(v_A)_{PAR}$

THEN.. $\Delta v_A = \frac{1}{\sqrt{2}}(v_A)_{PAR} - (v_A)_{PAR}$

$= \left(\frac{1}{\sqrt{2}} - 1\right)\left(10.1314\ \frac{\text{km}}{\text{s}}\right)$

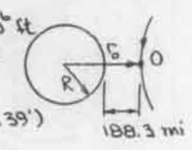
$= -2.97\ \frac{\text{km}}{\text{s}}$

$\therefore |\Delta v_A| = 2.97\ \frac{\text{km}}{\text{s}}$ ◀

12.100

GIVEN: TRAJECTORY OF GALILEO SPACECRAFT ABOUT THE EARTH; AT THE POINT OF CLOSEST APPROACH, $v = 46.2\times10^3\ \frac{\text{ft}}{\text{s}}$, ALTITUDE = 188.3 mi

FIND: ϵ AT POINT OF CLOSEST APPROACH



FIRST NOTE.. $R = 3960$ mi $= 20.9088\times10^6$ ft

AND $r_0 = (3960 + 188.3)$ mi $= 4148.3$ mi $= 21.9030\times10^6$ ft

HAVE.. $\frac{1}{r} = \frac{GM}{h^2}(1 + \epsilon\cos\theta)$ EQ. (12.39')

AT POINT O, $r = r_0$, $\theta = 0$, $h = h_0 = r_0 v_0$

ALSO.. $GM = gR^2$ EQ. (12.30)

THEN.. $\frac{1}{r_0} = \frac{gR^2}{(r_0 v_0)^2}(1 + \epsilon)$

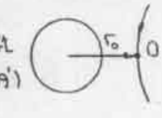
OR $\epsilon = \dfrac{r_0 v_0^2}{gR^2} - 1 = \dfrac{(21.9030\times10^6\,\text{ft})(46.2\times10^3\,\frac{\text{ft}}{\text{s}})^2}{(32.2\,\frac{\text{ft}}{\text{s}^2})(20.9088\times10^6\,\text{ft})^2} - 1$

OR $\epsilon = 2.32$ ◀

12.101

GIVEN: TRAJECTORY OF GALILEO SPACECRAFT ABOUT IO; AT THE POINT OF CLOSEST APPROACH, $r_0 = 1750$ mi, $v_0 = 49.4\times10^3\ \frac{\text{ft}}{\text{s}}$; $M_{Io} = 0.01496\,M_{EARTH}$

FIND: ϵ AT POINT OF CLOSEST APPROACH



FIRST NOTE.. $r_0 = 1750$ mi $= 9.24\times10^6$ ft

$R_{EARTH} = 3960$ mi $= 20.9088\times10^6$ ft

HAVE.. $\frac{1}{r} = \frac{GM}{h^2}(1 + \epsilon\cos\theta)$ EQ. (12.39')

AT POINT O, $r = r_0$, $\theta = 0$, $h = h_0 = r_0 v_0$

ALSO.. $GM_{Io} = G(0.01496\,M_{EARTH})$

$= 0.01496\,gR_{EARTH}^2$ USING EQ (12.30)

THEN.. $\frac{1}{r_0} = \frac{0.01496\,gR_{EARTH}^2}{(r_0 v_0)^2}(1 + \epsilon)$

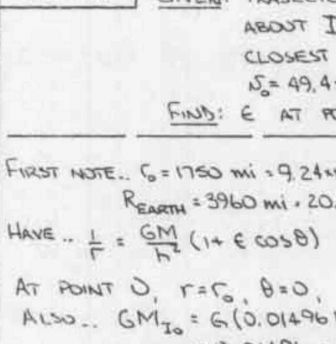
OR $\epsilon = \dfrac{r_0 v_0^2}{0.01496\,gR_{EARTH}^2} - 1 = \dfrac{(9.24\times10^6\,\text{ft})(49.4\times10^3\,\frac{\text{ft}}{\text{s}})^2}{0.01496(32.2\,\text{ft/s}^2)(20.9088\times10^6\,\text{ft})^2} - 1$

OR $\epsilon = 106.1$ ◀

12.102

GIVEN: ELLIPTIC ORBIT OF A SATELLITE ABOUT A PLANET OF MASS M

DERIVE: $\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2}$



HAVE.. $\frac{1}{r} = \frac{GM}{h^2}(1 + \epsilon\cos\theta)$ EQ. (12.39')

NOW.. AT A: $r = r_0$, $\theta = 0$: $\therefore \frac{1}{r_0} = \frac{GM}{h^2}(1 + \epsilon)$ (1)

AT B: $r = r_1$, $\theta = 180°$: $\therefore \frac{1}{r_1} = \frac{GM}{h^2}(1 - \epsilon)$ (2)

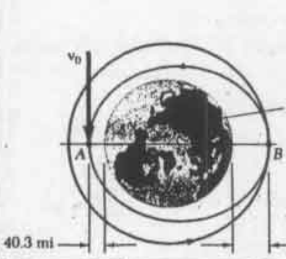
THEN (1) + (2) $\Rightarrow$ $\frac{1}{r_0} + \frac{1}{r_1} = \frac{GM}{h^2}[(1+\epsilon) + (1-\epsilon)]$

OR $\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2}$ Q.E.D. ◀

12.103

GIVEN: ELLIPTIC AND CIRCULAR ORBITS OF THE SPACE SHUTTLE ABOUT THE EARTH; v_0 PERPENDICULAR TO r_0

FIND: (a) v_0 (b) Δv_B



FIRST NOTE.. $r_A = (3960 + 40.3)\text{ mi} = 4000.3 \text{ mi}$
$= 21.1216 \times 10^6 \text{ ft}$
$r_B = (3960 + 336)\text{ mi} = 4296 \text{ mi}$
$= 22.6829 \times 10^6 \text{ ft}$
$R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$

(a) FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR THE ELLIPTIC ORBIT AB..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h^2}$$

NOW.. $h = h_A = r_A v_0$ $\quad GM = gR^2$ [EQ. (12.30)]

THEN.. $\frac{1}{r_A} + \frac{1}{r_B} = \frac{2gR^2}{(r_A v_0)^2}$

OR $v_0 = \frac{R}{r_A}\left(\frac{2g}{\frac{1}{r_A} + \frac{1}{r_B}}\right)^{1/2}$

$$= \frac{3960 \text{ mi}}{4000.3 \text{ mi}}\left(\frac{2 \times 32.2 \text{ ft/s}}{\frac{1}{21.1216 \times 10^6 \text{ ft}} + \frac{1}{22.6829 \times 10^6 \text{ ft}}}\right)^{1/2}$$

$= 26{,}272 \frac{\text{ft}}{\text{s}}$

OR $v_0 = 26.3 \times 10^3 \frac{\text{ft}}{\text{s}}$ ◀

(b) FOR THE ELLIPTIC ORBIT AB HAVE–

$h = h_A = h_B: \quad r_A v_0 = r_B (v_B)_{AB}$

THEN.. $(v_B)_{AB} = \frac{4000.3 \text{ mi}}{4296 \text{ mi}} \times 26{,}272 \frac{\text{ft}}{\text{s}}$
$= 24{,}464 \frac{\text{ft}}{\text{s}}$

FOR THE CIRCULAR ORBIT, USE EQ. (12.44)

$$(v_B)_{CIRC} = \sqrt{\frac{gR^2}{r_B}} = 20.9088 \times 10^6 \text{ ft}\left(\frac{32.2 \text{ ft/s}^2}{22.6829 \times 10^6 \text{ ft}}\right)^{1/2}$$
$= 24{,}912 \frac{\text{ft}}{\text{s}}$

FINALLY.. $(v_B)_{CIRC} = (v_B)_{AB} + \Delta v_B$

OR $\Delta v_B = (24{,}912 - 24{,}464)\text{ ft/s}$

OR $\Delta v_B = 448 \frac{\text{ft}}{\text{s}}$ ◀

12.104

GIVEN: A PLANET OF RADIUS R AND A SPACE PROBE IN A CIRCULAR ORBIT ABOUT THE PLANET AT AN ALTITUDE αR AND HAVING A VELOCITY v_0; ELLIPTIC ORBIT, WHERE $v = \beta v_0$, $\beta < 1$

FIND: β_{MIN} SO THAT THE PROBE DOES NOT CRASH

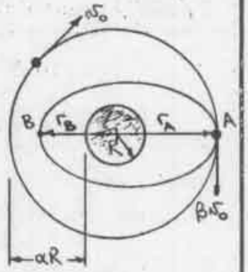
FOR THE CIRCULAR ORBIT -- $v_0 = \sqrt{\frac{GM}{r_A}}$ [EQ. (12.44)]

WHERE $r_A = R + \alpha R = R(1+\alpha)$

THEN.. $GM = v_0^2 R(1+\alpha)$

(CONTINUED)

12.104 continued



FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR THE ELLIPTIC ORBIT..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h^2}$$

NOW.. $h = h_A = r_A (v_A)_{AB}$
$= [R(1+\alpha)](\beta v_0)$

THEN..

$$\frac{1}{R(1+\alpha)} + \frac{1}{r_B} = \frac{2v_0^2 R(1+\alpha)}{[R(1+\alpha)\beta v_0]^2}$$
$$= \frac{2}{\beta^2 R(1+\alpha)}$$

NOW.. β_{MIN} CORRESPONDS TO $r_B \to R$. THEN..

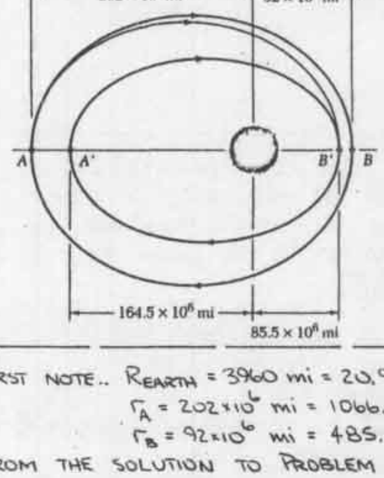
$$\frac{1}{R(1+\alpha)} + \frac{1}{R} = \frac{2}{\beta_{MIN}^2 R(1+\alpha)}$$

OR $\beta_{MIN} = \sqrt{\frac{2}{2+\alpha}}$ ◀

12.105

GIVEN: ELLIPTIC ORBITS AB AND A'B' OF A SPACECRAFT ABOUT THE SUN AND THE ELLIPTIC TRANSFER ORBIT AB'; $M_{SUN} = (332.8 \times 10^3) M_{EARTH}$

FIND: (a) v_A (ON AB)
(b) $|\Delta v_A|$ AND $|\Delta v_B|$



FIRST NOTE.. $R_{EARTH} = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$
$r_A = 202 \times 10^6 \text{ mi} = 1066.56 \times 10^9 \text{ ft}$
$r_B = 92 \times 10^6 \text{ mi} = 485.76 \times 10^9 \text{ ft}$

FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR ANY ELLIPTIC ORBIT ABOUT THE SUN..

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2GM_{SUN}}{h^2}$$

(a) FOR THE ELLIPTIC ORBIT AB HAVE..

$r_1 = r_A, \quad r_2 = r_B, \quad h = h_A = r_A v_A$

ALSO.. $GM_{SUN} = G[(332.8 \times 10^3) M_{EARTH}]$
$= gR_{EARTH}^2 (332.8 \times 10^3)$ USING EQ. (12.30)

THEN.. $\frac{1}{r_A} + \frac{1}{r_B} = \frac{2gR_{EARTH}^2 (332.8 \times 10^3)}{(r_A v_A)^2}$

OR $v_A = \frac{R_{EARTH}}{r_A}\left(\frac{665.6 \times 10^3}{\frac{1}{r_A} + \frac{1}{r_B}}\right)^{1/2}$

$$= \frac{3960 \text{ mi}}{202 \times 10^6 \text{ mi}}\left(\frac{665.6 \times 10^3 \times 32.2 \text{ ft/s}^2}{\frac{1}{1066.56 \times 10^9 \text{ ft}} + \frac{1}{485.76 \times 10^9 \text{ ft}}}\right)^{1/2}$$

$= 52{,}431 \frac{\text{ft}}{\text{s}}$

OR $v_A = 52.4 \times 10^3 \frac{\text{ft}}{\text{s}}$ ◀

(CONTINUED)

12.105 continued

(b) FROM PART (a) HAVE

$$2GM_{SUN} = (r_A v_A)^2\left(\frac{1}{r_A} + \frac{1}{r_B}\right)$$

THEN, FOR ANY OTHER ELLIPTIC ORBIT ABOUT THE SUN HAVE..

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{(r_A v_A)^2\left(\frac{1}{r_A} + \frac{1}{r_B}\right)}{h^2}$$

FOR THE ELLIPTIC TRANSFER ORBIT AB' HAVE..

$r_1 = r_A,\ r_2 = r_{B'},\ h = h_{TR} = r_A (v_A)_{TR}$

THEN.. $$\frac{1}{r_A} + \frac{1}{r_{B'}} = \frac{(r_A v_A)^2\left(\frac{1}{r_A} + \frac{1}{r_B}\right)}{[r_A (v_A)_{TR}]^2}$$

OR $$(v_A)_{TR} = v_A\left(\frac{\frac{1}{r_A} + \frac{1}{r_B}}{\frac{1}{r_A} + \frac{1}{r_{B'}}}\right)^{1/2} = v_A\left(\frac{1 + \frac{r_A}{r_B}}{1 + \frac{r_A}{r_{B'}}}\right)^{1/2}$$

$$= \left(52{,}431\ \tfrac{ft}{s}\right)\left(\frac{1 + \frac{202}{92}}{1 + \frac{202}{85.5}}\right)^{1/2}$$

$$= 51{,}113\ \tfrac{ft}{s}$$

NOW.. $h_{TR} = (h_A)_{TR} = (h_{B'})_{TR}:\ r_A (v_A)_{TR} = r_{B'} (v_{B'})_{TR}$

THEN $$(v_{B'})_{TR} = \frac{202\times10^6\ mi}{85.5\times10^6\ mi} \times 51{,}113\ \tfrac{ft}{s} = 120{,}758\ \tfrac{ft}{s}$$

FOR THE ELLIPTIC ORBIT A'B' HAVE..

$r_1 = r_{A'},\ r_2 = r_{B'},\ h = r_{B'} v_{B'}$

THEN.. $$\frac{1}{r_{A'}} + \frac{1}{r_{B'}} = \frac{(r_A v_A)^2\left(\frac{1}{r_A} + \frac{1}{r_B}\right)}{(r_{B'} v_{B'})^2}$$

OR $$v_{B'} = v_A \frac{r_A}{r_{B'}}\left(\frac{\frac{1}{r_A} + \frac{1}{r_B}}{\frac{1}{r_{A'}} + \frac{1}{r_{B'}}}\right)^{1/2}$$

$$= \left(52{,}431\ \tfrac{ft}{s}\right)\frac{202\times10^6\ mi}{85.5\times10^6\ mi}\left(\frac{\frac{1}{202\times10^6} + \frac{1}{92\times10^6}}{\frac{1}{164.5\times10^6} + \frac{1}{85.5\times10^6}}\right)^{1/2}$$

$$= 116{,}862\ \tfrac{ft}{s}$$

FINALLY.. $(v_A)_{TR} = v_A + \Delta v_A$

OR $\Delta v_A = (51{,}113 - 52{,}431)\ \tfrac{ft}{s}$

OR $|\Delta v_A| = 1318\ \tfrac{ft}{s}$ ◀

AND $v_{B'} = (v_{B'})_{TR} + \Delta v_B$

OR $\Delta v_{B'} = (116{,}862 - 120{,}758)\ \tfrac{ft}{s} = -3896\ \tfrac{ft}{s}$

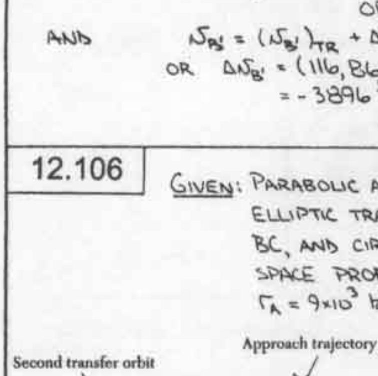
OR $|\Delta v_B| = 3900\ \tfrac{ft}{s}$ ◀

12.106

GIVEN: PARABOLIC APPROACH TRAJECTORY, ELLIPTIC TRANSFER ORBITS AB AND BC, AND CIRCULAR ORBIT OF A SPACE PROBE ABOUT MARS;

$r_A = 9\times10^3$ km, $r_B = 180\times10^3$ km; $M_{MARS} = 0.1074\, M_{EARTH}$

FIND: (a) $|\Delta v_A|$
(b) $|\Delta v_B|$
(c) $|\Delta v_C|$



(CONTINUED)

12.106 continued

(a) FOR THE PARABOLIC APPROACH TRAJECTORY, POINT A IS THE POINT OF CLOSEST APPROACH. THEN, FROM PAGE 709 OF THE TEXT HAVE

$$(v_A)_{PAR} = \sqrt{\frac{2GM_{MARS}}{r_A}}$$

NOW.. $GM_{MARS} = G(0.1074\, M_{EARTH})$
$= 0.1074\, g R_{EARTH}^2$ USING EQ. (12.30)

THEN.. $$(v_A)_{PAR} = R_{EARTH}\left(\frac{2\times0.1074\, g}{r_A}\right)^{1/2} = (6.37\times10^6\ m)\left(\frac{0.2148\times9.81\ m/s^2}{9\times10^6\ m}\right)^{1/2}$$

$$= 3082.3\ \tfrac{m}{s}$$

FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR ANY ELLIPTIC ORBIT ABOUT MARS..

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2GM_{MARS}}{h^2} \qquad (1)$$

FROM ABOVE.. $2GM_{MARS} = r_A[(v_A)_{PAR}]^2$

THEN.. FOR THE ELLIPTIC TRANSFER ORBIT AB..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A[(v_A)_{PAR}]^2}{h_{AB}^2}$$

WHERE $h_{AB} = (h_A)_{AB} = r_A (v_A)_{AB}$

THEN.. $$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A[(v_A)_{PAR}]^2}{[r_A(v_A)_{AB}]^2}$$

OR $$(v_A)_{AB} = \frac{(v_A)_{PAR}}{\sqrt{r_A}}\left(\frac{1}{\frac{1}{r_A} + \frac{1}{r_B}}\right)^{1/2} = (v_A)_{PAR}\left(\frac{1}{1 + \frac{r_A}{r_B}}\right)^{1/2}$$

$$= \left(3082.3\ \tfrac{m}{s}\right)\left(\frac{1}{1 + \frac{9\times10^3\ km}{180\times10^3\ km}}\right)^{1/2}$$

$$= 3008.0\ \tfrac{m}{s}$$

FINALLY.. $(v_A)_{AB} = (v_A)_{PAR} + \Delta v_A$

OR $\Delta v_A = (3008.0 - 3082.3)\ \tfrac{m}{s}$

OR $|\Delta v_A| = 74.3\ \tfrac{m}{s}$ ◀

(b) FOR THE ELLIPTIC TRANSFER ORBIT AB..

$h_{AB} = (h_A)_{AB} = (h_B)_{AB}:\ r_A (v_A)_{AB} = r_B (v_B)_{AB}$

THEN.. $$(v_B)_{AB} = \frac{9\times10^3\ km}{180\times10^3\ km} \times 3008.0\ \tfrac{m}{s} = 150.40\ \tfrac{m}{s}$$

NOW APPLY EQ. (1) TO THE SECOND ELLIPTIC TRANSFER ORBIT BC AND USE

$$h_{BC} = r_B (v_B)_{BC}$$

THEN.. $$\frac{1}{r_B} + \frac{1}{r_C} = \frac{r_A[(v_A)_{PAR}]^2}{[r_B(v_B)_{BC}]^2}$$

OR $$(v_B)_{BC} = \frac{(v_A)_{PAR}}{r_B}\left(\frac{r_A}{\frac{1}{r_B} + \frac{1}{r_C}}\right)^{1/2}$$

$$= \frac{3082.3\ \frac{m}{s}}{180\times10^3\ km}\left(\frac{9\times10^3\ km}{\frac{1}{180\times10^3\ km} + \frac{1}{4\times10^3\ km}}\right)^{1/2}$$

$$= 101.62\ \tfrac{m}{s}$$

FINALLY.. $(v_B)_{BC} = (v_B)_{AB} + \Delta v_B$

OR $\Delta v_B = (101.62 - 150.40)\ \tfrac{m}{s}$

OR $|\Delta v_B| = 48.8\ \tfrac{m}{s}$ ◀

(c) FOR THE ELLIPTIC TRANSFER ORBIT BC..

$h_{BC} = (h_B)_{BC} = (h_C)_{BC}:\ r_B (v_B)_{BC} = r_C (v_C)_{BC}$

THEN.. $$(v_C)_{BC} = \frac{180\times10^3\ km}{4\times10^3\ km} \times 101.62\ \tfrac{m}{s} = 4572.9\ \tfrac{m}{s}$$

FOR THE CIRCULAR ORBIT HAVE..

$$(v_C)_{CIRC} = \sqrt{\frac{GM_{MARS}}{r_C}} \qquad [\text{EQ. } (12.44)]$$

(CONTINUED)

12.106 continued

RECALLING FROM PART (a) THAT $(v_A)_{PAR} = \sqrt{\frac{2GM_{MARS}}{r_A}}$ HAVE

$$(v_C)_{CIRC} = (v_A)_{PAR}\left(\frac{r_A}{2r_C}\right)^{1/2}$$
$$= (3082.3\ \tfrac{m}{s})\left(\frac{9\times10^3\ km}{2\times4\times10^3\ km}\right)^{1/2}$$
$$= 3269.3\ \tfrac{m}{s}$$

FINALLY .. $(v_C)_{CIRC} = (v_C)_{BC} + \Delta v_C$

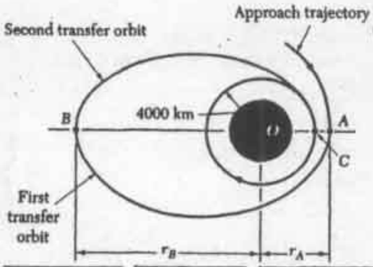
OR $\Delta v_C = (3269.3 - 4572.9)\ \frac{m}{s}$

OR $|\Delta v_C| = 1304\ \frac{m}{s}$ ◀

12.107

GIVEN: PARABOLIC APPROACH TRAJECTORY, ELLIPTIC TRANSFER ORBITS AB AND BC, AND CIRCULAR ORBIT OF A SPACE PROBE ABOUT MARS; $M_{MARS} = 0.1074\ M_{EARTH}$; $r_A = 9\times10^3$ km, $\Delta v_A = -440\ \frac{m}{s}$

FIND: (a) r_B (b) $|\Delta v_B|$ AND $|\Delta v_C|$



(a) FOR THE PARABOLIC APPROACH TRAJECTORY, POINT A IS THE POINT OF CLOSEST APPROACH. THEN, FROM PAGE 709 OF THE TEXT HAVE

$$(v_A)_{PAR} = \sqrt{\frac{2GM_{MARS}}{r_A}}$$

NOW .. $GM_{MARS} = G(0.1074\ M_{EARTH})$
$= 0.1074\ gR^2_{EARTH}$ USING EQ. (12.30)

THEN .. $(v_A)_{PAR} = R_{EARTH}\left(\frac{2\times0.1074 g}{r_A}\right)^{1/2}$
$$= (6.37\times10^6\ m)\left(\frac{0.2148\times9.81\ m/s^2}{9\times10^6\ m}\right)^{1/2}$$
$$= 3082.3\ \tfrac{m}{s}$$

NOW .. $(v_A)_{AB} = (v_A)_{PAR} + \Delta v_A = (3082.3 - 440)\ \frac{m}{s}$
$= 2642.3\ \frac{m}{s}$

FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR ANY ELLIPTIC ORBIT ABOUT MARS ..

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{2GM_{MARS}}{h^2} \qquad (1)$$

FROM ABOVE .. $2GM_{MARS} = r_A[(v_A)_{PAR}]^2$

THEN .. FOR THE ELLIPTIC TRANSFER ORBIT AB ..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A[(v_A)_{PAR}]^2}{h^2_{AB}}$$

WHERE $h_{AB} = (h_A)_{AB} = r_A(v_A)_{AB}$

THEN .. $\frac{1}{r_A} + \frac{1}{r_B} = \frac{r_A[(v_A)_{PAR}]^2}{[r_A(v_A)_{AB}]^2}$
$$= \left[\frac{(v_A)_{PAR}}{(v_A)_{AB}}\right]^2 \frac{1}{r_A}$$

OR $\frac{1}{r_B} = \frac{1}{r_A}\left\{\left[\frac{(v_A)_{PAR}}{(v_A)_{AB}}\right]^2 - 1\right\} = \frac{1}{9\times10^3\ km}\left[\left(\frac{3082.3\ \frac{m}{s}}{2642.3\ \frac{m}{s}}\right)^2 - 1\right]$

OR $r_B = 24.946\times10^3$ km

OR $r_B = 24.9\times10^3$ km ◀

(b) FOR THE ELLIPTIC TRANSFER ORBIT AB ..

$h_{AB} = (h_A)_{AB} = (h_B)_{AB}$: $r_A(v_A)_{AB} = r_B(v_B)_{AB}$

(CONTINUED)

12.107 continued

THEN .. $(v_B)_{AB} = \frac{9\times10^3\ km}{24.946\times10^3\ km}\times 2642.3\ \frac{m}{s}$
$= 953.3\ \frac{m}{s}$

NOW APPLY EQ. (1) TO THE SECOND ELLIPTIC TRANSFER ORBIT BC AND USE

$$h_{BC} = r_B(v_B)_{BC}$$

THEN .. $\frac{1}{r_B} + \frac{1}{r_C} = \frac{r_A[(v_A)_{PAR}]^2}{[r_B(v_B)_{BC}]^2}$

OR $(v_B)_{BC} = \frac{(v_A)_{PAR}}{r_B}\left(\frac{r_A}{\frac{1}{r_B}+\frac{1}{r_C}}\right)^{1/2}$
$$= \frac{3082.3\ \frac{m}{s}}{24.946\times10^3\ km}\left(\frac{9\times10^3\ km}{\frac{1}{24.946\times10^3\ km} + \frac{1}{4\times10^3\ km}}\right)^{1/2}$$
$$= 688.2\ \tfrac{m}{s}$$

THEN .. $(v_B)_{BC} = (v_B)_{AB} + \Delta v_B$

OR $\Delta v_B = (688.2 - 953.3)\ \frac{m}{s}$

OR $|\Delta v_B| = 265\ \frac{m}{s}$ ◀

NOW .. FOR THE ELLIPTIC TRANSFER ORBIT BC ..

$h_{BC} = (h_B)_{BC} = (h_C)_{BC}$: $r_B(v_B)_{BC} = r_C(v_C)_{BC}$

THEN .. $(v_C)_{BC} = \frac{24.946\times10^3\ km}{4\times10^3\ km}\times 688.2\ \frac{m}{s}$
$= 4292.0\ \frac{m}{s}$

FOR THE CIRCULAR ORBIT HAVE ..

$$(v_C)_{CIRC} = \sqrt{\frac{GM_{MARS}}{r_C}} \qquad [\text{EQ. } (12.44)]$$

RECALLING FROM PART (a) THAT $(v_A)_{PAR} = \sqrt{\frac{2GM_{MARS}}{r_A}}$ HAVE

$$(v_C)_{CIRC} = (v_A)_{PAR}\left(\frac{r_A}{2r_C}\right)^{1/2}$$
$$= (3082.3\ \tfrac{m}{s})\left(\frac{9\times10^3\ km}{2\times4\times10^3\ km}\right)^{1/2}$$
$$= 3269.3\ \tfrac{m}{s}$$

FINALLY .. $(v_C)_{CIRC} = (v_C)_{BC} + \Delta v_C$

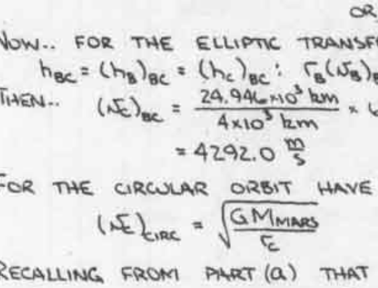
OR $\Delta v_C = (3269.3 - 4292.0)\ \frac{m}{s}$

OR $|\Delta v_C| = 1023\ \frac{m}{s}$ ◀

12.108

GIVEN: ELLIPTIC TRANSFER ORBIT AB OF PROBLEM 12.106; $r_A = 9\times10^3$ km, $r_B = 180\times10^3$ km

FIND: t_{AB}



FROM THE SOLUTION TO PROBLEM 12.106 HAVE

$$(v_A)_{AB} = 3008.0\ \tfrac{m}{s}$$

FROM EQ. (12.45) IT FOLLOWS THAT

$$t_{AB} = \tfrac{1}{2}(\tau_{ELLIPSE})_{AB} = \frac{\pi ab}{h_{AB}}$$

WHERE $a = \frac{1}{2}(r_A + r_B) = \frac{1}{2}(9\times10^3 + 180\times10^3) = 94.5\times10^3$ km

AND $b = \sqrt{r_A r_B} = [(9\times10^3)(180\times10^3)]^{1/2} = 40.249\times10^3$ km

ALSO .. $h_{AB} = r_A(v_A)_{AB} = (9\times10^6\ m)(3008.0\ \frac{m}{s}) = 27.072\times10^9\ \frac{m^2}{s}$

(CONTINUED)

12.108 continued

THEN.. $t_{AB} = \dfrac{\pi(94.5\times10^6 \text{ m})(40.249\times10^6 \text{ m})}{27.072\times10^9 \frac{\text{m}^2}{\text{s}}}$

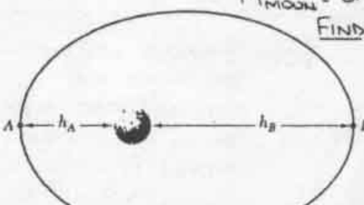
$= 441.384\times10^3 \text{ s}$

OR $t_{AB} = 122 \text{ h } 36 \text{ MIN } 24 \text{ S}$

12.109

GIVEN: ELLIPTIC ORBIT OF THE CLEMENTINE SPACECRAFT ABOUT THE MOON; $h_A = 400$ km, $h_B = 2940$ km; $R_{MOON} = 1737$ km, $M_{MOON} = 0.01230\, M_{EARTH}$

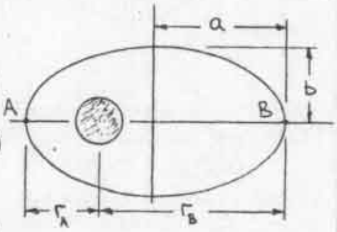
FIND: PERIODIC TIME τ



FIRST NOTE..

$r_A = (1737+400) = 2137$ km

$r_B = (1737+2940) = 4677$ km



NOW.. $\tau = \dfrac{2\pi ab}{h}$ EQ.(12.45)

WHERE $a = \frac{1}{2}(r_A + r_B)$
$= \frac{1}{2}(2137+4677)$ km
$= 3407$ km

AND $b = \sqrt{r_A r_B}$

FROM THE SOLUTION TO PROBLEM 12.102 HAVE..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM_{MOON}}{h^2}$$

NOW.. $GM_{MOON} = G(0.01230 M_{EARTH})$
$= 0.01230\, gR^2_{EARTH}$ USING EQ.(12.30)

THEN.. $h^2 = \dfrac{2(0.01230\, gR^2_{EARTH})}{\frac{1}{r_A}+\frac{1}{r_B}} = \dfrac{0.01230\, gR^2_{EARTH}}{\frac{r_A+r_B}{r_A r_B}}$

$= \dfrac{b^2}{2a}(2\times0.01230\, gR^2_{EARTH})$

OR $h = bR_{EARTH}\left(\dfrac{0.01230\, g}{a}\right)^{1/2}$

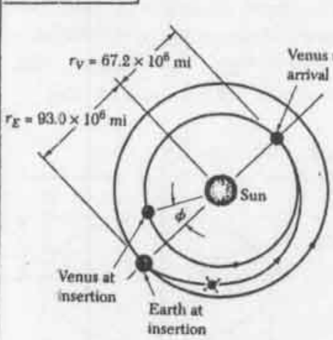
THEN.. $\tau = \dfrac{2\pi ab}{bR_{EARTH}\left(\frac{0.01230g}{a}\right)^{1/2}} = \dfrac{2\pi a^{3/2}}{R_{EARTH}(0.01230g)^{1/2}}$

$= \dfrac{2\pi(3407\times10^3 \text{ m})^{3/2}}{(6.37\times10^6 \text{ m})(0.01230\times9.81 \text{ m/s}^2)^{1/2}}$

$= 17.8571\times10^3$ s

OR $\tau = 4 \text{ h } 57 \text{ MIN } 37 \text{ S}$

12.110



GIVEN: ORBITS OF VENUS AND THE EARTH AND THE ELLIPTIC TRANSFER ORBIT OF A SPACE PROBE; $M_{SUN} = 332.8\times10^3 M_{EARTH}$

FIND: ϕ, THE RELATIVE POSITION OF VENUS WITH RESPECT TO THE EARTH AT THE TIME OF INSERTION

FIRST DETERMINE THE TIME t_{PROBE} FOR THE PROBE TO TRAVEL FROM THE EARTH TO VENUS. NOW..

$$t_{PROBE} = \tfrac{1}{2}\tau_{TR}$$

WHERE τ_{TR} IS THE PERIODIC TIME OF THE ELLIPTIC TRANSFER ORBIT. APPLYING KEPLER'S THIRD LAW TO THE ORBITS ABOUT THE SUN OF THE EARTH AND THE PROBE OBTAIN..

$$\frac{\tau^2_{TR}}{\tau^2_{EARTH}} = \frac{a^3_{TR}}{a^3_{EARTH}}$$

WHERE $a_{TR} = \frac{1}{2}(r_E + r_V) = \frac{1}{2}(93\times10^6 + 67.2\times10^6)$ mi
$= 80.1\times10^6$ mi

AND $a_{EARTH} \approx r_E$ (NOTE: $\epsilon_{EARTH} = 0.0167$)

THEN.. $t_{PROBE} = \frac{1}{2}\left(\dfrac{a_{TR}}{r_E}\right)^{3/2}\tau_{EARTH}$
$= \frac{1}{2}\left(\dfrac{80.1\times10^6 \text{ mi}}{93.0\times10^6 \text{ mi}}\right)^{3/2}(365.25 \text{ DAYS})$
$= 145.977$ DAYS
$= 12.6124\times10^6$ s

IN TIME t_{PROBE}, VENUS TRAVELS THROUGH THE ANGLE θ_V GIVEN BY

$$\theta_V = \dot{\theta}_V t_{PROBE} = \frac{v_V}{r_V} t_{PROBE}$$

ASSUMING THAT THE ORBIT OF VENUS IS CIRCULAR (NOTE: $\epsilon_{VENUS} = 0.0068$). THEN, FOR A CIRCULAR ORBIT..

$$v_V = \sqrt{\frac{GM_{SUN}}{r_V}} \qquad [\text{EQ.}(12.44)]$$

NOW.. $GM_{SUN} = G(332.8\times10^3 M_{EARTH})$
$= 332.8\times10^3(gR^2_{EARTH})$ USING EQ.(12.30)

THEN.. $\theta_V = \dfrac{t_{PROBE}}{r_V}\left[\dfrac{332.8\times10^3 gR^2_{EARTH}}{r_V}\right]^{1/2}$
$= t_{PROBE} R_{EARTH} \dfrac{(332.8\, g\times10^3)^{1/2}}{r_V^{3/2}}$

WHERE $R_{EARTH} = 3960 \text{ mi} = 20.9088\times10^6$ ft

AND $r_V = 67.2\times10^6 \text{ mi} = 354.816\times10^9$ ft

THEN.. $\theta_V = (12.6124\times10^6 \text{ s})(20.9088\times10^6 \text{ ft})\dfrac{(332.8\times10^3\times32.2 \text{ ft/s}^2)^{1/2}}{(354.816\times10^9 \text{ ft})^{3/2}}$
$= 4.0845$ RAD
$= 234.02°$

FINALLY.. $\phi = \theta_V - 180°$
$= 234.02° - 180°$

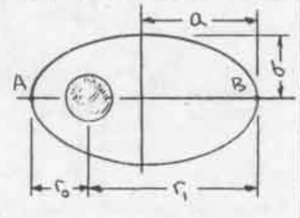
OR $\phi = 54.0°$

12.111

<u>GIVEN</u>: ELLIPTIC ORBIT ABOUT THE SUN OF THE COMET HYAKUTAKE; $\epsilon = 0.999887$, $r_{MIN} = 0.230 R_E$; $R_E = a_{EARTH}$ FOR THE EARTH'S ORBIT ABOUT THE SUN

<u>FIND</u>: τ FOR THE COMET

USING EQ. (12.39') HAVE FOR ANY ELLIPTIC ORBIT ABOUT THE SUN--



$$\frac{1}{r} = \frac{GM_{SUN}}{h^2}(1 + \epsilon\cos\theta)$$

AT A, $\theta = 0$:

$$\frac{1}{r_0} = \frac{GM_{SUN}}{h^2}(1+\epsilon) \quad (1)$$

AT B, $\theta = 180°$: $\frac{1}{r_1} = \frac{GM_{SUN}}{h^2}(1-\epsilon) \quad (2)$

FORMING $\frac{(1)}{(2)} \Rightarrow \frac{\frac{1}{r_0}}{\frac{1}{r_1}} = \frac{1+\epsilon}{1-\epsilon}$ OR $r_1 = \frac{1+\epsilon}{1-\epsilon} r_0$

NOW-- $a = \frac{1}{2}(r_0 + r_1) = \frac{1}{2}(r_0 + \frac{1+\epsilon}{1-\epsilon} r_0) = \frac{r_0}{1-\epsilon}$

APPLYING KEPLER'S THIRD LAW TO THE ORBITS ABOUT THE SUN OF THE EARTH AND THE COMET HAVE--

$$\frac{\tau^2_{COMET}}{\tau^2_{EARTH}} = \frac{a^3_{COMET}}{a^3_{EARTH}}$$

FROM ABOVE-- $a_{COMET} = \frac{(r_0)_{COMET}}{1-\epsilon_{COMET}} = \frac{(r_{MIN})_{COMET}}{1-\epsilon_{COMET}} = \frac{0.230 R_E}{1-\epsilon_{COMET}}$

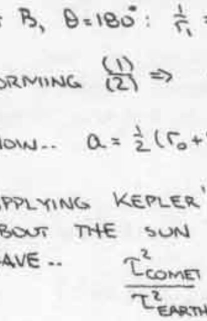
AND $a_{EARTH} = R_E$

THEN-- $\frac{\tau^2_{COMET}}{\tau^2_{EARTH}} = \left(\frac{\frac{0.230 R_E}{1-\epsilon_{COMET}}}{R_E}\right)^3 = \left(\frac{0.230}{1-\epsilon_{COMET}}\right)^3$

OR $\tau_{COMET} = \left(\frac{0.230}{1-0.999887}\right)^{3/2}(1\ yr)$

OR $\tau_{COMET} = 91.8 \times 10^3\ yr$ ◀

12.112



<u>GIVEN</u>: PARABOLIC TRAJECTORY OF THE GALILEO SPACECRAFT ABOUT THE EARTH; $v_A = 10.42\ \frac{km}{s}$

<u>FIND</u>: t_{BC}

HAVE-- $\frac{1}{r} = \frac{GM}{h^2}(1 + \epsilon\cos\theta)$ EQ. (12.39')

FOR A PARABOLIC TRAJECTORY, $\epsilon = 1$

NOW-- AT A, $\theta = 0$: $\frac{1}{r_A} = \frac{GM}{h^2}(1+1)$ OR $r_A = \frac{h^2}{2GM}$

AT C, $\theta = 90°$: $\frac{1}{r_C} = \frac{GM}{h^2}(1+0)$ OR $r_C = \frac{h^2}{GM}$

$\therefore r_C = 2r_A$

AS THE SPACECRAFT TRAVELS FROM B TO C, THE AREA SWEPT OUT IS THE PARABOLIC AREA BAC. THUS,

(CONTINUED)

12.112 continued

AREA SWEPT OUT $= A_{BAC} = \frac{4}{3}(r_A)(r_C) = \frac{8}{3}r_A^2$

NOW-- $\frac{dA}{dt} = \frac{1}{2}h$, WHERE $h =$ CONSTANT

THEN $A = \frac{1}{2}ht$ OR $t_{BC} = \frac{2A_{BAC}}{h}$ $\quad h = r_A v_A$

$$= \frac{2 \times \frac{8}{3} r_A^2}{r_A v_A} = \frac{16}{3}\frac{r_A}{v_A}$$

$$= \frac{16}{3}\frac{7330\ km}{10.42\ km/s}$$

$$= 3751.8\ s$$

OR $t_{BC} = 1\ h\ 2\ min\ 32\ s$ ◀

12.113

<u>GIVEN</u>: PARABOLIC APPROACH TRAJECTORY AND CIRCULAR ORBIT ABOUT VENUS OF A SPACE PROBE; $R = 6052$ km, $M_{VENUS} = 4.87 \times 10^{24}$ kg

<u>FIND</u>: t_{BC}

FROM THE SOLUTION TO PROBLEM 12.99 HAVE--

$(v_A)_{PAR} = 10\,131.4\ \frac{m}{s}$

AND $(v_A)_{CIRC} = \frac{1}{\sqrt{2}}(v_A)_{PAR} = 7164.0\ \frac{m}{s}$

ALSO, $r_A = (6052 + 280)\ km = 6332\ km$

FOR THE PARABOLIC TRAJECTORY BA HAVE

$$\frac{1}{r} = \frac{GM_V}{h^2_{BA}}(1 + \epsilon\cos\theta) \quad [EQ. (12.39')]$$

WHERE $\epsilon = 1$. NOW--

AT A, $\theta = 0$: $\frac{1}{r_A} = \frac{GM_V}{h^2_{BA}}(1+1)$ OR $r_A = \frac{h^2_{BA}}{2GM_V}$

AT B, $\theta = -90°$: $\frac{1}{r_B} = \frac{GM_V}{h^2_{BA}}(1+0)$ OR $r_B = \frac{h^2_{BA}}{GM_V}$

$\therefore r_B = 2r_A$

AS THE PROBE TRAVELS FROM B TO A, THE AREA SWEPT OUT IS THE SEMIPARABOLIC AREA DEFINED BY VERTEX A AND POINT B. THUS,

$(\text{AREA SWEPT OUT})_{BA} = A_{BA} = \frac{2}{3}(r_A)(r_B) = \frac{4}{3}r_A^2$

NOW-- $\frac{dA}{dt} = \frac{1}{2}h$, WHERE $h =$ CONSTANT

THEN $A = \frac{1}{2}ht$ OR $t_{BA} = \frac{2A_{BA}}{h_{BA}}$ $\quad h_{BA} = r_A v_A$

$$= \frac{2 \times \frac{4}{3}r_A^2}{r_A v_A} = \frac{8}{3}\frac{r_A}{v_A}$$

$$= \frac{8}{3}\frac{6332 \times 10^3\ m}{10\,131.4\ m/s}$$

$$= 1666.63\ s$$

FOR THE CIRCULAR TRAJECTORY AC,

$$t_{AC} = \frac{\pi/2\ r_A}{(v_A)_{CIRC}} = \frac{\pi}{2}\frac{6332 \times 10^3\ m}{7164.0\ m/s} = 1388.37\ s$$

FINALLY-- $t_{BC} = t_{BA} + t_{AC}$

$= (1666.63 + 1388.37)\ s$

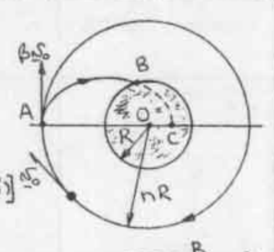
$= 3055.0\ s$

OR $t_{BC} = 50\ min\ 55\ s$ ◀

12.114

GIVEN: CIRCULAR ORBIT OF RADIUS nR OF A SPACE PROBE HAVING VELOCITY v_0 ABOUT A PLANET OF RADIUS R; AT POINT A, VELOCITY IS REDUCED TO βv_0 $(\beta < 1)$ SO THAT PROBE IMPACTS AT POINT B

FIND: $\measuredangle AOB$ IN TERMS OF n AND β



HAVE FOR THE CIRCULAR ORBIT

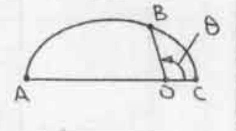
$$v_0 = \sqrt{\frac{GM}{nR}} \quad [\text{EQ. }(12.44)]$$

OR $GM = nRv_0^2$

FOR THE ELLIPTIC ORBIT ABC

$$\frac{1}{r} = \frac{GM}{h_{ABC}^2}(1+\epsilon\cos\theta) \quad [\text{EQ. }(12.39')]$$

WHERE
$$h_{ABC} = (h_A)_{ABC} = r_A(v_A)_{ABC} = (nR)(\beta v_0)$$



THEN..
$$\frac{1}{r} = \frac{nRv_0^2}{(nR\beta v_0)^2}(1+\epsilon\cos\theta) = \frac{1}{nR\beta^2}(1+\epsilon\cos\theta)$$

NOTING THAT POINT C IS THE PERIGEE OF THE ELLIPTIC IMPACT TRAJECTORY SO THAT ANGLE θ IS DEFINED AS SHOWN, HAVE..

AT A, $\theta = 180°$: $\quad \frac{1}{nR} = \frac{1}{nR\beta^2}(1-\epsilon)$

OR $\epsilon = 1-\beta^2$

AT B: $\quad \frac{1}{R} = \frac{1}{nR\beta^2}(1+\epsilon\cos\theta) = \frac{1}{nR\beta^2}[1+(1-\beta^2)\cos\theta]$

OR $\cos\theta = \frac{n\beta^2-1}{1-\beta^2}$

NOW.. $\measuredangle AOB = 180° - \theta$

SO THAT $\cos(180° - \measuredangle AOB) = \frac{n\beta^2-1}{1-\beta^2}$

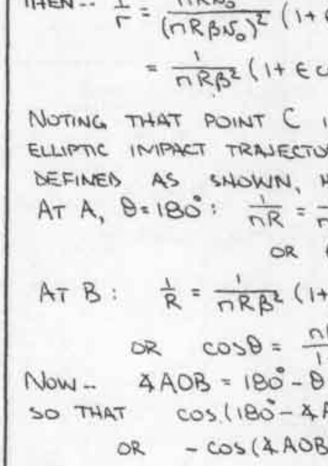
OR $-\cos(\measuredangle AOB) = \frac{n\beta^2-1}{1-\beta^2}$

OR $\measuredangle AOB = \cos^{-1}\frac{1-n\beta^2}{1-\beta^2}$ ◀

12.115

GIVEN: ELLIPTIC ORBIT AND ELLIPTIC IMPACT TRAJECTORY OF LUNAR ORBITER 2;
$M_{MOON} = 0.01230 M_{EARTH}$

FIND: $|\Delta v_B|$ FOR IMPACT AT POINT C



FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR THE ELLIPTIC ORBIT AB..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM_{MOON}}{h_{AB}^2}$$

WHERE $h_{AB} = (h_B)_{AB} = r_B(v_B)_{AB}$

AND
$$GM_{MOON} = G(0.01230\,M_{EARTH}) = 0.01230\,gR_{EARTH}^2 \quad \text{USING EQ. (12.30)}$$

THEN..
$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2(0.01230\,gR_{EARTH}^2)}{[r_B(v_B)_{AB}]^2}$$

(CONTINUED)

12.115 continued

OR
$$(v_B)_{AB} = \frac{R_{EARTH}}{r_B}\left(\frac{0.0246\,g}{\frac{1}{r_A}+\frac{1}{r_B}}\right)^{1/2}$$
$$= \frac{6.37\times10^6\text{ m}}{3600\times10^3\text{ m}}\left(\frac{0.0246\times9.81\text{ m/s}^2}{\frac{1}{1790\times10^3\text{ m}}+\frac{1}{3600\times10^3\text{ m}}}\right)^{1/2}$$
$$= 950.43\ \frac{\text{m}}{\text{s}}$$

FOR THE ELLIPTIC IMPACT TRAJECTORY HAVE..

$$\frac{1}{r} = \frac{GM_{MOON}}{h_{BC}^2} + C\cos\theta \quad [\text{EQ. }(12.39)]$$

WHERE $h_{BC} = (h_B)_{BC} = r_B(v_B)_{BC}$

NOTING THAT POINT B IS THE APOGEE OF THIS TRAJECTORY, HAVE

AT B, $\theta = 180°$: $\quad \frac{1}{r_B} = \frac{GM_{MOON}}{h_{BC}^2} - C$

OR $C = \frac{GM_{MOON}}{h_{BC}^2} - \frac{1}{r_B}$

AT C, $\theta = -70°$: $\quad \frac{1}{R} = \frac{GM_{MOON}}{h_{BC}^2} + C\cos(-70°)$

OR $C = \frac{1}{\cos 70°}\left(\frac{1}{R} - \frac{GM_{MOON}}{h_{BC}^2}\right)$

THEN..
$$\frac{GM_{MOON}}{h_{BC}^2} - \frac{1}{r_B} = \frac{1}{\cos 70°}\left(\frac{1}{R} - \frac{GM_{MOON}}{h_{BC}^2}\right)$$

OR
$$h_{BC}^2 = \frac{GM_{MOON}(1+\cos 70°)}{\frac{1}{R}+\frac{\cos 70°}{r_B}}$$

OR
$$(v_B)_{BC} = \frac{R_{EARTH}}{r_B}\left[\frac{0.01230\,g\,(1+\cos 70°)}{\frac{1}{R}+\frac{\cos 70°}{r_B}}\right]^{1/2}$$

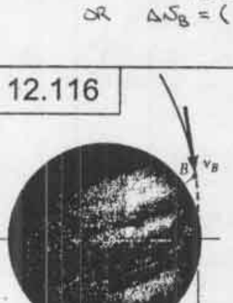
$$(v_B)_{BC} = \frac{6.37\times10^6\text{ m}}{3600\times10^3\text{ m}}\left[\frac{0.01230(9.81\ \frac{\text{m}}{\text{s}^2})(1+\cos 70°)}{\frac{1}{1737\times10^3\text{ m}}+\frac{\cos 70°}{3600\times10^3\text{ m}}}\right]^{1/2}$$
$$= 869.43\ \frac{\text{m}}{\text{s}}$$

FINALLY.. $(v_B)_{BC} = (v_B)_{AB} + \Delta v_B$

OR $\Delta v_B = (869.43 - 950.43)\ \frac{\text{m}}{\text{s}}$

OR $|\Delta v_B| = 81.0\ \frac{\text{m}}{\text{s}}$ ◀

12.116



GIVEN: HYPERBOLIC TRAJECTORY OF A PROBE, $\epsilon = 1.031$; ALTITUDE AT B = 450 km, v_B ∡ 82.9°; FOR JUPITER $R = 71.492\times10^3$ km, $M = 1.9\times10^{27}$ kg

FIND: (a) $\measuredangle AOB$
(b) v_B

FIRST NOTE.. $r_B = (71.492\times10^3 + 450)\text{ km} = 71.942\times10^3\text{ km}$

(a) HAVE.. $\frac{1}{r} = \frac{GM_J}{h^2}(1+\epsilon\cos\theta) \quad [\text{EQ. }(12.39')]$

AT A, $\theta = 0$: $\quad \frac{1}{r_A} = \frac{GM_J}{h^2}(1+\epsilon)$

OR $\frac{h^2}{GM_J} = r_A(1+\epsilon)$

AT B, $\theta = \theta_B = \measuredangle AOB$: $\quad \frac{1}{r_B} = \frac{GM_J}{h^2}(1+\epsilon\cos\theta_B)$

OR $\frac{h^2}{GM_J} = r_B(1+\epsilon\cos\theta_B)$

THEN.. $r_A(1+\epsilon) = r_B(1+\epsilon\cos\theta_B)$

OR
$$\cos\theta_B = \frac{1}{\epsilon}\left[\frac{r_A}{r_B}(1+\epsilon) - 1\right] = \frac{1}{1.031}\left[\frac{70.8\times10^3\text{ km}}{71.942\times10^3\text{ km}}(1+1.031) - 1\right] = 0.96873$$

(CONTINUED)

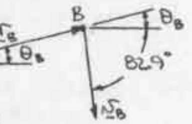
12.116 continued

OR $\theta_B = 14.3661^\circ$ $\therefore \measuredangle AOB = 14.37^\circ$

(b) FROM ABOVE... $h^2 = GM_J r_B (1 + \epsilon \cos\theta_B)$

WHERE $h = \frac{1}{m} |\mathbf{r}_B \times m\mathbf{v}_B| = r_B v_B \sin\phi$

$\phi = (\theta_B + 82.9^\circ) = 97.2661^\circ$



THEN...

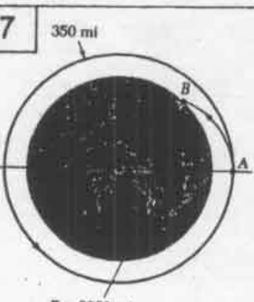
$(r_B v_B \sin\phi)^2 = GM_J r_B (1 + \epsilon\cos\theta_B)$

OR $v_B = \frac{1}{\sin\phi}\left[\frac{GM_J}{r_B}(1+\epsilon\cos\theta_B)\right]^{1/2}$

$$= \frac{1}{\sin 97.2661^\circ}\left\{\frac{66.73\times10^{-12}\frac{m^3}{kg\cdot s^2}\times 1.9\times10^{27}\,kg}{71.942\times10^6\,m} \times [1 + (1.031)(0.96873)]\right\}^{1/2}$$

OR $v_B = 59.8\,\frac{km}{s}$

12.117



GIVEN: CIRCULAR ORBIT AND THE ELLIPTIC DESCENT TRAJECTORY OF A SPACE SHUTTLE; $\Delta v_A = -500$ ft/s; ALTITUDE AT B = 75 mi

FIND: $\measuredangle AOB$

FIRST NOTE... $R = 3960$ mi $= 20.9088\times10^6$ ft

$r_A = (3960 + 350)$ mi $= 4310$ mi $= 22.7568\times10^6$ ft

$r_B = (3960 + 75)$ mi $= 4035$ mi

FOR THE CIRCULAR ORBIT HAVE

$$v_{CIRC} = \sqrt{\frac{gR^2}{r_A}} \qquad [EQ. (12.44)]$$

$$= 20.9088\times10^6\,ft\left(\frac{32.2\,ft/s^2}{22.7568\times10^6\,ft}\right)^{1/2} = 24{,}871\,\frac{ft}{s}$$

NOW... $(v_A)_{AB} = v_{CIRC} + \Delta v_A = (24{,}871 - 500)\frac{ft}{s} = 24{,}371$ ft/s

FOR THE ELLIPTIC DESCENT TRAJECTORY HAVE...

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta \qquad [EQ. (12.39)]$$

NOTING THAT POINT A IS AT THE APOGEE OF THIS TRAJECTORY, HAVE...

AT A, $\theta = 180^\circ$: $\frac{1}{r_A} = \frac{GM}{h^2} - C$ OR $C = \frac{GM}{h^2} - \frac{1}{r_A}$

AT B, $\theta = \theta_B = 180^\circ - \measuredangle AOB$: $\frac{1}{r_B} = \frac{GM}{h^2} + C\cos\theta_B$ OR $C = \frac{1}{\cos\theta_B}\left(\frac{1}{r_B} - \frac{GM}{h^2}\right)$

THEN... $\frac{GM}{h^2} - \frac{1}{r_A} = \frac{1}{\cos\theta_B}\left(\frac{1}{r_B} - \frac{GM}{h^2}\right)$

OR $\cos\theta_B = \dfrac{\frac{1}{r_B} - \frac{GM}{h^2}}{\frac{GM}{h^2} - \frac{1}{r_A}}$

NOW... $h = (h_A)_{AB} = r_A (v_A)_{AB}$

AND $GM = gR^2$ EQ. (12.30)

FROM ABOVE... $gR^2 = r_A (v_{CIRC})^2$ [EQ. (12.44)]

THEN... $\frac{GM}{h^2} = \frac{r_A (v_{CIRC})^2}{[r_A (v_A)_{AB}]^2} = \frac{1}{r_A}\left[\frac{v_{CIRC}}{(v_A)_{AB}}\right]^2$

(CONTINUED)

12.117 continued

SO THAT $\cos\theta_B = \dfrac{\frac{1}{r_B} - \frac{1}{r_A}\left[\frac{v_{CIRC}}{(v_A)_{AB}}\right]^2}{\frac{1}{r_A}\left[\frac{v_{CIRC}}{(v_A)_{AB}}\right]^2 - \frac{1}{r_A}} = \dfrac{\frac{r_A}{r_B} - \left[\frac{v_{CIRC}}{(v_A)_{AB}}\right]^2}{\left[\frac{v_{CIRC}}{(v_A)_{AB}}\right]^2 - 1}$

$$= \frac{\frac{4310\,mi}{4035\,mi} - \left(\frac{24{,}871\,ft/s}{43{,}371\,ft/s}\right)^2}{\left(\frac{24{,}871\,ft/s}{24{,}371\,ft/s}\right)^2 - 1}$$

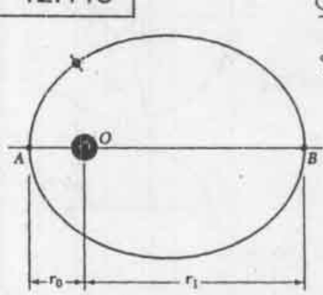
$= 0.64411$

OR $\theta_B = 49.901^\circ$

FINALLY... $\measuredangle AOB = 180^\circ - 49.901^\circ$

OR $\measuredangle AOB = 130.1^\circ$

12.118



GIVEN: ELLIPTIC ORBIT OF A SATELLITE AS SHOWN

SHOW: $\frac{1}{\rho} = \frac{1}{2}\left(\frac{1}{r_0} + \frac{1}{r_1}\right)$ WHERE $\rho = \rho_A = \rho_B$

FROM THE SOLUTION TO PROBLEM 12.102 HAVE...

$$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2} \qquad \text{WHERE } h = h_A = r_0 v_A$$

CONSIDER THE SATELLITE AT POINT A...

$\bigcirc \xrightarrow{F_A} \; = \; \bigcirc \xrightarrow{m(a_A)_n}$ $\quad +\Sigma F_n = ma_n$: $F_A = m\frac{v_A^2}{\rho}$

NOW... $F_A = G\frac{Mm}{r_0^2}$ [EQ. (12.28)]

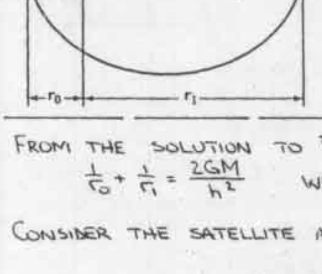
THEN... $m\frac{v_A^2}{\rho} = G\frac{Mm}{r_0^2}$

OR $GM = \frac{1}{\rho}(v_A^2 r_0^2) = \frac{1}{\rho}h^2$

FINALLY... $\frac{1}{r_0} + \frac{1}{r_1} = \frac{2(\frac{1}{\rho}h^2)}{h^2}$

OR $\frac{1}{\rho} = \frac{1}{2}\left(\frac{1}{r_0} + \frac{1}{r_1}\right)$ Q.E.D.

12.119



GIVEN: ELLIPTIC ORBIT OF A SATELLITE AS SHOWN; FOR COMET HYAKUTAKE, $r_0 = 0.230 R_E$, $\epsilon = 0.999887$; $R_E = 149.6\times10^6$ km

FIND: (a) ϵ IN TERMS OF r_0 AND r_1

(b) r_1 FOR COMET HYAKUTAKE

(a) HAVE... $\frac{1}{r} = \frac{GM}{h^2}(1 + \epsilon\cos\theta)$ EQ. (12.39')

AT A, $\theta = 0$: $\frac{1}{r_0} = \frac{GM}{h^2}(1+\epsilon)$ OR $\frac{h^2}{GM} = r_0(1+\epsilon)$

AT B, $\theta = 180^\circ$: $\frac{1}{r_1} = \frac{GM}{h^2}(1-\epsilon)$ OR $\frac{h^2}{GM} = r_1(1-\epsilon)$

THEN... $r_0(1+\epsilon) = r_1(1-\epsilon)$ OR $\epsilon = \frac{r_1 - r_0}{r_1 + r_0}$

(b) FROM ABOVE... $r_1 = \frac{1+\epsilon}{1-\epsilon} r_0$

WHERE $r_0 = 0.230 R_E$ (CONTINUED)

12.119 continued

THEN.. $r_1 = \frac{1+0.999887}{1-0.999887} \times 0.230(149.6\times10^9\text{ m})$

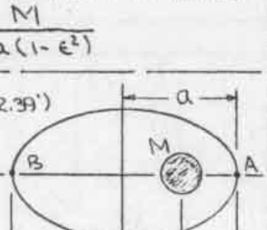
OR $r_1 = 609\times10^{12}\text{ m}$ ◀

NOTE: $r_1 \approx 4070 R_E$ OR $r_1 \approx 0.064$ LIGHT YEARS

12.120

GIVEN: ELLIPTIC ORBIT OF SEMIMAJOR AXIS a AND ECCENTRICITY ϵ OF A SATELLITE ABOUT A PLANET OF MASS M

SHOW: $h = \sqrt{GMa(1-\epsilon^2)}$



HAVE.. $\frac{1}{r} = \frac{GM}{h^2}(1+\epsilon\cos\theta)$ EQ. (12.39')

AT A, $\theta=0$: $\frac{1}{r_0} = \frac{GM}{h^2}(1+\epsilon)$

OR $r_0 = \frac{h^2}{GM}\frac{1}{1+\epsilon}$

AT B, $\theta=180°$: $\frac{1}{r_1} = \frac{GM}{h^2}(1-\epsilon)$

OR $r_1 = \frac{h^2}{GM}\frac{1}{1-\epsilon}$

THEN.. $r_0 + r_1 = \frac{h^2}{GM}\frac{1}{1+\epsilon} + \frac{h^2}{GM}\frac{1}{1-\epsilon} = \frac{h^2}{GM}\frac{2}{1-\epsilon^2}$

NOW.. $a = \frac{1}{2}(r_0+r_1)$

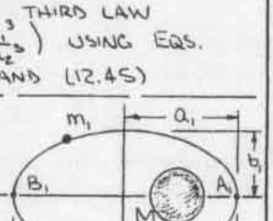
SO THAT $2a = \frac{h^2}{GM}\frac{2}{1-\epsilon^2}$

OR $h = \sqrt{GMa(1-\epsilon^2)}$ Q.E.D. ◀

12.121

GIVEN: TWO ELLIPTIC ORBITS OF SEMIMAJOR AXES a_1 AND a_2 ABOUT A BODY OF MASS M; PERIODIC TIMES τ_1 AND τ_2 OF TWO SATELLITES IN THE ELLIPTIC ORBITS

DERIVE: KEPLER'S THIRD LAW $\left(\frac{\tau_1^2}{\tau_2^2} = \frac{a_1^3}{a_2^3}\right)$ USING EQS. (12.39) AND (12.45)



CONSIDER THE ELLIPTIC ORBIT OF SATELLITE 1. NOW

$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta$ EQ. (12.39)

THEN, FOR ORBIT 1..

AT A_1, $\theta=0$: $\frac{1}{(r_A)_1} = \frac{GM}{h_1^2} + C_1$

AT B_1, $\theta=180°$: $\frac{1}{(r_B)_1} = \frac{GM}{h_1^2} - C_1$

THEN.. $\frac{1}{(r_A)_1} + \frac{1}{(r_B)_1} = \left(\frac{GM}{h_1^2}+C_1\right) + \left(\frac{GM}{h_1^2} - C_1\right)$

OR $\frac{(r_A)_1 + (r_B)_1}{(r_A)_1(r_B)_1} = \frac{2GM}{h_1^2}$

NOW $a_1 = \frac{1}{2}[(r_A)_1 + (r_B)_1]$ $\quad b_1 = \sqrt{(r_A)_1(r_B)_1}$

THEN.. $\frac{2a_1}{b_1^2} = \frac{2GM}{h_1^2}$

OR $h_1 = b_1\sqrt{\frac{GM}{a_1}}$

NOW.. $\tau = \frac{2\pi ab}{h}$ EQ. (12.45)

FOR ORBIT 1.. $\tau_1 = \frac{2\pi a_1 b_1}{b_1\sqrt{\frac{GM}{a_1}}} = \frac{2\pi}{\sqrt{GM}}a_1^{3/2}$

(CONTINUED)

12.121 continued

SIMILARLY, FOR THE ORBIT OF SATELLITE 2..

$\tau_2 = \frac{2\pi}{\sqrt{GM}}a_2^{3/2}$

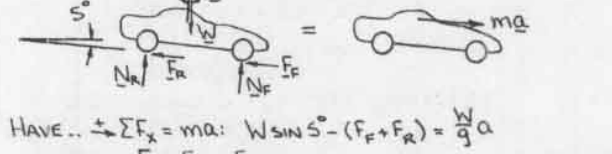
THEN.. $\frac{\tau_1}{\tau_2} = \frac{\frac{2\pi}{\sqrt{GM}}a_1^{3/2}}{\frac{2\pi}{\sqrt{GM}}a_2^{3/2}}$

OR $\frac{\tau_1^2}{\tau_2^2} = \frac{a_1^3}{a_2^3}$ Q.E.D. ◀

12.122

GIVEN: AUTOMOBILE OF WEIGHT 3000 lb MOVING DOWN A 5° INCLINE; $v_0 = 50\ \frac{\text{mi}}{\text{h}}$; AT $t=0$, $F_{BRAKE} = 1200$ lb IS APPLIED

FIND: x WHEN $v=0$



HAVE.. $\xrightarrow{+} \Sigma F_x = ma$: $W\sin5° - (F_F + F_R) = \frac{W}{g}a$

WHERE $F_F + F_R = F_{BRAKE}$

THEN.. $a = (32.2\ \frac{\text{ft}}{\text{s}^2})\left(\sin5° - \frac{1200\text{ lb}}{3000\text{ lb}}\right) = -10.0736\ \frac{\text{ft}}{\text{s}^2}$

FOR THIS UNIFORMLY DECELERATED MOTION HAVE..

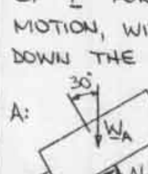
$v^2 = v_0^2 + 2a(x - \overset{0}{\cancel{x_0}})$

WHERE $v_0 = 50\ \frac{\text{mi}}{\text{h}} = 73.333$ ft/s

THEN WHEN $v=0$.. $0 = (73.333\ \frac{\text{ft}}{\text{s}})^2 + 2(-10.0736\ \frac{\text{ft}}{\text{s}^2})x$

OR $x = 267$ ft ◀

12.123

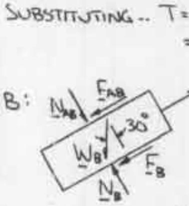


GIVEN: $m_A = 30$ kg, $m_B = 15$ kg; $\mu_s = 0.15$, $\mu_k = 0.10$; $\theta = 30°$, $P = 250$ N

FIND: (a) $\underline{a}_A$ (b) T

FIRST DETERMINE IF THE BLOCKS WILL MOVE FOR THE GIVEN VALUE OF $\underline{P}$. THUS, SEEK THE VALUE OF $\underline{P}$ FOR WHICH THE BLOCKS ARE IN IMPENDING MOTION, WITH THE IMPENDING MOTION OF A DOWN THE INCLINE.

A:



$\nearrow\Sigma F_y = 0$: $N_{AB} - W_A\cos30° = 0$

OR $N_{AB} = m_A g\cos30°$

NOW.. $F_{AB} = \mu_s N_{AB}$

$= 0.15 m_A g\cos30°$

$\nearrow\Sigma F_x = 0$: $T - P + F_{AB} - W_A\sin30° = 0$

OR $T = P + m_A g(\sin30° - 0.15\cos30°)$

SUBSTITUTING.. $T = P + (30\text{ kg})(9.81\ \frac{\text{m}}{\text{s}^2})(\sin30° - 0.15\cos30°)$

$= (P + 108.919)$ N

B:

$\nwarrow\Sigma F_y = 0$: $N_B - N_{AB} - W_B\cos30° = 0$

OR $N_B = g\cos30°(m_A + m_B)$

NOW.. $F_B = \mu_s N_B$

$= 0.15g\cos30°(m_A + m_B)$

$\nearrow\Sigma F_x = 0$: $T - F_{AB} - F_B - W_B\sin30° = 0$

OR $T = m_B g\sin30° + 0.15 m_A g\cos30° + 0.15 g\cos30°(m_A + m_B)$

(CONTINUED)

12.123 continued

OR $T = g\,[m_B \sin 30^\circ + 0.15(2m_A + m_B)\cos 30^\circ]$

$= (9.81\ \frac{m}{s^2})[(15\ kg)\sin 30^\circ + 0.15(2\times30+15)\ kg \times \cos 30^\circ]$

$= 169.152\ N$

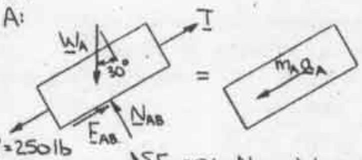
THEN -- $169.152\ N = (P + 108.919)\ N$

OR $P = 60.2\ N$ FOR IMPENDING MOTION OF A DOWNWARD. SINCE $P < 250\ N$, THE BLOCKS WILL MOVE, WITH A MOVING DOWNWARD.

NOW CONSIDER THE MOTION OF THE BLOCKS.

(a)

A:



$+\nearrow \Sigma F_y = 0:\ N_{AB} - W_A \cos 30^\circ = 0$

OR $N_{AB} = m_A g \cos 30^\circ$

SLIDING: $F_{AB} = \mu_k N_{AB}$

$= 0.1\, m_A g \cos 30^\circ$

$+\swarrow \Sigma F_x = m_A a_A:\ -T + P - F_{AB} + W_A \sin 30^\circ = m_A a_A$

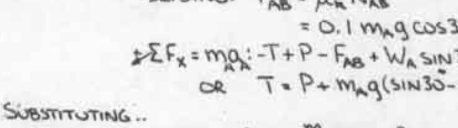
OR $T = P + m_A g(\sin 30^\circ - 0.1\cos 30^\circ) - m_A a_A$

SUBSTITUTING..

$T = 250\ N + (30\ kg)[(9.81\ \frac{m}{s^2})(\sin 30^\circ - 0.1\cos 30^\circ) - a_A]$

$= (371.663 - 30a_A)\ N \qquad (1)$

B:



$+\nwarrow \Sigma F_y = 0:\ N_B - N_{AB} - W_B \cos 30^\circ = 0$

OR $N_B = g\cos 30^\circ (m_A + m_B)$

SLIDING: $F_B = \mu_k N_B$

$= 0.1\, g\cos 30^\circ (m_A + m_B)$

$+\nearrow \Sigma F_x = m_B a_B:\ T - F_{AB} - F_B - W_B \sin 30^\circ = m_B a_B$

OR $T = m_B g \sin 30^\circ + 0.1 m_A g \cos 30^\circ + 0.1 g \cos 30^\circ (m_A + m_B) + m_B a_B$

$= g[m_B \sin 30^\circ + 0.1(2m_A + m_B)\cos 30^\circ] + m_B a_B$

$= (9.81\ \frac{m}{s^2})[(15\ kg)\sin 30^\circ + 0.1(2\times30+15)\ kg \times \cos 30^\circ]$

$+ (15\ kg)a_B$

$= (137.293 + 15a_B)\ N \qquad (2)$

EQUATING THE EXPRESSIONS FOR T [EQS. (1) AND (2)] AND NOTING THAT $a_A = a_B$ --

$371.663 - 30a_A = 137.293 + 15a_A$

OR $a_A = 5.2082\ \frac{m}{s^2}$

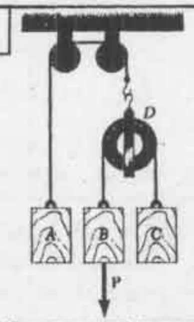
$\therefore\ \mathbf{a}_A = 5.21\ \frac{m}{s^2} \measuredangle 30^\circ$ ◀

(b) SUBSTITUTING INTO EQ (1)..

$T = (371.663 - 30 \times 5.2082)\ N$

OR $T = 215\ N$ ◀

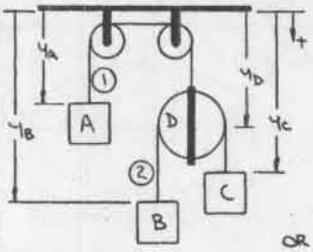
12.124



GIVEN: $W_A = 20\ lb$, $W_B = W_C = 10\ lb$; AT $t = 0$, $v = 0$; AT $t = 2\ s$, $\Delta y_B = 8\ ft \downarrow$

FIND: (a) P

(b) T_{AD}



FROM THE DIAGRAM..

CORD 1: $y_A + y_D =$ CONSTANT

THEN.. $v_A + v_D = 0$

AND $a_A + a_D = 0$

CORD 2: $(y_B - y_D) + (y_C - y_D)$ = CONSTANT

THEN.. $v_B + v_C - 2v_D = 0$

AND $a_B + a_C - 2a_D = 0$

OR.. $2a_A + a_B + a_C = 0 \qquad (1)$

NOW.. HAVE UNIFORMLY ACCELERATED MOTION BECAUSE ALL OF THE FORCES ARE CONSTANT. THEN..

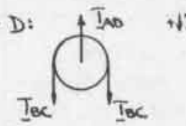
$y_B = (y_B)_0 + (v_B)_0^{\,0} t + \frac{1}{2}a_B t^2$

AT $t = 2\ s$, $\Delta y_B = 8\ ft$: $8\ ft = \frac{1}{2}a_B (2\ s)^2$

OR $\mathbf{a}_B = 4\ ft/s^2 \downarrow$

(a)

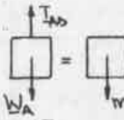
PULLEY D:



$+\downarrow \Sigma F_y = m_D^{\,0} a_D:\ 2T_{BC} - T_{AD} = 0$

OR $T_{AD} = 2T_{BC}$

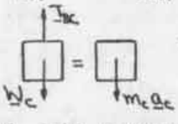
BLOCK A:



$+\downarrow \Sigma F_y = m_A a_A:\ W_A - T_{AD} = \frac{W_A}{g} a_A$

OR $a_A = g(1 - \frac{T_{AD}}{W_A})$

BLOCK C:



$+\downarrow \Sigma F_y = m_C a_C:\ W_C - T_{BC} = \frac{W_C}{g} a_C$

OR $a_C = g(1 - \frac{\frac{1}{2}T_{AD}}{W_C})$

SUBSTITUTING THE EXPRESSIONS FOR a_A AND a_C INTO EQ. (1)..

$2g(1 - \frac{T_{AD}}{W_A}) + a_B + g(1 - \frac{T_{AD}}{2W_C}) = 0$

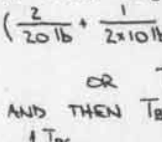
OR $(\frac{2}{W_A} + \frac{1}{2W_C})T_{AD} = 3 + \frac{a_B}{g}$

THEN.. $(\frac{2}{20\ lb} + \frac{1}{2\times10\ lb})T_{AD} = 3 + \frac{4\ ft/s^2}{32.2\ ft/s^2}$

OR $T_{AD} = 20.828\ lb$

AND THEN $T_{BC} = 10.414\ lb$

BLOCK B:



$+\downarrow \Sigma F_y = m_B a_B:\ P + W_B - T_{BC} = \frac{W_B}{g} a_B$

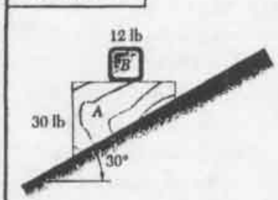
OR $P = T_{BC} + W_B(\frac{a_B}{g} - 1)$

SUBSTITUTING -- $P = 10.414\ lb + (10\ lb)(\frac{4\ ft/s^2}{32.2\ ft/s^2} - 1)$

OR $P = 1.656\ lb$ ◀

(b) HAVE FROM ABOVE.. $T_{AD} = 20.8\ lb$ ◀

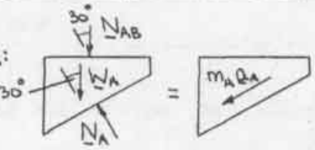
12.125



GIVEN: BLOCKS A AND B AS SHOWN; AT $t=0$, $v=0$; NEGLECT FRICTION

FIND: (a) $\underline{a}_A$ AT $t=0$
(b) $\underline{a}_{B/A}$ AT $t=0$

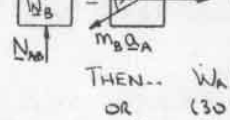
(a)
BLOCK A:



$\nearrow \Sigma F_x = m_A a_A$: $N_{AB}\sin 30° + W_A \sin 30° = \frac{W_A}{g} a_A$

OR $N_{AB} = W_A\left(\frac{a_A}{g\sin 30°} - 1\right) = W_A\left(\frac{2a_A}{g} - 1\right)$

BLOCK B: FIRST NOTE THAT $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$ WHERE $\underline{a}_{B/A}$ IS DIRECTED PARALLEL TO THE TOP SURFACE OF BLOCK A.



$+\downarrow \Sigma F_y = m_B a_B$: $W_B - N_{AB} = \frac{W_B}{g} a_A \sin 30°$

OR $N_{AB} = W_B\left(1 - \frac{a_A}{2g}\right)$

THEN.. $W_A\left(\frac{2a_A}{g} - 1\right) = W_B\left(1 - \frac{a_A}{2g}\right)$

OR $(30\text{ lb})\left(\frac{2a_A}{g} - 1\right) = (12\text{ lb})\left(1 - \frac{a_A}{2g}\right)$

OR $a_A = \frac{7}{11}g = \frac{7}{11}(32.2\text{ ft/s}^2) = 20.491\text{ ft/s}^2$

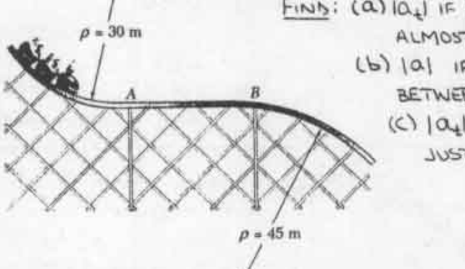
$\therefore \underline{a}_A = 20.49\ \frac{\text{ft}}{\text{s}^2} \measuredangle 30°$ ◀

(b) FOR BLOCK B.. $\xrightarrow{+} \Sigma F_x = m_B a_x$: $0 = m_B a_{B/A} - m_B a_A \cos 30°$

OR $a_{B/A} = (20.491\text{ ft/s}^2)\cos 30°$

OR $\underline{a}_{B/A} = 17.75\ \frac{\text{ft}}{\text{s}^2} \rightarrow$ ◀

12.126

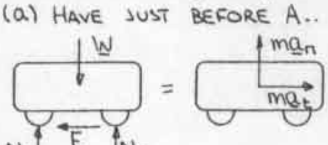


GIVEN: $v_0 = 72\ \frac{\text{km}}{\text{h}}$; SLIDING: $\mu_k = 0.25$

FIND: (a) $|a_t|$ IF THE CAR IS ALMOST AT A
(b) $|a|$ IF THE CAR IS BETWEEN A AND B
(c) $|a_t|$ IF THE CAR IS JUST PAST B

FIRST NOTE.. $v_0 = 72\ \frac{\text{km}}{\text{h}} = 20\ \frac{\text{m}}{\text{s}}$

(a) HAVE JUST BEFORE A.. $\rho_A = 30$ m



$+\uparrow \Sigma F_n = m a_n$: $(N_F + N_R) - W = m\frac{v_0^2}{\rho_A}$

OR $(N_F + N_R) = m\left(g + \frac{v_0^2}{\rho_A}\right)$

SLIDING: $F = \mu_k(N_F + N_R) = 0.25\, m\left(g + \frac{v_0^2}{\rho_A}\right)$

$\xrightarrow{+} \Sigma F_t = m a_t$: $-F = m a_t$

OR $-0.25\, m\left(g + \frac{v_0^2}{\rho_A}\right) = m a_t$

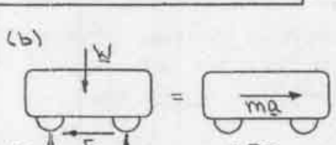
OR $a_t = -0.25\left[9.81\ \frac{\text{m}}{\text{s}^2} + \frac{(20\text{ m/s})^2}{30\text{ m}}\right]$

OR $|a_t| = 5.79\ \frac{\text{m}}{\text{s}^2}$ ◀

(CONTINUED)

12.126 continued

(b)



$+\uparrow \Sigma F_y = 0$: $(N_F + N_R) - W = 0$

OR $(N_F + N_R) = mg$

SLIDING: $F = \mu_k(N_F + N_R) = 0.25\, mg$

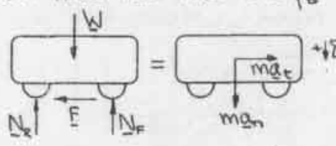
$\xrightarrow{+} \Sigma F_x = ma$: $-F = ma$

OR $-0.25\, mg = ma$

OR $a = -0.25\,(9.81\text{ m/s}^2)$

OR $|a| = 2.45\ \frac{\text{m}}{\text{s}^2}$ ◀

(C) HAVE JUST PAST B.. $\rho_B = 45$ m



$+\downarrow \Sigma F_n = m a_n$: $W - (N_F + N_R) = m\frac{v_0^2}{\rho_B}$

OR $(N_F + N_R) = m\left(g - \frac{v_0^2}{\rho_B}\right)$

SLIDING: $F = \mu_k(N_F + N_R) = 0.25\, m\left(g - \frac{v_0^2}{\rho_B}\right)$

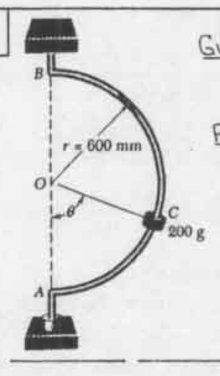
$\xrightarrow{+} \Sigma F_t = m a_t$: $-F = m a_t$

OR $-0.25\, m\left(g - \frac{v_0^2}{\rho_B}\right) = m a_t$

OR $a_t = -0.25\left[9.81\ \frac{\text{m}}{\text{s}^2} - \frac{(20\text{ m/s})^2}{45\text{ m}}\right]$

OR $|a_t| = 0.230\ \frac{\text{m}}{\text{s}^2}$ ◀

12.127

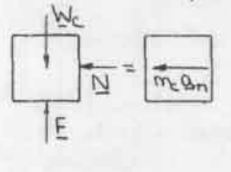


GIVEN: 0.2-kg COLLAR C ON ROD AB; $\dot{\phi}_{AB} = 6$ RAD/s

FIND: $(\mu_s)_{MIN}$ IF C IS NOT TO SLIDE ON AB WHEN
(a) $\theta = 90°$
(b) $\theta = 75°$
(c) $\theta = 45°$

FIRST NOTE.. $v_C = (r\sin\theta)\dot{\phi}_{AB} = (0.6\text{ m})(6\ \frac{\text{RAD}}{\text{s}})\sin\theta = (3.6\ \frac{\text{m}}{\text{s}})\sin\theta$

(a) WITH $\theta = 90°$, $v_C = 3.6\ \frac{\text{m}}{\text{s}}$



$+\uparrow \Sigma F_y = 0$: $F - W_C = 0$

OR $F = m_C g$

NOW.. $F = \mu_s N$

OR $N = \frac{1}{\mu_s} m_C g$

$\xleftarrow{+} \Sigma F_N = m_C a_n$: $N = m_C\frac{v_C^2}{r}$

OR $\frac{1}{\mu_s} m_C g = m_C \frac{v_C^2}{r}$

OR $\mu_s = \frac{gr}{v_C^2} = \frac{(9.81\ \frac{\text{m}}{\text{s}^2})(0.6\text{ m})}{(3.6\ \frac{\text{m}}{\text{s}})^2}$

OR $(\mu_s)_{MIN} = 0.454$ ◀

THE DIRECTION OF THE IMPENDING MOTION IS DOWNWARD ◀

(b) AND (C)

FIRST OBSERVE THAT FOR AN ARBITRARY VALUE OF θ IT IS NOT KNOWN WHETHER THE IMPENDING MOTION WILL BE UPWARD OR DOWNWARD. TO CONSIDER BOTH POSSIBILITIES FOR EACH VALUE OF θ, LET

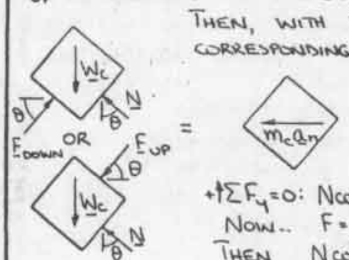
F_{DOWN} CORRESPONDS TO IMPENDING MOTION DOWNWARD

(CONTINUED)

12.127 continued

F_{UP} CORRESPOND TO IMPENDING MOTION UPWARD

THEN, WITH THE "TOP SIGN" CORRESPONDING TO F_{DOWN}, HAVE..



$+\uparrow \Sigma F_y = 0$: $N\cos\theta \pm F\sin\theta - W_c = 0$

NOW.. $F = \mu_s N$

THEN $N\cos\theta \pm \mu_s N\sin\theta - m_c g = 0$

OR $N = \dfrac{m_c g}{\cos\theta \pm \mu_s \sin\theta}$

AND $F = \dfrac{\mu_s m_c g}{\cos\theta \pm \mu_s \sin\theta}$

$\xleftarrow{+} \Sigma F_n = m_c a_n$: $N\sin\theta \mp F\cos\theta = m_c \dfrac{v_c^2}{\rho}$ $\quad \rho = r\sin\theta$

SUBSTITUTING FOR N AND F..

$$\frac{m_c g}{\cos\theta \pm \mu_s \sin\theta}\sin\theta \mp \frac{\mu_s m_c g}{\cos\theta \pm \mu_s\sin\theta}\cos\theta = m_c\frac{v_c^2}{r\sin\theta}$$

OR $\dfrac{\tan\theta}{1\pm\mu_s\tan\theta} \mp \dfrac{\mu_s}{1\pm\mu_s\tan\theta} = \dfrac{v_c^2}{gr\sin\theta}$

OR $\mu_s = \pm \dfrac{\tan\theta - \frac{v_c^2}{gr\sin\theta}}{1 + \frac{v_c^2}{gr\sin\theta}\tan\theta}$

NOW.. $\dfrac{v_c^2}{gr\sin\theta} = \dfrac{[(3.6\ \frac{m}{s})\sin\theta]^2}{(9.81\ \frac{m}{s^2})(0.6\text{m})\sin\theta} = 2.2018\sin\theta$

THEN.. $\mu_s = \pm\dfrac{\tan\theta - 2.2018\sin\theta}{1+2.2018\sin\theta\tan\theta}$

(b) $\theta = 75°$

$$\mu_s = \pm\frac{\tan 75° - 2.2018\sin 75°}{1+2.2018\sin 75°\tan 75°} = \pm 0.1796$$

THEN.. DOWNWARD: $\mu_s = +0.1796$

UPWARD: $\mu_s < 0$ .. NOT POSSIBLE

$\therefore \quad (\mu_s)_{MIN} = 0.1796$ ◀

THE DIRECTION OF THE IMPENDING MOTION IS DOWNWARD ◀

(c) $\theta = 45°$

$$\mu_s = \pm\frac{\tan 45° - 2.2018\sin 45°}{1+2.2018\sin 45°\tan 45°} = \pm(-0.218)$$

THEN.. DOWNWARD: $\mu_s < 0$.. NOT POSSIBLE

UPWARD: $\mu_s = 0.218$

$\therefore \quad (\mu_s)_{MIN} = 0.218$ ◀

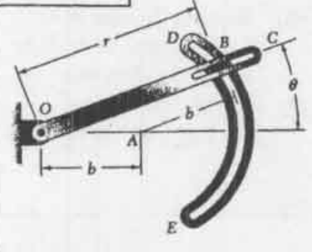
THE DIRECTION OF THE IMPENDING MOTION IS UPWARD ◀

NOTE: WHEN $\tan\theta - 2.2018\sin\theta = 0$

OR $\theta = 62.988°$

$\mu_s = 0$. THUS, FOR THIS VALUE OF θ FRICTION IS NOT NECESSARY TO PREVENT THE COLLAR FROM SLIDING ON THE ROD.

12.128



GIVEN: $W_B = \frac{1}{4}$ lb, b = 20 IN.; WHEN $\theta = 20°$, $\dot\theta = 15\ \frac{\text{RAD}}{\text{S}}$, $\ddot\theta = 250\ \frac{\text{RAD}}{\text{S}^2}$

FIND: (a) F_r AND F_θ ON PIN B WHEN $\theta = 20°$

(b) $\underline{P}$ AND $\underline{Q}$ WHEN $\theta = 20°$, WHERE $\underline{P}$ IS DUE TO OC AND $\underline{Q}$ IS DUE TO DE

KINEMATICS

FROM THE DRAWING OF THE SYSTEM HAVE –

$r = 2b\cos\theta$

THEN.. $\dot r = -(2b\sin\theta)\dot\theta$

AND $\ddot r = -2b(\ddot\theta\sin\theta + \dot\theta^2\cos\theta)$

NOW.. $a_r = \ddot r - r\dot\theta^2 = -2b(\ddot\theta\sin\theta + \dot\theta^2\cos\theta) - (2b\cos\theta)\dot\theta^2$

$= -2b(\ddot\theta\sin\theta + 2\dot\theta^2\cos\theta)$

$= -2(\frac{20}{12}\text{ft})[(250\ \frac{\text{RAD}}{\text{S}^2})\sin 20° + 2(15\ \frac{\text{RAD}}{\text{S}})^2\cos 20°]$

$= -1694.56\ \text{ft/s}^2$

AND $a_\theta = r\ddot\theta + 2\dot r\dot\theta = (2b\cos\theta)\ddot\theta + 2(-2b\dot\theta\sin\theta)\dot\theta$

$= 2b(\ddot\theta\cos\theta - 2\dot\theta^2\sin\theta)$

$= 2(\frac{20}{12}\text{ft})[(250\ \frac{\text{RAD}}{\text{S}^2})\cos 20° - 2(15\ \frac{\text{RAD}}{\text{S}})^2\sin 20°]$

$= 270.05\ \text{ft/s}^2$

KINETICS

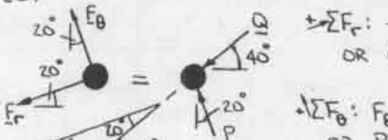
(a) HAVE.. $F_r = ma_r = \dfrac{\frac{1}{4}\text{lb}}{32.2\ \text{ft/s}^2}\times(-1694.56\ \frac{\text{ft}}{\text{s}^2}) = -13.1565$ lb

OR $F_r = -13.16$ lb ◀

AND $F_\theta = ma_\theta = \dfrac{\frac{1}{4}\text{lb}}{32.2\ \text{ft/s}^2}\times(270.05\ \frac{\text{ft}}{\text{s}^2}) = 2.0967$ lb

OR $F_\theta = 2.10$ lb ◀

(b)



$\xleftarrow{+} \Sigma F_r$: $-F_r = -Q\cos 20°$

OR $Q = \frac{1}{\cos 20°}(13.1565\text{ lb})$

$= 14.0009$ lb

$+\nwarrow \Sigma F_\theta$: $F_\theta = P - Q\sin 20°$

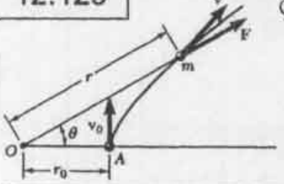
OR $P = (2.0967 + 14.0009\sin 20°)$ lb

$= 6.89$ lb

$\therefore$ $\underline{P} = 6.89$ lb $\measuredangle 70°$ ◀

$\underline{Q} = 14.00$ lb $\measuredangle 40°$ ◀

12.129



GIVEN: CENTRAL FORCE $\underline{F}$ AND PATH SHOWN; $r = r_0/\cos 2\theta$; AT $t = 0$, $\underline{v} = \underline{v}_0$, $\theta = 0$, $\underline{r} = \underline{r}_0$

FIND: v_r AND v_θ AS FUNCTIONS OF θ

HAVE.. $r = \dfrac{r_0}{\cos 2\theta}$

THEN.. $\dot r = \dfrac{2r_0\sin 2\theta}{\cos^2 2\theta}\dot\theta$

NOW.. $\underline{v} = \dot r\underline{e}_r + r\dot\theta\underline{e}_\theta$

SO THAT AT $t = 0$.. $v_0 = r_0\dot\theta_0$

FROM EQ. (12.27): $r^2\dot\theta = r_0^2\dot\theta_0$

$= r_0 v_0$

OR $\dot\theta = \dfrac{r_0 v_0}{r^2} = r_0 v_0\left(\dfrac{\cos 2\theta}{r_0}\right)^2 = \dfrac{v_0}{r_0}\cos^2 2\theta$

THEN $\dot r = \dfrac{2r_0\sin 2\theta}{\cos^2 2\theta}\left(\dfrac{v_0}{r_0}\cos^2 2\theta\right) = 2v_0\sin 2\theta$

(CONTINUED)

12.129 continued

NOW $v_r = \dot{r}$

$$\text{OR } v_r = 2v_0 \sin 2\theta$$

AND $v_\theta = r\dot{\theta} = \frac{r_0}{\cos 2\theta} \times \frac{v_0}{r_0} \cos^2 2\theta$

$$\text{OR } v_\theta = v_0 \cos 2\theta$$

12.130

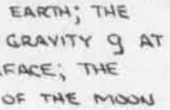
GIVEN: RADIUS r OF THE MOON'S ORBIT; RADIUS R OF THE EARTH; THE ACCELERATION OF GRAVITY g AT THE EARTH'S SURFACE; THE PERIODIC TIME τ OF THE MOON

SHOW: $r = f(R, g, \tau)$

FIND: r KNOWING THAT $\tau = 27.3$ DAYS

HAVE.. $F = G\frac{Mm}{r^2}$ [EQ. (12.28)]

AND $F = F_n = ma_n = m\frac{v^2}{r}$



THEN $G\frac{Mm}{r^2} = m\frac{v^2}{r}$

OR $v^2 = \frac{GM}{r}$

NOW $GM = gR^2$ EQ. (12.30)

SO THAT $v^2 = \frac{gR^2}{r}$ OR $v = R\sqrt{\frac{g}{r}}$

FOR ONE ORBIT.. $\tau = \frac{2\pi r}{v} = \frac{2\pi r}{R\sqrt{\frac{g}{r}}}$

$$\text{OR } r = \left(\frac{g\tau^2 R^2}{4\pi^2}\right)^{1/3} \quad \text{Q.E.D.}$$

NOW.. $\tau = 27.3$ DAYS $= 2.35872 \times 10^6$ s

$R = 3960$ mi $= 20.9088 \times 10^6$ ft

SI: $r = \left[\frac{9.81\frac{m}{s^2} \times (2.35872\times10^6 \text{ s})^2 \times (6.37\times10^6 \text{ m})^2}{4\pi^2}\right]^{1/3}$

$= 382.81 \times 10^6$ m

$$\text{OR } r = 383 \times 10^3 \text{ km}$$

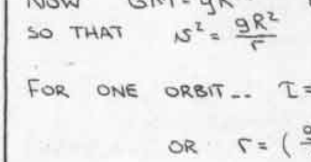
U.S. CUSTOMARY UNITS:

$r = \left[\frac{32.2\frac{ft}{s^2} \times (2.35872\times10^6 \text{ s})^2 \times (20.9088\times10^6 \text{ ft})^2}{4\pi^2}\right]^{1/3}$

$= 1256.52 \times 10^6$ ft

$$\text{OR } r = 238 \times 10^3 \text{ mi}$$

12.131



GIVEN: $m = 0.25$ kg; $k = 6\frac{N}{m}$, $(L_0)_{sp} = 0.5$ m; AT $t=0$, $\dot{\theta}_0 = 16\frac{\text{RAD}}{s}$, COLLAR IS AT A; NEGLECT FRICTION AND m_{ROD}

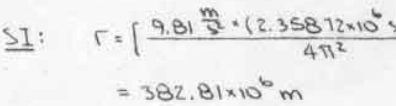
FIND: (a) $(v_B)_\theta$

(b) $(a_B)_r$ AND $(a_B)_\theta$

(c) $(a_{COLLAR/ROD})_B$

FIRST NOTE.. $F_{sp} = k[(L_0)_{sp} - r]$

AT B: $(F_{sp})_B = 6\frac{N}{m}(0.5 - 0.4)\text{m} = 0.6$ N



(a) AFTER THE CORD IS CUT, THE ONLY HORIZONTAL FORCE ACTING ON THE COLLAR IS DUE TO THE

(CONTINUED)

12.131 continued

SPRING. THUS, ANGULAR MOMENTUM ABOUT THE SHAFT IS CONSERVED.

$\therefore r_A m(v_A)_\theta = r_B m(v_B)_\theta$ WHERE $(v_A)_\theta = r_A\dot{\theta}_0$

THEN -- $(v_B)_\theta = \frac{0.1 \text{ m}}{0.4 \text{ m}}\left[(0.1\text{ m})(16\frac{\text{RAD}}{s})\right]$

$$\text{OR } (v_B)_\theta = 0.400\frac{m}{s}$$

(b) HAVE $F_\theta = 0$ $\therefore (a_B)_\theta = 0$

NOW.. $\xrightarrow{+}\Sigma F_r = ma_r$: $(F_{sp})_B = m(a_B)_r$

OR $(a_B)_r = \frac{0.6 \text{ N}}{0.25 \text{ kg}}$

$$\text{OR } (a_B)_r = 2.40\frac{m}{s^2}$$

(c) HAVE.. $a_r = \ddot{r} - r\dot{\theta}^2$

NOW.. $a_{COLLAR/ROD} = \ddot{r}$ AND $\dot{\theta}_B = \frac{(v_B)_\theta}{r_B}$

THEN.. AT B: $(a_{COLLAR/ROD})_B = 2.40\frac{m}{s^2} + (0.4\text{ m})\left(\frac{0.400\frac{m}{s}}{0.4\text{ m}}\right)^2$

$$\text{OR } (a_{COLLAR/ROD})_B = 2.80\frac{m}{s^2}$$

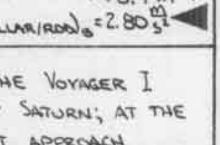
12.132

GIVEN: TRAJECTORY OF THE VOYAGER I SPACECRAFT ABOUT SATURN; AT THE POINT OF CLOSEST APPROACH, $r = 185\times10^3$ km, $v = 21.0$ km/s; FOR THE CIRCULAR ORBIT OF THE MOON TETHYS, $r = 295\times10^3$ km, $v = 11.35\times10^3$ km/s

FIND: ϵ AT THE POINT OF CLOSEST APPROACH OF VOYAGER I

FOR A CIRCULAR ORBIT

$v = \sqrt{\frac{GM}{r}}$ EQ. (12.44)

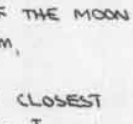


FOR THE ORBIT OF TETHYS..

$GM = r_T v_T^2$

FOR VOYAGER'S TRAJECTORY HAVE..

$\frac{1}{r} = \frac{GM}{h^2}(1 + \epsilon\cos\theta)$



WHERE $h = r_0 v_0$

AT O, $r = r_0$, $\theta = 0$

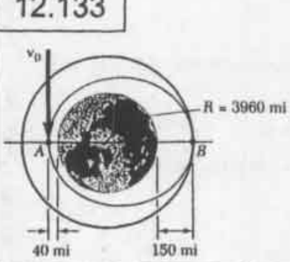
THEN -- $\frac{1}{r_0} = \frac{GM}{(r_0 v_0)^2}(1 + \epsilon)$

OR $\epsilon = \frac{r_0 v_0^2}{GM} - 1 = \frac{r_0 v_0^2}{r_T v_T^2} - 1$

$= \frac{185\times10^3 \text{ km}}{295\times10^3 \text{ km}} \times \left(\frac{21.0 \text{ km/s}}{11.35 \text{ km/s}}\right)^2 - 1$

$$\text{OR } \epsilon = 1.147$$

12.133



GIVEN: ELLIPTIC AND CIRCULAR ORBITS OF THE SHUTTLE COLUMBIA ABOUT THE EARTH

FIND: (a) t_{AB}
(b) τ_{CIRC}

FIRST NOTE.. $R = 3960 \text{ mi} = 20.9088 \times 10^6 \text{ ft}$

$r_A = (3960 + 40)\text{ mi} = 4000 \text{ mi} = 21.120 \times 10^6 \text{ ft}$

$r_B = (3960 + 150)\text{ mi} = 4110 \text{ mi} = 21.7008 \times 10^6 \text{ ft}$

(a) THE PERIODIC TIME τ OF AN ELLIPTIC ORBIT IS

$$\tau = \frac{2\pi ab}{h} \qquad [\text{EQ. }(12.45)]$$

$$\therefore\ t_{AB} = \tfrac{1}{2}\tau = \frac{\pi ab}{h_{AB}}$$

WHERE $a = \frac{1}{2}(r_A + r_B) = \frac{1}{2}(21.120 \times 10^6 + 21.7008 \times 10^6)\text{ ft}$
$= 21.4104 \times 10^6 \text{ ft}$

$b = \sqrt{r_A r_B} = [(21.120 \times 10^6 \text{ ft})(21.7008 \times 10^6 \text{ ft})]^{1/2}$
$= 21.4084 \times 10^6 \text{ ft}$

FROM THE SOLUTION TO PROBLEM 12.102, HAVE FOR THE ELLIPTIC ORBIT..

$$\frac{1}{r_A} + \frac{1}{r_B} = \frac{2GM}{h_{AB}^2}$$

NOW.. $GM = gR^2 \qquad [\text{EQ. }(12.30)]$

SO THAT

$$h_{AB} = \left(\frac{2gR^2}{\frac{1}{r_A} + \frac{1}{r_B}}\right)^{1/2} = \left[\frac{2(32.2 \frac{\text{ft}}{\text{s}^2})(20.9088 \times 10^6 \text{ ft})^2}{\frac{1}{21.120 \times 10^6 \text{ ft}} + \frac{1}{21.7008 \times 10^6 \text{ ft}}}\right]^{1/2}$$

$$= 548.95 \times 10^9 \ \frac{\text{ft}^2}{\text{s}}$$

FINALLY.. $t_{AB} = \dfrac{\pi(21.4104 \times 10^6 \text{ ft})(21.4084 \times 10^6 \text{ ft})}{548.95 \times 10^9 \text{ ft}^2/\text{s}}$

$= 2623.2 \text{ s}$

OR $t_{AB} = 43 \text{ MIN } 43 \text{ s}$ ◀

(b) FOR THE CIRCULAR ORBIT

$$\tau_{CIRC} = \frac{2\pi r_B}{v_{CIRC}}$$

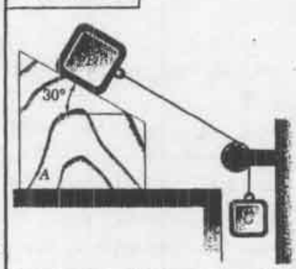
WHERE $v_{CIRC} = \sqrt{\dfrac{gR^2}{r_B}} \qquad [\text{EQ. }(12.44)]$

THEN.. $\tau_{CIRC} = \dfrac{2\pi r_B^{3/2}}{R\sqrt{g}} = \dfrac{2\pi(21.7008 \times 10^6 \text{ ft})^{3/2}}{(20.9088 \times 10^6 \text{ ft})(32.2 \text{ ft/s}^2)^{1/2}}$

$= 5353.5 \text{ s}$

OR $\tau_{CIRC} = 1 \text{ h } 29 \text{ MIN } 13 \text{ s}$ ◀

12.C1



GIVEN: $m_A = 20$ kg, $m_B = 10$ kg, $m_C = 2$ kg; $t = 0$, $v = 0$; $\mu \geq 0$

FIND: a_A AND $a_{B/A}$ FOR $\mu \geq 0$ USING $\Delta\mu = 0.01$ WHILE $a_A > 0$ AND $\Delta\mu = 0.1$ WHILE $a_{B/A} > 0$

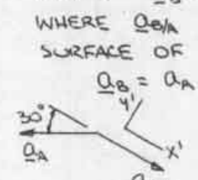
ANALYSIS

KINEMATICS

HAVE.. $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

WHERE $\underline{a}_{B/A}$ IS DIRECTED ALONG THE INCLINED SURFACE OF A. THEN..

$$\underline{a}_B = a_A(-\cos 30°\,\underline{i}' - \sin 30°\,\underline{j}') + a_{B/A}\,\underline{i}'$$

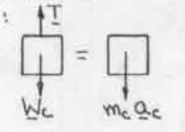


ALSO, SINCE THE CORD IS OF CONSTANT LENGTH

$a_C = (a_B)_{x'}$
$= a_{B/A} - a_A \cos 30°$

KINETICS

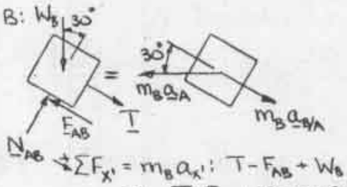
C:



$+\uparrow \Sigma F_y = m_C a_C$: $T - W_C = -m_C a_C$

OR $T = m_C(g - a_C)$
$= m_C(g - a_{B/A} + a_A \cos 30°)$ -- (2)

B:



$\searrow \Sigma F_{x'} = m_B a_{x'}$: $T - F_{AB} + W_B \sin 30° = m_B a_{B/A} - m_B a_A \cos 30°$

OR $T - F_{AB} + 10g \sin 30° = 10 a_{B/A} - 10 a_A \cos 30°$ (3)

$+\nearrow \Sigma F_{y'} = m_B a_{y'}$: $N_{AB} - W_B \cos 30° = -m_B a_A \sin 30°$

OR $N_{AB} = 10g \cos 30° - 10 a_A \sin 30°$ (4)

SLIDING: $F_{AB} = \mu N_{AB}$

OR $F_{AB} = 10\mu(g \cos 30° - a_A \sin 30°)$ (5)

SUBSTITUTING EQS. (2) AND (5) INTO EQ. (3)..

$2(g - a_{B/A} + a_A \cos 30°) - 10\mu(g \cos 30° - a_A \sin 30°) + 10g \sin 30°$
$= 10 a_{B/A} - 10 a_A \cos 30°$

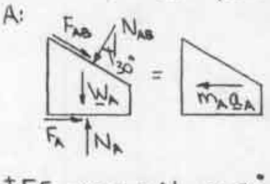
OR $g(1 - 5\mu \cos 30° + 5 \sin 30°)$
$= 6 a_{B/A} - a_A(5\mu \sin 30° + 6 \cos 30°)$ (6)

NOTE: BLOCK A WILL NOT MOVE ($a_A = 0$) BEFORE BLOCKS B AND C WILL NOT MOVE ($a_{B/A} = a_B = 0$). THEREFORE, THE SYSTEM WILL REMAIN AT REST WHEN

$g(1 - 5\mu \cos 30° + 5 \sin 30°) = 0$

OR $\mu \geq 0.808$ FOR NO MOTION

A:



$\overset{+}{\leftarrow} \Sigma F_x = m_A a_A$: $N_{AB} \sin 30° - F_A - F_{AB} \cos 30° = m_A a_A$

OR $N_{AB}(\sin 30° - \mu \cos 30°) - F_A = 20 a_A$ (7)

(CONTINUED)

12.C1 continued

$+\uparrow \Sigma F_y = 0$: $N_A - N_{AB}\cos 30^\circ - F_{AB}\sin 30^\circ - W_A = 0$

OR $N_A = N_{AB}(\cos 30^\circ + \mu \sin 30^\circ) + 20g$ (8)

SLIDING: $F_A = \mu N_A$

OR $F_A = \mu N_{AB}(\cos 30^\circ + \mu \sin 30^\circ) + 20\mu g$ (9)

SUBSTITUTING EQ. (9) INTO EQ. (7)..

$$N_{AB}(\sin 30^\circ - \mu\cos 30^\circ) - \mu N_{AB}(\cos 30^\circ + \mu \sin 30^\circ) - 20\mu g = 20a_A$$

OR $N_{AB}[(1-\mu^2)\sin 30^\circ - 2\mu\cos 30^\circ] - 20\mu g = 20a_A$

SUBSTITUTING FOR N_{AB} [EQ. (4)]..

$$(10g\cos 30^\circ - 10a_A\sin 30^\circ)[(1-\mu^2)\sin 30^\circ - 2\mu\cos 30^\circ] - 20\mu g = 20a_A$$

LET $A = (1-\mu^2)\sin 30^\circ - 2\mu\cos 30^\circ$

THEN.. $g(A\cos 30^\circ - 2\mu) = (2 + A\sin 30^\circ)a_A$

OR $a_A = \dfrac{A\cos 30^\circ - 2\mu}{2 + A\sin 30^\circ}\,g$ (10)

NOTE: BLOCK A WILL REMAIN AT REST WHEN

$g(A\cos 30^\circ - 2\mu) = 0$

OR $[(1-\mu^2)\sin 30^\circ - 2\mu\cos 30^\circ]\cos 30^\circ - 2\mu = 0$

OR $(\tfrac{1}{2}\sin 60^\circ)\mu^2 + 2(1+\cos^2 30^\circ)\mu - \tfrac{1}{2}\sin 60^\circ = 0$

OR $\mu \geq 0.12188$ FOR BLOCK A TO REMAIN AT REST

NOW.. REWRITE EQ. (6) AS

$$a_{B/A} = \tfrac{1}{6}[g(1 - 5\mu\cos 30^\circ + 5\sin 30^\circ) + a_A(5\mu\sin 30^\circ + 6\cos 30^\circ)] \quad (11)$$

WHICH REDUCES TO

$$a_{B/A} = \tfrac{g}{6}(1 - 5\mu\cos 30^\circ + 5\sin 30^\circ) \quad (12)$$

WHEN $a_A = 0$

OUTLINE OF PROGRAM

INPUT INITIAL VALUE OF μ: $\mu = 0$

COMPUTE A: $A = (1-\mu^2)\sin 30^\circ - 2\mu\cos 30^\circ$

COMPUTE a_A: $a_A = \dfrac{A\cos 30^\circ - 2\mu}{2 + A\sin 30^\circ}\,g$

WHILE $a_A > 0$

  COMPUTE $a_{B/A}$:

  $a_{B/A} = \tfrac{1}{6}[g(1 - 5\mu\cos 30^\circ + 5\sin 30^\circ) + a_A(5\mu\sin 30^\circ + 6\cos 30^\circ)]$

  PRINT THE VALUES OF μ, a_A, AND $a_{B/A}$

  UPDATE μ: $\mu = \mu + 0.01$

INCREASE μ TO THE NEXT TENTH:

  $\mu = \tfrac{1}{10}[\text{INTEGER VALUE}(10\mu)] + 0.1$

COMPUTE $a_{B/A}$:

  $a_{B/A} = \tfrac{g}{6}(1 - 5\mu\cos 30^\circ + 5\sin 30^\circ)$

WHILE $a_{B/A} > 0$

  PRINT THE VALUES OF μ AND $a_{B/A}$

  UPDATE μ: $\mu = \mu + 0.1$

(CONTINUED)

12.C1 continued

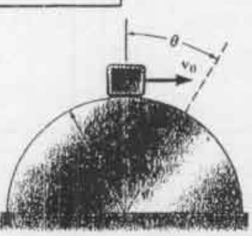
PROGRAM OUTPUT

μ	accel. of A, m/s²	accel. of B wrt A, m/s²
0.00	1.888	7.358
0.01	1.742	7.167
0.02	1.594	6.975
0.03	1.445	6.780
0.04	1.295	6.582
0.05	1.143	6.382
0.06	0.989	6.179
0.07	0.833	5.973
0.08	0.676	5.764
0.09	0.518	5.553
0.10	0.357	5.339
0.11	0.195	5.122
0.12	0.031	4.901

For those values of μ for which the wedge is at rest

μ	accel. of B wrt A, m/s²
0.20	4.307
0.30	3.599
0.40	2.891
0.50	2.183
0.60	1.475
0.70	0.767
0.80	0.059

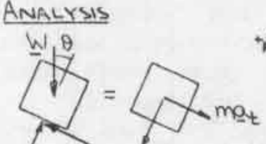
12.C2



GIVEN: $W = 1\,\text{lb}$, $v = 10\,\frac{\text{ft}}{\text{s}}$; $0 \leq \mu_k \leq 0.4$

FIND: θ AT WHICH THE BLOCK LEAVES THE SURFACE; $\mu = 0, 0.05, 0.10, \ldots, 0.4$

ANALYSIS



$+\swarrow \Sigma F_n = ma_n$: $W\cos\theta - N = m\frac{v^2}{\rho}$

OR $N = m(g\cos\theta - \frac{v^2}{\rho})$

SLIDING: $F = \mu_k N = \mu_k m(g\cos\theta - \frac{v^2}{\rho})$

$+\searrow \Sigma F_t = ma_t$: $W\sin\theta - F = ma_t$

OR $a_t = g\sin\theta - \frac{1}{m}F$

SUBSTITUTING FOR F.. $a_t = g(\sin\theta - \mu_k\cos\theta) + \mu_k\frac{v^2}{\rho}$

NOW.. $a_t = \frac{dv}{dt}$ .. $\frac{dv}{dt} = g(\sin\theta - \mu_k\cos\theta) + \mu_k\frac{v^2}{\rho}$ (1)

ALSO.. $v = r\dot{\theta}$ OR $\frac{d\theta}{dt} = \frac{1}{\rho}v$ (2)

THUS, DIFFERENTIAL EQUATIONS (1) AND (2) DEFINE THE MOTION OF THE BLOCK.

AS THE BLOCK LEAVES THE SURFACE, $N \to 0$.

THUS, $g\cos\theta - \frac{v^2}{\rho} = 0$

DEFINES THE VALUE OF θ AT WHICH THE BLOCKS LEAVES THE SURFACE.

OUTLINE OF PROGRAM

FOR EACH VALUE OF μ_k

  DEFINE THE INITIAL VALUES OF v AND θ

  USE THE MODIFIED EULER METHOD (SEE THE SOLUTION TO PROBLEM 11.C3) WITH A STEP

(CONTINUED)

12.C2 continued

SIZE $\Delta t = 0.01$ s TO NUMERICALLY INTEGRATE THE EQUATIONS

$$\frac{dv}{dt} = g(\sin\theta - \mu_k\cos\theta) + \mu_k\frac{v^2}{\rho}$$
$$\frac{d\theta}{dt} = \frac{1}{\rho}v$$

WHERE $\rho = 5$ ft.

COMPUTE N_1 AND N_2: $N_1 = \cos\theta_1 - \frac{v_1^2}{g\rho}$
$N_2 = \cos\theta_2 - \frac{v_2^2}{g\rho}$

WHERE θ_1 AND v_1 ARE THE VALUES OF θ AND THE VELOCITY, RESPECTIVELY, AT THE BEGINNING OF A TIME INTERVAL, AND θ_2 AND v_2 ARE THE VALUES AT THE END OF THE TIME INTERVAL.

IF $N_2 > 0$, UPDATE v AND θ: $v_1 = v_2$; $\theta_1 = \theta_2$

IF $N_2 < 0$, USE LINEAR INTERPOLATION TO DETERMINE THE VALUE OF θ AT WHICH $N = 0$:

$$\theta = \theta_1 + \frac{0 - N_1}{N_2 - N_1}(\theta_2 - \theta_1)$$

PRINT THE VALUES OF μ AND θ

PROGRAM OUTPUT

μ	θ
0.00	29.11°
0.05	29.61°
0.10	30.16°
0.15	30.72°
0.20	31.33°
0.25	31.96°
0.30	32.63°
0.35	33.35°
0.40	34.11°

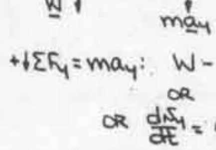
12.C3

GIVEN: BLOCK OF MASS m AND SPRING OF CONSTANT k; AT $t = 0$, $v = 0$ AND THE SPRING IS HORIZONTAL AND UNSTRETCHED

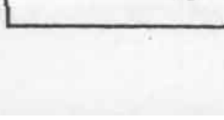
FIND: (a) r AND v WHEN THE BLOCK PASSES UNDER THE PIVOT O

(b) k/m WHEN $r_0 = 1$ m SO THAT $v \to$ WHEN THE BLOCK PASSES UNDER O

ANALYSIS



FIRST NOTE.. $r = \sqrt{x^2+y^2}$
$\cos\theta = \frac{x}{r}$ $\sin\theta = \frac{y}{r}$
$F_{sp} = k(r - r_0)$



$\xrightarrow{+} \Sigma F_x = ma_x$: $-F_{sp}\cos\theta = ma_x$
OR $a_x = -\frac{k}{m}(r - r_0)\cos\theta$
OR
$$\frac{dv_x}{dt} = -\frac{k}{m}\left(\sqrt{x^2+y^2} - r_0\right)\left(\frac{x}{\sqrt{x^2+y^2}}\right) \quad (1)$$

$+\downarrow \Sigma F_y = ma_y$: $W - F_{sp}\sin\theta = ma_y$
OR $a_y = g - \frac{k}{m}(r - r_0)\sin\theta$
OR
$$\frac{dv_y}{dt} = g - \frac{k}{m}\left(\sqrt{x^2+y^2} - r_0\right)\left(\frac{y}{\sqrt{x^2+y^2}}\right) \quad (2)$$

(CONTINUED)

12.C3 continued

ALSO.. $\frac{dx}{dt} = v_x$ (3) $\frac{dy}{dt} = v_y$ (4)

THEREFORE, DIFFERENTIAL EQUATIONS (1)-(4) DEFINE THE MOTION OF THE MASS.

NOW.. $v = \sqrt{v_x^2 + v_y^2}$ AND $\theta_v = \tan^{-1}\frac{v_y}{v_x}$

DEFINE THE MAGNITUDE AND DIRECTION, RESPECTIVELY, OF THE VELOCITY.

OUTLINE OF PROGRAM

INPUT VALUE OF k/m

INPUT UNSTRETCHED LENGTH OF THE SPRING r_0

INPUT SYSTEM OF UNITS

DEFINE THE INITIAL CONDITIONS:

$x_1 = r_0$, $y_1 = 0$; $(v_x)_1 = 0$, $(v_y)_1 = 0$

USE THE MODIFIED EULER METHOD (SEE THE SOLUTION TO PROBLEM 11.C3) WITH A STEP SIZE $\Delta t = 0.001$ s TO NUMERICALLY INTEGRATE THE EQUATIONS

$$\frac{dv_x}{dt} = -\frac{k}{m}\left(\sqrt{x^2+y^2} - r_0\right)\left(\frac{x}{\sqrt{x^2+y^2}}\right)$$
$$\frac{dv_y}{dt} = g - \frac{k}{m}\left(\sqrt{x^2+y^2} - r_0\right)\left(\frac{y}{\sqrt{x^2+y^2}}\right)$$
$$\frac{dx}{dt} = v_x$$
$$\frac{dy}{dt} = v_y$$

WHEN $x_1 > 0$ AND $x_2 < 0$

COMPUTE r_1 AND r_2: $r_1 = \sqrt{x_1^2 + y_1^2}$ $r_2 = \sqrt{x_2^2 + y_2^2}$

COMPUTE v_1 AND θ_{v_1}: $v_1 = \sqrt{(v_x)_1^2 + (v_y)_1^2}$
$(\theta_v)_1 = \tan^{-1}\frac{(v_y)_1}{(v_x)_1}$

COMPUTE v_2 AND θ_{v_2}: $v_2 = \sqrt{(v_x)_2^2 + (v_y)_2^2}$
$(\theta_v)_2 = \tan^{-1}\frac{(v_y)_2}{(v_x)_2}$

WHERE $(\ )_1$ AND $(\ )_2$ DENOTE VALUES AT THE BEGINNING AND END, RESPECTIVELY, OF A TIME INTERVAL.

USE LINEAR INTERPOLATION TO DETERMINE THE VALUES OF r, v, AND θ_v AT $x = 0$:

$$r = r_1 + \frac{0 - x_1}{x_2 - x_1}(r_2 - r_1)$$
$$v = v_1 + \frac{0 - x_1}{x_2 - x_1}(v_2 - v_1)$$
$$\theta_v = (\theta_v)_1 + \frac{0 - x_1}{x_2 - x_1}\left[(\theta_v)_2 - (\theta_v)_1\right]$$

PRINT THE VALUES OF k/m, r_0, r, v, AND θ_v

PROGRAM OUTPUT

(a)
```
k/m =  15.00 /s²
Unstretched length of the spring = 1 m

    X1 = 0.001 m      X2 = -.002 m
    r = 2.765 m
    v = 2.740 m/s
    Angle v forms with the horizontal = -6.19°



k/m =  20.00 /s²
Unstretched length of the spring = 1 m

    X1 = 0.001 m      X2 = -.002 m
    r = 2.372 m
    v = 2.983 m/s
    Angle v forms with the horizontal =  0.93°
```

(CONTINUED)

12.C3 continued

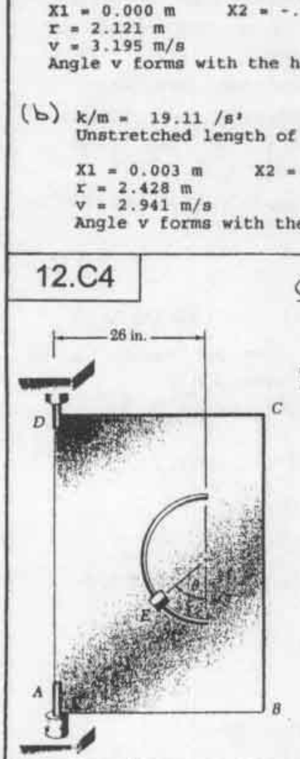
```
k/m =  25.00 /s²
Unstretched length of the spring = 1 m

X1 = 0.000 m      X2 = -.003 m
r = 2.121 m
v = 3.195 m/s
Angle v forms with the horizontal =  4.62°
```

(b)
```
k/m =  19.11 /s²
Unstretched length of the spring = 1 m

X1 = 0.003 m      X2 = -.000 m
r = 2.428 m
v = 2.941 m/s
Angle v forms with the horizontal = -0.00°
```

12.C4



GIVEN: $\mu_s = 0.35$; $\dot{\phi}_{ABCD} = 14\ \frac{\text{RAD}}{\text{S}}$, $2\ \frac{\text{RAD}}{\text{S}}$; $r_{SLOT} = 10$ IN.; $W_E = 0.8$ lb

FIND: RANGE OF VALUES OF θ FOR WHICH THE BLOCK DOES NOT SLIDE

ANALYSIS

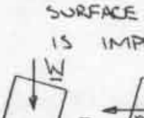
FIRST NOTE.. $\rho = \frac{1}{12}(26 - 10\sin\theta)\ \text{ft} = \frac{1}{6}(13 - 5\sin\theta)\ \text{ft}$

$v_E = \rho\dot{\phi} \quad (\dot{\phi} = \dot{\phi}_{ABCD})$

THEN $a_n = \frac{v_E^2}{\rho} = \rho\dot{\phi}^2 = \frac{1}{6}(13 - 5\sin\theta)\dot{\phi}^2 \quad \left(\frac{\text{ft}}{\text{s}^2}\right)$

NOW CONSIDER THE FOLLOWING FOUR CASES.

CASE 1: THE BLOCK IS RESTING ON THE INNER SURFACE OF THE SLOT; DOWNWARD MOTION IS IMPENDING $(0 \le \theta \le 90°)$



$+\uparrow \Sigma F_y = 0$: $F\sin\theta + N\cos\theta - W = 0$

$\overset{+}{\leftarrow} \Sigma F_n = ma_n$: $F\cos\theta - N\sin\theta = \frac{W}{g}\frac{v_E^2}{\rho}$

THEN.. $F = W(\sin\theta + \frac{v_E^2}{g\rho}\cos\theta)$

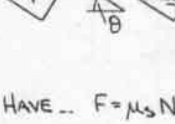
AND.. $N = W(\cos\theta - \frac{v_E^2}{g\rho}\sin\theta)$

HAVE.. $F = \mu_s N$

THEN.. $W(\sin\theta + \frac{v_E^2}{g\rho}\cos\theta) = \mu_s \cdot W(\cos\theta - \frac{v_E^2}{g\rho}\sin\theta)$

OR $[6g\sin\theta + \dot{\phi}^2(13 - 5\sin\theta)\cos\theta]$
$= 0.35[6g\cos\theta - \dot{\phi}^2(13 - 5\sin\theta)\sin\theta]$ (1)

CASE 2: THE BLOCK IS RESTING AGAINST THE OUTER SURFACE OF THE SLOT; DOWNWARD MOTION IS IMPENDING $(0 \le \theta \le 90°)$



$+\uparrow \Sigma F_y = 0$: $F\sin\theta - N\cos\theta - W = 0$

$\overset{+}{\leftarrow} \Sigma F_n = ma_n$: $F\cos\theta + N\sin\theta = \frac{W}{g}\frac{v_E^2}{\rho}$

THEN.. $F = W(\sin\theta + \frac{v_E^2}{g\rho}\cos\theta)$

AND.. $N = W(-\cos\theta + \frac{v_E^2}{g\rho}\sin\theta)$

HAVE.. $F = \mu_s N$

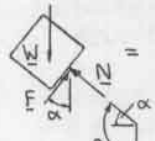
(CONTINUED)

12.C4 continued

THEN.. $W(\sin\theta + \frac{v_E^2}{g\rho}\cos\theta) = \mu_s \times W(-\cos\theta + \frac{v_E^2}{g\rho}\sin\theta)$

OR $[6g\sin\theta + \dot{\phi}^2(13 - 5\sin\theta)\cos\theta]$
$= 0.35[-6g\cos\theta + \dot{\phi}^2(13 - 5\sin\theta)\sin\theta]$ (2)

CASE 3: THE BLOCK IS RESTING AGAINST THE OUTER SURFACE OF THE SLOT; DOWNWARD MOTION IS IMPENDING $(90° \le \theta \le 180°)$



$+\uparrow \Sigma F_y = 0$: $F\cos\alpha + N\sin\alpha - W = 0$

$\overset{+}{\leftarrow} \Sigma F_n = ma_n$: $-F\sin\alpha + N\cos\alpha = \frac{W}{g}\frac{v_E^2}{\rho}$

THEN.. $F = W(\cos\alpha - \frac{v_E^2}{g\rho}\sin\alpha)$

AND.. $N = W(\sin\alpha + \frac{v_E^2}{g\rho}\cos\alpha)$

HAVE.. $F = \mu_s N$

THEN.. $W(\cos\alpha - \frac{v_E^2}{g\rho}\sin\alpha) = \mu_s \cdot W(\sin\alpha + \frac{v_E^2}{g\rho}\cos\alpha)$

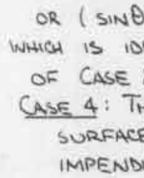
NOW $\alpha = \theta - 90°$

SUBSTITUTING.. $[\cos(\theta - 90°) - \frac{v_E^2}{g\rho}\sin(\theta - 90°)]$
$= \mu_s[\sin(\theta - 90°) + \frac{v_E^2}{g\rho}\cos(\theta - 90°)]$

OR $(\sin\theta + \frac{v_E^2}{g\rho}\cos\theta) = \mu_s(-\cos\theta + \frac{v_E^2}{g\rho}\sin\theta)$

WHICH IS IDENTICAL TO THE DEFINING EQUATION OF CASE 2

CASE 4: THE BLOCK IS RESTING AGAINST THE OUTER SURFACE OF THE SLOT; UPWARD MOTION IS IMPENDING $(90° \le \theta \le 180°)$



$+\uparrow \Sigma F_y = 0$: $-F\cos\alpha + N\sin\alpha - W = 0$

$\overset{+}{\leftarrow} \Sigma F_n = ma_n$: $F\sin\alpha + N\cos\alpha = \frac{W}{g}\frac{v_E^2}{\rho}$

THEN.. $F = W(-\cos\alpha + \frac{v_E^2}{g\rho}\sin\alpha)$

AND.. $N = W(\sin\alpha + \frac{v_E^2}{g\rho}\cos\alpha)$

HAVE.. $F = \mu_s N$

THEN.. $W(-\cos\alpha + \frac{v_E^2}{g\rho}\sin\alpha) = \mu_s \cdot W(\sin\alpha + \frac{v_E^2}{g\rho}\cos\alpha)$

NOW $\alpha = \theta - 90°$

SUBSTITUTING.. $[-\cos(\theta - 90°) + \frac{v_E^2}{g\rho}\sin(\theta - 90°)]$
$= \mu_s[\sin(\theta - 90°) + \frac{v_E^2}{g\rho}\cos(\theta - 90°)]$

OR $(-\sin\theta - \frac{v_E^2}{g\rho}\cos\theta) = \mu_s(-\cos\theta + \frac{v_E^2}{g\rho}\sin\theta)$

WHICH IS THE SAME AS THE DEFINING EQUATION OF CASE 1 AFTER MULTIPLYING BOTH SIDES OF THE EQUATION BY −1.

IT IS NEXT NECESSARY TO SOLVE EQS. (1) AND (2) FOR θ. EACH OF THESE EQUATIONS CAN BE EXPRESSED AS f(θ), AND THEN THE VALUES OF θ FOR WHICH f(θ) = 0 CAN BE DETERMINED. SUBSTITUTING FOR g (32.2 ft/s²) AND THEN SIMPLIFYING, FIND..

$f_1(\theta) = (67.62 - 13\dot{\phi}^2)\cos\theta - (4.55\dot{\phi}^2 + 193.2)\sin\theta$
$\quad + 1.75\dot{\phi}^2\sin^2\theta + 2.5\dot{\phi}^2\sin 2\theta$

$f_2(\theta) = -(67.62 + 13\dot{\phi}^2)\cos\theta + (4.55\dot{\phi}^2 - 193.2)\sin\theta$
$\quad - 1.75\dot{\phi}^2\sin^2\theta + 2.5\dot{\phi}^2\sin 2\theta$

NOTE: FOR THOSE VALUES OF θ FOR WHICH THE BLOCK IS AT REST WITH RESPECT TO THE PLATE,

$F_{MAX} = \mu_s N \ge F$

WHERE N AND F ARE GIVEN ABOVE FOR EACH OF THE CASES. ALSO, $f(\theta) = F_{MAX} - F$

(CONTINUED)

12.C4 continued

OUTLINE OF PROGRAM

INPUT VALUE OF $\dot{\phi}$

CONSIDER CASES 1 AND 4

FOR VALUES OF θ FROM 0 TO 179° IN INCREMENTS OF 1°

COMPUTE $f_1(\theta)$:

$$f_1(\theta) = (67.62 - 13\dot{\phi}^2)\cos\theta - (4.55\dot{\phi}^2 + 193.2)\sin\theta + 1.75\dot{\phi}^2\sin^2\theta + 2.5\dot{\phi}^2\sin 2\theta$$

COMPUTE $f_1(\theta + 1°)$

COMPUTE $f_1(\theta) \times f_1(\theta + 1°)$ TO DETERMINE IF A ROOT LIES BETWEEN θ AND $(\theta + 1°)$

IF $f_1(\theta) \times f_1(\theta + 1°) \le 0$, SOLVE $f_1(\theta)$ FOR θ USING NEWTON'S METHOD (SEE THE SOLUTION TO PROBLEM 11.C4)

PRINT THE VALUE OF θ_{ROOT} AND WHETHER $F_{MAX} - F$ AT θ IS $\ge$ OR ≤ 0

CONSIDER CASES 2 AND 3

FOR VALUES OF θ FROM 0 TO 179° IN INCREMENTS OF 1°

COMPUTE $f_2(\theta)$:

$$f_2(\theta) = -(67.62 + 13\dot{\phi}^2)\cos\theta + (4.55\dot{\phi}^2 - 193.2)\sin\theta - 1.75\dot{\phi}^2\sin^2\theta + 2.5\dot{\phi}^2\sin 2\theta$$

COMPUTE $f_2(\theta + 1°)$

COMPUTE $f_2(\theta) \times f_2(\theta + 1°)$

IF $f_2(\theta) \times f_2(\theta + 1°) \le 0$, SOLVE $f_2(\theta)$ FOR θ USING NEWTON'S METHOD

PRINT THE VALUE OF θ_{ROOT} AND WHETHER $F_{MAX} - F$ AT θ IS $\ge$ OR ≤ 0

PROGRAM OUTPUT

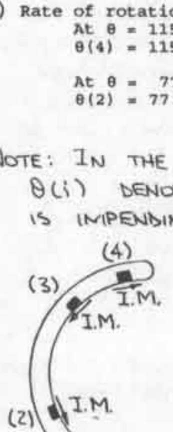
(a)
```
Rate of rotation =  2 rad/s
     At θ =   4°, F(max) - F >= 0
     θ(1) =  4.68°

     At θ = 148°, F(max) - F =< 0
     θ(3) = 148.57
```

(b)
```
Rate of rotation = 14 rad/s
     At θ = 115°, F(max) - F >= 0
     θ(4) = 115.91

     At θ =  77°, F(max) - F =< 0
     θ(2) = 77.63°
```

NOTE: IN THE ABOVE OUTPUT, THE i IN $\theta(i)$ DENOTES THE CASE FOR WHICH MOTION IS IMPENDING.



12.C5

GIVEN: TWO POINTS ON THE TRAJECTORY OF A SPACECRAFT: θ_1 AND θ_2 OR r_2 AND THE RADIAL DISTANCE TO AND THE VELOCITY AT THE APOGEE OR THE PERIGEE

FIND: TIME t FOR THE SPACECRAFT TO TRAVEL BETWEEN THE POINTS

(a) B AND C OF PROB. 12.115; $v_B = 869.4 \frac{m}{s}$

(b) A AND B OF PROB. 12.117; $v_A = 24{,}371 \frac{ft}{s}$

ANALYSIS

HAVE.. $\frac{1}{r} = \frac{GM}{h^2}(1 + \epsilon\cos\theta)$ [EQ. (12.39')]

WHERE $h = r_{APOGEE}v_{APOGEE} = r_{PERIGEE}v_{PERIGEE} = r_{AP}v_{AP}$

$\theta_{APOGEE} = 180°$ $\theta_{PERIGEE} = 0$

$GM = G\left(\frac{M}{M_{EARTH}}\right)M_{EARTH} = \left(\frac{M}{M_{EARTH}}\right)gR^2_{EARTH}$

THEN.. $\frac{1}{r_{AP}} = \frac{\left(\frac{M}{M_{EARTH}}\right)gR^2}{(r_{AP}v_{AP})^2}(1 + \epsilon\cos\theta_{AP})$

OR $\epsilon = \frac{1}{\cos\theta_{AP}}\left[\frac{r_{AP}v_{AP}^2}{\left(\frac{M}{M_{EARTH}}\right)gR^2_{EARTH}} - 1\right]$

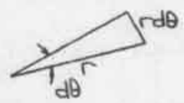
THUS, THE ECCENTRICITY OF THE TRAJECTORY CAN BE DETERMINED.

FROM PAGE 698 OF THE TEXT HAVE..

$$\frac{dA}{dt} = \frac{1}{2}h$$

WHERE h IS A CONSTANT. THEN..

$$t = \frac{2}{h}\int_{\theta_1}^{\theta_2} dA$$



WHERE $dA = \frac{1}{2}(r)(r\,d\theta)$

OR $\Delta A = \frac{1}{2}r^2\Delta\theta$

AND $t = \frac{1}{h}\sum r^2\Delta\theta$

WHERE r IS GIVEN BY EQ. (12.39').

OUTLINE OF PROGRAM

SET VALUE OF $\Delta\theta$: $\Delta\theta = 0.05°$

INPUT UNITS AND CONSTANTS

INPUT WHETHER VALUES ARE KNOWN AT THE APOGEE OR THE PERIGEE

SET VALUE OF θ_{AP}: $\theta_{AP} = 0$ (PERIGEE)

$\theta_{AP} = 180°$ (APOGEE)

INPUT THE DISTANCE r_{AP} TO AND THE VELOCITY v_{AP} AT THE APOGEE OR THE PERIGEE

INPUT THE VALUE OF θ_1 FOR THE FIRST POINT ON THE TRAJECTORY

INPUT WHETHER THE SECOND POINT ON THE TRAJECTORY IS DEFINED BY THE VALUE OF θ_2 (CASE 1) OR BY THE VALUE OF THE RADIAL DISTANCE r_2 (CASE 2)

INPUT M/M_{EARTH}

COMPUTE THE ECCENTRICITY ϵ OF THE TRAJECTORY:

$$\epsilon = \frac{1}{\cos\theta_{AP}}\left[\frac{r_{AP}v_{AP}^2}{\left(\frac{M}{M_{EARTH}}\right)gR^2_{EARTH}} - 1\right]$$

(CONTINUED)

12.C5 continued

CASE 1:

INPUT THE VALUE OF θ_2

IF $\theta_2 < \theta_1$, SET $\Delta\theta = -\Delta\theta$

FOR VALUES OF θ FROM θ_1 TO $\theta_2 - \Delta\theta$ IN INCREMENTS OF $\Delta\theta$

UPDATE AREA A:

$$A = A + \frac{|\Delta\theta|}{2}\left[\frac{(\frac{M}{M_{EARTH}})gR_{EARTH}^2}{(r_{AP}v_{AP})^2}(1+\epsilon\cos\theta)\right]^{-2}$$

COMPUTE TIME t: $t = \frac{2A}{r_{AP}v_{AP}}$

PRINT THE VALUES OF $r_{AP}, v_{AP}, \theta_1, \theta_2$, AND t

CASE 2:

INPUT THE VALUE OF r_2

SET THE INITIAL VALUE OF θ: $\theta = \theta_1$

WHILE $r > r_2$ IF $r_1 > r_2$ OR WHILE $r < r_2$ IF $r_1 < r_2$

COMPUTE r: $r = \left[\frac{(\frac{M}{M_{EARTH}})gR_{EARTH}^2}{(r_{AP}v_{AP})^2}(1+\epsilon\cos\theta)\right]^{-1}$

UPDATE AREA A: $A = A + \frac{1}{2}r^2\Delta\theta$

UPDATE θ: $\theta = \theta + \Delta\theta$

COMPUTE TIME t: $t = \frac{2A}{r_{AP}v_{AP}}$

PRINT THE VALUES OF $r_{AP}, v_{AP}, \theta_1, r_2$, AND t

PROGRAM OUTPUT

(a)

```
The radial distance to and the velocity at the apogee are,
    respectively, 3600 km and .8694 km/s
θ1 = 180°       θ2 = 290°
Time t =  1 h 18 min 29 s
```

(b)

```
The radial distance to and the velocity at the apogee are,
    respectively, 4310 mi and 24371 ft/s
θ1 = 180°       r2 =  4035 mi
Time t =  0 h 33 min 30 s
```

13.1

GIVEN: MASS OF SATELLITE, $m = 1500\ kg$

SPEED OF SATELLITE, $v = 22.9 \times 10^3\ km/h$

FIND: KINETIC ENERGY, T

$$v = 22.9 \times 10^3\ km/h = 6.36 \times 10^3\ m/s$$

$$T = \frac{1}{2} m v^2 = \frac{1}{2}(1500\ kg)(6.36 \times 10^3\ m/s)^2$$

$$T = 30.337 \times 10^9\ N \cdot m$$

NOTE: ACCELERATION OF GRAVITY HAS NO EFFECT ON THE MASS OF THE SATELLITE.

$$T = 30.3\ GJ$$

13.2

GIVEN: WEIGHT OF SATELLITE, $W = 870\ lb$

SPEED OF SATELLITE, $v = 12{,}500\ mi/h$

FIND: KINETIC ENERGY, T

$$v = (12{,}500\ mi/h)(h/3600\ s)(5280\ ft/mi)$$

$$v = 18{,}333\ ft/s$$

$$\text{MASS OF SATELLITE} = (870\ lb)/(32.2\ ft/s^2)$$

$$m = 27.019\ lb \cdot s^2/ft$$

$$T = \frac{1}{2} m v^2 = \frac{1}{2}(27.09)(18{,}333)^2$$

$$T = 4.5405 \times 10^9\ lb \cdot ft$$

NOTE: ACCELERATION OF GRAVITY HAS NO EFFECT ON THE MASS OF THE SATELLITE

$$T = 4.54 \times 10^9\ lb \cdot ft$$

13.3

GIVEN: WEIGHT OF STONE, $W = 5\ lb$

VELOCITY OF STONE, $v = 80\ ft/s$

ACCELERATION OF GRAVITY ON THE MOON, $g_m = 5.31\ ft/s^2$

FIND: (a) KINETIC ENERGY, T

HEIGHT h, FROM WHICH STONE WAS DROPPED

(b) T AND h ON THE MOON

(a) ON THE EARTH

$$T = \frac{1}{2} m v^2 = \frac{1}{2}\left(\frac{5\ lb}{32.2\ ft/s^2}\right)(80\ ft/s)^2$$

$$T = 496.89\ lb \cdot ft$$

$$T = 497\ lb \cdot ft$$

$T_1 + U_{1-2} = T_2 \quad T_1 = 0,\ U_{1-2} = Wh = (5\ lb)(h),\ T_2 = 497\ lb \cdot ft$

$$Wh = T_2 \quad h = \frac{5\ lb}{497\ lb \cdot ft} = 99.4\ ft$$

$$h = 99.4\ ft$$

(b) ON THE MOON

MASS IS UNCHANGED

THUS T IS UNCHANGED $\quad T = 497\ lb \cdot ft$

WEIGHT ON THE MOON IS: $W_m = m g_m = \frac{(5\ lb)}{(32.2\ ft/s^2)}(5.31\ ft/s^2)$

$$W_m = 0.8245\ lb$$

$$h_m = \frac{T_2}{W_m} = \frac{(497\ lb \cdot ft)}{(0.8245\ lb)} = 602.7\ ft$$

$$h_m = 603\ ft$$

13.4

GIVEN: MASS OF STONE, $m = 4\ kg$

VELOCITY OF STONE, $v = 25\ m/s$

ACCELERATION OF GRAVITY ON THE MOON, $g_m = 1.62\ m/s^2$

FIND:

(a) KINETIC ENERGY, T

HEIGHT h, FROM WHICH THE STONE WAS DROPPED

13.4 continued

(b) T AND h ON THE MOON

(a) ON THE EARTH

$$T = \frac{1}{2} m v^2 = \frac{1}{2}(4\ kg)(25\ m/s)^2 = 1250\ N \cdot m$$

$$T = 1250\ J$$

$$W = mg = (4\ kg)(9.81\ m/s^2) = 39.240\ N$$

$T_1 + U_{1-2} = T_2 \quad T_1 = 0 \quad U_{1-2} = Wh \quad T_2 = 39.240\ N$

$$h = \frac{T_2}{W} = \frac{(1250\ N \cdot m)}{(39.240\ N)} = 31.855\ m$$

$$h = 31.9\ m$$

(b) ON THE MOON

MASS IS UNCHANGED, $m = 4\ kg$

THUS T IS UNCHANGED $\quad T = 1250\ J$

WEIGHT ON THE MOON IS, $W_m = m g_m = (4\ kg)(1.62\ m/s^2)$

$$W_m = 6.48\ N$$

$$h_m = \frac{T}{W_m} = \frac{(1250\ N \cdot m)}{(6.48\ N)} = 192.9\ m$$

$$h_m = 192.9\ m$$

13.5

GIVEN: DISTANCE $d = 120\ m$

$\mu_s = 0.75$, NO SLIPPING

60 % OF WEIGHT ON FRONT WHEELS

40 % OF WEIGHT ON REAR WHEELS

FIND: MAXIMUM THEORETICAL SPEED AT 120 m STARTING FROM REST

(a) FOR FRONT WHEEL DRIVE

(b) FOR REAR WHEEL DRIVE

(a) FRONT WHEEL DRIVE

SINCE 60 % OF WEIGHT IS DISTRIBUTED ON FRONT WHEELS, THE MAXIMUM FORCE TO MOVE THE CAR IS $F = \mu_s N = (0.75)(0.6 W) = 0.450\ mg$

FOR 120 m $\quad U_{1-2} = (0.450\ mg)(120\ m) = 54\ mg$

$T_1 = 0$

$$T_1 + U_{1-2} = T_2$$

$$0 + 54\ mg = \frac{1}{2} m v_2^2$$

$$v_2^2 = (2)(54 g) = (108)(9.81\ m/s^2)$$

$$v_2^2 = 1059.5$$

$$v_2 = 32.55\ m/s$$

$$v_2 = 117.2\ km/h$$

(b) REAR WHEEL DRIVE

USE SAME SOLUTION AS FOR (a) EXCEPT THAT 40 % WEIGHT IS DISTRIBUTED ON REAR WHEELS

$$F = \mu_s N = (0.75)(0.40 W) = 0.3\ mg$$

FOR 120 m $\quad U_{1-2} = (0.3\ mg)(120\ m) = 36\ mg$

$T_1 = 0$

$$T_1 + U_{1-2} = T_2$$

$$0 + 36\ mg = \frac{1}{2} m v_2^2$$

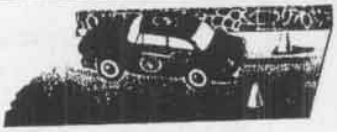
$$v_2^2 = (2)(36)(g) = (72)(9.81\ m/s^2) = 706.32$$

$$v_2 = 26.58\ m/s$$

$$v_2 = 95.7\ km/h$$

NOTE: THE CAR IS TREATED AS A PARTICLE IN THIS PROBLEM. THE WEIGHT DISTRIBUTION IS ASSUMED TO BE THE SAME FOR STATIC AND DYNAMIC CONDITIONS. COMPARE WITH SAMPLE PROBLEM 16.1 WHERE THE VEHICLE IS TREATED AS A RIGID BODY.

13.6



GIVEN: 1320 ft DRAG RACE TRACK, CAR STARTS FROM REST
CARS' FRONT WHEELS OFF THE GROUND FOR FIRST 60 ft
WHEELS ROLL WITHOUT SLIPPING FOR REMAINING 1260 ft WITH 60 % OF WEIGHT ON REAR WHEELS
$\mu_k = 0.60$, $\mu_s = 0.85$, NO AIR OR ROLLING RESISTANCE

FIND: (a) SPEED OF THE CAR AT END OF FIRST 60 ft
(b) MAXIMUM THEORETICAL SPEED AT FINISH LINE

(a) FIRST 60 ft: REAR WHEELS SKID TO GENERATE THE MAXIMUM FORCE. SINCE ALL THE WEIGHT IS ON THE REAR WHEELS THIS FORCE IS:

$$F = \mu_k N = (0.60)(W)$$

$T_1 = 0 \qquad T_2 = \frac{1}{2}\frac{W}{g}v_{60}^2$

FOR FIRST 60 ft

$$U_{1-2} = (F)(60\text{ ft}) = (0.6)(W)(60) = 36W$$
$$T_1 + U_{1-2} = T_2$$
$$36W = \frac{1}{2}\frac{W}{g}v_{60}^2$$
$$v_{60}^2 = 2318.4$$
$$v_{60} = 48.15\text{ ft/s}$$
$$v_{60} = 32.8\text{ mi/h}$$

(b) FOR 1320 ft REAR WHEELS SKID FOR FIRST 60 ft AND ROLL WITH SLIDING IMPENDING FOR REMAINING 1260 ft WITH 60 % OF THE WEIGHT ON THE REAR (DRIVE) WHEELS. THE MAXIMUM FORCE GENERATED IS:

FIRST 60 ft $\quad F_1 = (.6)(W)$ AS IN (a)

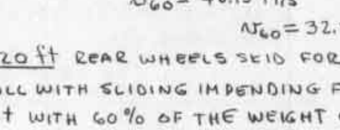
REMAINING 1260 ft $\quad F_2 = \mu_s N = (0.85)(0.60)(W) = 0.510\,W$

$T_1 = 0 \qquad T_2 = \frac{1}{2}\frac{W}{g}v_{1320}^2$

$$U_{1-2} = (.6)(W)(60) + (.510)(W)1260$$
$$= (36 + 642.6)W = 678.6\,W$$
$$0 + 678.6W = \frac{1}{2}\frac{W}{g}v_{1320}^2$$
$$v_{1320}^2 = 43702$$
$$v_{1320} = 209.05\text{ ft/s}$$
$$v_{1320} = 142.5\text{ mi/h}$$

SEE NOTE FOR PROB. 13.5 FOR DISCUSSION OF WEIGHT DISTRIBUTION

13.7



GIVEN: 1320 ft DRAG RACE TRACK, CAR STARTS FROM REST.
CARS' FRONT WHEELS OFF THE GROUND AND REAR WHEELS SKID FOR FIRST 60 ft
SPEED AT END OF FIRST 60 ft IS 36 mi/h.
WHEELS ROLL WITH SLIPPING IMPENDING FOR REMAINING 1260 ft, WITH 75 % OF THE WEIGHT ON REAR (DRIVE) WHEELS.
$\mu_k = 0.80\mu_s$
NO AIR OR ROLLING RESISTANCE

13.7 continued

FIND: SPEED OF CAR AT END OF RACE

FIRST 60 ft: SINCE ALL THE CARS' WEIGHT IS ON THE REAR WHEELS WHICH SKID, THE FORCE MOVING THE CAR IS

$$F = \mu_k N = (\mu_k)(W)$$
$$v_{60} = (36\text{ mi/h})(88\text{ ft/s})/(60\text{ mi/h})$$
$$v_{60} = 52.8\text{ ft/s}$$

$T_1 = 0 \qquad T_2 = \frac{1}{2}mv_{60}^2 = \frac{1}{2}\left(\frac{W}{g}\right)(52.8\text{ ft/s})^2 = (1393.9)\left(\frac{W}{g}\right)$

$$U_{1-2} = (F)(60\text{ ft}) = (\mu_k)(W)(60\text{ ft})$$
$$T_1 + U_{1-2} = T_2$$
$$0 + 60\mu_k W = (1393.9)\left(\frac{W}{g}\right)$$
$$\mu_k = \frac{(1393.9)}{(60)(32.2)} = 0.72149$$

FOR 1320 ft FORCE MOVING THE CAR IS
FOR FIRST 60 ft, $F_1 = (\mu_k)(W) = (0.72149)W$
FOR REMAINING 1260 ft, WITH 75 % OF WEIGHT ON REAR (DRIVE) WHEELS AND IMPENDING SLIDING,

$F_2 = (\mu_s)(0.75)W \qquad \mu_s = \mu_k/(0.80) = (0.72149)/0.80)$

$F_2 = (0.90186)(.75)W = 0.6764 \qquad \mu_s = 0.90186$

$T_1 = 0 \qquad T_2 = \frac{1}{2}\left(\frac{W}{g}\right)(v_{1320})^2$

$$U_{1-2} = (F_1)(60\text{ ft}) + F_2(1260\text{ ft})$$
$$= (0.72149)(W)(60\text{ ft}) + (0.6764)(W)(1260\text{ ft})$$
$$= 43.29W + 852.3W = 895.55W$$
$$T_1 + U_{1-2} = T_2$$
$$0 + 895.55W = \frac{1}{2}\left(\frac{W}{g}\right)(v_{1320})^2$$
$$v_{1320}^2 = (2g)(895.55) = (2)(32.2\text{ ft/s}^2)(895.55)$$
$$v_{1320}^2 = 57{,}673 \qquad v_{1320} = 240.2\text{ ft/s}$$

SEE NOTE FOR PROB 13.5 $\qquad v_{1320} = 163.7\text{ mi/h}$

13.8

GIVEN: 400 m DRAG RACE TRACK, CAR STARTS FROM REST
FRONT WHEELS OFF THE GROUND AND REAR WHEELS SKID FOR FIRST 20 m.
WHEELS ROLL WITH SLIPPING IMPENDING FOR REMAINING 380 m, WITH 80 % OF THE WEIGHT ON THE REAR DRIVE WHEELS
PEAK SPEED AT END OF THE RACE, = 270 km/h
$\mu_k = 0.75\mu_s$

FIND:
(a) COEFFICIENT OF STATIC FRICTION, μ_s
(b) SPEED AT THE END OF THE FIRST 20 m

(a) FORCE MOVING THE CAR FOR THE FIRST 20 m, WITH ALL OF THE WEIGHT ON THE REAR DRIVE WHEELS AND THE WHEELS SKIDDING,

$\mu_k = 0.75\mu_s \qquad F_1 = \mu_k N = \mu_k W = (0.75)(\mu_s)mg$

FORCE MOVING THE CAR FOR REMAINING 380 m WITH 80 % OF THE WEIGHT ON THE REAR (DRIVE) WHEELS AND SLIPPING IMPENDING (CONTINUED)

13.8 continued

$F_2 = \mu_s(0.80)(W) = \mu_s(0.80)(W) = \mu_s(.80)mg$

$T_1 = 0 \quad v_{400} = (270 \frac{km}{h})(\frac{1000\,m}{km})/(3600 \frac{s}{h})$

$v_{400} = 75\,m/s$

$T_2 = \frac{1}{2} m v_{400}^2 = \frac{1}{2} m (75)^2 = 2812.5\,m$

$U_{1-2} = F_1(20m) + F_2(380m)$

$U_{1-2} = (\mu_s)(.75)mg(20m) + (\mu_s)(.80)mg(380)$

$U_{1-2} = 15\mu_s mg + 304\mu_s mg = 319\mu_s mg$

$T_1 + U_{1-2} = T_2$

$0 + 319\mu_s mg = 2812.5\,m$

$\mu_s = (2812.5)/(319)(9.81) = 0.8987$

$\mu_s = 0.899$

(b) FOR FIRST 20 m

$\mu_k = (0.75)(\mu_s) = 0.6741$

$F_1 = \mu_k N = (0.6741)(mg)$

$T_1 = 0 \quad T_2 = \frac{1}{2} m v_{20}^2$

$U_{1-2} = (0.6741)(mg)(20m) = 13.481\,mg$

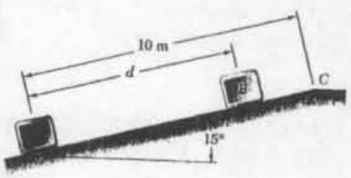
$0 + (13.481)(mg) = \frac{1}{2} m v_{20}^2$

$v_{20}^2 = (2)(13.481)(9.81) = 264.5$

$v_{20} = 16.26\,m/s$

SEE NOTE FOR P13.5 $\qquad v_{20} = 58.6\,km/h$

13.9



GIVEN: v AT C = 0

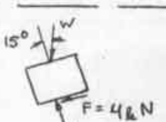
$\mu_k = 0.12$

FIND:

(a) INITIAL v AT A

(b) v AS PACKAGE RETURNS TO A

(a)



UP THE PLANE, FROM A TO C, $v_C = 0$

$T_A = \frac{1}{2} m v_A^2, \quad T_C = 0$

$U_{A-C} = (-W\sin 15° - F)(10m)$

$\Sigma F = 0 \quad N - W\cos 15° = 0$

$N = W\cos 15°$

$F = \mu_k N = 0.12\,W\cos 15°$

$U_{A-C} = -W(\sin 15° + 0.12\cos 15°)(10m)$

$T_A + U_{A-C} = T_C \qquad \frac{1}{2}\frac{W}{g} v_A^2 - W(\sin 15° + 0.12\cos 15°)(10m)$

$v_A^2 = (2)(9.81)(\sin 15° + 0.12\cos 15°)(10m)$

$v_A^2 = 73.5 \qquad v_A = 8.57\,m/s$

(b) DOWN THE PLANE FROM C TO A

$T_C = 0 \quad T_A = \frac{1}{2} m v_A^2 \quad U_{C-A} = (W\sin 15° - F)10$

(F REVERSES DIRECTION)

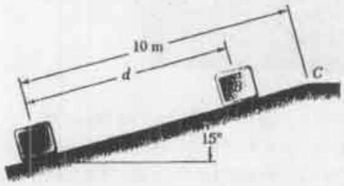
$T_C + U_{C-A} = T_A \qquad 0 + W(\sin 15° - 0.12\cos 15°)(10m) = \frac{1}{2} m v_A^2$

$v_A^2 = (2)(9.81)(\sin 15° - 0.12\cos 15°)(10m)$

$v_A^2 = 28.039$

$v_A = 5.30\,m/s$

13.10



GIVEN: v AT A = 8 m/s

$\mu_k = 0.12$

FIND:

(a) DISTANCE d PACKAGE MOVES UP THE PLANE

(b) VELOCITY v_A, AS PACKAGE RETURNS TO A.

(a) UP THE PLANE FROM A TO B

$T_A = \frac{1}{2} m v_A^2 = \frac{1}{2}\frac{W}{g}(8\,m/s)^2 = 32\frac{W}{g} \qquad T_B = 0$

$U_{A-B} = (-W\sin 15° - F)d \quad F = \mu_k N = 0.12N$

$\Sigma F = 0 \quad N - W\cos 15° = 0 \quad N = W\cos 15°$

$U_{A-B} = -W(\sin 15 + 0.12\cos 15°)d = -Wd(0.3747)$

$T_A + U_{A-B} = T_B \qquad 32\frac{W}{g} - Wd(0.3743) = 0$

$d = (32)/(9.81)(.3747)$

$d = 8.70\,m$

(b) DOWN THE PLANE FROM B TO A (F REVERSES DIRECTION)

$T_A = \frac{1}{2}\frac{W}{g} v_A^2 \quad T_B = 0 \qquad d = 8.72\,m/s$

$U_{B-A} = (W\sin 15° - F)d = W(\sin 15° - 0.12\cos 15°)(8.70\,m)$

$U_{B-A} = 1.245\,W$

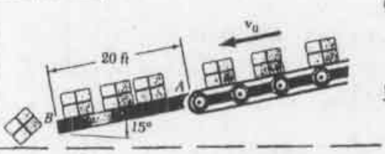
$T_B + U_{B-A} = T_A \qquad 0 + 1.245\,W = \frac{1}{2}\frac{W}{g} v_A^2$

$v_A^2 = (2)(9.81)(1.245) = 253.9$

$v_A = 4.94\,m/s$

$v_A = 4.94\,m/s$

13.11

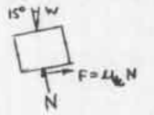


GIVEN: AT A, $v = v_0$

FOR AB, $\mu_k = 0.40$

AT B, v = 8 ft/s

FIND: v_0



$T_A = \frac{1}{2} m v_0^2 \qquad T_B = \frac{1}{2} m v_B^2 = \frac{1}{2} m (8\,ft/s)^2$

$T_B = 32\,m$

$U_{A-B} = (W\sin 15° - \mu_k N)(20\,ft)$

$\Sigma F = 0 \quad N - W\cos 15° = 0$

$N = W\cos 15°$

$U_{A-B} = W(\sin 15° - 0.40\cos 15°)(20\,ft)$

$U_{A-B} = -(2.551)(W) = -2.551\,mg$

$T_A + U_{A-B} = T_B$

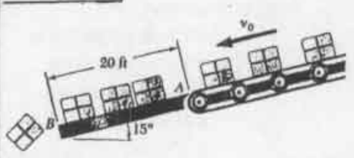
$\frac{1}{2} m v_0^2 - 2.551\,mg = 32\,m$

$v_0^2 = (2)(32 + (2.551)(32.2\,ft/s^2))$

$v_0^2 = 228.29$

$v_0 = 15.11\,ft/s$

13.12



GIVEN: AT A, $v = v_0$
AT B, $v = 0$
FOR AB, $\mu_k = 0.40$

FIND: v_0

$T_A = \frac{1}{2} m v_0^2 \quad T_B = 0$

$U_{A-B} = (W \sin 15° - \mu_k N)(20 \text{ ft})$

$\Sigma F = 0 \quad N - \cos 15° = 0$

$N = W \cos 15°$

$U_{A-B} = W(\sin 15° - 0.40 \cos 15°)(20 \text{ ft})$

$U_{A-B} = -(2.551)(W) = -2.551\, mg$

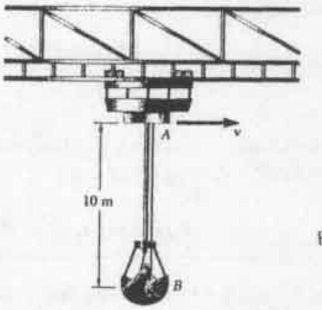
$T_A + U_{A-B} = T_B \quad \frac{1}{2} m v_0^2 - 2.551\, mg = 0$

$v_0^2 = (2)(2.551)(32.2 \text{ ft/s}^2)$

$v_0^2 = 164.28$

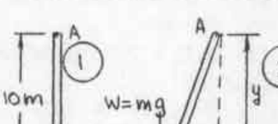
$v_0 = 12.82 \text{ ft/s}$
DOWN TO THE LEFT ◀

13.13



GIVEN: CRANE MOVES AT VELOCITY, v AND STOPS SUDDENLY. BUCKET IS TO SWING NO MORE THAN 4 M HORIZONTALLY

FIND: MAXIMUM ALLOWABLE VELOCITY v



$v_1 = v$
$v_2 = 0$

$T_1 = \frac{1}{2} m v^2$

$T_2 = 0$

$U_{1-2} = -mgh \qquad d = 4 \text{ m}$

$\overline{AB}^2 = (10 \text{ m})^2 = d^2 + y^2 = (4 \text{ m})^2 + y^2$

$y^2 = 100 - 16 = 84 \quad y = \sqrt{84}$

$h = 10 - y = 10 - \sqrt{84} = 0.8349 \text{ m}$

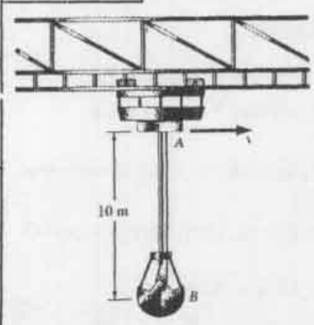
$U_{1-2} = -m(9.81)(0.8349) = -0.8190\, m$

$T_1 + U_{1-2} = T_2 \quad \frac{1}{2} m v^2 - 0.8190\, m = 0$

$v^2 = (2)(0.8190) = 16.38$

$v = 4.05 \text{ m/s}$ ◀

13.14



GIVEN: CRANE MOVES AT VELOCITY $v = 3$ m/s AND STOPS SUDDENLY

FIND: MAXIMUM HORIZONTAL DISTANCE d MOVED BY THE BUCKET

REFER TO FREE BODY DIAGRAM IN P.13.13

$v_1 = v = 3 \text{ m/s} \quad T_1 = \frac{1}{2} m v^2 = \frac{1}{2} m (3 \text{ m})^2 = 4.5\, m$

$T_2 = 0$

$U_{1-2} = -mgh$

$T_1 + U_{1-2} = T_2 \quad 4.5\, m - mgh = 0$

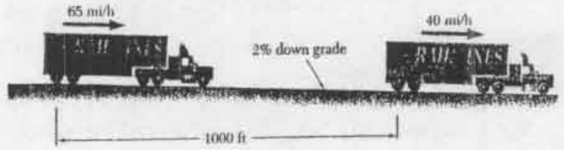
$h = \frac{4.5}{9.81} = 0.4587$

$\overline{AB}^2 = (10)^2 = d^2 + y^2 = d^2 + (10 - 0.4587)^2$

$100 = d^2 + 91.04$

$d^2 = 8.96 \qquad d = 2.99 \text{ m}$ ◀

13.15



GIVEN: CAB WEIGHT, $W_C = 4000$ lb
TRAILER WEIGHT, $W_T = 12{,}000$ lb, 2% GRADE
70% BRAKING FORCE SUPPLIED BY TRAILER
30% BRAKING FORCE SUPPLIED BY CAB

FIND:
(a) AVERAGE BRAKING FORCE TO SLOW DOWN FROM 65 mi/h TO 40 mi/h AS SHOWN
(b) AVERAGE FORCE BETWEEN CAB AND TRAILER

(a) CAB-TRAILER SYSTEM

$v_1 = 65 \text{ mi/h} = 95.33 \text{ ft/s}$
$v_2 = 40 \text{ mi/h} = 58.67 \text{ ft/s}$

$T_1 = \frac{1}{2}(m_T + m_C) v_1^2 = \frac{1}{2}(m_T + m_C)(95.33 \text{ ft/s})^2$

$T_1 = (4{,}544)(m_T + m_C)$

$T_2 = \frac{1}{2}(m_T + m_C)(v_2^2) = \frac{1}{2}(m_T + m_C)(58.67 \text{ ft/s})^2$

$T_2 = (1721)(m_T + m_C)$

$T_1 + U_{1-2} = T_2 \qquad U_{1-2} = -1000 F_B + (W_T + W_C)(20 \text{ ft})$

$4544(m_T + m_C) - 1000 F_B + (W_T + W_C)(20 \text{ ft}) = 1721(m_T + m_C)$

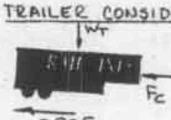
$F_B = \left[(4544 - 1721)\frac{(16{,}000)}{(32.2)} + (16{,}000)(20)\right]\frac{1}{(1000)} = 1722.7$

$F_B = 1723 \text{ lb}$ ◀

(CONTINUED)

13.15 continued

(b) TRAILER CONSIDERED SEPARATELY



$T_1 = \frac{1}{2}(m_T)(95.33)^2 = 4544\, m_T$

$T_2 = \frac{1}{2}(m_T)(58.67)^2 = 1721\, m_T$

$T_1 + U_{1-2} = T_2 \quad 4544\, m_T - 1000(F_C + .70F_B) + 20W_T = 1721\, m_T$

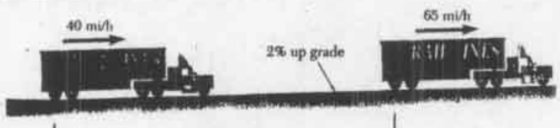
FROM (a) $F_B = 1722.7$

$1000F_C = (4544 - 1721)\frac{(12{,}000)}{(32.2)} - (700)(1722.7) + (20)(12{,}000)$

$F_C = (1052) - 1205.9 + 240 = 86.1$ lb

SEE NOTE FOR P13.5 $\qquad F_C = 86.1$ lb(C)

13.16



GIVEN: CAB WEIGHT, $W_C = 4000$ lb; TRAILER WEIGHT, $W_T = 12{,}000$ lb

2% UP GRADE

FIND (a) AVERAGE FORCE ON THE WHEELS TO SPEED UP, F

(b) AVERAGE FORCE IN THE COUPLING

(a) CAB-TRAILER SYSTEM

$\sin^{-1} 2/100$

$v_1 = 40$ mi/h $= 58.67$ ft/s

$v_2 = 65$ mi/h $= 95.33$ ft/s

$T_1 = \frac{1}{2}(m_T + m_C)(v_1)^2 = \frac{1}{2}(m_T + m_C)(58.67)^2 = 1721(m_T + m_C)$

$T_2 = \frac{1}{2}(m_T + m_C)(v_2)^2 = \frac{1}{2}(m_T + m_C)(95.33)^2 = 4544(m_T + m_C)$

$T_1 + U_{1-2} = T_2 \quad U_{1-2} = (1000)(F) - (1000)(2/100)(W_T + W_C)$

$1721(m_T + m_C) + 1000F - 20(W_T + W_C) = 4544(m_T + m_C)$

$1000F = (4544 - 1721)\frac{(16{,}000)}{(32.2)} + 20(16{,}000)$

$F = 1403. + 320 = 1723.$ lb

$F = 1723$ lb

(b) TRAILER CONSIDERED SEPARATELY

$T_1 = \frac{1}{2}(m_T)(58.67)^2 = 1721\, m_T$

$T_2 = \frac{1}{2}(m_T)(95.33)^2 = 4544\, m_T$

$T_1 + U_{1-2} = T_2 \quad 1721\, m_T + 1000F_C - (1000)\frac{(2)}{100}W_T = 4544\, m_T$

$1000F_C = (4544 - 1721)\frac{(12{,}000)}{(32.2)} + (20)(12000)$

$F_C = 1052 + 240 = 1292$ lb $\qquad F_C = 1292$ lb(T)

SEE NOTE FOR P.13.5

13.17

GIVEN: 2000-kg CAB

8000-kg TRAILER

LEVEL GROUND.

TRUCK COMES TO A STOP IN 1200 m.

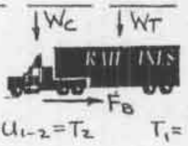
60% OF BRAKING FORCE FROM TRAILER

40% OF BRAKING FORCE FROM CAB.

FIND: (a) AVERAGE BRAKING FORCE

(b) AVERAGE FORCE IN THE COUPLING

(a) TRAILER AND CAB



$v_1 = (90\text{ km/h})\left(\frac{1000\text{ km}}{\text{m}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$

$v_1 = 25$ m/s $\quad v_2 = 0$

$T_1 + U_{1-2} = T_2 \quad T_1 = \frac{1}{2}(m_T + m_C)(v_1)^2 = (\frac{1}{2})(10{,}000\text{ kg})(25\text{ m/s})^2$

$T_1 = 3125 \times 10^3$ N-m $\qquad T_2 = 0$

$3125 \times 10^3 - (1200\text{ m})(F_B) = 0$

$F_B = \frac{(3125 \times 10^3\text{ N}\cdot\text{m})}{(1200\text{ m})} = 2604$ N·m

$F_B = 2.60$ kN

(b) CAB CONSIDERED SEPERATELY

$T_1 = \frac{1}{2}m_C(v_1)^2 = (1000)(25\text{ m/s})^2$

$T_1 = 625 \times 10^3$ N·m, $T_2 = 0$

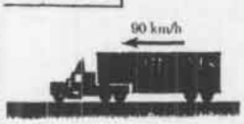
$T_1 + U_{1-2} = T_2 \quad F_B = 2604$ N·m (FROM (a)).

$625 \times 10^3 - (0.40)(2604)(1200) + (F_C)(1200) = 0$

$F_C = (0.40)(2604) - \frac{625}{1.2} = 1042 - 521$

SEE NOTE FOR P13.5 $\qquad F_C = 521$ N (C)

13.18



GIVEN: 2000 kg CAB, 8000 kg TRAILER

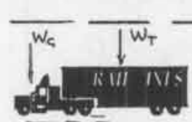
AVERAGE BRAKING FORCE 3000N

LEVEL GROUND

FIND: (a) DISTANCE x, TO COME TO A STOP

(b) FORCE IN COUPLING F_C (TRAILER BRAKES FAIL)

(a) TRAILER AND CAB



$v_1 = 25$ m/s

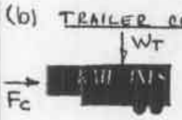
$T_1 = \frac{1}{2}(m_T + m_C)(25)^2 = 3125 \times 10^3$ J

$T_2 = 0 \qquad U_{1-2} = F_B x$

$T_1 + U_{1-2} = T_2 \quad 3125 \times 10^3 - (3000)x = 0$

$x = 1042$ m

(b) TRAILER CONSIDERED SEPARATELY



$T_1 = \frac{1}{2}m_T(25)^2 = (4000)(625)$

$T_1 = 2500 \times 10^3$ J

$T_2 = 0$

$T_1 + U_{1-2} = T_2$

$2500 \times 10^3 - (F_C)(x) = 0$

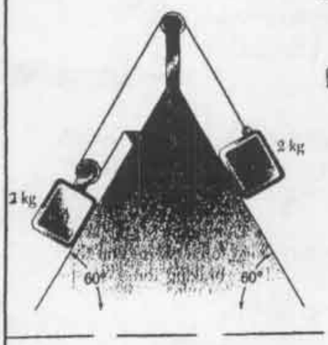
FROM (a) $x = 1042$ m

$2500 \times 10^3 - F_C(1042) = 0$

$F_C = \frac{2500 \times 10^3}{1042} = 2399.2$ N

SEE NOTE FOR P13.5 $\qquad F_C = 2.40$ kN(C)

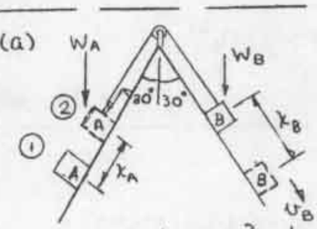
13.19



GIVEN:
BLOCKS RELEASED FROM REST; NO FRICTION

FIND:
(a) VELOCITY OF BLOCK B AFTER IT HAS MOVED 2 m.
(b) TENSION IN THE CABLE.

(a)



KINEMATICS $x_B = 2x_A$
$v_B = 2v_A$

A AND B
ASSUME B MOVES DOWN
$v_1 = 0 \quad T_1 = 0$

$$T_2 = \tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = \tfrac{1}{2}(2\,kg)\left(\frac{v_B^2}{4} + v_B^2\right)$$

$$T_2 = \tfrac{5}{4}v_B^2$$

$$U_{1-2} = -m_A g(\cos 30°)(x_A) + m_B g(\cos 30°)x_B$$

$x_B = 2\,m$
$x_A = 1\,m$

$$U_{1-2} = (2)(9.81)\left(\frac{\sqrt{3}}{2}\right)[-1+2]$$

$$U_{1-2} = 16.99\ J$$

SINCE WORK IS POSITIVE BLOCK B DOES MOVE DOWN

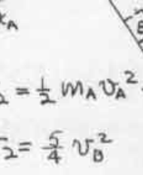
$T_1 + U_{1-2} = T_2$

$$0 + 16.99 = \tfrac{5}{4}v_B^2$$

$$v_B^2 = 13.59$$

$$v_B = 3.69\ m/s$$
DOWN TO THE RIGHT ◀

(b)



B ALONE

$v_1 = 0 \quad T_1 = 0$
$v_2 = 3.69\,m/s$, (FROM (a))

$$T_2 = \tfrac{1}{2}m_B v_2^2 = \tfrac{1}{2}(2)(3.69)^2 = 13.59\ J$$

$$U_{1-2} = (m_B g)(\cos 30°)(x_B) - (T)(x_B)$$

$$U_{1-2} = \left[(2\,kg)(9.81\,m/s^2)\left(\frac{\sqrt{3}}{2}\right) - (T)\right](2\,m)$$

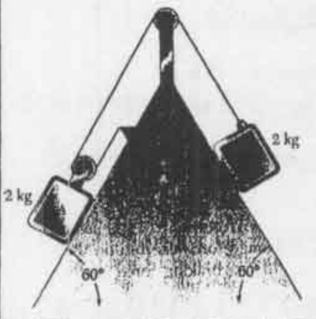
$$U_{1-2} = 33.98 - 2T$$

$T_1 + U_{1-2} = T_2 \qquad 0 + 33.98 - 2T = 13.59$

$$2T = 33.98 - 13.59 = 20.39$$

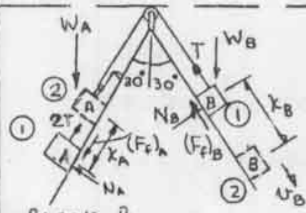
$$T = 10.19\ N$$ ◀

13.20



GIVEN:
BLOCKS RELEASED FROM REST; FRICTION
$\mu_s = 0.30,\ \mu_k = 0.20$

FIND:
(a) VELOCITY OF BLOCK B AFTER IT HAS MOVED 2 m.
(b) TENSION IN THE CABLE.



CHECK AT ① TO SEE IF BLOCKS MOVE. WITH MOTION IMPENDING AT B DOWNWARD DETERMINE REQUIRED FRICTION FORCE AT A FOR EQUILIBRIUM

BLOCK B

$$\nearrow \Sigma F = N_B - (m_B g)(\sin 30°) = 0$$

$$N_B = (2g)(\tfrac{1}{2}) = g$$

$$\nwarrow \Sigma F = T - (m_B g)(\cos 30°) + (F_B)_f = 0$$

$$(F_B)_f = \mu_s N_B = (0.30)(g)$$

$$T = (2g)(\sqrt{3}/2) - (0.30)g$$

$$T = (\sqrt{3} - 0.30)(g) \qquad ①$$

BLOCK A

$$\nwarrow \Sigma F = N_A - (m_A g)(\sin 30°) = 0$$

$$N_A = (2g)(\tfrac{1}{2}) = g$$

$$\nearrow \Sigma F = 2T - (m_A g)(\cos 30°) - (F_A)_f = 0$$

$$(F_A)_f = 2T - (2g)(\sqrt{3}/2) \qquad ②$$

SUBSTITUTE T FROM ① INTO ②

$$(F_A)_f = (2)(\sqrt{3} - 0.30)(g) - \sqrt{3}g$$

REQ. FOR EQUIL $\quad (F_A)_f = (\sqrt{3} - 0.60)g = 1.132\,g$

MAX FRICTION THAT CAN BE DEVELOPED AT A $= \mu_s N_A = 0.3g$

SINCE $0.3g < 1.132g$; BLOCKS MOVE

(a) A AND B

$(F_A)_f = \mu_k N_B = (0.20)g \quad (F_A)_f = \mu_k N_A = (0.20)g$

KINEMATICS $x_B = 2x_A \quad v_B = 2v_A$

$$v_1 = 0,\ T_1 = 0,\ T_2 = \tfrac{1}{2}m_A v_A^2 + \tfrac{1}{2}m_B v_B^2 = (\tfrac{1}{2})(2\,kg)\left(\frac{v_B^2}{4} + v_B^2\right)$$

$$T_2 = \tfrac{5}{4}v_B^2$$

$$U_{1-2} = -m_A g(\cos 30°)(x_A) + m_B g(\cos 30°)x_B - (F_A)_f(x_A) - (F_B)_f(x_B)$$

$x_B = 2\,m,\ x_A = 1\,m$

$$U_{1-2} = [-(2\,kg)(\sqrt{3}/2)(1\,m) + (2\,kg)(\sqrt{3}/2)(2\,m) - (0.20)(1\,m) - (0.20)(2\,m)][9.81\,m/s^2]$$

$$U_{1-2} = [(1.732) - (0.6)][9.81] = 11.105\ J$$

$T_1 + U_{1-2} = T_2$

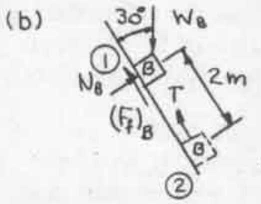
$$0 + 11.105 = 1.25v_B^2$$

$$v_B^2 = 8.88$$

$$v_B = 2.98\ m/s$$
DOWN TO THE RIGHT ◀

(CONTINUED)

13.20 continued

(b) 

B ALONE

$v_1 = 0 \quad T_1 = 0$

$v_2 = 2.98$ m/s (FROM (a))

$T_2 = \frac{1}{2} m_B v_B^2 = (\frac{1}{2})(2)(2.98)^2$

$N_B = m_B g \sin 30° = g$ N $\qquad T_2 = 8.88$ J

$U_{1-2} = m_B g (\cos 30°)(2) - (T)(2) - (F_B)_f(2)$

$U_{1-2} = (2 kg)(9.81 m/s^2)(\frac{\sqrt{3}}{2})(2m) - 2T - (0.2)(g N)(2m)$

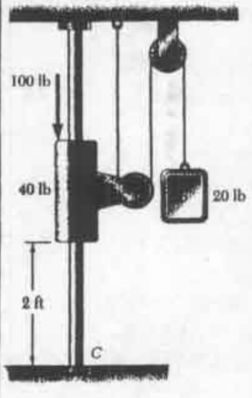
$U_{1-2} = 2\sqrt{3} g - 2T - 0.6g$

$T_1 + U_{1-2} = T_2 \qquad 0 + 2\sqrt{3} g - 2T - 0.4g = 8.88$

$2T = (2\sqrt{3} - 0.4)(g) - 8.88 = 21.179$

$T = 10.59$ N

13.21



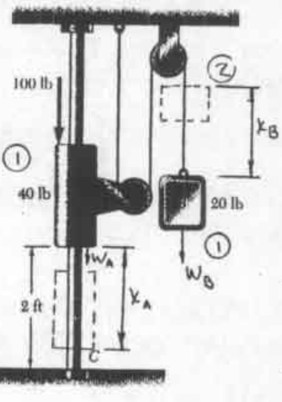
GIVEN:

SYSTEM AT REST WHEN 100 lb FORCE IS APPLIED TO. NO FRICTION. IGNORE PULLEYS MASS

FIND:

(a) VELOCITY, v_A OF A JUST BEFORE IT HITS C

(b) v_A IF COUNTERWEIGHT B IS REPLACED BY A 20-lb DOWNWARD FORCE



KINEMATICS

$x_B = 2x_A$

$v_B = 2v_A$

(a) BLOCKS A AND B

$T_1 = 0$

$T_2 = \frac{1}{2} m_B v_B^2 + \frac{1}{2} m_A v_A^2$

$T_2 = \frac{1}{2}(20 lb/32.2 ft/s^2)(2v_A)^2 + \frac{1}{2}(40 lb/32.2 ft/s^2)(v_A)^2$

$T_2 = (60/32.2)(v_A)^2$

$U_{1-2} = (100)(x_A) + (W_A)(x_A) - (W_B)(x_B)$

$U_{1-2} = (100 lb)(2ft) + (40 lb)(2ft) - (20 lb)(4ft)$

$U_{1-2} = 200 + 80 - 80 = 200$ lb·ft

(CONTINUED)

13.21 continued

$T_1 + U_{1-2} = T_2 \qquad 0 + 200 = (60/32.2) v_A^2$

$v_A^2 = 107.33$

$v_A = 10.36$ ft/s

(b) SINCE THE 20 lb WEIGHT AT B IS REPLACED BY A 20 lb FORCE THE KINETIC ENERGY AT ② IS $T_2 = \frac{1}{2} m_A v_A^2 = \frac{1}{2}(40/g) v_A^2 \quad T_1 = 0$

THE WORK DONE IS THE SAME AS IN PART (a)

$U_{1-2} = 200$ lb·ft

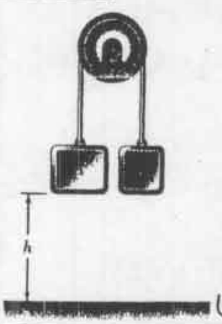
$T_1 + U_{1-2} = T_2$

$0 + 200 = (20/g) v_A^2$

$v_A^2 = 322$

$v_A = 17.94$ ft/s

13.22



GIVEN:

$m_A = 11$ kg $\quad m_B = 5$ kg

$h = 2$ m

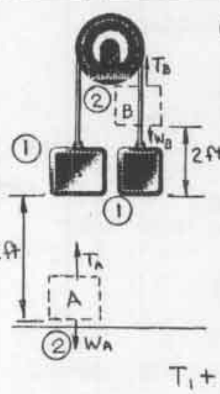
SYSTEM RELEASED FROM REST

$v_A = 3$ m/s JUST BEFORE HITTING THE GROUND

FIND:

(a) ENERGY, E_P, DISSIPATED IN FRICTION

(b) TENSION IN EACH PORTION OF CORD



(a) $v_1 = 0 \quad T_1 = 0$ $\quad$ ENERGY DISSIPATED

$v_2 = v_A = 3$ m/s $= v_B$

$T_2 = \frac{1}{2}(m_A + m_B) v_2^2$

$T_2 = (\frac{16}{2} kg)(3 m/s)^2 = 72$ J

$U_{1-2} = m_A g(2) - m_B g(2) - E_P$

$U_{1-2} = (6 kg)(9.81 m/s^2)(2m) - E_P$

$U_{1-2} = 117.72 - E_P$

$T_1 + U_{1-2} = T_2$

$0 + 117.72 - E_P = 72$

$E_P = 117.72 - 72 = 45.7$ J

(b) BLOCK A

$T_1 = 0 \quad T_2 = \frac{1}{2} m_A v_2^2 = (\frac{11}{2} kg)(3 m/s)^2 = 49.5$ J

$U_{1-2} = (m_A g - T_A)(2) = [(11 kg)(9.81 m/s^2) - T_A](2m)$

$U_{1-2} = 215.82 - 2T_A$

$T_1 + U_{1-2} = T_2 \qquad 0 + 215.82 - 2T_A = 49.5$

$T_A = 83.2$ N

BLOCK B

$T_1 = 0 \quad T_2 = \frac{1}{2} m_B v_2^2 = (\frac{5}{2} kg)(3 m/s)^2 = 22.5$ J

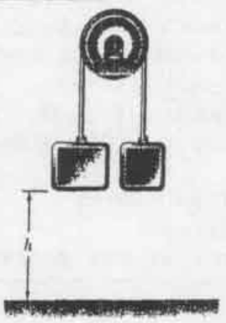
$U_{1-2} = -m_B g(2) + T_B(2) = -(5 kg)(9.81 m/s^2)(2m) + 2T_B$

$T_1 + U_{1-2} = T_2 \qquad U_{1-2} = -98.1 + 2T_B$

$0 - 98.1 + 2T_B = 22.5$

$T_B = 60.3$ N

13.23



GIVEN:

$W_A = 20\,lb$; $W_B = 8\,lb$

$h = 1.5\,ft$

SYSTEM RELEASED FROM REST

BLOCK A HITS THE GROUND WITHOUT REBOUND

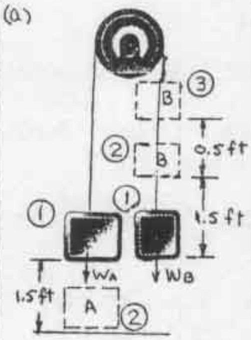
BLOCK B REACHES A HEIGHT OF 3.5 ft

FIND:

(a) v_A JUST BEFORE BLOCK A HITS THE GROUND

(b) ENERGY, E_p, DISSIPATED BY THE PULLEY IN FRICTION

(a)



v_B AT ② $= v_A$ AT ② JUST BEFORE IMPACT

FROM ② TO ③; BLOCK B

$T_3 = 0$

$T_2 = \frac{1}{2} m_B v_B^2$

$T_2 = \frac{1}{2}(8\,lb/32.2\,ft/s^2)\,v_B^2$

$T_2 = 0.1242\,v_B^2$

TENSION IN THE CORD IS ZERO

THUS $U_{2-3} = (8\,lb)(0.5\,ft)$

$U_{2-3} = 4\,lb\cdot ft$

$T_2 + U_{2-3} = T_3$

$0.1242\,v_B^2 = 4$

$v_B^2 = 32.2 = v_A^2$

$v_A = 5.68\,ft/s$ ◀

(b) FROM ① TO ②

BLOCKS A AND B

$T_1 = 0 \quad T_2 = \frac{1}{2}(m_A + m_B)\,v_2^2$

JUST BEFORE IMPACT, $v_2 = v_B = v_A = 5.68\,ft/s$

$T_2 = \frac{1}{2}(28\,lb/32.2\,ft/s^2)(5.68)^2$

$T_2 = 14\,lb\cdot ft$

$U_{1-2} = (W_A)(1.5) - (W_B)(1.5) - E_p$

(E_p = ENERGY DISSIPATED BY PULLEY)

$U_{1-2} = (12\,lb)(1.5\,ft) - E_p = 18 - E_p$

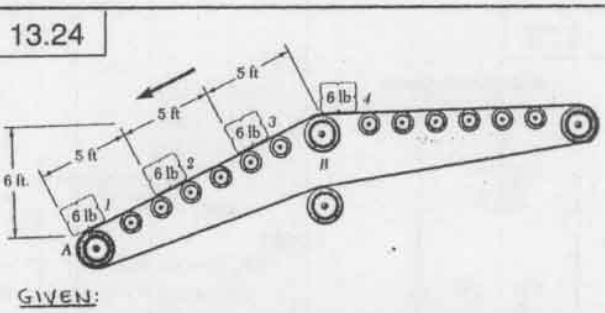
$T_1 + U_{1-2} = T_2$

$0 + 18 - E_p = 14$

$-E_p = 14 - 18$

$E_p = 4.00\,ft\cdot lb$ ◀

13.24



GIVEN:

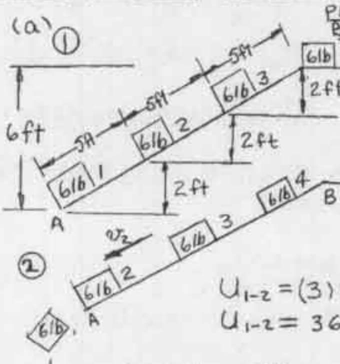
CONVEYOR IS DISENGAGED, PACKAGES HELD BY FRICTION AND SYSTEM IS RELEASED FROM REST. NEGLECT MASS OF BELT AND ROLLERS. PACKAGE 1 LEAVES THE BELT AS PACKAGE 4 COMES ONTO THE BELT.

FIND:

(a) VELOCITY OF PACKAGE 2 AS IT LEAVES THE BELT AT A

(b) VELOCITY OF PACKAGE 3 AS IT LEAVES THE BELT AT A.

(a) PACKAGE 1 FALLS OFF THE BELT AND 2, 3, 4 MOVE DOWN



$\frac{6}{3} = 2\,ft$

$T_2 = 3\left[\frac{1}{2} m v_2^2\right]$

$T_2 = \frac{3}{2}(6\,lb/32.2\,ft/s^2)\,v_2^2$

$T_2 = 0.2795\,v_2^2$

$U_{1-2} = (3)(W)(2) = (3)(6\,lb)(2\,ft)$

$U_{1-2} = 36\,lb\cdot ft$

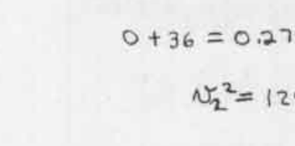
$T_1 + U_{1-2} = T_2$

$0 + 36 = 0.2795\,v_2^2$

$v_2^2 = 128.8$

$v_2 = 11.35\,ft/s$ ◀

(b)



PACKAGE 2 FALLS OFF THE BELT AND ITS ENERGY IS LOST TO THE SYSTEM AN 3 AND 4 MOVE DOWN 2 ft.

$T_2' = (2)\left[\frac{1}{2} m v_2^2\right]$

$T_2' = (6\,lb/32.2\,ft/s^2)(128.8)$

$T_2' = 24\,lb\cdot ft$

$T_3 = (2)\left[\frac{1}{2} m v_3^2\right]$

$T_3 = (6\,lb/32.2\,ft/s^2)(v_3^2)$

$T_3 = 0.18634\,v_3^2$

$U_{2-3} = (2)(W)(2) = (2)(6\,lb)(2\,ft)$

$U_{2-3} = 24\,lb\cdot ft$

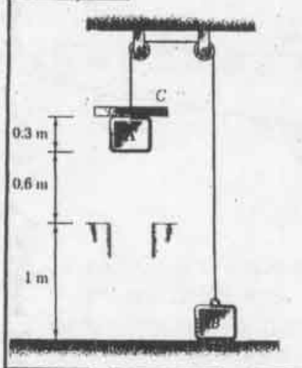
$T_2' + U_{2-3} = T_3$

$24 + 24 = 0.18634\,v_3^2$

$v_3^2 = 257.6$

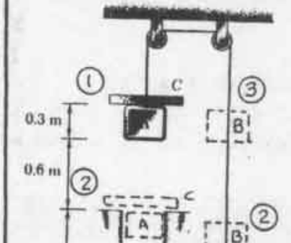
$v_3 = 16.05\,ft/s$ ◀

13.25



GIVEN:
$m_A = 4\,kg$
$m_B = 5\,kg$
$m_C = 3\,kg$
SYSTEM RELEASED FROM REST

FIND:
v_A, JUST BEFORE IT STRIKES THE GROUND



POSITION 1 TO POSITION 2

$v_1 = 0 \quad T_1 = 0$

AT 2 BEFORE C IS REMOVED FROM THE SYSTEM

$T_2 = \frac{1}{2}(m_A + m_B + m_C)v_2^2$

$T_2 = \frac{1}{2}(12\,kg)v_2^2 = 6v_2^2$

$U_{1-2} = (m_A + m_C - m_B)g(.9m)$

$U_{1-2} = (4+3-5)(g)(.9m) = (2kg)(9.81 m/s^2)(.9m)$

$U_{1-2} = 17.658\,J$

$T_1 + U_{1-2} = T_2$

$0 + 17.658 = 6v_2^2$

$v_2^2 = 2.943$

AT POSITION 2, COLLAR C IS REMOVED FROM THE SYSTEM

POSITION 2 TO POSITION 3

$T_2' = \frac{1}{2}(m_A + m_B)v_2^2 = (\frac{9}{2}kg)(2.943)$

$T_2' = 13.244\,J$

$T_3 = \frac{1}{2}(m_A + m_B)(v_3)^2 = \frac{9}{2}v_3^2$

$U_{2'-3} = (m_A - m_B)(g)(0.7m)$
$= (-1kg)(9.81 m/s^2)(0.7m)$

$U_{2'-3} = -6.867\,J$

$T_2' + U_{2-3} = T_3$

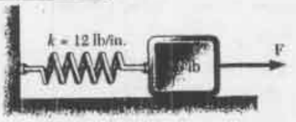
$13.244 - 6.867 = 4.5v_3^2$

$v_3^2 = 1.417$

$v_A = v_3 = 1.190\,m/s$

$v_A = 1.190\,m/s$

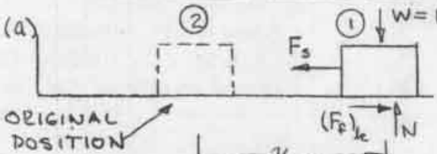
13.26



GIVEN:
$\mu_s = 0.60,\ \mu_k = 0.40$
FORCE F IS SLOWLY APPLIED UNTIL THE TENSION IN THE SPRING IS 20 lb AND THEN RELEASED

FIND:
(a) VELOCITY OF BLOCK AS IT RETURNS TO ITS ORIGINAL POSITION
(b) THE MAXIMUM VELOCITY OF THE BLOCK

(a)



FIND INITIAL POSITION x_0, OF THE BLOCK AT 1

$k = 12\,lb/in = 144\,lb/ft$

AT 1, $F_s = 20\,lb \quad F_s = kx_0 \quad 20\,lb = (144\,lb/ft)x_0$

$x_0 = 20/144 = 0.1389\,ft$

$T_1 = 0,\ T_2 = \frac{1}{2}\left(\frac{W}{g}\right)v_2^2 = \left(\frac{1}{2}\right)(10\,lb/32.2\,ft/s^2)\,v_2^2$

$T_2 = 0.1553\,v_2^2$

$U_{1-2} = \int_{x_0}^{0} -F_s\,dx + (F_f)_k(-x_0); \quad F_s = kx = 144x$

$(F_f)_k = \mu_k N$

$U_{1-2} = \left[\frac{-144x^2}{2}\right]_{x_0}^{0} + (F_f)_k(-x_0) \quad (F_f)_k = (0.4)(10) = 4\,lb$

$U_{1-2} = (72\,lb/ft)(0.1389\,ft)^2 + (4\,lb)(-0.1389\,ft)$

$U_{1-2} = 1.389 - 0.5556 = 0.8335\,lb{\cdot}ft$

$T_1 + U_{1-2} = T_2 \qquad 0 + 0.8335 = 0.1553\,v_2^2$

$v_2^2 = 5.367$

$v_2 = 2.32\,ft/s$

AT ORIGINAL POSITION, $v = 2.32\,ft/s$

(b) FOR ANY POSITION 2 AT A DISTANCE X TO THE RIGHT OF THE ORIGNAL POSITION 2

$T_1 = 0 \quad T_2 = \frac{1}{2}\left(\frac{W}{g}\right)(v_{2'})^2 = 0.1553\,v_{2'}^2$

$U_{1-2'} = \int_{x_0}^{x} -F_s\,dx + \int_{x_0}^{x}(F_f)_k\,dx \qquad x_0 = 0.1389$

$U_{1-2'} = \left[\frac{-144}{2}x^2\right]_{x_0}^{x} + (F_f)_k(x - x_0) \qquad (F_f)_k = 4\,lb$

$T_1 + U_{1-2'} = T_2 \quad 0 + (72\,lb/ft)[(0.1389)^2 - x^2] + (4\,lb)(x - 0.1389) = 0.1553\,v_{2'}^2$

MAX v_1 WHEN $\frac{dv_1}{dx} = 0$

$-144x + 4 = 0$

MAX v_1 WHEN $x = 0.027778\,m$

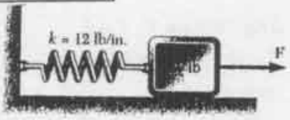
$0.1553\,v_{max}^2 = (72)[(0.1389)^2 - (0.02778)^2] + (4)(.02778 - 0.1389)$

$0.1553\,v_{max}^2 = 1.3336 - 0.4445 = 0.8891$

$v_{max}^2 = 5.725$

$v_{max} = 2.39\,ft/s$

13.27



GIVEN:

$\mu_s = 0.60$, $\mu_k = 0.40$

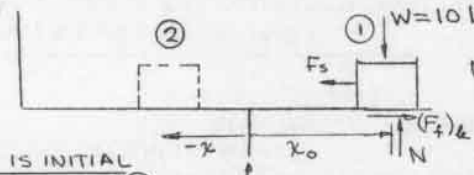
FORCE F IS SLOWLY APPLIED UNTIL THE TENSION IN THE SPRING IS 20 lb AND THEN RELEASED

FIND:

(a) DISTANCE THE BLOCK MOVES TO THE LEFT BEFORE COMING TO A STOP

(b) WHETHER THE BLOCK THEN MOVES BACK TO THE RIGHT.

(a)



$N = 10\text{ lb}$

x_0 IS INITIAL POSITION AT ①

ORIGINAL POSITION

BLOCK HAS A VELOCITY TO THE LEFT AS IT REACHES ITS ORIGINAL POSITION (SEE P 13.26)

$k = 12\text{ lb/in} = 144\text{ lb/ft}$

$T_1 = 0 \quad T_2 = 0$

$F_s = 144x$

$(F_f)_k = \mu_k N$

$(F_f)_k = (0.4)(10)$

$(F_f)_k = 4\text{ lb}$

$$U_{1-2} = \int_{x_0}^{-x} -F_s\,dx + \int_{x_0}^{-x} (F_f)_k\,dx$$

$$U_{1-2} = -\frac{144}{2} x^2 \Big|_{x_0}^{-x} + (F_f)_k(-x - x_0)$$

$$U_{1-2} = -72(x^2 - x_0^2) - 4(x + x_0)$$

$T_1 + U_{1-2} = T_2 \qquad 0 - 72(x - x_0)(x + x_0) - 4(x + x_0) = 0$

$$-72(x - x_0) - 4 = 0$$

$$-72x = 4 - 72x_0$$

AT ① $F_s = 20\text{ lb}$

$F_s = k x_0 = 144 x_0$

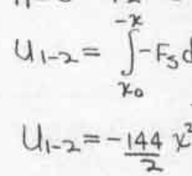
$x_0 = \frac{20}{144} = 0.1389 \qquad x = 0.0833\text{ ft}$

TOTAL DISTANCE MOVED TO THE LEFT $= x_0 + x$

$x_0 + x = 0.1389 + 0.0833$

$x_0 + x = 0.222\text{ ft}$ ◀

(b)



IF F_s AT 2 IS LARGER THAN THE MAXIMUM POSSIBLE STATIC FRICTION FORCE, THEN BLOCK WILL MOVE TO THE RIGHT

$N = 10\text{ lb}$

FROM (a) WITH $x = 0.0833\text{ ft}$

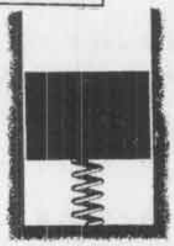
$F_s = (144)(0.0833) = 12\text{ lb}$

$(F_f)_s = \mu_s N = (0.60)(10) = 6\text{ lb}$

SINCE $F_s > (F_f)_s$

BLOCK MOVES TO THE RIGHT ◀

13.28



GIVEN:

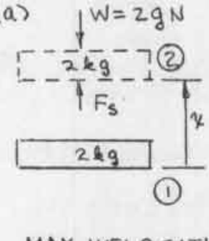
3 kg BLOCK RESTS ON 2 kg BLOCK WHICH IS NOT ATTACHED TO A SPRING OF CONSTANT 40 N/m UPPER BLOCK IS SUDDENLY REMOVED

FIND:

(a) v_{max} OF 2 kg BLOCK

(b) MAXIMUM HEIGHT, h, REACHED BY THE 2 kg BLOCK

(a)



AT THE INITIAL POSITION ① THE FORCE IN THE SPRING EQUALS THE WEIGHT OF BOTH BLOCKS, i.e. 5g N THUS AT A DISTANCE x THE FORCE IN THE SPRING IS $F_s = 5g - kx$

$F_s = 5g - 40x$

MAX VELOCITY OF THE 2 kg BLOCK OCCURS WHILE THE SPRING IS IS STILL IN CONTACT WITH THE BLOCK.

$T_1 = 0 \quad T_2 = \frac{1}{2} m v^2 = \left(\frac{1}{2}\right)(2\text{ kg})(v^2) = v^2$

$$U_{1-2} = \int_0^x (5g - 40x)\,dx - 2gx = 3gx - 20x^2$$

$T_1 + U_{1-2} = T_2 \qquad 0 + 3gx - 20x^2 = v^2 \qquad (1)$

MAX v WHEN $\frac{dv}{dx} = 0 = 3g - 40x$

$x(\text{MAX } v) = \frac{3g}{40}\text{ m}$

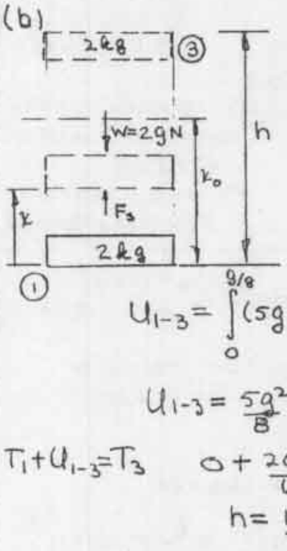
SUBSTITUTE IN (1)

$x(\text{MAX } v) = 0{,}7358\text{ m}$

$$v_{MAX}^2 = (3)(9.81)(0.7358) - (20)(0.7358)^2$$
$$= 21.65 - 10.83 = 10.83$$

$v_{max} = 3.29\text{ m/s}$ ◀

(b)



x_0 = INITIAL COMPRESSION OF THE SPRING

$x_0 = \frac{(2g + 3g)}{40} = \frac{g}{8}\text{ m}$

$F_s = 5g - 40x$

$T_1 = 0 \quad T_3 = 0$

$$U_{1-3} = \int_0^{g/8} (5g - 40x)\,dx - 2gh$$

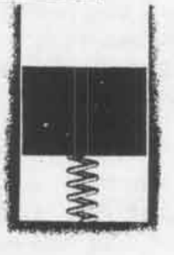
$$U_{1-3} = \frac{5g^2}{8} - \frac{20g^2}{64} - 2gh$$

$T_1 + U_{1-3} = T_3 \qquad 0 + \frac{20g^2}{64} - 2gh = 0$

$$h = \frac{10g}{64} = \frac{(10)(9.81)}{64}$$

$h = 1.533\text{ m}$ ◀

13.29



GIVEN:
3 kg BLOCK RESTS ON A 2 kg BLOCK WHICH IS ATTACHED TO A SPRING OF 40 N/m WHEN UPPER BLOCK IS SUDDENLY REMOVED

FIND:
(a) v_{max} OF 2 kg BLOCK
(b) MAXIMUM HEIGHT h REACHED BY 2 kg BLOCK

(a) SEE SOLUTION TO (a) OF P13.28

$v_{MAX} = 3.29$ m/s

(b) REFER TO FIGURE IN (b) OF P13.28

$T_1 = 0 \quad T_3 = 0$

$$U_{1-3} = \int_0^h (5g - 40x)\,dx - 2gh$$

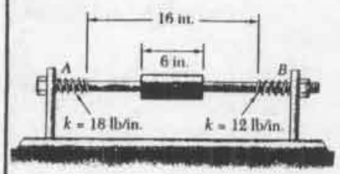
SINCE THE SPRING REMAINS ATTACHED TO THE 2 kg BLOCK THE INTEGRATION MUST BE CARRIED OUT THROUGHOUT THE TOTAL DISTANCE h.

$$T_1 + U_{1-3} = T_2 \qquad 0 + 5gh - 20h^2 - 2gh = 0$$

$$h = \frac{3g}{20} = \frac{(3)(9.81)}{20}$$

$$h = 1.472 \text{ m}$$

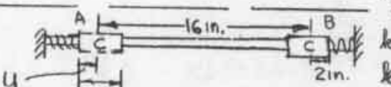
13.30



GIVEN:
$W_C = 8$ lb
COLLAR C COMPRESSES SPRING AT B 2 in. AND IS RELEASED

FIND:
(a) DISTANCE TRAVELED BY COLLAR WITH NO FRICTION.
(b) SAME AS (a) WITH FRICTION, $\mu_k = 0.35$

(a)



$k_B = 144$ lb/ft
$k_A = 216$ lb/ft

SINCE COLLAR C LEAVES THE SPRING AT B AND THERE IS NO FRICTION IT MUST ENGAGE THE SPRING AT A

$T_A = 0 \quad T_B = 0$

$$U_{A\text{-}B} = \int_0^{2/12} k_B x\,dx - \int_0^{y} k_A x\,dx$$

$$U_{A\text{-}B} = \left(\frac{144 \text{ lb/ft}}{2}\right)\left(\frac{2}{12}\text{ ft}\right)^2 - \left(\frac{216 \text{ lb/ft}}{2}\right)(y)^2$$

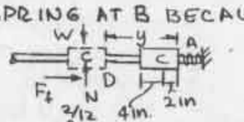
$$T_A + U_{A\text{-}B} = T_B \qquad 0 + 2 - 108y^2 = 0$$

$$y = 0.1361 \text{ ft} = 1.633 \text{ in.}$$

TOTAL DISTANCE = $2 + 16 - (6 - 1.633) = 13.63$ in.

13.30 continued

(b) ASSUME THAT C DOES NOT REACH THE SPRING AT B BECAUSE OF FRICTION



$N = W = 6$ lb
$F_f = (0.35)(8 \text{ lb}) = 2.80$ lb
$T_A = T_D = 0$

$$U_{A\text{-}D} = \int_0^{2/12} 144x\,dx - F_f(y) = 2 - 2.80y$$

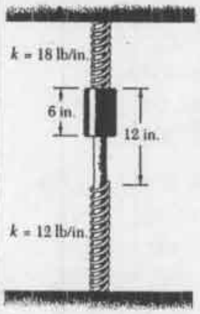
$$T_A + U_{A\text{-}D} = T_D \qquad 0 + 2 - 2.80y = 0$$

$$y = 0.714 \text{ ft} = 8.57 \text{ in.}$$

THE COLLAR MUST TRAVEL $16 - 6 + 2 = 12$ in. BEFORE IT ENGAGES THE SPRING AT B. SINCE $y = 8.57$ in. IT STOPS BEFORE ENGAGING THE SPRING AT B

TOTAL DISTANCE = 8.57 in.

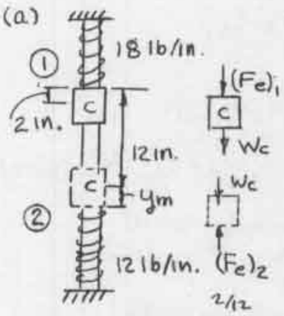
13.31



GIVEN:
$W_C = 6$ lb.
UPPER SPRING IS COMPRESSED 2 in AND COLLAR C IS RELEASED

FIND:
(a) y_m, THE MAXIMUM DEFLECTION OF THE LOWER SPRING
(b) v_m, THE MAXIMUM VELOCITY OF THE COLLAR

(a)



SPRING CONSTANTS
18 lb/in. = 216 lb/ft
12 lb/in. = 144 lb/ft

MAXIMUM DEFLECTION AT ② WHEN VELOCITY OF COLLAR C IS ZERO
$v_2 = 0 \quad T_2 = 0$
$v_1 = 0 \quad T_1 = 0$

$$U_{1-2} = U_e + U_g = \int_0^{2/12} (F_e)_1\,dx - \int_0^{y_m} (F_e)_2\,dx + W_c(1 + y_m)$$

$$U_{1-2} = \left(\frac{216}{2}\text{ lb/ft}\right)\left(\frac{1}{6}\text{ ft}\right)^2 - \left(\frac{144 \text{ lb/ft}}{2}\right)(y_m^2) + 6\text{ lb}(1 + y)$$

$$U_{1-2} = 3 - 72y_m^2 + 6 + 6y_m = -72y_m^2 + 6y_m + 6$$

$$T_1 + U_{1-2} = T_2 \qquad 0 - 72y_m^2 + 6y_m + 6 = 0$$

$$y_m = \frac{1}{3}\text{ ft} = 4.00 \text{ in.}$$

(b) MAXIMUM VELOCITY OCCURS AS THE LOWER SPRING IS COMPRESSED A DISTANCE y'

$$T_1 = 0 \quad T_2 = \frac{1}{2}m_c v^2 = \frac{1}{2}\left(\frac{6}{g}\right)v^2 = \left(\frac{3\text{ lb}}{32.2 \text{ ft/s}^2}\right)v^2$$

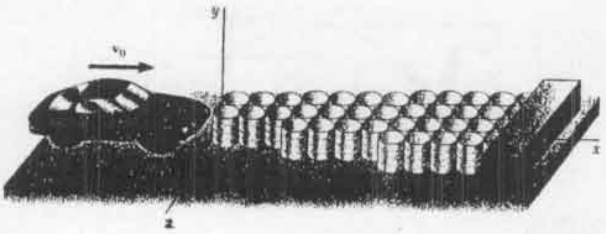
$$T_1 + U_{1-2} = T_2 \qquad 0 - 72y'^2 + 6y' + 6 = (0.09317)v^2$$

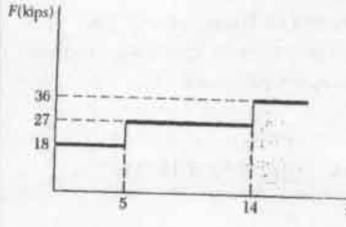
$$\frac{dv^2}{dy'} = 0 \qquad -144y' + 6 = 0;\ y' = 0.041667 \text{ ft}$$

SUBSTITUTE $y' = 0.041667$ ft

$$-0.125 + 0.250 + 6 = 0.09317v_m^2;\ v_m = 811 \text{ ft/s} = 923 \text{ m/s}$$

13.32

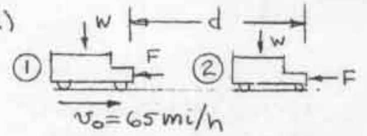




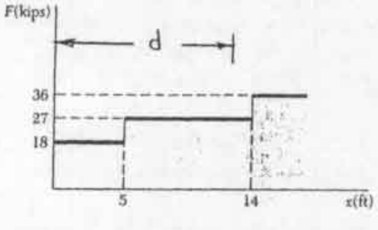
GIVEN:
$v_0 = 65\ mi/h$
FORCE FROM CUSHION AS SHOWN
NEGLECT FRICTION
$W = 2250\ lb$

FIND:
(a) DISTANCE d, FOR AUTOMOBILE TO COME TO REST
(b) MAXIMUM DECELERATION, a_D

(a)



$65\ mi/h = 95.3\ ft/s$



ASSUME AUTO STOPS IN $5 < d < 14$ ft

$v_1 = 95.33\ ft/s \qquad T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}\left(\frac{2250\ lb}{32.2\ ft/s^2}\right)(95.3\ ft/s)^2$

$T_1 = 317{,}530\ lb\cdot ft = 317.53\ k\cdot ft$

$v_2 = 0 \quad T_2 = 0$

$U_{1-2} = (18k)(5ft) + (27k)(d-5)$
$= 90 + 27d - 135 = 27d - 45 \quad k\cdot ft$

$T_1 + U_{1-2} = T_2$

$317.53 = 27d - 45$

$d = 13.43\ ft$ ◀

ASSUMPTION THAT $d < 14$ ft IS OK

(b) MAXIMUM DECELERATION OCCURS WHEN F IS LARGEST. FOR $d = 13.43$ ft, $F = 27k$.

THUS $F = m a_D$

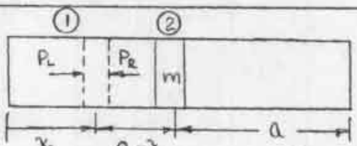
$(27{,}000\ lb) = \left(\frac{2250\ lb}{32.2\ ft/s^2}\right)(a_D)$

$a_D = 386\ ft/s^2$ ◀

13.33

GIVEN:
PISTON AREA A
PISTON MASS m
INITIAL PRESSURE P
PRESSURE VARIES INVERSELY WITH VOLUME. PISTON MOVED $a/2$ AND RELEASED

FIND:
VELOCITY OF THE PISTON AS IT RETURNS TO THE CENTER



PRESSURES VARY INVERSELY AS THE VOLUME

$\frac{P_L}{P} = \frac{Aa}{Ax} \qquad P_L = \frac{Pa}{x}$

$\frac{P_R}{P} = \frac{Aa}{A(2a-x)} \qquad P_R = \frac{Pa}{(2a-x)}$

INITIALLY AT ①
$v = 0 \quad x = \frac{a}{2}$
$T_1 = 0$

AT ②, $x = a$, $T_2 = \frac{1}{2} m v^2$

$U_{1-2} = \int_{a/2}^{a} (P_L - P_R) A\,dx = \int_{a/2}^{a} paA\left[\frac{1}{x} - \frac{1}{2a-x}\right] dx$

$U_{1-2} = paA\left[\ln x + \ln(2a-x)\right]_{a/2}^{a}$

$U_{1-2} = paA\left[\ln a + \ln a - \ln(a/2) - \ln(3a/2)\right]$

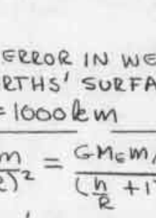
$U_{1-2} = paA\left[\ln a^2 - \ln 3a^2/4\right] = paA \ln(4/3)$

$T_1 + U_{1-2} = T_2 \qquad 0 + paA\ln(4/3) = \frac{1}{2} m v^2$

$v^2 = \frac{2paA\ln(4/3)}{m} = 0.5754\frac{paA}{m}$

$v = 0.759\sqrt{\frac{paA}{m}}$ ◀

13.34



GIVEN:
ACCELERATION OF GRAVITY g_0 AT EARTHS' SURFACE

FIND:
ACCELERATION OF GRAVITY g_h AT HEIGHT h ABOVE THE EARTHS' SURFACE IN TERMS OF g_0, h, R.
AND ERROR IN WEIGHT AT h IF WEIGHT AT EARTHS' SURFACE IS USED FOR (a) h = 1 km
(b) h = 1000 km

$F = \frac{GM_E m}{(h+R)^2} = \frac{GM_E m/R^2}{(\frac{h}{R}+1)^2} = m g_h$

AT EARTHS' SURFACE (h = 0) $GM_E m/R^2 = m g_0$

$GM_E/R^2 = g_0 \qquad g_h = \frac{GM_E/R^2}{(\frac{h}{R}+1)^2}$

THUS $\quad g_h = \frac{g_0}{(\frac{h}{R}+1)^2}$ ◀

(CONTINUED)

13.34 continued

$R = 6370\ km$

AT ALTITUDE h, "TRUE" WEIGHT $F = mg_h = W_T$

ASSUMED WEIGHT $W_o = mg_o$

$$ERROR = E = \frac{W_o - W_T}{W_o} = \frac{mg_o - mg_h}{mg_o} = \frac{g_o - g_h}{g_o}$$

$$g_h = \frac{g_o}{(\frac{h}{R}+1)^2} \qquad E = \frac{g_o - \frac{g_o}{(1+\frac{h}{R})^2}}{g_o} = \left[1 - \frac{1}{(1+\frac{h}{R})^2}\right]$$

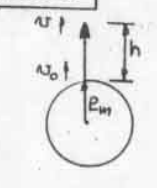
(a) $h = 1\ km \quad P = 100E = 100\left[1 - \frac{1}{(1+\frac{1}{6370})^2}\right]$

$$P = 0.0314\ \%$$

(b) $h = 1000\ km \quad P = 100\left[1 - \frac{1}{(1+\frac{1000}{6370})^2}\right]$

$$P = 25.3\ \%$$

13.35



GIVEN:

VELOCITY AT MOONS SURFACE $= v_o$

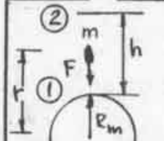
VELOCITY AT HEIGHT $h = v$

RADIUS OF THE MOON, R_M

ACCELERATION OF GRAVITY ON THE MOONS' SURFACE, g_m

FIND:

FORMULA FOR h_n/h_u, WHERE h_n IS FOUND USING NEWTONS LAW OF GRAVITATION AND h_u IS FOUND USING A UNIFORM GRAVITATIONAL FIELD



NEWTONS LAW OF GRAVITATION

$$T_1 = \frac{1}{2}mv_o^2, \quad T_2 = \frac{1}{2}mv^2$$

$$U_{1-2} = \int_{R_m}^{R_m+h_n} (-F_n)dr \qquad F_n = \frac{mg_m R_m^2}{r^2}$$

$$U_{1-2} = -mg_m R_m^2 \int_{R_m}^{R_m+h_n} \frac{dr}{r^2}$$

$$U_{1-2} = mg_m R_m^2\left(\frac{1}{R_m} - \frac{1}{R_m+h_n}\right)$$

$T_1 + U_{1-2} = T_2$

$$\frac{1}{2}mv_o^2 + mg_m\left(R_m - \frac{R_m}{R_m+h_n}\right) = \frac{1}{2}mv^2$$

$$h_n = \frac{(v_o^2 - v^2)}{2g_m}\left[\frac{R_m}{R_m - \frac{(v_o^2-v^2)}{2g_m}}\right] \qquad (1)$$

UNIFORM GRAVITATIONAL FIELD

$$T_1 = \frac{1}{2}mv_o^2 \quad T_2 = mv^2$$

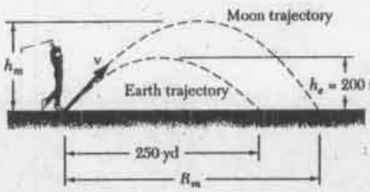
$$U_{1-2} = \int_{R_m}^{R_m+h_u} (-F_u dr) = -mg_m(R_m + h_u - R_m) = -mg\,h_u$$

$T_1 + U_{1-2} = T_2 \quad \frac{1}{2}mv_o^2 - mg_m h_u = \frac{1}{2}mv^2$

$$h_u = (v_o^2 - v^2)/2g_m \qquad (2)$$

DIVIDE (1) BY (2) $\quad \frac{h_n}{h_u} = \frac{1}{1 - (v_o^2 - v^2)/(2g_m R_m)}$

13.36



GIVEN:

EARTH TRAJECTORY AS SHOWN

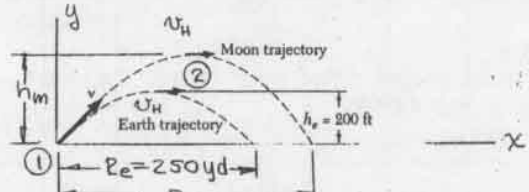
MAGNITUDE AND DIRECTION OF v ON THE EARTH IS THE SAME ON THE MOON

TRAJECTORY IS A PARABOLA

$g_m = 0.165\, g_e$

FIND:

RANGE R_m OF THE BALL ON THE MOON



SOLVE FOR h_m

AT MAXIMUM HEIGHT THE TOTAL VELOCITY IS THE HORIZONTAL COMPONENT OF THE VELOCITY WHICH IS CONSTANT AND THE SAME IN BOTH CASES

$$T_1 = \frac{1}{2}mv^2 \quad T_2 = \frac{1}{2}mv_H^2$$

$$U_{1-2} = -mg_e h_e \quad \text{EARTH}$$

$$U_{1-2} = -mg_m h_m \quad \text{MOON}$$

EARTH $\frac{1}{2}mv^2 - mg_e h_e = \frac{1}{2}mv_H^2$

MOON $\frac{1}{2}mv^2 - mg_m h_m = \frac{1}{2}mv_H^2$

SUBTRACTING $-g_e h_e + g_m h_m = 0 \quad \frac{h_m}{h_e} = \frac{g_e}{g_m}$

$$h_m = (200\ ft)\left(\frac{g_e}{0.165 g_e}\right) = 1212\ ft$$

EQUATION OF A PARABOLA $(y-h) = -C\left(x - \frac{R}{2}\right)^2$

$(y - h_e) = -C_e\left(x - \frac{R_e}{2}\right)^2$ EARTH

$(y - h_m) = -C_m\left(x - \frac{R_m}{2}\right)^2$ MOON

AT $x = 0$, $\underline{v}$ IS THE SAME, THUS $\frac{dy}{dx}$ IS THE SAME

$$\left.\frac{dy}{dx}\right|_{x=0} = C_e R_e = C_m R_m \qquad \frac{C_e}{C_m} = \frac{R_m}{R_e}$$

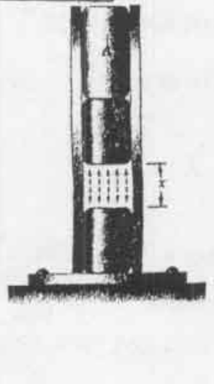
AT $x = 0$, $y = 0 \quad h_e = C_e \frac{R_e^2}{4} \quad h_m = C_m \frac{R_m^2}{4}$

$$\frac{h_m}{h_e} = \frac{C_m R_m^2}{C_e R_e^2} = \frac{R_m}{R_e}$$

$$\frac{h_m}{h_e} = \frac{g_e}{g_m} = \frac{R_m}{R_e} \qquad R_m = (g_e/(0.165 g_e))(250\ yd)$$

$$R_m = 1515\ yd$$

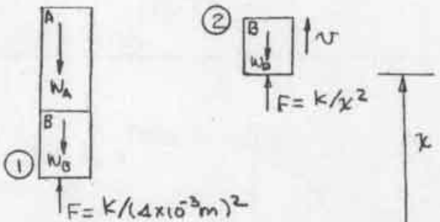
13.37



GIVEN:
$m_A = 300\text{-g}$ (NON MAGNETIC)
$m_B = 200\text{-g}$ (MAGNETIC)
$x = 4\,mm$, INITIALLY
REPELLING FORCE BETWEEN B AND C IS
$F = k/x^2$
BLOCK A IS SUDDENLY REMOVED. NO AIR RESISTANCE

FIND:
(a) MAXIMUM VELOCITY, v_m OF B
(b) MAXIMUM ACCELERATION a_m OF B

(a)



CALCULATE K $\quad \Sigma F = (m_A + m_B)g - k/(4\times10^{-3} m)^2$

$k = (4\times10^{-3} m)^2 (0.5\,kg)(g\,m/s^2)$

$k = 8\times10^{-6}\, g\ N\text{-}m$

$v_1 = 0 \quad T_1 = 0 \quad v_2 = v \quad T_2 = \frac{1}{2} m_B v^2 = 0.1 v^2\, N.m$

$$U_{1-2} = \int_{①}^{②} (F - m_B g)\, dx$$

$$U_{1-2} = \int_{4\times10^{-3}}^{x} \left(\frac{8\times10^{-6} g}{x^2} - 0.2g\right) dx$$

$T_1 + U_{1-2} = T_2$

$$0 + \int_{4\times10^{-3}}^{x} \left(\frac{8\times10^{-6} g}{x^2} - 0.2g\right) dx = 0.1 v^2$$

FOR MAX v, $\dfrac{d(0.1v^2)}{dx} = 0$

THUS

$$\frac{8\times10^{-6} g}{x^2} - 0.2g = 0$$

AT v_{MAX}, $x = 0.00632\, m$

$$0 + \int_{.004}^{.00632} \left(\frac{8\times10^{-6} g}{x^2} - 0.2g\right) dx = 0.1 v^2_{MAX}$$

$$0 + \left[-\frac{(8\times10^{-6})(9.81)}{x} - 0.2(9.81)x\right]_{.004\,m}^{.00632\,m} = 0.1 v^2_{MAX}$$

$v_{MAX} = 0.1628\, m/s$

$v_M = 162.8\, mm/s$ ◀

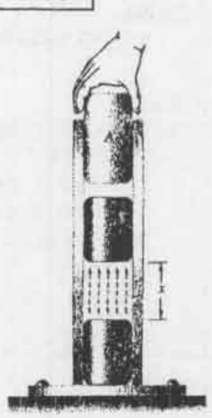
(b) MAXIMUM ACCELERATION AT $x = 0.004\, m$
WHEN ΣF ARE THE GREATEST

$\Sigma F = k/x^2 - W_B = m_B a$

$(8\times10^{-6})(9.81)/(0.004)^2 - (0.2)(9.81) = (0.2) a_M$

$a_M = 14.72\, m/s^2$ ◀

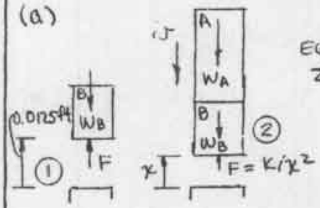
13.38



GIVEN:
$W_B = 0.4\text{-lb}$ (MAGNETIC)
$W_A = 0.6\text{-lb}$ (NON-MAGNETIC)
$x = 0.15\, in.$ INITIALLY
REPELLING FORCE BETWEEN B AND C IS
$F = k/x^2$; NO AIR RESISTANCE
BLOCK A IS PLACED ON BLOCK B AND RELEASED

FIND:
(a) MAXIMUM VELOCITY OF A AND B
(b) MAXIMUM DEFLECTION OF A AND B

(a)



CALCULATE K
EQUILIBRIUM AT ①

$\Sigma F = k/x_1^2 - W_B = 0$

$k = x_1^2 W_B$

$x_1 = 0.15\, in. = 0.0125\, ft$

$k = (.0125\, ft)^2 (0.4\, lb)$

$k = 0.0000625\, ft^2 \cdot lb$

$v_1 = 0 \quad T_1 = 0 \quad v_2 = v \quad T_2 = \frac{1}{2}(m_A + m_B) v^2$

$T_2 = \frac{1}{2}\left(\frac{1\,lb}{32.2\, ft/s^2}\right) v^2$

$T_2 = 0.01553\, v^2$

$$U_{1-2} = \int_{0.0125}^{x} [F - (W_A + W_B)]\, dx$$

$T_1 + U_{1-2} = T \qquad 0 + \displaystyle\int_{0.0125}^{x} \left[\frac{.0000625}{x^2} - 1\right] dx = 0.01553\, v^2$

FOR MAX v, $\dfrac{d(.001553 v^2)}{dx} = 0$

AT v_M, $\quad \dfrac{0.0000625}{x^2} - 1 = 0 \quad x = 0.007906\, ft$

$$\int_{0.0125}^{0.007906} \left[\frac{0.0000625}{x^2} - 1\right] dx = 0.01553\, v_m^2$$

$$\left[-\frac{0.0000625}{x} - x\right]_{.0125}^{.007906} = 0.01553\, v_m^2$$

$v_m^2 = 0.10876$

$v_m = 0.3298\, ft/s$

$v_m = 3.96\, in/s$ ◀

(b) MAXIMUM DEFLECTION WHEN $v = 0$

$T_1 = 0 \quad T_2 = 0$

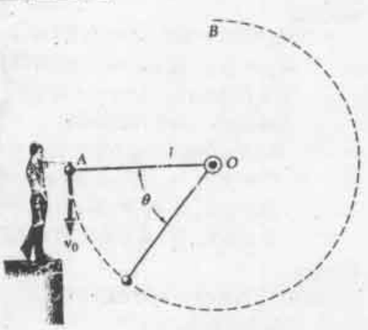
$$0 + \int_{.0125}^{x} \left[\frac{0.0000625}{x^2} - 1\right] dx = 0$$

$$-0.0000625\left[\frac{1}{x} - \frac{1}{0.0125}\right] - x + 0.0125 = 0$$

$x = 0.005\, ft$

MAXIMUM DEFLECTION $= 0.0125 - 0.005 = 0.0075\, ft$
$= 0.090\, in.$ ◀

13.39

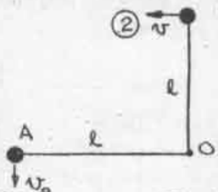


GIVEN:

v_0, AS SHOWN

FIND:

SMALLEST v_0 FOR THE SPHERE TO REACH B, IF
(a) AO IS A ROPE
(b) AO IS A ROD



$T_1 = \frac{1}{2} m v_0^2$

$T_2 = \frac{1}{2} m v^2$

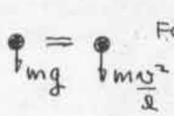
$U_{1-2} = -mgl$

$T_1 + U_{1-2} = T_2 \quad \frac{1}{2} m v_0^2 - mgl = \frac{1}{2} m v^2$

$v_0^2 = v^2 + 2gl$

NEWTONS' LAW AT ②

(a)

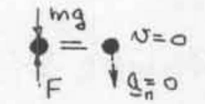


FOR MINIMUM v, TENSION IN THE CORD MUST BE ZERO.
THUS, $v^2 = gl$

$v_0^2 = v^2 + 2gl = 3gl$

$$v_0 = \sqrt{3gl}$$

(b)



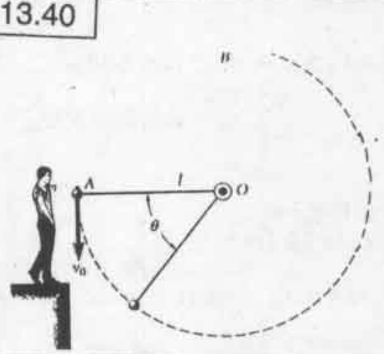
FORCE IN THE ROD CAN SUPPORT THE WEIGHT SO THAT v CAN BE ZERO

$F = mg$

THUS $\quad v_0^2 = 0 + 2gl$

$$v_0 = \sqrt{2gl}$$

13.40

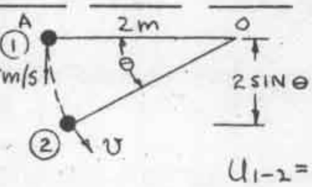


GIVEN:

$v_0 = 5$ m/s
$l = 2$ m
TENSION = 2W WHEN ROPE BREAKS

FIND:

VALUE OF θ WHEN ROPE BREAKS



$T_1 = \frac{1}{2} m v_0^2 = \frac{1}{2} m (5)^2$

$T_1 = 12.5\,m$

$T_2 = \frac{1}{2} m v^2$

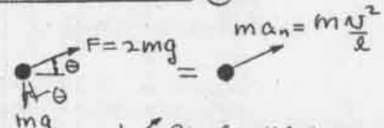
$U_{1-2} = mg(2)\sin\theta$

13.40 continued

$T_1 + U_{1-2} = T_2 \quad 12.5m + 2mg\sin\theta = \frac{1}{2} m v^2$

$25 + 4g\sin\theta = v^2 \quad$ (a)

NEWTONS' LAW AT ②



$+\nearrow\ 2mg - mg\sin\theta = m\frac{v^2}{l} = m\frac{v^2}{2}$

$v^2 = 4g - 2g\sin\theta \quad$ (b)

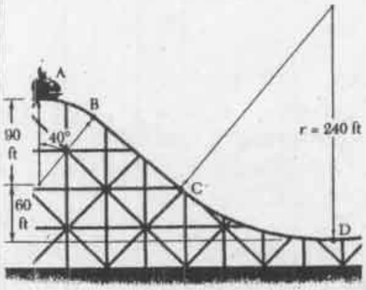
SUBSTITUTE FOR v^2 FROM EQ (b) INTO EQ. (a)

$25 + 4g\sin\theta = 4g - 2g\sin\theta$

$\sin\theta = \frac{(4)(9.81) - 25}{(6)(9.81)} = 0.2419$

$$\theta = 14.00°$$

13.41



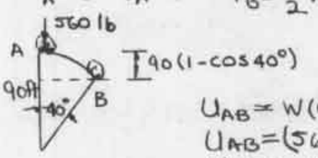
GIVEN:

$v_A = 0$
WEIGHT OF CAR AND OCCUPANTS = 560 lb

FIND:

NORMAL FORCE N, AS CAR REACHES B

$v_A = 0 \quad T_A = 0 \quad T_B = \frac{1}{2} m v_B^2 = \frac{1}{2}\left(\frac{560}{g}\right) v_B^2 = \frac{280}{g} v_B^2$



$U_{AB} = W(90)(1 - \cos 40°)$

$U_{AB} = (560\text{ lb})(90\text{ ft})(0.234)$

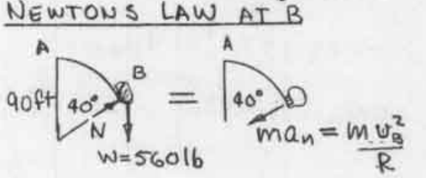
$U_{AB} = 11791$ ft·lb

$T_A + U_{AB} = T_A \quad 0 + 11791 = \frac{280}{g} v_B^2$

$v_B^2 = \frac{(11791\text{ ft·lb})(32.2\text{ ft/s}^2)}{(280\text{ lb})}$

$v_B^2 = 1356\text{ ft}^2/\text{s}^2$

NEWTONS LAW AT B



$+\nearrow\ N - W\cos 40° = -\frac{m v_B^2}{R}\ ; \quad v_B^2 = 1356\text{ ft}^2/\text{s}^2$

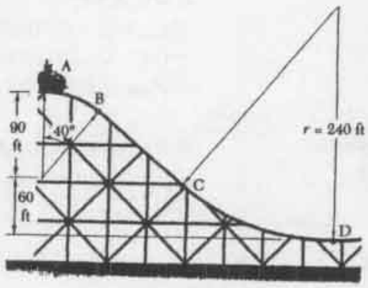
$N = (560\text{ lb})(\cos 40°) - \frac{(560\text{ lb})(1356\text{ ft}^2/\text{s}^2)}{(32.2\text{ ft/s}^2)(90\text{ ft})}$

$N = 429 - 262 = 167.0$ lb

$$N = 167.0\text{ lb}$$

13.42

GIVEN: $v_A = 0$, CAR AND OCCUPANTS WEIGH 560 lb.



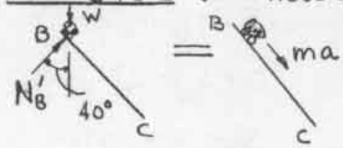
FIND: MAXIMUM, N_{MAX} AND MINIMUM, N_{MIN} NORMAL FORCE ON THE CAR AS IT GOES FROM A TO D

NORMAL FORCE AT B

SEE SOLUTION TO PROB. 13.41, $N_B = 167.0$ lb

NEWTONS LAW

FROM B TO C (CAR MOVES IN A STRAIGHT LINE)

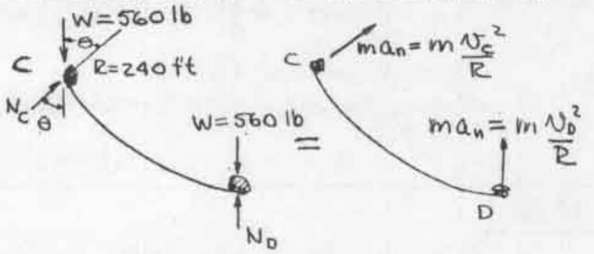


$$+\nearrow \; N_B' - W\cos 40 = 0$$

$$N_B' = (560)\cos 40°$$

$$N_B' = 429 \text{ lb}$$

AT C AND D (CAR IN THE CURVE AT C)



AT C

$$+\nearrow \; N_C - W\cos\theta = \frac{W}{g}\frac{v_C^2}{R}$$

$$N_C = 560\left(\cos\theta + \frac{v_C^2}{gR}\right)$$

AT D

$$+\uparrow \; N_D - W = +\frac{W}{g}\frac{v_D^2}{R}$$

$$N_D = 560\left(1 + \frac{v_D^2}{gR}\right)$$

SINCE $v_D > v_C$ AND $\cos\theta < 1$, $N_D > N_C$

WORK AND ENERGY FROM A TO D

$v_A = 0$, $T_A = 0$ $\quad T_D = \frac{1}{2}\frac{W}{g}v_D^2 = \frac{280}{g}v_D^2$

$$U_{A-B} = W(90+60) = (560 \text{ lb})(150 \text{ ft})$$

$$U_{A-B} = 84000 \text{ lb·ft}$$

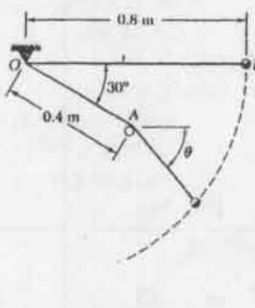
$T_A + U_{A-B} = T_B \qquad 0 + 84000 = 280\, v_D^2$

$$\frac{v_D^2}{g} = 300$$

$$N_D = 560\left(1 + \frac{v_D^2}{gR}\right) = 560\left(1 + \frac{300}{240}\right) = 1260 \text{ lb}$$

$$N_{MIN} = N_B = 167.0 \text{ lb};\; N_{MAX} = N_D = 1260 \text{ lb}$$

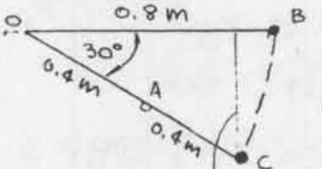
13.43



GIVEN: SPHERE RELEASED FROM REST AT B, ($v_B = 0$)

FIND: TENSION IN THE CORD,
(a) JUST BEFORE IT COMES IN CONTACT WITH THE PEG
(b) JUST AFTER CONTACT WITH PEG

VELOCITY OF THE SPHERE AS THE CORD CONTACTS A



$v_B = 0 \quad T_B = 0$

$T_C = \frac{1}{2} m v_C^2$

$$U_{B-C} = (mg)(0.4)$$

$$(0.8)(\sin 30°) = 0.4$$

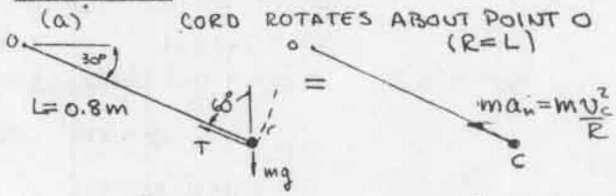
$T_B + U_{B-C} = T_C$

$$0 + 0.4\,mg = \frac{1}{2} m v_C^2$$

$$v_C^2 = (0.8)(g)$$

NEWTONS LAW

(a) CORD ROTATES ABOUT POINT O (R=L)

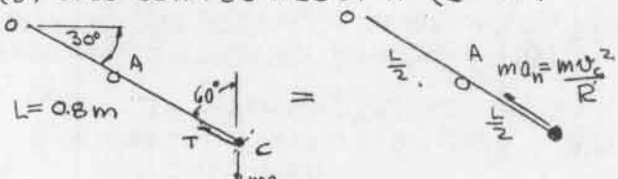


$$+\nwarrow \; T - mg(\cos 60°) = m\frac{v_C^2}{L}$$

$$T = mg(\cos 60°) + \frac{m(0.8)g}{0.8}$$

$$T = \frac{3}{2}mg \qquad T = 1.5\,mg$$

(b) CORD ROTATES ABOUT A ($R = L/2$)



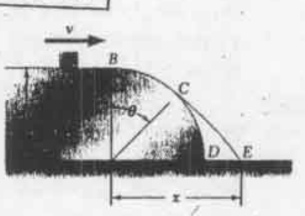
$$T - mg(\cos 60°) = \frac{m v_C^2}{L/2}$$

$$T = mg/2 + m(0.8)(g)/0.4$$

$$T = \left(\frac{1}{2} + 2\right)mg = \frac{5}{2}mg$$

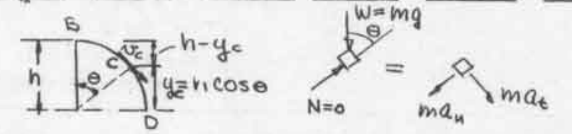
$$T = 2.5\,mg$$

13.44



GIVEN:
$v = 8$ ft/s
$h = 3$ ft

FIND:
(a) θ, ANGLE AT WHICH BLOCK LEAVES THE SURFACE
(b) x



BLOCK LEAVES SURFACE AT C WHEN THE NORMAL FORCE $N=0$

$+\swarrow\ mg\cos\theta = ma_n$

$g\cos\theta = \dfrac{v_c^2}{h}$

$v_c^2 = gh\cos\theta = gy$ (1)

WORK-ENERGY PRINCIPLE

(a) $T_B = \frac{1}{2}mv^2 = \frac{1}{2}m(8)^2 = 32m$

$T_c = \frac{1}{2}mv_c^2 \quad U_{B-C} = W(h-y) = mg(h-y_c)$

$T_B + U_{B-C} = T_c$

$32m + mg(h-y) = \frac{1}{2}mv_c^2$

USE EQ (1) $\quad 32 + g(h-y_c) = \frac{1}{2}gy_c$ (2)

$32 + gh = \frac{3}{2}gy_c$

$y_c = (32+gh)/(\frac{3}{2}g)$

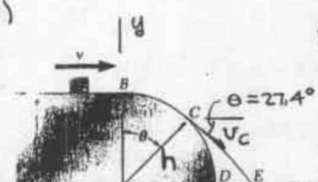
$y = (32+(32.2)(3))/\frac{3}{2}(32.2)$

$y = 2.6625$ ft (3)

$y_c = h\cos\theta \quad \cos\theta = \dfrac{y_c}{h} = \dfrac{2.6625}{3} = 0.8875$

$\theta = 27.4°$

(b)



FROM (1) AND (3)

$v_c = \sqrt{gy}$

$v_c = \sqrt{(32.2)(2.6625)}$

$v_c = 9.259$ ft/s

AT C: $(v_c)_x = v_c\cos\theta = (9.259)(\cos 27.4°) = 8.220$ ft/s

$(v_c)_y = -v_c\sin\theta = -(9.259)(\sin 27.4°) = 4.261$ ft/s

$y = y_c + (v_c)_y t - \frac{1}{2}gt^2 = 2.6625 - 4.261t - 16.1t^2$

AT E: $y_E = 0 \quad t^2 + 0.2647t - 0.1654 = 0$

$t = 0.2953$ s

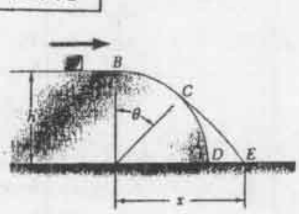
AT E:

$x = h(\sin\theta) + (v_c)_x t = (3)(\sin 27.4°) + (8.220)(.2953)$

$x = 1.381 + 2.427 = 3.808$ ft

$x = 3.81$ ft

13.45



GIVEN:
$h = 2.5$ m
BLOCK LEAVES THE SURFACE WHEN $\theta = 40°$

FIND:
INITIAL SPEED, v

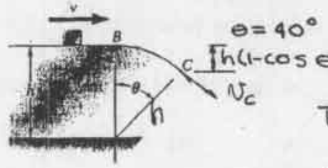
← SEE LEFT; BLOCK LEAVES THE SURFACE WHEN $N=0 \quad g\cos\theta = \dfrac{v_c^2}{h}$

$h = 2.5$ m, $\theta = 40°$

THUS $v_c^2 = gh\cos\theta = (9.81)(2.5)(\cos 40°)$

$v_c^2 = 18.79$

WORK-ENERGY PRINCIPLE



$T_B = \frac{1}{2}mv^2$

$T_c = \frac{1}{2}mv_c^2 = \frac{1}{2}m(18.79)$

$T_c = 9.395m$

$U_{B-C} = mgh(1-\cos\theta)$

$T_B + U_{B-C} = T_c$

$\frac{1}{2}mv^2 + mgh(1-\cos\theta) = \frac{1}{2}mv_c^2$

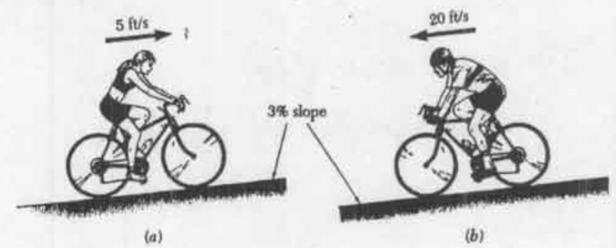
$v^2 = -2gh(1-\cos\theta) + 18.79$

$v^2 = -2(9.81\ m/s^2)(2.5m)(1-\cos 40°) + 18.79$

$v^2 = 7.315$

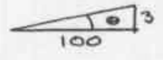
$v_c = 2.70$ m/s

13.46



GIVEN:
(a) $v = 5$ ft/s, UP 3% SLOPE
BICYCLE WEIGHT, $W_B = 15$ lb
WOMANS WEIGHT $W_W = 120$ lb
(b) $v = 20$ ft/s, DOWN 3% SLOPE, BRAKING
BICYCLE WEIGHT, $W_B = 18$ lb
MANS' WEIGHT, $W_M = 180$

FIND:
(a) POWER DEVELOPED BY THE WOMAN, P_W
(b) POWER DISSIPATED BY THE BRAKES, P_B

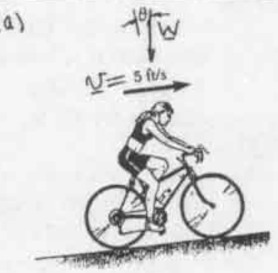


$\tan\theta = \dfrac{3}{100} \quad \theta = 1.718°$

(CONTINUED)

13.46 continued

(a)



(a)

$W = W_B + W_W = 15 + 120$

$W = 135\ \text{lb}$

$P_W = \underline{W} \cdot \underline{v} = (W \sin\theta)(v)$

$P_W = (135)(\sin 1.718°)(5)$

$P_W = 20.24\ \text{ft·lb/s}$

$P_W = 20.2\ \text{ft·lb/s}$ ◀

(b)

(b)

$W = W_B + W_m = 18 + 180$

$W = 198\ \text{lb}$

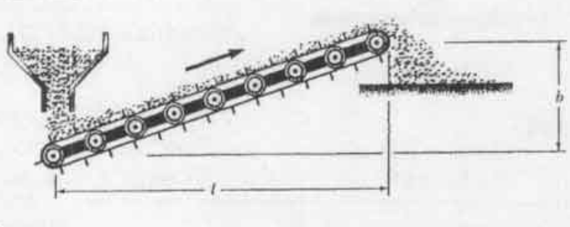
BRAKES MUST DISSIPATE THE POWER GENERATED BY THE BIKE AND THE MAN GOING DOWN THE SLOPE AT 20 ft/s

$P_B = \underline{W} \cdot \underline{v} = (W \sin\theta)(v)$

$P_B = (198)(\sin 1.718)(20)$

$P_B = 118.7\ \text{ft·lb/s}$ ◀

13.47



GIVEN:

(a) MASS FLOW RATE, m(kg/h), l(m), b(m)

(b) MASS FLOW RATE, W(TONS/H), l(ft), b(ft)

MOTOR EFFICIENCY, η

FIND:

(a) POWER P IN kW

(b) POWER IN hp

(a) MATERIAL IS LIFTED TO A HEIGHT b AT A RATE, $(m\ \text{kg/h})(g\ \text{m/s}^2) = [mg\ (\text{N/h})]$

THUS $\dfrac{\Delta U}{\Delta t} = \dfrac{[mg(\text{N/h})][b(\text{m})]}{(3600\ \text{s/h})} = \left(\dfrac{mg\,b}{3600}\right)\dfrac{\text{N·m}}{\text{s}}$

$1000\ \text{N·m/s} = 1\ \text{kW}$

THUS, INCLUDING MOTOR EFFICIENCY η

$$P(\text{kW}) = \frac{mg\,b\ (\text{N·m/s})}{(3600)\left(\frac{1000\ \text{N·m/s}}{\text{kW}}\right)(\eta)}$$

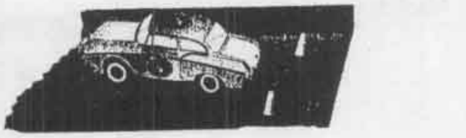
$$P(\text{kW}) = 0.278 \times 10^{-6}\,\frac{mgb}{\eta}$$ ◀

(b) $\dfrac{\Delta U}{\Delta t} = \dfrac{[W(\text{TONS/H})(2000\ \text{lb/TON})][b(\text{ft})]}{3600\ \text{s/h}}$

$= \dfrac{Wb}{1.8}\ \text{ft·lb/s}\ ;\ 1\ \text{hp} = 550\ \text{ft·lb/s}$

WITH η, $\text{hp} = \left[\dfrac{W \cdot b}{1.8}(\text{ft·lb/s})\right]\left[\dfrac{1\ \text{hp}}{550\ \text{ft·lb/s}}\right]\left[\dfrac{1}{\eta}\right] = \dfrac{1.010 \times 10^{-3}\,Wb}{\eta}$ ◀

13.48



GIVEN:

2000 lb CAR REAR WHEEL DRIVE, SKIDS FOR FIRST 60 ft WITH FRONT WHEELS OFF THE GROUND, $\mu_k = 0.60$

ROLLS WITH SLIDING IMPENDING FOR REMAINING 1260 ft WITH 60% OF ITS WEIGHT ON REAR WHEELS, $\mu_s = 0.85$

FIND:

(a) HP DEVELOPED AT END OF 60 ft PORTION OF THE RACE

(b) HP DEVELOPED AT THE END OF THE RACE

(a) FIRST 60 ft (CALCULATE VELOCITY AT 60 ft)

FORCE GENERATED BY REAR WHEELS $= \mu_k W$ SINCE CAR SKIDS. THUS $F = (0.6)(2000\ \text{lb})$

$F = 1200\ \text{lb}$

WORK AND ENERGY $\quad T_1 = 0 \quad T_2 = \frac{1}{2}\frac{W}{g}v_{60}^2 = \frac{1000}{g}v_{60}^2$

$T_1 + U_{1-2} = T_2$

$U_{1-2} = (F)(60\ \text{ft}) = (1200\ \text{lb})(60\ \text{ft}) = 72{,}000\ \text{lb·ft}$

$T_1 + U_{1-2} = T_2 \quad 0 + 72{,}000 = \dfrac{1000}{g}v_{60}^2$

$v_{60}^2 = (72)(32.2) = 2318.4$

$v_{60} = 48.15\ \text{ft/s}$

$\text{POWER} = F \cdot v_{60}$

$P = (1200\ \text{lb})(48.15\ \text{ft/s})$

$P = 57780\ \text{ft·lb/s}$

$1\ \text{hp} = 550\ \text{ft·lb/s}$

$\text{hp} = \dfrac{(57780\ \text{ft·lb/s})}{\left(550\ \frac{\text{ft·lb/s}}{\text{hp}}\right)} = 105.1$ ◀

(b) END OF RACE (CALCULATE VELOCITY AT 1320 ft)

FOR FIRST 60 ft, FORCE GENERATED BY REAR WHEELS $F_s = 1200$ lb (SEE (a))

FOR REMAINING 1260 ft WITH 60% OF WEIGHT ON REAR WHEELS, THE FORCE GENERATED AT IMPENDING SLIDING IS $\mu_s(.60)(W) = (0.85)(0.60)(2000)$

$F_I = 1020\ \text{lb}$

$T_1 + U_{1-2} = T_2 \quad T_1 = 0 \quad T_2 = \frac{1}{2}\frac{W}{g}v_{1320}^2 = \frac{1000}{g}v_{1320}^2$

$U_{1-2} = (F_s)(60\ \text{ft}) + (F_I)(1260\ \text{ft})$

$U_{1-2} = (1200\ \text{lb})(60\ \text{ft}) + (1020\ \text{lb})(1260\ \text{ft})$

$U_{1-2} = 1{,}357{,}200\ \text{lb·ft}$

$0 + 1{,}357{,}200 = \dfrac{1000}{g}v_{1320}^2$

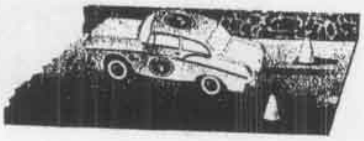
$v_{1320} = 209\ \text{ft/s}$

$\text{POWER} = F_I \cdot v_{1320}$

$P = (1020\ \text{lb})(209\ \text{ft/s}) = 213{,}230\ \frac{\text{ft·lb}}{\text{s}}$

$\text{hp} = \dfrac{(213{,}200\ \text{ft·lb/s})}{(550\ \text{ft·lb/s/hp})} = 388$ ◀

13.49



GIVEN:

1000 kg CAR, REAR WHEEL DRIVE
SKIDS FOR FIRST 20 m, WITH FRONT WHEELS OFF THE GROUND, $\mu_k = 0.68$
ROLLS WITH SLIDING IMPENDING FOR REMAINING 380 m WITH 80 % OF ITS WEIGHT ON REAR WHEELS, $\mu_s = 0.90$

FIND:

(a) POWER DEVELOPED AT END OF 20 m (IN kW & hp)
(b) POWER DEVELOPED AT END OF THE RACE (kW & hp)

(a) FIRST 20 m (CALCULATE VELOCITY AT 20 m)
FORCE GENERATED BY REAR WHEELS $= \mu_k W$
SINCE CAR SKIDS. THUS $F_s = (0.68)(1000)(g)$

$F_s = (0.68)(1000\,kg)(9.81\,m/s^2) = 6670.8\,N$

WORK AND ENERGY $T_1 = 0$, $T_2 = \frac{1}{2} m v_{20}^2 = 500 v_{20}^2$

$T_1 + U_{1-2} = T_2$

$U_{1-2} = (20\,m)(F_s) = (20\,m)(6670.8\,N)$

$U_{1-2} = 133420\,J$

$0 + 133{,}420 = 500 v_{20}^2$

$v_{20}^2 = 133{,}420/500 = 266.83$

$v_{20} = 16.335\,m/s$

POWER $= (F_s)(v_{20}) = (6670.8\,N)(16.335\,m/s)$

POWER $= 108{,}970\,J/s = 108.97\,kJ/s$

$1\,kJ/s = 1\,kW$

$1\,hp = 0.7457\,kW$ POWER $= 109.0\,kJ/s = 109.0\,kW$ ◄

POWER $= \dfrac{(109.0\,kW)}{(0.7457\,kW/hp)} = 146.2\,hp$ ◄

(b) END OF RACE (CALCULATE VELOCITY AT 400 m)
FOR REMAINING 380 m, WITH 80% OF WEIGHT ON REAR WHEELS
THE FORCE GENERATED AT IMPENDING SLIDING IS $(\mu_s)(0.80)(mg)$

$F_I = (0.90)(0.80)(1000\,kg)(9.81\,m/s^2)$

$F_I = 7063.2\,N$

WORK AND ENERGY, FROM 20 m (2) TO 28 m (3)

$v_2 = 16.335\,m/s$ (FROM PART (a))

$T_2 = \frac{1}{2}(1000\,kg)(16.335\,m/s)^2$

$T_2 = 133420\,J$

$T_3 = \frac{1}{2} m v_{300}^2 = 500\,v_{30}^2$

$U_{2-3} = (F_I)(380\,m) = (7063.2\,N)(380\,m)$

$U_{2-3} = 2{,}684{,}000\,J$

$T_2 + U_{2-3} = T_3$

$(133{,}420\,J) + (2{,}684{,}000\,J) = 500 v_{30}^2$

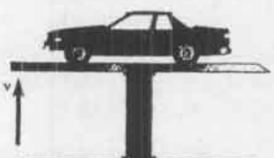
$v_{30} = 75.066\,m/s$

POWER $= (F_I)(v_{30}) = (7063.2\,N)(75.066\,m/s)$
$= 530{,}200\,J$

kW POWER $= 530{,}200\,J = 530\,kW$ ◄

hp POWER $= \dfrac{530\,kW}{(0.7457\,kW/hp)} = 711\,hp$ ◄

13.50



GIVEN:

CAR MASS, $M_c = 1200\,kg$
LIFT MASS, $M_L = 300\,kg$
SYSTEM RISES 2.8 M IN 15 S.

FIND:

(a) AVERAGE POWER OUTPUT OF PUMP, $(P_p)_A$
(b) AVERAGE ELECTRIC POWER $(P_E)_A$ WITH $\eta = 82\%$

(a) $(P_p)_A = (F)(v_A) = (m_c + m_L)(g)(v_A)$

$v_A = S/t = (2.8\,m)/(15\,s) = 0.18667\,m/s$

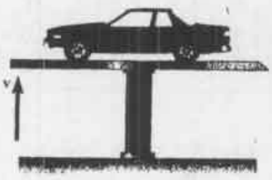
$(P_p)_A = [(1200\,kg) + (300\,kg)](9.81\,m/s^2)(0.18667\,m/s)$

$(P_p)_A = 2747\,J = 2.75\,kW$ ◄

(b) $(P_E)_A = (P_p)/\eta = (2.75\,kW)/(0.82)$

$(P_E)_A = 3.35\,kW$ ◄

13.51



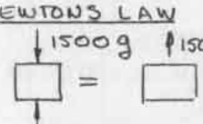
GIVEN:

CAR MASS, $M_c = 1200\,kg$
LIFT MASS, $M_L = 500\,kg$
PEAK VELOCITY AT MID HEIGHT IN 7.5 S INCREASING UNIFORMLY. VELOCITY DECREASES UNIFORMLY TO 0, IN ANOTHER 7.5 S
PEAK PUMP POWER, $P = 6\,kW$, WHEN VELOCITY IS MAXIMUM

FIND:

MAXIMUM LIFTING FORCE, F

NEWTONS LAW

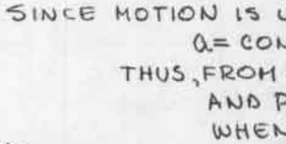


$Mg = (M_c + M_L)g = (1200 + 300)g$

$Mg = 1500g$

$+\uparrow \Sigma F = F - 1500g = 1500a$ (1)

SINCE MOTION IS UNIFORMLY ACCELERATED
$a =$ CONSTANT
THUS, FROM (1), F IS CONSTANT
AND PEAK POWER OCCURS WHEN THE VELOCITY IS A MAXIMUM AT 7.5 S.



$a = \dfrac{v_{MAX}}{7.5\,s}$

$P = (6000\,W) = (F)(v_{MAX})$

$v_{MAX} = (6000)/F$

THUS $a = (6000)/(7.5)(F)$ (2)

SUBSTITUTE (2) INTO (1)

$F - 1500g = (1500)(6000)/(7.5)(F)$

$F^2 - (1500\,kg)(9.81\,m/s^2)F - \dfrac{(1500\,kg)(6000\,N\cdot m/s)}{(7.5\,s)} = 0$

$F^2 - 14715F - 1.2\times10^6 = 0$

$F = 14{,}800\,N$ $F = 14.8\,kN$ ◄

13.52 GIVEN:

W = 100 TONS
P = 400 hp
v = 50 mi/h, CONSTANT

FIND:

(a) F_R, FORCE NEEDED TO OVERCOME AXLE FRICTION, ROLLING RESISTANCE AND AIR RESISTANCE

(b) ΔP, ADDITIONAL hp TO MAINTAIN THE SAME SPEED UP A 1-PERCENT GRADE

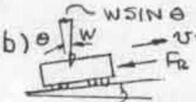
(a) $P = 400\text{ hp} = (550\ \frac{ft\cdot lb}{s}/hp)(400\ hp) = 220{,}000\ \frac{ft\cdot lb}{s}$

$v = 50\text{ mi/h} = 73.33\text{ ft/s}$

$P = F_R \cdot v$

$F_R = P/v = (220{,}000\ \frac{ft\cdot lb}{s})/(73.33\ \frac{ft}{s})$

$F_R = 3000\text{ lb}$ ◀

(b) 

$W = (100\text{ TONS})(2000\text{ lb/TON})$
$W = 200{,}000\text{ lb}$

$\theta = \tan^{-1}\frac{1}{100} = 0.573°$

$\Delta P = W\sin\theta \cdot v$

$\Delta P = (200{,}000\text{ lb})(\sin .573°)(73.33\text{ ft/s})$

$\Delta P = 146{,}667\text{ ft}\cdot\text{lb/s}$

$\Delta P = 267\text{ hp}$ ◀

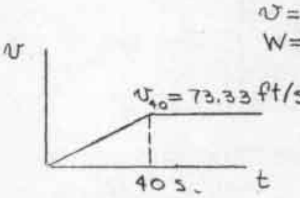
13.53 GIVEN:

W = 600 TONS
UNIFORM ACCELERATION FROM 0 TO 50 mi/h IN 40 s
CONSTANT 50 mi/h AFTER 40 s
HORIZONTAL TRACK
F_R, FRICTION AND ROLLING RESISTANCE = 3000 lb
NEGLECT AIR RESISTANCE

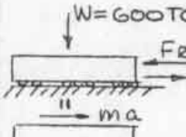
FIND:

P, POWER REQUIRED AS A FUNCTION OF TIME t.

$v = 50\text{ mi/h} = 73.33\text{ ft/s}$
$W = (600\text{ TONS})(2000\text{ lb/TON})$
$W = 1{,}200{,}000\text{ lb}$



FOR UNIFORM MOTION
$a = v/t_{40} = (73.33\text{ ft/s})/(40\text{ s})$
$a = 1.833\text{ ft/s}^2$
$v = 1.833t$



$\Sigma F = F - F_R = ma = \frac{W}{g}a$

$F = (3000\text{ lb}) + \frac{(1{,}200{,}000\text{ lb})}{(32.2\text{ ft/s}^2)}(1.833\text{ ft/s}^2)$

$F = 71{,}311\text{ lb}$

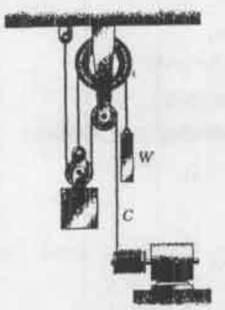
$P = F\cdot v = (71{,}311)(1.833t) = 130{,}710t\ \frac{lb\cdot ft}{s}$

$P = 130{,}710t/550 = 238t$ (hp)

FOR $t < 40$ s $\quad P = 238t$ hp ◀

FOR $t > 40$ s $\quad P = \frac{(3000)(73.3)}{(550)} = 400$ hp ◀

13.54



GIVEN:

M_E = 3000 kg, ELEVATOR MASS
M_W = 1000 kg, COUNTERWEIGHT

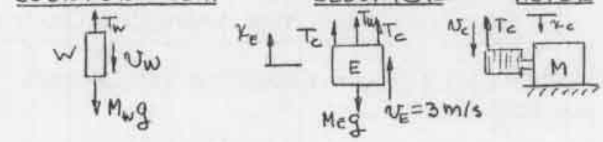
FIND:

(a) P(kW), DELIVERED BY MOTOR WHEN VELOCITY OF E, v_E = 3 m/s DOWN AND CONSTANT ($a_E = 0$)

(b) P(kW) WHEN v_E = 3 m/s UPWARD, a_E = 0.5 m/s² DOWN

(a) ACCELERATION = 0

COUNTERWEIGHT ELEVATOR MOTOR



$+\uparrow \Sigma F = T_W - M_W g = 0$
$T_W = (1000\text{ kg})(9.81\text{ m/s}^2)$
$T_W = 9810\text{ N}$

$+\uparrow \Sigma F = 2T_C + T_W - M_E g = 0$
$2T_C = (-9810\text{ N}) + (3000\text{ kg})(9.81\frac{m}{s^2})$

$T_C = 9810\text{ N}$

KINEMATICS

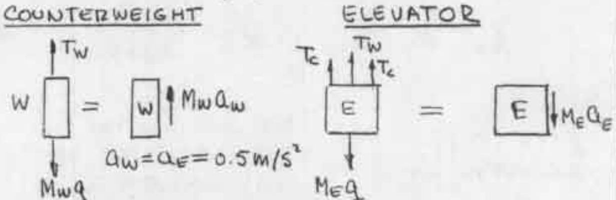
$2x_E = x_C \quad 2\dot{x}_E = \dot{x}_C \quad v_C = 2v_E = 6\text{ m/s}$

$P = T_C \cdot v_C = (9810\text{ N})(6\text{ m/s}) = 58{,}860\text{ J/s}$

$P(\text{kW}) = 58.9$ ◀

(b) $a_E = 0.5\text{ m/s}^2\uparrow$, $v_E = 3\text{ m/s}\downarrow$

COUNTERWEIGHT ELEVATOR



COUNTERWEIGHT $\Sigma F = Ma$

$\Sigma F = T_W - M_W g = M_W a_W$

$T_W = (1000\text{ kg})[(9.81\text{ m/s}^2) + (0.5\text{ m/s}^2)]$

$T_W = 10310\text{ N}$

ELEVATOR $\Sigma F = Ma$

$+\uparrow \Sigma F = 2T_C + T_W - M_E g = -M_E a_E$

$2T_C = (3000\text{ kg})[(9.81\text{ m/s}^2 - (0.5\text{ m/s}^2)] - 10310\text{ N}$

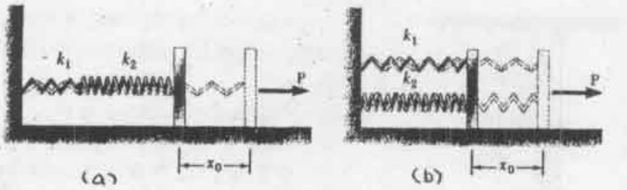
$T_C = 8810\text{ N} \quad v_C = 6\text{ m/s}$ [SEE (a)]

$P = T_C \cdot v_C = (8810\text{ N})(6\text{ m/s})$

$P = 52{,}860\text{ J} = 52.860\text{ kJ} = 52.86\text{ kW}$

$P(\text{kW}) = 52.9$ ◀

13.55



<u>GIVEN:</u>

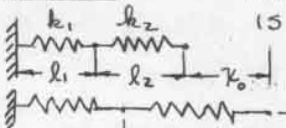
P CAUSES DEFLECTION x_0, IS SLOWLY APPLIED
(a) SPRINGS k_1 AND k_2 IN SERIES
(b) SPRINGS k_1 AND k_2 IN PARALLEL

<u>FIND:</u>

SINGLE EQUIVALENT SPRING k_e WHICH CAUSES THE SAME DEFLECTION

SYSTEM IS IN EQUILIBRIUM IN DEFLECTED x_0 POSITION.

<u>CASE (a)</u> FORCE IN BOTH SPRINGS IS THE SAME = P



$x_0 = x_1 + x_2$

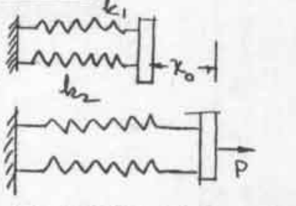
$x_0 = \dfrac{P}{k_e}$

$x_1 = \dfrac{P}{k_1} \quad x_2 = \dfrac{P}{k_2}$

THUS $\dfrac{P}{k_e} = \dfrac{P}{k_1} + \dfrac{P}{k_2}$

$\dfrac{1}{k_e} = \dfrac{1}{k_1} + \dfrac{1}{k_2} \qquad k_e = \dfrac{k_1 k_2}{k_1 + k_2}$

<u>CASE (b)</u>



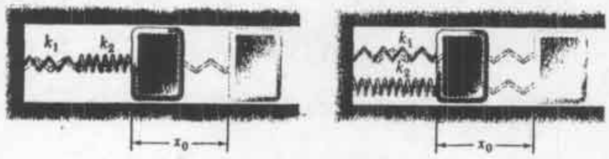
DEFLECTION IN BOTH SPRINGS IS THE SAME = x_0

$P = k_1 x_0 + k_2 x_0$
$P = (k_1 + k_2) x_0$
$P = k_e x_0$

EQUATING THE TWO EXPRESSIONS FOR
$P = (k_1+k_2)x_0 = k_e x_0$

$k_e = k_1 + k_2$

13.56



<u>GIVEN:</u>

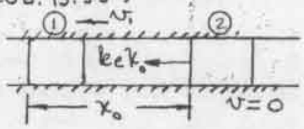
BLOCK OF MASS m
BLOCK MOVED TO x_0 AND RELEASED FROM REST.

<u>FIND:</u>

MAXIMUM VELOCITY, v_{MAX}

13.56 continued

WE WILL USE AN EQUIVALENT SPRING CONSTANT k_e (SEE PROB. 13.55)



CHOOSE ① AT INITIAL UNDEFLECTED POSITION
$V_1 = 0 \quad T_1 = \frac{1}{2} m v_1^2$
CHOOSE ② AT x_0 WHERE $v = 0$
$V_2 = \frac{1}{2} k_e x_0^2 \quad T_2 = 0$

$T_1 + V_1 = T_2 + V_2 \quad 0 + \frac{1}{2} m v_1^2 = \frac{1}{2} k_e x_0^2 + 0$

THUS $v_1 = v_{MAX} = x_0 \sqrt{\dfrac{k_e}{m}}$

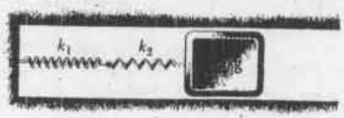
<u>CASE (a)</u>

$k_e = \dfrac{k_1 k_2}{k_1 + k_2} \qquad v_{MAX} = x_0 \sqrt{\dfrac{k_1 k_2}{m(k_1+k_2)}}$

<u>CASE (b)</u>

$k_e = k_1 + k_2 \qquad v_{MAX} = x_0 \sqrt{\dfrac{k_1 + k_2}{m}}$

13.57



<u>GIVEN:</u>

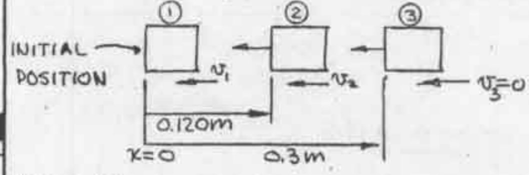
$k_1 = 12\,kN/m$
$k_2 = 8\,kN/m$
$m = 16\,kg$
INITIAL POSITION, 300mm TO RIGHT, $v = 0$

<u>FIND:</u>

(a) MAXIMUM VELOCITY, v_{MAX}
(b) VELOCITY 120 mm FROM INITIAL POSITION

USE EQUIVALENT SPRING CONSTANT (SEE P 13.55)
FORM SPRINGS IN SERIES, $k_e = \dfrac{k_1 k_2}{k_1 + k_2}$

$k_e = \dfrac{(12)(8)}{(12+8)} = 4.8\,kN/m$



(a) AT ①, SPRING DEFLECTION, 0
$V_1 = 0, \ T_1 = \frac{1}{2} m v_1^2 = 8 v_1^2$
THUS $v_1 = v_{MAX}$

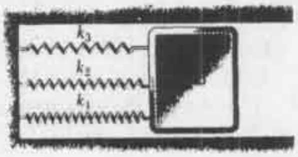
AT ③, $v_3 = 0 \quad T_3 = 0$
$V_3 = \frac{1}{2} k_e x_3^2 = \left(\frac{4800}{2}\right)(0.3)^2 = 216\ N\cdot m$

$T_1 + V_1 = T_3 + V_3$
$8 v_{max}^2 + 0 = 0 + 216$
$v_{MAX}^2 = 27$
$v_{MAX} = 5.20\,m/s$

(b) $T_2 = \frac{1}{2} m v_2^2 = 8 v_2^2$
$V_2 = \frac{1}{2} k_e x_2^2 = \left(\frac{4800}{2}\right)(0.120)^2 = 34.56\ N\cdot m$

$T_2 + V_2 = T_3 + V_3 \quad 8v_2^2 + 34.56 = 0 + 216$
$v_2^2 = 22.68 \qquad v_2 = 4.76\,m/s$

13.58

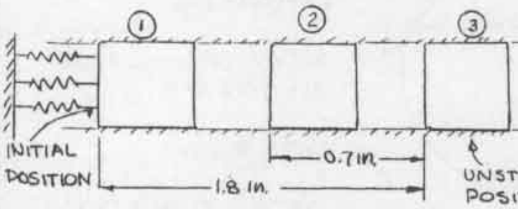


GIVEN:
$W = 6\text{ lb}$
$k_1 = 5\text{ lb/in.}$
$k_2 = 10\text{ lb/in.}$
$k_3 = 20\text{ lb/in}$
INITIAL DISPLACEMENT,
$x_0 = 1.8\text{ in.}$ TO LEFT FROM UNSTRETCHED POSITION
$v_0 = 0$

FIND:
(a) MAXIMUM VELOCITY, v_{MAX}
(b) VELOCITY AT 0.7 in. FROM INITIAL POSTION

EQUIVALENT $k_e = k_1 + k_2 + k_3$ (SEE P13.55 (b))

$$k_e = 5 + 10 + 20 = 35\text{ lb/in} = 420\text{ lb/ft}$$



(a) MAXIMUM VELOCITY OCCURS AT ③ WHERE THE SPRINGS ARE UNSTRETCHED

$$T_3 = \frac{1}{2} m v_{MAX}^2 = \frac{3}{g} v_{MAX}^2 \qquad V_3 = 0$$

$$T_1 = 0 \qquad V_1 = \frac{1}{2} k_e x_0^2 = \left(\frac{420\text{ lb/ft}}{2}\right)\left(\frac{1.8\text{ in.}}{12\text{ in./ft}}\right)^2$$

$$V_1 = 4.725\text{ lb}\cdot\text{ft}$$

$T_1 + V_1 = T_3 + V_3$

$$0 + 4.725 = \frac{3}{g} v_{MAX}^2 + 0$$

$$v_{MAX}^2 = \frac{(32.2\text{ ft/s}^2)(4.725\text{ lb}\cdot\text{ft})}{3\text{ lb}} = 50.715$$

$$v_{MAX} = 7.12\text{ ft/s} \longleftrightarrow$$

(b) $T_2 = \frac{1}{2} m v_2^2 = \frac{6}{2g} v_2^2 = \frac{3}{g} v_2^2$

$$V_2 = \frac{1}{2} k_e x_2^2 = \frac{420\text{ lb/ft}}{2}\left(\frac{.7\text{ in.}}{12\text{ in./ft}}\right)^2$$

$$V_2 = 0.7146\text{ lb}\cdot\text{ft}$$

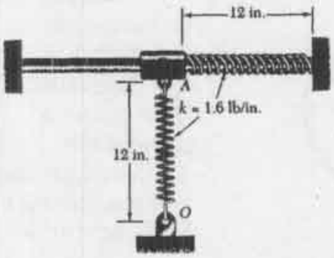
$T_1 + V_1 = T_2 + V_2$

$$0 + 4.725 = \frac{3}{g} v_2^2 + 0.7146$$

$$v_2^2 = \frac{(32.2\text{ ft/s}^2)(4.010\text{ lb}\cdot\text{ft})}{(3\text{ lb})} = 43.05$$

$$v_2 = 6.56\text{ ft/s} \longleftrightarrow$$

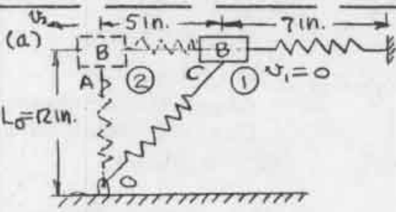
13.59



GIVEN:
$W_B = 10\text{ lb}$
COLLAR B PUSHED TO RIGHT, $x_0 = 5\text{ in.}$ AND RELEASED.
UNDEFORMED LENGTH OF EACH SPRING, $L_0 = 12\text{ in}$
$k = 1.6\text{ lb/in.}$ FOR EACH SPRING

FIND:
(a) MAXIMUM VELOCITY, v_{MAX}
(b) MAXIMUM ACCELERATION, a_{MAX}

(a)



MAXIMUM VELOCITY OCCURS AT A WHERE THE COLLAR IS PASSING THROUGH ITS EQUILIBRIUM POSITION

POSITION ①

$$T_1 = 0 \qquad k = (1.6\text{ lb/in.})(12\text{ in./ft}) = 19.2\text{ lb/ft}$$

$$L_{OC} = \sqrt{5^2 + 12^2} = 13\text{ in}$$

$$\Delta L_{OC} = 13\text{ in.} - 12\text{ in.} = 1\text{ in.} = \frac{1}{12}\text{ ft.}$$

$$\Delta L_{AC} = 5\text{ in.} = \frac{5}{12}\text{ ft}$$

$$V_1 = \frac{1}{2} k (\Delta L_{OC})^2 + \frac{1}{2} k (\Delta L_{AC})^2 = \left(\frac{19.2}{2}\frac{\text{lb}}{\text{ft}}\right)\left[\left(\frac{1}{12}\text{ ft}\right)^2 + \left(\frac{5}{12}\text{ ft}\right)^2\right]$$

$$V_1 = 1.733\text{ lb}\cdot\text{ft}$$

POSITION ②

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}\left(\frac{10}{g}\right) v_{MAX}^2 = \frac{5}{g} v_{MAX}^2$$

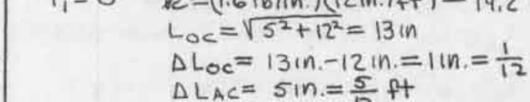
$V_2 = 0$ (BOTH SPRINGS ARE UNSTRETCHED)

$$T_1 + V_1 = T_2 + V_2 \qquad 0 + 1.733 = \frac{5}{g} v_{MAX}^2 + 0$$

$$v_{MAX}^2 = \frac{(1.733\text{ lb}\cdot\text{ft})(32.2\text{ ft/s}^2)}{(5\text{ lb})} = 11.16\ \frac{\text{ft}^2}{\text{s}^2}$$

$$v_{MAX} = 3.34\text{ ft/s}$$

(b) MAXIMUM ACCELERATION OCCURS AT C WHERE THE HORIZONTAL FORCE ON THE COLLAR IS A MAXIMUM



$$\Sigma F = ma \qquad F_1 \cos\theta + F_2 = m a_{MAX}$$

$$k \Delta L_{OC} \cos\theta + k \Delta L_{AC} = m a_{MAX}$$

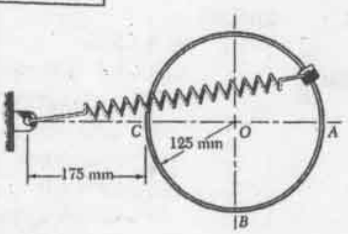
$$(19.2\text{ lb/ft})\left[\left(\frac{1}{12}\text{ ft}\right)\left(\frac{5}{13}\right) + \left(\frac{5}{12}\text{ ft}\right)\right] = \frac{10\text{ lb}}{g} a_{MAX}$$

$$8.615 = \frac{10}{g} a$$

$$a_{MAX} = \frac{(8.615\text{ lb})(32.2\text{ ft/s}^2)}{(10\text{ lb})}$$

$$a_{MAX} = 27.7\text{ ft/s}^2$$

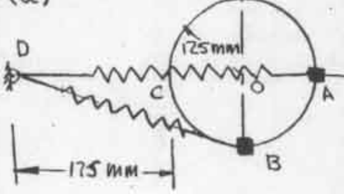
13.60



GIVEN:
MASS OF COLLAR
$m = 1.5\,kg.$
$k = 400\,N/m.$
UNDEFORMED LENGTH OF SPRING, $L_o = 150mm$
COLLAR RELEASED FROM REST AT A

FIND:
(a) VELOCITY OF THE COLLAR AT B, v_B
(b) VELOCITY OF THE COLLAR AT C, v_C

(a) VELOCITY AT B



$v_A = 0 \quad T_A = 0$
$\Delta L_{AD} = L_{AD} - L_o$
$\Delta L_{AD} = 425mm - 150mm$
$\Delta L_{AD} = 275mm = 0.275\,m$
$V_A = \frac{1}{2}k(\Delta L_{AD})^2$
$V_A = \frac{1}{2}(400\,N/m)(0.275\,m)^2$
$V_A = 15.125\,J$

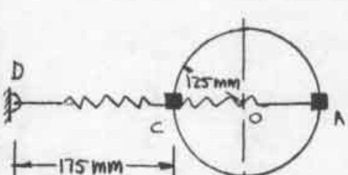
$T_B = \frac{1}{2}mv_B^2 = \left(\frac{1.5\,kg}{2}\right)(v_B^2) = (0.75)\,v_B^2$
$L_{BD} = (300^2mm + 125^2mm)^{\frac{1}{2}} = 325mm$
$\Delta_{BD} = L_{BD} - L_o = (325mm - 150mm) = 175mm = 0.175\,m$
$V_B = \frac{1}{2}k(\Delta_{BD})^2 = \frac{1}{2}(400\,N/m)(.175\,m)^2 = 6.125\,J$

$T_A + V_A = T_B + V_B \qquad 0 + 15.125 = 0.75\,v_B^2 + 6.125$

$v_B^2 = \frac{(15.125 - 6.125)}{(0.75)} = 12.00\,\frac{m^2}{s^2}$

$v_B = 3.46\,\frac{m}{s}$

(b) VELOCITY AT C

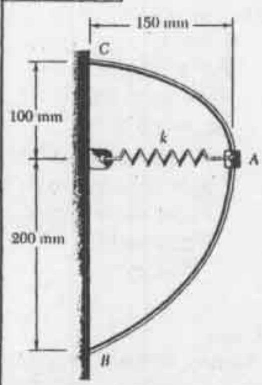


$T_A = 0$
$V_A = 15.125\,J$ (SEE (a))

$T_C = \frac{1}{2}mv_C^2 = \frac{1}{2}(1.5\,kg)v_C^2 = 0.75\,v_C^2$
$\Delta L_{DC} = L_o - L_{DC} = (150mm - 175m) = -25mm$
$V_C = \frac{1}{2}k(\Delta L_{DC})^2 = \frac{1}{2}(400\,N/m)(-0.025\,m)^2$
$V_C = 0.125\,J$
$T_A + V_A = T_C + V_C$
$0 + 15.125 = 0.75\,v_C^2 + 0.125$
$v_C^2 = 15/0.75 = 20$

$v_C = 4.47\,m/s$

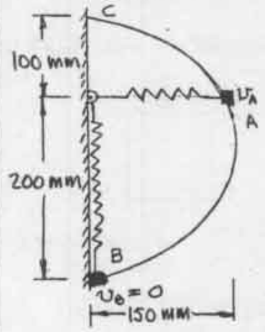
13.61



GIVEN:
HORIZONTAL PLANE
MASS OF COLLAR, $m = 500\,g$
UNDEFORMED LENGTH OF SPRING, $L_o = 80mm$
$k = 400\,kN/m$

FIND:
(a) VELOCITY AT A, v_A FOR VELOCITY AT B = 0
(b) VELOCITY AT C, v_C

(a) VELOCITY AT A



$T_A = \frac{1}{2}mv_A^2 = \left(\frac{0.5\,kg}{2}\right)v_A^2$
$T_A = (0.25)\,v_A^2$

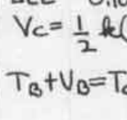
$\Delta L_A = 0.150\,m - 0.080\,m$
$\Delta L_A = 0.070\,m$
$V_A = \frac{1}{2}k(\Delta L_A)^2$
$V_A = \frac{1}{2}(400 \times 10^3\,N/m)(0.070\,m)^2$
$V_A = 980\,J$

$v_B = 0 \quad T_B = 0$
$\Delta L_B = 0.200\,m - 0.080\,m = 0.120\,m$
$V_B = \frac{1}{2}k(\Delta L_B)^2 = \frac{1}{2}(400 \times 10^3\,N/m)(0.120\,m)^2$
$V_B = 2880\,J$

$T_A + V_A = T_B + V_B \qquad 0.25\,v_A^2 + 980 = 0 + 2880$
$v_A^2 = (2880 - 980)/(0.25)$
$v_A^2 = 7600\,m^2/s^2$

$v_A = 87.2\,m/s$

(b) VELOCITY AT C



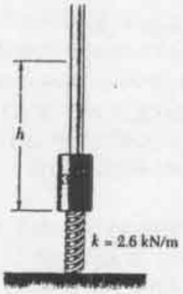
SINCE SLOPE AT B IS POSITIVE THE COMPONENT OF THE SPRING FORCE F_P, PARALLEL TO THE ROD, CAUSES THE BLOCK TO MOVE BACK TOWARD A
$T_B = 0$, $V_B = 2880\,J$ (FROM PART (a))

$T_C = \frac{1}{2}mv_C^2 = \left(\frac{0.5\,kg}{2}\right)v_C^2 = 0.25\,v_C^2$

$\Delta L_C = 0.100\,m - 0.080\,m = 0.020\,m$
$V_C = \frac{1}{2}k(\Delta L_C)^2 = \frac{1}{2}(400 \times 10^3\,N/m)(0.020\,m)^2 = 80.0\,J$
$T_B + V_B = T_C + V_C \quad 0 + 2880 = 0.25\,v_C^2 + 80.0$
$v_C^2 = 11200\,m^2/s^2$

$v_C = 105.8\,m/s$

13.62



GIVEN:

MASS OF COLLAR, $m = 3\ kg$

VERTICAL ROD, $k = 2.6\ kN/m$

MASS IS PUSHED DOWN 150 mm FROM ITS EQUILIBRIUM POSITION AND RELEASED

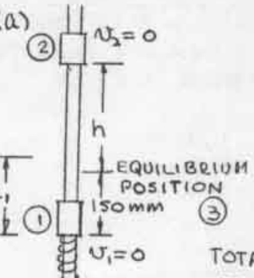
SPRING IS UNATTACHED

FIND:

(a) MAXIMUM HEIGHT h ABOVE EQUILIBRIUM POSITION.

(b) MAXIMUM VELOCITY OF THE COLLAR, v_{MAX}

(a)



MAXIMUM HEIGHT IS REACHED WHEN $v_2 = 0$

THUS $T_1 = T_2 = 0$

$$V = V_g + V_e$$

POSITION ①

$$(V_g)_1 = 0$$

TOTAL SPRING DEFLECTION FROM UNDEFLECTED SPRING POSITION, x_1

$$x_1 = mg/k + 0.150$$

$$x_1 = mg/k + 0.150 = \frac{(3\,kg)(9.81\,m/s^2)}{(2.6\times10^3\,N/m)} + 0.150\,m$$

$$x_1 = 0.01132 + 0.150 = 0.1613\,m$$

$$(V_e)_1 = \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}(2.6\times10^3\,N/m)(0.1613\,m)^2 = 33.83\,J$$

$$V_1 = 0 + 33.83 = 33.83\,J$$

POSITION ②

$$(V_g)_2 = mg(0.150 + h) = 3g(0.150 + h)$$

$(V_e)_2 = 0$ (SPRING IS NOT ATTACHED TO THE COLLAR)

$$T_1 + V_1 = T_2 + V_2 \quad 0 + (V_g)_1 + (V_e)_1 = 0 + (V_g)_2 + (V_e)_2$$

$$0 + 0 + 33.83 = 0 + 3g(0.150 + h) + 0$$

$$h = \frac{33.83\,J}{(3\,kg)(9.81\,m/s^2)} - (0.150\,m) = 0.9995\,m$$

$$h = 1000\,mm$$

(b) MAXIMUM VELOCITY OCCURS WHEN THE ACCELERATION $= 0$, i.e. AT EQUILIBRIUM AT POSITION ③

$$T_3 = \tfrac{1}{2}mv_3^2 = \tfrac{1}{2}(3)v_{MAX}^2 = 1.5\,v_{MAX}^2$$

$$V_3 = (V_g)_3 + (V_e)_3 = mg(0.150) + \tfrac{1}{2}k(x_1 - 0.150)^2$$

$$V_3 = (3\,kg)(9.81\,\tfrac{m}{s^2})(0.150\,m) + \tfrac{1}{2}(2.6\times10^3\,N/m)(0.1613 - 0.150\,m)^2$$

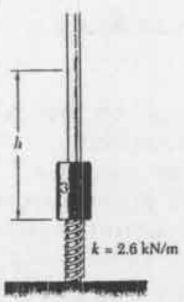
$$V_3 = 4.415\,J + 0.1660\,J = 4.581\,J$$

$$T_1 + V_1 = T_3 + V_3 \quad 0 + 33.83 = 1.5\,v_{MAX}^2 + 4.581$$

$$v_{MAX}^2 = (29.249)/1.5 = 19.50\,m^2/s^2$$

$$v_{MAX} = 4.42\,m/s$$

13.63



GIVEN:

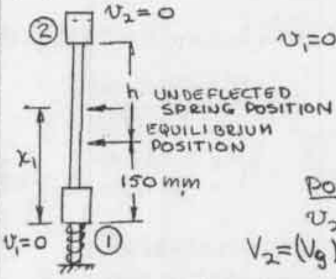
SAME AS P13.62 AT LEFT EXCEPT THAT THE SPRING IS ATTACHED TO THE COLLAR

FIND:

(a) MAXIMUM HEIGHT h ABOVE THE EQUILIBRIUM POSITION

(b) MAXIMUM VELOCITY OF THE COLLAR, v_{MAX}

(a)



POSITION ①

$v_1 = 0$, $T_1 = 0$

$V_1 = 33.83\,J$

(SAME AS P13.62 AT LEFT)

POSITION ②.

$v_2 = 0 \quad T_2 = 0$

$$V_2 = (V_g)_2 + (V_e)_2$$

$$(V_g)_2 = mg(h + 0.150) = 3g(h + 0.150)$$

$$(V_e)_2 = \tfrac{1}{2}k[h - (x_1 - 0.150)]^2 \qquad x_1 = 0.1613\,m$$

(P.13.62 AT LEFT)

$$(V_e)_2 = \tfrac{1}{2}(2.6\times10^3\,N/m)(h - 0.0113)^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 33.83 = 3g(h + 0.150) + (1.3\times10^3)(h - 0.0113)^2$$

$$33.83 = 29.4\,h + 4.415 + 1.3\times10^3 h^2 - 29.4\,h + 0.166\times10^{-3}$$

$$h^2 = (33.83 - 4.415 - 0.166\times10^{-3})/1.3\times10^3$$

$$h^2 = 22.499$$

$$h = 0.1500\,m$$

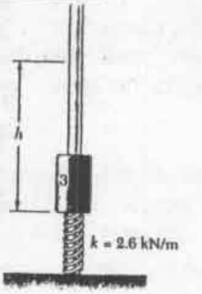
$$h = 150\,mm$$

(b) MAXIMUM VELOCITY

← SEE (b) AT LEFT

$$v_{MAX} = 4.42\,m/s$$

13.64



GIVEN:

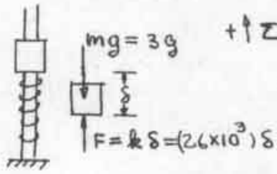
$m = 3\ kg$, $k = 2.6\ kN/m$

FIND:

(a) COMPRESSION OF SPRING FROM UNDEFORMED POSITION IF COLLAR COMES TO EQUILIBRIUM

(b) MAXIMUM COMPRESSION IF COLLAR IS SUDDENLY RELEASED

(a) COLLAR IS IN EQUILIBRIUM



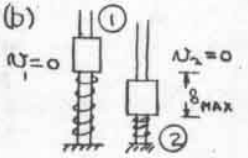
$+\uparrow \Sigma F = (2.6\times10^3\ N/m)\delta - 3g = 0$

$$\delta = \frac{(3\ kg)(9.81\ m/s^2)}{(2.6\times10^3\ N/m)}$$

$\delta = 0.01132\ m$

$\delta = 11.32\ mm$

(b)



MAXIMUM COMPRESSION OCCURS WHEN VELOCITY AT ② IS ZERO

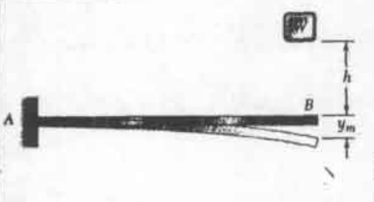
$T_1 = 0 \quad V_1 = 0$

$T_2 = 0 \quad V_2 = -mg\delta_{MAX} + \frac{1}{2}k\delta_{MAX}^2$

$$\delta_{MAX} = \frac{(2)(3\ kg)(9.81\ m/s^2)}{(2.6\times10^3\ N/m)} = 0.02264\ m$$

$\delta = 22.6\ mm$

13.65



GIVEN:

STATIC DEFLECTION, y_{ST} IS PROPORTIONAL TO W.

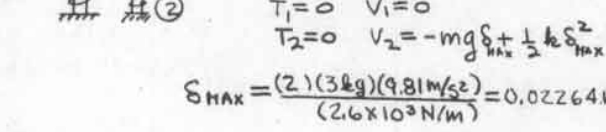
FIND:

y_m, WHEN W IS DROPPED FROM h

DENOTE BY k AN EQUIVALENT SPRING CONSTANT

STATIC DEFLECTION OF BEAM IS THEN $y_{ST} = \frac{W}{k}$ (1)

DROP W FROM HEIGHT h



$T_1 = 0 \quad V_1 = Wh \qquad T_2 = 0 \quad V_2 = -Wy_m + \frac{1}{2}ky_m^2$

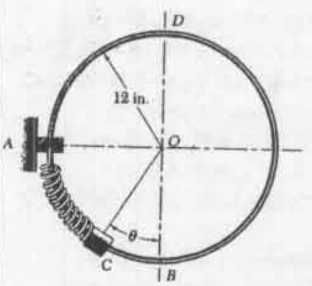
$T_1 + V_1 = T_2 + V_2 \qquad 0 + Wh = 0 - Wy_m + \frac{1}{2}ky_m^2$

FROM EQUATION (1), $W = ky_{ST}$.

$ky_{ST}(h + y_m) = \frac{1}{2}ky_m^2$

$y_m^2 - 2y_{ST}y_m - 2y_{ST}h = 0 \qquad y_m = y_{ST}\left(1 + \sqrt{\frac{2h}{y_{ST}}}\right)$

13.66



GIVEN:

VERTICAL PLANE

SPRING, $k = 3\ lb/ft$, UNDEFORMED LENGTH AB, IS UNATTACHED TO COLLAR, $W = 8\ oz$.

$\theta = 30°$ ($v = 0$)

NO FRICTION

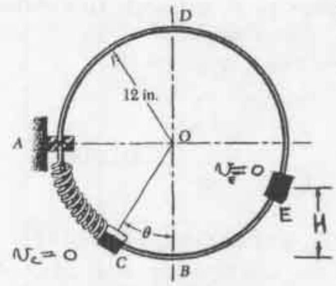
FIND:

(a) MAXIMUM HEIGHT H ABOVE B REACHED BY THE COLLAR

(b) MAXIMUM VELOCITY, v_{MAX} OF THE COLLAR.

(a)

MAXIMUM HEIGHT ABOVE B IS REACHED WHEN THE VELOCITY AT E IS ZERO



$T_C = 0$

$T_E = 0$

$V = V_e + V_g$

POINT C

$\theta = 30° = \pi/6\ RAD$

$R = 12\ in. = 1\ ft$

$\Delta L_{BC} = (1\ ft)(\frac{\pi}{6}\ RAD)$

$\Delta L_{BC} = \frac{\pi}{6}\ ft$

$(V_C)_e = \frac{1}{2}k(\Delta L_{BC})^2$

$(V_C)_e = \frac{1}{2}(3\ lb/ft)(\frac{\pi}{6}\ ft)^2 = 0.4112\ lb\cdot ft$

$(V_C)_g = WR(1 - \cos\theta) = \frac{(8\ oz)}{(16\ oz/lb)}(1\ ft)(1 - \cos 30°)$

$(V_C)_g = 0.06699\ lb\cdot ft$

POINT E

$(V_E)_e = 0$ (SPRING IS UNATTACHED)

$(V_E)_g = WH = \left(\frac{8}{16}\right)(H) = \frac{H}{2}\ (lb\cdot ft)$

$T_C + V_C = T_E + V_E$

$0 + 0.4112 + 0.06699 = 0 + 0 + \frac{H}{2}$

$H = 0.956\ ft$

(b) THE MAXIMUM VELOCITY IS AT B WHERE THE POTENTIAL ENERGY IS ZERO, $v_B = v_{MAX}$

$T_C = 0 \qquad V_C = 0.4112 + .06699 = 0.4782\ lb\cdot ft$

$T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}\left(\frac{1}{2}\ lb/32.2\ ft/s^2\right)v_{MAX}^2$

$T_B = 0.07640\ v_{MAX}^2$

$V_B = 0$

$T_C + V_C = T_B + V_B \qquad 0 + 0.4782 = (0.0764)v_{MAX}^2$

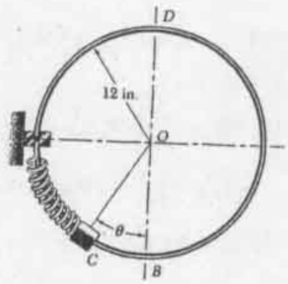
$v_{MAX}^2 = 61.59\ ft^2/s^2$

$v_{MAX} = 7.85\ ft/s$

13.67

GIVEN:

VERTICAL PLANE.
SPRING, $k = 3\,\text{lb/ft}$
OF UNDEFORMED LENGTH AB IS UNATTACHED TO COLLAR C
OF WEIGHT $W = 8\,\text{oz}$
IS RELEASED FROM REST AT AN ANGLE θ
NO FRICTION

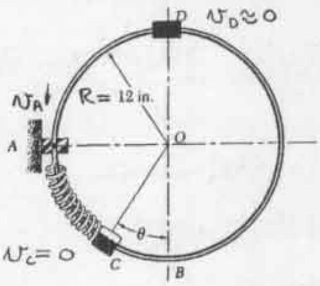


FIND:

(a) SMALLEST VALUE OF θ FOR WHICH THE COLLAR WILL REACH POINT A

(b) VALUE OF THE VELOCITY AT IT REACHES A.

(a) SMALLEST ANGLE θ OCCURS WHEN THE VELOCITY AT D IS CLOSE TO ZERO



$v_C = 0 \quad v_D = 0$

$T_C = 0 \quad T_D = 0$

$V = V_e + V_g$

POINT C

$\Delta L_{BC} = (1\,\text{ft})(\theta) = \theta\,\text{ft}$

$(V_C)_e = \frac{1}{2}k(\Delta L_{BC})^2$

$(V_C)_e = \frac{3}{2}\theta^2$

$R = 12\,\text{in.} = 1\,\text{ft}$

$(V_C)_g = WR(1-\cos\theta)$

$(V_C)_g = \left(\frac{8\,\text{oz}}{16\,\text{oz/lb}}\right)(1\,\text{ft})(1-\cos\theta)$

$(V_C)_g = \frac{1}{2}(1-\cos\theta)$

$V_C = (V_C)_e + (V_C)_g = \frac{3}{2}\theta^2 + \frac{1}{2}(1-\cos\theta)$

POINT D

$(V_D)_e = 0$ (SPRING IS UNATTACHED)

$(V_D)_g = W(2R) = (2)(0.5\,\text{lb})(1\,\text{ft}) = 1\,\text{lb}\cdot\text{ft}$

$T_C + V_C = T_D + V_D \quad 0 + \frac{3}{2}\theta^2 + \frac{1}{2}(1-\cos\theta) = 1$

$(1.5)\theta^2 - (0.5)\cos\theta = 0.5$

BY TRIAL $\theta = 0.7592\,\text{RAD}$

$\theta = 43.5°$ ◀

(b) VELOCITY AT A

POINT D

$v_D = 0 \quad T_D = 0 \quad V_D = 1\,\text{lb}\cdot\text{ft}$ (SEE PART (a))

POINT A

$T_A = \frac{1}{2}mv_A^2 = \frac{1}{2}\frac{(0.5\,\text{lb})}{(32.2\,\text{ft/s}^2)}v_A^2$

$T_A = 0.007640\,v_A^2$

$V_A = (V_A)_g = W(R) = (0.5\,\text{lb})(1\,\text{ft}) = 0.5\,\text{lb}\cdot\text{ft}$

$T_A + V_A = T_D + V_D$

$0.007640\,v_A^2 + 0.5 = 0 + 1$

$v_A^2 = 64.4\,\text{ft}^2/\text{s}^2$

$v_A = 8.02\,\text{ft/s}$ ◀

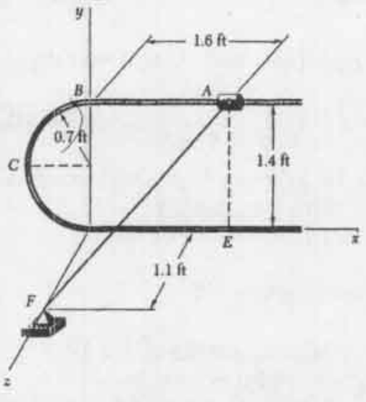
13.68

GIVEN:

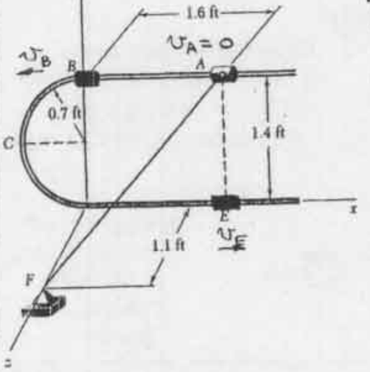
COLLAR, $W = 2.7\,\text{lb}$
UNDEFORMED LENGTH OF ELASTIC CORD
$L_0 = 0.9\,\text{ft}$
$k = 5\,\text{lb/ft}$
$v_A = 0$

FIND:

SPEED OF COLLAR
(a) AT B
(b) AT E



(a)



$L_{AF} = \sqrt{(1.6)^2 + (1.4)^2 + (1.1)^2}$

$L_{AF} = 2.394\,\text{ft}$

$L_{BF} = \sqrt{(1.4)^2 + (1.1)^2}$

$L_{BF} = 1.780\,\text{ft}$

$L_{FE} = \sqrt{(1.6)^2 + (1.1)^2}$

$L_{FE} = 1.942\,\text{ft}$

$V = V_e + V_g$

(a) SPEED AT B

$v_A = 0, \; T_A = 0$

$(V_A)_e = \frac{1}{2}k(\Delta L_{AF})^2$

POINT A

$\Delta L_{AF} = L_{AF} - L_0 = 2.394 - 0.9$

$\Delta L_{AF} = 1.494\,\text{ft}$

$(V_A)_e = \frac{1}{2}(5\,\text{lb/ft})(1.494\,\text{ft})^2$

$(V_A)_e = 5.580\,\text{lb}\cdot\text{ft}$

$(V_A)_g = (W)(1.4) = (2.7\,\text{lb})(1.4\,\text{ft}) = 3.78\,\text{lb}\cdot\text{ft}$

$V_A = (V_A)_e + (V_A)_g = 5.580 + 3.78 = 9.360\,\text{lb}\cdot\text{ft}$

POINT B $\quad T_B = \frac{1}{2}mv_B^2 = \frac{1}{2}\left(\frac{2.7\,\text{lb}}{32.2\,\text{lb/ft}^2}\right)v_B^2$

$T_B = 0.04193\,v_B^2$

$(V_B)_e = \frac{1}{2}k(\Delta L_{BF})^2 \qquad \Delta L_{BF} = L_{BF} - L_0 = 1.780 - 0.9$

$\Delta L_{BF} = 0.880\,\text{ft}$

$(V_B)_e = \frac{1}{2}(5\,\text{lb/ft})(0.880\,\text{ft})^2 = 1.936\,\text{lb}\cdot\text{ft}$

$(V_B)_g = (W)(1.4) = (2.7\,\text{lb})(1.4\,\text{ft}) = 3.78\,\text{lb}\cdot\text{ft}$

$V_B = (V_B)_e + (V_B)_g = 1.936 + 3.78 = 5.716\,\text{lb}\cdot\text{ft}$

$T_A + V_A = T_B + V_B$

$0 + 9.360 = 0.04193\,v_B^2 + 5.716$

$v_B^2 = (3.644)/(0.04193)$

$v_B^2 = 86.91\,\text{ft}^2/\text{s}^2$

(CONTINUED) $\qquad v_B = 9.32\,\text{ft/s}$ ◀

13.68 continued

(b) SPEED AT E

POINT A $T_A = 0$ $V_A = 9.360$ lb·ft (FROM PART(a))

POINT E
$$T_E = \frac{1}{2} m v_E^2 = \frac{1}{2}\left(\frac{2.7\text{ lb}}{32.2\text{ ft/s}^2}\right) v_E^2 = 0.04193 v_E^2$$

$(V_E)_e = \frac{1}{2} k (\Delta L_{FE})^2$ $\Delta L_{FE} = L_{FE} - L_0 = 1.942 - 0.900$

$\Delta L_{FE} = 1.042$ ft

$(V_E)_e = \frac{1}{2}(5\text{ lb/ft})(1.042\text{ ft})^2 = 2.714$ lb·ft

$(V_E)_g = 0$ $V_E = 2.714$ lb·ft

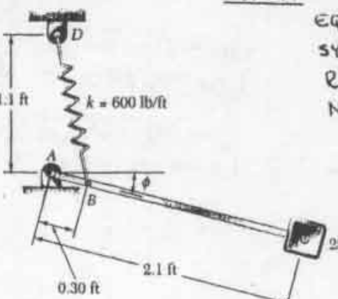
$T_A + V_A = T_E + V_E$ $0 + 9.360 = 0.04193 v_E^2 + 2.714$

$v_E^2 = 6.6456/0.04193$

$v_E^2 = 158.49$ ft²/s²

$v_E = 12.59$ ft/s

13.69



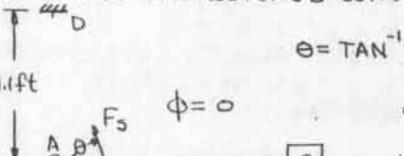
GIVEN:

EQUILIBRIUM FOR $\phi = 0°$
SYSTEM RELEASED FROM REST WHEN $\phi = 90°$
NEGLECT WEIGHT OF ROD

FIND:

VELOCITY OF BLOCK C AS IT PASSES THROUGH $\phi = 0$

FIND THE UNSTRETCHED LENGTH OF THE SPRING



$\theta = \tan^{-1}\frac{1.1}{0.3} = 1.3045$ RAD

$\theta = 74.745°$

$L_{BD} = \sqrt{(1.1)^2 + .3^2}$

$L_{BD} = 1.140$ ft

EQUILIBRIUM $\Sigma M_A = (0.3)(F_S \sin\theta) - (25)(2.1) = 0$

$$F_S = \frac{(25\text{ lb})(2.1\text{ ft})}{(0.3\text{ ft})(\sin 74.745°)} = 181.39\text{ lb}$$

$F_S = k\,\Delta L_{BD}$

181.39 lb $= (600$ lb/ft$)(\Delta L_{BD})$

$\Delta L_{BD} = 0.30232$ ft

UNSTRETCHED LENGTH $L_0 = L_{BD} - \Delta L_{BD}$

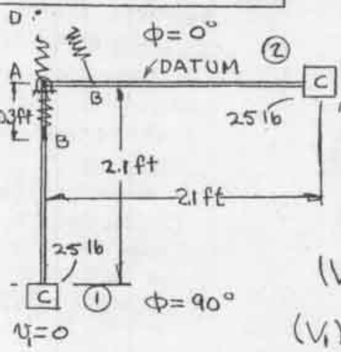
$L_0 = 1.140 - 0.3023 = 0.83768$ ft

SPRING ELONGATION, $\Delta L'_{BD}$, WHEN $\phi = 90°$

$\Delta L'_{BD} = (1.1\text{ ft} + 0.3\text{ ft}) - L_0$

$\Delta L'_{BD} = 1.4\text{ ft} - 0.8377\text{ ft} = 0.56232$ ft

13.69 continued



AT ① ($\phi = 90°$)

$v_1 = 0$, $T_1 = 0$

$V_1 = (V_1)_e + (V_1)_g$

$(V_1)_e = \frac{1}{2} k (\Delta L'_{BD})^2$

$(V_1)_e = \frac{1}{2}(600\text{ lb/ft})(0.56232\text{ ft})^2$

$(V_1)_e = 94.86$ lb·ft

$(V_1)_g = -(25\text{ lb})(2.1\text{ ft}) = -52.5$ lb·ft

$V_1 = 94.86 - 52.5 = 42.36$ lb·ft

AT ② ($\phi = 0°$)

$(V_2)_e = \frac{1}{2} k (\Delta L_{BD})^2 = \frac{1}{2}(600\,\frac{\text{lb}}{\text{ft}})(0.3023\text{ ft})^2$

$(V_2)_e = 27.42$ lb·ft

$(V_2)_g = 0$ $V_2 = 27.42$ lb·ft

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}\left(\frac{25\text{ lb}}{32.2\text{ ft/s}^2}\right) v_2^2 = 0.3882 v_2^2$$

$T_1 + V_1 = T_2 + V_2$

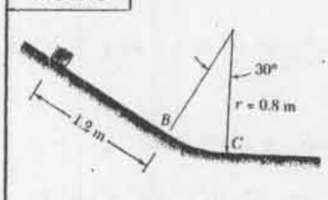
$0 + 42.36 = 0.3882 v_2^2 + 27.42$

$v_2^2 = (14.941)/(0.3882)$

$v_2^2 = 38.48$ ft²/s²

$v_2 = 6.20$ ft/s

13.70

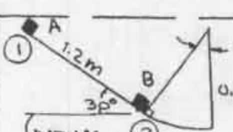


GIVEN:

30.0-g PELLET RELEASED FROM REST. NO FRICTION

FIND:

FORCE ON PELLET,
(a) JUST BEFORE B
(b) IMMEDIATELY AFTER B.



VELOCITY AT ②

$v_1 = 0$ $T_1 = 0$ $V_1 = mg(1.2)(\sin 30°)$

$V_1 = (.03\text{ kg})(9.81\,\frac{\text{m}}{\text{s}^2})(1.2\text{ m})(\frac{1}{2})$

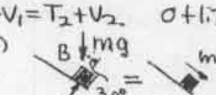
$V_1 = 1.766$ J

$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}(.03\text{ kg})(v_2^2) = 0.015 v_2^2$

$V_2 = 0$

$T_1 + V_1 = T_2 + V_2$ $0 + 1.766 = 0.015 v_2^2 + 0$ $v_2^2 = 117.7\,\frac{\text{m}^2}{\text{s}^2}$

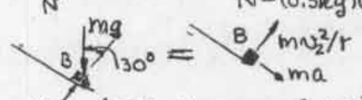
(a)



$\Sigma F = N - .03g\cos 30° = 0$

$N = (0.03\text{ kg})(9.81\,\frac{\text{m}}{\text{s}^2})(\frac{\sqrt{3}}{2}) = .255$ N

(b)

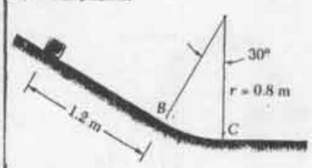


$\Sigma F = N - .03g\cos 30° = mv^2/R = \frac{(0.03\text{ kg})(117.72\text{ m}^2/\text{s}^2)}{(0.8\text{ m})}$

$N = .255 + 4.41 = 4.66$ N

$N = 4.66$ N

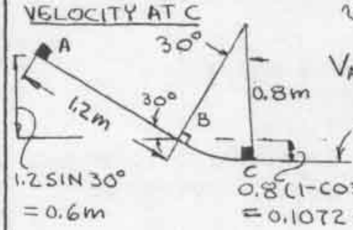
13.71



GIVEN:
300g PELLET RELEASED FROM REST. NO FRICTION

FIND:
FORCE FROM THE SURFACE ON THE PELLET,
(a) JUST BEFORE C
(b) JUST AFTER C

VELOCITY AT C



$v_A = 0 \quad T_A = 0$

$V_A = (0.3\,kg)(9.81\,\frac{m}{s^2})(0.7072\,m)$

$V_A = 2.081\,J$

$T_C = \frac{1}{2} m v_C^2$

$T_C = 0.15 v_C^2$

$V_C = 0$

$T_A + V_A = T_C + V_C$

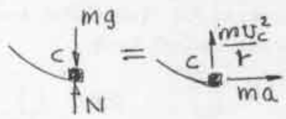
$0 + 2.081 = 0.15 v_C^2 + 0$

$v_C^2 = 13.873\ m^2/s^2$

(a) NORMAL FORCE JUST BEFORE C

PELLET IS IN THE CURVE ($a_n = v_C^2/r$)

$+\uparrow \Sigma F = m v_C^2/r$



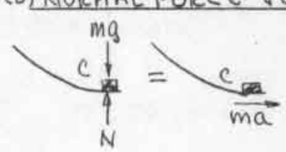
$N - mg = \frac{m v_C^2}{r}$

$N = m\left(g + \frac{v_C^2}{r}\right)$

$N = (0.3\,kg)\left(9.81\,\frac{m}{s^2} + \frac{13.873\,m^2/s^2}{(0.8\,m)}\right)$

$N = (0.3)(9.81 + 17.34) = 8.15\,N$

(b) NORMAL FORCE JUST AFTER C

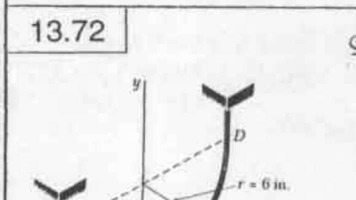


PELLET IS ON THE STRAIGHT SECTION OF THE SURFACE ($a_n = 0$)

$+\uparrow \Sigma F = N - mg = 0$

$N = 0.3g = 2.94\,N$

13.72



GIVEN:
$W = 1.2\,lb$, RELEASED FROM REST AT B
$k = 1.8\,lb/in.$
UNSTRETCHED SPRING LENGTH
$L_0 = 8\,in.$

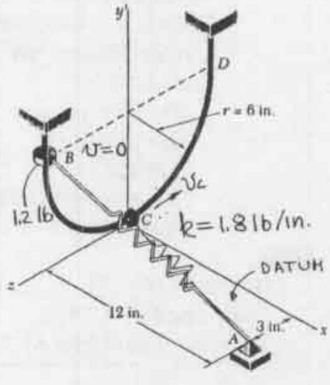
FIND:
(a) SPEED OF THE COLLAR AT C
(b) FORCE EXERTED BY THE ROD ON THE COLLAR AT C.

$L_{AB} = \sqrt{12^2 + 6^2 + 3^2} = 13.748\,in.$ $\quad L_{AC} = \sqrt{12^2 + 3^2} = 12.369\,in.$

13.72 continued

$k = 1.8\,lb/in. = 21.6\,lb/ft$

(a) SPEED AT C



AT B

$v_B = 0 \quad T_B = 0$

$V_B = (V_B)_e + (V_B)_g$

$\Delta L_{AB} = 13.748\,in. - 8\,in.$

$\Delta L_{AB} = 5.748\,in. = 0.479\,ft$

$(V_B)_e = \frac{1}{2} k (\Delta L_{AB})^2$

$(V_B)_e = \frac{1}{2}(21.6\,\frac{lb}{ft})(0.479\,ft)^2$

$(V_B)_e = 2.478\,lb\cdot ft$

$(V_B)_g = Wr = (1.2\,lb)(\frac{6}{12}\,ft) = 0.600\,lb\cdot ft$

$V_B = (V_B)_e + (V_B)_g = 2.478 + 0.600 = 3.078\,lb\cdot ft$

AT C

$T_C = \frac{1}{2} m v_C^2 = \frac{1}{2}\left(\frac{1.2\,lb}{32.2\,ft/s^2}\right) v_C^2$

$T_C = 0.018634\, v_C^2$

$(V_C)_e = \frac{1}{2} k (\Delta L_{AC})^2$

$\Delta L_{AC} = 12.369\,in. - 8\,in. = 4.369\,in. = 0.364\,ft$

$(V_C)_e = \frac{1}{2}(21.6\,lb/ft)(0.364)^2 = 1.4316\,lb\cdot ft$

$(V_C)_g = 0$

$(V_C) = (V_C)_e + (V_C)_g = 1.4316 + 0 = 1.4316\,lb\cdot ft$

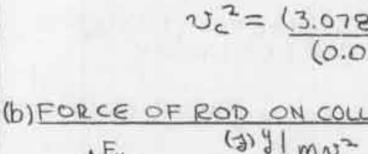
$T_B + V_B = T_C + V_C$

$0 + 3.078 = 0.018634\, v_C^2 + 1.4316$

$v_C^2 = \frac{(3.078 - 1.4316)}{(0.018634)} = 88.36\,\frac{ft^2}{s^2}$

$v_C = 9.40\,\frac{ft}{s}$

(b) FORCE OF ROD ON COLLAR AT C



$F_z = 0$ (NO FRICTION)

$\underline{F} = F_x \underline{i} + F_y \underline{j}$

$\theta = \tan^{-1}\frac{3}{12} = 14.04°$

$F_e = (k\,\Delta L_{AC})(\cos\theta\,\underline{i} + \sin\theta\,\underline{k})$

$\underline{F}_e = (21.6)(0.364)(\cos 14.04°\,\underline{i} + \sin 14.04°\,\underline{k})$

$\underline{F}_e = 7.628\,\underline{i} + 1.9069\,\underline{k}$ (lb)

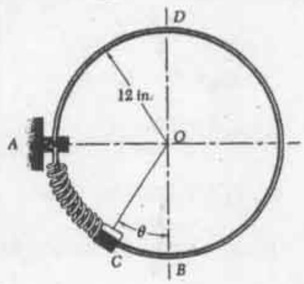
$\Sigma \underline{F} = (F_x + 7.628)\underline{i} + (F_y - 1.2)\underline{j} + 1.906\underline{k} = \frac{mv^2}{r}\underline{j} + ma\underline{i}$

$F_x + 7.628\,lb = 0 \quad F_y = 1.2\,lb + \frac{(1.2\,lb)(88.36\,ft^2/s^2)}{(32.2\,ft/s^2)(0.5\,ft)}$

$F_x = -7.628\,lb$

$F_y = 7.78\,lb \qquad \underline{F} = -7.63\,lb\,\underline{i} + 7.78\,lb\,\underline{j}$

13.73



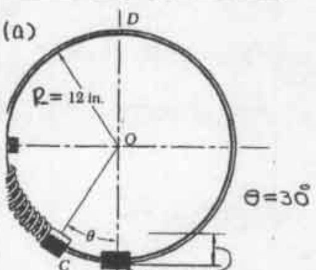
GIVEN:

VERTICAL PLANE
SPRING, $k = 3\,lb/ft$
UNDEFORMED LENGTH = ARC AB, UNATTACHED TO COLLAR.
COLLAR WEIGHT $W = 8\,oz.$
$\theta = 30°$.
COLLAR RELEASED FROM REST AT C.

FIND:

(a) VELOCITY AT B, V_c
(b) FORCE ON THE COLLAR FROM ROD AT B

(a)



$\theta = 30°$

$v_c = 0,\ T_c = 0$

$T_B = \frac{1}{2} m v_B^2$

$T_B = \frac{1}{2}\frac{(8\,oz)}{(16\,oz/lb)(32.2\,ft/s^2)} v_B^2$

$T_B = 0.07764\, v_B^2$

$V_c = (V_c)_e + (V_c)_g$

$\text{ARC } BC = \Delta L_{BC} = R\theta$

$\Delta L_{BC} = (1\,ft)(30°)\frac{(\pi)}{180°}$

$\Delta L_{BC} = 0.5236\,ft$

$(V_c)_e = \frac{1}{2} k (\Delta L_{BC})^2$

$(V_c)_e = \frac{1}{2}(3\,lb/ft)(0.5236\,ft)^2 = 0.4112\,lb\cdot ft$

$(V_c)_g = WR(1-\cos\theta) = \frac{(8\,oz)}{(16\,oz/lb)}(1\,ft)(1-\cos 30°)$

$(V_c)_g = 0.06699\,lb\cdot ft$

$V_c = (V_c)_e + (V_c)_g = 0.4112 + 0.06699 = 0.4782\,lb\,ft$

$V_B = (V_B)_e + (V_B)_g = 0 + 0 = 0$

$T_c + V_c = T_B + V_B \qquad 0 + 0.4782 = 0.07764\, v_B^2$

$v_B^2 = 61.59\,ft^2/s^2$

$v_B = 7.85\,ft/s$

(b)

$W = 0.5\,lb$, F_R $\quad = \quad$ $m v_B^2/R$, ma

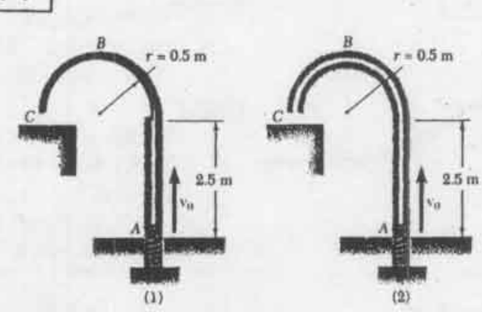
$+\uparrow \Sigma F = F_R - W = m v_B^2/R$

$F_R = 0.5\,lb + \frac{(0.5\,lb)}{(32.2\,ft/s^2)}\frac{(61.59\,ft^2/s^2)}{(1\,ft)}$

$F_R = 0.5\,lb + 0.9564\,lb = 1.456\,lb$

$F_R = 1.456\,lb$

13.74



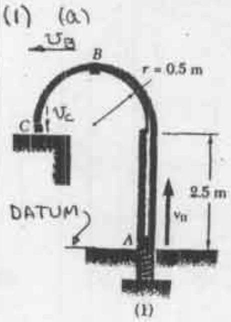
GIVEN:

PACKAGE, MASS $m = 200\text{-}g$
INITIAL VELOCITY, v_0
FRICTION LESS TUBE
(1) TUBE IS OPEN ALONG CIRCULAR ARC
(2) TUBE IS CLOSED THROUGHOUT

FIND:

(a) SMALLEST VELOCITY v_0 FOR PACKAGE TO REACH POINT C
(b) FORCE EXERTED BY THE PACKAGE ON THE TUBE.

(1) (a)



THE SMALLEST VELOCITY AT B WILL OCCUR WHEN THE FORCE EXERTED BY THE TUBE ON THE PACKAGE IS ZERO.

B: $N = 0$, $mg = 0.2\,g$ $\quad = \quad$ $\frac{m v_B^2}{r}$

$+\downarrow \Sigma F = 0 + mg = \frac{m v_B^2}{r}$

$v_B^2 = gr = (9.81\,m/s^2)(0.5\,m)$

$v_B^2 = 4.905\,m^2/s^2$

AT A

$T_A = \frac{1}{2} m v_0^2 \quad V_A = 0$

AT B

$T_B = \frac{1}{2} m v_B^2 = \frac{1}{2} m\,(4.905) = 2.453\,m$

$V_B = mg\,(2.5 + 0.5) = 3mg$

$T_A + V_A = T_B + V_B$

$\frac{1}{2} m v_0^2 + 0 = 2.453m + 3mg$

$v_0^2 = 2[(2.453) + 3(9.81)] = 63.77$

$v_0 = 7.99\,m/s$

AT C

$T_c = \frac{1}{2} m v_c^2 \quad V_c = mg\,(2.5\,m)$

$T_A + V_A = T_c + V_c$

$\frac{1}{2} m v_0^2 + 0 = \frac{1}{2} m v_c^2 + 2.5mg$

$v_c^2 = [63.77 - (5.0)(9.81)]$

$v_c^2 = 14.72\,m^2/s^2$

(b)

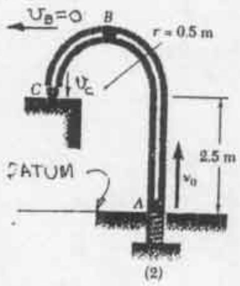
N_c, mg $\quad = \quad$ $\frac{m v_c^2}{r}$, ma

$\overset{+}{\rightarrow} \Sigma F = N_c = \frac{m v_c^2}{r} = \frac{(0.2\,kg)(14.72\,m^2/s^2)}{(0.5\,m)}$

(PACKAGE ON TUBE) $N_c = 5.89\,N \leftarrow$

13.74 continued

(2) (a)



THE VELOCITY AT B CAN BE NEARLY EQUAL TO ZERO SINCE THE WEIGHT OF THE PACKAGE IS SUPPORTED BY THE TUBE.

THUS, $v_B=0 \quad T_B=0$

$V_B=mg(2.5m+0.5m)$

$V_B=3mg$

$T_A=\frac{1}{2}mv_0^2 \quad V_A=0$

$T_B+V_B=T_A+V_A \qquad 0+3mg=\frac{1}{2}mv_0^2+0$

$v_0^2=6g$

$v_0=7.67\frac{m}{s}$

(b)

$T_C=\frac{1}{2}mv_C^2 \quad V_C=mg(2.5m)$

$T_A+V_A=T_C+V_C \qquad \frac{1}{2}mv_0^2+0=\frac{1}{2}mv_C^2+2.5mg$

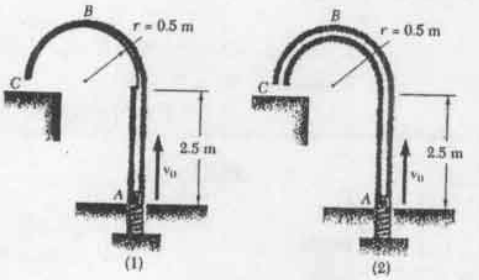
$v_C^2=6g-5g=9.81\,m^2/s^2$

N_C, mg $=$ $\frac{mv_C^2}{r}$, ma $\quad \xrightarrow{+}\Sigma F=N_C=mv_C^2/r$

$N_C=(0.2kg)(9.81m/s^2)/(0.5m)$

PACKAGE ON TUBE, $N_C=3.92N$

13.75



GIVEN:

VELOCITY AT C, $<3.5\,m/s$ (REQUIRED)

FIND:

(a) LOOP (2) BUT NOT LOOP (1) CAN SATISFY REQUIREMENT THAT $v_C<3.5m/s$

(b) LARGEST ALLOWABLE VELOCITY v_0 WHEN LOOP (2) IS USED AND $v_C<3.5m/s$.

(a) LOOP (1). THE SMALLEST ALLOWABLE VELOCITY AT B WILL OCCUR WHEN THE FORCE EXERTED BY THE TUBE ON THE PACKAGE IS ZERO

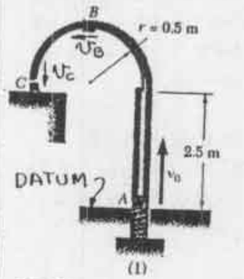
B $N=0$, $mg=0.2g$ $=$ $\frac{mv_B^2}{r}$

$+\downarrow\Sigma F=0+mg=mv_B^2/r$

$v_B^2=gr=(9.81m/s^2)(0.5m)=4.905\,m^2/s^2$

$v_B=2.215\,m/s$

13.75 continued



THE VELOCITY AT B CANNOT B LESS THAN 2.215m/s IF THE PACKAGE IS TO MAINTAIN CONTACT WITH THE TUBE

FOR v_C TO BE AS SMALL AS POSSIBLE, V_B MUST BE AS SMALL AS POSSIBLE; THAT IS $v_B=2.215m/s$

AT B

$T_B=\frac{1}{2}mv_B^2=\frac{1}{2}m(2.215)^2$

$T_B=2.453\,m$

$V_B=mg(2.5+0.5)=3mg$

ATC

$T_C=\frac{1}{2}mv_C^2$

$V_C=2.5mg$

$T_B+V_B=T_C+V_C$

$2.453m+3mg=\frac{1}{2}mv_C^2+2.5mg$

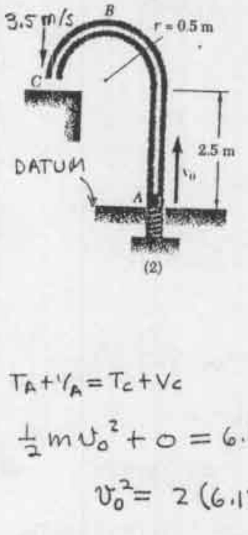
$v_C^2=2[2.453+0.5(9.81m/s^2)]$

$v_C^2=14.72\,m^2/s^2$

$v_C=3.836\,m/s>3.5m/s$

THUS, LOOP (1) CANNOT MEET THE REQUIREMENT

(b) LOOP (2)



AT A

$T_A=\frac{1}{2}mv_0^2$

$V_A=0$

AT C

$v_C=3.5\,m/s$

$T_C=\frac{1}{2}m(3.5)^2$

$T_C=6.125m$

$V_C=2.5mg$

$T_A+V_A=T_C+V_C$

$\frac{1}{2}mv_0^2+0=6.125m+2.5mg$

$v_0^2=2(6.125+2.5g)=61.3\,m^2/s^2$

$v_0=7.83\,m/s$

NOTE:

A LARGER VELOCITY AT A WOULD RESULT IN A VELOCITY AT C, GREATER THAN 3.5 m/s

13.76

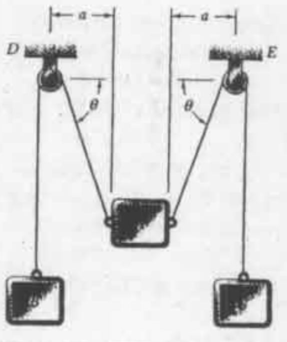
GIVEN:

ALL PACKAGES HAVE THE SAME WEIGHT AND $v=0$ WHEN $\theta=0$.

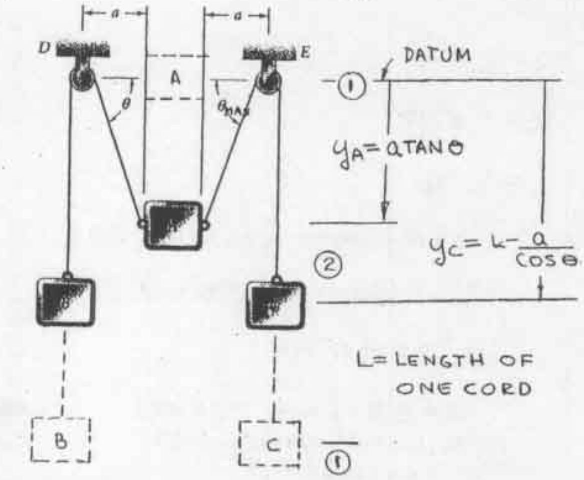
FIND:

(a) MAXIMUM θ

(b) TENSION IN THE CORD, F, WHEN $\theta=\theta_{MAX}$



(a) θ_{MAX} WHEN $v=0$



(a)

POSITION ① $\theta=0$

$v_A=v_B=v_C=0 \quad T_1=0$

$V_1=-2W(L-a)$

POSITION ② $v_A=v_B=v_C=0 \quad T_2=0$

$\theta=\theta_{MAX}$

$V_2=-W_A\,a\,\mathrm{TAN}\,\theta_{MAX}-(W_B+W_C)\left(L-\dfrac{a}{\mathrm{COS}\,\theta_{MAX}}\right)$

$V_2=-W\left(a\,\mathrm{TAN}\,\theta_{MAX}+2L-\dfrac{2a}{\mathrm{COS}\,\theta_{MAX}}\right)$

$T_1+V_1=T_2+V_2$

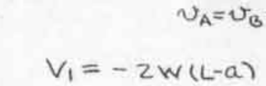
$0-2W(\cancel{L}-a)=0-W\left(a\,\mathrm{TAN}\,\theta_{MAX}+2\cancel{L}-\dfrac{2a}{\mathrm{COS}\,\theta_{MAX}}\right)$

$-2\,\mathrm{COS}\,\theta_{MAX}=\mathrm{SIN}\,\theta_{MAX}-2$

$\mathrm{SIN}\,\theta_{MAX}+2\,\mathrm{COS}\,\theta_{MAX}=2$

BY TRIAL $\qquad \theta_{MAX}=53.1°$ ◀

(b) AT $\theta=53.1°$

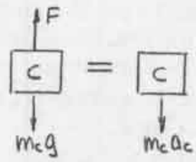


$+\uparrow \Sigma F=2F\,\mathrm{SIN}\,\theta-m_A g=m_A a_A$

$2F\,\mathrm{SIN}\,53.1°-W_A=\dfrac{W_A}{g}a_A$

$1.600F-W_A=\dfrac{W_A}{g}a_A \qquad (1)$

13.76 continued



$+\uparrow \Sigma F=F-m_C g=-m_C a_C$

$F-W_C=-\dfrac{W_C}{g}a_C \qquad (2)$

KINEMATICS $\qquad a_A=\ddot{y}_A \quad a_C=\ddot{y}_C$

$y_A=a\,\mathrm{TAN}\,\theta \qquad y_C=L-\dfrac{a}{\mathrm{COS}\,\theta}$

$\dot{y}_A=-a\,\mathrm{SEC}^2\theta\,\dot{\theta}$

LET $f_A(\theta)=-a\,\mathrm{SEC}^2\theta \qquad \dot{y}_C=\dfrac{a\,\mathrm{TAN}\,\theta}{\mathrm{COS}\,\theta}\dot{\theta}$

$\ddot{y}_A=f_A'(\theta)\dot{\theta}+f_A(\theta)\ddot{\theta} \qquad$ LET $f_C=\dfrac{a\,\mathrm{TAN}\,\theta}{\mathrm{COS}\,\theta}$

$\ddot{y}_C=f_C'\dot{\theta}+f_C(\theta)\ddot{\theta}$

AT θ_{MAX}, $\dot{\theta}=0$, $\theta_{MAX}=53.1°$

THUS $\dfrac{\ddot{y}_A}{\ddot{y}_C}=\dfrac{f_A(53.1°)}{f_C(53.1°)}=\dfrac{-a\,\mathrm{SEC}^2(53.1°)}{a\,\mathrm{TAN}(53.1°)/\mathrm{COS}\,53.1°}$

$\dfrac{a_A}{a_C}=1.250 \qquad a_A=1.250a_C \qquad (3)$

REPLACE a_A IN (1) BY $1.250a_C$ FROM (3)

$W_A=W_B=W_C=W$

(1) $\quad 1.600F-W=\dfrac{W}{g}(1.250a_C)$

(2) $\quad F-W=-\dfrac{W}{g}a_C$

$1.600F-W=-1.250(F-W)$

$2.850F=2.250W$

$F=0.789W$ ◀

13.77

GIVEN:

$W_A=2\,\mathrm{lb}$

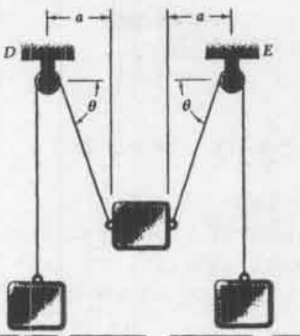
$W_B=W_C=3\,\mathrm{lb}$

$v=0$, WHEN $\theta=0$

FIND:

(a) MAXIMUM θ

(b) TENSION F AT θ_{MAX}



REFER TO FIGURE IN P13.76 (a) AT LEFT

(a) $\theta=0 \quad T_1=0 \quad V_1=-(W_B+W_C)(L-a)=-6(L-a)$

$\theta=\theta_{MAX} \quad T_2=0 \quad V_2=-2a\,\mathrm{TAN}\,\theta_{MAX}-6\left(L-\dfrac{a}{\mathrm{COS}\,\theta_{MAX}}\right)$

$T_1+V_1=T_2+V_2$

$0-6(\cancel{L}-a)=0-2a\,\mathrm{TAN}\,\theta_{MAX}-6\left(\cancel{L}-\dfrac{a}{\mathrm{COS}\,\theta_{MAX}}\right)$

$-6\,\mathrm{COS}\,\theta_{MAX}=2\,\mathrm{SIN}\,\theta_{MAX}-6$

BY TRIAL $\qquad \theta_{MAX}=36.9°$ ◀

(b) REFER TO (b) PROB 13.76

$2F\,\mathrm{SIN}\,36.9°-W_A=\dfrac{W_A}{g}a_A \qquad 1.201F-2=\dfrac{2}{g}a_A \qquad (1)$

$F-W_C=-\dfrac{W_C}{g}a_C \qquad F-3=-\dfrac{3}{g}a_C \qquad (2)$

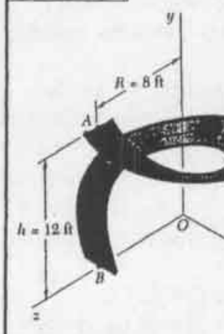
KINEMATICS $\dfrac{a_A}{a_C}=\dfrac{\ddot{y}_A}{\ddot{y}_C}=\dfrac{\mathrm{SEC}^2 36.9}{\mathrm{TAN}\,36.9/\mathrm{COS}\,36.9}=1.665 \qquad (3)$

SOLVE (1), (2) AND (3) FOR F

$F=2.31\,\mathrm{lb}$ ◀

***13.78**

GIVEN:



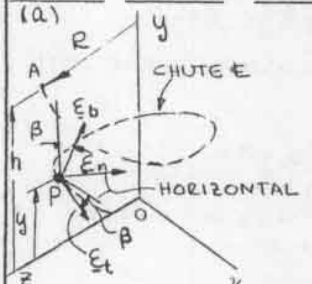
PACKAGES RELEASED FROM REST AT A CHUTE IS BANKED SO THAT PACKAGES DO NOT TOUCH ITS EDGES. NO FRICTION PACKAGE WEIGHT, $W = 20\text{ lb}$. CHUTE IS A HELIX WITH PRINCIPAL NORMAL HORIZONTAL AND DIRECTED TOWARD y AXIS.

FIND:

(a) ANGLE ϕ FORMED BY THE NORMAL TO THE SURFACE OF THE CHUTE AN THE PRINCIPAL NORMAL

(b) MAGNITUDE AN DIRECTION OF THE CHUTE ON THE PACKAGE AT B

(a)



AT POINT A

$v_A = 0 \quad T_A = 0$

$V_A = mgh$

AT ANY POINT P

$T_P = \frac{1}{2} m v^2$

$V_P = Wy = mgy$

e_n, ALONG PRINCIPAL NORMAL, HORIZONTAL AND DIRECTED TOWARD y AXIS

e_t, TANGENT TO CENTERLINE OF THE CHUTE

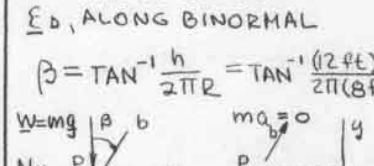
e_b, ALONG BINORMAL

$T_A + V_A = T_P + V_P$

$0 + mgh = \frac{1}{2} m v^2 + mgy$

$v^2 = 2g(h-y)$

$$\beta = \tan^{-1}\frac{h}{2\pi R} = \tan^{-1}\frac{(12\text{ ft})}{2\pi(8\text{ ft})} \qquad \beta = 13.427°$$



$ma_b = 0$
SINCE $a_b = 0$

NOTE: FRICTION IS ZERO

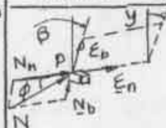
$$\Sigma F_t = ma_t \quad mg\sin\beta = ma_t \quad a_t = g\sin\beta$$

$$\Sigma F_b = ma_b \quad N_b - W\cos\beta = 0 \quad N_B = W\cos\beta$$

$$\Sigma F_n = ma_n \quad N_n = \frac{mv^2}{\rho} = \frac{m2g(h-y)}{\rho} = 2W\frac{(h-y)}{\rho}$$

THE TOTAL NORMAL FORCE IS THE RESULTANT OF N_b AND N_m, LIES IN THE b-m PLANE AND FORMS ANGLE ϕ WITH m AXIS.

***13.78 continued**



$$\tan\phi = N_b/N_n$$

$$\tan\phi = W\cos\beta \Big/ \frac{2(W(h-y)}{\rho}$$

$$\tan\phi = (\rho/2(h-y)\cos\beta$$

GIVEN: $\rho = R\left[1 + \left(\frac{h}{2\pi R}\right)^2\right] = R(1+\tan^2\beta) = \frac{R}{\cos^2\beta}$

THUS:

$$\tan\phi = \frac{\rho}{2(h-y)}\cos\beta = \frac{R}{2(h-y)\cos\beta}$$

$$\tan\phi = \frac{8\text{ ft}}{2(12-y)\cos 14.327°} = \frac{4.113}{12-y}$$

$$\text{OR } \cot\phi = 0.243(12-y)$$

(b) At POINT B $\quad y = 0 \quad$ FOR x, y, z AXES WE WRITE, WITH $W = 20\text{ lb}$

$$N_x = N_b\sin\beta = W\cos\beta\sin\beta = (20\text{ lb})\cos 14.327°\sin 14.327°$$

$$N_x = 4.517\text{ lb}$$

$$N_y = N_b\cos\beta = W\cos^2\beta = (20\text{ lb})\cos^2 14.327°$$

$$N_y = 18.922\text{ lb}$$

$$N_z = -N_n = -2W\frac{h-y}{\rho} = -2W\frac{h-y}{R/\cos^2\beta}$$

$$N_z = 2(20\text{ lb})\frac{(12\text{ ft}-0)}{8\text{ ft}}\cos^2 14.327° \qquad N_z = -56.765\text{ lb}$$

$$N = \sqrt{(4.517)^2 + (18.922)^2 + (-56.765)^2} \qquad N = 60.0\text{ lb}$$

$$\cos\theta_x = \frac{N_x}{N} = \frac{4.517}{60} \qquad \theta_x = 85.7°$$

$$\cos\theta_y = \frac{N_y}{N} = \frac{18.922}{60} \qquad \theta_y = 71.6°$$

$$\cos\theta_z = \frac{N_z}{N} = -\frac{56.742}{60} \qquad \theta_z = 161.1°$$

***13.79**

GIVEN:

$\mathbf{F}(x,y,z)$ IS CONSERVATIVE

SHOW THAT:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x} \qquad \frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \qquad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$$

FOR A CONSERVATIVE FORCE, EQ (13.22) MUST BE SATISFIED

$$F_x = -\frac{\partial V}{\partial x} \quad F_y = -\frac{\partial V}{\partial y} \quad F_z = -\frac{\partial V}{\partial z}$$

WE NOW WRITE $\frac{\partial F_x}{\partial y} = -\frac{\partial^2 V}{\partial x\partial y} \qquad \frac{\partial F_y}{\partial x} = -\frac{\partial^2 V}{\partial y\partial x}$

SINCE $\frac{\partial^2 V}{\partial x\partial y} = \frac{\partial^2 V}{\partial y\partial x}$:

$$\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}$$

WE OBTAIN IN A SIMILAR WAY

$$\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y} \qquad \frac{\partial F_z}{\partial x} = \frac{\partial F_x}{\partial z}$$

***13.80** GIVEN:

$\underline{F} = (yz\underline{i} + zx\underline{j} + xy\underline{k})/xyz$

SHOW:

(a) $\underline{F}$ IS A CONSERVATIVE FORCE

FIND:

(b) THE POTENTIAL FUNCTION ASSOCIATED WITH $\underline{F}$

(a) $F_x = yz/xyz \quad F_y = zx/xyz$

$$\frac{\partial F_x}{\partial y} = \frac{\partial(1/x)}{\partial y} = 0 \quad \frac{\partial F_y}{\partial x} = \frac{\partial(1/y)}{\partial x} = 0$$

THUS $\dfrac{\partial F_x}{\partial y} = \dfrac{\partial F_y}{\partial x}$

THE OTHER TWO EQUATIONS DERIVED IN PROB. 13.80 ARE CHECKED IN A SIMILAR WAY

(b) RECALL THAT $F_x = -\frac{\partial V}{\partial x}, F_y = -\frac{\partial V}{\partial y}, F_z = -\frac{\partial V}{\partial z}$

$$F_x = \frac{1}{x} = -\frac{\partial V}{\partial x} \quad V = -\ln x + f(y,z) \quad (1)$$

$$F_y = \frac{1}{y} = -\frac{\partial V}{\partial y} \quad V = -\ln y + g(z,x) \quad (2)$$

$$F_z = \frac{1}{z} = -\frac{\partial V}{\partial z} \quad V = -\ln z + h(x,y) \quad (3)$$

EQUATING (1) AND (2)

$$-\ln x + f(y,z) = -\ln y + g(z,x)$$

THUS $f(y,z) = -\ln y + k(z) \quad (4)$

$g(z,x) = -\ln x + k(z) \quad (5)$

EQUATING (2) AND (3)

$$-\ln z + h(x,y) = -\ln y + g(z,x)$$

$$g(z,x) = -\ln z + l(x)$$

FROM (5) $g(z,x) = -\ln x + k(z)$

THUS

$$k(z) = -\ln z$$

$$l(x) = -\ln x$$

FROM (4)

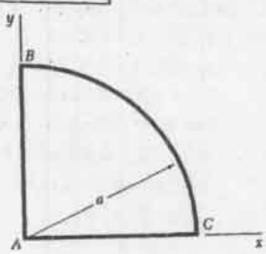
$$f(y,z) = -\ln y - \ln z$$

SUBSTITUTE FOR $f(y,z)$ IN (1)

$$V = -\ln x - \ln y - \ln z$$

$$V = -\ln xyz + C$$

***13.81**



GIVEN:

PARTICLE $P(x,y)$ ACTED UPON BY FORCE $\underline{F}$

FIND:

WHETHER $\underline{F}$ IS A CONSERVATIVE FORCE, AND COMPUTE THE WORK OF F WHEN $P(x,y)$ DESCRIBES A PATH ABCA, CLOCKWISE FOR,

(a) $\underline{F} = ky\underline{i}$

(b) $\underline{F} = k(y\underline{i} + x\underline{j})$

(a) $F_x = ky \quad F_y = 0 \quad \dfrac{\partial F_x}{\partial y} = k \quad \dfrac{\partial F_y}{\partial x} = 0$

THUS $\dfrac{\partial F_x}{\partial y} \neq \dfrac{\partial F_y}{\partial x}$ $\underline{F}$ IS NOT CONSERVATIVE

$$U_{ABCA} = \int_{ABCA} \underline{F} \cdot d\underline{r} = \int_A^B ky\underline{i} \cdot dy\underline{j} + \int_B^C ky\underline{i} \cdot (dx\underline{i} + dy\underline{j}) + \int_C^A ky\underline{i} \cdot dx\underline{j}$$

$\int_A^B = 0$, $\underline{F}$ IS PERPENDICULAR TO THE PATH

$$\int_B^C ky\underline{i} \cdot (dx\underline{i} + dy\underline{j}) = \int_B^C ky\,dx$$

FROM B TO C THE PATH IS A QUARTER CIRCLE WITH ORIGIN AT A.

THUS $x^2 + y^2 = a^2$

$$y = \sqrt{a^2 - x^2}$$

ALONG BC $\displaystyle\int_B^C ky\,dx = \int_0^a k\sqrt{a^2 - x^2}\,dx$

$$= \frac{\pi k a^2}{4}$$

$$\int_C^A ky\underline{i} \cdot dx\underline{j} = 0 \quad (y = 0 \text{ ON } CA)$$

$$U_{ABCA} = \int_A^B + \int_B^C + \int_C^A = 0 + \frac{\pi k a^2}{4} + 0$$

$$U_{ABCA} = \frac{\pi k a^2}{4}$$

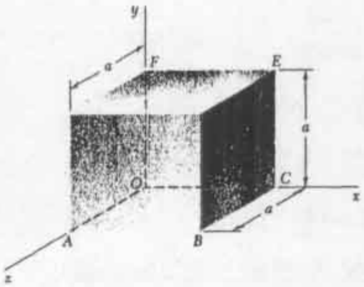
(b) $F_x = ky \quad F_y = kx \quad \dfrac{\partial F_x}{\partial y} = k, \dfrac{\partial F_y}{\partial x} = k$

$\dfrac{\partial F_x}{\partial y} = \dfrac{\partial F_y}{\partial x}$, $\underline{F}$ IS CONSERVATIVE

SINCE ABCA IS A CLOSED LOOP AND $\underline{F}$ IS CONSERVATIVE,

$$U_{ABCA} = 0$$

* 13.82



GIVEN:
POTENTIAL FUNCTION $V(x,y,z) = -(x^2+y^2+z^2)^{1/2}$ ASSOCIATED WITH FORCE $\underline{P}$.

FIND:
(a) x, y, z COMPONENTS OF $\underline{P}$
(b) WORK DONE BY $\underline{P}$ FROM O TO D BY INTEGRATING ALONG THE PATH OABD, U_{OABD}. SHOW THAT $U_{OABD} = -\Delta V_{OD}$

(a) $$P_x = -\frac{\partial V}{\partial x} = -\frac{\partial - (x^2+y^2+z^2)^{1/2}}{\partial x} = x(x^2+y^2+z^2)^{-1/2}$$

$$P_y = -\frac{\partial V}{\partial y} = -\frac{\partial - (x^2+y^2+z^2)^{1/2}}{\partial y} = y(x^2+y^2+z^2)^{-1/2}$$

$$P_z = -\frac{\partial V}{\partial z} = -\frac{\partial - (x^2+y^2+z^2)^{1/2}}{\partial z} = z(x^2+y^2+z^2)^{-1/2}$$

(b) $U_{OABD} = U_{OA} + U_{AB} + U_{BD}$

O-A P_y AND P_x ARE PERPENDICULAR TO O-A AND DO NO WORK
ALSO, ON O-A $x=y=0$ AND $P_z = 1$

THUS $U_{O-A} = \int_0^a P_z\,dz = \int_0^a dz = a$

A-B P_z AND P_y ARE PERPENDICULAR TO A-B AND DO NO WORK
ALSO ON A-B $y=0$, $z=a$ AND $P_x = x/(x^2+a^2)^{1/2}$

THUS $U_{A-B} = \int_0^a \frac{x\,dx}{(x^2+a^2)^{1/2}} = a(\sqrt{2}-1)$

B-D P_x AND P_z ARE PERPENDICULAR TO B-D AND DO NO WORK
ON B-D $x=a$, $z=a$ $P_y = y/(y^2+2a^2)^{1/2}$

THUS $U_{BD} = \int_0^a \frac{y}{(y^2+2a^2)^{1/2}}dy = (y^2+2a^2)^{1/2}\Big|_0^a$

$$U_{BD} = (a^2+2a^2)^{1/2} - (2a^2)^{1/2} = a(\sqrt{3}-\sqrt{2})$$

$U_{OABD} = U_{O-A} + U_{AB} + U_{BD} = a + a(\sqrt{2}-1) + a(\sqrt{3}-\sqrt{2})$

$$U_{OABD} = a\sqrt{3}$$

$\Delta V_{OD} = V(a,a,a) - V(0,0,0) = -(a^2+a^2+a^2)^{1/2} - 0$

$$\Delta V_{OD} = -a\sqrt{3}$$

THUS $U_{OABD} = -\Delta V_{OD}$

* 13.83

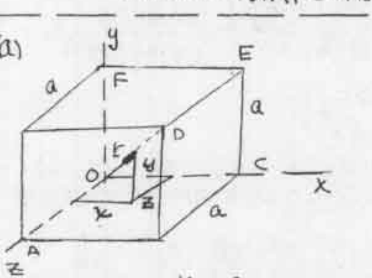
REFER TO FIG. P13.82 ON THE LEFT

GIVEN:
FROM SOLUTION TO (a) OF PROB. 13.82
$$\underline{P} = \frac{x\underline{i} + y\underline{j} + z\underline{k}}{(x^2+y^2+z^2)^{1/2}}$$

FIND:
(a) WORK DONE BY $\underline{P}$ ALONG THE DIAGONAL OD

VERIFY:
(b) THAT WORK DONE AROUND THE CLOSED PATH OABDO IS ZERO.

(a)



$$U_{OD} = \int_0^D \underline{P}\cdot d\underline{r}$$

$$\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$$

$$d\underline{r} = dx\,\underline{i} + dy\,\underline{j} + dz\,\underline{k}$$

$$\underline{P} = \frac{x\underline{i} + y\underline{j} + z\underline{k}}{(x^2+y^2+z^2)^{1/2}}$$

ALONG THE DIAGONAL $x = y = z$

THUS $\underline{P}\cdot d\underline{r} = \frac{3x}{(3x^2)^{1/2}} = \sqrt{3}$

$$U_{O\text{-}D} = \int_0^a \sqrt{3}\,dx = \sqrt{3}\,a$$

$$U_{OD} = \sqrt{3}\,a$$

(b)
$U_{OABDO} = U_{OABD} + U_{DO}$

FROM PROB 13.82

$U_{OABD} = \sqrt{3}\,a$ AT LEFT

THE WORK DONE FROM D TO O ALONG THE DIAGONAL IS THE NEGATIVE OF THE WORK DONE FROM O TO D

$U_{DO} = -U_{OD} = -\sqrt{3}\,a$ (PART (a))

THUS
$$U_{OABDO} = \sqrt{3}\,a - \sqrt{3}\,a = 0$$

*13.84

GIVEN:

$$\underline{F} = (x\underline{i}+y\underline{j}+z\underline{k})/(x^2+y^2+z^2)^{3/2}$$

PROVE:
(a) $\underline{F}$ IS CONSERVATIVE

FIND:
(b) THE POTENTIAL FUNCTION $V(x,y,z)$ ASSOCIATED WITH $\underline{F}$

(a) $F_x = x/(x^2+y^2+z^2)^{3/2}$ $\quad F_y = y/(x^2+y^2+z^2)^{3/2}$

$$\frac{\partial F_x}{\partial y} = \frac{x(-\frac{3}{2})(2y)}{(x^2+y^2+z^2)^{\frac{5}{2}}} \qquad \frac{\partial F_y}{\partial x} = \frac{y(-\frac{3}{2})2y}{(x^2+y^2+z^2)^{\frac{5}{2}}}$$

THUS $\dfrac{\partial F_x}{\partial y} = \dfrac{\partial F_y}{\partial x}$

THE OTHER TWO EQUATIONS DERIVED IN PROB. 13.79 ARE CHECKED IN A SIMILAR FASHION

(b) RECALLING THAT $F_x = -\frac{\partial V}{\partial x}$, $F_y = -\frac{\partial V}{\partial y}$, $F_z = -\frac{\partial V}{\partial z}$

$F_x = -\frac{\partial V}{\partial x}$ $\qquad V = -\int x/(x^2+y^2+z^2)^{3/2}\,dx$

$$V = (x^2+y^2+z^2)^{-\frac{1}{2}} + f(y,z)$$

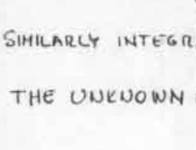
SIMILARLY INTEGRATING $\frac{\partial V}{\partial y}$ AND $\frac{\partial V}{\partial z}$ SHOWS THAT THE UNKNOWN FUNCTION $f(x,y)$ IS A CONSTANT

$$V = \frac{1}{(x^2+y^2+z^2)^{1/2}}$$

13.85

GIVEN:
3600-kg LAUNCHED FROM A CIRCULAR ORBIT AT 300 km ABOVE THE EARTH.
ALTITUDE OF GEOSYNCHRONOUS (CIRCULAR) ORBIT = 35770 km

FIND:
(a) ENERGY NEEDED TO PLACE THE SATELLITE INTO GEOSYNCHRONOUS ORBIT FROM 300 km
(b) ENERGY NEEDED TO PLACE THE SATELLITE INTO A GEOSYNCHRONOUS ORBIT FROM THE EARTH (EXCLUDE AIR RESISTANCE



GEOSYNCHRONOUS ORBIT

$r_2 = 6370\text{ km} + 35770\text{ km} = 42.140\times10^6\text{ m}$

ORBIT AT 300 km

$r_1 = 6370\text{ km} + 300\text{ km} = 6.67\times10^6\text{ m}$

$R_E = 6370\text{ km}$

FOR ANY CIRCULAR ORBIT OF RADIUS r THE TOTAL ENERGY $E = T+V = \frac{1}{2}mv^2 - \frac{GMm}{r}$

M = MASS OF THE EARTH

m = 3600 kg = SATELLITE MASS

13.85 continued

NEWTON'S SECOND LAW

$F = ma_n$: $\quad \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad v^2 = \frac{GM}{r}$

$T = \frac{1}{2}mv^2 = m\frac{GM}{2r} \qquad V = -\frac{GMm}{r}$

$E = T+V = \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} = -\frac{1}{2}\frac{GMm}{r}$

$GM = gR_E^2 \qquad E = -\frac{1}{2}\frac{gR_E^2 m}{r}$

$$E = -\frac{1}{2}\frac{(9.81\text{ m/s}^2)(6370\times10^3\text{ m})^2(3600\text{ kg})}{r}$$

$$E = -\frac{716.15\times10^{15}}{r}\text{ (N·m)}$$

FOR A GEOSYNCHRONOUS ORBIT ($r_2 = 42.140\times10^6$ m)

$$E_{GS} = \frac{-716\times10^{15}}{42.140\times10^6} = -17.003\times10^9\text{ J} = -17.003\text{ GJ}$$

(a) AT 300 km ($r_1 = 6.67\times10^6$ m)

$$E_{300} = -\frac{716\times10^{15}}{6.67\times10^6} = -107.42\times10^9\text{ J} = -107.42\text{ GJ}$$

ADDITIONAL ENERGY $\Delta E_{300} = E_{GS} - E_{300}$

$$\Delta E_{300} = -17.003 + 107.42$$

$$\Delta E_{300} = 90.4\text{ GJ}$$

(b) LAUNCH FROM THE EARTH ($R_E = 6370$ km)

AT LAUNCH PAD $E_E = V = -\frac{GMm}{R_E} = -\frac{gR_E^2 m}{R_E}$

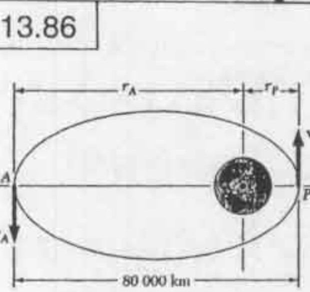
$$E_E = -(9.81\text{ m/s}^2)(6370\times10^3\text{ m})(3600\text{ kg})$$

$$E_E = -224.96\times10^9\text{ J} = -224.96\text{ GJ}$$

ADDITIONAL ENERGY $\Delta E_E = E_{GS} - E_E$

$$\Delta E_E = -17.003 + 224.96 = 208\text{ GJ}$$

13.86

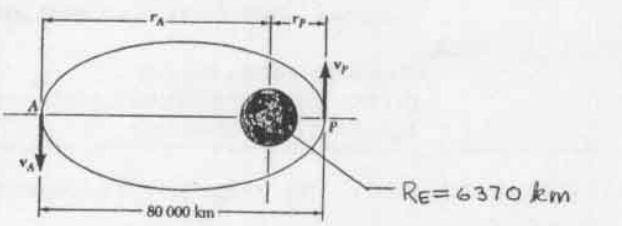


GIVEN:
$v_A/v_P = r_P/r_A$
$r_A + r_P = 80000$ km
ELLIPTIC ORBIT

FIND:
ENERGY PER UNIT MASS E/m REQUIRED TO PLACE THE SATELLITE IN ORBIT

DETERMINE THE TOTAL ENERGY PER UNIT MASS FOR THE ELLIPTIC ORBIT AND SUBTRACT FROM IT THE ENERGY PER UNIT MASS ON THE EARTH TO GET THE ENERGY PER UNIT MASS NEEDED FOR PROPULSION. (EXCLUDING AIR RESISTANCE, THE WEIGHT OF THE BOOSTER ROCKET AND MANEUVERING.)

13.86 continued



$R_E = 6370$ km

TOTAL ENERGY PER UNIT MASS FOR THE ORBIT

$E_0 = T_A + V_A = T_P + V_P$

$$E_0/m = \frac{v_A^2}{2} - \frac{GM}{r_A} = \frac{v_P^2}{2} - \frac{GM}{r_P} \qquad (1)$$

$$v_A^2\left(1 - \frac{v_P^2}{v_A^2}\right) = 2GM\left(\frac{1}{r_A} - \frac{1}{r_P}\right)$$

$v_A/v_P = r_P/r_A$ (GIVEN)

$$v_A^2\left(1 - \frac{r_A^2}{r_P^2}\right) = 2GM\left(\frac{r_P - r_A}{r_A r_P}\right)$$

$$v_A^2 \frac{(r_P - r_A)(r_P + r_A)}{r_P^2} = 2GM\frac{(r_P - r_A)}{r_A r_P}$$

$$v_A^2 = 2GM\frac{r_P}{r_A}\left(\frac{1}{r_P + r_A}\right) \qquad (2)$$

SUBSTITUTING v_A IN (2) IN (1)

$$E_0/m = GM\frac{r_P}{r_A}\left(\frac{1}{r_P + r_A}\right) - \frac{GM}{r_A}$$

$$E_0/m = GM\,\frac{1}{r_A}\left[\frac{r_P - (r_P + r_A)}{r_P + r_A}\right] = -\frac{GM}{r_P + r_A}$$

$GM = gR_E^2 = (9.81\ \text{m/s}^2)(6370 \times 10^3\ \text{m})$

$r_P + r_A = 80.000 \times 10^3$ m (GIVEN)

$$E_0/m = \frac{-(9.81\ \text{m/s}^2)(6370 \times 10^3\ \text{m})^2}{80000 \times 10^3\ \text{m}}$$

$$E_0/m = 4.9765 \times 10^6\ \frac{\text{N-m}}{\text{kg}} = -4.9765\ \frac{\text{MJ}}{\text{kg}}$$

TOTAL ENERGY PER UNIT MASS ON THE EARTH

$E_E = T_E + V_E \quad v_E = 0 \quad T_E = 0 \quad V_E = -\frac{mGM}{R_E}$

$E_E/m = -\frac{gR_E^2}{R_E} = -(9.81\ \text{m/s}^2)(6370 \times 10^3\ \text{m})$

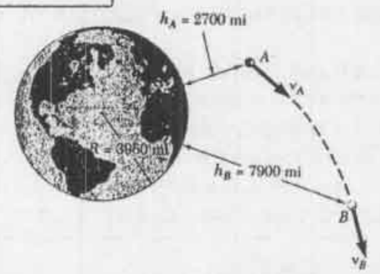
$E_E/m = -62.490 \times 10^6\ \frac{\text{N-m}}{\text{kg}} = -62.49$ MJ/kg

ENERGY PER UNIT MASS NEEDED FOR PROPULSION, $E_P/m = E_0/m - E_E/m$

$E_P/m = -4.9765$ MJ/kg $+ 62.490$ MJ/kg

$$E_P/m = 57.5\ \frac{\text{MJ}}{\text{kg}}$$

13.87



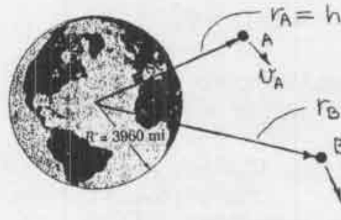
GIVEN:

h_A AND h_B AS SHOWN

$v_A = 20.2 \times 10^3\ \frac{\text{mi}}{\text{h}}$

FIND:

v_B



$r_A = h_A + R = 2700\ \text{mi} + 3960\ \text{mi}$

$r_A = 6660$ mi

$r_B = h_B + R = 7900\ \text{mi} + 3960\ \text{mi}$

$r_B = 11860$ mi

AT A, $v_A = 20.2 \times 10^3\ \frac{\text{mi}}{\text{h}} = 29627$ ft/s

$T_A = \frac{1}{2}m(29627\ \text{ft})^2 = 438.87 \times 10^6\, m$

$V_A = -\frac{GMm}{r_A} = -\frac{gR^2 m}{r_A}$

$r_A = 6660\ \text{mi} = 35.165 \times 10^6$ ft

$R = 3960\ \text{mi} = 20.909 \times 10^6$ ft

$$V_A = -\frac{(32.2\ \text{ft/s}^2)(20.909 \times 10^6\ \text{ft})^2}{(35.165 \times 10^6\ \text{ft})} m = -400.3 \times 10^6\, m$$

AT B

$T_B = \frac{1}{2}mv_B^2$

$V_B = -\frac{GMm}{r_B} = \frac{-gR^2 m}{r_B}$

$r_B = 11860\ \text{mi} = 62.621 \times 10^6$ ft

$$V_B = -\frac{(32.2\ \text{ft/s}^2)(20.909 \times 10^6)^2}{(62.621 \times 10^6\ \text{ft})} m$$

$V_B = -224.8 \times 10^6\, m$

$T_A + V_A = T_B + V_B$

$438.87 \times 10^6 m - 400.3 \times 10^6 m = \frac{1}{2}mv_B^2 - 224.8 \times 10^6 m$

$v_B^2 = 2[438.87 \times 10^6 - 400.3 \times 10^6 + 224.8 \times 10^6]$

$v_B^2 = 526.75 \times 10^6\ \text{ft}^2/\text{s}^2$

$v_B = 22.951 \times 10^3\ \text{ft/s} = 15.65 \times 10^3$ mi/h

$$v_B = 15.65 \times 10^3\ \frac{\text{mi}}{\text{h}}$$

13.88

GIVEN:

LUNAR EXCURSION MODULE (LEM)

FIND:

ENERGY PER POUND NEEDED TO ESCAPE MOON'S GRAVITATIONAL FIELD STARTING FROM
(a) MOON'S SURFACE
(b) CIRCULAR ORBIT 50 mi. ABOVE THE MOON'S SURFACE

NOTE: $GM_{MOON} = 0.0123\, GM_{EARTH}$

BY EQ. 12.30 $\quad GM_{MOON} = 0.0123\, g R_E^2$

AT ∞ DISTANCE FROM MOON: $r_2=\infty$, ASSUME $v_2=0$

$$E_2 = T_2+V_2 = 0 - \frac{GM_M m}{\infty} = 0-0=0$$

(a) ON SURFACE OF MOON $\quad R_M = 1081\text{ mi} = 5.7077\times10^6\text{ ft}$

$v_1=0 \quad T_1=0 \qquad R_E = 3960\text{ mi} = 20.909\times10^6\text{ ft}$

$V_1 = -\frac{GM_M m}{R_M} \qquad E_1 = T_1+V_1 = 0 - \frac{0.0123\, g R_E^2 m}{R_M}$

$$E_1 = -\frac{(0.0123)(32.2\text{ ft/s}^2)(20.909\times10^6\text{ ft})^2}{(5.7077\times10^6\text{ ft})} m$$

W_E = WEIGHT OF LEM ON THE EARTH

$$E_1 = \left(-30.336\times10^6 \frac{\text{ft}^2}{\text{s}^2}\right) m \quad m = \frac{W_E}{g}$$

$$E_1 = \left(-\frac{30.336\times10^6}{32.2\text{ ft/s}^2}\text{ ft}^2/\text{s}^2\right) W_E$$

$$\Delta E = E_2 - E_1 = 0 + \left(942.1\times10^3 \frac{\text{ft}\cdot\text{lb}}{\text{lb}}\right) W_E$$

ENERGY PER POUND: $\quad \frac{\Delta E}{W_E} = 942\times10^3 \frac{\text{ft}\cdot\text{lb}}{\text{lb}}$

(b)

$r_1 = R_M + 50\text{ mi}$

$r_1 = (1081\text{ mi} + 50\text{ mi}) = 1131\text{ mi} = 5.9717\times10^6\text{ ft}$

NEWTON'S SECOND LAW:

$$F = ma_n: \ \frac{GM_M m}{r_1^2} = m\frac{v_1^2}{r_1}$$

$$v_1^2 = \frac{GM_m}{r_1} \quad T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m\frac{GM_m}{r_1}$$

$$V_1 = -\frac{GM_M m}{r_1} \qquad E_1 = T_1+V_1 = \frac{1}{2}\frac{GM_m m}{r_1} - \frac{GM_M m}{r_1}$$

$$E_1 = -\frac{1}{2}\frac{GM_m m}{r_1} = -\frac{1}{2}\frac{0.0123\, g R_E^2 m}{r_1}$$

$$E_1 = -\frac{1}{2}\frac{(0.0123)(32.2\text{ ft/s}^2)(20.909\times10^6\text{ ft})^2}{5.9717\times10^6\text{ ft}} m$$

$$E_1 = \frac{(14.498\times10^6\text{ ft}^2/\text{s}^2)W_E}{(32.2\text{ ft/s}^2)} = 450.2\times10^3 \frac{\text{ft}\cdot\text{lb}}{\text{lb}} W_E$$

$$\Delta E = E_2 - E_1 = 0 + 450.2\times10^3 \frac{\text{ft}\cdot\text{lb}}{\text{lb}} W_E$$

ENERGY PER POUND

$$\frac{\Delta E}{W_E} = 450\times10^3 \frac{\text{ft}\cdot\text{lb}}{\text{lb}}$$

13.89

GIVEN:

SATELLITE OF MASS m
CIRCULAR ORBIT OF RADIUS r ABOUT EARTH

FIND:

(a) ITS POTENTIAL ENERGY
(b) ITS KINETIC ENERGY
(c) ITS TOTAL ENERGY

(a) POTENTIAL ENERGY $\quad V = -\frac{GMm}{r} = -\frac{gR^2m}{r}$ + CONSTANT

CHOOSING THE CONSTANT (cf EQ 13.17)
SO THAT V=0 FOR r=R:

$$V = mgR\left(1-\frac{R}{r}\right)$$

(b) KINETIC ENERGY

NEWTON'S SECOND LAW

$$F = ma_n: \ \frac{GMm}{r^2} = m\frac{v^2}{r}$$

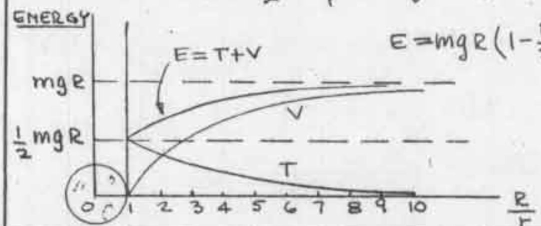
$$v^2 = \frac{GM}{r} = \frac{gR^2}{r}$$

$$T = \frac{1}{2}mv^2 \quad T = \frac{1}{2}\frac{mgR^2}{r}$$

(c) TOTAL ENERGY

$$E = T+V = \frac{1}{2}m\frac{gR^2}{r} + mg\left(1-\frac{R}{r}\right)$$

$$E = mgR\left(1-\frac{R}{2r}\right)$$



13.90

GIVEN:

SATELLITE IN A CIRCULAR ORBIT

FIND:

ENERGY REQUIRED TO PLACE IT INTO ORBIT AT (a) 600 km, (b) 6000 km

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BEFORE LAUNCHING: $r_1 = R = 6.37\times10^6$ m ; $v_1 = 0$

$$E_1 = T_1+V_1 = 0 - \frac{GMm}{R} = -\frac{gR^2m}{R} = -mgR$$

IN CIRCULAR ORBIT OF RADIUS r_2: $\quad$ [cf. EQ 12.30]

NEWTON'S SECOND LAW

$$F = ma_n \quad \frac{GMm}{r_2^2} = m\frac{v_2^2}{r_2}$$

$$v_2^2 = \frac{GM}{r_2} = \frac{gR^2}{r_2}$$

$$E_2 = T_2+V_2 = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2}$$

$$E_2 = \frac{1}{2}m\frac{gR^2}{r_2} - \frac{gR^2m}{r_2} = -\frac{1}{2}\frac{gR^2m}{r_2}$$

ENERGY IMPARTED IS

$$\Delta E = E_2 - E_1 = -\frac{1}{2}\frac{gR^2m}{r_2} - (-mgR) = Rmg\left(1-\frac{R}{2r_2}\right)$$

ENERGY PER KG IS

$$\Delta E/m = Rg\left(1-\frac{R}{2r_2}\right)$$

(a) $r_2 = 6370 + 600 = 6970$ km

$$\Delta E/m = (6.37\times10^6)(9.81)\left(1-\frac{(6370)}{2(6970)}\right) = 33.9\ \frac{\text{MJ}}{\text{kg}}$$

(b) $r_2 = 6370 + 6000 = 12370$ km

$$\Delta E/m = (6.37\times10^6)(9.81)\left(1-\frac{6370}{2(12370)}\right) = 46.4\ \frac{\text{MJ}}{\text{kg}}$$

13.91 GIVEN:

EQ (13.17'), $V_g = -\frac{WR^2}{r}$

DISTANCE ABOVE EARTH'S SURFACE, y

SHOW:

(a) $V_g = Wy$ (FIRST ORDER APPROXIMATION)

DERIVE

(b) A SECOND ORDER APPROXIMATION

$V_g = -\frac{WR^2}{r}$ SETTING $r = R + y$: $V_g = -\frac{WR^2}{R+y} = -\frac{WR}{1+\frac{y}{R}}$

$$V_g = -WR\left(1+\frac{y}{R}\right)^{-1} = -WR\left[1 + \frac{(-1)}{1}\frac{y}{R} + \frac{(-1)(-2)}{1\cdot 2}\left(\frac{y}{R}\right)^2 + \cdots\right]$$

WE ADD THE CONSTANT WR, WHICH IS EQUIVALENT TO CHANGING THE DATUM FROM $r = \infty$ TO $r = R$:

$$V_g = WR\left[\frac{y}{R} - \left(\frac{y}{R}\right)^2 +\right]$$

(a) FIRST ORDER APPROXIMATION:

$$V_g = WR\left(\frac{y}{R}\right) = Wy \quad \text{[EQ 13.16]}$$

(b) SECOND ORDER APPROXIMATION:

$$V_g = WR\left[\frac{y}{R} - \left(\frac{y}{R}\right)^2\right]$$

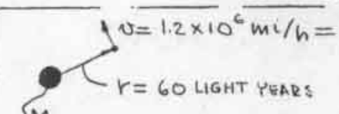
$$V_g = Wy - \frac{Wy^2}{R}$$

13.92 GIVEN:

CELESTIAL BODY IN CIRCULAR ORBIT, RADIUS $r = 60$ LIGHT YEARS VELOCITY $v = 1.2\times10^6$ mi/h ABOUT A POINT OF MASS, M_B

FIND:

RATIO M_B/M_S, WHERE M_S IS THE MASS OF THE SUN

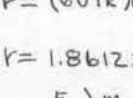


1 LIGHT YEAR IS THE DISTANCE TRAVELED BY LIGHT IN ONE YEAR

SPEED OF LIGHT = 186,300 mi/s

$$r = (60\,\text{YR})\left(186{,}300\frac{\text{mi}}{\text{s}}\right)\left(5280\frac{\text{ft}}{\text{mi}}\right)\left(365\frac{\text{DAYS}}{\text{YR}}\right)\left(24\frac{\text{h}}{\text{DAY}}\right)\left(3600\frac{\text{s}}{\text{h}}\right)$$

$r = 1.8612\times10^{18}$ ft



NEWTONS SECOND LAW

$$F = \frac{GM_Bm}{r^2} = m\frac{v^2}{r}$$

$$M_B = \frac{rv^2}{G}$$

$$GM_{EARTH} = gR_{EARTH}^2 = \left(32.2\frac{\text{ft}}{\text{s}^2}\right)\left(3960\,\text{mi}\times5280\frac{\text{ft}}{\text{mi}}\right)^2 = 14.077\times10^{15}\ (\text{ft}^3/\text{s}^2)$$

$M_{SUN} = 330{,}000\,M_E$: $GM_{SUN} = 330{,}000\,GM_{EARTH}$

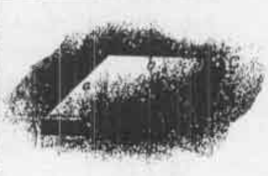
$$GM_{SUN} = (330{,}000)(14.077\times10^{15}) = 4.645\times10^{21}\ \text{ft}^3/\text{s}^2$$

$G = 4.645\times10^{21}/M_{SUN}$

$$M_B = \frac{rv^2}{G} = rv^2M_{SUN}/4.645\times10^{21}$$

$$M_B/M_{SUN} = \frac{(1.8612\times10^{18})(1.76\times10^6)^2}{4.645\times10^{21}} = 1.241\times10^9$$

13.93 GIVEN:



FRICTIONLESS PLATE FIRMLY ATTACHED TO A HORIZONTAL PLANE

CORD ABC ATTACHED TO THE PLATE AT A AND TO A SPHERE AT C

v_0 = INITIAL VELOCITY OF SPHERE CAUSES IT TO MAKE A COMPLETE CIRCUIT AND RETURN TO C

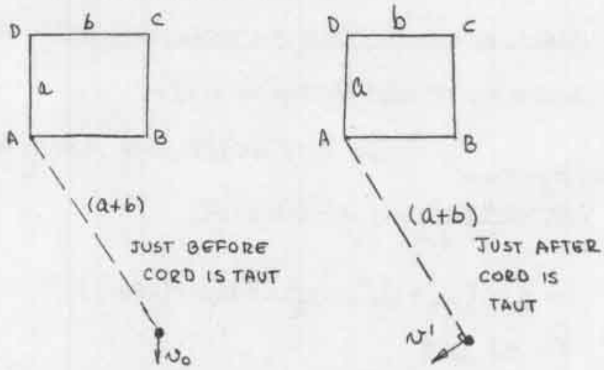
FIND:

VELOCITY OF THE SPHERE AS IT STRIKES C IF,

(a) v_0 IS PARALLEL TO BC

(b) v_0 IS PERPENDICULAR TO BC.

(a) v_0 PARALLEL TO BC



ANGULAR MOMENTUM IS CONSERVED ABOUT A

$bv_0 = (a+b)v'$ $\qquad v' = \frac{bv_0}{(a+b)}$

AS THE SPHERE CONTINUES ITS CIRCUIT TO POINT C ITS VELOCITY IS ALWAYS PERPENDICULAR TO THE CORD AND ENERGY IS CONSERVED

THUS $v_C = v'$

$$v_C = \frac{bv_0}{(a+b)}$$

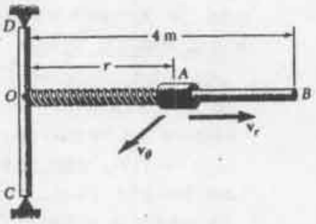
(b) v_0 PERPENDICULAR TO BC

AS THE SPHERE MAKES A COMPLETE CIRCUIT AROUND THE PLATE ITS VELOCITY IS ALWAYS PERPENDICULAR TO THE CORD AND ENERGY IS CONSERVED

THUS $v_C = v_0$

$$v_C = v_0$$

13.94



GIVEN:

$k = 750$ N/m

UNDEFORMED SPRING LENGTH, $r_0 = 1.5$ m

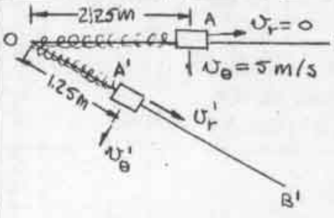
COLLAR MASS, $M = 2.4$ kg

INITIALLY, $r = 2.25$ m, $v_\theta = 5$ m/s

$v_r = 0$

FIND:

v_r' AND v_θ' WHEN $r = 1.25$ m



CONSERVATION OF ANGULAR MOMENTUM (ABOUT O)

$$(2.25\,m)(m)(5\,m/s) = (1.25\,m)(M)(v_\theta')$$

$$v_\theta' = (2.25)(5)/(1.25) = 9.00\,\frac{m}{s}$$

NO FRICTION

CONSERVATION OF ENERGY

$$T + V = T' + V'$$

$$T = \tfrac{1}{2} m (v_r^2 + v_\theta^2) = \frac{(2.4\,kg)}{2}\left(0 + (5\,m/s)^2\right)$$

$$T = 30.0\ J$$

$$V = \tfrac{1}{2} k (r - r_0)^2 = \tfrac{1}{2}(750\,N/m)(2.25\,m - 1.5\,m)^2$$

$$V = 210.9\ J$$

$$v_\theta' = 9.00\ m/s\ ,\ v_r'$$

$$T' = \tfrac{1}{2} m (v_r'^2 + v_\theta'^2) = \frac{(2.4\,kg)}{2}\left(v_r'^2 + (9.00\,m/s)^2\right)$$

$$T' = 1.2\, v_r'^2 + 97.2$$

$$V' = \tfrac{1}{2} k (r' - r_0)^2 = \tfrac{1}{2}(750\,N/m)(1.25\,m - 1.5\,m)^2$$

$$V' = 23.44\ J$$

$$T + V = T' + V'$$

$$30 + 210.9 = 1.2\, v_r'^2 + 97.2 + 23.44$$

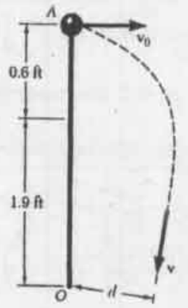
$$1.2\, v_r'^2 = 120.26$$

$$v_r'^2 = 100.22$$

$$v_r' = 10.01\ m/s$$

$$v_r' = 10.01\,\frac{m}{s}$$

13.95



GIVEN:

ELASTIC CORD FIXED AT O

$k = 10$ lb/ft

UNDEFORMED LENGTH, $L_0 = 1.9$ ft

WEIGHT OF BALL, $W = 1.5$ lb

HORIZONTAL FRICTIONLESS PLANE

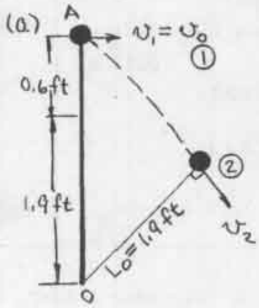
INITIAL VELOCITY v_0 PERPENDICULAR T OA

FIND:

(a) SMALLEST ALLOWABLE v_0 IF CORD DOES NOT BECOME SLACK

(b) CLOSEST DISTANCE d FOR v_0' EQUAL TO HALF VALUE FOR v_0 FOUND IN (a)

(a)



THE CORD WILL NOT GO SLACK IF v_2 IS PERPENDICULAR TO THE UNDEFORMED CORD LENGTH, L_0, AT ②

CONSERVATION OF ANGULAR MOMENTUM

$$2.5\, v_1 = 1.9\, v_2$$

$$v_2 = \frac{2.5}{1.9} v_1 = 1.3158\, v_0$$

CONSERVATION OF ENERGY

POINT ① $\quad v_1 = v_0 \quad T_1 = \tfrac{1}{2}\frac{W}{g} v_0^2 = \frac{0.75}{g} v_0^2$

$$V_1 = \tfrac{1}{2} k (L - L_0)^2 = \tfrac{1}{2}(10\,lb/ft)(2.5\,ft - 1.9\,ft)^2$$

$$V_1 = 1.800\ lb\cdot ft$$

POINT ② $\quad T_2 = \tfrac{1}{2}\frac{W}{g} v_2^2 = \frac{0.75}{g} v_2^2$

$$\Delta L = 0 \quad V = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{0.75}{g} v_0^2 + 1.800 = \frac{0.75}{g} v_2^2 + 0$$

FROM CONS. OF ANG. MOM. $\quad v_2 = 1.3158\, v_0$

$$\frac{0.75}{g} v_0^2\left[(1.3158)^2 - 1\right] = 1.800$$

$$v_0^2 = \frac{(1.8\,lb\cdot ft)(32.2\,ft/s^2)}{(0.75\,lb)(0.7313)}$$

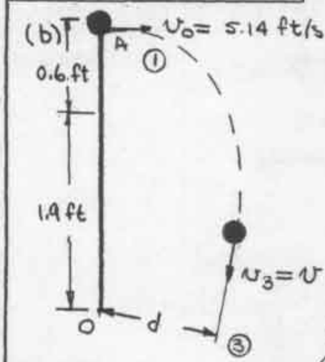
$$v_0^2 = 105.67\,\frac{ft^2}{s^2}$$

$$v_0 = 10.28\,\frac{ft}{s}$$

(CONTINUED)

13.95 continued

(b)



THE BALL TRAVELS IN A STRAIGHT LINE AFTER THE CORD GOES SLACK

CONS. OF ANG MOMENTUM

$(2.5)(5.14) = d v$

$d = \dfrac{12.85}{v}$

CONS OF ENERGY

$v_1 = 5.14$ ft/s

POINT ① $T_1 = \frac{1}{2}\frac{W}{g}v_1^2 = \frac{1}{2}\frac{(1.5\text{ lb})}{(32.2\text{ ft/s}^2)}(5.14\text{ ft/s})^2$

$T_1 = 0.6154$ ft·lb

$V_1 = \frac{1}{2}k(L-L_0)^2 = \frac{1}{2}(10\text{ lb/ft})(2.5\text{ ft} - 1.9\text{ ft})^2 = 1.800$ lb·ft

POINT ③ $T_3 = \frac{1}{2}\frac{W}{g}v_3^2 = \frac{0.75}{g}v^2$

$V_3 = 0$

$T_1 + V_1 = T_3 + V_3$

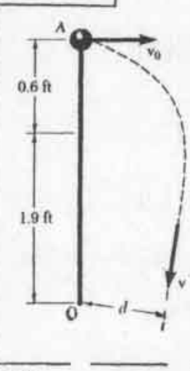
$0.6154 + 1.800 = \frac{0.75}{g}v^2 + 0$

$v = 10.18$ ft/s

FROM CONS. OF ANG. MOM.

$d = \dfrac{12.85}{v} = \dfrac{12.85}{10.18} = 1.262$ ft

13.96



GIVEN:

ELASTIC CORD FIXED AT O

$k = 10$ lb/ft

UNDEFORMED LENGTH $L_0 = 1.9$ ft

WEIGHT OF BALL, $W = 1.5$ lb

HORIZONTAL FRICTIONLESS PLANE

v_0 PERPENDICULAR TO OA

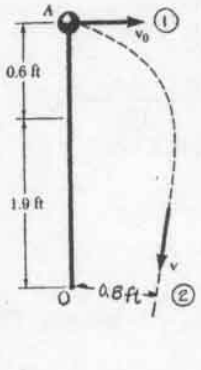
$d = 0.8$ ft AFTER CORD BECOMES SLACK

FIND:

(a) INITIAL SPEED v_0

(b) MAXIMUM SPEED, v_M

(a)



CONSERVATION OF ANGULAR MOMENTUM ABOUT O

$2.5 v_0 = 0.8 v$

$v = 3.125 v_0$

CONSERVATION OF ENERGY

POINT ①

$v_1 = v_0 \quad T_1 = \frac{1}{2}\frac{W}{g}v_0^2 = \frac{0.75}{g}v_0^2$

$V_1 = \frac{1}{2}k(L_1 - L_0)^2 = \frac{1}{2}(10\text{ lb/ft})(2.5\text{ ft} - 1.9\text{ ft})^2$

$V_1 = 1.800$ lb·ft

13.96 continued

POINT ② $v_2 = v \quad T_2 = \frac{1}{2}\frac{W}{g}v^2 = \frac{0.75}{g}v^2$

$V_2 = 0$ (CORD IS SLACK)

$T_1 + V_1 = T_2 + V_2$

$\frac{0.75}{g}v_0^2 + 1.800 = \frac{0.75}{g}v^2 + 0$

FROM CONS. OF ANG. MOM., $v = 3.125 v_0$

$\frac{0.75}{g}v_0^2[(3.125)^2 - 1] = 1.800$

$v_0^2 = \dfrac{(1.800\text{ lb·ft})(32.2\text{ ft/s}^2)}{(0.75\text{ lb})(8.7656)}$

$v_0^2 = 8.816$ ft²/s²

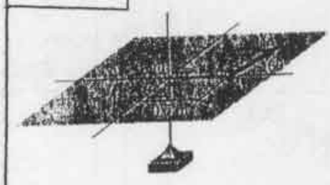
$v_0 = 2.97 \frac{\text{ft}}{\text{s}}$ ◀

(b) MAXIMUM VELOCITY OCCURS WHEN THE BALL IS AT ITS MINIMUM DISTANCE FROM O, (WHEN $d = 0.8$ ft)

$v_m = 3.125 v_0 = (3.125)(2.97) = 9.28$ ft/s

$v_m = 9.28 \frac{\text{ft}}{\text{s}}$ ◀

13.97



GIVEN:

SPHERE OF MASS, $m = 0.6$ kg

FORCE BETWEEN A AND O DIRECTED TOWARD O OF MAGNITUDE $F = (80/r^2)$ N

$v_A = 20$ m/s

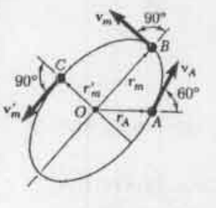
HORIZONTAL FRICTIONLESS PLANE

FIND:

(a) MAXIMUM AND MINIMUM DISTANCES FROM O

(b) CORRESPONDING VALUES OF THE SPEED

(a)



THE FORCE EXERTED ON THE SPHERE PASSES THROUGH O. ANGULAR MOMENTUM ABOUT O IS CONSERVED

MINIMUM VELOCITY IS AT B WHERE THE DISTANCE FROM O IS MAXIMUM

MAXIMUM VELOCITY IS A C WHERE DISTANCE FROM O IS MINIMUM

$r_A m v_A \sin 60 = r_m m v_m$

$(0.5\text{ m})(0.6\text{ kg})(20\text{ m/s})\sin 60 = r_m (0.6\text{ kg}) v_m$

$v_m = 8.66/r_m \qquad (1)$

CONSERVATION OF ENERGY

AT POINT A $T_A = \frac{1}{2}m v_A^2 = \frac{1}{2}(0.6\text{ kg})(20\text{ m/s})^2 = 120$ J

$V = \int F dr = \int \frac{80}{r^2}dr = -\frac{80}{r}, \quad V_A = \frac{-80}{0.5} = -160$ J

AT POINT B $T_B = \frac{1}{2}m v_m^2 = \frac{1}{2}(0.6\text{ kg})v_m^2 = 0.3 v_m^2$

(AND POINT C)

$V_B = -\dfrac{80}{r_m}$

$T_A + V_A = T_B + V_B$

$120 - 160 = 0.3 v_m^2 - \dfrac{80}{r_m} \qquad (2)$

SUBSTITUTE (1) INTO (2) $\quad -40 = (0.3)\left(\dfrac{8.66}{r_m}\right)^2 - \dfrac{80}{r_m}$

$r_m^2 - 2 r_m + 0.5625 = 0$ (CONTINUED)

13.97 continued

$r_m' = 0.339\,m$ AND $r_m = 1.661\,m$

$r_{MAX} = 1.661\,m$

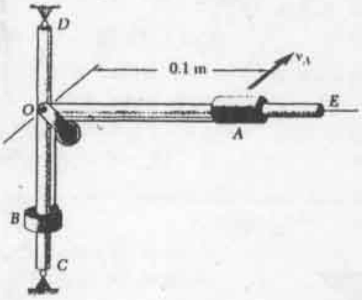
$r_{MIN} = 0.339\,m$

(b) SUBSTITUTE r_m' AND r_m FROM RESULTS OF PART (a) INTO (1) TO GET CORRESPONDING MAXIMUM AND MINIMUM VALUES OF THE SPEED

$v_m' = \frac{8.66}{0.339} = 25.6\,\frac{m}{s}$ $\quad v_{MAX} = 25.6\,\frac{m}{s}$

$v_m = \frac{8.66}{1.661} = 5.21\,\frac{m}{s}$ $\quad v_{MIN} = 5.21\,\frac{m}{s}$

13.98



GIVEN:

$m_A = 1.8\,kg$

$m_B = 0.7\,kg$

INITIALLY, $v_A = 2.1\,\frac{m}{s}$ AND $v_B = 0$

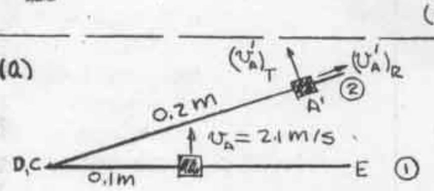
A STOP IS SUDDENLY REMOVE AT B

FIND:

(a) v_A', WHEN m_A IS 0.2 m FROM O

(b) v_A', WHEN $v_B = 0$

(a)



CONSERVATION OF ANGULAR MOMENTUM ABOUT DC

$(0.1\,m)(m_A)(v_A) = (0.2\,m)(m_A)(v_A')_T$

$(v_A')_T = \left(\frac{0.1}{0.2}\right)(2.1\,\frac{m}{s}) = 1.05\,m/s$

CONSERVATION OF ENERGY

① $v_A = 2.1\,m/s$ $\quad T_1 = \frac{1}{2}(1.8\,kg)(2.1\,m/s)^2 = 3.969\,J$

$v_B = 0$,

CHOOSE DATUM FOR B AT ITS INITIAL POSITION AND NOTE THAT THE POTENTIAL ENERGY OF A DOES NOT CHANGE THUS WE TAKE $V_1 = 0$

② $(v_A')_T = 1.050\,m/s$ $\quad (v_A')_R = v_B'$ (KINEMATICS)

$T_2 = \frac{1}{2} m_A[(v_A')_T^2 + (v_A')_R^2] + \frac{1}{2} m_B (v_B')^2$

$T_2 = \frac{1}{2}(1.8\,kg)\left[(1.050\,m/s)^2 + (v_A')_R^2\right] + \frac{1}{2}(0.7\,kg)(v_A')_R^2$

$T_2 = 0.9923 + 1.25(v_A')_R^2$

$V_2 = m_B g\,(0.1\,m) = (0.7\,kg)(9.81\,m/s^2)(0.1\,m) = 0.6867\,J$

$T_1 + V_1 = T_2 + V_2$ $\quad 3.969 + 0 = 0.9923 + 1.25(v_A')_R^2 + 0.6867$

$(v_A')_R^2 = 1.832\,\frac{m^2}{s^2}$; $(v_A')_R = 1.354\,\frac{m}{s}$

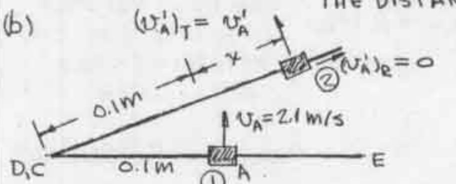
$v_A' = \sqrt{(v_A')_T^2 + (v_A')_R^2} = [(1.05)^2 + (1.354)^2]^{\frac{1}{2}} = 1.713\,\frac{m}{s}$

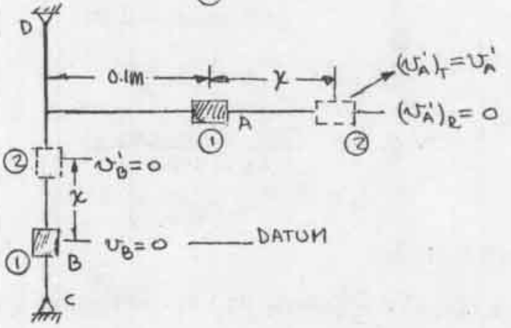
$v_A' \measuredangle \theta$ $\quad \theta = \tan^{-1}(v_A')_T/(v_A')_R = \tan^{-1}\frac{1.05}{1.354} = 37.8°$

13.98 continued

WHEN B COMES TO REST THE DISTANCE x MOVED BY A AND B IS UNKNOWN

(b)





CONSERVATION OF ANGULAR MOMENTUM ABOUT DC

$(0.1\,m)(m_A)(v_A) = (0.1\,m + x\,m)(m_A)(v_A')_T$

KINEMATICS $\quad (v_A')_R = (v_B') = 0$, THUS $(v_A')_T = v_A'$

$v_A = 2.1\,m/s$

$(0.1)(2.1) = (0.1 + x)\,v_A'$ $\qquad x = \frac{0.21}{v_A'} - 0.1$

CONSERVATION OF ENERGY

AT ① $\quad v_A = 2.1\,m/s$

$v_B = 0$ $\qquad T_1 = \frac{1}{2} m_A v_A^2 = \frac{1}{2}(1.8\,kg)(2.1)^2$

$T_1 = 3.969\,J$

$V_1 = 0$

AT ② $\quad v_B' = 0$, $(v_A')_R = 0$

$T_2 = \frac{1}{2} m_A v_A'^2 = 0.9\,v_A'^2$

$V_2 = m_B g\,x = (0.7\,kg)(9.81\,\frac{m}{s^2})\,x$

$V_2 = 6.867\,x$

$T_1 + V_1 = T_2 + V_2$

$3.969 + 0 = 0.9\,v_A'^2 + 6.867\,x$

FROM CONSERVATION OF ANG MOM. $\quad x = \frac{0.21}{v_A'} - 0.1$

THUS $\quad 3.969 = 0.9\,v_A'^2 + (6.867)\left(\frac{0.21 - 0.1\,v_A'}{v_A'}\right)$

$3.969\,v_A' = 0.9\,v_A'^3 + 1.442 - 0.6867\,v_A'$

$4.6557\,v_A' = 0.9\,v_A'^3 + 1.442$

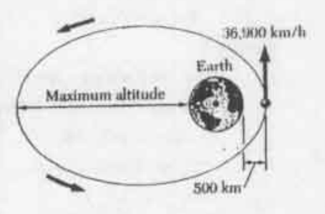
$5.173\,v_A' = v_A'^3 + 1.602$

BY TRIAL $\qquad v_A' = 0.316\,m/s$

13.99

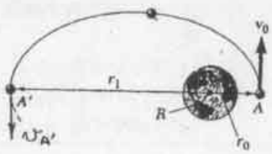
GIVEN:

SATELLITE LAUNCHED AS SHOWN



FIND:

MAXIMUM ALTITUDE, USING CONSERVATION OF ENERGY AND CONSERVATION OF MOMENTUM



$R = 6370$ km

$r_0 = 500$ km $+ 6370$ km

$r_0 = 6870$ km

$= 6.87 \times 10^6$ m

$v_0 = 36{,}900$ km/h

$= \dfrac{36.9 \times 10^6 \text{ m}}{3.6 \times 10^3 \text{ s}}$

$= 10.25 \times 10^3$ m/s

CONSERVATION OF ANGULAR MOMENTUM

$r_0 m v_0 = r_1 m v_{A'} \qquad r_0 = r_{MIN},\ r_1 = r_{MAX}$

$v_{A'} = \left(\dfrac{r_0}{r_1}\right) v_0 = \left(\dfrac{6.870 \times 10^6}{r_1}\right)(10.25 \times 10^3)$

$v_{A'} = \dfrac{70.418 \times 10^9}{r_1} \qquad (1)$

CONSERVATION OF ENERGY

POINT A

$v_0 = 10.25 \times 10^3 \ \frac{\text{m}}{\text{s}}$

$T_A = \frac{1}{2} m v_0^2 = \frac{1}{2} m (10.25 \times 10^3)^2$

$T_A = (m)(52.53 \times 10^6)(\text{J})$

$V_A = -\dfrac{GMm}{r_0} \qquad GM = gR^2 = (9.81 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})^2$

$GM = 398 \times 10^{12} \text{ m}^3/\text{s}^2$

$r_0 = 6.87 \times 10^6$ m

$V_A = -\dfrac{(398 \times 10^{12} \text{ m}^3/\text{s}^2)\, m}{(6.87 \times 10^6 \text{ m})} = -57.93 \times 10^6 m \ (\text{J})$

POINT A'

$T_{A'} = \frac{1}{2} m v_{A'}^2$

$V_{A'} = -\dfrac{GMm}{r_1} = -\dfrac{398 \times 10^{12} m}{r_1} \ (\text{J})$

$T_A + V_A = T_{A'} + V_{A'}$

$52.53 \times 10^6 m - 57.93 \times 10^6 m = \frac{1}{2} m v_{A'}^2 - \dfrac{398 \times 10^{12} m}{r_1}$

SUBSTITUTING FOR $v_{A'}$ FROM (1)

$-5.402 \times 10^6 = \dfrac{(70.418 \times 10^9)^2}{(2)(r_1)^2} - \dfrac{398 \times 10^{12}}{r_1}$

$-5.402 \times 10^6 = \dfrac{(2.4793 \times 10^{21})}{r_1^2} - \dfrac{398 \times 10^{12}}{r_1}$

$(5.402 \times 10^6) r_1^2 - (398 \times 10^{12}) r_1 + 2.4793 \times 10^{21} = 0$

$r_1 = 66.7 \times 10^6$ m, 6.87×10^6 m

$r_{MAX} = 66{,}700$ km

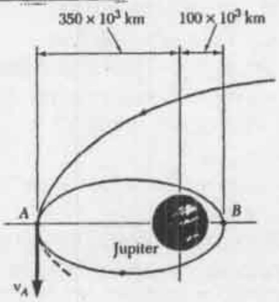
13.100

GIVEN:

$v_A = 26.9$ km/s

MASS OF JUPITER

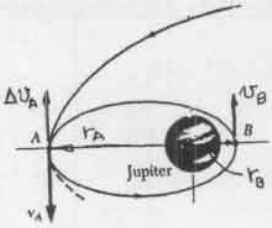
$M_J = 319 M_E$



FIND:

Δv_A, TO BRING THE SPACE CRAFT TO WITHIN 100×10^3 km AT B

CONSERVATION OF ENERGY



POINT A

$T_A = \frac{1}{2} m (v_A - \Delta v_A)^2$

$V_A = -\dfrac{GM_J m}{r_A}$

$GM_J = 319\, GM_E = 319\, g R_E^2$

$R_E = 6.37 \times 10^6$ m

$GM_J = (319)(9.81 \ \frac{\text{m}}{\text{s}^2})(6.37 \times 10^6 \text{ m})^2$

$GM_J = 126.98 \times 10^{15} \ \frac{\text{m}^3}{\text{s}^2}$

$r_A = 350 \times 10^6$ m $\qquad V_A = -\dfrac{(126.98 \times 10^{15} \text{ m}^3/\text{s}^2)\, m}{(350 \times 10^6 \text{ m})}$

$V_A = -(362.8 \times 10^6) m$

POINT B

$T_B = \frac{1}{2} m v_B^2$

$V_B = -\dfrac{GM_J m}{r_B} = -\dfrac{(126.98 \times 10^{15} \text{ m}^3/\text{s}^2)\, m}{(100 \times 10^6 \text{ m})}$

$V_B = -(1269.8 \times 10^6) m$

$T_A + V_A = T_B + V_B$

$\frac{1}{2} m (v_A - \Delta v_A)^2 - 362.8 \times 10^6 m = \frac{1}{2} m v_B^2 - 1269.8 \times 10^6 m$

$(v_A - \Delta v_A)^2 - v_B^2 = -1814 \times 10^6 \qquad (1)$

CONSERVATION OF ANGULAR MOMENTUM

$r_A = 350 \times 10^6$ m $\quad r_B = 100 \times 10^6$ m

$r_A m (v_A - \Delta v_A) = r_B m v_B$

$v_B = \left(\dfrac{r_A}{r_B}\right)(v_A - \Delta v_A) = \left(\dfrac{350}{100}\right)(v_A - \Delta v_A) \qquad (2)$

SUBSTITUTE v_B IN (2) INTO (1)

$(v_A - \Delta v_A)^2 [1 - (3.5)^2] = -1814 \times 10^6$

$(v_A - \Delta v_A)^2 = 1612.4 \times 10^6 \quad (v_A - \Delta v_A) = \mp 12.698 \times 10^3 \ \frac{\text{m}}{\text{s}}$

(TAKE + ROOT, − ROOT REVERSES FLIGHT DIRECTION)

$v_A = 26.9 \times 10^3 \ \frac{\text{m}}{\text{s}}$ (GIVEN)

$\Delta v_A = (26.9 \times 10^3 \ \frac{\text{m}}{\text{s}} - 12.698 \times 10^3 \ \frac{\text{m}}{\text{s}})$

$\Delta v_A = 14.20$ km/s

13.101

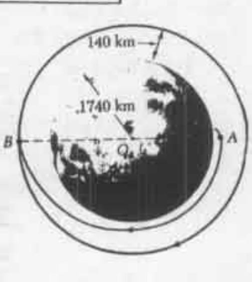
GIVEN:

AT ENGINE SHUTOFF AT A

$r_A = 1740 + = 1748\ km$

AT B, $r_B = 1740 + 140 = 1880\ km$

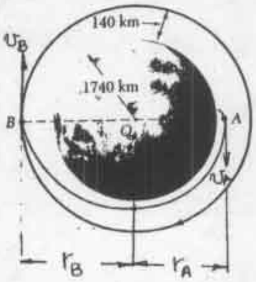
COMMAND MODULE IN A CIRCULAR ORBIT



FIND:

(a) SPEED AT A AT ENGINE SHUTOFF.

(b) RELATIVE VELOCITY LEM APPROACHES COMMAND MODULE AT A



CONSERVATION OF ANG. MOMENTUM

$m r_A v_A = m r_B v_B$

$v_B = \frac{r_A}{r_B} v_A = \frac{1748}{1880} v_A$

$v_B = 0.9298\, v_A \qquad (1)$

CONSERVATION OF ENERGY

AT POINT A $\quad T_A = \frac{1}{2} m v_A^2 \quad V_A = -\frac{G M_{MOON} m}{r_A}$

$M_{MOON} = 0.0123\, M_{EARTH}$

$G M_{MOON} = 0.0123\, G M_{EARTH} = 0.0123\, g R_{EARTH}^2$

$G M_{MOON} = (0.0123)(9.81 \frac{m}{s^2})(6.37 \times 10^6 m)^2$

$G M_{MOON} = 4.896 \times 10^{12} \frac{m^3}{s^2} \qquad r_A = 1748 \times 10^3\ m$

$V_A = -\frac{(4.896 \times 10^{12} m^3/s^2) m}{(1748 \times 10^3 m)} = -2.801 \times 10^6 m$

AT POINT B $\quad T_B = \frac{1}{2} m v_B^2 \quad r_B = 1880 \times 10^3\ m$

$V_B = -\frac{G M_{MOON} m}{r_B} = -\frac{(4.896 \times 10^{12} m^3/s^2) m}{(1880 \times 10^3 m)} = -2.604 \times 10^6 m$

$T_A + V_A = T_B + V_B;\ \frac{1}{2} m v_A^2 - 2.801 \times 10^6 m = \frac{1}{2} m v_B^2 - 2.604 \times 10^6 m$

$v_A^2 = v_B^2 + 393.3 \times 10^3 \ (\frac{m^2}{s^2}) \qquad (2)$

(a) SPEED AT A

SUBSTITUTE v_B IN (1) INTO (2)

$v_A^2 [1 - (0.9298)^2] = 393.3 \times 10^3$

$v_A^2 = 2.903 \times 10^6 \quad v_A = 1.704 \times 10^3 \frac{m}{s} \quad v_A = 1704 \frac{m}{s}$

(b) AT POINT B

FROM (1) AND RESULT IN (a) $v_B = (0.9298)(1704)$

$v_B = 1584.0 \frac{m}{s}$

COMMAND MODULE IS IN CIRCULAR ORBIT, $r_B = 1.88 \times 10^6\ m$

(EQ 12.44)

$v_{circ} = \sqrt{\frac{G M_{MOON}}{r_B}} = \sqrt{\frac{4.896 \times 10^{12}}{1.88 \times 10^6}} = 1613.8 \frac{m}{s}$

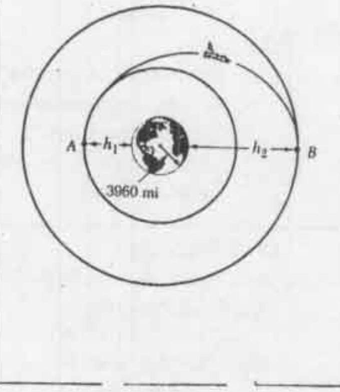
RELATIVE VELOCITY $= v_{circ} - v_B = 1613.8 - 1584.0 = 29.8 \frac{m}{s}$

13.102

GIVEN:

$h_1 = 200\ mi.$

$h_2 = 500\ mi.$

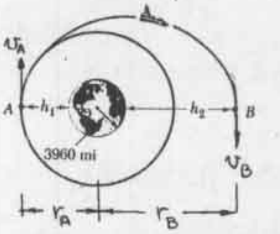


FIND:

FOR A SPACECRAFT TRANSFERRING FROM A CIRCULAR ORBIT TO A CIRCULAR ORBIT AT B

(a) INCREASES IN SPEED AT A AND B.

(b) TOTAL ENERGY PER UNIT MASS TO EXECUTE THE TRANSFER



ELLIPTICAL ORBIT BETWEEN A AND B

CONS. OF ANG. MOMENTUM

$m r_A v_A = m r_B v_B$

$v_A = \frac{r_B}{r_A} v_B = \frac{23.549}{21.965} v_B$

$r_A = 3960\ mi + 200\ mi = 4160\ mi$

$r_A = 21.965 \times 10^6\ ft \qquad v_A = 1.0721\, v_B \quad (1)$

$r_B = 3960\ mi + 500\ mi = 4460\ mi$

$r_B = 23.549 \times 10^6\ ft \qquad R = (3960)(5280) = 20.909 \times 10^6\ ft$

CONSERVATION OF ENERGY $\quad GM = gR^2 = (32.2 \frac{ft}{s^2})(20.909 \times 10^6 ft)^2$

$GM = 14.077 \times 10^{15}\ ft^3/s^2$

POINT A:

$T_A = \frac{1}{2} m v_A^2 \quad V_A = -\frac{GMm}{r_A} = -\frac{(14.077 \times 10^{15}) m}{(21.965 \times 10^6)}$

$V_A = 640.89 \times 10^6 m$

POINT B:

$T_B = \frac{1}{2} m v_B^2 \quad V_B = \frac{-GMm}{r_B} = \frac{(14.077 \times 10^{15}) m}{(23.549 \times 10^6)} = -597.79 \times 10^6 m$

$T_A + V_A = T_B + V_B$

$\frac{1}{2} m v_A^2 - 640.89 \times 10^6 m = \frac{1}{2} m v_B^2 - 597.79 \times 10^6 m$

$v_A^2 - v_B^2 = 86.219 \times 10^6$

FROM (1) $\quad v_A = 1.0721\, v_B \quad v_B^2 ((1.0721)^2 - 1) = 86.219 \times 10^6$

$v_B^2 = 576.98 \times 10^6\ ft^2/s^2$

$v_A = (1.0721)(24.028 \times 10^3) \qquad v_B = 24{,}020.4\ ft/s$

$v_A = 25752.6\ ft/s$

CIRCULAR ORBIT AT A AND B

(EQ. 12.44) $(v_A)_c = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.077 \times 10^{15}}{21.965 \times 10^6}} = 25316\ ft/s$

$(v_B)_c = \sqrt{\frac{GM}{r_B}} = \sqrt{\frac{14.077 \times 10^{15}}{23.549 \times 10^6}} = 24450\ ft/s.$

(a) INCREASES IN SPEED AT A AND AT B

$\Delta v_A = v_A - (v_A)_c = 25753 - 25316 = 437\ ft/s$

$\Delta v_B = (v_B)_c - v_B) = 24449 - 24020 = 429\ ft/s$

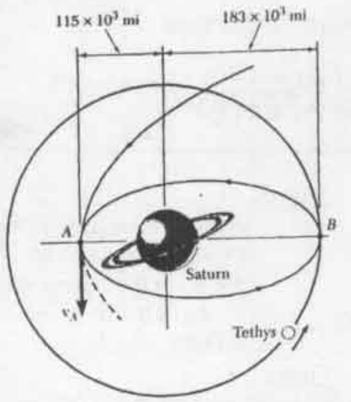
(b) TOTAL ENERGY PER UNIT MASS

$E/m = \frac{1}{2}[v_A^2 - (v_A)_c^2 + (v_B)_c^2 - (v_B)^2]$

$E/m = \frac{1}{2}[(25753)^2 - (25313)^2 + (24450)^2 - (24020)^2]$

$E/m = 21.6 \times 10^6\ ft^2/s^2$

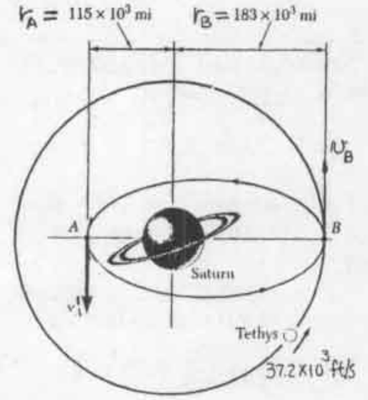
13.103



GIVEN:
$v_A = 68.8\times10^3$ ft/s
$v_{THS} = 37.2\times10^3$ ft/s IN CIRCULAR ORBIT

FIND:
(a) DECREASE IN SPEED, Δv_A OF A SPACECRAFT AT A TO ACHIEVE AN ELLIPTICAL ORBIT THROUGH A AND B
(b) THE SPEED v_B OF THE SPACECRAFT AS IT REACHES B

(a)



$r_A = 607.2\times10^6$ ft
$r_B = 966.2\times10^6$ ft

v_A' = SPEED OF SPACECRAFT IN THE ELLIPTICAL ORBIT AFTER ITS SPEED HAS BEEN DECREASED

ELLIPTICAL ORBIT BETWEEN A AND B
CONSERVATION OF ENERGY

POINT A $\quad T_A = \frac{1}{2} m v_A'^2 \quad V_A = -\frac{GM_{SAT}m}{r_A}$

M_{SA} = MASS OF SATURN, DETERMINE GM_{SA} FROM THE SPEED OF TETHYS IN ITS CIRCULAR ORBIT

(EQ 12.44) $\quad v_{circ} = \sqrt{\frac{GM_{SAT}}{r}} \quad GM_{SAT} = r_B v_{circ}^2$

$$GM_{SAT} = (966.2\times10^6 \text{ ft})(37.2\times10^3 \text{ ft/s})^2 = 1.337\times10^{18} \text{ ft}^3/\text{s}^2$$

$$V_A = -\frac{(1.337\times10^{18} \text{ ft}^3/\text{s}^2)\,m}{(607.2\times10^6 \text{ ft})} = -2.202\times10^9 m$$

POINT B $\quad T_B = \frac{1}{2} m v_B^2 \quad V_B = \frac{-GM_{SAT}m}{r_B} = -\frac{(1.337\times10^{18} \text{ ft}^3/\text{s}^2)\,m}{(966.2\times10^6 \text{ ft})}$

$V_B = -1.384\times10^9$

$T_A + V_A = T_B + V_B; \quad \frac{1}{2}\cancel{m}v_A'^2 - 2.202\times10^9\cancel{m} = \frac{1}{2}\cancel{m}v_B^2 - 1.384\times10^9\cancel{m}$

$v_A'^2 - v_B^2 = 1.636\times10^9$

CONSERVATION OF ANGULAR MOMENTUM

$r_A m v_A' = r_B m v_B \quad v_B = \frac{r_A}{r_B} v_A' = \frac{607.2\times10^6}{966.2\times10^6} v_A' = 0.6284 v_A'$

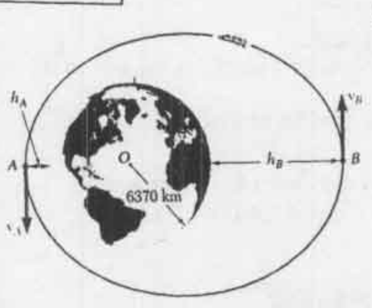
$v_A'^2[1-(0.6284)^2] = 1.636\times10^9 \quad v_A' = 52005$ ft/s

$\Delta v_A = v_A - v_A' = 68800 - 52005 = 16\,795$ ft/s ◀

(b)

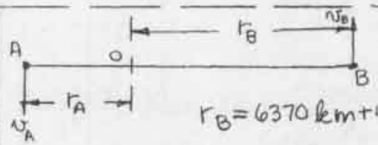
$v_B = \frac{r_A}{r_B} v_A' = (0.6284)(52005) = 32700$ ft/s ◀

13.104



GIVEN:
$h_A = 2400$ km
$h_B = 9600$ km

FIND:
SPEED, v_A



$r_A = 6370$ km $+ 2400$ km
$r_A = 8770$ km

$r_B = 6370$ km $+ 9600$ km $= 15970$ km

CONSERVATION OF MOMENTUM $\quad r_A m v_A = r_B m v_B$

$$v_B = \frac{r_A}{r_B} v_A = \frac{8770}{15970} v_A = 0.5492 v_A \quad (1)$$

CONSERVATION OF ENERGY

$T_A = \frac{1}{2} m v_A^2 \quad V_A = -\frac{GMm}{r_A} \quad T_B = \frac{1}{2} m v_B^2 \quad V_B = -\frac{GMm}{r_B}$

$$GM = gR^2 = (9.81 \tfrac{m}{s^2})(6370\times10^3 m)^2 = 398.1\times10^{12} \tfrac{m^3}{s^2}$$

$$V_A = -\frac{(398.1\times10^{12})m}{8770\times10^3} = 45.39\times10^6 m$$

$$V_B = -\frac{(398.1\times10^{12})m}{(15970\times10^3)} = -24.93\,m$$

$T_A + V_A = T_B + V_B \quad \frac{1}{2}\cancel{m}v_A^2 - 45.39\times10^6\cancel{m} = \frac{1}{2}\cancel{m}v_B^2 - 24.93\times10^6 \cancel{m} \; (2)$

SUBSTITUTE FOR v_B IN (2) FROM (1)

$v_A^2[1-(0.5492)^2] = 40.92\times10^6$

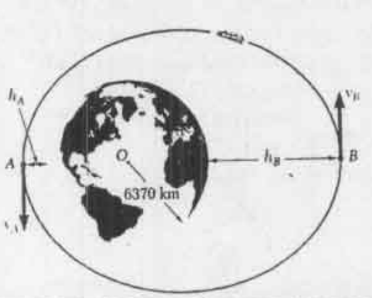
$v_A^2 = 58.59\times10^6 \frac{m^2}{s^2}$

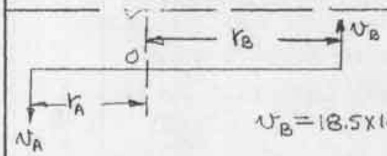
$v_A = 7.65\times10^3 \frac{m}{s} \quad v_A = 27.6\times10^3$ km/h ◀

13.105

GIVEN:
$v_A = 26.3\times10^3$ km/h
$v_B = 18.5\times10^3$ km/h

FIND:
ALTITUDE, h_B





$v_A = 26.3\times10^3$ km/h
$v_A = 7.31\times10^3$ m/s
$v_B = 18.5\times10^3$ km/h $= 5.14\times10^3$ m/s

CONSERVATION OF MOMENTUM $\quad r_A m v_A = r_B m v_B$

$r_A v_A = r_B v_B \quad r_A = \frac{v_B}{v_A} r_B = \frac{18.5}{26.3} r_B$

$r_A = 0.7034\, r_B \quad (1)$

13.105 continued

CONSERVATION OF ENERGY

$T_A = \frac{1}{2} m v_A^2 \quad T_A = \frac{1}{2} m (7.31\times10^3)^2 = 26.69\times10^6 m$

$T_B = \frac{1}{2} m v_B^2 \quad T_B = \frac{1}{2} m (5.14\times10^3)^2 = 13.20\times10^6 m$

$V_A = -\frac{GMm}{r_A} \quad GM = gR^2 = (9.81 \frac{m}{s})(6370\times10^3)^2$

$GM = 398.1\times10^{12}\ m^3/s^2$

$V_A = -\frac{398.1\times10^{12}}{r_A}$

$V_B = -\frac{GMm}{r_B} = -\frac{398.1\times10^{12}}{r_B}$

$T_A + V_A = T_B + V_B$

$26.69\times10^6 \cancel{m} - \frac{398.1\times10^{12}}{r_A}\cancel{m} = 13.20\times10^6 \cancel{m} - \frac{398.1\times10^{12}\cancel{m}}{r_B}$

SUBSTITUTE FOR r_A FROM (1)

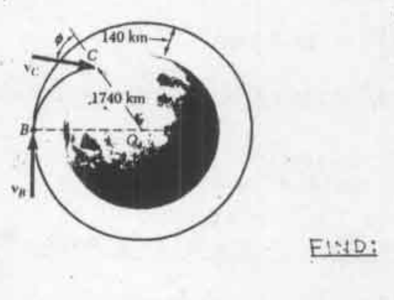
$\frac{398.1\times10^{12}}{r_B}\left[\frac{1}{(0.7034)} - 1\right] = 13.49\times10^6$

$\frac{1}{r_B} = 80.37\times10^{-9}$

$r_B = 12.442\times10^6\ m = 12442\ km$

$h_B = r_B - R = 12442\ km - 6370\ km = 6070\ km$

*13.106

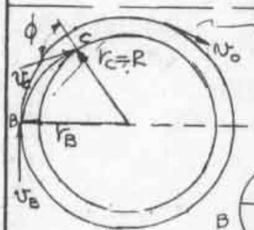


GIVEN:

COMMAND MODULE IN CIRCULAR ORBIT AT AN ALTITUDE OF 140 km. ATTACHED LEM CAST ADRIFT AT RELATIVE VELOCITY OF 200 m/s

FIND:

v_C AND ϕ AS THE LEM HITS THE MOON



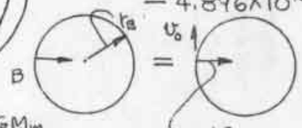
COMMAND MODULE IN CIRCULAR ORBIT

$r_B = 1740 + 140 = 1880\ km = 1.88\times10^6 m$

$GM_{MOON} = 0.0123 GM_{EARTH} = 0.0123\, gR^2$

$= 0.0123(9.81)(6.37\times10^6)^2$

$= 4.896\times10^{12}\ m^3/s^2$



$R = 1740\ km \qquad \frac{GM_m}{r_B^2} \qquad m\frac{v_0^2}{r_B}$

$\Sigma F = ma_n \quad \frac{GM_m m}{r_B^2} = m\frac{v_0^2}{r_B} \quad v_0 = \sqrt{\frac{GM_m}{r_B}} = \sqrt{\frac{4.896\times10^{12}}{1.88\times10^6}}$

$v_0 = 1614\ m/s \quad v_B = 1614 - 200 = 1414\ m/s$

CONSERVATION OF ENERGY BETWEEN B AND C

$\frac{1}{2} m v_B^2 - \frac{GM_m m}{r_B} = \frac{1}{2} m v_C^2 - \frac{GM_m m}{r_C} \qquad r_C = R$

$v_C^2 = v_B^2 + \frac{2GM_m}{r_B}\left(\frac{r_B}{R} - 1\right)$

$v_C^2 = (1414\ m/s)^2 + 2\frac{(4.896\times10^{12} m^3/s^2)}{(1.88\times10^6 m)}\left(\frac{1.88\times10^6}{1.74\times10^6} - 1\right)$

$v_C^2 = 1.999\times10^6 + 0.4191\times10^6 = 2.418\times10^6\ \frac{m^2}{s^2}$

$v_C = 1555\ \frac{m}{s}$

*13.106 continued

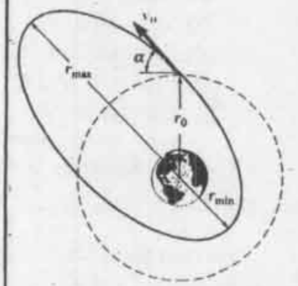
CONSERVATION OF ANGULAR MOMENTUM

$r_B m v_B = R m v_C \sin\phi$

$\sin\phi = \frac{r_B v_B}{r_C v_C} = \frac{(1.88\times10^6 m)(1414 \frac{m}{s})}{(1.74\times10^6 m)(1555 \frac{m}{s})} = 0.98249$

$\phi = 79.26^\circ \qquad \phi = 79.3^\circ$

13.107



GIVEN:

SATELLITE PROJECTED AT VELOCITY v_0 AT AN ANGLE α WITH ITS INTENDED CIRCULAR ORBIT

FIND:

r_{MAX} AND r_{MIN}

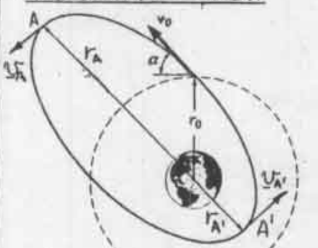
v_0 FOR CIRCULAR ORBIT OF RADIUS r_0

$F = ma_n \quad \frac{GMm}{r_0^2} = m\frac{v_0^2}{r_0}$

$v_0^2 = \frac{GM}{r_0}$

BUT v_0 FORMS AN ANGLE α WITH THE INTENDED CIRCULAR PATH

FOR ELLIPTIC ORBIT



CONS. OF ANG. MOMENTUM

$r_0 m v_0 \cos\alpha = r_A m v_A$

$v_A = \left(\frac{r_0}{r_A}\cos\alpha\right) v_0 \qquad (1)$

CONS. OF ENERGY

$\frac{1}{2} m v_0^2 - \frac{GMm}{r_0} = \frac{1}{2} m v_A^2 - \frac{GMm}{r_A}$

$v_0^2 - v_A^2 = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$

SUBSTITUTE FOR v_A FROM (1)

$v_0^2\left[1 - \left(\frac{r_0}{r_A}\right)^2 \cos^2\alpha\right] = \frac{2GM}{r_0}\left(1 - \frac{r_0}{r_A}\right)$

BUT $v_0^2 = \frac{GM}{r_0}$, THUS $1 - \left(\frac{r_0}{r_A}\right)^2 \cos^2\alpha = 2\left(1 - \frac{r_0}{r_A}\right)$

$\cos^2\alpha\left(\frac{r_0}{r_A}\right)^2 - 2\left(\frac{r_0}{r_A}\right) + 1 = 0$

SOLVING FOR $\frac{r_0}{r_A}$

$\frac{r_0}{r_A} = \frac{+2 \pm \sqrt{4 - 4\cos^2\alpha}}{2\cos^2\alpha} = \frac{1 \pm \sin\alpha}{1 - \sin^2\alpha}$

$r_A = \frac{(1+\sin\alpha)(1-\sin\alpha)}{1 \pm \sin\alpha} r_0 = (1 \mp \sin\alpha) r_0$

ALSO VALID FOR POINT A'

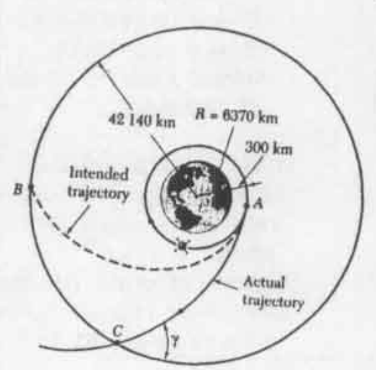
THUS

$r_{MAX} = (1+\sin\alpha) r_0 \quad r_{MIN} = (1-\sin\alpha) r_0$

13.108

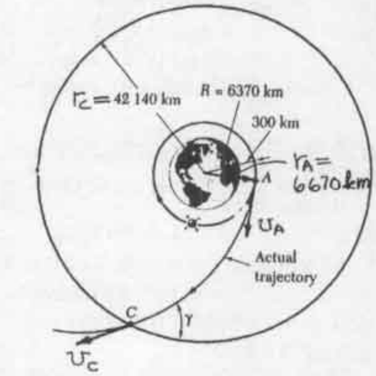
GIVEN:

COMMUNICATION SATELLITE AT A IS LAUNCHED WITH A VELOCITY RELATIVE TO A SPACE PLATFORM IN CIRCULAR ORBIT OF $(v_A)_R = 3.44$ km/s



FIND:

ANGLE γ AT WHICH THE SATELLITE CROSSES THE CIRCULAR ORBIT AT C.



$R = 6370$ km

$r_A = 6370\text{ km} + 300\text{ km}$

$r_A = 6.67 \times 10^6$ m

$r_C = 42.14 \times 10^6$ m

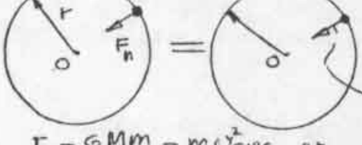
$GM = gR^2$

$GM = (9.81\ \frac{m}{s^2})(6.37 \times 10^6\ m)^2$

$GM = 398.1 \times 10^{12}\ m^3/s^2$

FOR ANY CIRCULAR ORBIT

$F_n = ma_n = \frac{m v_{circ}^2}{r}$



$F_n = \frac{GMm}{r^2} = m\frac{v_{circ}^2}{r} \qquad v_{circ} = \sqrt{\frac{GM}{r}}$

VELOCITY AT A

$(v_A)_{circ} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{398.1 \times 10^{12}\ \frac{m^3}{s^2}}{(6.67 \times 10^6\ m)}} = 7.726 \times 10^3\ \frac{m}{s}$

$v_A = (v_A)_{circ} + (v_A)_R = 7.726 \times 10^3 + 3.44 \times 10^3 = 11.165 \times 10^3\ \frac{m}{s}$

VELOCITY AT C

CONSERVATION OF ENERGY $\quad T_A + V_A = T_C + V_C$

$\frac{1}{2}\cancel{m} v_A^2 - \frac{GM\cancel{m}}{r_A} = \frac{1}{2}\cancel{m} v_C^2 - \frac{GM\cancel{m}}{r_C}$

$v_C^2 = v_A^2 + 2GM\left(\frac{1}{r_C} - \frac{1}{r_A}\right) = (11.165 \times 10^3)^2 + 2(398.1 \times 10^{12})\left(\frac{1}{42.14 \times 10^6} - \frac{1}{6.67 \times 10^6}\right)$

$v_C^2 = 124.67 \times 10^6 - 100.48 \times 10^6 = 24.19 \times 10^6\ \frac{m^2}{s^2}$

$v_C = 4.919 \times 10^3$ m/s

CONSERVATION OF ANGULAR MOMENTUM

$r_A m v_A = r_C m v_C \cos\gamma$

$\cos\gamma = \frac{r_A v_A}{r_C v_C} = \frac{(6.67 \times 10^6)(11.165 \times 10^3)}{(42.14 \times 10^6)(4.919 \times 10^3)},$

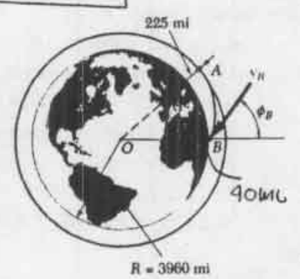
$\cos\gamma = 0.35926$

$\gamma = 68.9°$ ◀

13.109

GIVEN:

VEHICLE IN CIRCULAR ORBIT AT ALTITUDE OF 225 MI. SPEED DECREASED AT A SO THAT IT REACHES ALTITUDE AT B OF 40 MI AT AN ANGLE $\phi_B = 60°$

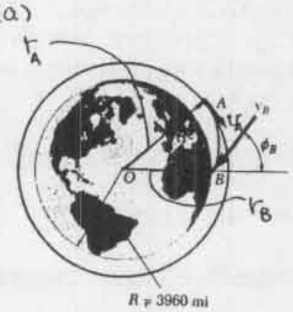


FIND:

(a) v_A, AS VEHICLE LEAVES ITS CIRCULAR ORBIT

(b) v_B

(a)



$r_A = 3960\text{ mi} + 225\text{ mi} = 4185\text{ mi}$

$r_A = 4185\text{ mi} \times 5280\frac{ft}{mi} = 22097 \times 10^3$ ft

$r_B = 3960\text{ mi} + 40\text{ mi} = 4000\text{ mi}$

$r_B = 4000 \times 5280 = 21120 \times 10^3$ ft

$R = 3960\text{ mi} = 20909 \times 10^3$ ft

$GM = gR^2 = (32.2\ \frac{ft}{s^2})(20909 \times 10^3\ ft)^2$

$GM = 14.077 \times 10^{15}\ ft^3/s^2$

CONSERVATION OF ENERGY

$T_A = \frac{1}{2} m v_A^2 \quad V_A = -\frac{GMm}{r_A} = -\frac{14.077 \times 10^{15}}{22097 \times 10^3} m = -637.1 \times 10^6 m$

$T_B = \frac{1}{2} m v_B^2 \quad V_B = -\frac{GMm}{r_B} = -\frac{14.077 \times 10^{15}}{21120 \times 10^3} m = -666.5 \times 10^6 m$

$T_A + V_A = T_B + V_B$

$\frac{1}{2}\cancel{m} v_A^2 - 637.1 \times 10^6 \cancel{m} = \frac{1}{2}\cancel{m} v_B^2 - 666.5 \times 10^6 \cancel{m}$

$v_A^2 = v_B^2 - 58.94 \times 10^6 \qquad (1)$

CONSERVATION OF ANGULAR MOMENTUM

$r_A m v_A = r_B m v_B \sin\phi_B$

$v_B = \frac{(r_A)\ v_A}{(r_B)(\sin\phi_B)} = \frac{4185}{4000}\left(\frac{1}{\sin 60°}\right) v_A$

$v_B = 1.208\ v_A \qquad (2)$

SUBSTITUTE v_B FROM (2) IN (1)

$v_A^2 = (1.208\ v_A)^2 - 58.94 \times 10^6$

$v_A^2[(1.208)^2 - 1] = 58.94 \times 10^6$

$v_A^2 = 128.27 \times 10^6\ ft^2/s^2$

(a) $\qquad v_A = 11.32 \times 10^3$ ft/s ◀

(b) FROM (2)

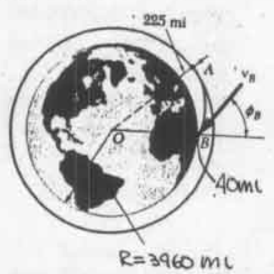
$v_B = 1.208\ v_A = 1.208\,(11.32 \times 10^6) = 13.68 \times 10^3$ ft/s

$v_B = 13.68 \times 10^3$ ft/s ◀

*13.110

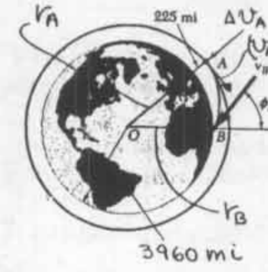
GIVEN:

VEHICLE AT A IN CIRCULAR ORBIT IS GIVEN AN INCREMENTAL VELOCITY Δv_A TOWARD O. ALTITUDES AS SHOWN ENERGY EXPENDITURE IS 50% OF THAT USED IN PROB 13.109



FIND:

v_B AND ϕ_B



$r_A = 3960\ mi + 225\ mi$

$r_A = 4185\ mi = 22.097\times10^6\ ft$

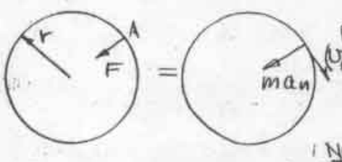
$r_B = 3960\ mi + 40\ mi = 4000\ mi$

$r_B = 21.120\times10^6\ ft$

$GM = gR^2 = (32.2\ \frac{ft}{s^2})[(3960)(5280)\ ft]^2$

$GM = 14.077\times10^{15}\ ft^3/s^2$

VELOCITY IN CIRCULAR ORBIT AT 225 MI ALTITUDE



$F = \frac{GMm}{r_A^2}$

$a_n = \frac{m(v_A)^2_{circ}}{r_A^2}$

NEWTONS SECOND LAW

$F = ma_n \quad \frac{GMm}{r_A^2} = m\frac{(v_A)^2_{circ}}{r_A}$

$(v_A)_{circ} = \sqrt{\frac{GM}{r_A}} = \sqrt{\frac{14.077\times10^{15}}{22.097\times10^6}} = 25.24\times10^3\ ft/s$

ENERGY EXPENDITURE

FROM PROB. 13.109 $v_A = 11.32\times10^3\ ft/s$

ENERGY, $\Delta E_{109} = \frac{1}{2}m(v_A)^2_{circ} - \frac{1}{2}mv_A^2$

$\Delta E_{109} = \frac{1}{2}m(25.24\times10^3)^2 - \frac{1}{2}m(11.32\times10^3)^2$

$\Delta E_{109} = 254.46\times10^6\ m\ ft\cdot lb$ (1)

$\Delta E_{110} = (0.50)\Delta E_{109} = (254.46\times10^6 m)/2\ ft\cdot lb$

THUS, ADDITIONAL KINETIC ENERGY AT A IS

$\frac{1}{2}m(\Delta v_A)^2 = \Delta E_{110} = (254.46\times10^6 m)/2$ (1)

CONSERVATION OF ENERGY BETWEEN A AND B

$T_A = \frac{1}{2}m[(v_A)^2_{circ} + (\Delta v_A)^2] \quad V_A = -\frac{GMm}{r_A}$

$T_B = \frac{1}{2}mv_B^2 \quad V_B = -\frac{GMm}{r_A}$

$T_A + V_A = T_B + V_B$

$\frac{1}{2}m(25.24\times10^3)^2 + \frac{254.46\times10^6}{2}m - \frac{14.077\times10^{15}\ m}{22.097\times10^6} =$

$\frac{1}{2}mv_B^2 - \frac{14.077\times10^{15} m}{21.120\times10^6}$

$v_B^2 = 637.4\times10^6 + 254.96\times10^6 - 1274.1\times10^6 + 1333\times10^6$

$v_B^2 = 950.4\times10^3$

$v_B = 30.8\times10^3\ ft/s$

CONSERVATION OF ANGULAR MOMENTUM BETWEEN A AND B

$r_A m(v_A)_{circ} = r_B m v_B \sin\phi_B$

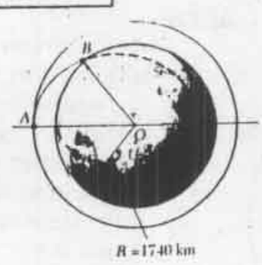
$\sin\phi_B = \left(\frac{r_A}{r_B}\right)\frac{(v_A)_{circ}}{(v_B)} = \frac{(4185)(25.24\times10^3)}{(4000)(30.8\times10^3)} = 0.8565$

$\phi_B = 58.9°$

13.111

GIVEN:

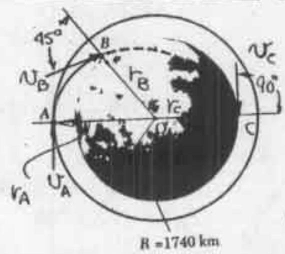
LEM AT AN ALTITUDE OF 140 km IS SET ADRIFT FROM A CIRCULAR ORBIT AND ITS SPEED IS REDUCED



FIND:

(a) SHALLEST REDUCTION OF SPEED TO MAKE SURE THE LEM WILL HIT THE MOON

(b) THE REDUCTION IN SPEED WHICH WILL CAUSE THE LEM TO HIT THE MOON AT 45°



$r_A = 1740\ km + 140\ km = 1880\times10^3\ m$

$r_B = r_C = R = 1740\ km = 1740\times10^3\ m$

$GM_{MOON} = 0.0123\ GM_E$

$= 0.0123\ gR_E^2$

$= (0.0123)(9.81\ m/s^2)(6.37\times10^6\ m)^2$

$GM_{MOON} = 4.896\times10^{12}\ m^3/s^2$

VELOCITY IN A CIRCULAR ORBIT AT 140 km ALTITUDE

$v_{circ}\sqrt{\frac{GM_{MOON}}{r_A}} = \sqrt{\frac{4.896\times10^{12}\ m^3/s^2}{1880\times10^3\ m}} = 1.6138\times10^3\ \frac{m}{s}$

(a) AN ELLIPTIC TRAJECTORY BETWEEN A AND C, WHERE THE LEM IS JUST TANGENT TO THE SURFACE OF THE MOON, WILL GIVE THE SMALLEST REDUCTION OF SPEED AT A WHICH WILL CAUSE IMPACT

CONSERVATION OF ENERGY (A AND C)

$T_A = \frac{1}{2}mv_A^2 \quad V_A = -\frac{GM_M m}{r_A} = -\frac{4.896\times10^{12}\ m}{1880\times10^3} = -2.604\times10^6 m$

$T_C = \frac{1}{2}mv_C^2 \quad V_C = -\frac{GM_M m}{r_C} = \frac{4.896\times10^{12} m}{1740\times10^3} \quad -2.814\times10^6 m$

$T_A + V_A = T_C + V_C \quad \frac{1}{2}mv_A^2 - 2.604\times10^6 m = \frac{1}{2}mv_C^2 - 2.814\times10^6 m$

$v_A^2 = v_C^2 - 419.1\times10^3$ (1)

CONSERVATION OF ANGULAR MOMENTUM (A AND C)

$r_A m v_A = r_C m v_C$

$v_C = \frac{r_A}{r_C}v_A = \frac{1880}{1740}v_A = 1.0805 v_A$ (2)

REPLACE v_C IN (1) BY (2)

$v_A^2 = (1.0805\, v_A)^2 - 419.1\times10^3$

$v_A^2[(1.0805)^2 - 1] = 419.1\times10^3 \quad v_A^2 = 2.502\times10^6$

$v_A = 1582\ m$

$\Delta v_A = (v_A)_{circ} - v_A = 1614 - 1582 = 31.5\ \frac{m}{s}$

(b) CONSERVATION OF ENERGY (A AND B)

SINCE $r_B = r_C$ CONS OF ENERGY IS THE SAME AS BETWEEN A AND C.

THUS FROM (1) $\quad v_A^2 = v_B^2 - 419.1\times10^3$ (1')

CONSERVATION OF ANGULAR MOMENTUM (A AND B)

$r_A m v_A = r_B m v_B \sin\phi \quad \phi = 45°$

$v_B = \frac{r_A v_A}{r_B \sin\phi} = \frac{1880 v_A}{1740 \sin 45°} = 1.528\, v_A$ (3)

REPLACE v_B IN (1') BY (3)

$v_A^2 = (1.528\, v_A^2) - 419\times10^3$

$v_A^2 = 313.98\times10^3 \quad v_A = 560\ \frac{m}{s}$

$\Delta v_A = (v_A)_{circ} - v_A = 1614 - 560 = 1053\ \frac{m}{s}$

*13.112

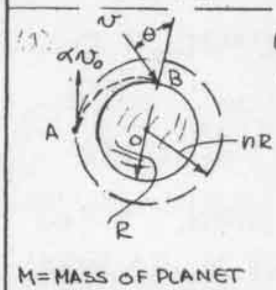
GIVEN:

SPACE PROBE IN CIRCULAR ORBIT OF RADIUS nR, WITH VELOCITY v_0 ABOUT A PLANET OF RADIUS R.

SHOW THAT:

(a) PROBE WILL HIT THE PLANET AT AN ANGLE θ WITH THE VERTICAL, IF ITS VELOCITY IS REDUCED TO αv_0 WHERE $\alpha = \sin\theta\sqrt{\frac{2(n-1)}{n^2-\sin^2\theta}}$

(b) PROBE WILL MISS THE PLANET IF $\alpha > \sqrt{\frac{2}{1+n}}$



(a) CONSERVATION OF ENERGY

AT A $\quad T_A = \frac{1}{2}m(\alpha v_0)^2$

$$V_A = -\frac{GMm}{nR}$$

AT B

$$T_B = \frac{1}{2}mv^2$$

$$V_B = -\frac{GMm}{R}$$

M = MASS OF PLANET

m = MASS OF PROBE

$T_A + V_A = T_B + V_B$

$$\frac{1}{2}\cancel{m}(\alpha v_0)^2 - \frac{GM\cancel{m}}{nR} = \frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{R} \quad (1)$$

CONSERVATION OF ANGULAR MOMENTUM

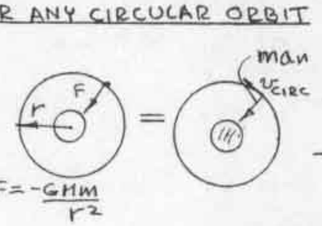
$$nRm\alpha v_0 = Rmv\sin\theta$$

$$v = \frac{n\alpha v_0}{\sin\theta} \quad (2)$$

REPLACE v IN (1) BY (2)

$$(\alpha v_0)^2 - \frac{2GM}{nR} = \left(\frac{n\alpha v_0}{\sin\theta}\right)^2 - \frac{2GM}{R} \quad (3)$$

FOR ANY CIRCULAR ORBIT



$F = -\frac{GMm}{r^2}$

$$a_n = \frac{v^2}{r}$$

NEWTONS SECOND LAW

$$-\frac{GMm}{r^2} = \frac{m(v)^2_{circ}}{r}$$

$$v_{circ} = \sqrt{\frac{GM}{r}}$$

FOR $r = nR \quad v_0 = v_{circ} = \sqrt{\frac{GM}{nR}}$

SUBSTITUTE FOR v_0 IN (3)

$$\alpha^2\frac{GM}{nR} - \frac{2GM}{nR} = \frac{n^2\alpha^2}{\sin^2\theta}\left(\frac{GM}{nR}\right) - \frac{2GM}{R}$$

$$\alpha^2\left[1 - \frac{n^2}{\sin^2\theta}\right] = 2(1-n)$$

$$\alpha^2 = \frac{2(1-n)(\sin^2\theta)}{(\sin^2\theta - n^2)} = \frac{2(n-1)\sin^2\theta}{(n^2 - \sin^2\theta)}$$

$$\alpha = \sin\theta\sqrt{\frac{2(n-1)}{n^2-\sin^2\theta}} \quad \text{(QED)}$$

(b) PROBE WILL JUST MISS THE PLANET IF $\theta \geqslant 90°$,

$$\alpha = \sin 90°\sqrt{\frac{2(n-1)}{n^2-\sin^2 90°}} = \sqrt{\frac{2}{n+1}}$$

NOTE: $n^2 - 1 = (n-1)(n+1)$

13.113

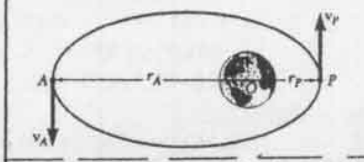
GIVEN:

v_P AND v_A AS SHOWN

SHOW THAT:

$$v_A^2 = \frac{2GM}{r_A + r_P}\frac{r_P}{r_A}$$

$$v_P^2 = \frac{2GM}{r_A + r_P}\frac{r_A}{r_P}$$



CONSERVATION OF ANGULAR MOMENTUM

$$r_A m v_A = r_P m v_P \qquad v_A = \frac{r_P}{r_A}v_P \quad (1)$$

CONSERVATION OF ENERGY

$$\frac{1}{2}mv_P^2 - \frac{GMm}{r_P} = \frac{1}{2}mv_A^2 - \frac{GMm}{r_A} \quad (2)$$

SUBSTITUTING FOR v_A FROM (1) INTO (2)

$$v_P^2 - \frac{2GM}{r_P} = \left(\frac{r_P}{r_A}\right)^2 v_P^2 - \frac{2GM}{r_A}$$

$$\left(1 - \left(\frac{r_P}{r_A}\right)^2\right)v_P^2 = 2GM\left(\frac{1}{r_P} - \frac{1}{r_A}\right)$$

$$\frac{r_A^2 - r_P^2}{r_A^2}v_P^2 = 2GM\frac{r_A - r_P}{r_A r_P}$$

WITH $r_A^2 - r_P^2 = (r_A - r_P)(r_A + r_P)$

$$v_P^2 = \frac{2GM}{r_A + r_P}\frac{r_A}{r_P} \quad (3)$$

EXCHANGING SUBSCRIPTS P AND A

$$v_A^2 = \frac{2GM}{r_A + r_P}\frac{r_P}{r_A} \quad \text{(QED)}$$

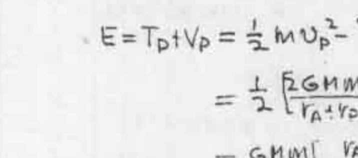
13.114

GIVEN:

EARTH SATELLITE OF MASS m DESCRIBING AN ELLIPTIC ORBIT r_A IS MAXIMUM AND r_P IS MINIMUM DISTANCES TO EARTHS CENTER

SHOW THAT:

TOTAL ENERGY $E = -\frac{GMm}{r_A + r_P}$, WHERE M = MASS OF THE EARTH



SEE SOLUTION TO PROB 13.113 (ABOVE) FOR DERIVATION OF EQUATION (3)

$$v_P^2 = \frac{2GM}{(r_A + r_P)}\frac{r_A}{r_P}$$

TOTAL ENERGY AT POINT P IS

$$E = T_P + V_P = \frac{1}{2}mv_P^2 - \frac{GMm}{r_P}$$

$$= \frac{1}{2}\left[\frac{2GMm}{r_A + r_P}\frac{r_A}{r_P}\right] - \frac{GMm}{r_P}$$

$$= GMm\left[\frac{r_A}{r_P(r_A + r_P)} - \frac{1}{r_P}\right] = GMm\frac{(r_A - r_P - r_P)}{r_P(r_A + r_P)}$$

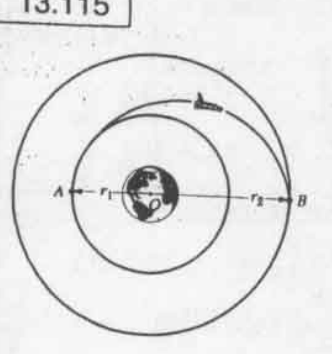
$$E = -\frac{GMm}{r_A + r_P}$$

NOTE: RECALL THAT GRAVITATIONAL POTENTIAL OF A SATELLITE IS DEFINED AS BEING ZERO AT AN INFINITE DISTANCE FROM THE EARTH

13.115

GIVEN:

SPACECRAFT OF MASS m IN CIRCULAR ORBIT OF RADIUS r_1 ABOUT THE EARTH

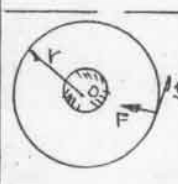


SHOW THAT:

(a) ADDITIONAL ENERGY ΔE TO TRANSFER IT TO A CIRCULAR ORBIT OF LARGER RADIUS r_2 IS $\Delta E = \dfrac{GMm(r_2 - r_1)}{2r_1 r_2}$

(b) AMOUNTS OF ENERGY AT A AND B ARE

$\Delta E_A = \dfrac{r_2}{r_1+r_2}\Delta E$, $\Delta E_B = \dfrac{r_1}{r_1+r_2}\Delta E$



(a) FOR A CIRCULAR ORBIT OF RADIUS r

$F = ma_n$: $\dfrac{GMm}{r^2} = m\dfrac{v^2}{r}$

$v^2 = \dfrac{GM}{r}$

$E = T + V = \frac{1}{2}mv^2 - \dfrac{GMm}{r} = -\dfrac{1}{2}\dfrac{GMm}{r}$ (1)

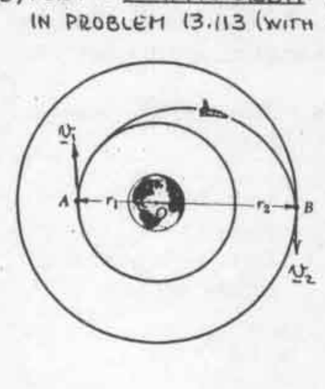
THUS ΔE REQUIRED TO PASS FROM CIRCULAR ORBIT OF RADIUS r_1 TO CIRCULAR ORBIT OF RADIUS r_2 IS

$$\Delta E = E_1 - E_2 = -\frac{1}{2}\frac{GMm}{r_1} + \frac{1}{2}\frac{GMm}{r_2}$$

$$\Delta E = \frac{GMm(r_2 - r_1)}{2r_1 r_2} \quad (2)\ (\text{Q.E.D})$$

(b) FOR AN ELLIPTIC ORBIT WE RECALL EQ(3) DERIVED IN PROBLEM 13.113 (WITH $v_p = v_1$)

$$v_1^2 = \frac{2GM}{r_1+r_2}\frac{r_2}{r_1}$$



AT POINT A: INITIALLY SPACECRAFT IS IN A CIRCULAR ORBIT OF RADIUS r_1

$v_{circ}^2 = \dfrac{GM}{r_1}$

$T_{circ} = \frac{1}{2}mv_{circ}^2 = \frac{1}{2}m\dfrac{GM}{r_1}$

AFTER THE SPACECRAFT ENGINES ARE FIRED AND IT IS PLACED ON A SEMI-ELLIPTIC PATH AB, WE RECALL

$$v_1^2 = \frac{2GM}{(r_1+r_2)}\cdot\frac{r_2}{r_1}$$

AND

$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}m\frac{2GMr_2}{r_1(r_1+r_2)}$$

AT POINT A, THE INCREASE IN ENERGY IS

$$\Delta E_A = T_1 - T_{circ} = \frac{1}{2}m\frac{2GMr_2}{r_1(r_1+r_2)} - \frac{1}{2}m\frac{GM}{r_1}$$

$$\Delta E_A = \frac{GMm(2r_2 - r_1 - r_2)}{2r_1(r_1+r_2)} = \frac{GMm(r_2 - r_1)}{2r_1(r_1+r_2)}$$

$$\Delta E_A = \frac{r_2}{r_1+r_2}\left[\frac{GMm(r_2-r_1)}{2r_1r_2}\right]$$

RECALL EQ (2): $\Delta E_A = \dfrac{r_2}{(r_1+r_2)}\Delta E$ (Q.E.D)

A SIMILAR DERIVATION AT POINT B YIELDS, $\Delta E_B = \dfrac{r_1}{(r_1+r_2)}\Delta E$ (QED)

13.116

GIVEN:

MISSILE FIRED FROM THE GROUND WITH VELOCITY v_0 AT AN ANGLE ϕ_0 WITH THE VERTICAL, REACHES A MAXIMUM ALTITUDE αR WHERE R IS THE RADIUS OF THE EARTH

SHOW THAT:

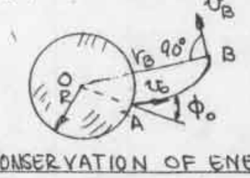
$$\text{(a)}\ \sin\phi_0 = (1+\alpha)\sqrt{1 - \frac{\alpha}{1+\alpha}\left(\frac{v_{esc}}{v_0}\right)^2}$$

WHERE v_{esc} = ESCAPE VELOCITY

FIND:

(b) RANGE OF ALLOWABLE VALUES OF v_0

(a)



$r_A = R$

CONSERVATION OF ANG. MOM.

$Rmv_0\sin\phi_0 = r_B m v_B$

$r_B = R + \alpha R = (1+\alpha)R$

$$v_B = \frac{Rv_0\sin\phi_0}{(1+\alpha)R} = \frac{v_0\sin\phi_0}{(1+\alpha)} \quad (1)$$

CONSERVATION OF ENERGY

$T_A + V_A = T_B + V_B$ $\quad \frac{1}{2}mv_0^2 - \dfrac{GMm}{R} = \frac{1}{2}mv_B^2 - \dfrac{GMm}{(1+\alpha)R}$

$$v_0^2 - v_B^2 = \frac{2GMm}{R}\left(1 - \frac{1}{1+\alpha}\right) = \frac{2GMm}{R}\left(\frac{\alpha}{1+\alpha}\right)$$

SUBSTITUTE FOR v_B FROM (1)

$$v_0^2\left(1 - \frac{\sin^2\phi_0}{(1+\alpha)^2}\right) = \frac{2GMm}{R}\left(\frac{\alpha}{1+\alpha}\right)$$

FROM EQ.(12.43): $v_{esc}^2 = \dfrac{2GM}{R}$

$$v_0^2\left[1 - \frac{\sin^2\phi_0}{(1+\alpha)^2}\right] = v_{esc}^2\left(\frac{\alpha}{1+\alpha}\right)$$

$$\frac{\sin^2\phi_0}{(1+\alpha)^2} = 1 - \left(\frac{v_{esc}}{v_0}\right)^2\frac{\alpha}{1+\alpha} \quad (2)$$

$$\sin\phi_0 = (1+\alpha)\sqrt{1 - \frac{\alpha}{1+\alpha}\left(\frac{v_{esc}}{v_0}\right)^2} \quad \text{Q.E.D.}$$

(b) ALLOWABLE VALUES OF v_0 (FOR WHICH MAXIMUM ALTITUDE $= \alpha R$)

$$0 \le \sin^2\phi_0 \le 1$$

FOR $\sin\phi_0 = 0$, FROM (2)

$$0 = 1 - \left(\frac{v_{esc}}{v_0}\right)^2\frac{\alpha}{1+\alpha}$$

$$v_0 = v_{esc}\sqrt{\frac{\alpha}{1+\alpha}}$$

FOR $\sin\phi_0 = 1$, FROM 2

$$\frac{1}{(1+\alpha)^2} = 1 - \left(\frac{v_{esc}}{v_0}\right)^2\frac{\alpha}{1+\alpha}$$

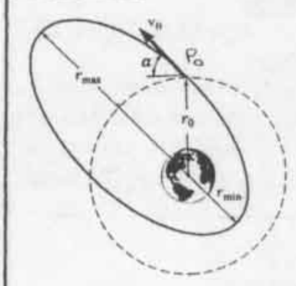
$$\left(\frac{v_{esc}}{v_0}\right)^2 = \frac{1}{\alpha}\left(1 + \alpha - \frac{1}{1+\alpha}\right) = \frac{1 + 2\alpha + \alpha^2 - 1}{\alpha(1+\alpha)} = \frac{2+\alpha}{1+\alpha}$$

$$v_0 = v_{esc}\sqrt{\frac{1+\alpha}{2+\alpha}}$$

$$v_{esc}\sqrt{\frac{\alpha}{1+\alpha}} \le v_0 \le v_{esc}\sqrt{\frac{1+\alpha}{2+\alpha}}$$

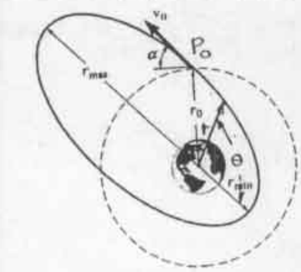
*13.117

GIVEN:



FROM PROB. 13.107
$r_{min} = r_0(1-\sin\alpha)$
$r_{max} = r_0(1+\sin\alpha)$

SHOW THAT:
INTENDED CIRCULAR ORBIT AND RESULTING ELLIPTIC ORBIT INTERSECT AT THE ENDS OF THE MINOR AXIS OF THE ELLIPTIC ORBIT AT P_0



IF THE POINT OF INTERSECTION P_0 OF THE CIRCULAR AND ELLIPTIC ORBITS IS AT AN END OF THE MINOR AXIS, THEN v_0 IS PARALLEL TO THE MAJOR AXIS. THIS WILL BE THE CASE ONLY IF $\alpha + 90° = \theta_0$, THAT IS IF $\cos\theta_0 = -\sin\alpha$. WE MUST THEREFORE PROVE THAT

$$\cos\theta_0 = -\sin\alpha \quad (1)$$

WE RECALL FROM EQ (12.39):

$$\frac{1}{r} = \frac{GM}{h^2} + C\cos\theta \quad (2)$$

WHEN $\theta = 0$, $r = r_{min}$ AND $r_{min} = r_0(1-\sin\alpha)$

$$\frac{1}{r_0(1-\sin\alpha)} = \frac{GM}{h^2} + C \quad (3)$$

FOR $\theta = 180°$, $r = r_{max} = r_0(1+\sin\alpha)$

$$\frac{1}{r_0(1+\sin\alpha)} = \frac{GM}{h^2} - C \quad (4)$$

ADDING (3) AND (4) AND DIVIDING BY 2:

$$\frac{GM}{h^2} = \frac{1}{2r_0}\left(\frac{1}{1-\sin\alpha} + \frac{1}{1+\sin\alpha}\right) = \frac{1}{r_0\cos^2\alpha}$$

SUBTRACTING (4) FROM (3) AND DIVIDING BY 2:

$$C = \frac{1}{2r_0}\left(\frac{1}{1-\sin\alpha} - \frac{1}{1+\sin\alpha}\right) = \frac{1}{2r_0}\frac{2\sin\alpha}{1-\sin^2\alpha}$$

$$C = \frac{\sin\alpha}{r_0\cos^2\alpha}$$

SUBSTITUTE FOR $\frac{GM}{h^2}$ AND C INTO EQ (2)

$$\frac{1}{r} = \frac{1}{r_0\cos^2\alpha}(1+\sin\alpha\cos\theta) \quad (5)$$

LETTING $r = r_0$ AND $\theta = \theta_0$ IN EQ (5), WE HAVE

$$\cos^2\alpha = 1 + \sin\alpha\cos\theta_0$$

$$\cos\theta_0 = \frac{\cos^2\alpha - 1}{\sin\alpha} = -\frac{\sin^2\alpha}{\sin\alpha} = -\sin\alpha$$

THIS PROVES THE VALIDITY OF EQ (1) AND THUS P_0 IS AN END OF THE MINOR AXIS OF THE ELLIPTIC ORBIT

*13.118

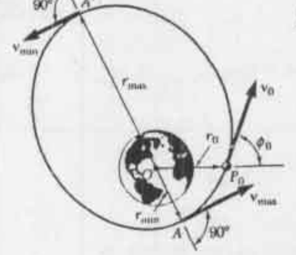
GIVEN:
SPACE VEHICLE UNDER GRAVITATIONAL ATTRACTION OF A PLANET OF MASS M (FIG. 13.15, SHOWN BELOW)

FIND:
(a) TOTAL ENERGY PER UNIT MASS, E/m, IN TERMS OF r_{min} AND v_{max} AND THE ANGULAR MOMENTUM PER UNIT MASS h.

DERIVE:
(b) $\frac{1}{r_{min}} = \frac{GM}{h^2}\left[1+\sqrt{1+\frac{2E}{m}\left(\frac{h}{GM}\right)^2}\right]$

SHOW THAT:
(c) ECCENTRICITY $\varepsilon = \sqrt{1+\frac{2E}{m}\left(\frac{h}{GM}\right)^2}$

(d) TRAJECTORY IS a,
HYPERBOLA IF $E>0$
ELLIPSE IF $E=0$
PARABOLA IF $E<0$



(a) POINT A
ANGULAR MOMENTUM PER UNIT MASS

$$h = \frac{H_O}{m} = \frac{r_{min}\, m v_{max}}{m}$$

$$h = r_{min} v_{max} \quad (1)$$

(b) ENERGY PER UNIT MASS

$$E/m = \frac{1}{m}(T+V)$$

$$E/m = \frac{1}{m}\left(\frac{1}{2}mv_{max}^2 - \frac{GMm}{r_{min}}\right) = \frac{1}{2}v_{max}^2 - \frac{GM}{r_{min}} \quad (2)$$

(b) FROM EQ. (1): $v_{max} = h/r_{min}$. SUBSTITUTING INTO (2)

$$E/m = \frac{1}{2}\frac{h^2}{r_{min}^2} - \frac{GM}{r_{min}}$$

$$\left(\frac{1}{r_{min}}\right)^2 - \frac{2GM}{h^2}\cdot\frac{1}{r_{min}} - \frac{2(E/m)}{h^2} = 0$$

SOLVING THE QUADRATIC: $\frac{1}{r_{min}} = \frac{GM}{h^2} + \sqrt{\left(\frac{GM}{h^2}\right)^2 + \frac{2(E/m)}{h^2}}$

REARRANGING

$$\frac{1}{r_{min}} = \frac{GM}{h^2}\left[1+\sqrt{1+\frac{2E}{m}\left(\frac{h}{GM}\right)^2}\right] \quad (3)$$

(c) ECCENTRICITY OF THE TRAJECTORY

EQ (12.39') $\frac{1}{r} = \frac{GM}{h^2}(1+\varepsilon\cos\theta)$

WHEN $\theta = 0$, $\cos\theta = 1$ AND $r = r_{min}$, THUS

$$\frac{1}{r_{min}} = \frac{GM}{h^2}(1+\varepsilon) \quad (4)$$

COMPARING (3) AND (4) $\quad \varepsilon = \sqrt{1+\frac{2E}{m}\left(\frac{h}{GM}\right)^2} \quad (5)$

(d) RECALLING DISCUSSION ON PAGES 708, 709 AND IN VIEW OF EQ. (5)

1. HYPERBOLA IF $\varepsilon > 1$, THAT IS IF $E>0$
2. PARABOLA IF $\varepsilon = 1$, THAT IS IF $E=0$
3. ELLIPSE IF $\varepsilon < 1$, THAT IS IF $E<0$

NOTE: FOR CIRCULAR ORBIT $\varepsilon = 0$ AND
$1+\frac{2E}{m}\left(\frac{h}{GM}\right)^2 = 0$ OR $E = -\left(\frac{GM}{h}\right)^2\frac{m}{2}$

BUT FOR CIRCULAR ORBIT $v^2 = \frac{GM}{r}$ AND $h^2 = v^2r^2 = GMr$
THUS $E = -\frac{1}{2}\frac{m(GM)^2}{GMr} = -\frac{1}{2}\frac{GMm}{r}$ (CHECKS WITH (1) FOUND IN 13.115)

13.119

GIVEN:

PARTICLE MASS $m = 1.6$ kg, ACTED UPON BY A FORCE $\underline{F} = (10\sin 2t)\underline{i} + (12\cos 2t)\underline{j}$ (F IN NEWTONS, t IN SECONDS)

VELOCITY OF PARTICLE, $v_0 = 0$ AT $t = 0$

FIND:

MAGNITUDE AND DIRECTION OF $\underline{v}$ AT $t = 4$ s

$$m\underline{v}_0 + \int_0^4 \underline{F}\,dt = m\underline{v}_4 \qquad \text{FOR } t = 4 \text{ SEC}, \quad v_0 = 0$$

$$0 + \int_0^4 [(10\sin 2t)\underline{i} + (12\cos 2t)\underline{j}]\,dt = 1.6\underline{v}_4$$

$$0 + \left[-5\cos 2t\,\underline{i} + 6\sin 2t\,\underline{j}\right]_0^4 = 1.6\underline{v}_4$$

$$(-5\cos 8 + \cos 0)\underline{i} + (6\sin 8 - 0)\underline{j} = 1.6\underline{v}_4$$

$$\underline{v}_4 = 3.58\underline{i} + 3.71\underline{j}$$

$$|\underline{v}_4| = 5.16 \text{ m/s}$$

$$\theta = \tan^{-1}\frac{3.71}{3.58} = 46.0^\circ$$

13.120

GIVEN:

5-lb PARTICLE ACTED UPON BY A FORCE $\underline{F} = -2t^2\underline{i} + (3-t)\underline{j}$ (F IN POUNDS AND t IN SECONDS)

VELOCITY OF THE PARTICLE IS $\underline{v}_0 = (10 \text{ ft/s})\underline{i}$ AT $t = 0$.

FIND:

(a) TIME AT WHICH VELOCITY IS PARALLEL TO THE y-AXIS

(b) THE CORRESPONDING VELOCITY

(a) $m\underline{v}_0 + \int_0^t \underline{F}\,dt = m(v_x\underline{i} + v_y\underline{j})$

BUT $v_x = 0$, IF VELOCITY IS PARALLEL TO y-AXIS

$$\left(\frac{5}{g}\right)10\underline{i} + \int_0^t [-2t^2\underline{i} + (3-t)\underline{j}]\,dt = \frac{5}{g}v_y\underline{j}$$

$$\frac{50}{g}\underline{i} - \left(\frac{2t^3}{3}\right)\underline{i} + \left(3t - \frac{t^2}{2}\right)\underline{j} = \frac{5}{g}v_y\underline{j} \qquad (1)$$

$$\left(\frac{50}{g} - \frac{2t^3}{3}\right)\underline{i} + \left(3t - \frac{t^2}{2}\right)\underline{j} = \frac{5}{g}v_y\underline{j}$$

SINCE THE X COMPONENT OF THE VELOCITY IS ZERO

$$\frac{50}{32.2} - \frac{2t^3}{3} = 0 \qquad t^3 = 2.329$$

$$t = 1.326 \text{ s}$$

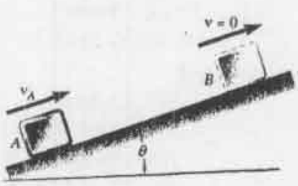
(b) SUBSTITUTE $t = 1.326$ IN (1)

$$0\underline{i} + \left[3(1.326) - \frac{(1.326)^2}{2}\right]\underline{j} = \frac{5}{32.2}v_y\underline{j}$$

$$3.098\underline{j} = 0.1553 v_y\underline{j}$$

$$v_y = 19.95 \text{ ft/s}$$

13.121



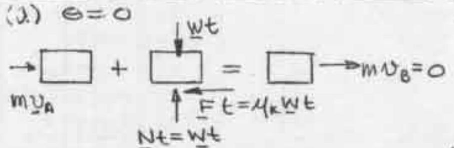
GIVEN:

$v_A = 30 \frac{\text{ft}}{\text{s}}$, $\mu_k = 0.30$

FIND:

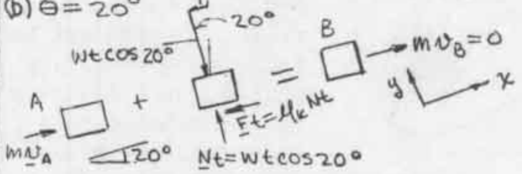
TIME FOR THE BLOCK TO REACH B WHERE $v_B = 0$, IF (a) $\theta = 0$, (b) $\theta = 20°$

(a) $\theta = 0$



$$\frac{W}{g}v_A - \mu_k Wt = 0 \qquad t = \frac{v_A}{g\mu_k} = \frac{(30\frac{\text{ft}}{\text{s}})}{(32.2\frac{\text{ft}}{\text{s}^2})(.30)} = 3.11 \text{ s}$$

(b) $\theta = 20°$



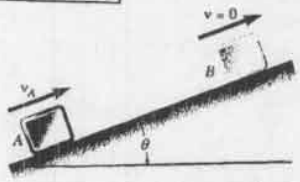
IMPULSE-MOMENTUM IN X DIRECTION

$$+\nearrow \ \frac{W}{g}v_A - \mu_k Wt\cos 20° - Wt\sin 20° = 0$$

$$t = \frac{v_A}{g(\mu_k\cos 20° + \sin 20°)} = \frac{30 \text{ ft/s}}{(32.2 \text{ ft/s}^2)(.30\cos 20° + \sin 20°)}$$

$$t = 1.493 \text{ s}$$

13.122



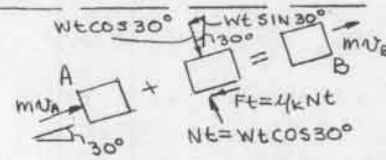
GIVEN:

$v_A = 10$ m/s, $\mu_k = 0.30$

$\theta = 30°$

FIND:

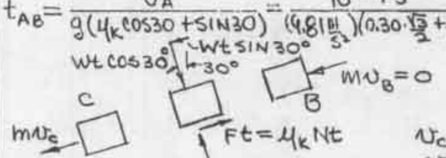
TIME FOR THE BLOCK TO REACH $v = 10$ m/s DOWN AND TO THE LEFT



UP THE PLANE TO B

$$\overset{+}{\nearrow} \ \frac{W}{g}v_A - \mu_k W t_{AB}\cos 30° - Wt_{AB}\sin 30 = 0$$

$$t_{AB} = \frac{v_A}{g(\mu_k\cos 30 + \sin 30)} = \frac{10 \text{ m/s}}{(9.81\frac{\text{m}}{\text{s}^2})(0.30\cdot\frac{\sqrt{3}}{2} + \frac{1}{2})} = 1.342 \text{ s}$$



DOWN THE PLANE TO C

$v_C = 10$ m/s $\measuredangle 30°$

$v_B = 0$

$$\overset{+}{\nearrow} \ -\frac{W}{g}v_B + \mu_k W t_{BC}\cos 30° - Wt_{BC}\sin 30° = -\frac{W}{g}v_C$$

$$t_{BC} = \frac{10 \text{ m/s}}{g(\sin 30° - \mu_k\cos 30°)} = \frac{10 \text{ m/s}}{(9.81\frac{\text{m}}{\text{s}^2})(\frac{1}{2} - .3(\frac{\sqrt{3}}{2}))} = 4.244 \text{ s}$$

$$t = t_{AB} + t_{BC} = 1.342 + 4.244 = 5.59 \text{ s}$$

13.123

GIVEN:

REAR (DRIVE) WHEELS OF A CAR SLIP FOR FIRST 60 ft WITH FRONT WHEELS JUST OFF THE GROUND. $\mu_k = 0.60$

WHEELS ROLL WITHOUT SLIPPING FOR THE REMAINING 1260 ft WITH 60 % OF THE WEIGHT ON THE REAR WHEELS. $\mu_s = 0.85$

IGNORE AIR AND ROLLING RESISTANCE

FIND:

(a) SHORTEST TIME FOR THE CAR TO TRAVEL THE FIRST 60 ft STARTING FROM REST

(b) MINIMUM TIME FOR THE CAR TO RUN THE WHOLE RACE

(a) FIRST 60 ft

VELOCITY AT 60 ft REAR WHEELS SKID TO GENERATE THE MAXIMUM FORCE RESULTING IN MAXIMUM VELOCITY AND MINIMUM TIME SINCE ALL THE WEIGHT IS ON THE REAR WHEELS THIS FORCE IS $F = \mu_k N = 0.60\,W$

WORK AND ENERGY: $T_0 + U_{0-60} = T_{60}$

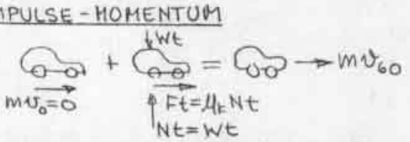
$T_0 = 0 \quad U_{0-60} = (F)(60) \quad T_{60} = \frac{1}{2} m v_{60}^2$

$0 + (\mu_k W)(60) = \frac{1}{2}\frac{W}{g} v_{60}^2$

$v_{60}^2 = (2)(0.60)(60\text{ ft})(32.2\text{ ft/s}^2)$

$v_{60} = 48.15\text{ ft/s}$

IMPULSE - MOMENTUM



$\xrightarrow{+}\ 0 + \mu_k W t_{0-60} = \frac{W}{g} v_{60} \qquad v_{60} = 48.15\text{ ft/s}$

$t_{0-60} = \dfrac{48.15\text{ ft/s}}{(0.60)(32.2\text{ ft/s}^2)}$

$t_{0-60} = 2.49\text{ s}$

(b) FOR THE WHOLE RACE

THE MAXIMUM FORCE ON THE WHEELS FOR THE FIRST 60 ft IS $F = \mu_k W = 0.60\,W$

FOR REMAINING 1260 ft THE MAXIMUM FORCE IF THERE IS NO SLIDING AND 60 % OF THE WEIGHT IS ON THE REAR (DRIVE) WHEELS IS

$F = \mu_s(0.60)W = (0.85)(0.60)\,W = 0.510\,W$

VELOCITY AT 1320 ft

WORK AND ENERGY $T_0 + U_{0-60} + U_{60-1320} = T_{1320}$

$T_0 = 0 \quad U_{0-60} = (0.60\,W)(60\text{ ft}),\ U_{60-1320} = (0.510\,W)(1260\text{ ft})$

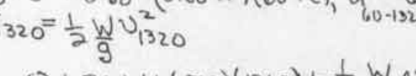
$T_{1320} = \frac{1}{2}\frac{W}{g} v_{1320}^2$

$0 + 36\,W + (.510)(1260)W = \frac{1}{2}\frac{W}{g} v_{1320}^2$

$v_{1320} = 209.0\text{ ft/s}$

IMPULSE-MOMENTUM

FROM 60 ft TO 1320 ft



$F = \mu_s N = 0.510\,W$

$v_{60} = 48.15\text{ ft/s}$

$v_{1320} = 209\text{ ft/s}$

$\left(\frac{W}{g}\right)(48.15) + 0.510\,W t_{60-1320} = \frac{W}{g}(209);\ t_{60-1320} = 9.795\text{ s}$

$t_{0-1320} = t_{0-60} + t_{60-1320} = 2.49 + 9.80 = 12.29\text{ s}$

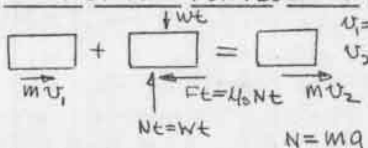
13.124

GIVEN:

TRUCK ON LEVEL ROAD TRAVELING AT 90 km/h BRAKES ARE APPLIED TO SLOW IT TO 30 km/h ANTI SKID BRAKING SYSTEM LIMITS BRAKING FORCE SO THAT WHEELS ARE AT IMPENDING SLIDING. $\mu_s = 0.65$

FIND:

SHORTEST TIME FOR TRUCK TO SLOW DOWN



$v_1 = 90\text{ km/h} = 25\text{ m/s}$

$v_2 = 30\text{ km/h} = 8.33\text{ m/s}$

$N = mg$

$mv_1 - \mu_s Nt = m v_2$

$m(25\text{ m/s}) - (0.65)m(9.81\frac{\text{m}}{\text{s}^2})t = m(8.33\text{ m/s})$

$t = \dfrac{25 - 8.33}{(0.65)(9.81)} = 2.61\text{ s}$

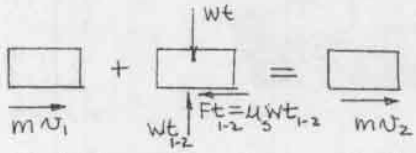
13.125

GIVEN:

TRAIN DECREASES SPEED FROM 200 km/h TO 90 km/h AT A CONSTANT RATE IN 12 S.

FIND:

SMALLEST ALLOWABLE COEFFICIENT OF FRICTION IF A TRUNK IS NOT TO SLIDE



$t_{1-2} = 12\text{ s}$

$v_1 = 200\text{ km/h} = 55.56\text{ m/s} \qquad v_2 = 90\text{ km/h} = 25.0\text{ m/s}$

$\xrightarrow{+}\ \cancel{m}v_1 - \mu_s \cancel{m} g\, t_{1-2} = \cancel{m} v_2$

$(55.56\,\frac{\text{m}}{\text{s}}) - \mu_s(9.81\frac{\text{m}}{\text{s}^2})(12\text{ s}) = 25\text{ m/s}$

$\mu_s = \dfrac{(55.56 - 25.0)}{(9.81)(12)} = 0.2596 \qquad \mu_s = 0.260$

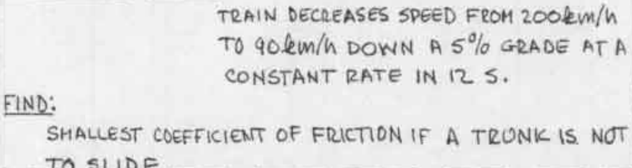
13.126

GIVEN:

TRAIN DECREASES SPEED FROM 200 km/h TO 90 km/h DOWN A 5% GRADE AT A CONSTANT RATE IN 12 S.

FIND:

SMALLEST COEFFICIENT OF FRICTION IF A TRUNK IS NOT TO SLIDE



$v_1 = 200\text{ km/h} = 55.56\,\frac{\text{m}}{\text{s}}$

$v_2 = 90\text{ km/h} = 25.0\,\frac{\text{m}}{\text{s}}$

$t_{1-2} = 12\text{ s}$

$\theta = \tan^{-1}.05 = 2.86°$

$\xrightarrow{+}\ \cancel{m}v_1 - \mu_s \cancel{m} g\, t_{1-2}\cos\theta + \cancel{m} g t_{1-2}\sin\theta = \cancel{m} v_2$

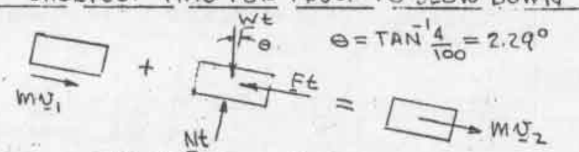
$(55.56\,\frac{\text{m}}{\text{s}}) - \mu_s(9.81\,\frac{\text{m}}{\text{s}^2})(12\text{ s})(\cos 2.86°) + (9.81\,\frac{\text{m}}{\text{s}^2})(12\text{ s})(\sin 2.86°) = 25\,\frac{\text{m}}{\text{s}}$

$\mu_s = \dfrac{55.56 - 25.0 + (9.81)(12)(\sin 2.86°)}{(9.81)(12)(\cos 2.86°)} = 0.310$

13.127

GIVEN:
TRUCK SLOWS FROM 60 mi/h TO 20 mi/h DOWN A 4% GRADE WITH ITS WHEELS JUST ABOUT TO SLIDE. $\mu_s = 0.60$

FIND:
SHORTEST TIME FOR TRUCK TO SLOW DOWN



$\theta = \tan^{-1}\frac{4}{100} = 2.29°$

$$m\underline{v}_1 + \sum \text{IMP}_{1-2} = m\underline{v}_2$$

$$+\searrow \quad mv_1 + Wt\sin\theta - Ft = mv_2$$

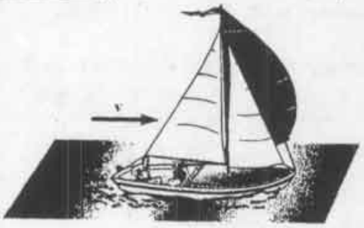
$v_1 = 60\text{ mi/h} = 88\text{ ft/s}$ $\quad N = W\cos\theta \quad W = mg$

$v_2 = 20\text{ mi/h} = 29.33\text{ ft/s}$ $\quad F = \mu_s N = \mu_s W\cos\theta$

$$(m)(88\tfrac{ft}{s}) + (m)(32.2\tfrac{ft}{s^2})(t)(\sin 2.29°) - (0.60)(m)(32.2\tfrac{ft}{s^2})(\cos 2.29°)(t) = (m)(29.33\tfrac{ft}{s})$$

$$t = \frac{88.0 - 29.33}{32.2[(0.60)\cos 2.29° - \sin 2.29°]} = 3.26\text{ s}$$

13.128

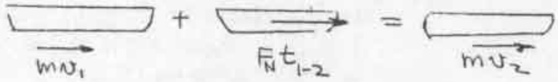


GIVEN:
INITIAL BOAT SPEED = $v_1 = 8$ mi/h.
BOAT SPEED 10 SEC AFTER SPINNAKER IS RAISED = $v_2 = 12$ mi/h.
$W = 980$ lb

FIND:
NET FORCE PROVIDED BY THE SPINNAKER OVER THE 10 SEC. INTERVAL

$v_1 = 8\text{ mi/h} = 11.73\text{ ft/s}$ $\qquad t_{1-2} = 10\text{ sec}$

$v_2 = 12\text{ mi/h} = 17.60\text{ ft/s}$



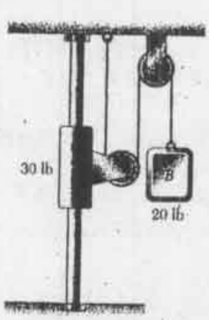
$$mv_1 + \text{IMP}_{1-2} = mv_2$$

$$m(11.73\text{ ft/s}) + F_N(10\text{ s}) = m(17.60\text{ ft/s})$$

$$F_N = \frac{(980\text{ lb})(17.60\text{ ft/s} - 11.73\text{ ft/s})}{(32.2\text{ ft/s}^2)(10\text{ s})} = 17.86\text{ lb}$$

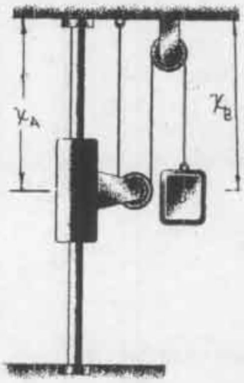
NOTE:
F_N IS THE NET FORCE PROVIDED BY THE SAILS. THE FORCE ON THE SAILS IS ACTUALLY GREATER AND INCLUDES THE FORCE NEEDED TO OVERCOME THE WATER RESISTANCE ON THE HULL.

13.129



GIVEN:
SYSTEM RELEASED FROM REST.

FIND:
TIME FOR A TO REACH A VELOCITY OF 2 ft/s



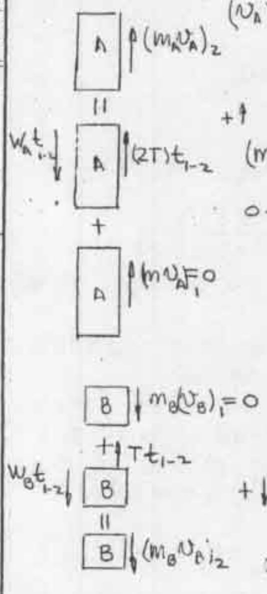
KINEMATICS
LENGTH OF CABLE IS CONSTANT

$L = 2x_A + x_B$

$\frac{dL}{dt} = 2v_A + v_B = 0$

$v_B = -2v_A$

$(v_A)_2 = 2\text{ ft/s}$



COLLAR A

$m_A = \frac{W_A}{g} = \frac{30}{g}$

$$+\uparrow \quad (m_A v_A)_1 + (2T)(t_{1-2}) - W_A t_{1-2} = m(v_A)_2$$

$$0 + (2T - 30)t_{1-2} = \left(\frac{30}{g}\right)(2)$$

$$(T - 15)t_{1-2} = \frac{30}{g} \qquad (1)$$

COLLAR B

$m_B = \frac{W_B}{g} = \frac{20}{g}$

$(v_B)_2 = 2(v_A)_2 = 4\text{ ft/s}\downarrow$

$$+\downarrow \quad (m_B v_A)_1 - T(t_{1-2}) + W_B(t_{1-2}) = (m_B v_B)_2$$

$$0 + (20 - T)(t_{1-2}) = \frac{20}{g}(4) \qquad (2)$$

ADD EQ. (1) AND (2) (ELIMINATING T)

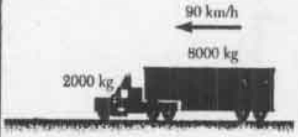
$$(20 - 15)(t_{1-2}) = \frac{(30+80)}{g} = \frac{110}{g}$$

$$t_{1-2} = \frac{22}{32.2} = 0.683\text{ s}$$

$t = 0.683$ sec.

13.130

GIVEN:

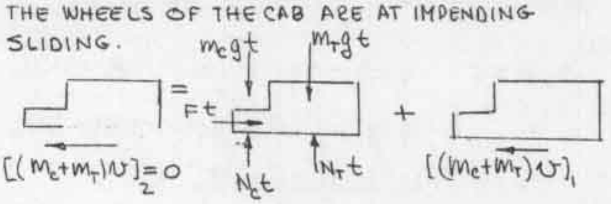


$m_C = 2000$ kg
$m_T = 8000$ kg
INITIAL $v = 90$ km/h
FINAL $v = 0$
TRAILER BRAKES FAIL
$\mu_s = 0.65$

FIND:
(a) SHORTEST TIME FOR RIG TO COME TO A STOP
(b) FORCE ON THE COUPLING DURING THIS TIME

$v = 90$ km/h $= 25$ m/s

(a) THE SHORTEST TIME FOR THE RIG TO COME TO A STOP WILL BE WHEN THE FRICTION FORCE ON THE WHEELS IS MAXIMUM. THE DOWNWARD FORCE EXERTED BY THE TRAILER ON THE CAB IS ASSUMED TO BE ZERO. SINCE THE TRAILER BRAKES FAIL ALL OF THE BRAKING FORCE IS SUPPLIED BY THE WHEELS OF THE CAB, WHICH IS MAXIMUM WHEN THE WHEELS OF THE CAB ARE AT IMPENDING SLIDING.



$Ft_{1-2} = \mu_s N_C t_{1-2}$ $\quad N_C = m_C g = (2000)g$

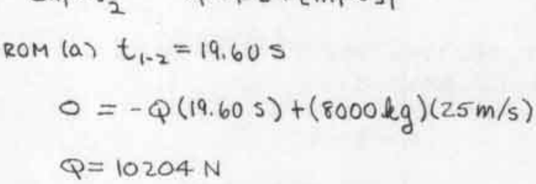
$Ft_{1-2} = (0.65)(2000)gt$

$\overset{+}{\leftarrow}$ $[(m_C+m_T)v]_2 = -Ft + [(m_C+m_T)v]_1$

$0 = -(0.65)(2000\,kg)(9.81\,m/s^2)(t_{1-2}) = (10000\,kg)(25\frac{m}{s})$

$t_{1-2} = 19.60\,s$ ◀

(b) FOR THE TRAILER



$\overset{+}{\leftarrow}$

$(m_T v)_2 = -Q t_{1-2} + [m_T v]_1$

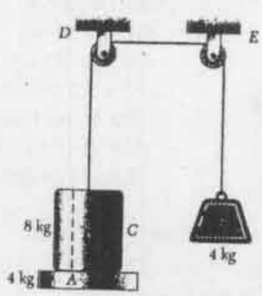
FROM (a) $t_{1-2} = 19.60\,s$

$0 = -Q(19.60\,s) + (8000\,kg)(25\,m/s)$

$Q = 10204$ N

$Q = 10.20$ kN (C) ◀

13.131

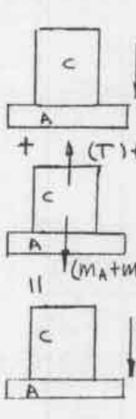


GIVEN:

$m_A = 4$ kg
$m_B = 4$ kg
$m_C = 8$ kg
SYSTEM IS RELEASED FROM REST

FIND:
(a) VELOCITY OF BLOCK B AFTER 8 SEC.
(b) FORCE EXERTED BY C ON A

(a) BLOCKS A AND C

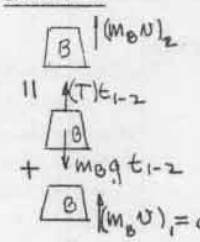


$\downarrow +$

$[(m_A+m_C)v]_1 - T(t_{1-2}) + (m_A+m_C)g t_{1-2} = [(m_A+m_C)v]_2$

$0 + (12g - T)(0.8) = 12v$ $\quad$ (1)

BLOCK B



$\uparrow +$

$[m_B v]_1 + (T)t_{1-2} - m_B g t_{1-2} = (m_B v)_2$

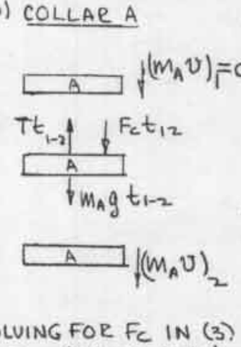
$0 + (T - 4g)(0.8) = 4v$ $\quad$ (2)

ADDING (1) AND (2), (ELIMINATING T)

$(12g - 4g)(0.8) = (12+4)v$

$v = \dfrac{(8\,kg)(9.81\,m/s^2)(0.8\,s)}{16\,kg} = 3.92\,\frac{m}{s}$ $\quad v_B = 3.92\,\frac{m}{s}$ ◀

(b) COLLAR A $\quad \downarrow +$



$0 + (F_C + m_A g - T)t_{1-2} = (m_A v)_2$ (3)

FROM EQ (2) WITH $v = 3.92\,\frac{m}{s}$

$T = \dfrac{4v}{0.8} + 4g$

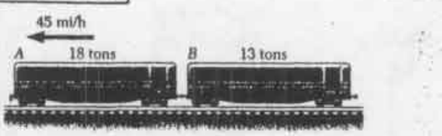
$T = \dfrac{(4\,kg)(3.92\frac{m}{s})}{(0.8\,s)} + (4\,kg)(9.81\frac{m}{s^2})$

$T = 58.84$ N

SOLVING FOR F_C IN (3)

$F_C = \dfrac{(4\,kg)(3.92\frac{m}{s})}{(0.8\,s)} - (4\,kg)(9.81\frac{m}{s^2}) + 58.84\,N = 39.2\,N$ ◀

13.132



GIVEN:

$W_A = 18$ TONS

$W_B = 13$ TONS

INITIAL VELOCITY $v = 45$ mi/h

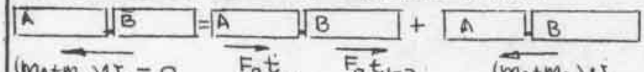
BRAKING FORCE APPLIED TO EACH CAR, $F_B = 4300$ lb

FIND:

(a) TIME REQUIRED FOR THE TRAIN TO STOP

(b) THE FORCE IN THE COUPLING AS THE TRAIN SLOWS

(a) ENTIRE TRAIN $v_1 = 45$ mi/h $= 66$ ft/s

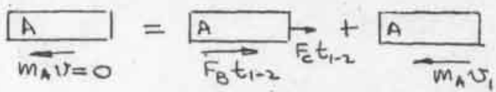


$W_A + W_B = 18 + 13 = 31$ TONS $= 62000$ lb

$$\overset{+}{\leftarrow}\quad 0 = -(4300+4300)t_{1-2} + \frac{62{,}000}{g}(66)$$

$$t_{1-2} = \frac{(62000\,\text{lb})(66\,\text{ft/s})}{(32.2\,\text{ft/s}^2)(8600\,\text{lb})} = 14.78\ \text{s}$$

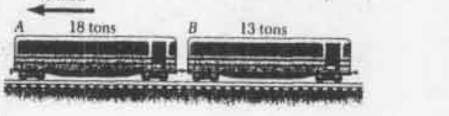
(b) CAR A $W_A = 18$ TONS $= 36000$ lb, $t_{1-2} = 14.78$ s



$$\overset{+}{\leftarrow}\quad 0 - [(4300\,\text{lb}) + F_c][14.78\,\text{s}] = \frac{(36{,}000\,\text{lb})}{(32.2\,\text{ft/s}^2)}(66\,\text{ft/s})$$

$F_c = 692.5$ lb $\qquad F_c = 693$ lb T

13.133



GIVEN:

$W_A = 18$T, $W_B = 13$T

INITIAL VELOCITY $v = 45$ mi/h

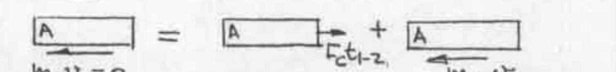
BRAKING FORCE $F_B = 4300$ lb APPLIED TO B BUT NOT TO A.

FIND:

(a) TIME REQUIRED FOR THE TRAIN TO STOP

(b) FORCE IN THE COUPLING AS THE TRAIN SLOWS

(a) ENTIRE TRAIN $v_1 = 45$ mi/h $= 66$ ft/s

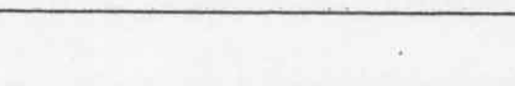


$W_A + W_B = 18 + 13 = 31$ TONS $= 62000$ lb

$$\overset{+}{\leftarrow}\ 0 = -(4300\,\text{lb})t_{1-2} + \frac{(62000\,\text{lb})}{(32.2\,\text{ft/s}^2)}(66\,\text{ft/s})$$

$t_{1-2} = 29.55$ s $\qquad t_{1-2} = 29.6$ s

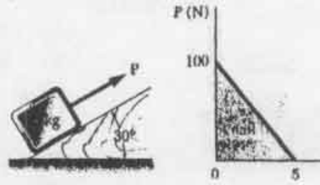
(b) CAR A



$$\overset{+}{\leftarrow}\quad 0 = -F_c(t_{1-2}) + m_A v_1 \qquad t_{1-2} = 29.55\ \text{s}$$

$$F_c = \frac{(36000\,\text{lb})(66\,\text{ft/s})}{(32.2\,\text{ft/s}^2)(29.55\,\text{s})} = 2497\,\text{lb} \qquad F_c = 2500\,\text{lb T}$$

13.134



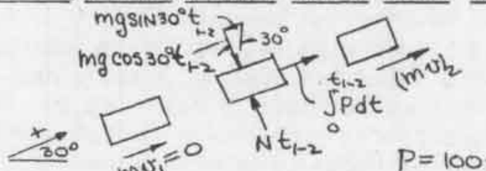
GIVEN:

6-kg BLOCK ACTED UPON BY P AS SHOWN IS INITIALLY AT REST. NO FRICTION

FIND:

(a) VELOCITY AT $t = 5$ s

(b) TIME AT WHICH THE VELOCITY IS ZERO



$P = 100 - 20t$

(a)

$$t_{1-2} = 5\ \text{s} \qquad (mv)_1 - mg\sin 30° t_{1-2} + \int_0^5 P\,dt = (mv)_2$$

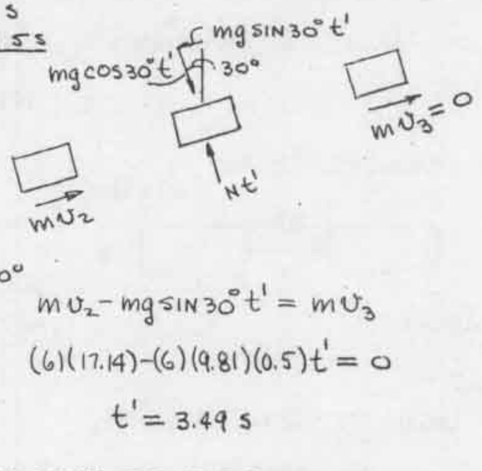
$$0 - (6)(g)(0.5)(5) + \int_0^5 (100 - 20t)\,dt = 6v_2$$

$$(-2.5)(9.81) + \frac{(100)(5) - (10)(5)^2}{6} = v_2$$

$$v_2 = 17.14\ \frac{\text{m}}{\text{s}}$$

(b) AT $t = 5$ s, $v_2 = 17.14$ m/s (FROM (a)). AFTER $t = 5$ s, $P = 0$. DENOTE BY t' THE TIME FOR THE BLOCK TO COME TO REST AFTER $t = 5$ s

AFTER $t = 5$ s



$$mv_2 - mg\sin 30° t' = mv_3$$

$$(6)(17.14) - (6)(9.81)(0.5)t' = 0$$

$$t' = 3.49\ \text{s}$$

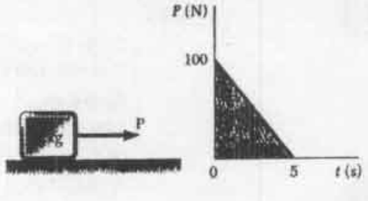
THE TOTAL TIME FOR THE BLOCK TO COME TO REST IS

$$t = 5 + t'$$

$$t = 5 + 3.49 = 8.49\ \text{s}$$

$$t = 8.49\ \text{s}$$

13.135



GIVEN:

A 6-kg BLOCK IS ACTED UPON BY THE FORCE P AS SHOWN AND IS INITIALLY AT REST.

COEFFICIENTS OF FRICTION, $\mu_s = 0.60$ $\mu_k = 0.45$

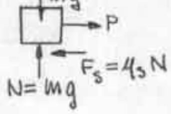
FIND:

(a) VELOCITY OF THE BLOCK AT $t = 5$ s

(b) MAXIMUM VELOCITY OF THE BLOCK

(a)

CHECK TO SEE IF THE BLOCK MOVES WHEN P IS APPLIED



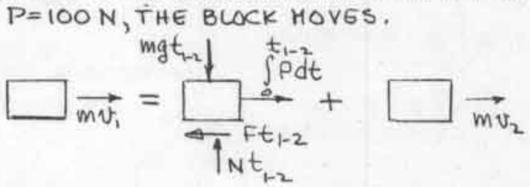
$\Sigma F = 0$ $P - \mu_s N = 0$

$P = \mu_s mg$

$P = (0.60)(6\,kg)(9.81\,\frac{m}{s^2})$

$P = 35.3$ N

SINCE 35.3 N IS LESS THAN THE INITIAL VALUE OF P = 100 N, THE BLOCK MOVES.



$P = 100 - 20t$ $t_{1-2} = 5$ s $F = \mu_k mg = (0.45)(6)(g)$

$$mv_1 = \int_0^{t_{1-2}} P\,dt - F t_{1-2} + m v_2$$

$$0 = \int_0^5 (100 - 20t)\,dt - (0.45)(6)(9.81)(5) = 6 v_2$$

$$0 = 500 - 250 - 132.4 + 6 v_2$$

$$v_2 = 19.59 \text{ m/s}$$

(b) DETERMINE TIME AT WHICH THE VELOCITY IS A MAXIMUM, WHICH MUST OCCUR AT $t < 5$ s

$$0 = \int_0^t (100 - 20t)\,dt - (0.45)(6)(9.81)\,t + 6v \quad (1)$$

$$\frac{dv}{dt} = 0\ ; \quad 100 - 20t - 26.49 = 0$$

$t = 3.68$ s WHEN v IS MAXIMUM

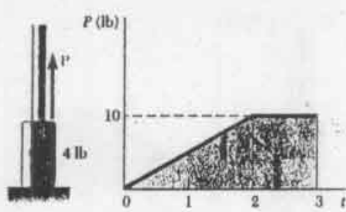
SUBSTITUTE $t = 3.68$ s IN EQ.(1)

$$0 = (100)(3.68) - 10(3.68)^2 - 97.47 + 6 v_{MAX}$$

$$v_{MAX} = \frac{134.67}{6} = 22.45 \text{ m/s}$$

$$v_{MAX} = 22.5 \text{ m/s}$$

13.136



GIVEN:

A 4-lb BLOCK IS ACTED UPON BY THE FORCE P AS SHOWN AND IS INITIALLY AT REST

NO FRICTION

FIND:

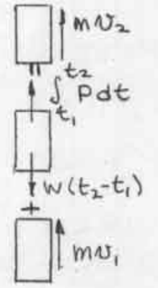
(a) VELOCITY AT $t = 2$ s

(b) VELOCITY AT $t = 3$ s

THE BLOCK DOES NOT MOVE UNTIL $P = 4$ lb

FROM $t = 0$ TO $t = 2$ s $P = 5t$

THUS, THE BLOCK STARTS TO MOVE WHEN $t = 4/5 = 0.8$ s



(a) FOR $0 < t < 2$ s

$P = 5t$

$\uparrow +$ $t_1 = 0.8$ s $t_2 = 2$ s, $v_1 = 0$

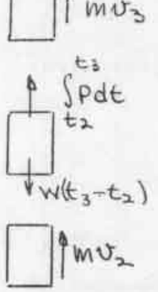
$$mv_1 + \int_{t_1}^{t_2} P\,dt - W(t_2 - t_1) = m v_2$$

$$0 + \int_{0.8}^{5} 5t\,dt - 4(2 - 0.8) = \frac{4}{g} v_2$$

$$v_2 = \frac{(32.2\,\frac{ft}{s^2})}{4\,(lb)}\left[\left(\frac{5}{2}\,\frac{lb}{s}\right)\left[(2s)^2 - (0.8\,s)^2\right] - (4\,lb)(2s - 0.8s)\right]$$

$v_2 = 28.98$ ft/s $\qquad v_2 = 29.0$ ft/s

(b) FROM $t = 2$ s TO $t = 3$ s



$v_2 = 29.0$ ft/s, FROM (a)

$P = 10$ lb $2\text{s} \le t \le 3\text{s}$

$t_2 = 2$ s $t_3 = 3$ s

$$m v_2 + \int_{t_2}^{t_3} P\,dt - W(t_3 - t_2) = m v_3$$

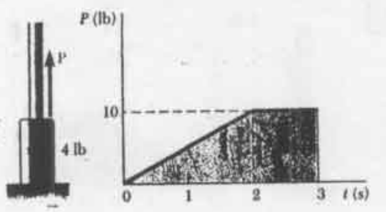
$$\left(\frac{4}{g}\right)(29.0) + \int_2^3 10\,dt - 4\,(3 - 2) = \frac{4}{g} v_3$$

$$v_3 = (29.0\,ft/s) + \frac{(32.2\,ft/s^2)}{(4\,lb)}\left[(6\,lb)(1\,s)\right] = v_3$$

$$v_3 = 29.0 + 48.3 = 77.3 \text{ ft/s}$$

$$v_3 = 77.3 \text{ ft/s}$$

13.137



GIVEN:

COLLAR INITIALLY AT REST IS ACTED UPON B A FORCE P(lb) AS SHOWN. NO FRICTION

FIND:

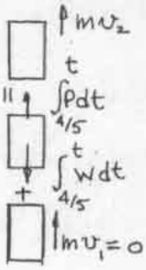
(a) THE MAXIMUM VELOCITY OF THE COLLAR, v_{MAX}

(b) THE TIME WHEN THE VELOCITY IS ZERO.

(a) DETERMINE TIME AT WHICH COLLAR STARTS TO MOVE

$P = 5t \quad 0 < t < 2s$

COLLAR MOVES WHEN $P = 4lb$ OR $t = \frac{P}{5} = \frac{4}{5}s$



$$mv_1 + \int_{4/5}^{t} P dt - \int_{4/5}^{t} W dt = mv_2$$

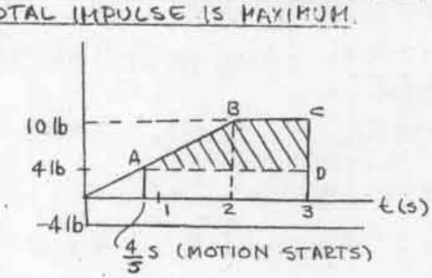
FOR $t < 2s \quad P = 5t$ (lb)

$2s < t < 3s \quad P = 10$ lb

$t > 3s \quad P = 0$

FOR $t < 3s \quad W = 4lb$

THE MAXIMUM VELOCITY OCCURS WHEN THE TOTAL IMPULSE IS MAXIMUM.



$$AREA_{ABCD} = MAX\ IMPULSE = \tfrac{1}{2}(6lb)(\tfrac{6}{5}s) + (6lb)(1s)$$

$$AREA_{ABCD} = 9.6\ lb \cdot s$$

$$0 + 9.6\ lb \cdot s = \frac{(4\ lb)}{(32.2\ ft/s^2)} v_{MAX}$$

$$v_{MAX} = 77.3\ ft/s$$

(b) VELOCITY IS ZERO WHEN TOTAL IMPULSE IS ZERO AT $t + \Delta t$

FOR $\frac{4}{5}s < t < 3s$, IMPULSE $= 9.6 (lb \cdot s)$, PART (a)

FOR Δt BEYOND 3 S IMPULSE $= -4\Delta t\ (lb \cdot s)$

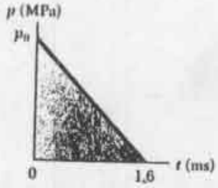
THUS

$$TOTAL\ IMPULSE = 0 = 9.6 - 4\Delta t$$

$$\Delta t = 2.4 s$$

TIME TO ZERO VELOCITY $t = 3s + 2.4s = 5.4s$

13.138



GIVEN:

20-g BULLET

10 MM DIAMETER RIFLE BARREL

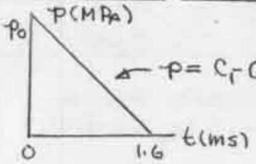
EXIT VELOCITY OF THE BULLET = 700 m/s

TIME BULLET TO EXIT = 1.6 MS

VARIATION OF PRESSURE AS SHOWN

FIND:

p_0



AT $t = 0 \quad p = p_0 = C_1 - C_2(0)$

$C_1 = p_0$

AT $t = 1.6 \times 10^{-3}s \quad p = 0$

$0 = C_1 - C_2(1.6 \times 10^{-3}s)$

$C_2 = p_0 / (1.6 \times 10^{-3}s)$

$$mv_1 = 0 \; + \; A\int_0^{1.6\times10^{-3}s} p dt \; = \; mv_2$$

$m = 20 \times 10^{-3} kg$

$$0 + A\int_0^{1.6\times10^{-3}s} p\, dt = mv_2$$

$A = \frac{\pi (10^{-3})^2}{4}$

$A = 78.54 \times 10^{-6} m^2$

$$0 + A\int_0^{1.6\times10^{-3}s} (C_1 - C_2 t) dt = \frac{20 \times 10^{-3}}{g}$$

$$(78.54 \times 10^{-6} m^2)\left[(C_1)(1.6 \times 10^{-3}s) - \frac{(C_2)(1.6 \times 10^{-3}s)^2}{2}\right] = (20 \times 10^{-3} kg)(700\ m/s)$$

$$1.6 \times 10^{-3} C_1 - 1.280 \times 10^{-6} C_2 = 178.25 \times 10^3$$

$$(1.6 \times 10^{-3}\ m^2 \cdot s) p_0 - \frac{(1.280 \times 10^{-6}\ m^2 s^2)}{(1.6 \times 10^{-3} s)} p_0 = 178.25 \times 10^3 \frac{kg \cdot m}{s}$$

$$p_0 = 222.8 \times 10^6\ N/m^2$$

$$p_0 = 223\ MPa$$

13.139

GIVEN:

25-g BULLET, 10 mm DIA. RIFLE BARREL

EXIT VELOCITY = 520 m/s

TIME FOR BULLET TO EXIT = 1.44 ms

PRESSURE MODEL

$p(t) = (950\ MPa)(e^{-t/(0.16 ms)})$

FIND:

% ERROR IF GIVEN EQUATION FOR $p(t)$ IS USED TO CALCULATE THE EXIT VELOCITY

$$mv_1 = 0 \; + \; A\int_0^t p dt \; = \; mv_2$$

$A = \frac{\pi (10^{-3})^2}{4} = 78.54 \times 10^{-6} m^2$

$$0 + (78.54 \times 10^{-6} m^2) \int_0^{1.44\times10^{-3}s} (950 \times 10^6 \tfrac{N}{m^2})(e^{-t/0.16\times10^{-3}}) dt = (25 \times 10^{-3} kg) v_2$$

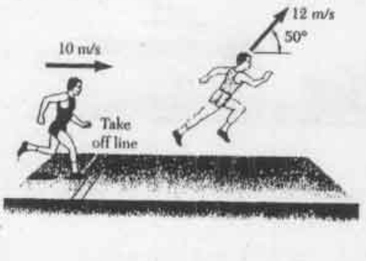
$$(78.54 \times 10^{-6})(950 \times 10^6)(-0.16 \times 10^{-3})(e^{-1.44/0.16} - 1) = 25 \times 10^{-3} v_2$$

$$v_2 = 477.46\ m/s$$

$$ERROR = 477.46 - 520 = -42.54\ m/s$$

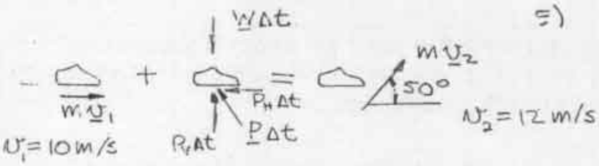
$$\%\ ERROR = 100(-42.54/520) = 8.18\%$$

13.140



GIVEN:
INITIAL VELOCITY AT TAKEOFF $= 10\ m/s$.
VELOCITY AFTER TAKEOFF $= 12\ m/s$ AT $50°$
IMPACT TIME $= 0.18\ s$.

FIND:
VERTICAL COMPONENT OF THE AVERAGE IMPULSIVE FORCE ON ATHLETES FOOT FROM THE GROUND. (IN TERMS OF HIS WEIGHT W)



$m\underline{v}_1 + (\underline{P} - \underline{W})\Delta t = m\underline{v}_2 \quad \Delta t = 0.18\ s$

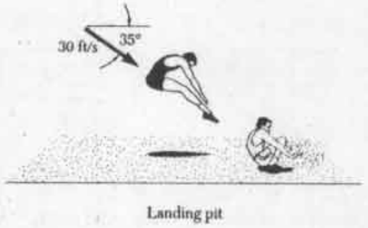
VERTICAL COMPONENTS

$0 + (P_V - W)(0.18) = (W/g)(12)(\sin 50°)$

$P_V = W + \frac{(12)(\sin 50°)}{(9.81)(0.18)} W$

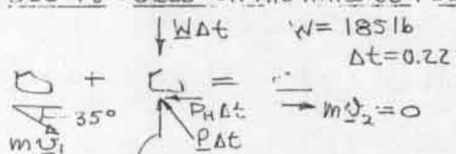
$P_V = 6.21\ W$

13.141



GIVEN:
VELOCITY BEFORE LANDING $= 30\ ft/s$ AT $35°$
IMPACT TIME BEFORE COMING TO A STOP $= 0.22\ s$
WEIGHT $= 185\ lb$

FIND:
HORIZONTAL COMPONENT OF THE AVERAGE IMPULSIVE FORCE ON THE ATHLETES FEET



$W = 185\ lb$
$\Delta t = 0.22\ s$

$v_1 = 30\ ft/s$

$m\underline{v}_1 + (\underline{P} - \underline{W})\Delta t = m\underline{v}_2$

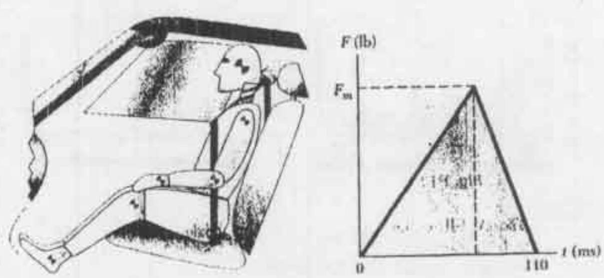
HORIZONTAL COMPONENTS

$\frac{W}{g}(30)(\cos 35°) - P_H(0.22) = 0$

$P_H = \frac{(185\ lb)(30\ ft/s)(\cos 35°)}{(32.2\ ft/s^2)(0.22\ s)} = 641.7\ lb$

$P_H = 642\ lb$

13.142

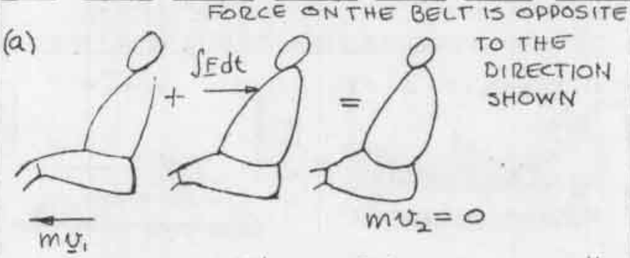


GIVEN:
AUTOMOBILE TRAVELING AT 45 mi/h COMES TO A STOP IN 110 ms.
FORCE ACTING ON MAN AS SHOWN
MAN'S WEIGHT = 200 lb

FIND:
(a) AVERAGE IMPULSIVE FORCE EXERTED ON THE BELT AS SHOWN
(b) MAXIMUM FORCE F_m EXERTED ON THE BELT

FORCE ON THE BELT IS OPPOSITE TO THE DIRECTION SHOWN

(a)



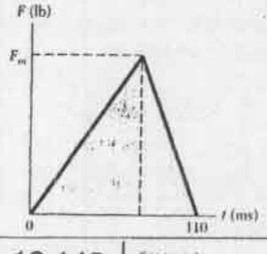
$v_1 = 45\ mi/h = 66\ ft/s$, $W = 200\ lb$

$m\underline{v}_1 - \int \underline{F}\, dt = m\underline{v}_2 \qquad \int F dt = F_{AVE}\Delta t$

$\frac{(200\ lb)(66\ ft/s)}{(32.2\ ft/s^2)} - F_{AVE}(0.110\ s) = 0 \qquad \Delta t = 0.110\ s$

$F_{AVE} = \frac{(200)(66)}{(32.2)(.110)} = 3727\ lb \qquad F_{AVE} = 3730\ lb$

(b)



IMPULSE = AREA UNDER F-t DIAGRAM $= \frac{1}{2} F_m (.110\ s)$

FROM (a), IMPULSE $= F_{AVE}\Delta t = (3727\ lb)(.110\ s)$

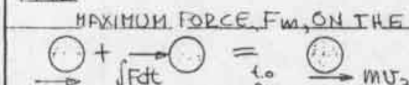
$\frac{1}{2} F_m (.110) = (3727)(.110)$

$F_m = 7450\ lb$

13.143

GIVEN:
1.6 OZ. GOLF BALL HAS A VELOCITY OF 125 ft/s AFTER IMPACT
DURATION OF IMPACT $= t_0 = 0.5\ ms$
FORCE DURING IMPACT $F = F_m \sin(\pi t/t_0)$

FIND:
MAXIMUM FORCE, F_m, ON THE BALL

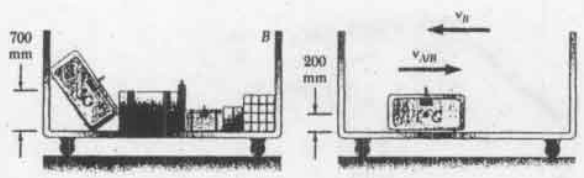


$v_2 = 125\ ft/s$

$m\underline{v}_1 + \int_0^{t_0} \underline{F} dt = m\underline{v}_2$

$0 + \int_0^{0.5\times10^{-3}} F_m \sin\frac{\pi t}{0.5\times10^{-3}} dt = \frac{(1.6/16)(125)}{(32.2)} \qquad F_m = 1220\ lb$

13.144



GIVEN:

15-kg SUITCASE A

40-kg LUGGAGE CARRIER B

INITIAL VELOCITY OF CARRIER, $v_B = 0.8\frac{m}{s}$

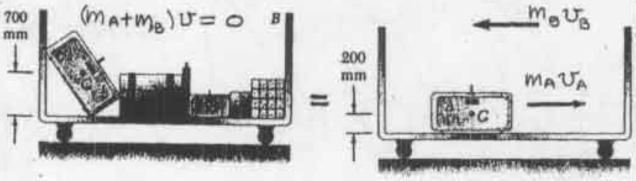
FIND:

(a) $v_{A/B}$

(b) v_B, AFTER THE SUITCASE HITS THE RIGHT SIDE OF THE CARRIER WITHOUT REBOUND

(c) ENERGY LOST BY THE IMPACT OF THE SUITCASE ON THE FLOOR OF THE CARRIER

(a) SINCE THERE ARE NO EXTERNAL FORCES ACTING ON THE SYSTEM OF THE SUITCASE A AND THE LUGGAGE CARRIER B, IN THE HORIZONTAL DIRECTION, LINEAR MOMENTUM IS CONSERVED



$\xrightarrow{+}$ $(m_A+m_B)v = m_A v_A + m_B v_B$

$v = 0 \quad v_B = -0.8\text{ m/s} \quad v_A = v_{A/B} + v_B$

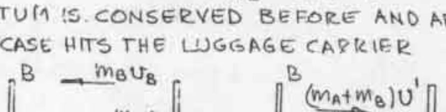
$m_B = 40\text{ kg} \quad m_A = 15\text{ kg}$

$0 = (15\text{ kg})(v_{A/B} - 0.8\text{ m/s}) + 40\text{ kg}(-.8\text{ m/s})$

$$v_{A/B} = \frac{(40\text{ kg})(0.8\text{ m/s})}{(15\text{ kg})} + 0.8\text{ m/s} = 2.93\text{ m/s}$$

$v_{A/B} = 2.93\text{ m/s} \rightarrow$

(b) MOMENTUM IS CONSERVED BEFORE AND AFTER THE SUITCASE HITS THE LUGGAGE CARRIER



$\xrightarrow{+}$ $m_A v_A + m_B v_B = (m_A+m_B)v'$

$$v' = \frac{m_A v_A + m_B v_B}{(m_A+m_B)}$$

FROM (a)

$v_A = v_{A/B} + v_B = 2.93 - 0.8 = 2.13\text{ m/s}$

$v' = (15)(2.13) - (40)(0.8) = 0 \quad v' = 0$

(c) BEFORE SUITCASE FALLS, $E_1 = m_A g(.7\text{m})$

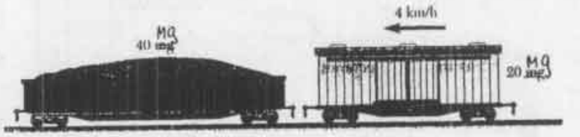
AFTER SUITCASE HITS THE BOTTOM OF THE CARRIER $E_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + m_A g(0.200\text{m})$

ENERGY LOST, $\Delta E_L = E_1 - E_2 \qquad E_1 = 15\,g\,(.7)$

$\Delta E_L = (15)(9.81)(0.7) - \frac{1}{2}(15)(2.13)^2 - \frac{1}{2}(40)(0.8)^2 - (15)(9.81)(0.2)$

$\Delta E_L = 26.7\text{ J}$

13.145



GIVEN:

BEFORE COUPLING, 20-Mg CAR IS TRAVELING AT 4 km/h AS SHOWN

40-Mg CAR HAS ITS WHEELS LOCKED

$\mu_k = 0.30$, 40-Mg CAR ONLY

FIND:

(a) VELOCITY OF BOTH CARS IMMEDIATELY AFTER COUPLING

(b) THE TIME FOR BOTH CARS TO COME TO REST

(a) THE MOMENTUM OF THE SYSTEM CONSISTING OF THE TWO CARS IS CONSERVED IMMEDIATELY BEFORE AND AFTER COUPLING.

[40 Mg] [20 Mg] = [40 Mg] [20 Mg]

$\overleftarrow{40v = 0} \quad \overleftarrow{(20)(4)} \qquad \overleftarrow{(20+40)v'}$

BEFORE COUPLING $\qquad$ AFTER COUPLING

$\xleftarrow{+}$ $\Sigma mv = \Sigma mv'$

$0 + (20\text{ Mg})(4\text{ km/h}) = (20\text{ Mg} + 40\text{ Mg})(v')$

$$v' = \frac{(20)(4)}{(20+40)} = 1.333\text{ km/h}$$

(b) AFTER COUPLING

[60 Mg] = [60 Mg] + [60 Mg]

$\overleftarrow{60v_2 = 0} \qquad \overrightarrow{\int F_f\,dt} \qquad \overleftarrow{60v_1}$

THE FRICTION FORCE ACTS ONLY ON THE 40 Mg CAR SINCE ITS WHEELS ARE LOCKED THUS,

$F_f = \mu_k N_{40} = (0.30)(40\times10^3\text{ kg})(9.81\frac{m}{s^2})$

$F_f = 117.72\times10^3\text{ N}$

FROM (a) $v_1 = v' = 1.333\text{ km/h} = 0.3704\text{ m/s}$

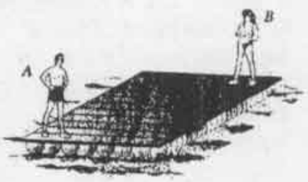
IMPULSE MOMENTUM

$$\Sigma m v_1 + \int_0^t F_f\,dt = \Sigma m v_2$$

$$(60\times10^3\text{ kg})(.3704\text{ m/s}) - \int_0^t (117.72\times10^3\text{ N})dt = 0$$

$$t = \frac{(60\times10^3)(.3704)}{(117.72\times10^3)} = 0.1888\text{ s}$$

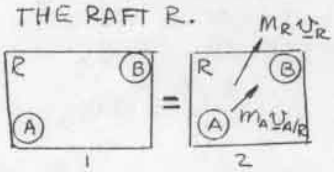
13.146



GIVEN:
$W_A = 190\ lb$
$W_B = 125\ lb$
RAFT $W_R = 300\ lb$
$v_{A/R} = 2\ ft/s$
TOWARD B, AFTER THE RAFT BREAKS LOOSE FROM ITS ANCHOR.

FIND:
(a) SPEED OF THE RAFT, v_R, IF B DOES NOT MOVE
(b) SPEED v_B OF B. IF THE RAFT IS NOT TO MOVE

(a) THE SYSTEM CONSISTS OF A AND B AND THE RAFT R.



MOMENTUM IS CONSERVED

$$(\Sigma m\underline{v})_1 = (\Sigma m\underline{v})_2$$

$$0 = m_A \underline{v}_A + m_B \underline{v}_B + m_R \underline{v}_R \qquad (1)$$

$\underline{v}_A = \underline{v}_{A/R} + \underline{v}_R \quad \underline{v}_B = \underline{v}_{B/R} + \underline{v}_R \quad v_{B/R} = 0$

$\underline{v}_A = 2\ ft/s \overset{B}{\underset{A}{\nearrow}} + \underline{v}_R \qquad \underline{v}_B = \underline{v}_R$

$$0 = m_A [2 \overset{B}{\underset{A}{\nearrow}} + \underline{v}_R] + m_B \underline{v}_R + m_R \underline{v}_R$$

$$\underline{v}_R = \frac{-2\, m_A}{(m_A + m_B + m_R)} = -\frac{(2\ ft/s)(190\ lb)}{(190\ lb + 125\ lb + 300\ lb)}$$

$$v_R = 0.618\ ft/s$$ ◀

(b) FROM EQ (1)

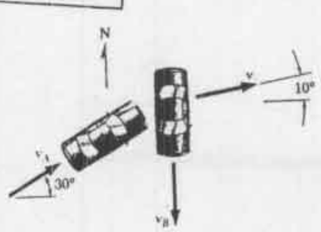
$$0 = m_A v_A + m_B v_B + 0 \quad (v_R = 0)$$

$$v_B = -\frac{m_A v_A}{m_B} \qquad v_A = v_{A/R} + \cancel{v_R}^{\,0} = 2\ ft/s$$

$$v_B = -\frac{(2\ ft/s)(190\ lb)}{(125\ lb)} = 3.04\ ft/s$$

$$v_B = 3.04\ ft/s$$ ◀

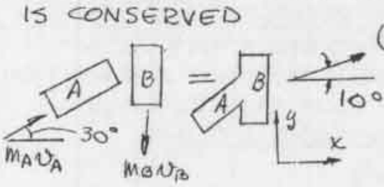
13.147



GIVEN:
$m_A = 1500\ kg$
$m_B = 1200\ kg$
BOTH CARS TOGETHER, SKID AT 10° NORTH OF EAST AFTER IMPACT

FIND:
(a) WHO WAS GOING FASTER
(b) SPEED OF THE FASTER CAR IF SLOWER CAR WAS GOING AT 50 km/h

(a) TOTAL MOMENTUM OF THE TWO CARS IS CONSERVED



$$\Sigma mv,\ x: \quad m_A v_A \cos 30° = (m_A + m_A) v \cos 10 \qquad (1)$$

$$\Sigma mv,\ y: \quad m_A v_A \sin 30° - m_B v_B = (m_A + m_B) v \sin 10° \qquad (2)$$

DIVIDING (1) INTO (2)

$$\frac{\sin 30°}{\cos 30°} - \frac{m_B v_B}{m_A v_A \cos 30°} = \frac{\sin 10°}{\cos 10°}$$

$$\frac{v_B}{v_A} = \frac{(\tan 30° - \tan 10°)(m_A \cos 30°)}{m_B}$$

$$\frac{v_B}{v_A} = (0.4010)\frac{(1500)}{(1200)}\cos 30°$$

$$\frac{v_B}{v_A} = 0.434 \qquad v_A = 2.30\, v_B$$

THUS, A WAS GOING FASTER ◀

(b) SINCE v_B WAS THE SLOWER CAR
$v_B = 50\ km/h$

$$v_A = (2.30)(50) = 115.2\ km/h$$ ◀

13.148

GIVEN:

MOTHER AND CHILD TRAVELING AT 7.2 km/h INITIALLY. $M_M = 55$ kg $M_C = 20$ kg

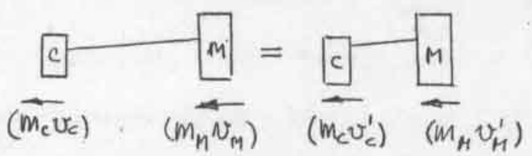
CHILD'S SPEED DECREASES TO 3.6 km/h IN 3 S AS THE MOTHER PULLS ON THE ROPE

FIND:

(a) MOTHERS SPEED AT THE END OF THE 3 S INTERVAL

(b) AVERAGE VALUE OF THE TENSION IN THE ROPE DURING THE 3 S INTERVAL

(a) CONSIDER MOTHER AND CHILD AS A SINGLE SYSTEM. ASSUMING THE FRICTION FORCE ON THE SKIS IS NEGLIGIBLE MOMENTUM IS CONSERVED



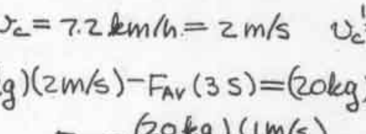
$$m_c v_c + m_M v_M = m_c v_c' + m_M v_M'$$

$v_c = v_M = 7.2$ km/h $\quad v_c' = 3.6$ km/h

$$(20)(7.2) + (55)(7.2) = 20(3.6) + (55)(v_M')$$

$$v_M' = 8.51 \text{ km/h}$$

(b) CHILD ALONE



$$t = 3 \text{ s}$$

$$m_c v_c - F_{Av} t = m_c v_c'$$

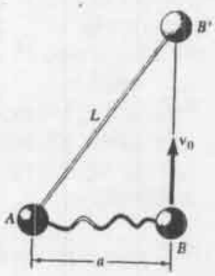
$$v_c = 7.2 \text{ km/h} = 2 \text{ m/s} \quad v_c' = 3.6 \text{ km/h} = 1 \text{ m/s}$$

$$(20 \text{ kg})(2 \text{ m/s}) - F_{Av}(3 \text{ s}) = (20 \text{ kg})(1 \text{ m/s})$$

$$F_{Av} = \frac{(20 \text{ kg})(1 \text{ m/s})}{(3 \text{ s})} = 6.67 \text{ kg}\cdot\text{m/s}^2$$

$$F_{Av} = 6.67 \text{ N}$$

13.149



GIVEN:

A AND B ON A HORIZONTAL FRICTIONLESS PLANE ARE ATTACHED BY AN INEXTENSIBLE CORD OF LENGTH L

MASS OF A = MASS OF B

$v_B = v_0$, $v_A = 0$ INITIALLY

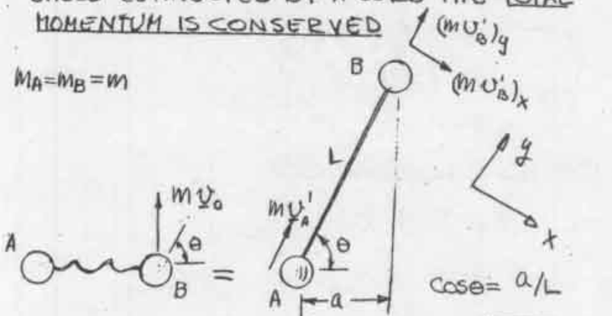
FIND:

(a) v_A' AND v_B' AFTER THE CORD BECOMES TAUT

(b) THE ENERGY LOST AS THE CORD BECOMES TAUT

(a) FOR THE SYSTEM CONSISTING OF BOTH BALLS CONNECTED BY A CORD THE TOTAL MOMENTUM IS CONSERVED

$M_A = M_B = m$



$\cos\theta = a/L$

$\sin\theta = \dfrac{\sqrt{L^2 - a^2}}{L}$

$$\Sigma m\underline{v} = \Sigma m\underline{v}'$$

$$x: \quad -mv_0 \cos\theta = m(v_B')_x \qquad (1)$$

$$(v_B')_x = -v_0 \cos\theta = -v_0 \frac{a}{L}$$

$$y: \quad mv_0 \sin\theta = mv_A' + m(v_B')_y \qquad (2)$$

SINCE THE CORD IS INEXTENSIBLE

$$v_A' = (v_B')_y \qquad (3)$$

THUS FROM (2) $\quad v_0 \sin\theta = 2v_A'$

$$v_A' = (v_0/2L)\sqrt{L^2 - a^2}$$

FROM (3)

$$(v_B')_y = v_A' = (v_0/2L)\sqrt{L^2 - a^2}$$

$$v_B' = \sqrt{(v_B')_x^2 + (v_B')_y^2} = v_0\sqrt{\frac{a^2}{L^2} + \frac{(L^2 - a^2)}{4L^2}}$$

$$v_B' = (v_0/2L)\sqrt{L^2 + 3a^2}$$

(b)

INITIAL $T = \frac{1}{2} m v_0^2$

$$T' = \tfrac{1}{2} m(v_A')^2 + \tfrac{1}{2} m(v_B')^2 = \tfrac{1}{2} m(v_0/2L)^2\left[(L^2 - a^2) + (L^2 + 3a^2)\right]$$

$$T' = \tfrac{1}{2}(mv_0^2/4L^2)(2L^2 + 2a^2) = (mv_0^2/4L^2)(L^2 + a^2)$$

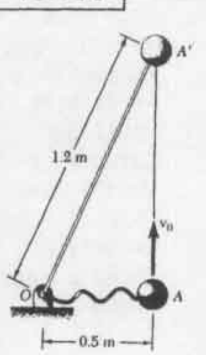
$$\Delta T = T - T' = \tfrac{1}{2} mv_0^2 - (mv_0^2/4L^2)(L^2 + a^2)$$

$$\Delta T = (mv_0^2/4L^2)(L^2 - a^2)$$

13.150

GIVEN:

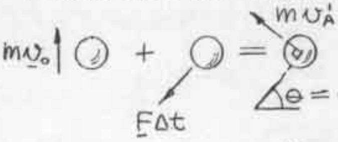
2-kg SPHERE CONNECTED BY AN INEXTENSIBLE CORD OF LENGTH 1.2M TO POINT O ON A HORIZONTAL FRICTIONLESS PLANE INITIAL VELOCITY v_o PEPPENDICULAR TO OA



FIND:

MAXIMUM ALLOWABLE v_o IF IMPULSE OF THE FORCE EXERTED ON THE CORD IS NOT TO EXCEED 3N·S.

FOR THE SPHERE AT A' IMMEDIATELY BEFORE AND AFTER THE CORD BECOMES TAUT



$\theta = \cos^{-1}(.5)/(1.2) = 65.38°$

$m\underline{v}_o + \underline{F}\Delta t = m\underline{v}_A'$

$+\angle\theta \quad mv_o \sin\theta - F\Delta t = 0 \qquad F\Delta t = 3\text{ N·S}$

$m = 2\text{-kg}$

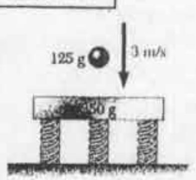
$v_0 = \dfrac{3\text{ N·S}}{(2\text{ kg})(\sin 65.38°)}$

$\text{kg} = \dfrac{\text{N·S}^2}{\text{m}} \qquad v_o = 1.650\text{ m/s}$

13.151

GIVEN:

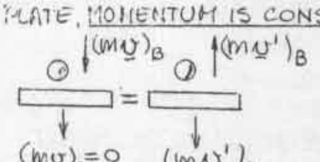
MASSES AND INITIAL VELOCITY OF THE BALL AS SHOWN. NO ENERGY LOST IN THE IMPACT



FIND:

(a) VELOCITY OF THE BALL IMMEDIATELY AFTER IMPACT
(b) IMPULSE OF THE FORCE EXERTED BY THE PLATE ON THE BALL

(a) FOR THE SYSTEM WHICH IS THE BALL AND THE PLATE, MOMENTUM IS CONSERVED



ALL FORCES ARE NON-IMPULSIVE EXCEPT THE EQUAL AND OPPOSITE FORCES BETWEEN THE PLATE AND THE BALL

$+\downarrow \; (mv)_B = -(mv')_B + (mv')_p$

$(0.125\text{ kg})(3\text{ m/s}) = (-0.125\text{ kg})(v_B') + (0.250\text{ kg})\, v_p'$

$v_p' = 0.5 v_B' + 1.5 \qquad (1)$

SINCE THERE IS NO ENERGY LOST THE KINETIC ENERGY OF THE SYSTEM IS CONERVED

(CONTINUED)

13.151 continued

BEFORE IMPACT, $T = \frac{1}{2} m_B v_B^2 = \frac{1}{2}(0.125\text{ kg})(3\text{ m/s})^2 = 0.563\text{ J}$

AFTER IMPACT $T' = \frac{1}{2} m_B (v_B')^2 + \frac{1}{2} m_p (v_p')^2$

SUBSTITUTE FOR v_p' FROM (1)

$T' = \frac{1}{2}(0.125\text{ kg})(v_B')^2 + (\frac{1}{2})(0.250\text{ kg})[0.5 v_B' + 1.5]^2$

$T' = 0.09375 (v_B')^2 + 0.1875 v_B + 0.2813$

$T = T' \quad 0.563 = 0.09375 (v_B')^2 + 0.1875 v_B + 0.2813$

$v_B'^2 + 2v_B' - 3 = 0$

$v_B' = \dfrac{-2 \mp \sqrt{4+12}}{2} = -1 \mp 2 = -3\downarrow, +1\uparrow$

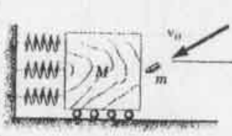
$(v_B' = -3\text{ m/s BEFORE IMPACT}) \qquad v_B' = 1\text{ m/s}\uparrow$

(b) BALL ALONE

$+\uparrow \; (0.125\text{ kg})(-3\text{ m/s}) + F\Delta t = (0.125\text{ kg})(1\text{ m/s})$

$F\Delta t = 0.5\text{ N·S}\uparrow$

13.152



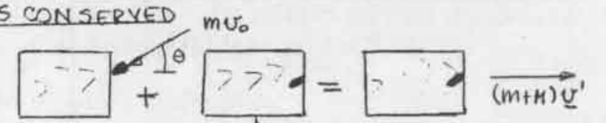
GIVEN:

BULLET FIRED INTO THE BLOCK AS SHOWN

FIND:

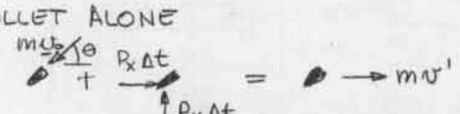
HORIZONTAL AND VERTICAL COMPONENTS OF THE IMPULSE ON THE BULLET.

FOR THE SYSTEM WHICH IS THE BULLET AND THE BLOCK, MOMENTUM IN THE HORIZONTAL DIRECTION IS CONSERVED



$\xrightarrow{+} \quad -mv_o\cos\theta = (M+m)v' \qquad v' = \dfrac{-mv_o\cos\theta}{(M+m)}$

BULLET ALONE



$\xrightarrow{+} \; -mv_o\cos\theta + P_x\Delta t = mv'$

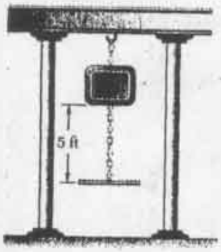
$P_x\Delta t = mv_o\cos\theta\left[1 - \dfrac{M}{M+m}\right]$

$P_x\Delta t = \dfrac{mM}{m+M} v_o\cos\theta$

$+\uparrow \; -mv_o\sin\theta + P_y\Delta t = 0$

$P_y\Delta t = mv_o\sin\theta$

13.153



GIVEN:

RIGID BEAM WEIGHS 240 lb
BLOCK WEIGHS 60 lb
INITIAL VELOCITY OF THE BLOCK = 0 AND IT IS DROPPED FROM 5 ft.

FIND:

INITIAL IMPULSE EXERTED ON THE CHAIN AND THE ENERGY ABSORBED BY THE CHAIN IF THE SUPPORTING COLUMNS ARE,
(a) RIGID, (b) EQUIVALENT TO TWO ELASTIC SPRINGS

VELOCITY OF THE BLOCK JUST BEFORE IMPACT

$T_1 = 0 \quad V_1 = Wh = (60\text{ lb})(5\text{ ft}) = 300\text{ lb}\cdot\text{ft}$

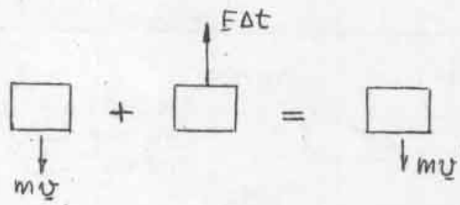
$T_2 = \frac{1}{2}mv^2 \quad V_2 = 0$

$T_1 + V_1 = T_2 + V_2$

$$0 + 300 = \frac{1}{2}\left(\frac{60}{g}\right)v^2$$

$$v = \sqrt{(600)(32.2)/60} = 17.94\text{ ft/s}$$

(a) RIGID COLUMNS



$+\uparrow \; -mv + F\Delta t = 0 \qquad \left(\frac{60}{g}\right)(17.94) = F\Delta t$

$F\Delta t = 33.43\text{ lb}\cdot\text{s} \uparrow$ ON THE BLOCK

$F\Delta t = 33.4\text{ lb}\cdot\text{s}$ ◀

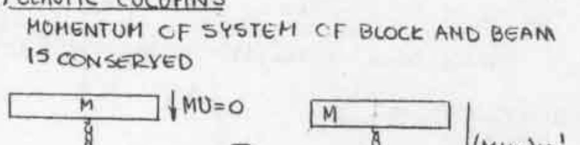
ALL OF THE KINETIC ENERGY OF THE BLOCK IS ABSORBED BY THE CHAIN

$$T = \frac{1}{2}\left(\frac{60}{g}\right)(17.94)^2 = 300\text{ ft}\cdot\text{lb}$$

$E = 300\text{ ft}\cdot\text{lb}$ ◀

(b) ELASTIC COLUMNS

MOMENTUM OF SYSTEM OF BLOCK AND BEAM IS CONSERVED



$mv = (M+m)v' \quad v' = \frac{m}{(m+M)}v = \frac{60}{300}(17.94\text{ ft/s})$

$v' = 3.59\text{ ft/s}$

REFERRING TO FIGURE IN PART (a)

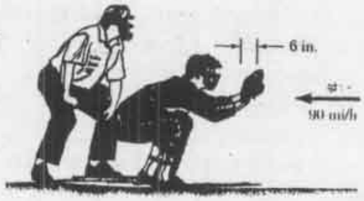
$-mv + F\Delta t = -mv'$

$F\Delta t = m(v - v') = (60/g)(17.94 - 3.59) = 26.7\text{ lb}\cdot\text{s}$ ◀

$E = \frac{1}{2}mv^2 - \frac{1}{2}mv'^2 - \frac{1}{2}Mv'^2 = \frac{60}{2g}\left[(17.94)^2 - (3.59)^2\right] - \frac{240}{2g}(3.59)^2$

$E = 240\text{ ft}\cdot\text{lb}$ ◀

13.154



GIVEN:

$W_B = 5\text{-oz}$
INITIAL SPEED OF THE BALL = 90 mi/h
AVERAGE SPEED OF THE GLOVE DURING IMPACT = 30 ft/s OVER A 6 in. DISTANCE

FIND:

AVERAGE IMPULSIVE FORCE EXERTED ON THE PLAYERS HAND

$v = 90\text{ mi/h} = 132\text{ ft/s}$

$m = \frac{5}{16}/g$

$$t = \frac{d}{v_{AV}} = \frac{(6/12)}{30} = (1/60)\text{ s}$$

$\xrightarrow{+} \; 0 - F_{AV}t + mv \qquad F_{AV} = \frac{Wv}{gt}$

$$F_{AV} = \frac{mv}{t} = \frac{(5/16\text{ lb})(132\text{ ft/s})}{(32.2\text{ ft/s}^2)(1/60\text{ s})} = 76.9\text{ lb}$$ ◀

13.155

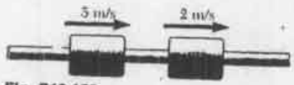


Fig. P13.155

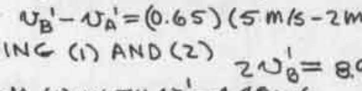
GIVEN:

IDENTICAL COLLARS WITH VELOCITIES AS SHOWN.
$e = 0.65$, $m = 1.2$ kg
NO FRICTION

FIND:

(a) v_A' AND v_B' AFTER IMPACT
(b) ENERGY LOST DURING IMPACT

(a) TOTAL MOMENTUM IS CONSERVED



$m_A = m_B = m$
$m = 1.2$ kg

$\xrightarrow{+} \; mv_A + mv_B = mv_A' + mv_B'$

$(+5\text{ m/s}) + (+2\text{ m/s}) = v_A' + v_B'$

$7\text{ m/s} = v_A' + v_B' \qquad (1)$

RELATIVE VELOCITIES ALONG LINE OF IMPACT

$v_B' - v_A' = e(v_A - v_B) \qquad e = 0.65$

$v_B' - v_A' = (0.65)(5\text{ m/s} - 2\text{ m/s}) = 1.95\text{ m/s} \quad (2)$

ADDING (1) AND (2) $\quad 2v_B' = 8.95 \quad v_B' = 4.48\ \frac{\text{m}}{\text{s}} \rightarrow$ ◀

FROM (1) WITH $v_B' = 4.48$ m/s

$v_A' = 7\text{ m/s} - 4.48\text{ m/s} = 2.53\text{ m/s} \rightarrow$ ◀

(b) ENERGY LOST DURING IMPACT

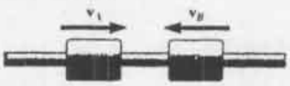
$E_L = T_A + T_B - T_A' - T_B'$

$E_L = \frac{1}{2}(1.2\text{ kg})\left[5^2 + 2^2 - (4.475)^2 - (2.525)^2\right]$

$E_L = 1.559\text{ N}\cdot\text{m}$ ◀

13.156

GIVEN:



IDENTICAL COLLARS MOVE TOWARD EACH OTHER WITH VELOCITIES SHOWN
$e=0$

SHOW THAT:

(a) AFTER IMPACT THE COMMON VELOCITY $v'=(1/2)(v_A-v_B)$

(b) THE ENERGY LOSS IS $\frac{1}{4}m(v_A+v_B)^2$

(a) $\xrightarrow{v_A}$ [A] $\xleftarrow{v_B}$ [B] = $\xrightarrow{v'}$ [A | B] $m_A=m_B=m$, $e=0$

CONSERVATION OF TOTAL MOMENTUM

$\xrightarrow{+}$ $mv_A-mv_B=2mv'$

$$v'=\frac{1}{2}(v_A-v_B)$$

(b) ENERGY LOSS

$$E_L=T_A+T_B-(T_A'+T_B')$$

$$E_L=\frac{1}{2}m(v_A^2+v_B^2)-\frac{1}{2}m(v'^2+v'^2)$$

FROM (a)

$$v'=\frac{1}{2}(v_A-v_B)$$

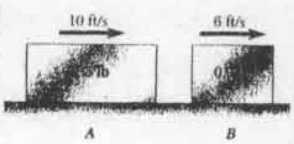
$$E_L=\frac{1}{2}m(v_A^2+v_B^2)-\frac{1}{2}m\left[\frac{1}{2}(v_A-v_B)^2\right]$$

$$E_L=\frac{1}{2}m(v_A^2+v_B^2)-\frac{1}{4}m(v_A^2-2v_Av_B+v_B^2)$$

$$E_L=\frac{1}{4}m[v_A^2+2v_Av_B+v_B^2]=\frac{1}{4}m(v_A+v_B)^2$$

13.157

GIVEN:



INITIAL VELOCITIES AS SHOWN
$W_A=1.5\text{lb}$, $W_B=0.9\text{ lb}$
AFTER IMPACT
$v_B'=10.5\text{ft/s}\rightarrow$
NO FRICTION

FIND:

e, THE COEFFICIENT OF RESTITUTION

THE TOTAL MOMENTUM IS CONSERVED

$v_A=10\text{ ft/s}$, $v_B=6\text{ft/s}$, v_A', $v_B'=10.5\text{ft/s}$

[1.5 lb] A [0.9 lb] B = [1.5 lb] A [0.9 lb] B

$\xrightarrow{+}$ $m_Av_A+m_Bv_B=m_Av_A'+m_Bv_B'$

$$\frac{(1.5\text{lb})}{g}(10\text{ ft/s})+\frac{(0.9\text{ lb})}{g}(6\text{ft/s})=\frac{(1.5\text{lb})}{g}(v_A')+\frac{(0.9\text{ lb})}{g}(10.5\text{ft/s})$$

$$v_A'=\frac{15+5.4-9.45}{1.5}=7.30\text{ft/s}$$

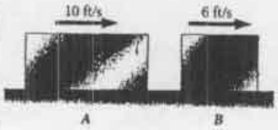
COEFFICIENT OF RESTITUTION

$$e=\frac{v_B'-v_A'}{v_A-v_B}=\frac{10.5-7.30}{10-6}=0.800$$

$$e=0.800$$

13.158

GIVEN:



INITIAL VELOCITIES AS SHOWN
$W_A=1.5\text{lb}$ $W_B=0.9\text{lb}$
$e=0.75$
NO FRICTION

FIND:

(a) AFTER IMPACT v_A' AND v_B'

(b) ENERGY LOSS DUE TO THE IMPACT

(a) THE TOTAL MOMENTUM IS CONSERVED

$v_A=10\text{ft/s}$, $v_B=6\text{ft/s}$, v_A', v_B'

[1.5 lb] A [0.9 lb] B = [1.5 lb] A [0.9 lb] B

$\xrightarrow{+}$ $m_Av_A+m_Bv_B=m_Av_A'+m_Bv_B'$

$$\frac{(1.5\text{lb})}{g}(10\text{ft/s})+\frac{(0.9\text{lb})}{g}(6\text{ft/s})=\frac{(1.5\text{lb})}{g}v_A'+\frac{(0.9\text{lb})}{g}v_B'$$

$$15+5.4=20.4=1.5v_A'+0.9v_B' \qquad (1)$$

RELATIVE VELOCITIES

$$(v_A-v_B)e=(v_B'-v_A')$$

$$(10-6)(0.75)=v_B'-v_A'$$

$$v_B'-v_A'=3 \qquad (2)$$

SOLVING (1) AND (2) SIMULTANEOUSLY

$$v_B'=10.38\text{ft/s}\rightarrow$$

$$v_A'=7.38\text{ft/s}\rightarrow$$

(b) ENERGY LOSS

$$E_L=\frac{1}{2}m_Av_A^2+\frac{1}{2}m_Bv_B^2-\frac{1}{2}m_Av_A'^2-\frac{1}{2}m_Bv_B'^2$$

$$E_L=\frac{1}{2g}\left[(1.5)(10)^2+(0.9)(6)^2-(1.5)(7.375)^2-(0.9)(10.375)^2\right]$$

$$E_L=\frac{1}{(2)(32.2\text{ft/s}^2)}(150+32.4-81.585-96.876)(\text{lb}\cdot\text{ft/s}^2)$$

$$E_L=\frac{3.937}{(2)(32.2)}=0.06113 \qquad E_L=0.0611\text{ ft}\cdot\text{lb}$$

13.159

GIVEN:

INITIALLY $v_A=v_B=0$, $v_c=1.5\text{m/s}\leftarrow$
ALL CARS HAVE THE SAME WEIGHT
$e_{BC}=0.8$, $e_{AB}=0.5$

FIND:

v_A', v_B', v_c' AFTER ALL COLLISIONS

$m_A=m_B=m_C=m$

COLLISION BETWEEN B AND C

THE TOTAL MOMENTUM IS CONSERVED

$\xleftarrow{v_B'}$ [B] $\xleftarrow{v_c'}$ [C] = $v_B=0$ [B] $v_c=1.5\text{m/s}$ [C]

$\xleftarrow{+}$ $mv_B'+mv_c'=mv_B+mv_c$

$$v_B'+v_c'=0+1.5 \qquad (1)$$

(CONTINUED)

13.159 continued

RELATIVE VELOCITIES

$$(v_B - v_C)(e_{BC}) = (v_C' - v_B')$$

$$(-1.5)(0.8) = (v_C' - v_B')$$

$$-1.2 = v_C' - v_B' \qquad (2)$$

SOLVING (1) AND (2) SIMULTANEOUSLY

$$v_B' = 1.35 \text{ m/s}$$

$$v_C' = 0.15 \text{ m/s}$$

SINCE $v_B' > v_C'$, CAR B COLLIDES WITH CAR A

COLLISION BETWEEN A AND B

$\overleftarrow{v_A'}$ $\overleftarrow{v_B''}$ $\overleftarrow{v_A = 0}$ $\overleftarrow{v_B' = 1.35 \text{ m/s}}$

$\overset{+}{\leftarrow}$ [A] [B] = [A] [B]

$$m v_A' + m v_B'' = m v_A + m v_B'$$

$$v_A' + v_B'' = 0 + 1.35 \qquad (3)$$

RELATIVE VELOCITIES

$$(v_A - v_B')e_{AB} = (v_B'' - v_A')$$

$$(0 - 1.35)(0.5) = v_B'' - v_A'$$

$$v_A' - v_B'' = 0.675 \qquad (4)$$

SOLVING (3) AND (4) SIMULTANEOUSLY

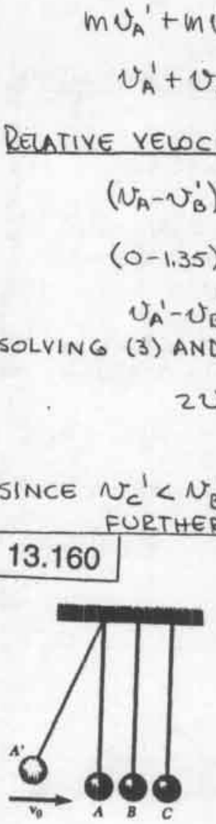
$$2v_A' = 1.35 + 0.675$$

$$v_A' = 1.013 \text{ m/s}$$

$$v_B'' = 0.338 \text{ m/s}$$

SINCE $v_C' < v_B'' < v_A'$ THERE ARE NO FURTHER COLLISIONS

13.160



GIVEN:

SPHERES A, B, C OF EQUAL WEIGHT INITIAL VELOCITY OF A IS v_0 AND B AND C ARE AT REST. e IS THE SAME FOR ALL SPHERES

FIND:

(a) v_A' AND v_B' AFTER THE FIRST COLLISION

(b) v_B'' AND v_C' AFTER THE SECOND COLLISION

(c) FOR n SPHERES, THE VELOCITY v_n' AFTER IT IS HIT FOR THE FIRST TIME

(d) USING THE RESULT FROM PART (c) THE VELOCITY OF THE LAST SPHERE FOR $n=6$, $e=0.95$

(a) FIRST COLLISION (BETWEEN A AND B)

THE TOTAL MOMENTUM IS CONSERVED

$\overrightarrow{v_A = v_0}$ $\overrightarrow{v_B = 0}$ $\overrightarrow{v_A'}$ $\overrightarrow{v_B'}$

○○ = ○○ (A B = A B)

$$m v_A + m v_B = m v_A' + m v_B'$$

$$\overset{+}{\rightarrow} \quad v_0 = v_A' + v_B' \qquad (1)$$

13.160 continued

RELATIVE VELOCITIES

$$(v_A - v_B)e = (v_B' - v_A')$$

$$v_0 e = v_B' - v_A' \qquad (2)$$

SOLVING EQUATIONS (1) AND (2) SIMULTANEOUSLY

$$v_A' = v_0(1-e)/2$$

$$v_B' = v_0(1+e)/2$$

(b) SECOND COLLISION (BETWEEN B AND C)

THE TOTAL MOMENTUM IS CONSERVED

$\overrightarrow{v_B'}$ $\overrightarrow{v_C = 0}$ $\overrightarrow{v_B''}$ $\overrightarrow{v_C'}$

Ⓑ Ⓒ = Ⓑ Ⓒ

$$m v_B' + m v_C = m v_B'' + m v_C'$$

USING THE RESULT FROM (a) FOR v_B'

$$v_0(1+e)/2 + 0 = v_B'' + v_C' \qquad (3)$$

RELATIVE VELOCITIES

$$(v_B' - 0)e = v_C' - v_B''$$

SUBSTITUTING AGAIN FOR v_B' FROM (a)

$$v_0 \frac{(1+e)}{2}(e) = v_C' - v_B'' \qquad (4)$$

SOLVING EQUATIONS (3) AND (4) SIMULTANEOUSLY

$$v_C' = \tfrac{1}{2}\left[v_0(1+e)/2 + v_0(1+e)(e)/2\right]$$

$$v_C' = v_0(1+e)^2/4$$

$$v_B'' = v_0(1-e^2)/4$$

(c) FOR n SPHERES

$\overrightarrow{v_{n-1}}$ $\overrightarrow{v_n}$ $\overrightarrow{v_{n-1}'}$ $\overrightarrow{v_n'}$

○ ○ = ○ ○ ($n-1$ n $n-1$ n)

n BALLS

$n-1^{\text{TH}}$ COLLISION

WE NOTE FROM THE ANSWER TO PART (b), WITH $n = 3$

$$v_n' = v_3' = v_C' = v_0(1+e)^2/4$$

$$\text{OR } v_3' = v_0(1+e)^{(3-1)}/(2)^{(3-1)}$$

THUS FOR n BALLS

$$v_n' = v_0(1+e)^{(n-1)}/2^{(n-1)}$$

(d) FOR $n = 6$ AND $e = 0.95$

FROM THE ANSWER TO PART (c) WITH $n = 6$

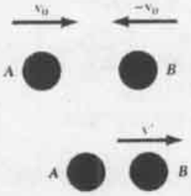
$$v_6' = v_0(1+0.95)^{(6-1)}/2^{(6-1)}$$

$$v_6' = 0.881 v_0$$

$$v_6' = 0.881 v_0$$

13.161

GIVEN:



$m_A = 3\,kg$

INITIAL VELOCITIES OF DISKS A AND B ARE EQUAL AND OPPOSITE OF MAGNITUDE v_0

AFTER IMPACT $v_A' = 0$

$e = 0.5$. NO FRICTION

FIND:

(a) m_B

(b) RANGE OF VALUES FOR m_B IF e IS UNKNOWN

(a) TOTAL MOMENTUM CONSERVED

$v_A = v_0 \quad v_B = -v_0 \quad v_A' = 0 \quad v_B = v'$

A B = A B

$$\xrightarrow{+} \quad m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$(3kg)(v_0) + m_B(-v_0) = 0 + m_B v'$$

$$v' = 3v_0/m_B - v_0$$

$$v' = v_0(3/m_B - 1) \qquad (1)$$

RELATIVE VELOCITIES

$$\xrightarrow{+} \quad (v_A - v_B)e = (v_B' - v_A')$$

$$2v_0 e = v' - 0$$

$$v' = 2v_0 e \qquad (2)$$

SUBSTITUTE FOR v' IN EQUATION (1) FROM (2)

$$2v_0 e = v_0(3/m_B - 1) \qquad (3)$$

$e = 0.5 \qquad (2)(.5) = 3/m_B - 1$

$$m_B = 3/2\ kg$$

(b) FROM EQ. (3)

$$2e + 1 = 3/m_B$$

$$m_B = \frac{3}{(2e+1)}$$

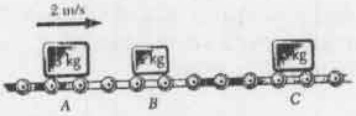
$e = 0 \qquad m_B = 3\,kg$

$e = 1 \qquad m_B = 1\,kg$

$$1kg < m_B < 3kg$$

13.162

GIVEN:



INITIALLY B AND C ARE AT REST, $v_A = 2\,m/s$, $e = 0.3$, MASSES AS SHOWN

FIND:

(a) v_C', AFTER A HITS B AND B HITS C

(b) v_A'', AFTER A HITS B THE SECOND TIME

(a) PACKAGES A AND B

$v_A = 2\,m/s \quad v_B = 0 \quad v_A' \quad v_B'$

$\xrightarrow{+}$ 8kg 4kg = 8kg 4kg

A B A B

TOTAL MOMENTUM CONSERVED

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$(8kg)(2m/s) + 0 = (8kg)v_A' + (4kg)v_B'$$

$$4 = 2v_A' + v_B' \qquad (1)$$

RELATIVE VELOCITIES

$$(v_A - v_B)e = (v_B' - v_A')$$

$$(2)(.3) = v_B' - v_A' \qquad (2)$$

SOLVING EQUATIONS (1) AND (2) SIMULTANEOUSLY

$v_A' = 1.133\ m/s \rightarrow$

$v_B' = 1.733\ m/s \rightarrow$

PACKAGES B AND C

$v_B' = 1.733\,m/s \quad v_C = 0 \quad v_B'' \quad v_C'$

4kg 6kg = 4kg 6kg

B C B C

$$\xrightarrow{+} \quad m_B v_B' + m_C v_C = m_B v_B'' + m_C v_C'$$

$$(4kg)(1.733\,m/s) + 0 = 4v_B'' + 6v_C'$$

$$6.932 = 4v_B'' + 6v_C' \qquad (3)$$

RELATIVE VELOCITIES

$$(v_B' - v_C)e = v_C' - v_B''$$

$$(1.733)(.3) = 0.5199 = v_C' - v_B'' \qquad (4)$$

SOLVING EQUATIONS (3) AND (4) SIMULTANEOUSLY

$v_C' = 0.901\ m/s \rightarrow$

$v_B'' = 0.381\ m/s \rightarrow$

(b) PACKAGES A AND B (SECOND TIME)

$v_A' = 1.133\,\frac{m}{s} \quad v_B'' = 0.381\,\frac{m}{s} \quad v_A'' \quad v_B'''$

8kg 4kg = 8kg 4kg

A B A B

TOTAL MOMENTUM CONSERVED

$$(8)(1.133) + (4)(0.381) = 8v_A'' + 4v_B'''$$

$$10.588 = 8v_A'' + 4v_B'' \qquad (5)$$

RELATIVE VELOCITIES

$$(v_A' - v_B'')e = v_B''' - v_A''$$

$$(1.133 - 0.381)(0.3) = 0.2256 = v_B''' - v_A'' \qquad (6)$$

SOLVING (5) AND (6) SIMULTANEOUSLY.

$v_A'' = 0.807\,m/s \rightarrow$

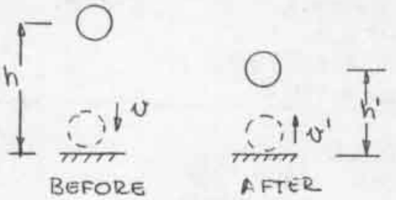
13.163

GIVEN:

BALL DROPPED FROM A HEIGHT OF 100-IN. ONTO A RIGID SURFACE. MUST REBOUND TO A HEIGHT $53 \text{ IN.} \le h' \le 58 \text{ IN}$

FIND:

RANGE OF ALLOWABLE VALUES OF e



UNIFORM ACCELERATED MOTION

$v = \sqrt{2gh}$

$v' = \sqrt{2gh'}$

COEFFICIENT OF RESTITUTION $\quad e = \dfrac{v'}{v}$

$$e = \sqrt{\frac{h'}{h}}$$

HEIGHT OF DROP $h = 100$ IN

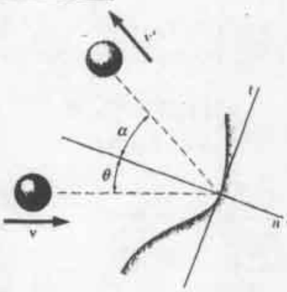
HEIGHT OF BOUNCE $53 \text{ IN.} \le h' < 58 \text{ IN}$

THUS

$$\sqrt{\frac{53}{100}} \le e \le \sqrt{\frac{58}{100}}$$

$$0.728 \le e \le 0.762$$

13.164

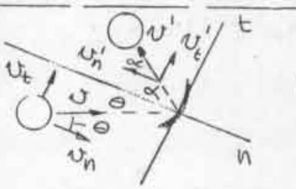


GIVEN:

BALLS HITS SURFACE AT AN ANGLE θ AND REBOUNDS AT A ANGLE α

SHOW:

$\alpha > \theta$ AND THAT % LOSS IN KINETIC ENERGY IS $100(1-e^2)\cos^2\theta$



MOMENTUM IN t DIRECTION IS CONSERVED (NO FRICTION)

$m v_t = m v_t'$

$$v \sin\theta = v' \sin\alpha \quad (1)$$

COEFFICIENT OF RESTITUTION (n-DIRECTION)

$$v_n e = v_n' \qquad v(\cos\theta)(e) = v'\cos\alpha \quad (2)$$

DIVIDE EQ (2) INTO EQ.(1)

$$\frac{\tan\theta}{\tan\alpha} = e$$

THUS

FOR $0 < e < 1$ $\tan\alpha > \tan\theta$ AND $\alpha > \theta$

% LOSS IN KINETIC ENERGY

SQUARING BOTH SIDES OF (1) AND (2) AND ADDING

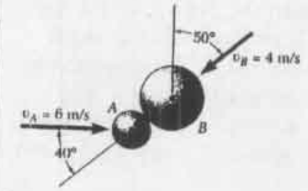
$$v^2(\sin^2\theta + e^2\cos^2\theta) = (v')^2$$

$$\Delta T = \frac{1}{2}m\left[v^2 - (v')^2\right] = \frac{1}{2}mv^2\left[1 - (\sin^2\theta + e^2\cos^2\theta)\right]$$

$$\Delta T = \frac{1}{2}mv^2\cos^2\theta\,(1-e^2)$$

$$\% \text{ LOSS} = 100\,\frac{\Delta T}{\frac{1}{2}mv^2} = 100(1-e^2)\cos^2\theta$$

13.165



GIVEN:

INITIAL VELOCITIES AS SHOWN

$m_A = 600\text{g}$

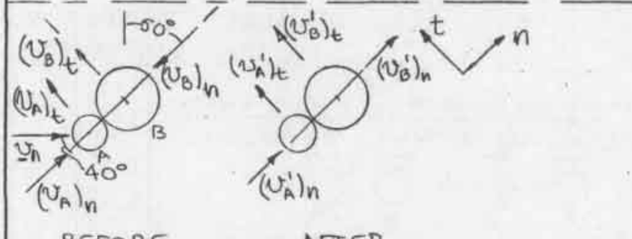
$m_B = 1\text{kg}$

$e = 0.8$

NO FRICTION

FIND:

$\underline{v}_A'$ AND $\underline{v}_B'$ AFTER IMPACT



$v_A = 6\text{m/s}$

$(v_A)_n = (6)(\cos 40°) = 4.596$ m/s

$(v_A)_t = -6(\sin 40°) = -3.857$ m/s

$v_B = (v_B)_n = -4\text{m/s}$

$(v_B)_t = 0$

t-DIRECTION

TOTAL MOMENTUM CONSERVED

$$m_A(v_A)_t + m_B(v_B)_t = m_A(v_B)_t + m_B(v_B')_t$$

$$(0.6\text{kg})(-3.857\text{m/s}) + 0 = (0.6\text{kg})(v_A')_t + (1\text{kg})(v_B')_t$$

$$-2.314\,\tfrac{\text{m}}{\text{s}} = 0.6(v_A')_t + (v_B')_t \quad (1)$$

BALL A ALONE MOMENTUM CONSERVED

$m_A(v_A)_t = m_A(v_A')_t \qquad -3.857 = (v_A')_t$

$$(v_A')_t = -3.857 \text{ m/s} \quad (2)$$

REPLACE $(v_A')_t$ IN (2) IN EQUATION (1)

$-2.314 = (0.6)(-3.857) + (v_B')_t$

$-2.314 = -2.314 + (v_B')_t$

$(v_B')_t = 0$

n-DIRECTION

RELATIVE VELOCITIES

$$[(v_A)_n - (v_B)_n]e = (v_B')_n - (v_A')_n$$

$$[(4.596) - (-4)](0.8) = (v_B')_n - (v_A')_n$$

$$6.877 = (v_B')_n - (v_A')_n \quad (3)$$

TOTAL MOMENTUM CONSERVED

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v_A')_n + m_B(v_B')_n$$

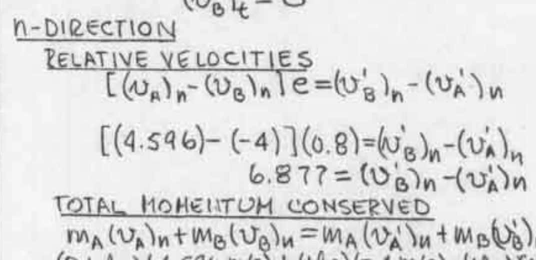
$$(0.6\text{kg})(4.596\text{ m/s}) + (1\text{kg})(-4\text{ m/s}) = (1\text{kg})(v_B')_n + (0.6\text{kg})(v_A')_n$$

$$-1.2424 = (v_B')_n + 0.6(v_A')_n \quad (4)$$

SOLVING EQ. (4) AND (3) SIMULTANEOUSLY

$(v_A')_n = -5.075$ m/s

$(v_B')_n = 1.802$ m/s



77.2°

$v_A' = 6.37$ m/s

$v_B' = 1.802$ m/s

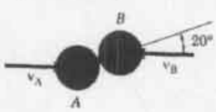
40°

13.166

GIVEN:

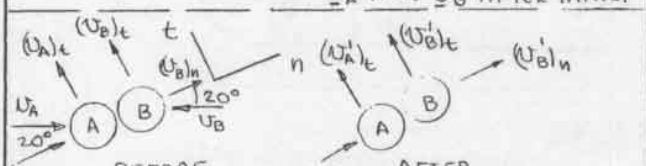
TWO IDENTICAL HOCKEY PUCKS WITH SPEEDS $v_A = v_B = 3\text{ m/s}$ IN THE DIRECTIONS SHOWN

$e = 1$



FIND:

v_A' AND v_B' AFTER IMPACT



$(v_A)_n = (3\text{ m/s})\cos 20° = 2.819\text{ m/s}$ $\quad m_A = m_B$

$(v_A)_t = (-3\text{ m/s})\sin 20° = -1.0261\text{ m/s}$

$(v_B)_n = (-3\text{ m/s})\cos 20° = -2.819\text{ m/s}$

$(v_B)_t = (3\text{ m/s})\sin 20° = 1.0261\text{ m/s}$

t-DIRECTION

MOMENTUM OF A IS CONSERVED

$m_A(v_A)_t = m_A(v_A')_t \qquad -1.0261 = (v_A')_t$

$(v_A')_t = -1.0261\ \frac{m}{s}$

MOMENTUM OF B IS CONSERVED

$m_B(v_B)_t = m_B(v_B')_t \qquad 1.0261 = (v_B')_t$

$(v_B')_t = 1.0261\ \frac{m}{s}$

n-DIRECTION

TOTAL MOMENTUM IS CONSERVED

$m_A(v_A)_n + m_B(v_B)_n = m_A(v_A')_n + m_B(v_B')_n$

$m_A = m_B$

$2.819 - 2.819 = (v_A')_n + (v_B')_n$

$(v_A')_n = -(v_B')_n$

RELATIVE VELOCITIES (COEFF. OF RESTITUTION)

$[(v_A)_n - (v_B)_n]e = (v_B')_n - (v_A')_n$

$e = 1$

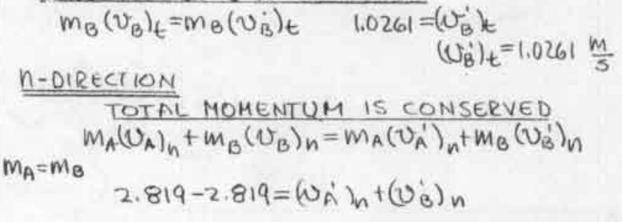
$[2.819 - (-2.819)](1) = (v_B')_n - (v_A')_n$

$(v_B')_n - (v_A')_n = 5.638$

$(v_A')_n = -(v_B')_n$

$2(v_A')_n = -5.638$

$(v_A')_n = -2.819\text{ m/s} \qquad (v_B')_n = 2.819\text{ m/s}$



$\theta = \tan^{-1}\frac{1.026}{2.819} = 20°$

$v_A' = 3\text{ m/s} \measuredangle 40°$ ◀

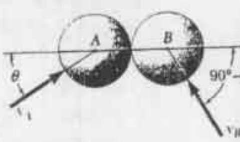
$v_B' = 3\text{ m/s} \angle 40°$ ◀

13.167

GIVEN:

TWO IDENTICAL SPHERES WITH INITIAL VELOCITIES PERPENDICULAR TO EACH OTHER

$e = 1$

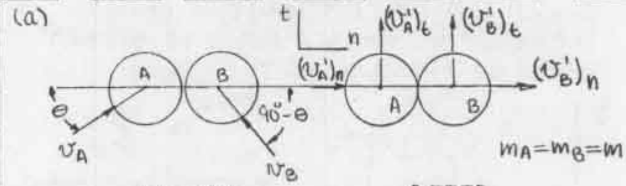


SHOW THAT:

(a) AFTER REBOUND THE VELOCITIES v_A' AND v_B' ARE ALSO PERPENDICULAR

(b) WITH $v_A = 30$ ft/s, $v_B = 40$ ft/s AND $\theta = 30°$ (SAMPLE PROB. 13.15) FIND v_A' AND v_B' AND THE ANGLE BETWEEN THEM. $e = 1$

(a)



$m_A = m_B = m$

t-DIRECTION

MOMENTUM OF A IS CONSERVED

$m v_A \sin\theta = m(v_A')_t$

$(v_A')_t = v_A \sin\theta$

MOMENTUM OF B IS CONSERVED

$m v_B \cos\theta = m(v_B')_t$

$(v_B')_t = v_B \cos\theta$

n-DIRECTION

TOTAL MOMENTUM IS CONSERVED

$m v_A \cos\theta - m v_B \sin\theta = m(v_A')_n + m(v_B')_n$

$(v_A')_n + (v_B')_n = v_A\cos\theta - v_B\sin\theta \quad (1)$

RELATIVE VELOCITIES (COEFF. OF RESTITUTION)

$e = 1 \qquad (v_B')_n - (v_A')_n = (1)(v_A\cos\theta + v_B\sin\theta) \quad (2)$

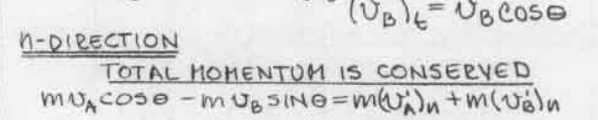
ADDING EQ. (1) AND (2)

$(v_B')_n = v_A\cos\theta$

$(1) - (2)$

$(v_A')_n = -v_B\sin\theta$

THUS, AFTER IMPACT



$\tan\alpha = \frac{v_A}{v_B} \qquad \tan\beta = \frac{v_A}{v_B}$ $\quad$ THUS $\alpha = \beta$ AND $v_A' \perp v_B'$ ◀

(b) USING THE RESULTS FROM (a)

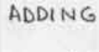
$v_A' = \sqrt{(v_A')_t^2 + (v_A')_n^2} = \sqrt{v_A^2\sin^2\theta + v_B^2\sin^2\theta}$

$v_A' = \sin 30°\sqrt{(30)^2 + (40)^2} = 25\text{ ft/s}$ ◀

$v_B' = \sqrt{(v_B')_t^2 + (v_B')_n^2} = \sqrt{v_B^2\cos^2\theta + v_A^2\cos^2\theta}$

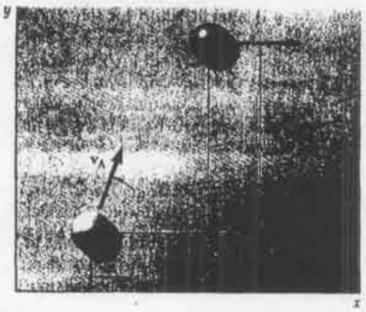
$v_B' = \cos 30°\sqrt{(40)^2 + (30)^2} = 43.3\text{ ft/s}$ ◀

$\alpha = \beta = \tan^{-1}\frac{v_A}{v_B} = \tan^{-1}\frac{30}{40} = 36.9°$



$\gamma = 180 - (\alpha + 90 - \beta)] = 90°$ ◀

13.168



GIVEN:

DIAMETER OF BALLS, $D = 2.37$ in

$v_A = 3$ ft/s

AFTER IMPACT, v_B' IN THE x DIRECTION

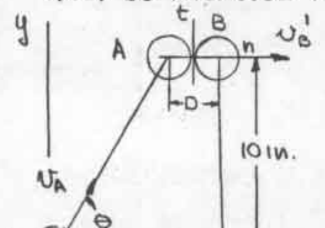
$e = 0.9$

FIND:

(a) θ

(b) v_B'

(a) SINCE v_B' IS IN THE x-DIRECTION AND (ASSUMING NO FRICTION) THE COMMON TANGENT BETWEEN A AND B AT IMPACT MUST BE PARALLEL TO THE y-AXIS



THUS

$$\tan\theta = \frac{10}{6-D}$$

$$\theta = \tan^{-1}\frac{10}{6-2.37} = 70.04^\circ$$

$$\theta = 70.0^\circ$$

(b) CONSERVATION OF MOMENTUM IN x (n) DIRECTION

$$m v_A \cos\theta + m(v_B)_n = m(v_A')_n + m v_B'$$

$$(3)(\cos 70.04) + 0 = (v_A')_n + v_B'$$

$$1.0241 = (v_A')_n + (v_B') \quad (1)$$

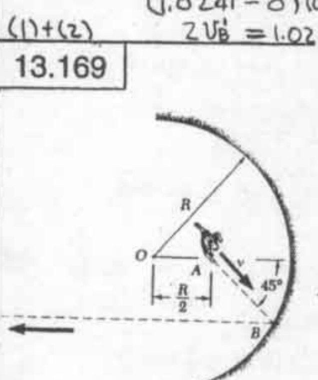
RELATIVE VELOCITIES IN THE n DIRECTION

$e = 0.9$ $\quad (v_A\cos\theta - (v_B)_n)e = v_B' - (v_A')_n$

$$(1.0241 - 0)(0.9) = v_B' - (v_A')_n \quad (2)$$

(1)+(2) $\quad 2v_B' = 1.0241(1.9), \quad v_B' = 0.972$ ft/s

13.169

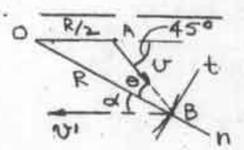


GIVEN:

BALL THROWN WITH VELOCITY v AS SHOWN. BALL REBOUNDS IN A DIRECTION PARALLEL TO OA

FIND:

COEFFICIENT OF RESTITUTION e BETWEEN THE BALL AND THE WALL



LAW OF SINES

$$\frac{\sin\theta}{R/2} = \frac{\sin 135^\circ}{R}$$

$$\theta = 20.705^\circ$$

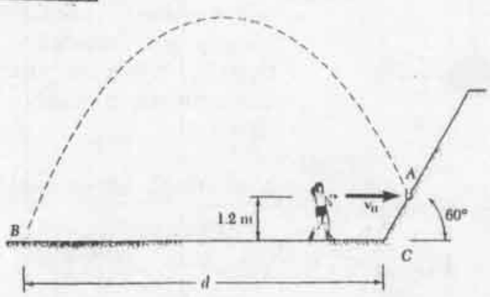
$$\alpha = 45^\circ - 20.705^\circ = 24.295^\circ$$

CONS. OF MOM FOR WALL IN t DIRECTION, $-v\sin\theta = -v'\sin\alpha$

COEFF. OF RESTITUTION IN n; $\quad v(\cos\theta)e = v'\cos\alpha$

DIVIDING, $\dfrac{\tan\theta}{e} = \tan\alpha \qquad e = \dfrac{\tan 20.705^\circ}{\tan 24.295^\circ} = 0.837$

13.170



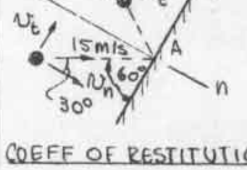
GIVEN:

INITIAL VELOCITY, $v_0 = 15$ m/s

$e = 0.9$

FIND:

DISTANCE d



MOMENTUM IN t DIRECTION IS CONSERVED

$$m v \sin 30^\circ = m v_t'$$

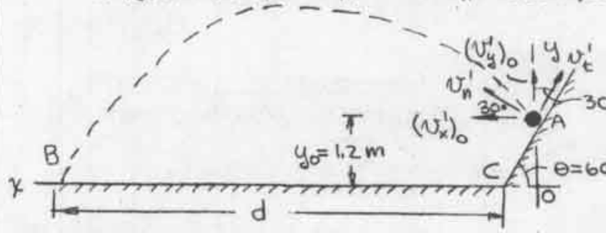
$$(15)(\sin 30^\circ) = v_t'$$

$$v_t' = 7.5 \text{ m/s}$$

COEFF OF RESTITUTION IN n-DIRECTION

$$(v\cos 30^\circ)e = v_n'$$

$$(15)(\cos 30^\circ)(0.9) = v_n' \qquad v_n' = 11.69 \text{ m/s}$$



WRITE v' IN TERMS OF X AND Y COMPONENTS

$$(v_x')_0 = v_n'\cos 30^\circ - v_t'\sin 30^\circ$$

$$(v_x')_0 = (11.69)(\cos 30^\circ) - (7.5)(\sin 30^\circ) = 6.374 \text{ m/s}$$

$$(v_y')_0 = v_n'\sin 30^\circ + v_t'\cos 30^\circ$$

$$(v_y')_0 = (11.69)(\sin 30^\circ) + (7.5)(\cos 30^\circ) = 12.340 \text{ m/s}$$

MOTION OF A PROJECTILE (ORIGIN AT O)

$$y = y_0 + (v_y')_0 t - (gt^2)/2$$

$$y = 1.2 + (12.340 \text{ m/s})t - (9.81 \text{ m/s}^2)t^2/2$$

TIME TO REACH POINT B ($y_B = 0$)

$$0 = 1.2 + 12.340t_B - (9.81/2)t_B^2$$

$$t_B = 2.610 \text{ s}$$

$$x = x_0 + (v_x')_0 t$$

$$x = 0 + 6.374t$$

$$x_B = (6.374)(t_B) = (6.374 \text{ m/s})(2.610 \text{ s})$$

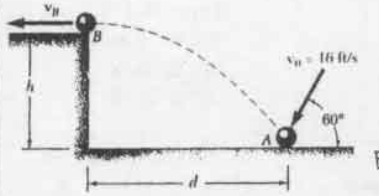
$$x_B = 16.63 \text{ m}$$

$$d = x_B - 1.2\cot 60^\circ = 15.94 \text{ m}$$

$$d = 15.94 \text{ m}$$

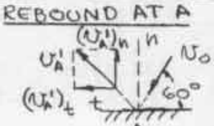
13.171

GIVEN:
INITIAL VELOCITY OF BALL AS SHOW AT A
$e = 0.6$
$\underline{v}_B$ IS HORIZONTAL



FIND:
(a) h AND d
(b) v_B

(a) REBOUND AT A



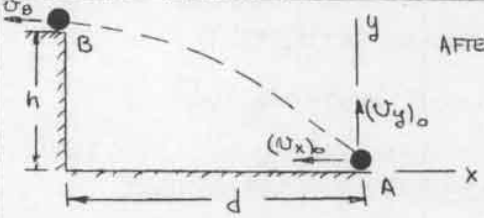
CONSERVATION OF MOMENTUM IN THE t-DIRECTION

$$m v_0 \cos 60° = m (v_A')_t$$
$$(v_A')_t = (16 \text{ ft/s})(\cos 60°) = 8.00 \text{ ft/s}$$

COEFF. OF RESTITUTION IN THE n-DIRECTION

$$(-(v_A)_n - 0)e = 0 - (v_A')_n$$
$$(16 \text{ ft/s})(\sin 60°)(0.6) = (v_A')_n$$
$$(v_A')_n = 8.314 \text{ ft/s}$$

PROJECTILE MOTION BETWEEN A AND B



$$(v_x)_0 = -(v_A')_t = -8.00 \text{ ft/s}$$
$$(v_y)_0 = (v_A')_n = 8.314 \text{ ft/s}$$

X DIRECTION
$$x = (v_x)_0 t = -8t, \quad v_x = -8 \text{ ft/s}$$

y DIRECTION
$$y = (v_y)_0 t - \tfrac{1}{2} g t^2 = 8.314t - (16.1)t^2$$
$$v_y = (v_y)_0 - gt = 8.314 - 32.2t$$

AT B, $(v_B)_y = 0$
$$(v_B)_y = 0 = 8.314 - 32.2\, t_{A-B}$$
$$t_{A-B} = 0.2582 \text{ s}$$

$y_B = h$
$$h = (8.314)t_{A-B} - (16.1)t_{A-B}^2$$
$$h = (8.314)(0.2582) - (16.1)(0.2582)^2$$
$$h = 1.073 \text{ ft}$$

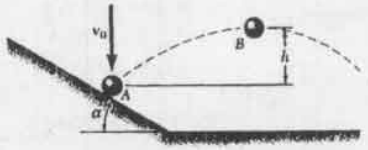
$$x_B = -d = -8t_{A-B}$$
$$d = (8)(0.2582) = 2.065 \text{ ft}$$
$$d = 2.07 \text{ ft}$$

(b) $v_B = (v_x)_0 = -8.00$ ft/s

$$v_B = -8.00 \text{ ft/s}$$

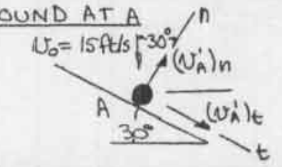
13.172

GIVEN:
$v_0 = 15$ ft/s
$\alpha = 30°$
$e = 0.8$



FIND:
h

REBOUND AT A



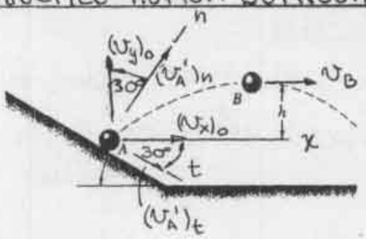
CONSERVATION OF MOMENTUM IN THE t-DIRECTION

$$m v_0 \sin 30° = m (v_A')_t$$
$$(v_A')_t = (15 \text{ ft/s})(\sin 30°) = 7.5 \text{ ft/s}$$

RELATIVE VELOCITIES IN THE n-DIRECTION

$$(-v_0 \cos 30° - 0)e = 0 - (v_A')_n$$
$$(v_A')_n = (0.8)(15 \text{ ft/s})(\cos 30°)$$
$$(v_A')_n = 10.392 \text{ ft/s}$$

PROJECTILE MOTION BETWEEN A AND B



$$(v_x)_0 = (v_A')_t \cos 30° + (v_A')_n \sin 30°$$
$$(v_x)_0 = (7.5)(\cos 30°) + (10.392)\sin 30°$$
$$(v_x)_0 = 11.691 \text{ ft/s}$$

$$(v_y)_0 = -(v_A')_t \sin 30° + (v_A')_n \cos 30°$$
$$(v_y)_0 = -(7.5)(\sin 30°) + (10.392)\cos 30°$$
$$(v_y)_0 = 5.2497 \text{ ft/s}$$

X DIRECTION $\quad x = (v_x)_0 t \quad v_x = (v_x)_0$

$$x = 11.69t \quad v_x = 11.69 \text{ ft/s} = v_B$$

y DIRECTION $\quad y = (v_y)_0 t - \tfrac{1}{2} g t^2$

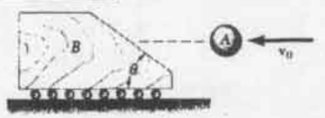
$$v_y = (v_y)_0 - gt$$

AT A $\quad v_y = 0 = (v_y)_0 - g t_{AB}$

$$t_{AB} = (v_y)_0 / g = \frac{5.2497 \text{ ft/s}}{32.2 \text{ ft/s}^2}$$
$$t_{A-B} = 0.1630 \text{ s}$$

AT B
$$y = h = (v_y)_0 t_{A-B} - \frac{g t_{A-B}^2}{2}$$
$$h = (5.2497)(.1630) - (16.1)(.1630)^2 = 0.428 \text{ ft}$$

13.173

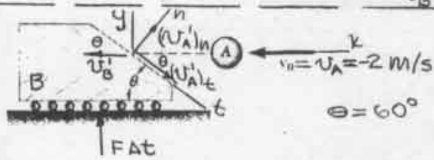


GIVEN:

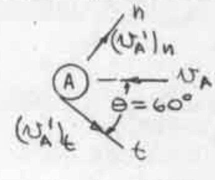
$m_A = 1.2$ kg
$m_B = 4.8$ kg
$v_0 = 2$ m/s
$e = 1,\ \theta = 60°$

FIND:

v_B' AFTER IMPACT



A ALONE MOMENTUM CONSERVED IN t-DIRECTION



$$m_A v_A \cos 60° = m_A (v_A')_t$$

$$(v_A')_t = -(2\text{m/s})(.5) = -1 \text{ m/s}$$

A AND B TOTAL MOMENTUM CONSERVED ALONG THE X-AXIS

$$m_A v_A + m_B v_B = m_A[(v_A')_t \cos\theta + (v_A')_n \sin\theta] - m_B v_B'$$

$$(1.2\text{kg})(-2\text{m/s}) + 0 = (1.2\text{kg})[(-1\tfrac{\text{m}}{\text{s}})(\cos 60°) + (v_A')_n(\sin 60°)] - 4.8\text{kg}(v_B')$$

$$-1.8 = 1.0392 (v_A')_n - 4.8 v_B' \qquad (1)$$

RELATIVE VELOCITIES IN THE n DIRECTION

$$[v_A \sin\theta - (v_B)_n]e = (v_B')_n - (v_A')_n$$

$(v_B)_n = 0$
$e = 1$
$v_A = -2$m/s
$\theta = 60°$

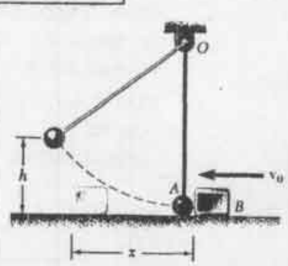
$$(-2)(\sin 60°)(1) = -v_B' \sin 60° - (v_A')_n$$

$$-1.732 = -0.866 v_B' - (v_A')_n \qquad (2)$$

SOLVING (1) AND (2) SIMULTANEOUSLY

$(v_A')_n = 1.184$ m/s $\qquad v_B' = 0.632$ m/s ←

13.174

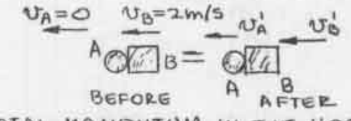


GIVEN:

$m_B = 1$ kg
$m_A = 0.5$ kg
$v_0 = 2$ m/s
$\mu_k = 0.6$
$e = 0.8$

FIND:

(a) MAX. HEIGHT h
(b) DISTANCE x TRAVELED BY THE BLOCK



VELOCITIES JUST AFTER IMPACT

TOTAL MOMENTUM IN THE HORIZONTAL DIRECTION IS CONSERVED

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$

$$0 + (1\text{kg})(2\text{m/s}) = (0.5\text{kg})(v_A') + (1\text{kg})(v_B')$$

$$4 = v_A' + 2v_B' \qquad (1)$$

RELATIVE VELOCITIES

$$(v_A - v_B)e = (v_B' - v_A')$$

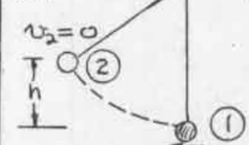
$$(0-2)(0.8) = v_B' - v_A'$$

$$-1.6 = v_B' - v_A' \qquad (2)$$

SOLVING (1) AND (2) SIMULTANEOUSLY

$$v_B' = 0.8 \text{ m/s} \qquad v_A' = 2.4 \text{ m/s}$$

(a)



CONSERVATION OF ENERGY

$T_1 = \frac{1}{2} m_A v_1^2 \qquad V_1 = 0$

$T_1 = \frac{1}{2} m_A (2.4\text{m/s})^2 = 2.88 m_A$

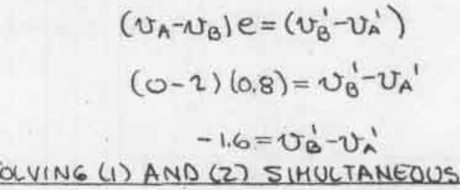
$T_2 = 0$

$V_2 = m_A g h$

$$T_1 + V_1 = T_2 + V_2 \qquad 2.88 \cancel{m_A} + 0 = 0 + \cancel{m_A}(9.81)$$

$$h = 0.294 \text{ m}$$

(b) WORK AND ENERGY



$$T_1 = \tfrac{1}{2} m_B v_1^2 = \tfrac{1}{2} m_B (0.8\text{m/s})^2 = 0.32 m_B \qquad T_2 = 0$$

$$U_{1-2} = -F_f x = \mu_k N x = \mu_k m_B g x = -(0.6)(m_B)(9.81)x$$

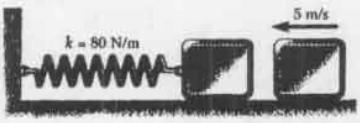
$$U_{1-2} = -5.886 m_B x$$

$$T_1 + U_{1-2} = T_2 \qquad 0.32 m_B - 5.886 m_B x = 0$$

$$x = 0.0544 \text{ m}$$

13.175

GIVEN:



$m_A = m_B = 1.5\,kg$

INITIALLY,

$v_A = 5\,m/s,\ v_B = 0$

NO FRICTION

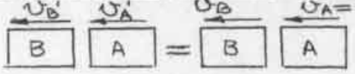
(1) $e = 1$

(2) $e = 0$

FIND:

(a) MAXIMUM DEFLECTION OF THE SPRING

(b) FINAL VELOCITY OF BLOCK A



PHASE I IMPACT

CONSERVATION OF TOTAL MOMENTUM

$\overset{+}{\leftarrow}\quad m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$m_A = m_B$

$$5 + 0 = v_A' + v_B' \qquad (1)$$

RELATIVE VELOCITIES

$$(v_A - v_B)e = (v_B' - v_A')$$

$$(5-0)e = v_B - v_A' \qquad (2)$$

ADDING (1) AND (2)

$$\frac{5(1+e)}{2} = v_B'$$

SUBTRACTING (2) FROM (1)

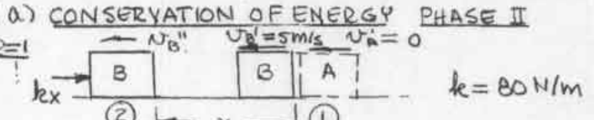
$$\frac{5(1-e)}{2} = v_A'$$

$e=1 \quad v_B' = 5\,m/s \quad v_A' = 0$

$e=0 \quad v_B' = 2.5\,m/s \quad v_A' = 2.5\,m/s$

(a) CONSERVATION OF ENERGY PHASE II

$e=1$



$T_1 = \frac{1}{2} m_B (v_B')^2 = \frac{1}{2}(1.5\,kg)(5\,m/s)^2 = 18.75\,J$

$V_1 = 0$

AT $x = x_{MAX}$, $T_2 = 0$; $V_2 = \frac{1}{2}k(x_{MAX})^2 = (40)(x_{MAX})^2$

$T_1 + V_1 = T_2 + V_2$

$$18.75 + 0 = 0 + 40x_{MAX}^2$$

$$e = 1 \qquad x_{MAX} = 0.685\,m$$

$e=0$ BOTH A AND B HAVE THE SAME VELOCITY INITIALLY AT ①, OF 2.5 m/s

THUS $T_1 = \frac{1}{2}(m_A + m_B)(v_A')^2 = \left(\frac{3\,kg}{2}\right)(2.5\,m/s)^2$

$T_1 = 9.375\,J \qquad V_1 = 0$

AT $x = x_{MAX}$ $\quad T_2 = 0 \quad V_2 = \frac{1}{2}kx_{MAX}^2 = 40x_{MAX}^2$

$T_1 + V_1 = T_2 + V_2 \qquad 9.375 + 0 = 40x_{MAX}^2$

$$e = 0 \qquad x_{MAX} = 0.484\,m$$

13.175 continued

(b) $e=1$, BLOCK B IS RETURNED TO POSITION ① WITH A VELOCITY OF 5 m/s → SINCE ENERGY IS CONSERVED, AND IMPACTS BLOCK A WHICH IS AT REST. IN THE IMPACT TOTAL MOMENTUM IS CONSERVED AND PHASE I IS REPEATED WITH THE VELOCITIES OF A AND B INTERCHANGED. THUS $v_A''' = 5\,m/s \rightarrow$ AND $v_B''' = 0$. SINCE THERE IS NO FRICTION THESE VELOCITIES ARE THE FINAL VELOCITIES OF A AND B

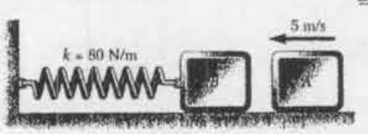
$$e = 1 \qquad v_A''' = 5\,m/s \rightarrow$$

$e=0$, BLOCKS A AND B ARE RETURNED TO POSITION ① WITH THE SAME VELOCITY OF 2.5 m/s → SINCE ENERGY IS CONSERVED. THERE IS NO ADDITIONAL IMPACT AND THE SPRING SLOWS BLOCK B DOWN AND A AND B SEPARATE WITH A CONTINUING WITH A VELOCITY OF 2.5 m/s TO THE RIGHT

$$e = 0 \qquad v_A''' = 2.5\,m/s \rightarrow$$

13.176

GIVEN:



$m_A = m_B = 1.5\,kg$

INITIALLY

$v_A = 5\,m/s,\ v_B = 0$

$\mu_k = 0.3,\ \mu_s = 0.5$

$e = 1$

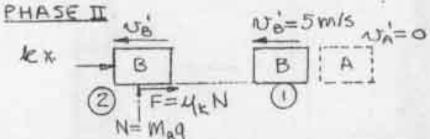
FIND:

FINAL POSITION OF (a) BLOCK A (b) BLOCK B

IMPACT SEE PHASE I OF PROB 13.175, $e=1$

AFTER IMPACT $v_A' = 0$, $v_B' = 5\,m/s$

PHASE II



MAXIMUM DEFLECTION, x_{MAX}, OF THE SPRING

WORK AND ENERGY

$T_1 = \frac{1}{2} m_B (v_B')^2 = \frac{1}{2}(1.5\,kg)(5\,m/s)^2 = 18.75\,J$

$T_2 = 0 \quad U_{1-2} = \int_0^{x_{MAX}} -kx\,dx - \int_0^{x_{MAX}} \mu_k m_B g\,dx$

$U_{1-2} = -\frac{1}{2}(80)(x_{MAX})^2 - (0.3)(1.5\,kg)(9.81\,\frac{m}{s^2})x_{MAX}$

$T_1 + U_{1-2} = T_2$

$$18.75 - 40x_{MAX}^2 - 4.4145x_{MAX} = 0$$

$$x_{MAX} = 0.632\,m$$

PHASE III RETURN OF B TO POSITION ① BEFORE IMPACT WITH A

$T_2 = 0,\ U_{2-1} = +\frac{1}{2}k(x_{MAX})^2 - \mu_k m_B g x_{MAX}$

$T_1 = \frac{1}{2} m_B v_B''^2 = (0.75)v_B''^2 \qquad U_{2-1} = (40)(0.632)^2 - (4.4145)(0.632)$

$U_{2-1} = 15.977 - 2.790 = 13.173$

$0 + 13.173 = (0.75)v_B''^2 \qquad v_B'' = 4.191\,m/s \rightarrow$

AFTER IMPACT WITH A AT POSITION ①

RELATIVE VELOCITIES

$$(v_B'' - v_A')(e) = (v_A'' - v_B''')$$

$$\overset{+}{\rightarrow}\quad (4.191 - 0)(1) = v_A'' - v_B''' \qquad (1)$$

(CONTINUED)

13.176 continued

<u>CONSERVATION OF TOTAL MOMENTUM AT ①</u>

$\xrightarrow{+}$ $m_A v_A' + m_B v_o'' = m_A v_A'' + m_B v_B'''$ $\quad m_A = m_B$

$0 + 4.191 = v_A'' + v_B'''$ (2)

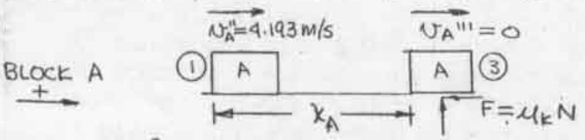
ADDING EQUATIONS (1) AND (2)

$2(4.191) = 2v_A''$

$v_A'' = 4.191$ m/s

FROM EQ. (2) $v_B''' = 4.191 - 4.191 = 0$

(a) <u>PHASE IV</u> (VELOCITY OF B = 0 AT ①)



$T_1 = \frac{1}{2} m_A (v_A'')^2 = (0.75\,kg)(4.191\,m/s)^2$ $\quad N = m_A g$

$T_1 = 13.173$ J $\quad U_{1\text{-}3} = -\mu_k m_B g x_A = -(0.3)(1.5\,kg)(9.81\,\frac{m}{s^2})x_A$

$U_{1\text{-}3} = -4.415\,x_A$

$T_3 = 0$

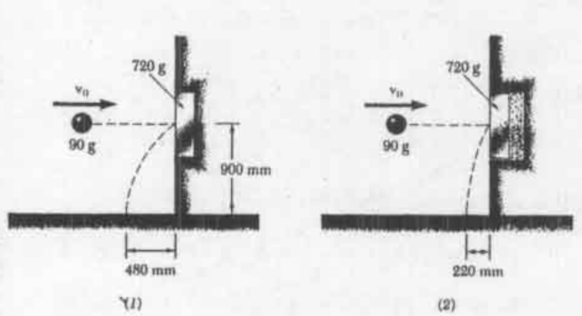
$T_1 + U_{1\text{-}3} = T_2$ $\quad 13.173 - 4.415\,x_A = 0$

FINAL POSITION OF A $\quad x_A = 2.98 \rightarrow$ m

(b) $v_B''' = 0$ AT IMPACT POINT AND THE SPRING IS UNDEFLECTED AT THIS POINT.

$x_B = 0$

13.177



<u>GIVEN:</u>

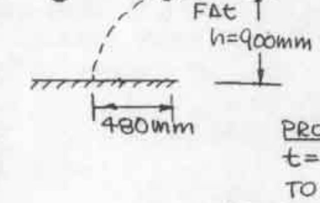
BALL REBOUNDS AS SHOWN IN FIGURES (1) AND (2). FOAM RUBBER BEHIND PLATE IN (2)

<u>FIND:</u>

(a) COEFFICIENT OF RESTITUTION e BETWEEN THE BALL AND THE PLATE

(b) <u>THE INITIAL VELOCITY v_0</u>

(a)



<u>FIG.(1), BALL ALONE</u>

<u>RELATIVE VELOCITIES</u>

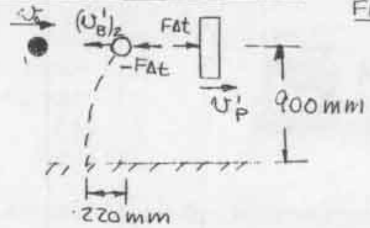
$v_0 e = (v_B')_1$

<u>PROJECTILE MOTION</u>

t = TIME FOR THE BALL TO HIT THE GROUND

$0.480\,m = v_0 e t$ (1)

13.177 continued



<u>FIG (2) BALL AND PLATE</u>

<u>RELATIVE VELOCITIES</u>

$\xrightarrow{+}$ $(v_B - v_P)e = v_P' + (v_B')_2$

$v_B = v_0 \quad v_P = 0$

$v_0 e = v_P' + (v_B')_2$ (2)

<u>CONSERVATION OF MOMENTUM</u>

$\xrightarrow{+}$ $m_B v_B + m_P v_P = m_B(-v_B')_2 + m_P(v_P')$

$(0.09\,kg)(v_0) + 0 = (0.09\,kg)(-v_B')_2 + (0.720\,kg)v_P'$

$v_0 = (-v_B')_2 + 8v_P'$ (3)

SOLVING (2) AND (3) SIMULTANEOUSLY FOR $(v_B')_2$

$(v_B')_2 = v_0 \frac{(8e-1)}{9}$

<u>PROJECTILE MOTION</u>

$0.220\,m = v_0 \frac{(8e-1)}{9} t$ (4)

DIVIDING EQ. (4) BY EQ. (3)

$\frac{0.220}{0.480} = \frac{8e-1}{9e}$

$4.125e = 8e - 1$

$e = 0.258$

(b) <u>FROM FIG (1)</u>

PROJECTILE MOTION

$h = \frac{1}{2} g t^2$

$0.900 = \frac{(9.81)}{2} t^2, \quad 1.80 = 9.81 t^2$ (5)

EQUATION (1)

$0.480 = v_0 e t$

$t = \frac{(0.480)}{(0.258)v_0} = \frac{1.860}{v_0}$

SUBSTITUTING FOR t IN (5)

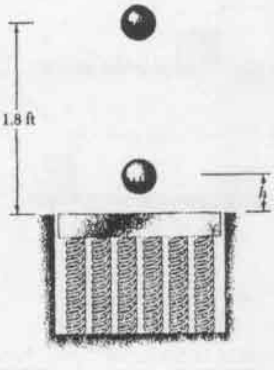
$1.800 = (9.81)\frac{(1.860)^2}{v_0^2}$

$v_0^2 = 18.855$

$v_0 = 4.34$ m/s

13.178

GIVEN:
$W_A = 1.3\ \text{lb}$
$W_B = 2.6\ \text{lb}$
$e = 0.8$



FIND:
(a) REBOUND HEIGHT h OF THE SPHERE A
(b) EQUIVALENT SPRING CONSTANT k IF THE MAXIMUM DEFLECTION OF THE PLATE IS $3h$

(a) VELOCITY OF A AND B AFTER IMPACT

$+\uparrow$ v_A, A, B $=$ A, B, v_A', v_B'
$v_B = 0$
BEFORE AFTER

INITIAL VELOCITY OF A (BEFORE IMPACT)

$v_A = 0$ (A) ① $T_1 = 0 \quad V_1 = (m_A g)(1.8\ \text{ft})$

1.8 ft $\qquad T_2 = \frac{1}{2} m_A v_A^2 \quad V_2 = 0$

$T_1 + V_1 = T_2 + V_2$

v_A (A) ② $\qquad 0 + 1.8 m_A g = \frac{1}{2} m_A v_A^2$

$v_A^2 = (2)(1.8\ \text{ft})(32.2\ \text{ft/s}^2) = 115.92\ \text{ft}^2/\text{s}^2$

$v_A = 10.77\ \text{ft/s} \downarrow$

VELOCITIES AFTER IMPACT

CONSERVATION OF MOMENTUM

$m_A(-v_A) + m_B(v_B) = m_A v_A' + m_B(-v_B')$

$\frac{1.3}{g}(-10.77) + 0 = \frac{1.3}{g} v_A' + \frac{2.6}{g}(-v_B')$

$-10.77 = v_A' - 2 v_B' \qquad (1)$

RELATIVE VELOCITIES

$(-v_A - v_B)e = (-v_B' - v_A')$

$(10.77)(0.8) = v_B' + v_A' \qquad (2)$

SOLVING (1) AND (2) SIMULTANEOUSLY

$v_A' = 2.15\ \text{ft/s} \qquad v_B' = 6.46\ \text{ft/s}$

REBOUND HEIGHT OF A (CONSERVATION OF ENERGY)

$v = 0$ (A) ③ $\quad T_2 = \frac{1}{2} m_A v_A'^2 \quad V_2 = 0$

$h \quad T_3 = 0 \quad V_3 = m_A g h$

$T_2 + V_2 = T_3 + V_3$

v_A' (A) ② $\quad \frac{1}{2} m_A v_A'^2 + 0 = 0 + m_A g h$

$h = \frac{1}{2}(2.15\ \text{ft/s})^2 / (32.2\ \text{ft/s}^2) = 0.0720\ \text{ft}$ ◀

(b) B ② $\quad T_2 = \frac{1}{2} m_B v_B'^2 \quad V_2 = 0$

$v_B' = 6.46\ \text{ft/s} \quad 3h = 0.216\ \text{ft} \quad T_4 = 0 \quad V_4 =$

$V_4 = \frac{1}{2} k (3h)^2$

B ④, $v_B'' = 0$, F_S

$T_2 + V_2 = T_4 + V_4$

$\frac{1}{2}\left(\frac{2.6\ \text{lb}}{32.2\ \text{ft/s}^2}\right)(6.46\ \text{ft/s})^2 = \frac{1}{2} k (0.216)^2$

$k = 72.2\ \text{lb/ft}$ ◀

13.179

GIVEN:
FIGURE AS SHOW IN 13.178 (LEFT)
$W_A = 1.3\ \text{lb}, W_B = 2.6\ \text{lb}$
EQUIV SINGLE SPRING $k = 5\ \frac{\text{lb}}{\text{in}}$

FIND:
(a) VALUE OF e FOR WHICH h IS A MAXIMUM
(b) CORRESPONDING VALUE OF h
(c) CORRESPONDING MAXIMUM DEFLECTION OF B

(a) INITIAL VELOCITY OF A (BEFORE IMPACT)

FROM SOLUTION TO PROB. 13.178, $v_A = 10.77\ \text{ft/s} \downarrow$

$v_A = 10.77\ \text{ft/s}$, A, B, $v_B = 0$ $=$ A, B, v_A', v_B'

CONSERVATION OF MOMENTUM

$+\uparrow \quad m_A(-v_A) + m_B v_B = m_A v_A' + m_B(-v_B')$

$\frac{1.3}{g}(-10.77) + 0 = \frac{1.3}{g} v_A' + \frac{2.6}{g}(-v_B')$

$-10.77 = v_A' - 2 v_B' \qquad (1)$

RELATIVE VELOCITIES

$+\uparrow \quad (-v_A - v_B)e = (-v_B' - v_A')$

$10.77 e = v_B' + v_A' \qquad (2)$

SOLVING (1) AND (2) SIMULTANEOUSLY FOR v_A'

$3 v_A' = (10.77)(2e - 1)$

h IS MAXIMUM WHEN v_A' IS MAXIMUM, THAT IS WHEN $e = 1$

$e = 1$ ◀

(b) FOR $e = 1$

$v_A' = 10.77/3 = 3.59\ \text{ft/s}$

FOR A ALONE

CONSERVATION OF ENERGY

$v_A = 0$ (A) ② $\quad T_1 = \frac{1}{2}\frac{W_A}{g}(v_A')^2 \quad V_1 = 0$

h

(A) ① $\quad T_2 = 0 \quad V_2 = W_A h$

$v_A' = 3.59\ \text{ft/s}$

$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2}\frac{W_A}{g}(v_A')^2 + 0 = 0 + W_A h$

$h = \frac{1}{2}\frac{(3.59\ \text{ft/s})^2}{(32.2\ \text{ft/s}^2)} = 0.200\ \text{ft}$ ◀

(c) FOR B ALONE

CONSERVATION OF ENERGY

v_B', B ①, x, B ③, F_S

FROM (a), EQ (2)

$v_B' = 10.77 e - v_A'$

$v_B' = (10.77)(1) - 3.59$

$v_B' = 7.18\ \text{ft/s}$

$T_1 = \frac{1}{2}\frac{W_B}{g}(v_B')^2 = \frac{1}{2}\frac{(2.6\ \text{lb})}{(32.2\ \text{ft/s}^2)}(7.18\ \text{ft/s})^2$

$T_1 = 2.08\ \text{ft}\cdot\text{lb} \qquad V_1 = 0$

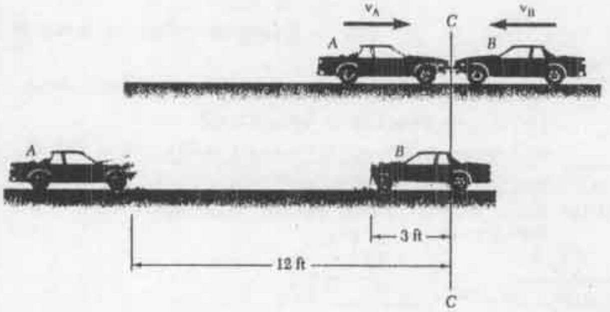
$T_3 = 0 \quad V_3 = \frac{1}{2} k x^2 \quad k = 5\ \text{lb/in.} = 60\ \text{lb/ft}$

$V_3 = \frac{1}{2}(60)(x)^2 = 30 x^2$

$T_1 + V_1 = T_3 + V_3$

$2.09 + 0 = 0 + 30 x^2 \qquad x = 0.263\ \text{ft}$ ◀

13.180



GIVEN:

CARS A AND B OF THE SAME MASS
BEFORE COLLISION $v_A = 5$ mi/h
BRAKES LOCKED, $\mu_k = 0.30$
CARS AT REST IN POSITION SHOWN

FIND:

(a) THE SPEED OF B, v_B, BEFORE IMPACT
(b) EFFECTIVE COEFF. OF RESTITUTION, e

(a) v_A' v_B' [A] [B] = [A] [B] $v_A = 5$ mi/h v_B

AFTER BEFORE

AT C

CONSERVATION OF TOTAL MOMENTUM

$\overset{+}{\leftarrow}$ $m_A = m_B = m$ 5 mi/h = 7.333 ft/s

$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$-7.33 + v_B = v_A' + v_B'$ (1)

WORK AND ENERGY

CAR A (AFTER IMPACT)

$v_A'' = 0$ $m_A g$ v_A' $T_1 = \frac{1}{2} m_A (v_A')^2$

(2) $F_f = \mu_k N$ (1) $T_2 = 0$

$N = m_A g$ 12' $U_{1-2} = F_f(12)$

$U_{1-2} = -\mu_k m_A g\,(12\text{ ft})$

$T_1 + U_{1-2} = T_2$

$\frac{1}{2} m_A (v_A')^2 - \mu_k m_A g\,(12) = 0$

$(v_A')^2 = (2)(12\text{ ft})(0.3)(32.2\text{ ft/s}^2) = 231.84\text{ ft}^2/\text{s}^2$

$v_A' = 15.226$ ft/s

CAR B (AFTER IMPACT)

$v_B'' = 0$ v_B' $T_1 = \frac{1}{2} m_B (v_B')^2$ $T_2 = 0$

(2) [B]   [B] (1) $U_{1-2} = \mu_k m_B g\,(3)$

F_f 3 ft $T_1 + U_{1-2} = T_2$

$\frac{1}{2} m_B (v_B')^2 - \mu_k m_B g\,(3)$

$v_B'^2 = (2)(3\text{ ft})(0.3)(32.2\text{ ft/s}^2)$

$(v_B')^2 = 57.96\text{ ft}^2/\text{s}^2$ $v_B' = 7.613$ ft/s

FROM (1) $v_B = 7.333 + v_A' + v_B' = 7.333 + 15.226 + 7.613$

$v_B = 30.2\text{ ft/s} = 20.6\ \frac{\text{mi}}{\text{h}}$

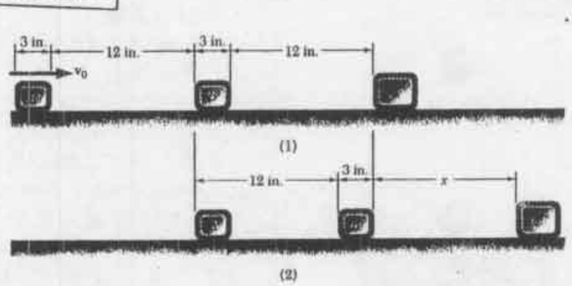
(b) RELATIVE VELOCITIES

$\overset{+}{\leftarrow}$ $(-v_A - v_B)e = v_B' - v_A'$

$(-7.333 - 30.2)\,e = 7.613 - 15.226$

$e = \frac{-(7.613)}{-(37.53)} = 0.2028$ $e = 0.203$

13.181



GIVEN:

$W_A = W_B = 0.8$ lb, $W_C = 2.4$ lb, $\mu_k = 0.30$
INITIALLY $v_A = v_0 = 15$ ft/s, $v_B = v_C = 0$
AFTER A STRIKES B AND B STRIKES C ALL BLOCKS COME TO REST AS SHOWN IN (2)

FIND:

(a) COEFF. OF RESTITUTION BETWEEN A AND B AND BETWEEN B AND C.
(b) THE DISPLACEMENT x OF BLOCK C

(a) WORK AND ENERGY

VELOCITY OF A JUST BEFORE IMPACT WITH B

$v_0 = 15$ ft/s v_A $T_1 = \frac{1}{2}\frac{W_A}{g} v_0^2$ $T_2 = \frac{1}{2}\frac{W_A}{g}(v_A^2)_2$

F_f 12 in.   (2) $U_{1-2} = -\mu_k W_A\,(1\text{ ft})$

(1) $F_f = \mu_k N_A = \mu_k W_A$ $T_1 + U_{1-2} = T_2$

$\frac{1}{2}\frac{W_A}{g} v_0^2 - \mu_k W_A(1) = \frac{1}{2}\frac{W_A}{g}(v_A^2)_2$

$(v_A^2)_2 = v_0^2 - 2\mu_k g = (15\text{ ft/s})^2 - 2(.3)(32.2\text{ ft/s}^2)(1\text{ ft})$

$(v_A^2)_2 = 205.68\text{ ft}^2/\text{s}^2$, $(v_A)_2 = 14.342$ ft/s

VELOCITY OF A AFTER IMPACT WITH B $(v_A')_2$

$(v_A')_2$ $v_A = 0$ $T_2 = \frac{1}{2}\frac{W_A}{g}(v_A'^2)_2$ $T_3 = 0$

F_f (3) $U_{2-3} = -\mu_k W_A\,(3/12)$

(2)   3 in. $T_2 + U_{2-3} = T_3$, $\frac{1}{2}\frac{W_A}{g}(v_A'^2)_2 - (\mu_k)(W_A/4) = 0$

$(v_A'^2)_2 = 2(.3)(32.2\text{ ft/s}^2)(\frac{1}{4}\text{ ft}) = 4.83\text{ ft}^2/\text{s}^2$

$(v_A')_2 = 2.198$ ft/s

CONSERVATION OF MOMENTUM AS A HITS B

$(v_A)_2$ $v_B = 0$ $(v_A')_2$ v_B' $(v_A)_2 = 14.342$ ft/s

[A] [B] = [A] [B] $(v_A')_2 = 2.198$ ft/s

$\overset{+}{\rightarrow}$

$m_A (v_A)_2 + m_B v_B = m_A (v_A')_2 + m_B v_B'$ $m_A = m_B$

$14.342 + 0 = 2.198 + v_B'$ $v_B' = 12.144$ ft/s

RELATIVE VELOCITIES (A AND B)

$\overset{+}{\rightarrow}$ $((v_A)_2 - v_B)\,e_{AB} = v_B' - (v_A')_2$

$(14.342 - 0)\,e_{AB} = 12.144 - 2.198$

$e_{AB} = 0.694$

WORK AND ENERGY

VELOCITY OF B JUST BEFORE IMPACT WITH C

$(v_B')_2$ $(v_B')_4$

[B]   [B] $T_2 = \frac{1}{2}\frac{W_B}{g}(v_B')_2^2 = \frac{W_B}{2g}(12.144)^2$

F_f (2)   12 in.   (4) $T_4 = \frac{1}{2}\frac{W_B}{g}(v_B')_4^2 = \frac{W_B}{2g}(v_B')_4^2$

$(v_B')_2 = 12.144$ ft/s $U_{2-4} = -\mu_k W_B(1\text{ ft}) = (0.3)W_B$

$F_f = \mu_k W_B$ $T_2 + U_{2-4} = T_4$, $\frac{(12.144)^2}{2g} - 0.3 = \frac{(v_B')_4^2}{2g}$

(CONTINUED) $(v_B')_4 = 11.321$ ft/s

13.181 continued

CONSERVATION OF MOMENTUM AS B HITS C

$(v_B')_4 \quad v_C=0 \quad (v_B'')_4 \quad v_C'$

$m_B=\frac{0.8}{g}$ $\quad m_C=\frac{2.4}{g}$

$(v_B')_4 = 11.321$ ft/s

$\xrightarrow{+}$ $m_B(v_B')_4+m_C v_C = m_B(v_B'')_4+m_C v_C'$

$\frac{0.8}{g}(11.321)+0=\frac{0.8}{g}(v_B'')_4+\left(\frac{2.4}{g}\right)(v_C')$

$11.321=(v_B'')_4+3v_C'$

VELOCITY OF B AFTER B HITS C, $(v_B'')_4 = 0$
(COMPARE FIGURE (1) AND FIGURE (2))
THUS $v_C'=3.774$ ft/s

RELATIVE VELOCITIES (B AND C)

$((v_B')_4-v_C)e_{BC}=v_C'-(v_B'')_4$

$(11.321-0)e_{BC}=3.774-0$

$e_{BC}=0.333$ ◀

(b) WORK AND ENERGY (BLOCK C)

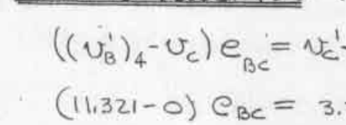
$v_C'=3.774$ ft/s $\quad v=0$ $\quad F_f$ $\quad$ (4) $\quad x \quad$ (5)

$T_4=\frac{1}{2}\frac{W_C}{g}(v_C')^2 \quad T_5=0 \quad U_{4-5}=-\mu_k W_C(x)$

$T_4+U_{4-5}=T_5 \quad \frac{1}{2}\frac{W_C}{g}(3.774)^2-(0.3)W_C(x)=0$

$x=\frac{(3.774)^2}{2(32.2)(0.3)}=0.737$ ft $\quad x=8.84$ in. ◀

13.182



GIVEN:
$W_A=W_B=W_C=W$
INITIALLY
$v_A=3$ ft/s
$v_C=v_B=0$
$e=0$
$\mu_K=0.20$

FIND:
(a) TIME REQUIRED FOR ALL BLOCKS TO REACH THE SAME VELOCITY
(b) THE TOTAL DISTANCE TRAVELLED BY EACH BLOCK DURING THE TIME FOUND IN (a)

(a) IMPACT BETWEEN A AND B

CONSERVATION OF MOMENTUM

$v_A=3$ ft/s $\quad v_C'=0$ $\quad v_B=v_C=0$ $\quad v_A' \quad v_B'$

$mv_A+mv_B+mv_C=mv_A'+mv_B'+mv_C'$

$3+0=v_A'+v_B'+0$

RELATIVE VELOCITIES $(e=0)$

$(v_A-v_B)e=v_B'-v_A'$

$0=v_B'-v_A'$

$v_A'=v_B'$

$3=2v_B'$

$v_B'=1.5$ ft/s

13.182 continued

v = FINAL (COMMON) VELOCITY

BLOCK C IMPULSE AND MOMENTUM

$mv_C=0 \quad + \quad W_C t,\ F_f t,\ Nt \quad = \quad mv$

$\xrightarrow{+}$ $W_C v_C+F_f t=\frac{W_C}{g}v \quad F_f=\mu_k W_C$

$0+(0.2)t=\frac{v}{g} \quad v=(0.2)gt \quad (1)$

BLOCK A AND B IMPULSE AND MOMENTUM

$mv_A+mv_B \quad + \quad Wt,\ Wt,\ \mu_k Wt,\ Wt,\ 3\mu_k Wt,\ 3Wt \quad = \quad 2mv$

$v_A'=v_B'=1.5$ ft/s

$2\frac{W}{g}(1.5)-4(0.2)Wt=2\frac{W}{g}v$

$1.5-0.4gt=v \quad (2)$

SUBSTITUTE v FROM (1) INTO (2)

$1.5-0.4gt=0.2gt$

$t=\frac{(1.5\text{ ft/s})}{0.6(32.2\text{ ft/s}^2)}=0.0776$ s ◀

(b) WORK AND ENERGY
FROM (1) $v=(0.2)(32.2)(.0776)=0.5$ ft/s

BLOCK C

$v=0 \quad v=0.5$ ft/s $\quad F_f=\mu_k W$ $\quad$ (1) $\quad x_C \quad$ (2)

$T_1=0 \quad T_2=\frac{1}{2}\frac{W}{g}(v)^2=\frac{W}{2g}(0.5)^2$

$U_{1-2}=F_f x_C=\mu_k W x_C=0.2Wx_C$

$T_1+U_{1-2}=T_2 \quad 0+(0.2)(W)x_C=\frac{1}{2}\frac{W}{g}v^2$

$x_C=\frac{(.5\text{ ft/s})^2}{0.2(2)(32.2\text{ ft/s}^2)}=0.01941$ ft

$x_C=0.01941$ ft ◀

BLOCKS A AND B

$v=1.5$ ft/s $\quad \mu_k W \quad \mu_k 3W \quad v=0.5$ ft/s $\quad$ (1) $\quad x_A \quad$ (2)

$T_1=\frac{1}{2}\left(\frac{2W}{g}\right)(1.5)^2=2.25W \quad T_2=\frac{1}{2}\left(\frac{2W}{g}\right)(0.5)^2=0.25W$

$U_{1-2}=-4\mu_k Wg\,x_A=-0.8Wg\,x_A$

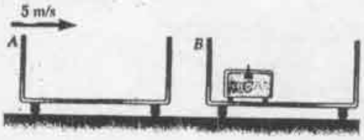
$T_1+U_{1-2}=T_2$

$2.25W-4(.2)W(32.2)x_A=0.25W$

$x_A=0.07764$ ft $\quad x_A=0.0776$ ft ◀

13.183

GIVEN:



MASS OF CARRIERS $m_A = m_B = 40\,kg$

MASS OF LUGGAGE $m_L = 15\,kg$

$e_{BC} = 0.30$

$e_{AB} = 0.80$

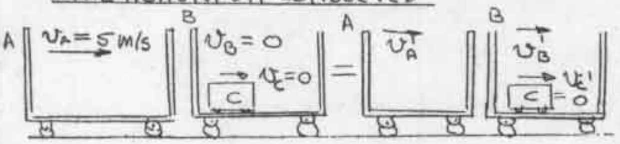
FIND:

(a) VELOCITY OF CARRIER B AFTER C HITS THE WALL OF B THE FIRST TIME

(b) THE TOTAL ENERGY LOST IN THE IMPACT BETWEEN B AND C.

(a) IMPACT BETWEEN A AND B

TOTAL MOMENTUM CONSERVED



$m_A v_A + m_B v_B = m_A v_A' + m_B v_B' \qquad m_A = m_B = 40\,kg$

$\xrightarrow{+} \quad 5\,m/s + 0 = v_A' + v_B' \qquad (1)$

RELATIVE VELOCITIES

$(v_A - v_B)e_{AB} = v_B' - v_A'$

$(5-0)(0.80) = v_B' - v_A' \qquad (2)$

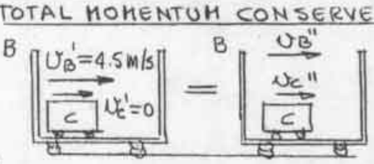
ADDING (1) AND (2)

$(5\,m/s)(1+.80) = 2v_B'$

$v_B' = 4.5\,m/s \longrightarrow$

IMPACT BETWEEN B AND C (AFTER A HITS B)

TOTAL MOMENTUM CONSERVED



$\xrightarrow{+}$

$m_B v_B' + m_C v_C' = m_B v_B'' + m_C v_C''$

$(40\,kg)(4.5\,m/s) + 0 = (40\,kg)v_B'' + (15\,kg)v_C''$

$4.5 = v_B'' + 0.375\,v_C'' \qquad (3)$

RELATIVE VELOCITIES

$(v_B' - v_C')e_{BC} = v_C'' - v_B''$

$(4.5 - 0)(0.30) = v_C'' - v_B'' \qquad (4)$

ADDING (4) AND (3)

$(4.5)(1+0.3) = (1.375)v_C''$

$v_C'' = 4.2545\,m/s$

(3) $\quad v_B'' = 4.5 - 0.375(4.2545) = 2.90\,m/s$ ◀

(b)

$\Delta T_L = (T_B' + T_C') - (T_B'' + T_C'')$

$T_B' = \frac{1}{2} m_B (v_B')^2 = \left(\frac{40}{2}\,kg\right)(4.5\,m/s)^2 = 405\,J$

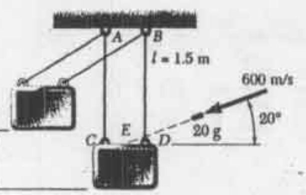
$T_C' = 0 \quad T_B'' = \frac{1}{2} m_B (v_B'')^2 = \left(\frac{40}{2}\,kg\right)(2.90)^2 = 168.72\,J$

$T_C'' = \frac{1}{2} m_C (v_C'')^2 = \left(\frac{15}{2}\,kg\right)(4.2545\,m/s)^2 = 135.76\,J$

$\Delta T_L = (405 + 0) - (168.72 + 135.76) = 100.5\,J \quad \Delta T_L = 100.5\,J$ ◀

13.184

GIVEN:



INITIAL VELOCITY OF 20g BULLET $= 600\,m/s$

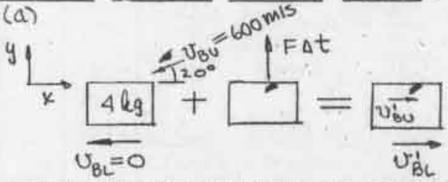
MASS OF BLOCK $= 4\,kg$

FIND:

(a) MAXIMUM HEIGHT h

(b) TOTAL IMPULSE BY THE CORDS ON THE BLOCK

(a)



BL = BLOCK

BU = BULLET

TOTAL MOMENTUM IN x IS CONSERVED

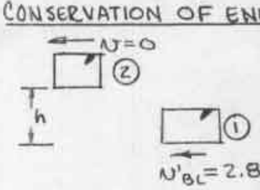
$\xrightarrow{+}$

$m_{BL} v_{BL} + m_{BU} v_{BU} \cos 20° = m_{BL} v_{BL}' + m_{BU} v_{BU}' \quad (v_{BL}' = v_{BU}')$

$0 + (.02\,kg)(-600\,m/s)(\cos 20) = (4.02\,kg)(v_{BL}')$

$v_{BL}' = 2.805\,m/s$

CONSERVATION OF ENERGY



$T_1 = \frac{1}{2}(m_{BL} + m_{BU})(v_{BL}')^2$

$T_1 = \left(\frac{4.02\,kg}{2}\right)(2.805\,m/s)^2$

$T_1 = 15.815\,J$

$V_1 = 0$

$T_2 = 0 \quad V_2 = (m_{BL} + m_{BU})g\,h$

$V_2 = (4.02\,kg)(9.81\,m/s^2)(h) = 39.44\,h$

$T_1 + V_1 = T_2 + V_2$

$15.815 + 0 = 0 + 39.44h$

$h = 0.401\,m$ ◀

(b) REFER TO FIGURE IN PART (a)

IMPULSE-MOMENTUM IN y DIRECTION

$\overset{y}{\uparrow}+ \quad m_{BU} v_{BU} \sin 20 + F\Delta t = (m_{BL} + m_{BU})(v_{BL})_y$

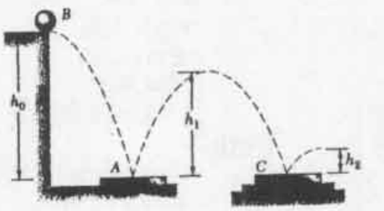
$(v_{BL})_y = 0$

$(0.02\,kg)(-600\,m/s)(\sin 20°) + F\Delta t = 0$

$F\Delta t = 4.10\,N\cdot s$ ◀

13.185

GIVEN:



MASS OF BALL $m_B = 70\,g$

$h_0 = 210\,g$

BALL DROPS FREELY FROM B

$h_2 = 0.25\,m$

$m_A = m_C = 210\,g$

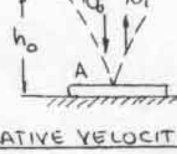
FOAM RUBBER SUPPORT AT C

FIND:

(a) COEFFICIENT OF RESTITUTION BETWEEN THE BALL AND THE PLATES

(b) THE HEIGHT h_1 OF THE BALL'S FIRST BOUNCE

(a) PLATE ON HARD GROUND (FIRST REBOUND)



CONS. OF ENERGY

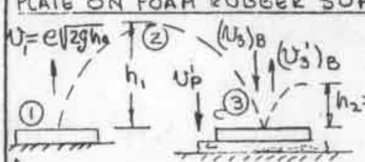
$$\tfrac{1}{2} m_B v_0^2 = m_B g h_0$$

$$v_0 = \sqrt{2g h_0}$$

RELATIVE VELOCITIES

$$v_0 e = v_1 \qquad v_1 = e\sqrt{2gh_0}$$

PLATE ON FOAM RUBBER SUPPORT AT C



CONSERVATION OF ENERGY

POINTS ① AND ③ $\quad V_1 = V_3 = 0$

$$\tfrac{1}{2} m_B v_1^2 = \tfrac{1}{2} m_B (v_3)_B^2$$

$$(v_3)_B = e\sqrt{2gh_0}$$

CONSERVATION OF MOMENTUM

$+\uparrow$ AT ③ $\quad m_B(-v_3)_B + m_P v_P = m_B (v_3')_B - m_P v_P'$

$$\frac{m_P}{m_B} = \frac{210}{70} = 3 \qquad -e\sqrt{2gh_0} = (v_3')_B - 3 v_P' \qquad (1)$$

RELATIVE VELOCITIES

$$[(-v_3)_B - (v_P)]e = -v_P' - (v_3')_B$$

$$e^2\sqrt{2gh_0} + 0 = v_P' + (v_3')_B \qquad (2)$$

MULTIPLY (2) BY 3 AND ADD TO (1)

$$4(v_3')_B = \sqrt{2gh_0}\,(3e^2 - e)$$

CONSERVATION OF ENERGY AT ③, $(v_3')_B = \sqrt{2gh_2}$

THUS $\quad 4\sqrt{2gh_2} = \sqrt{2gh_0}\,(3e^2 - e)$

$$4\sqrt{\frac{h_2}{h_0}} = 4\sqrt{\frac{0.25}{1.5}} = 3e^2 - e$$

$$3e^2 - e - 1.633 = 0 \qquad e = 0.923$$

(b) FROM (a), $v_1 = e\sqrt{2gh_0}$

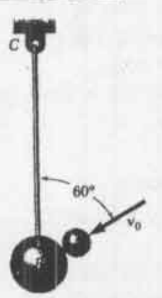
POINTS ① AND ② CONS. OF ENERGY $\quad \tfrac{1}{2} m_B v_1^2 = m_B g h_1$

$$\tfrac{1}{2} e^2 2g h_0 = g h_1$$

$$h_1 = (0.923)^2 (1.5) = 1.278\,m$$

13.186

GIVEN:



$m_B = 700\,g$, $m_A = 350\,g$

$e = 1$

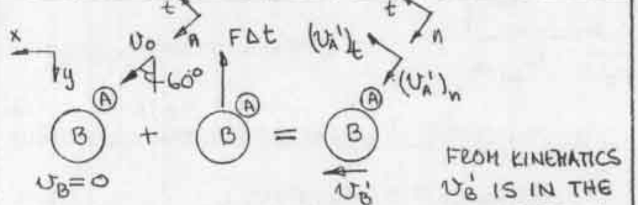
A STRIKES B WITH VELOCITY v_0 AT 60° AS SHOWN

CORD BC ATTACHED TO B IS INEXTENSIBLE

NO FRICTION

FIND:

VELOCITY OF EACH BALL AFTER IMPACT, CHECK THAT NO ENERGY IS LOST IN THE IMPACT



FROM KINEMATICS v_B' IS IN THE X DIRECTION

BALL A ALONE

MOMENTUM IN t DIRECTION IS CONSERVED

$$m_A (v_A)_t = m_A (v_A')_t, \quad (v_A)_t = 0$$

THUS $(v_A')_t = 0$ AND $(v_A')_n = v_A' \angle 60°$

BALLS A AND B

TOTAL MOMENTUM IN X DIRECTION CONSERVED

$$m_A v_0 \sin 60° = m_A (v_A')_n \sin 60° + m_B v_B'$$

$$(0.350)\tfrac{\sqrt{3}}{2} v_0 = (0.350)\tfrac{\sqrt{3}}{2} v_A' + 0.700 v_B'$$

$$v_0 = v_A' + 2.309 v_B' \qquad (1)$$

RELATIVE VELOCITIES (n-DIRECTION)

$$[(v_A)_n - (v_B)_n]e = (v_B')_n - (v_A')_n$$

$$[v_0 - 0](1) = v_B' \sin 60 - v_A'$$

$$v_0 = 0.866 v_B' - v_A' \qquad (2)$$

ADDING (1) AND (2)

$$2v_0 = (2.309 + 0.866) v_B'$$

$$v_B' = 0.630 v_0 \leftarrow$$

FROM (1) $v_A' = v_0 - (2.309)(.630 v_0) = -0.455 v_0$

$$v_A' = 0.455 v_0 \angle 30°$$

ENERGY

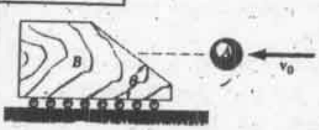
$$\Delta T = \tfrac{1}{2} m_A v_0^2 - \tfrac{1}{2} m_A v_A'^2 - \tfrac{1}{2} m_B v_B'^2$$

$$\Delta T = \tfrac{1}{2}\left[(0.350)(v_0^2 - (0.455 v_0)^2) - (0.700)(0.630 v_0)^2\right]$$

$$\Delta T = \tfrac{1}{2}\left[0.350(1 - 0.2065) - .700(0.3969)\right] v_0^2$$

$$\Delta T = \tfrac{1}{2}[0.278 - 0.278] v_0^2 = 0 \quad \text{(CHECK)}$$

13.187



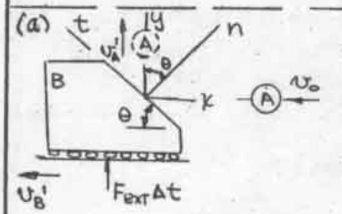
GIVEN:

$m_A = 700$ g
$m_B = 2.1$ kg
$e = 0.6$
SPHERE A REBOUNDS UP

FIND:

(a) ANGLE θ
(b) ENERGY LOST

(a)



MOMENTUM OF SPHERE A ALONE IS CONSERVED IN THE t-DIRECTION

$$m_A v_0 \cos\theta = m_A v_A' \sin\theta$$

$$v_0 = v_A' \tan\theta \qquad (1)$$

TOTAL MOMENTUM IS CONSERVED IN THE X-DIRECTION

$$m_B v_B + m_A v_0 = m_B v_B' + m_A (v_A')_x \qquad v_B = 0,\ (v_A')_x = 0$$

$$0 + 0.700\, v_0 = 2.1 v_B' + 0$$

$$v_B' = v_0/3 \qquad (2)$$

RELATIVE VELOCITIES IN THE n-DIRECTION

$$(-v_0 \sin\theta - 0)e = -v_B' \sin\theta - v_A' \cos\theta$$

$$(v_0)(0.6) = v_B' + v_A' \cot\theta \qquad (3)$$

SUBSTITUTING v_B' FROM (2) INTO (3)

$$0.6 v_0 = 0.333 v_0 + v_A' \cot\theta$$

$$0.267 v_0 = v_A' \cot\theta \qquad (4)$$

DIVIDE (4) INTO (1)

$$\frac{1}{0.267} = \frac{\tan\theta}{\cot\theta} = \tan^2\theta$$

$$\tan\theta = 1.935 \qquad \theta = 62.7°$$

(b) FROM (1) $v_0 = v_A' \tan\theta = v_A'(1.935)$

$$v_A' = 0.5168 v_0, \quad v_B' = v_0/3 \quad (2)$$

$$T_{LOST} = \tfrac{1}{2} m_A v_A^2 - \tfrac{1}{2}(m_A (v_A')^2 + m_B v_B^2)$$

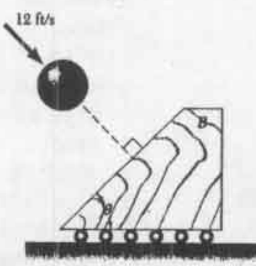
$$T_{LOST} = \tfrac{1}{2}(0.7)(v_0)^2 - \tfrac{1}{2}\left[(0.7)(0.5168 v_0)^2 + (2.1)(v_0/3)^2\right]$$

$$T_{LOST} = \tfrac{1}{2}[0.7 - 0.1870 - 0.2333]\, v_0^2$$

$$T_{LOST} = 0.1400 v_0^2 \text{ J}$$

$$T_{LOST} = 0.1400 v_0^2$$

13.188



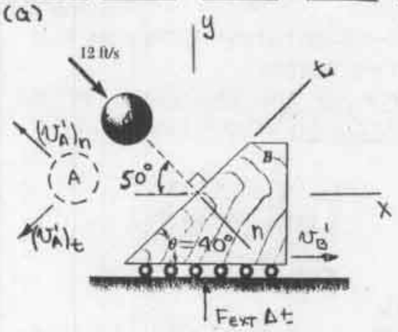
GIVEN:

$W_A = 3$ lb
$W_B = 9$ lb
$e = 0.50$
$\theta = 40°$
$v_A = 12$ ft/s

FIND:

(a) VELOCITIES AFTER IMPACT, $\underline{v}_A'$ AND $\underline{v}_B'$
(b) ENERGY LOST

(a)



MOMENTUM OF THE SPHERE A ALONE IS CONSERVED IN THE t-DIRECTION

$$m_A (v_A)_t = m_A (v_A')_t \qquad (v_A)_t = 0$$

$$(v_A')_t = 0 \qquad (v_A')_n = v_A' \quad \nwarrow 50°$$

TOTAL MOMENTUM IS CONSERVED IN THE X-DIRECTION

$$m_A v_A \cos 50° + m_B v_B = m_A(-v_A')\cos 50° + m_B v_B'$$

$$v_B = 0 \qquad v_A = 12 \text{ ft/s}$$

$$\left(\tfrac{3}{g}\right)(12)(\cos 50°) + 0 = \left(\tfrac{3}{g}\right)(-v_A')(\cos 50°) + \left(\tfrac{9}{g}\right) v_B'$$

$$(1) \qquad 23.140 = -1.9284 v_A' + 9 v_B'$$

RELATIVE VELOCITIES IN THE n-DIRECTION

$$(v_A - v_B)e = (v_B' \cos 50° + v_A') \qquad v_B = 0$$

$$(12 - 0)(0.5) = 0.6428 v_B' + v_A' \qquad v_A = 12 \text{ ft/s}$$

$$(2) \qquad 6 = 0.6428 v_B' + v_A' \qquad e = 0.50$$

SOLVING EQ. (1) AND (2) SIMULTANEOUSLY

$$v_B' = 3.39 \text{ ft/s} \rightarrow$$
$$v_A' = 3.82 \text{ ft/s} \nwarrow 50°$$

(b) $T_{LOST} = \tfrac{1}{2} m_A v_A^2 - \tfrac{1}{2}[m_A (v_A')^2 + m_B (v_B')^2]$

$$T_{LOST} = \tfrac{1}{2}\left[\left(\frac{3 \text{ lb}}{32.2 \text{ ft/s}^2}\right)\left((12 \text{ ft/s})^2 - (3.82 \text{ ft/s})^2\right) - \left(\frac{9 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(3.39 \text{ ft/s})^2\right]$$

$$T_{LOST} = \tfrac{1}{2}[12.064 - 3.212] = 4.42 \text{ ft·lb}$$

$$T_{LOST} = 4.42 \text{ ft·lb}$$

13.189

GIVEN:

$W_B = 12$ OZ.

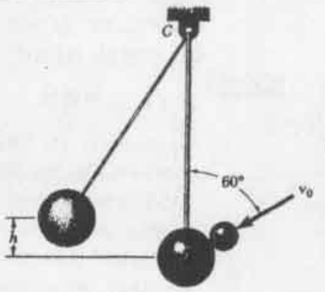
$W_A = 6$ OZ.

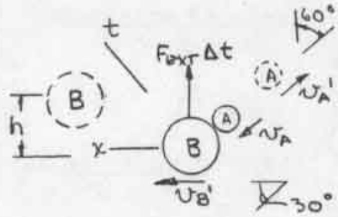
$v_0 = 4.8$ ft/s AT $60°$ AS SHOWN

$e = 1$

FIND:

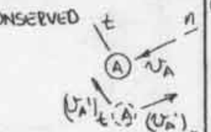
HEIGHT h REACHED BY BALL B





BALL A ALONE

MOMENTUM IN t-DIRECTION CONSERVED



$m_A (v_A)_t = m_A (v_A')_t$

$(v_A)_t = 0 = (v_A')_t$

THUS $(v_A')_n = v_A'$

TOTAL MOMENTUM IN THE X-DIRECTION IS CONSERVED

$$m_A v_A \sin 60° + m_B (v_B)_x = m_A(-v_A')\sin 60° + m_B v_B'$$

$$v_A = v_0 = 4.8 \text{ ft/s} \quad (v_B)_x = 0$$

$$\frac{(6/16)}{g}(4.8)(\sin 60°) + 0 = -\frac{(6/16)}{g}(v_A' \sin 60°) + \frac{(12/16)}{g} v_B'$$

$$4.1568 = -0.866\, v_A' + 2 v_B' \qquad (1)$$

RELATIVE VELOCITY IN THE n-DIRECTION

$$[-v_A - (v_B)_n]\, e = -v_B' \cos 30° - v_A'$$

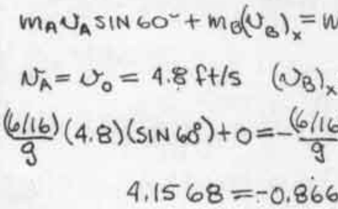
$$(-4.8 - 0)(1) = -0.866 v_B' - v_A' \qquad (2)$$

SOLVE EQ (1) AND (2) SIMULTANEOUSLY

$$v_B' = 3.0232 \text{ ft/s} \quad v_A' = 2.18 \text{ ft/s}$$

CONSERVATION OF ENERGY

BALL B



$$T_1 = \frac{1}{2} m_B (v_B')^2$$

$$T_1 = \frac{1}{2}\frac{W_B}{g}(3.0232)^2$$

$$V_1 = 0$$

$$T_2 = 0 \quad V_2 = W_B h$$

$$T_1 + V_1 = T_2 + V_2 \quad \frac{1}{2}\frac{\cancel{W_B}}{g}(3.0232)^2 = 0 + \cancel{W_B} h$$

$$h = \frac{(3.0232)^2}{(2)(32.2)} = 0.1419 \text{ ft} \qquad h = 0.1419 \text{ ft}$$

13.190

GIVEN:

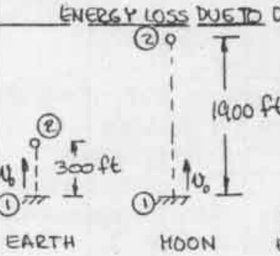
PELLET OF WEIGHT $W = 2$ OZ, SHOT VERTICALLY RISES TO $h_e = 300$ ft ON EARTH

$h_m = 1900$ ft ON THE MOON

ACCELERATION OF GRAVITY ON THE MOON, $g_m = 0.165 g_e$

FIND:

ENERGY LOSS DUE TO DRAG FOR PELLET ON THE EARTH



SINCE THE PELLET IS SHOT FROM THE SAME PISTOL THE INITIAL VELOCITY v_0, IS THE SAME ON THE MOON AND ON THE EARTH

WORK AND ENERGY

EARTH: $T_1 = \frac{1}{2} m v_0^2$

$$U_{1-2} = -m g_E (300 \text{ ft}) - E_L$$

(E_L = LOSS OF ENERGY DUE TO DRAG)

$$T_2 = 0$$

$$T_1 - 300 m g_E - E_L = 0 \qquad (1)$$

MOON:

$T_1 = \frac{1}{2} m v_0^2$

$U_{1-2} = -m g_m (1900)$

$T_2 = 0$

$T_1 - 1900 m g_m = 0 \quad (2)$

SUBTRACT (1) FROM (2)

$$-1900 m g_m + 300 m g_E + E_L = 0$$

$g_m = 0.165\, g_E$

$m = \frac{(2/16)}{g_E}$

$$E_L = (1900)\frac{(2/16)}{g_E}(0.165 g_E) - 300\frac{(2/16)}{g_E} g_E$$

$$E_L = 1.688 \text{ ft·lb}$$

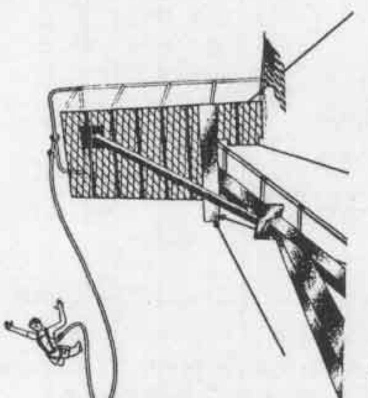
13.191

GIVEN:

130 ft TOWER

ELASTIC CABLE $L = 85$ ft UNSTRETCHED

CABLE IS TO STRETCH TO 100 ft WHEN A 600 lb WEIGHT ATTACHED TO IT IS DROPPED FROM THE TOWER

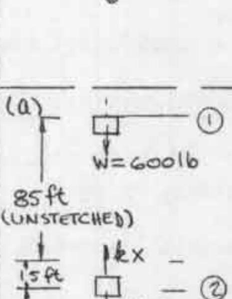


FIND:

(a) k FOR THE CABLE

(b) DISTANCE FROM THE GROUND WHEN A 186 lb MAN JUMPS

(a)



CONSERVATION OF ENERGY

$v_1 = 0 \quad T_1 = 0 \quad V_1 = 100\, W$

DATUM AT (2) $\quad V_1 = (100 \text{ ft})(600 \text{ lb})$

$$= 6 \times 10^4 \text{ ft·lb}$$

$v_2 = 0 \quad T_2 = 0$

$V_2 = V_g + V_e = 0 + \frac{1}{2} k (15 \text{ ft})^2$

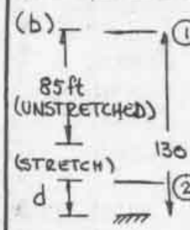
$T_1 + V_1 = T_2 + V_2$

$0 + 6 \times 10^4 = 0 + (112.5) k$

(CONTINUED)

$k = 533$ lb/ft

13.191 continued



FROM (a), $k = 533$ lb/ft

$T_1 = 0$

$V_1 = (186)(130-d)$

$T_2 = 0$

$V_2 = V_g + V_e = 0 + \frac{1}{2}(533)(130-85-d)^2$

$V_2 = (266.67)(45-d)^2$

d = DISTANCE FROM THE GROUND

$T_1 + V_1 = T_2 + V_2$

$0 + (186)(130-d) = 0 + (266.67)(45-d)^2$

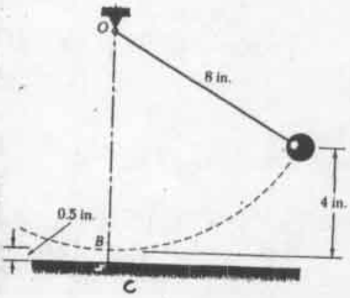
$266.7d^2 - 23815d + 516018 = 0$

$d = \dfrac{23815 \mp \sqrt{(23815)^2 - 4(266.7)(516018)}}{(2)(266.7)} = 36.99 \text{ ft}, \; 52.3 \text{ ft}$

DISCARD 52.3 ft (ASSMES CORD ACTS IN COMPRESSION WHEN REBOUND OCCURS)

$d = 37.0$ ft

13.192

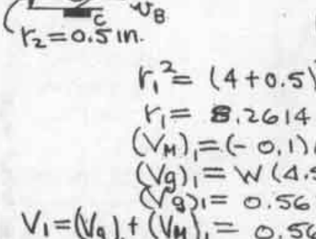


GIVEN:

2-OZ SPHERE A. MAGNET AT B EXERTS A FORCE $F = 0.1/r^2$ (lb, in.) SPHERE RELEASED FROM POSITION SHOWN

FIND:

SPEED OF A AS IT PASSES THROUGH B



POTENTIAL ENERGY OF THE MAGNET

$F = 0.1/r^2 = -dV_M/dr$

$V_M = -(0.1)/r$

$v_1 = 0 \quad T_1 = 0$

$V_1 = (V_M)_1 + (V_g)_1$

$(V_M)_1 = -0.1/r_1$

$r_1^2 = (4+0.5)^2 + [8^2 - (8-4)^2] = 68.25$

$r_1 = 8.2614$ in.

$(V_M)_1 = (-0.1)/(8.2614) = -0.012105$ lb·in

$(V_g)_1 = W(4.5 \text{ in.}) = (2/16 \text{ lb})(4.5 \text{ in})$

$(V_g)_1 = 0.5625$ lb·in

$V_1 = (V_g)_1 + (V_M)_1 = 0.5625 - 0.012105 = 0.5504$ lb·in

$V_1 = 0.045866$ lb·ft

$T_2 = \frac{1}{2} m v_B^2 = \frac{1}{2}\left(\frac{2/16 \text{ lb}}{32.2 \text{ ft/s}^2}\right) v_B^2 = 0.001941\, v_B^2$

$V_2 = (V_g)_2 + (V_M)_2 = \left(\frac{2}{16} \text{ lb}\right)(0.5 \text{ in.}) - \left(\frac{0.1}{0.5}\right) = -0.1375$ lb·in

$V_2 = -0.1375$ lb·in $= -0.011458$ lb·ft

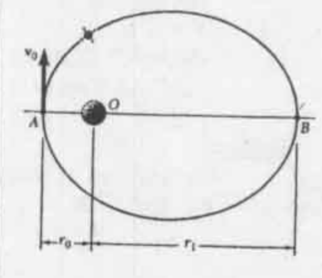
$T_1 + V_1 = T_2 + V_2$

$0 + 0.045866 = 0.001941 v_B^2 - 0.011458$

$v_B^2 = 29.53$ ft²/s²

$v_B = 5.43$ ft/s

13.193



GIVEN:

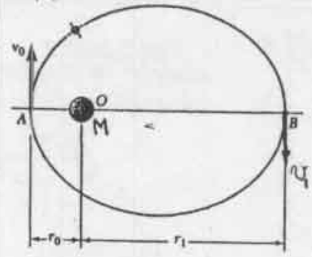
PLANET OF MASS M AT O SATELLITE IN AN ELLIPTICAL ORBIT

DERIVE:

$\frac{1}{r_0} + \frac{1}{r_1} = \frac{2GM}{h^2}$

WHERE h IS THE ANGULAR MOMENTUM

USE CONSERVATION OF ENERGY AND CONSERVATION OF ANGULAR MOMENTUM



ANGULAR MOMENTUM

$h = r_0 v_0 = r_1 v_1$

$v_0 = \frac{h}{r_0} \quad v_1 = \frac{h}{r_1} \quad (1)$

CONSERVATION OF ENERGY

$T_A = \frac{1}{2} m v_0^2$

$V_A = -\frac{GMm}{r_0}$

$T_B = \frac{1}{2} m v_1^2 \quad V_B = -\frac{GMm}{r_1}$

$T_A + V_A = T_B + V_B \quad \frac{1}{2} m v_0^2 - \frac{GMm}{r_0} = \frac{1}{2} m v_1^2 - \frac{GMm}{r_1}$

$v_0^2 - v_1^2 = 2GM\left[\frac{1}{r_0} - \frac{1}{r_1}\right] = 2GM\left[\frac{r_1 - r_0}{r_1 r_0}\right]$

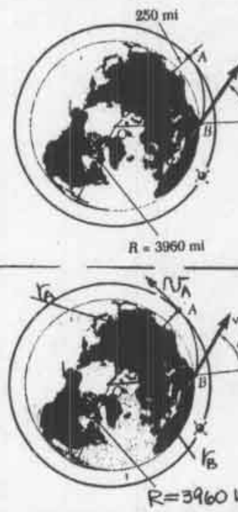
SUBSTITUTE FOR v_0 AND v_1 FROM (1)

$h^2\left[\frac{1}{r_0^2} - \frac{1}{r_1^2}\right] = 2GM\left[\frac{r_1 - r_0}{r_1 r_0}\right]$

$h^2\left[\frac{r_1^2 - r_0^2}{r_1^2 r_0^2}\right] = \frac{h^2}{r_1^2 r_0^2}(r_1 - r_0)(r_1 + r_0) = 2GM\left[\frac{r_1 - r_0}{r_1 r_0}\right]$

$h^2\left(\frac{1}{r_0} + \frac{1}{r_1}\right) = 2GM \qquad \left(\frac{1}{r_0} + \frac{1}{r_1}\right) = \frac{2GM}{h^2}$ (QED)

13.194

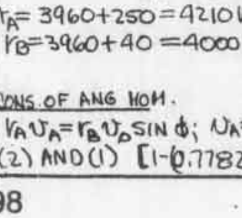


GIVEN:

SHUTTLE ALTITUDE AT B = 40 MI, $\phi_0 = 55°$, MUST BE TANGENT TO ORBIT AT POINT A AT AN ALTITUDE OF 250 MI. ENGINE TURNED OFF AT B

FIND:

v_0



CONSERVATION OF ENERGY

$T_B = \frac{1}{2} m v_0^2 \quad V_B = -\frac{GMm}{r_B}$

$T_A = \frac{1}{2} m v_A^2 \quad V_A = -\frac{GMm}{r_A}$

$GM = gR^2$ (EQ 12.30)

$T_A + V_A = T_B + V_B$

$\frac{1}{2} m v_0^2 - \frac{gR^2}{r_B} m = \frac{1}{2} m v_A^2 - \frac{gR^2}{r_A} m$

$r_A = 3960 + 250 = 4210$ mi $\quad v_A^2 = v_0^2 - \frac{2gR^2}{r_B}\left(1 - \frac{r_B}{r_A}\right)$

$r_B = 3960 + 40 = 4000$ mi $\quad v_A^2 = v_0^2 - \frac{2(32.2)(3960 \times 5280)^2}{(4000 \times 5280)}\left(1 - \frac{4000}{4210}\right)$

$v_A^2 = v_0^2 - 66.495 \times 10^6 \quad (1)$

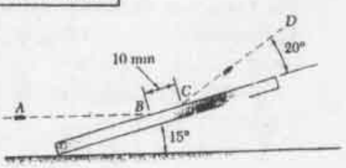
CONS. OF ANG MOM.

$r_A v_A = r_B v_0 \sin\phi$; $v_A = (4000/4210) v_0 \sin 55° = 0.77829 v_0 \; (2)$

(2) AND (1) $[1 - (0.77829)^2] v_0^2 = 66.495 \times 10^6 \quad v_0 = 12{,}990$ ft/s

13.195

GIVEN:



25-g BULLET INITIAL VELOCITY $v_1 = 600$ m/s, HORIZONTAL RICOCHET VELOCITY $v_2 = 400$ m/s AT 20°.

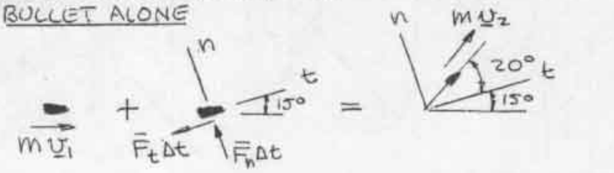
BULLET LEAVES A 10-mm SCRATCH ON THE PLATE AT AN AVERAGE SPEED OF 500 m/s

FIND:

THE MAGNITUDE AND DIRECTION OF THE AVERAGE IMPULSIVE FORCE EXERTED BY THE BULLET ON THE PLATE

IMPULSE AND MOMENTUM

BULLET ALONE



$$m\underline{v}_1 + \underline{F}\Delta t = m\underline{v}_2$$

t DIRECTION $\quad mv_1\cos 15° - F_t\Delta t = mv_2\cos 20°$

$$F_t\Delta t = (0.025\text{ kg})\,[600\text{ m/s}\cos 15° - 400\text{ m/s}\cos 20°]$$

$$F_t\Delta t = 5.092\text{ kg·m/s}$$

$$\Delta t = \frac{S_{BC}}{v_{AV}} = \frac{0.010\text{ m}}{500\text{ m/s}} = 20\times10^{-6}\text{ s}$$

$$F_t = (5.092\text{ kg·m/s})/(20\times10^{-6}\text{ s}) = 254.6\times10^3\ \frac{\text{kg·m}}{\text{s}^2}$$

$$F_t = 254.6\text{ kN}$$

n DIRECTION

$$-mv_1\sin 15° + F_n\Delta t = mv_2\sin 20°$$

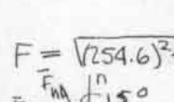
$$F_n\Delta t = (0.025\text{ kg})\,[600\text{ m/s}\sin 15° + 400\text{ m/s}\sin 20°]$$

$$F_n\Delta t = 7.3025\text{ kg·m/s} \qquad \Delta t = 20\times10^{-6}\text{ s}$$

$$F_n = (7.3025\text{ kg·m/s})/(20\times10^{-6}\text{ s}) = 365.1\times10^3\ \frac{\text{kg·m}}{\text{s}^2}$$

$$F_n = 365.1\text{ kN}$$

$$F = \sqrt{(254.6)^2 + (365.1)^2} = 445.1\text{ kN}$$



$$\theta = \tan^{-1}\frac{F_t}{F_n} = \tan^{-1}\frac{254.6}{365.1}$$

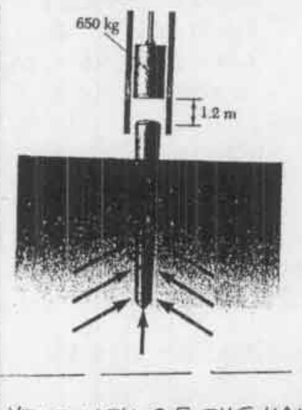
$$\theta = 34.9°$$

$$\alpha = 34.9 + 15° = 49.9°$$

FORCE OF THE BULLET ON THE PLATE $\quad F = 445\text{ kN}$ ∠49.9°

13.196

GIVEN:



650 kg HAMMER DROPS 1.2 m AND DRIVES A 140 kg PILE 110 mm INTO THE GROUND

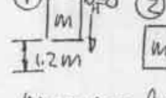
$e = 0$

FIND:

AVERAGE RESISTANCE OF THE GROUND TO PENETRATION

VELOCITY OF THE HAMMER AT IMPACT

CONSERVATION OF ENERGY



$T_1 = 0 \quad V_1 = mg\,(1.2\text{ m})$

$V_1 = (0.650\text{ kg})(9.81\frac{\text{m}}{\text{s}^2})(1.2\text{ m})$

$V_1 = 7.652\text{ J}$

$m = 0.650\text{ kg}$

$T_2 = \frac{1}{2}mv_H^2 = \frac{0.650}{2}v^2 = 0.325v_H^2$

$V_2 = 0$

$T_1 + V_1 = T_2 + V_2$

$$0 + 7.652 = 0.325v^2$$

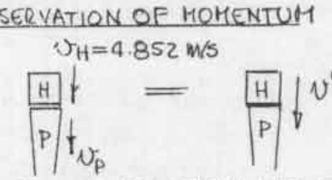
$$v^2 = 23.54\text{ m}^2/\text{s}^2$$

$$v = 4.852\text{ m/s}$$

VELOCITY OF PILE AFTER IMPACT

SINCE THE IMPACT IS PLASTIC ($e=0$), THE VELOCITY OF THE PILE AND HAMMER ARE THE SAME AFTER IMPACT

CONSERVATION OF MOMENTUM

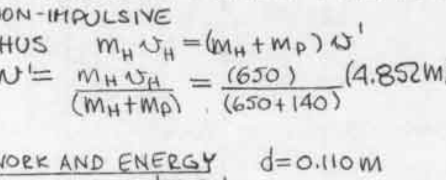


THE GROUND REACTION AND THE WEIGHTS ARE NON-IMPULSIVE

THUS $\quad m_Hv_H = (m_H + m_P)v'$

$$v' = \frac{m_Hv_H}{(m_H+m_P)} = \frac{(650)}{(650+140)}(4.852\text{ m/s}) = 3.992\text{ m/s}$$

WORK AND ENERGY $\quad d = 0.110\text{ m}$



$T_2 + U_{2\text{-}3} = T_3 \qquad T_2 = \frac{1}{2}(m_H + m_P)(v')^2$

$T_3 = 0 \qquad T_2 = \frac{1}{2}(650 + 140)(3.992)^2$

$T_2 = 6.295\times10^3\text{ J}$

$$U_{2\text{-}3} = (m_H + m_P)gd - F_{AV}d = (650+140)(9.81)(.110) - F_{AV}(.110)$$

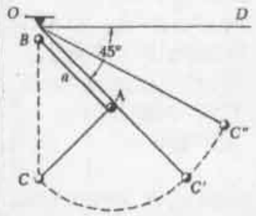
$$U_{2\text{-}3} = 852.49 - (0.110)F_{AV}$$

$T_2 + U_{2\text{-}3} = T_3$

$$6.295\times10^3 + 852.49 - (0.110)F_{AV} = 0$$

$$F_{AV} = (7147.5)/(0.110) = 64.98\times10^3\text{ N} \qquad F_{AV} = 65\text{ kN}$$

13.197



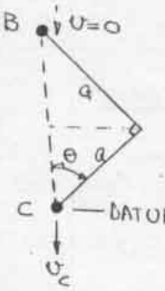
GIVEN:

SPHERE RELEASED FROM REST AT B. CORD OF LENGTH 2a BECOMES TAUT AT C

FIND:

VERTICAL DISTANCE FROM OD TO THE HIGHEST POINT C" REACHED BY THE SPHERE

VELOCITY AT POINT C (BEFORE THE CORD IS TAUT)
CONSERVATION OF ENERGY FROM B TO C.



$T_B = 0$

$V_B = mg(2)\left(\frac{\sqrt{2}}{2}\right)a = mga\sqrt{2}$

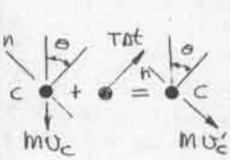
$T_C = \frac{1}{2}mv_C^2 \quad V_C = 0$

$T_B + V_B = T_C + V_C$

$0 + mga\sqrt{2} = \frac{1}{2}mv_C^2 + 0$

$v_C = \sqrt{2\sqrt{2}\,ga}$

VELOCITY AT C (AFTER THE CORD BECOMES TAUT)
LINEAR MOMENTUM PERPENDICULAR TO THE CORD IS CONSERVED.



$\theta = 45°$

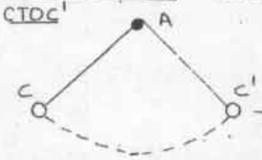
$+\ -mv_C \sin\theta = mv'_C$

$v'_C = \left(\sqrt{2\sqrt{2}}\right)\left(\frac{\sqrt{2}}{2}\right)\sqrt{ga}$

$v'_C = \sqrt{\sqrt{2}\,ga} = 2^{\frac{1}{4}}\sqrt{ga}$

NOTE: THE WEIGHT OF THE SPHERE IS A NON-IMPULSIVE FORCE

VELOCITY AT C' (CONSERVATION OF ENERGY)
C TO C'



$T_C = \frac{1}{2}m(v'_C)^2 \quad V_C = 0$

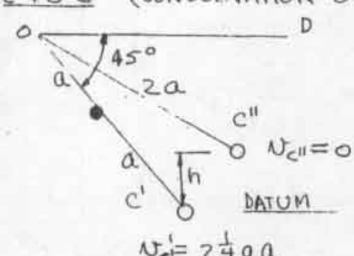
$T_{C'} = \frac{1}{2}m(v_{C'})^2 \quad V_{C'} = 0$

$T_C + V_C = T_{C'} + V_{C'}$

$\frac{1}{2}m(v'_C)^2 + 0 = \frac{1}{2}m(v_{C'})^2 + 0$

$v'_C = v'_{C'}$

C' TO C" (CONSERVATION OF ENERGY)



$v'_{C'} = 2^{\frac{1}{4}}\sqrt{ga}$

$T_{C'} = \frac{1}{2}m(v'_{C'})^2$

$T_{C'} = \frac{1}{2}m\left(2^{\frac{1}{4}}\sqrt{ga}\right)^2$

$T_{C'} = \frac{\sqrt{2}}{2}mga$

$V_{C'} = 0$

$T_{C''} = 0$

$V_{C''} = mgh$

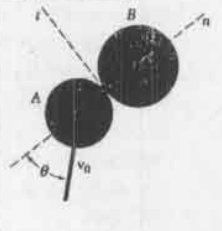
$T_{C'} + V_{C'} = T_{C''} + V_{C''}$

$\frac{\sqrt{2}}{2}mga + 0 = 0 + mgh$

$h = \frac{\sqrt{2}}{2}a$

$h = 0.707a$ ◀

13.198



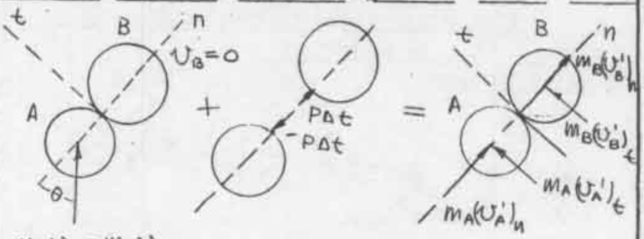
GIVEN:

m_A AND m_B SLIDING ON A FRICTIONLESS SURFACE
INITIALLY, $v_B = 0$
$v_A = v_0$ AT ANGLE θ
COEFFICIENT OF RESTITUTION, e

SHOW:

THAT n COMPONENT OF THE VELOCITY OF A AFTER IMPACT IS,

(a) POSITIVE IF $m_A > em_B$
(b) NEGATIVE IF $m_A < em_B$
(c) ZERO IF $m_A = em_B$



$m_A \underline{v}_A = m_A \underline{v}_0$

DISKS A AND B (TOTAL MOMENTUM CONSERVED)

$$m_A \underline{v}_A + m_B \underline{v}_B = m_A \underline{v}'_A + m_B \underline{v}'_B$$

NORMAL DIRECTION:

$$m_A v_0 \cos\theta + 0 = m_A (v'_A)_n + m_B (v'_B)_n \quad (1)$$

RELATIVE VELOCITIES

$$[v_A \cos\theta - (v_B)_n]e = (v'_B)_n - (v'_A)_n$$

$$v_0(\cos\theta)e = (v'_B)_n - (v'_A)_n \quad (2)$$

MULTIPLY (2) BY m_B AND SUBTRACT IT FROM (1)

$$v_0 \cos\theta (m_A - em_B) = (m_A + m_B)(v'_A)_n$$

$$(v'_A)_n = (v_0\cos\theta)\frac{(m_A - em_B)}{(m_A + m_B)} \quad (3)$$

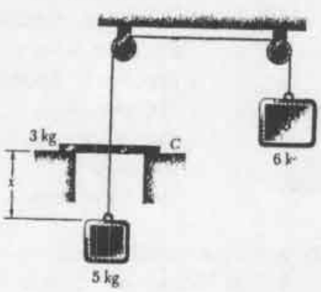
FROM EQUATION (3)

(a) $m_A > em_B$ $\quad (v'_A)_n$ POSITIVE ◀

(b) $m_A < em_B$ $\quad (v'_A)_n$ NEGATIVE ◀

(c) $m_A = em_B$ $\quad (v'_A)_n = 0$ ◀

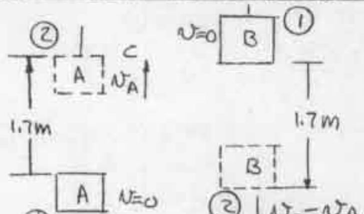
13.199



GIVEN:
MASSES A, B AND C AS SHOWN
INITIAL
$v_A = 0$, $x = 1.7$ m
IMPACT BETWEEN A AND C IS PLASTIC, $e = 0$

FIND:
(a) v_A', v_B', v_C' IMMEDIATELY AFTER A HITS C
(b) DISTANCE A AND C MOVE BEFORE STOPPING
(c) x AFTER ONE COMPLETE CYCLE

(a) VELOCITY OF A JUST BEFORE IT HITS C



CONS OF ENERGY
DATUM AT ①
POSITION ①
$(v_A)_1 = (v_B)_1 = 0$
$T_1 = 0$
$V_1 = 0$

POSITION ② $\quad T_2 = \frac{1}{2} m_A (v_A)^2 + \frac{1}{2} m_B v_B^2$

$v_A = v_B$ (KINEMATICS)

$T_2 = \frac{1}{2}(5+6)v_A^2 = \frac{11}{2} v_A^2$

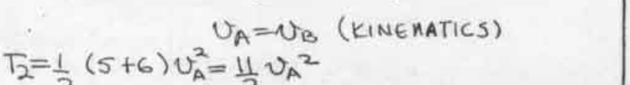
$V_2 = m_A g(1.7) - m_B g(1.7) = (5-6)(g)(1.7)$

$V_2 = -1.7g$

$T_1 + V_1 = T_2 + V_2 \qquad 0 + 0 = \frac{11}{2} v_A^2 - 1.7g$

$v_A^2 = \left(\frac{3.4}{11}\right)(9.81) = 3.032 \ \frac{m^2}{s^2} \qquad v_A = 1.741 \ \frac{m}{s}$

VELOCITY OF A AND C AFTER A HITS C



$v_A' = v_C'$ (PLASTIC IMPACT)

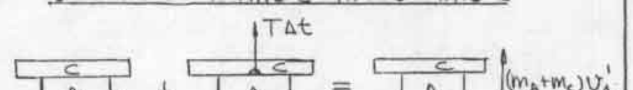
IMPULSE-MOMENTUM A AND C

$+\uparrow \quad m_A v_A + T\Delta t = (m_A + m_C) v_A'$

$(5)(1.741) + T\Delta t = 8 v_A' \qquad (1)$

$v_B = v_A$; $v_B' = v_A'$ (CORD REMAINS TAUT)

B ALONE



$m_B v_B = m_B v_A \qquad m_B v_A - T\Delta t = m_B v_A'$

$(6)(1.741) - T\Delta t = 6 v_A' \qquad (2)$

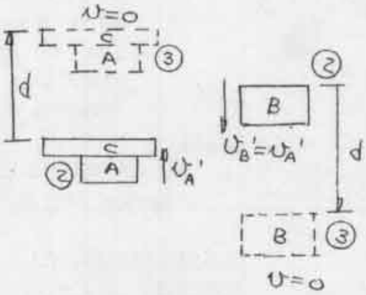
13.199 continued

ADDING EQUATIONS (1) AND (2), $11(1.741) = 14 v_A'$

$v_A' = 1.3679$ m/s

$v_A' = v_B' = v_C' = 1.368 \ \frac{m}{s}$

DISTANCE A AND C MOVE BEFORE STOPPING



CONS. OF ENERGY
DATUM AT ②

POSITION ②

$T_2 = \frac{1}{2}(m_A + m_B + m_C)(v_A')^2$

$T_2 = \left(\frac{14}{2}\right)(1.3681)^2$

$T_2 = 13.103$ J

$V_2 = 0$

POSITION ③

$T_3 = 0 \quad V_3 = (m_A + m_C) g d - m_B g d$

$V_3 = (8-6) g d = 2gd$

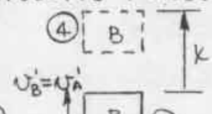
$T_2 + V_2 = T_3 + V_3$

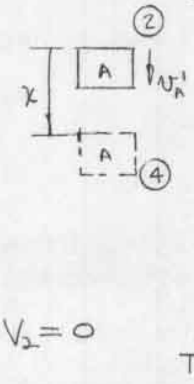
$13.103 + 0 = 0 + 2gd$

$d = (13.103)/(2)(9.81) = 0.6679$ m

$d = 0.668$ m

(b) AS THE SYSTEM RETURNS TO POSITION ②, AFTER STOPPING IN POSITION ③, ENERGY IS CONSERVED AND THE VELOCITIES OF A, B AN C BEFORE THE COLLAR AT C IS REMOVED, ARE THE SAME AS THEY WERE IN (a) ABOVE WITH THE DIRECTIONS REVERSED. THUS, $v_A' = v_C' = v_B' = 1.3679$ m/s. AFTER THE COLLAR C IS REMOVED THE VELOCITIES OF A AND B REMAIN TH SAME SINCE THERE IS NO IMPULSIVE FORCE ACTING ON EITHER.





CONSERVATION OF ENERGY
DATUM AT ②

$T_2 = \frac{1}{2}(m_A + m_B)(v_A')^2$

$T_2 = \frac{1}{2}(5+6)(1.3679)^2$

$T_2 = 10.291$ J

$V_2 = 0$

$T_4 = 0 \quad V_4 = m_B g x - m_A g x$

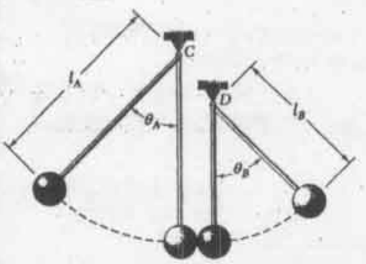
$V_4 = (6-5) g x$

$T_2 + V_2 = T_4 + V_4$

$10.291 + 0 = (1)(9.81) x$

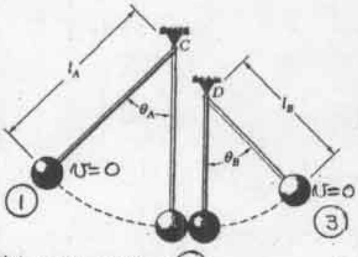
$x = 1.049$

13.200



GIVEN: SPHERE A IS RELEASED FROM REST AT AN ANGLE θ_A. SPHERE B IS AT REST, IS HIT BY A, AND RISES TO A MAXIMUM ANGLE $\theta_B = \theta_A$

FIND: θ_B IN TERMS OF l_B/l_A AND e.



CONSERVATION OF ENERGY ① → ②

DATUM AT ②

SPHERE A

POSITION ①

$v_A = 0 \quad T_1 = 0$

$V_1 = mg\,l_A(1-\cos\theta_A)$

$m_A = m_B = m$ ②

$\theta_A = \theta_B$

POSITION ②

$T_2 = \frac{1}{2} m v_A^2 \quad V_2 = 0$

$T_1 + V_1 = T_2 + V_2$

$$0 + mg\,l_A(1-\cos\theta_A) = \frac{1}{2} m v_A^2 + 0$$

$$v_A^2 = 2g l_A (1-\cos\theta_A) \qquad (1)$$

CONSERVATION OF MOMENTUM AT ②

$$\underset{m v_A \rightarrow \; v_B = 0}{(A)(B)} = \underset{m v_A' \rightarrow \; m v_B' \rightarrow}{(A)(B)}$$

$m v_A + m v_B = m v_A' + m v_B'$

$$v_A + 0 = v_A' + v_B' \qquad (2)$$

RELATIVE VELOCITIES AT ②

$$(v_A - v_B)e = v_B' - v_A' \qquad v_A e = v_B' - v_A' \qquad (3)$$

ADDING EQUATIONS (2) AND (3) AND SOLVING FOR v_B',

$$v_B' = \frac{1}{2}(1+e)v_A \qquad (4)$$

CONSERVATION OF ENERGY ② → ③

SPHERE B

POSITION ② $\quad T_2 = \frac{1}{2} m (v_B')^2 \quad V_2 = 0$

$T_3 = 0 \qquad V_3 = mg\,l_B(1-\cos\theta_B)$

$T_2 + V_2 = T_3 + V_3 \quad \frac{1}{2} m (v_B')^2 + 0 = 0 + mg\,l_B(1-\cos\theta_B)$

$$(v_B')^2 = 2g l_B (1-\cos\theta_B) \qquad (5)$$

SUBSTITUTE v_B' FROM EQ. (4) INTO EQ. (5)

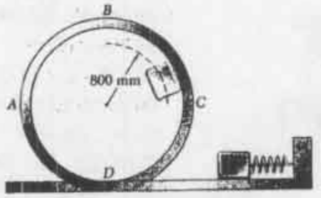
$$\frac{1}{4}(1+e)^2 v_A^2 = 2g l_B (1-\cos\theta_A) \qquad (6)$$

DIVIDE (1) INTO (6) AND SET $\theta_A = \theta_B$

$$\frac{1}{4}\frac{(1+e)^2 v_A^2}{v_A^2} = \frac{2g l_B(1-\cos\theta_B)}{2g l_A(1-\cos\theta_B)}$$

$$l_B/l_A = (1+e)^2/4$$

13.201

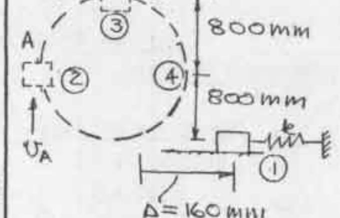


GIVEN: 300-g BLOCK SPRING OF CONSTANT $k = 600$ N/m IS INITIALLY COMPRESSED 160 mm WHEN THE BLOCK IS RELEASED. NO FRICTION

FIND: FORCE EXERTED BY THE LOOP ABCD ON THE BLOCK AS IT PASSES THROUGH.

(a) POINT A

(b) POINT B, (c) POINT C



VELOCITIES AT A AND B

CONSERVATION OF ENERGY, DATUM AT ①

POSITION ①

$v = 0 \quad T_1 = 0$

$V_1 = \frac{1}{2} k \Delta^2$

$V_1 = \frac{1}{2}(600\text{ N/m})(0.160\text{ m})^2$

$V_1 = 7.68$ J

POSITION ② $\quad T_2 = \frac{1}{2} m v_A^2 = \frac{1}{2}(0.3)v_A^2 = 0.15 v_A^2$

$V_2 = mg(0.800\text{m}) = (0.3\text{kg})(g)(0.8\text{m}) = 0.24g$

$T_1 + V_1 = T_2 + V_2 \quad 0 + 7.68 = 0.15 v_A^2 + 0.24g$

$$v_A^2 = \frac{7.68 - (0.24)(9.81)}{0.15}$$

$$v_A^2 = 35.50 \text{ m}^2/\text{s}^2$$

POSITION ③ $\quad T_3 = \frac{1}{2} m v_B^2 = \frac{1}{2}(0.3)v_B^2 = 0.15 v_B^2$

$V_3 = mg(1.6\text{m}) = (0.3)(g)(1.6) = 0.48g$

$T_1 + V_1 = T_2 + V_2 \quad 0 + 7.68 = 0.15 v_B^2 + 0.48g$

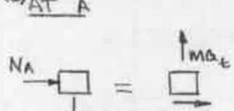
$$v_B^2 = \frac{7.68 - (0.48)(9.81)}{0.15} = 19.81 \frac{\text{m}^2}{\text{s}^2}$$

POSITION ④ SINCE $V_4 = V_2$ THE VELOCITY $v_A = v_C$

$v_C^2 = 35.50 \text{ m}^2/\text{s}^2$

NEWTONS SECOND LAW

(a) AT A



$\xrightarrow{+}$

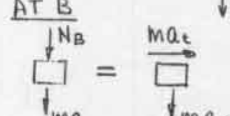
$\Sigma F_n = N_A = m a_n$

$a_n = \frac{v_A^2}{R} = \frac{(35.50 \text{ m}^2/\text{s}^2)}{(0.8\text{ m})}$

$N_A = (0.3\text{ kg})\frac{(35.50 \text{ m}^2/\text{s}^2)}{(0.8\text{m})}$

$N_A = 13.31$ N →

(b) AT B



$\downarrow +$

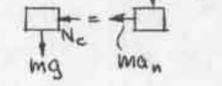
$\Sigma F_n = N_B + mg = m a_n$

$a_n = \frac{v_B^2}{R} = \frac{(19.81 \text{ m}^2/\text{s}^2)}{(0.8\text{ m})}$

$N_B = \frac{(0.3\text{ kg})(19.81 \text{ m}^2/\text{s}^2)}{(0.8\text{m})} - (0.3\text{kg})(9.81 \frac{\text{m}}{\text{s}^2})$

$N_B = 4.49$ N ↓

(c) AT (C)



$\xleftarrow{+} \quad \Sigma F_n = N_C = m a_n \quad a_n = \frac{v_C^2}{R}$

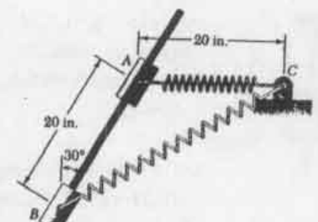
$N_C = (0.3\text{kg})(35.50 \text{ m}^2/\text{s}^2)/(0.8\text{m})$

$N_C = 13.31$ N

13.C1

GIVEN:

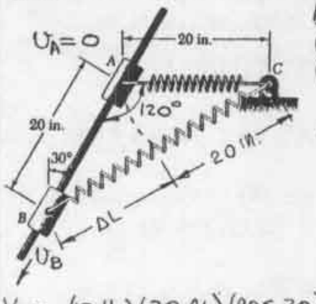
COLLAR $W_A = 12$ lb
SPRING IS UNSTRETCHED WHEN COLLAR IS AT A.
COLLAR RELEASED FROM REST AT A



FIND:

VELOCITY AT B FOR $k = 0.1$ lb/in TO 2.0 lb/in IN 0.1 lb/in INCREMENTS

WRITE EQUATION FOR v_B IN TERMS OF k



ANALYSIS

$(20+\Delta L)^2 = 20^2 + 20^2 - 2(20)^2\cos 120°$

$(20+\Delta L)^2 = 800 + 400 = 1200$

$\Delta L = 14.64$ in $= 1.220$ ft

CONSERVATION OF ENERGY

$v_A = 0 \quad T_A = \frac{1}{2} m v_A^2 = 0$

$T_B = \frac{1}{2} m v_B^2 = \frac{6}{g} v_B^2$

$V_A = 0$ (DATUM AT A)

$V_B = V_g + V_e$

$V_B = -(12\text{ lb})(\frac{20}{12}\text{ ft})(\cos 30) + \frac{1}{2}(k(\text{lb/in})(12\text{ in/ft})(1.220\text{ ft})^2$

$V_B = (-17.32 + 8.932k)(\text{lb}\cdot\text{ft})$, (INPUT k IN lb/in.)

$T_A + V_A = T_B + V_B \quad 0 + 0 = \left(\frac{6\text{ lb}}{32.2\text{ ft/s}^2}\right) v_B^2 - 17.32 + 8.932k$

$v_B = [92.95 - 47.933k]^{1/2}$ (ft/s) (1)

OUTLINE OF PROGRAM

INPUT k IN (1) IN lb/in IN 0.1 lb/in INCREMENTS AND STOP WHEN $k = 2.0$ lb/in

PRINT VALUES OF v_B (IN ft/s)

NOTE: COLLAR NEVER REACHES B FOR $k > (92.95)/(47.933) = 1.939$ lb/in.

PROGRAM OUTPUT

13.C1

K (LB/IN)	VELOCITY (FT/S)
0.10	9.39
0.20	9.13
0.30	8.86
0.40	8.59
0.50	8.31
0.60	8.01
0.70	7.71
0.80	7.39
0.90	7.06
1.00	6.71
1.10	6.34
1.20	5.95
1.30	5.54
1.40	5.08
1.50	4.59
1.60	4.03
1.70	3.39
1.80	2.58
1.90	1.37

13.C2

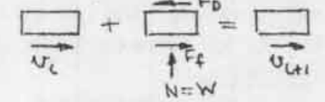
GIVEN:

CAR WEIGHT, $W = 2000$ lb
FOR FIRST 60 ft ALL WEIGHT IS ON THE REAR WHEELS WHICH ARE SLIPPING
FOR REMAINING 1260 ft, 60 % OF THE WEIGHT IS ON THE REAR WHEELS WITH SLIPPING IMPENDING.
$\mu_s = 0.60 \quad \mu_k = 0.85$
AERODYNAMIC DRAG $F_d = 0.0098 v^2$ WITH v IN ft/s AND F_d IN lb.

FIND:

VELOCITY AND ELAPSED TIME WITH AND WITHOUT DRAG. EVERY 5 ft FOR THE FIRST 60 ft AND EVERY 90 ft FOR THE REMAINING 1260 ft.

ANALYSIS USE WORK AND ENERGY IN INCREMENTS OF $\Delta x_i = 0.1$ ft, BETWEEN iTH AND i+1 TH INTERVAL TO GET v_{i+1} ($v_i = 0$ FOR $i = 0$)



$T_i + U_{(i)\to(i+1)} = T_{i+1} \quad \frac{1}{2} m v_i^2 - (F_D + F_f)\Delta x_i = \frac{1}{2} m v_{i+1}^2$

(1) $v_{i+1} = \left[v_i^2 + \frac{2g}{W}(F_f - F_D)\Delta x_i\right]^{\frac{1}{2}} \quad F_D = 0.0098 v_{iD}^2$

(2) $\Delta t_i = \frac{2\Delta x_i}{(v_i + v_{i+1})}$

FIRST 60 ft $F_f = \mu_k W = (0.85)W$

FOR REMAINING 1260 ft

$F_f = (0.60)\mu_s W = 0.36W$

$\Delta x_i = 0.1$ ft
$g = 32.2$ ft/s
$W = 2000$ lb

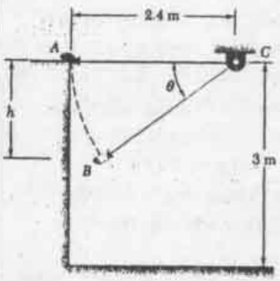
OUTLINE OF PROGRAM

IDENTIFY v_i AND v_{iD} AS THE VELOCITIES IN THE iTH INTERVAL WITHOUT AND WITH DRAG, WITH $v_{(i=1)} = 0$ AND $F_f = 0.85W$ USE A LOOP TO SOLVE FOR v_{i+1} AND TO SOLVE FOR t_i. SUM Δx_i TO FIND x_i AND SUM Δt_i TO FIND t_i. PRINT v_i, t_i, x_i AT 5 ft INTERVALS
REPEAT FOR REMAINING 1260 ft WITH $F_f = 0.36W$.
PRINT x_i, v_i, t_i AT 90 ft INTERVALS

13.C2

DISTANCE (FT)	V(FT/S) NO DRAG	T(S) NO DRAG	V(FT/S) DRAG	T(S) DRAG
5.	13.90	0.719	13.89	0.720
10.	19.66	1.017	19.64	1.018
15.	24.07	1.246	24.05	1.247
20.	27.80	1.439	27.76	1.440
25.	31.08	1.609	31.02	1.610
30.	34.05	1.762	33.97	1.764
35.	36.78	1.903	36.67	1.905
40.	39.31	2.035	39.19	2.037
45.	41.70	2.158	41.55	2.161
50.	43.95	2.275	43.78	2.278
55.	46.10	2.386	45.90	2.390
60.	48.15	2.492	47.92	2.496
60 FT. TO 1320 FT AT 90 FT. INTERVALS				
150.	72.65	3.984	71.76	4.000
240.	90.74	5.086	89.00	5.119
330.	105.78	6.002	103.02	6.057
420.	118.93	6.803	115.02	6.882
510.	130.77	7.524	125.60	7.630
600.	141.62	8.184	135.09	8.320
690.	151.70	8.798	143.71	8.966
780.	161.15	9.373	151.63	9.575
870.	170.08	9.917	158.95	10.155
960.	178.56	10.433	165.75	10.709
1050.	186.66	10.926	172.10	11.242
1140.	194.42	11.398	178.06	11.756
1230.	201.88	11.852	183.67	12.253
1320.	209.07	12.290	188.96	12.736

13.C3



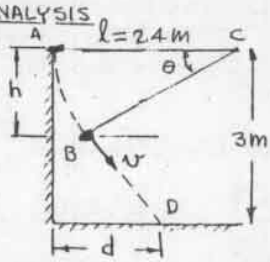
GIVEN:

5-kg BAG
ROPE = 2.4 M LONG
INITIAL VELOCITY ZERO

FIND:

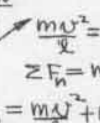
FOR VALUES OF MAXIMUM TENSION F_m FROM 40 TO 140 N IN 5-N INCREMENTS, THE
(a) DISTANCE h
(b) DISTANCE d FROM THE WALL TO THE POINT WHERE THE BAG HITS THE FLOOR.

ANALYSIS



BAG MOVES ALONG A CIRCULAR ARC AB UNTIL THE ROPE BREAKS (RADIUS, ℓ)

NEWTONS LAW



$\frac{mv^2}{\ell} = a_n$

$\Sigma F_n = m a_n$

$F_m = \frac{mv^2}{\ell} + mg\sin\theta$

CONSERVATION OF ENERGY

$v = \sqrt{2gh}$ (1)

$\ell \sin\theta = h$

$\sin\theta = \frac{h}{\ell}$ $\quad F_m = \frac{2mg}{\ell} h + \frac{mg}{\ell} h = \frac{3mg}{\ell} h$

$\theta = \sin^{-1}\frac{h}{\ell}$ (2) $\quad h = \frac{F_m \ell}{3mg}$ (3)

FROM B TO D (PROJECTILE TRAJECTORY)

$v_H = v\sin\theta \rightarrow \quad d = (\ell - \ell\cos\theta) + v_H t_D$ (4)

$v_v = v\cos\theta \downarrow \quad (3-h) = v_v t_D + g t_D^2/2$

(6) $\quad t_D = -\frac{v_v}{g} + \sqrt{\left(\frac{v_v}{g}\right)^2 + 2\frac{(3-h)}{g}}$ (5)

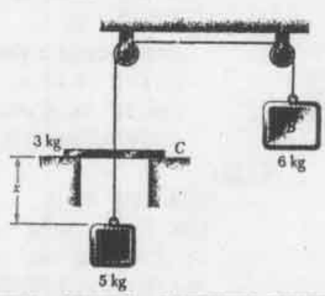
OUTLINE OF PROGRAM

WITH $\ell = 2.4$ m, $m = 5$ kg, $g = 9.81$ m/s² IN EQUATION (3), AND FOR F_m IN 5N INCREMENTS FROM 40 TO 140 N, SOLVE FOR h. FOR EACH h, SOLVE FOR v (EQ 1), AND θ (EQ 2). SOLVE FOR v_H AND v_v (EQ 6) AND WITH v_v AND h, SOLVE FOR t_D (EQ. 5) AND WITH θ, h, t_D SOLVE FOR d IN (EQ 4). PRINT h AND d FOR EACH F_m.

PROGRAM OUTPUT

FORCE (NEWTONS)	H (METERS)	d (METERS)
40	0.652	0.503
45	0.734	0.585
50	0.815	0.668
55	0.897	0.752
60	0.979	0.839
65	1.060	0.927
70	1.142	1.017
75	1.223	1.109
80	1.305	1.203
85	1.386	1.300
90	1.468	1.401
95	1.549	1.505
100	1.631	1.615
105	1.713	1.731
110	1.794	1.854
115	1.876	1.989
120	1.957	2.137
125	2.039	2.306
130	2.120	2.504
135	2.202	2.753
140	2.283	3.101

13.C4



GIVEN:

INITIALLY
$v_A = 0 \quad x = 1.7$ M
PLASTIC IMPACT BETWEEN A AND C
$e = 0$

FIND:

(a) TIME TO COMPLETE 10 COMPLETE CYCLES
(b) VALUE OF x AFTER THE 10TH CYCLE

ANALYSIS (FOR THE i^{TH} CYCLE)

REFER TO FIGURES IN THE SOLUTION TO PROB. 13.199

FROM ① TO ② CONSERVATION OF ENERGY (BEFORE IMPACT)

$T_1 = 0,\ V_1 = 0,\ T_2 = \frac{1}{2}(5+6)(v_A^2)_i = \frac{11}{2}(v_A^2)_i$

$V_2 = (5-6)g(x_i) = -g x_i$

$T_1 + V_1 = T_2 + V_2 \quad 0 + 0 = \frac{11}{2}(v_A^2)_i - g x_i,\ (v_A)_i = \sqrt{\frac{2}{11} g x_i}$ (1)

TIME $(t_{1-2})_i$

ACCELERATION FROM ① TO ② IS CONSTANT, THUS AVERAGE VELOCITY IS $(\bar{v}_A)_i = \frac{0 + (v_A)_i}{2}$

AND $(\bar{v}_A)_i = \frac{x_i}{(t_{1-2})_i} \quad (t_{1-2})_i = \frac{2x_i}{(v_A)_i} = \frac{2x_i}{\sqrt{\frac{2}{11} g x_i}}$

$(t_{1-2})_i = 1.498\sqrt{x_i}$ (2)

(AFTER IMPACT) AT ②

IMPULSE-MOMENTUM FOR A AND C

$5(v_A)_i + T\Delta t = (8)(v_A')_i$

IMPULSE-MOMENTUM FOR B

$6(v_A)_i - T\Delta t = 6(v_A')_i$

ADDING $\quad 11(v_A)_i = 14(v_A')_i \quad (v_A')_i = \frac{11}{14}(v_A)_i = \sqrt{\frac{11 g x_i}{98}}$ (3)

FROM ② TO ④, (SEE (D) IN SOLUTION TO PROB. 13.199)

CONSERVATION OF ENERGY DATUM AT ②

$T_2 = \frac{1}{2}(5+6)(v_A')^2 = \frac{11}{2} \cdot \frac{11 g x_i}{98} = \frac{121}{196} g x_i \quad V_2 = 0$

$T_4 = 0 \quad V_4 = m_B g x_{i+1} - m_A g x_{i+1} = (6-5) g x_{i+1} = g x_{i+1}$

$T_2 + V_2 = T_4 + V_4 \quad \frac{121 g}{196} x_i + 0 = 0 + g x_{i+1}$

$x_{i+1} = \frac{121}{196} x_i$ (4)

TIME ② TO ④ $(t_{2-4})_i$

$(t_{2-4})_i = \frac{2x_{i+1}}{(v_A')_i} = \frac{(2)(121)(x_i)}{(196)\sqrt{\frac{11 g x_i}{98}}} = 1.1766\sqrt{x_i}$ (5)

TIME FROM ② TO ③ AND FROM ③ TO ②

$T_2 = \frac{1}{2}(m_A + m_B + m_C)(v_A')_i^2 = 7\left(\frac{11 g x_i}{98}\right) = \frac{77 g}{98} x_i$

$T_3 = 0 \quad V_3 = (m_A + m_C) g d_i - m_B g d_i = (8-6) g d_i = 2 g d_i$

$T_2 + V_2 = T_3 + V_3 \quad \frac{77 g}{98} x_i + 0 = 0 + 2 g d_i$

$d_i = \frac{77}{196} x_i$

$(t_{2-3})_i = \frac{2 d_i}{(v_A')_i}$

$(t_{2-3})_i = 2\left(\frac{77}{196} x_i\right) \Big/ \sqrt{\frac{11 g x_i}{98}} = 0.7488\sqrt{x_i}$ (6)

$(t_{3-2})_i = (t_{3-4})_i = 0.7488\sqrt{x_i}$ (7)

(CONTINUED)

13.C4 continued

TOTAL TIME TO COMPLETE THE i^{TH} CYCLE

EQS. (2) + (6) + (7) + (5)

$$t_i = (t_{1-2})_i + (t_{2-3})_i + (t_{3-2})_i + (t_{2-4})_i$$

$$t_i = (1.498 + 0.7488 + 0.7488 + 1.1766)\sqrt{x_i}$$

$$t_i = 4.172\sqrt{x_i} \qquad (8)$$

OUTLINE OF PROGRAM

SET $x_1 = 1.7$ M $(i=1)$

(a) CALCULATE x_{i+1} FROM EQUATION (4) FOR $i=1$ TO $i=10$. FOR EACH VALUE OF x USE EQUATION (8) TO DETERMINE t FOR THE i^{th} CYCLE. SUM t's TO OBTAIN THE TOTAL TIME THROUGH THE 10^{TH} CYCLE.

(b) FOR $i=10$ OBTAIN x FOR THE TENTH CYCLE

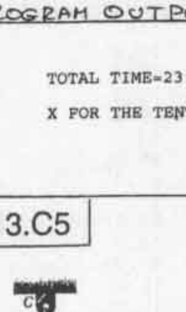
PRINT TOTALTIME AND x FOR THE 10^{TH} CYCLE.

PROGRAM OUTPUT

```
TOTAL TIME=23.1 SECONDS

X FOR THE TENTH CYCLE=0.01367 METERS
```

13.C5



GIVEN:

$m_B = 700$ g, $m_A = 350$ g

$v_0 = 6$ m/s $\quad v_B = 0$

$\theta_0 = 20°$ TO $150°$ IN $10°$ INCREMENTS

FIND:

v_A' AND v_B' AFTER IMPACT AND ENERGY LOST FOR,

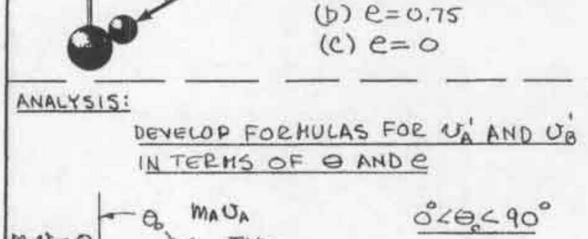
(a) $e = 1$

(b) $e = 0.75$

(c) $e = 0$

ANALYSIS:

DEVELOP FORMULAS FOR v_A' AND v_B' IN TERMS OF θ AND e



MOMENTUM OF A ALONE IS CONSERVED IN THE t DIRECTION

$$m_A(v_A)_t = m(v_A')_t$$

$(v_A)_t = 0$ THUS $(v_A')_t = 0$ AND v_A' IS ALONG THE n AXIS

KINEMATICS

$$(v_B')_x = v_B'$$

13.C5 continued

CONSERVATION OF MOMENTUM IN THE x DIRECTION FOR A AND B TOGETHER

$$m_A v_A \sin\theta_0 = m_A v_A' \sin\theta_0 + m_B v_B'$$

$m_A = 0.350$ kg $\quad m_B = 0.700$ kg $\quad v_A = v_0 = 6$ m/s

$$6\sin\theta_0 = v_A' \sin\theta_0 + 2v_B' \qquad (1)$$

RELATIVE VELOCITIES IN THE n DIRECTION

$$(v_A - 0)e = v_B' \sin\theta_0 - v_A' \qquad v_A = v_0 = 6 \text{ m/s}$$

$$6e = v_B' \sin\theta_0 - v_A' \qquad (2)$$

MULTIPLY (2) BY $\sin\theta_0$ AND ADD TO (1) TO GET v_B'

$$v_B' = \frac{6\sin\theta_0(1+e)}{(2+\sin^2\theta_0)} \qquad (3)$$

SUBSTITUTE (3) IN (2) FOR v_A'

$$v_A' = \frac{6\sin^2\theta_0 - 12e}{(2+\sin^2\theta_0)} \qquad (4)$$

FOR $\theta_0 \geq 90°$, $T\Delta t = 0$, AND BALL A AT A VELOCITY OF 6 m/s HITS BALL B WHICH IS AT 0 VELOCITY AND IS NOT CONSTRAINED BY THE CORD. THUS IF ONLY MAGNITUDES ARE CONSIDERED v_A' AND v_B' HAVE VALUES FOR $110° < \theta > 90°$ WHICH ARE THE AS FOR $\theta_0 = 90°$

ENERGY LOST

$$\Delta E = \frac{1}{2}m_A v_A^2 - \frac{1}{2}(m_A v_A'^2 + m_B v_B'^2)$$

$$\Delta E = \frac{1}{2}(0.350)\left[v_A^2 - v_A'^2\right] - \frac{1}{2}(0.700)v_B'^2 \qquad (5)$$

OUTLINE OF PROGRAM

INPUT θ_0 INTO EQUATIONS (3) AND (4) FROM $20°$ TO $90°$ IN INCREMENTS OF $5°$ FOR $e=1$, $e=0.75$ AND $e=0$ TO OBTAIN v_A' AND v_B'. SUBSTITUTE v_A' AND v_B' IN (5) TO OBTAIN ΔE. PRINT $e, \theta_0, v_A', v_B', \Delta E$

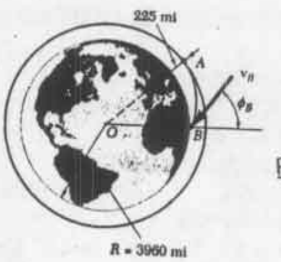
PROGRAM OUTPUT

13.C5

e	THETA (DEG)	VEL A (M\S)	VEL B (M\S)	% E LOST
1.00	20.	-5.337	1.939	0.0
1.00	30.	-4.667	2.667	0.0
1.00	40.	-3.945	3.196	0.0
1.00	50.	-3.278	3.554	0.0
1.00	60.	-2.727	3.779	0.0
1.00	70.	-2.325	3.911	0.0
1.00	80.	-2.081	3.979	0.0
1.00	90.	-2.000	4.000	0.0
0.75	20.	-3.920	1.696	41.3
0.75	30.	-3.333	2.333	38.9
0.75	40.	-2.702	2.797	36.3
0.75	50.	-2.118	3.109	33.8
0.75	60.	-1.636	3.307	31.8
0.75	70.	-1.284	3.422	30.4
0.75	80.	-1.071	3.482	29.5
0.75	90.	-1.000	3.500	29.2
0.00	20.	0.332	0.969	94.5
0.00	30.	0.667	1.333	88.9
0.00	40.	1.027	1.598	82.9
0.00	50.	1.361	1.777	77.3
0.00	60.	1.636	1.890	72.7
0.00	70.	1.838	1.956	69.4
0.00	80.	1.959	1.990	67.3
0.00	90.	2.000	2.000	66.7

VALUES FOR ANGLES OF 90 TO 150 DEGREES ARE THE SAME AS THOSE FOR 90 DEGREES

13.C6



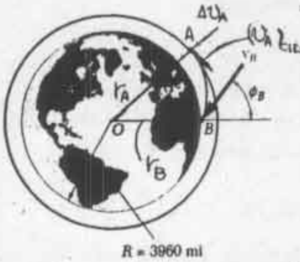
GIVEN:

INITIAL CIRCULAR ORBIT OF 225 MI ABOVE THE SURFACE OF THE EARTH. INCREMENTAL VELOCITY Δv_A TOWARD THE CENTER OF THE EARTH

FIND:

v_B AND ϕ_B AT AN ALTITUDE OF 40 MI FOR ENERGY EXPENDITURE OF 5 TO 100 % OF THAT USED IN PROB. 13.109 IN 5% INCREMENTS

ANALYSIS



CONSERVATION OF ENERGY

AT POINT A

$$T_A = \frac{1}{2} m[(v_A)^2_{circ} + (\Delta v_A)^2]$$

$$V_A = -\frac{GMm}{r_A}$$

AT POINT B

$$T_B = \frac{1}{2} m v_B^2$$

$$V_B = -\frac{GMm}{r_B}$$

$r_A = 3960 + 225 = 4185$ mi

$r_B = 3960 + 40 = 4000$ mi

$T_A + V_A = T_B + V_B \qquad (v_A)^2_{circ} = \frac{gR^2}{r_A}$

$$\frac{1}{2} m[(v_A)^2_{circ} + (\Delta v_A)^2] - \frac{GMm}{r_A} = \frac{1}{2} m v_B^2 - \frac{GMm}{r_B}$$

$$v_B^2 = (v_A)^2_{circ} + (\Delta v_A)^2 + 2GM\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \qquad (1)$$

ENERGY EXPENDITURE IN PROB. 13.109

LET v_A = VELOCITY AT A IN PROB. 13.109 TO BRING THE VEHICLE TO B AT $\phi = 60°$.

FROM 13.109, $v_A = 11.32 \times 10^3$ ft/s

ENERGY EXPENDITURE, $E = \frac{1}{2} m[(v_A)^2_{circ} - (v_A)^2]$ (2)

ENERGY EXPENDITURE IN THIS PROBLEM

$\frac{K E}{100} = \frac{1}{2} m (\Delta v_A)^2$, WHERE K IS THE % ENERGY USED IN PROB 13.109.

SOLVING FOR $(\Delta v_A)^2$ AND REPLACING E BY EQUATION (2)

$$(\Delta v_A)^2 = \frac{K}{100}\left[(v_A)^2_{circ} - (v_A)^2\right] \qquad (3)$$

EQUATION FOR v_B (SUBSTITUTE (3) INTO (1))

$$v_B = \left\{(v_A)^2_{circ} + \frac{K}{100}\left[(v_A)^2_{circ} - (v_A)^2\right] + 2GM\left[\frac{1}{r_B} - \frac{1}{r_A}\right]\right\}^{\frac{1}{2}} \qquad (4)$$

CONSTANTS: $(v_A)^2_{circ} = \frac{gR^2}{r_A} = \frac{(32.2)[(3960)(5280)]^2}{(4185)(5280)}$

$$(v_A)^2_{circ} = 637.07 \times 10^6 \ ft^2/s^2$$

FROM 13.109, $(v_A)^2 = (11.32 \times 10^3)^2 = 128.14 \times 10^6 \ ft^2/s^2$

$$2GM\left[\frac{1}{r_B} - \frac{1}{r_A}\right] = 2gR^2\left[\frac{1}{r_B} - \frac{1}{r_A}\right] = \frac{(2)(32.2)[(3960)(5280)]^2}{5280}\left[\frac{1}{4000} - \frac{1}{4185}\right]$$

(CONTINUED)

13.C6 continued

$$2GM\left[\frac{1}{r_B} - \frac{1}{r_A}\right] = 58.93 \times 10^6 \ ft^2/s^2$$

EQUATION FOR ϕ_B

CONSERVATION OF ANGULAR MOMENTUM

$$r_A (v_A)_{circ} = r_B v_B \sin\phi_B$$

$$\phi_B = \sin^{-1}[r_A (v_A)_{circ} / r_B (v_B)] \qquad (5)$$

OUTLINE OF PROGRAM

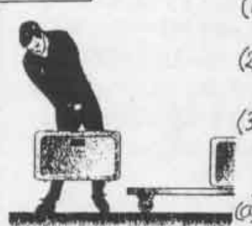
INPUT CONSTANTS INTO EQUATION (4) AND SOLVE FOR v_B FOR VALUES OF K OF 5% TO 100% AT INTERVALS OF 5%. FOR EACH VALUE OF v_B AND USING THE GIVEN CONSTANT VALUES OF $(v_A)_{circ}$, r_A AND r_B, USE EQUATION (5) TO SOLVE FOR ϕ_B. PRINT K, v_B AND ϕ_B.

PROGRAM OUTPUT

13.C6

K (%)	VB (FT/S)	PHI (DEGREES)
5.	26860.	79.5
10.	27329.	75.1
15.	27791.	71.8
20.	28245.	69.2
25.	28692.	67.0
30.	29132.	65.0
35.	29566.	63.3
40.	29993.	61.7
45.	30414.	60.3
50.	30830.	58.9
55.	31240.	57.7
60.	31644.	56.6
65.	32044.	55.5
70.	32438.	54.5
75.	32828.	53.6
80.	33214.	52.7
85.	33595.	51.8
90.	33971.	51.0
95.	34344.	50.3
100.	34712.	49.5

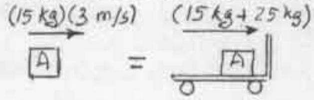
14.1



GIVEN:
(1) 15-kg SUITCASE A TOSSED WITH VELOCITY OF 3 m/s →.
(2) 20-kg SUITCASE B TOSSED WITH VELOCITY OF 2 m/s →.
(3) 25-kg CARRIER INITIALLY AT REST

FIND: FINAL VELOCITY OF CARRIER
(a) IF 15-kg SUITCASE IS TOSSED FIRST.
(b) IF 20-kg SUITCASE IS TOSSED FIRST.

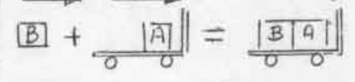
(a) 15-kg SUITCASE TOSSED ON CARRIER FIRST:
CONSERVATION OF MOMENTUM:



$(15)(3) = (40)\,v_1$

$v_1 = 1.125$ m/s

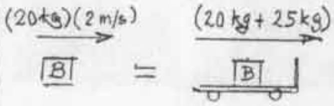
20-kg SUITCASE TOSSED NEXT:



$(20)(2) + (40)(1.125) = 60\,v_2$

$v_2 = 1.417$ m/s →

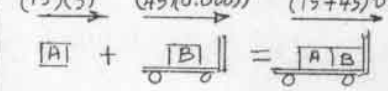
(b) 20-kg SUITCASE TOSSED ON CARRIER FIRST:
CONSERVATION OF MOMENTUM:



$(20)(2) = (45)\,v_1'$

$v_1' = 0.8889$ m/s

15-kg SUITCASE TOSSED NEXT:



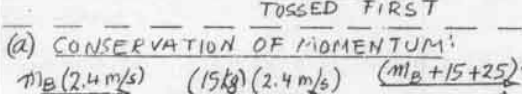
$(15)(3) + (45)(0.8889) = 60\,v_2'$

$v_2' = 1.417$ m/s →

14.2

GIVEN:
EMPLOYEE TOSSES TWO SUITCASES A AND B ON CARRIER WITH $v_0 = 2.4$ m/s →.
MASS OF SUITCASE A IS 15 kg
MASS OF CARRIER IS 25 kg
(a) FIND m_B KNOWING THAT $v_{FINAL} = 1.2$ m/s →.
(b) FIND v_{FINAL} IF B IS TOSSED FIRST

(a) CONSERVATION OF MOMENTUM:



$2.4(m_B + 15) = (m_B + 40)\,v_{FINAL}$ (1)

LET $v_{FINAL} = 1.2$ m/s:

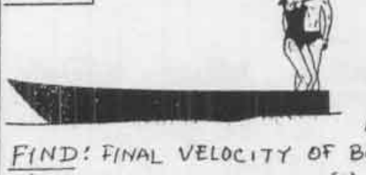
$2.4 m_B + 36 = 1.2\,m_B + 48$ $m_B = 10.00$ kg

(b) F.B.I. EQUATION IS STILL VALID
LET $m_B = 10.00$ kg IN EQ. (1):

$2.4(10+15) = (10+40)\,v_{FINAL}$

$v_{FINAL} = 1.200$ m/s

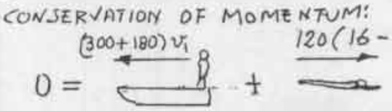
14.3



GIVEN:
180-lb MAN
120-lb WOMAN
300-lb BOAT
MAN AND WOMAN DIVE WITH 16 ft/s VELOCITY W/R BOAT.

FIND: FINAL VELOCITY OF BOAT IF
(a) WOMAN DIVES FIRST, (b) MAN DIVES FIRST.

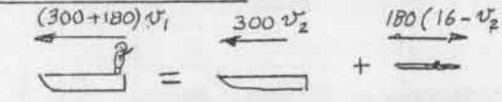
(a) WOMAN DIVES FIRST
CONSERVATION OF MOMENTUM:



$\xrightarrow{+}$ x-COMP: $-(300+180)v_1 + 120(16 - v_1) = 0$ (1)

$-600 v_1 + 1920 = 0$ $v_1 = 3.20$ ft/s

MAN DIVES NEXT:



$\xrightarrow{+}$ x-COMP: $-(300+180)v_1 = -300 v_2 + 180(16 - v_2)$ (2)

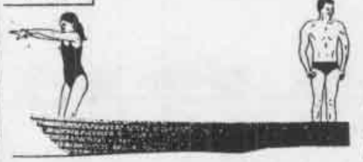
$-480(3.20) = -480 v_2 + 2880,$ $v_2 = 9.20$ ft/s ←

(b) MAN DIVES FIRST
SIMILAR ANALYSIS YIELDS THE FOLLOWING EQS.:

$-(300+120)v_1' + 180(16 - v_1') = 0$ $v_1' = 4.80$ ft/s

$-(300+120)v_1' = -300 v_2' + 120(16 - v_2'),$ $v_2' = 9.37$ ft/s ←

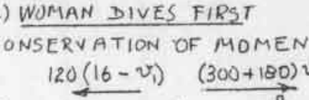
14.4



GIVEN:
180-lb MAN
120-lb WOMAN
300-lb BOAT
MAN AND WOMAN DIVE WITH 16 ft/s VELOCITY W/R BOAT IN OPPOSITE DIRECTIONS

FIND: FINAL VELOCITY OF BOAT IF
(a) WOMAN DIVES FIRST, (b) MAN DIVES FIRST

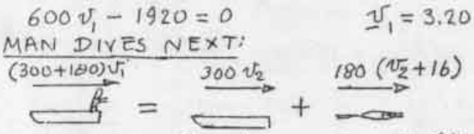
(a) WOMAN DIVES FIRST
CONSERVATION OF MOMENTUM:



$\xrightarrow{+}$ x COMP.: $-120(16 - v_1) + (300+180)v_1 = 0$ (1)

$600 v_1 - 1920 = 0$ $v_1 = 3.20$ ft/s

MAN DIVES NEXT:



$\xrightarrow{+}$ x COMP.: $480(3.20) = 300 v_2 + 180(v_2 + 16)$ (2)

$480 v_2 = -1344,$ $v_2 = -2.80$ $v_2 = 2.80$ ft/s ←

(b) MAN DIVES FIRST
SIMILAR ANALYSIS YIELDS THE FOLLOWING EQS.:

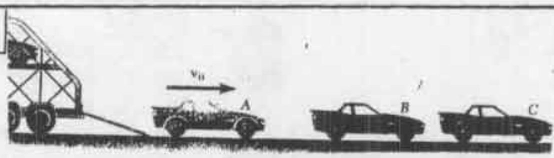
$+180(16 + v_1') + (300+120)v_1' = 0$ $v_1' = -4.80$ ft/s

$420(-4.80) = 300 v_2' + 120(v_2' - 16)$

$420 v_2' = -96$

$v_2' = -0.229$ ft/s $v_2' = 0.229$ ft/s ←

14.5



GIVEN: IDENTICAL CARS. $v_0 = 1.920$ m/s
AFTER A HITS B: $(v_B)_1 = 1.680$ m/s
AFTER B HITS C: $(v_B)_2 = 0.210$ m/s
AFTER A HITS B AGAIN: $(v_B)_3 = 0.23625$ m/s
FIND: (a) FINAL VELOCITIES OF A AND C, (b) COEFF. e

CONSERVATION OF MOMENTUM:

A HITS B

$m(1.920) = m(v_A)_1 + m(1.680)$

$(v_A)_1 = 1.920 - 1.680 = 0.240$ m/s

B HITS C

$m(1.680) = m(0.210) + m(v_C)_F$

$(v_C)_F = 1.680 - 0.210$

$(v_C)_F = 1.470$ m/s

A HITS B AGAIN

$m(0.240) + m(0.210) = m(v_A)_F + m(0.23625)$

$0.240 + 0.210 = (v_A)_F + 0.23625 \qquad (v_A)_F = 0.21375$ m/s

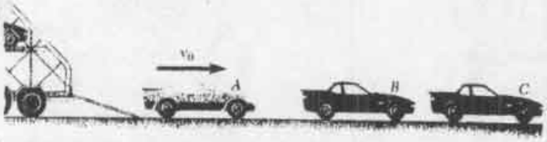
(a) $v_A = 0.214$ m/s →; $v_C = 1.470$ m/s →

(b) FIRST COLLISION: $e = \dfrac{1.680 - 0.240}{1.920} = 0.750$

SECOND COLLISION: $e = \dfrac{1.470 - 0.210}{1.680} = 0.750$

THIRD COLLISION: $e = \dfrac{0.23625 - 0.21375}{0.240 - 0.210} = 0.750$

14.6



GIVEN: IDENTICAL CARS. $v_0 = 2.00$ m/s
AFTER A HITS B: $(v_A)_1 = 0.400$ m/s
AFTER B HITS C: $v_C = 1.280$ m/s
AFTER A HITS B AGAIN: $(v_A)_2 = 0.336$ m/s
FIND: (a) v_B AFTER EACH COLLISION, (b) COEFF. e

CONSERVATION OF MOMENTUM:

A HITS B

$m(2.00) = m(0.400) + m(v_B)_1$

$(v_B)_1 = 1.600$ m/s →

B HITS C

$m(1.600) = m(v_B)_2 + m(1.280)$

$(v_B)_2 = 0.320$ m/s →

A HITS B AGAIN

$m(0.400) + m(0.320) = m(0.336) + m(v_B)_3$

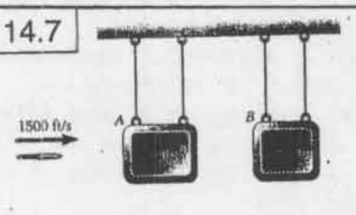
$0.400 + 0.320 = 0.336 + (v_B)_3 \qquad (v_B)_3 = 0.384$ m/s →

(b) FIRST COLLISION: $e = \dfrac{1.600 - 0.400}{2.00} = 0.600$

SECOND COLLISION: $e = \dfrac{1.280 - 0.320}{1.600} = 0.600$

THIRD COLLISION: $e = \dfrac{0.384 - 0.336}{0.400 - 0.320} = 0.600$

14.7



GIVEN:
BULLET FIRED THROUGH A AND BECOMES EMBEDDED IN B. BLOCKS MOVE WITH $v_A = 5$ ft/s AND $v_B = 9$ ft/s

FIND: (a) WEIGHT w OF BULLET
(b) VELOCITY OF BULLET BETWEEN A AND B.

CONSERVATION OF MOMENTUM:

(a) BULLET GOES THROUGH BLOCK A INTO BLOCK B:

$(w/g)(1500) = (6/g)(5) + [(4.95 + w)/g](9)$

$1500w = 30 + 44.5 + 9w$

$1491w = 74.5$

$w = 0.0500$ lb, $\quad w = 0.800$ oz

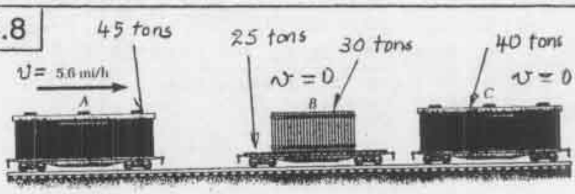
(b) BULLET GOES THROUG BLOCK A

$(0.05\text{ lb}/g)(1500) = (6/g)(5) + (0.05/g)v$

$75 = 30 + 0.05v$

$v = 900$ ft/s

14.8



GIVEN: CARS AND CONTAINER SHOWN.
CARS GET COUPLED AS THEY HIT EACH OTHER.

FIND:
VELOCITY OF CAR A AFTER EACH COUPLING, ASSUMING THAT CONTAINER
(a) DOES NOT SLIDE ON FLATCAR
(b) SLIDES AFTER FIRST COUPLING BUT HITS STOP BEFORE SECOND COUPLING
(c) SLIDES AFTER BOTH COUPLINGS

CONSERVATION OF MOMENTUM:

(a) CONTAINER DOES NOT SLIDE

$(45)(5.6)\ [45t] = (100)v_1\ [45t][55t] = (140)v_2\ [45t][55t][40t]$

$252 = 100v_1 = 140v_2$

$v_1 = 2.52$ mi/h

$v_2 = 1.800$ mi/h

(b) CONTAINER SLIDES AFTER 1st COUPLING, STOPS BEFORE 2nd

$(45)(5.6)\ [45t] = (70)v_1\ [45t][25t] = (140)v_2\ [45t][55t][40t]$

$252 = 70v_1 = 140v_2$

$v_1 = 3.60$ mi/h

$v_2 = 1.800$ mi/h

(c) CONTAINER SLIDES AND STOPS ONLY AFTER 2nd COUPLING

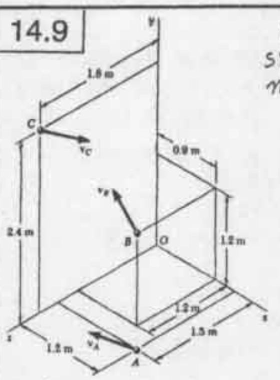
$(45)(5.6)\ [45t] = (70)v_1\ [45t][25t] = (110)v_2\ [45t][25t][40t]$

$252 = 70v_1 = 110v_2$

$v_1 = 3.60$ mi/h

$v_2 = 2.29$ mi/h

14.9



GIVEN:
SYSTEM OF PARTICLES WITH
$m_A = 3$ kg, $m_B = 4$ kg, $m_C = 5$ kg
AND VELOCITIES (m/s)
$\underline{v}_A = -4\underline{i} + 4\underline{j} + 6\underline{k}$
$\underline{v}_B = -6\underline{i} + 8\underline{j} + 4\underline{k}$
$\underline{v}_C = 2\underline{i} - 6\underline{j} - 4\underline{k}$

FIND:
ANGULAR MOMENTUM $\underline{H}_0$

$$\underline{H}_0 = \underline{r}_A \times m_A \underline{v}_A + \underline{r}_B \times m_B \underline{v}_B + \underline{r}_C \times m_C \underline{v}_C$$

USING DETERMINANT FORM FOR VECTOR PRODUCTS AND FACTORING MASSES:

$$\underline{H}_0 = (3\text{kg})\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.2\text{m} & 0 & 1.5\text{m} \\ -4\text{m/s} & 4\text{m/s} & 6\text{m/s} \end{vmatrix} + 4\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.9 & 1.2 & 1.2 \\ -6 & 8 & 4 \end{vmatrix} + 5\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 2.4 & 1.8 \\ 2 & -6 & -4 \end{vmatrix}$$

$$= -18\underline{i} - 39.6\underline{j} + 14.4\underline{k} - 19.2\underline{i} - 43.2\underline{j} + 57.6\underline{k} + 6\underline{i} + 18\underline{j} - 24\underline{k}$$

$$\underline{H}_0 = -(31.2\ \text{kg}\cdot\text{m}^2/\text{s})\underline{i} - (64.8\ \text{kg}\cdot\text{m}^2/\text{s})\underline{j} + (48.0\ \text{kg}\cdot\text{m}^2/\text{s})\underline{k}$$

14.10

GIVEN:
SYSTEM OF PARTICLES OF PROB. 14.9

FIND:
(a) POSITION VECTOR $\bar{\underline{r}}$ OF MASS CENTER G.
(b) LINEAR MOMENTUM OF SYSTEM.
(c) ANGULAR MOMENTUM $\underline{H}_G$ OF SYSTEM.
ALSO: VERIFY THAT ANSWERS TO PROBS. 14.9 AND 14.10 SATISFY EQUATION

$$\underline{H}_0 = \bar{\underline{r}} \times m\bar{\underline{v}} + \underline{H}_G$$

(a) EQ. (14.12):

$$m\bar{\underline{r}} = \sum m_i \underline{r}_i$$

$$(3+4+5)\bar{\underline{r}} = 3(1.2\underline{i} + 1.5\underline{k}) + 4(0.9\underline{i} + 1.2\underline{j} + 1.2\underline{k}) + 5(2.4\underline{j} + 1.8\underline{k})$$

$$12\bar{\underline{r}} = 7.2\underline{i} + 16.8\underline{j} + 18.3\underline{k}$$

$$\bar{\underline{r}} = (0.600\text{m})\underline{i} + (1.400\text{m})\underline{j} + (1.525\text{m})\underline{k}$$

(b) $\underline{L} = \sum m_i \underline{v}_i = 3(-4\underline{i} + 4\underline{j} + 6\underline{k}) + 4(-6\underline{i} + 8\underline{j} + 4\underline{k}) + 5(2\underline{i} - 6\underline{j} - 4\underline{k})$

$$\underline{L} = (-26.0\ \text{kg}\cdot\text{m/s})\underline{i} + (14.00\ \text{kg}\cdot\text{m/s})\underline{j} + (14.00\ \text{kg}\cdot\text{m/s})\underline{k}$$

(c) $\underline{H}_G = \underline{r}_{A/G} \times m_A \underline{v}_A + \underline{r}_{B/G} \times m_B \underline{v}_B + \underline{r}_{C/G} \times m_C \underline{v}_C$

WHERE $\underline{r}_{A/G} = \underline{r}_A - \bar{\underline{r}} = 1.2\underline{i} + 1.5\underline{k} - (0.6\underline{i} + 1.4\underline{j} + 1.525\underline{k})$
$= 0.6\underline{i} - 1.4\underline{j} - 0.025\underline{k}$

$\underline{r}_{B/G} = \underline{r}_B - \bar{\underline{r}} = 0.3\underline{i} - 0.2\underline{j} - 0.325\underline{k}$

$\underline{r}_{C/G} = \underline{r}_C - \bar{\underline{r}} = -0.6\underline{i} + \underline{j} + 0.275\underline{k}$,

$$\underline{H}_G = (3\text{kg})\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.6\text{m} & -1.4\text{m} & -0.025\text{m} \\ -4\text{m/s} & 4\text{m/s} & 6\text{m/s} \end{vmatrix} + 4\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.3 & -0.2 & -0.325 \\ -6 & 8 & 4 \end{vmatrix} + 5\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -0.6 & 1 & 0.275 \\ 2 & -6 & -4 \end{vmatrix}$$

$$= -24.9\underline{i} - 10.5\underline{j} - 9.6\underline{k} + 7.2\underline{i} + 3\underline{j} + 4.8\underline{k} - 11.75\underline{i} - 9.25\underline{j} + 8\underline{k}$$

$$\underline{H}_G = -(29.45\ \text{kg}\cdot\text{m}^2/\text{s})\underline{i} - (16.75\ \text{kg}\cdot\text{m}^2/\text{s})\underline{j} + (3.20\ \text{kg}\cdot\text{m}^2/\text{s})\underline{k}$$

(CONTINUED)

14.10 continued

WE COMPUTE $\bar{\underline{r}} \times m\bar{\underline{v}}$:

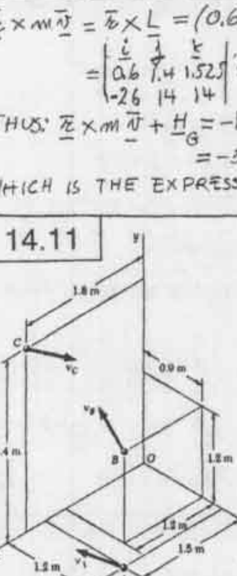
$$\bar{\underline{r}} \times m\bar{\underline{v}} = \bar{\underline{r}} \times \underline{L} = (0.6\underline{i} + 1.4\underline{j} + 1.525\underline{k}) \times (-26\underline{i} + 14\underline{j} + 14\underline{k})$$

$$= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.6 & 1.4 & 1.525 \\ -26 & 14 & 14 \end{vmatrix} = -1.75\underline{i} - 48.05\underline{j} + 44.8\underline{k}$$

THUS: $\bar{\underline{r}} \times m\bar{\underline{v}} + \underline{H}_G = -1.75\underline{i} - 48.05\underline{j} + 44.8\underline{k} - 29.45\underline{i} - 16.75\underline{j} + 3.20\underline{k}$
$= -31.2\underline{i} - 64.8\underline{j} + 48.0\underline{k}$

WHICH IS THE EXPRESSION OBTAINED FOR $\underline{H}_0$ IN PROB. 14.9.

14.11



GIVEN: SYSTEM OF PARTICLES WITH $m_A = 3$ kg, $m_B = 4$ kg, $m_C = 5$ kg
AND VELOCITIES (m/s)
$\underline{v}_A = -4\underline{i} + 4\underline{j} + 6\underline{k}$
$\underline{v}_B = v_x\underline{i} + v_y\underline{j} + 4\underline{k}$
$\underline{v}_C = 2\underline{i} - 6\underline{j} - 4\underline{k}$

FIND:
(a) v_x AND v_y FOR WHICH $\underline{H}_0$ IS PARALLEL TO z AXIS.
(b) CORRESPONDING $\underline{H}_0$.

$$\underline{H}_0 = \underline{r}_A \times m_A \underline{v}_A + \underline{r}_B \times m_B \underline{v}_B + \underline{r}_C \times m_C \underline{v}_C$$

$$= (3\text{kg})\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.2\text{m} & 0 & 1.5\text{m} \\ -4\text{m/s} & 4\text{m/s} & 6\text{m/s} \end{vmatrix} + 4\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0.9 & 1.2 & 1.2 \\ v_x & v_y & 4 \end{vmatrix} + 5\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 2.4 & 1.8 \\ 2 & -6 & -4 \end{vmatrix}$$

$$= -18\underline{i} - 39.6\underline{j} + 14.4\underline{k} + (19.2 - 4.8v_y)\underline{i} + (4.8v_x - 14.4)\underline{j} + (3.6v_y - 4.8v_x)\underline{k} + 6\underline{i} + 18\underline{j} - 24\underline{k}$$

$$\underline{H}_0 = (7.2 - 4.8v_y)\underline{i} + (-36 + 4.8v_x)\underline{j} + (-9.6 + 3.6v_y - 4.8v_x)\underline{k} \quad (1)$$

(a) FOR $\underline{H}_0$ TO BE // z AXIS:

$H_x = 7.2 - 4.8v_y = 0 \qquad H_y = -36 + 4.8v_x = 0$

$v_x = 7.50$ m/s, $v_y = 1.500$ m/s

(b) $\underline{H}_0 = H_z\underline{k} = (-9.6 + 3.6 \times 1.500 - 4.8 \times 7.50)\underline{k}$

$$\underline{H}_0 = -(40.2\ \text{kg}\cdot\text{m}^2/\text{s})\underline{k}$$

14.12

GIVEN: SAME SYSTEM OF PARTICLES WITH SAME VELOCITY DATA AS IN PROB. 14.11

FIND:
(a) v_x AND v_y FOR WHICH $\underline{H}_0$ IS PARALLEL TO y AXIS.
(b) CORRESPONDING $\underline{H}_0$.

SEE SOLUTION OF PROB. 14.11 FOR DERIVATION OF EQ. (1):

$$\underline{H}_0 = (7.2 - 4.8v_y)\underline{i} + (-36 + 4.8v_x)\underline{j} + (-9.6 + 3.6v_y - 4.8v_x)\underline{k}$$

(a) FOR $\underline{H}_0$ TO BE // y AXIS:

$H_x = 7.2 - 4.8v_y = 0 \qquad H_z = -9.6 + 3.6v_y - 4.8v_x = 0$

$v_y = 1.500$ m/s $\qquad -9.6 + 3.6(1.500) - 4.8v_x = 0$

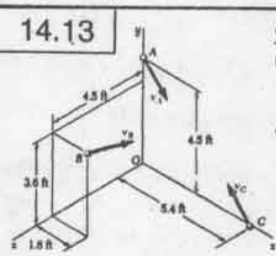
$v_x = -0.875$ m/s

$v_x = -0.875$ m/s, $v_y = 1.500$ m/s

(b) $\underline{H}_0 = H_y\underline{j} = [-36 + 4.8 \times (-0.875)]\underline{j}$

$$\underline{H}_0 = -(40.2\ \text{kg}\cdot\text{m}^2/\text{s})\underline{j}$$

14.13



GIVEN: SYSTEM OF PARTICLES WITH $W_A = 9.66$ lb, $W_B = 6.44$ lb, $W_C = 12.88$ lb AND VELOCITIES (ft/s)

$\underline{v}_A = 4\underline{i} + 2\underline{j} + 2\underline{k}$

$\underline{v}_B = 4\underline{i} + 3\underline{j}$

$\underline{v}_C = -2\underline{i} + 4\underline{j} + 2\underline{k}$

FIND: ANG. MOMENTUM $\underline{H}_O$.

$$\underline{H}_O = \underline{r}_A \times m_A \underline{v}_A + \underline{r}_B \times m_B \underline{v}_B + \underline{r}_C \times m_C \underline{v}_C$$

USING DETERMINANT FORM FOR VECTOR PRODUCTS AND FACTORING MASSES:

$$\underline{H}_O = \frac{9.66}{32.2}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4.5 & 0 \\ 4 & 2 & 2 \end{vmatrix} + \frac{6.44}{32.2}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.8 & 3.6 & 4.5 \\ 4 & 3 & 0 \end{vmatrix} + \frac{12.88}{32.2}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5.4 & 0 & 0 \\ -2 & 4 & 2 \end{vmatrix}$$

$$= 0.3(9\underline{i} - 18\underline{k}) + 0.2(-13.5\underline{i} + 18\underline{j} - 9\underline{k}) + 0.4(-10.8\underline{j} + 21.6\underline{k})$$

$$\underline{H}_O = -(0.720 \text{ ft}\cdot\text{lb}\cdot\text{s})\underline{j} + (1.440 \text{ ft}\cdot\text{lb}\cdot\text{s})\underline{k}$$ ◄

14.14

GIVEN: SYSTEM OF PARTICLES OF PROB. 14.13.

FIND: (a) POSITION VECTOR $\bar{\underline{r}}$ OF MASS CENTER G.
(b) LINEAR MOMENTUM OF SYSTEM.
(c) ANGULAR MOMENTUM $\underline{H}_G$ OF SYSTEM.
ALSO: VERIFY THAT ANSWERS TO PROBS. 14.13 AND 14.14 SATISFY EQUATION

$$\underline{H}_O = \bar{\underline{r}} \times m\bar{\underline{v}} + \underline{H}_G$$

(a) EQ. (14.12): $m\bar{\underline{r}} = \Sigma m_i \underline{r}_i$

WHERE $m_A = 0.3$, $m_B = 0.2$, $m_C = 0.4$, $m = 0.9$

$$0.9\bar{\underline{r}} = 0.3(4.5\underline{j}) + 0.2(1.8\underline{i} + 3.6\underline{j} + 4.5\underline{k}) + 0.4(5.4\underline{i})$$

$$\bar{\underline{r}} = (2.80 \text{ ft})\underline{i} + (2.30 \text{ ft})\underline{j} + (1.00 \text{ ft})\underline{k}$$ ◄

(b) $\underline{L} = \Sigma m_i \underline{v}_i = 0.3(4\underline{i} + 2\underline{j} + 2\underline{k}) + 0.2(4\underline{i} + 3\underline{j}) + 0.4(-2\underline{i} + 4\underline{j} + 2\underline{k})$

$$\underline{L} = (1.200 \text{ lb}\cdot\text{s})\underline{i} + (2.80 \text{ lb}\cdot\text{s})\underline{j} + (1.400 \text{ lb}\cdot\text{s})\underline{k}$$ ◄

(c) $\underline{H}_G = \underline{r}_{A/G} \times m_A \underline{v}_A + \underline{r}_{B/G} \times m_B \underline{v}_B + \underline{r}_{C/G} \times m_C \underline{v}_C$

WHERE

$\underline{r}_{A/G} = \underline{r}_A - \bar{\underline{r}} = 4.5\underline{j} - (2.8\underline{i} + 2.3\underline{j} + \underline{k}) = -2.8\underline{i} + 2.2\underline{j} - \underline{k}$

$\underline{r}_{B/G} = \underline{r}_B - \bar{\underline{r}} = 1.8\underline{i} + 3.6\underline{j} + 4.5\underline{k} - (2.8\underline{i} + 2.3\underline{j} + \underline{k}) = -\underline{i} + 1.3\underline{j} + 3.5\underline{k}$

$\underline{r}_{C/G} = \underline{r}_C - \bar{\underline{r}} = 5.4\underline{i} - (2.8\underline{i} + 2.3\underline{j} + \underline{k}) = 2.6\underline{i} - 2.3\underline{j} - \underline{k}$

$$\underline{H}_G = 0.3\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -2.8 & 2.2 & -1 \\ 4 & 2 & 2 \end{vmatrix} + 0.2\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & 1.3 & 3.5 \\ 4 & 3 & 0 \end{vmatrix} + 0.4\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2.6 & -2.3 & -1 \\ -2 & 4 & 2 \end{vmatrix}$$

$$= 0.3(6.4\underline{i} + 1.6\underline{j} - 14.4\underline{k}) + 0.2(-10.5\underline{i} + 14\underline{j} - 8.2\underline{k}) + 0.4(-0.6\underline{i} - 3.2\underline{j} + 5.8\underline{k})$$

$$\underline{H}_G = -(0.420 \text{ ft}\cdot\text{lb}\cdot\text{s})\underline{i} + (2.00 \text{ ft}\cdot\text{lb}\cdot\text{s})\underline{j} - (3.64 \text{ ft}\cdot\text{lb}\cdot\text{s})\underline{k}$$ ◄

COMPUTE $\bar{\underline{r}} \times m\bar{\underline{v}}$:

$$\bar{\underline{r}} \times m\bar{\underline{v}} = \bar{\underline{r}} \times \underline{L} = (2.8\underline{i} + 2.3\underline{j} + \underline{k}) \times (1.2\underline{i} + 2.8\underline{j} + 1.4\underline{k})$$

$$= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2.8 & 2.3 & 1 \\ 1.2 & 2.8 & 1.4 \end{vmatrix} = 0.420\underline{i} - 2.72\underline{j} + 5.08\underline{k}$$

THUS: $\bar{\underline{r}} \times m\bar{\underline{v}} + \underline{H}_G = 0.420\underline{i} - 2.72\underline{j} + 5.08\underline{k} + (-0.420)\underline{i} + 2\underline{j} - 3.64\underline{k}$

$$= -0.720\underline{j} + 1.440\underline{k}$$ ◄

WHICH IS THE EXPRESSION OBTAINED FOR $\underline{H}_O$ IN PROB. 14.13.

14.15

GIVEN: 900-lb SPACE VEHICLE WITH $\underline{v}_0 = (1200 \text{ ft/s})\underline{i}$ AS IT PASSES THROUGH O AT $t = 0$, IT EXPLODES INTO A(450 lb), B(300 lb), C(150 lb)

AT $t = 4$ s, POSITIONS OF A AND B ARE
A(3840 ft, −960 ft, −1920 ft)
B(6480 ft, 1200 ft, 2640 ft)

FIND: POSITION OF C AT THAT TIME

MOTION OF MASS CENTER:

SINCE THERE IS NO EXTERNAL FORCE,

$$\bar{\underline{r}} = \underline{v}_0 t = (1200 \text{ ft/s})\underline{i}(4 \text{ s}) = (4800 \text{ ft})\underline{i}$$

EQUATION (14.12)

$m\bar{\underline{r}} = \Sigma m_i \underline{r}_i$:

$$(900/g)(4800\underline{i}) = (450/g)(3840\underline{i} - 960\underline{j} - 1920\underline{k}) + (300/g)(6480\underline{i} + 1200\underline{j} + 2640\underline{k}) + (150/g)\underline{r}_C$$

$$150\underline{r}_C = (900 \times 4800 - 450 \times 3840 - 300 \times 6480)\underline{i} + (450 \times 960 - 300 \times 1200)\underline{j} + (450 \times 1920 - 300 \times 2640)\underline{k}$$

$$= 648{,}000\,\underline{i} + 72{,}000\,\underline{j} + 72{,}000\,\underline{k}$$

$$\underline{r}_C = (4320 \text{ ft})\underline{i} + (480 \text{ ft})\underline{j} + (480 \text{ ft})\underline{k}$$ ◄

14.16

GIVEN: 30-lb PASSES THROUGH O WITH VELOCITY $\underline{v}_0 = (120 \text{ ft/s})\underline{i}$ WHEN IT EXPLODES INTO FRAGMENTS A(12 lb) AND B(18 lb).
AT $t = 3$ s, POSITION OF A IS A(300 ft, 24 ft, −48 ft).

FIND: POSITION OF B AT THAT TIME

ASSUME: $a_y = -g = -32.2 \text{ ft/s}^2$

MOTION OF MASS CENTER:

IT MOVES AS IF PROJECTILE HAD NOT EXPLODED.

$$\bar{\underline{r}} = v_0 t\,\underline{i} - \tfrac{1}{2} g t^2 \underline{j}$$

$$= (120 \text{ ft/s})(3 \text{ s})\underline{i} - \tfrac{1}{2}(32.2 \text{ ft/s}^2)(3 \text{ s})^2 \underline{j}$$

$$= (360 \text{ ft})\underline{i} - (144.9 \text{ ft})\underline{j}$$

EQUATION (14.12):

$m\bar{\underline{r}} = \Sigma m_i \underline{r}_i$:

$m\bar{\underline{r}} = m_A \underline{r}_A + m_B \underline{r}_B$

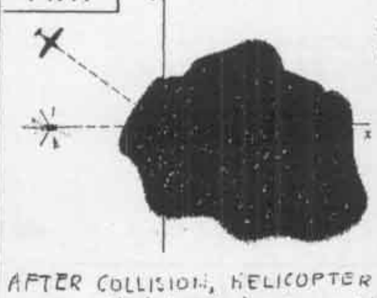
$$\frac{30}{g}(360\underline{i} - 144.9\underline{j}) = \frac{12}{g}(300\underline{i} + 24\underline{j} - 48\underline{k}) + \frac{18}{g}\underline{r}_B$$

$$18\underline{r}_B = (30 \times 360 - 12 \times 300)\underline{i} + (-30 \times 144.9 - 12 \times 24)\underline{j} + (12 \times 48)\underline{k}$$

$$= 7200\underline{i} - 4635\underline{j} + 576\underline{k}$$

$$\underline{r}_B = (400 \text{ ft})\underline{i} - (258 \text{ ft})\underline{j} + (32.0 \text{ ft})\underline{k}$$ ◄

14.17



GIVEN:
AIRPLANE: $m_A = 1500\,kg$
HELICOPTER: $m_H = 3000\,kg$
COLLIDE AT 1200 m ABOVE O.
4 MIN. BEFORE: HELICOPTER WAS 8.4 km WEST OF O; PLANE WAS 16 km WEST AND 12 km NORTH OF O.
AFTER COLLISION, HELICOPTER BREAKS INTO H_1 (1000 kg) AND H_2 (2000 kg)

FIND: POINT A WHERE WRECKAGE OF PLANE WILL BE FOUND, KNOWING THAT FRAGMENTS OF HELICOPTER WERE AT H_1(500m, −100m) AND H_2(600m, −500m).

MOTION OF MASS CENTER G.

AT COLLISION: $\underline{v}_H = \dfrac{(8400\,m)}{4(60s)}\,\underline{i} = (35.00\,m/s)\,\underline{i}$

$$\underline{v}_A = \frac{(16000\,m)\underline{i} - (12000\,m)\underline{j}}{4(60s)} = (66.67\,m/s)\underline{i} - (50\,m/s)\underline{j}$$

VELOCITY OF MASS CENTER:

$$(m_H + m_A)\bar{\underline{v}} = m_H \underline{v}_H + m_A \underline{v}_A$$

$$4500\,\bar{\underline{v}} = 3000(35.00\,\underline{i}) + 1500(66.67\,\underline{i} - 50\,\underline{j})$$

$$\bar{\underline{v}} = (45.556\,m/s)\,\underline{i} - (16.667\,m/s)\,\underline{j}$$

VERTICAL MOTION OF G:

$$h = \tfrac{1}{2}gt^2 \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1200\,m)}{9.81\,m/s^2}} = 15.641\,s$$

POSITION OF G AT TIME OF GROUND IMPACT:

$$\bar{\underline{r}} = \bar{\underline{v}}t = (45.556\,\underline{i} - 16.667\,\underline{j})(15.641)$$

$$\bar{\underline{r}} = (712.55\,m)\,\underline{i} - (260.69\,m)\,\underline{j} \qquad (1)$$

FROM EQ. (14.12):

$$(m_H + m_A)\bar{\underline{r}} = m_{H_1}\underline{r}_{H_1} + m_{H_2}\underline{r}_{H_2} + m_A\underline{r}_A \qquad (2)$$

$$4500(712.55\,\underline{i} - 260.69\,\underline{j}) = 1000(500\,\underline{i} - 100\,\underline{j}) + 2000(600\,\underline{i} - 500\,\underline{j}) + 1500\,\underline{r}_A$$

$$1.5\,\underline{r}_A = (4.5\times712.55 - 500 - 2\times600)\,\underline{i} + (-4.5\times260.69 + 100 + 2\times500)\,\underline{j}$$

$$\underline{r}_A = (1004\,m)\,\underline{i} - (48.7\,m)\,\underline{j}$$

14.18

GIVEN: SAME AS FOR PROB. 14.17.

FIND: POINT WHERE FRAGMENT H_2 WILL BE FOUND, KNOWING THAT WRECKAGE OF PLANE WAS FOUND AT A(1200 m, 80 m) AND FRAGMENT H_1 AT H_1(400 m, −200 m).

SEE SOLUTION OF PROB. 14.17 FOR DERIVATION OF

$$\bar{\underline{r}} = (712.55\,m)\,\underline{i} - (260.69\,m)\,\underline{j} \qquad (1)$$

$$(m_H + m_A)\bar{\underline{r}} = m_{H_1}\underline{r}_{H_1} + m_{H_2}\underline{r}_{H_2} + m_A\underline{r}_A \qquad (2)$$

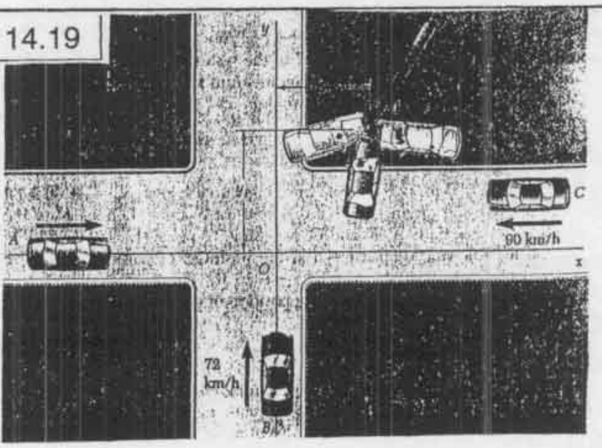
SUBSTITUTING DATA:

$$4500(712.55\,\underline{i} - 260.69\,\underline{j}) = 1000(400\,\underline{i} - 200\,\underline{j}) + 2000\,\underline{r}_{H_2} + 1500(1200\,\underline{i} + 80\,\underline{j})$$

$$2\underline{r}_{H_2} = (4.5\times712.55 - 400 - 1.5\times1200)\,\underline{i} + (-4.5\times260.69 + 200 - 1.5\times80)\,\underline{j}$$

$$\underline{r}_{H_2} = (503\,m)\,\underline{i} - (547\,m)\,\underline{j}$$

14.19



GIVEN: CARS A (1500 kg), B (1300 kg), AND C (1200 kg) WERE TRAVELING AS SHOWN WHEN CAR A HITS CAR B. AT THAT INSTANT CAR C IS AT $x_C = 10\,m$, $y_C = 3\,m$. CAR C HITS A AND B, AND ALL CARS HIT $P(x_P, y_P)$.

FIND: (a) TIME t FROM FIRST COLLISION TO STOP AT P.
(b) SPEED v_A OF CAR A

KNOWING THAT $x_P = 18\,m$, $y_P = 13.9\,m$

MOTION OF MASS CENTER

FINAL POSITION OF MASS CENTER OF SYSTEM IS THE SAME AS IF THE CARS HAD NOT COLLIDED AND HAD KEPT MOVING WITH THEIR ORIGINAL VELOCITIES.

$$(m_A + m_B + m_C)\underline{r}_P = m_A(v_A t)\underline{i} + m_B(v_B t)\underline{j} + m_C(x_C\underline{i} + y_C\underline{j} - v_C t\,\underline{i})$$

WHERE $v_B = 72\,km/h = 20\,m/s$, $v_C = 90\,km/h = 25\,m/s$

$$4000\,\underline{r}_P = 1500\,v_A t\,\underline{i} + 1300(20t)\underline{j} + 1200(10\underline{i} + 3\underline{j} - 25t\,\underline{i})$$

$$\underline{r}_P = (0.375\,v_A - 7.5)t\,\underline{i} + 3\,\underline{i} + 6.5t\,\underline{j} + 0.9\,\underline{j}$$

THUS: $x_P = (0.375\,v_A - 7.5)t + 3$, $y_P = 6.5t + 0.9$ (1)

(a) MAKING $y_P = 13.9\,m$: $13.9 = 6.5t + 0.9$

$$t = 2.00\,s$$

(b) MAKING $x_P = 18\,m$ AND $t = 2\,s$:

$$18 = (0.375\,v_A - 7.5)2 + 3 \qquad v_A = 40\,m/s = 144\,km/h$$

14.20

GIVEN: SAME AS FOR PROB. 14.19.

FIND: COORDINATES OF POLE P, KNOWING THAT $v_A = 129.6\,km/h$ AND THAT TIME FROM FIRST COLLISION TO STOP AT P IS $t = 2.4\,s$.

SEE SOLUTION OF PROB. 14.19 FOR DERIVATION OF

$$x_P = (0.375\,v_A - 7.5)t + 3, \quad y_P = 6.5t + 0.9 \qquad (1)$$

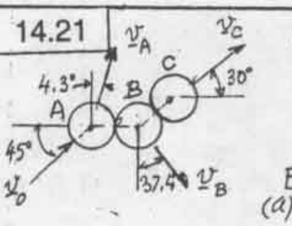
MAKING $v_A = 129.6\,km/h = 36\,m/s$ AND $t = 2.4\,s$ IN EQS. (1):

$$x_P = (0.375\times36 - 7.5)(2.4) + 3 = 17.40\,m$$

$$y_P = 6.5(2.4) + 0.9 = 16.50\,m$$

$$x_P = 17.40\,m, \; y_P = 16.50\,m$$

14.21



GIVEN: 3 BALLS OF SAME MASS. BALL A STRIKES B AND C WHICH ARE AT REST.
BEFORE IMPACT, $v_0 = 12$ ft/s
AFTER IMPACT, $v_C = 6.29$ ft/s
FIND:
(a) v_A, (b) v_B AFTER IMPACT

CONSERVATION OF LINEAR MOMENTUM

IN x DIRECTION:

$m(12\text{ ft/s})\cos 45° = m v_A \sin 4.3° + m v_B \sin 37.4° + m(6.29)\cos 30°$

$0.07498\, v_A + 0.60738\, v_B = 3.0380 \qquad (1)$

IN y DIRECTION:

$m(12\text{ ft/s})\sin 45° = m v_A \cos 4.3° - m v_B \cos 37.4° + m(6.29)\sin 30°$

$0.99719\, v_A - 0.79441\, v_B = 5.3403 \qquad (2)$

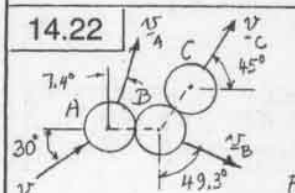
(a) MULTIPLY (1) BY 0.79441, (2) BY 0.60738, AND ADD:

$0.66524\, v_A = 5.6570 \qquad v_A = 8.50$ ft/s ◀

(b) MULTIPLY (1) BY 0.99719, (2) BY -0.07498, AND ADD:

$0.66524\, v_B = 2.6290 \qquad v_B = 3.95$ ft/s ◀

14.22



GIVEN: 3 BALLS OF SAME MASS. BALL A STRIKES B AND C WHICH ARE AT REST.
BEFORE IMPACT, $v_0 = 12$ ft/s
AFTER IMPACT, $v_C = 6.29$ ft/s.
FIND:
(a) v_A, (b) v_B AFTER IMPACT.

CONSERVATION OF LINEAR MOMENTUM

IN x DIRECTION:

$m(12\text{ ft/s})\cos 30° = m v_A \sin 7.4° + m v_B \sin 49.3° + m(6.29)\cos 45°$

$0.12880\, v_A + 0.75813\, v_B = 5.9446 \qquad (1)$

IN y DIRECTION:

$m(12\text{ ft/s})\sin 30° = m v_A \cos 7.4° - m v_B \cos 49.3° + m(6.29)\sin 45°$

$0.99167\, v_A - 0.65210\, v_B = 1.5523 \qquad (2)$

(a) MULTIPLY (1) BY 0.65210, (2) BY 0.75813, AND ADD:

$0.83581\, v_A = 5.0533 \qquad v_A = 6.05$ ft/s ◀

(b) MULTIPLY (1) BY 0.99167, (2) BY -0.12880, AND ADD:

$0.83581\, v_B = 5.6951 \qquad v_B = 6.81$ ft/s ◀

14.23

GIVEN: 3-kg BIRD FLYING 15 m ABOVE GROUND WITH $\underline{v}_B = (10\text{ m/s})\underline{i}$ IS HIT BY 50-g ARROW WITH $\underline{v}_A = (60\text{ m/s})\underline{j} + (80\text{ m/s})\underline{k}$.

FIND: DISTANCE FROM O UNDER POINT OF IMPACT TO P WHERE BIRD HITS THE GROUND.

CONSERVATION OF MOMENTUM:

$(3000\text{ g})(10\text{ m/s})\underline{i} + (50\text{ g})(60\underline{j} + 80\underline{k}) = (3050\text{ g})\,\bar{\underline{v}}$

VELOCITY OF BIRD AND ARROW AFTER IMPACT:

$\bar{\underline{v}} = (9.8361\text{ m/s})\underline{i} + (0.98361\text{ m/s})\underline{j} + (1.3115\text{ m/s})\underline{k}$

VERTICAL MOTION:

$y = y_0 + v_y t - \frac{1}{2} g t^2 \qquad$ MAKE $y = 0$:

$0 = 15\text{ m} + (0.98361\text{ m/s})t - \frac{1}{2}(9.81\text{ m/s}^2)t^2$

(CONTINUED)

14.23 continued

$t^2 - 0.20053t - 3.0581 = 0$

$t = \dfrac{0.20053 + \sqrt{(0.20053)^2 + 4(3.0581)}}{2} = 1.8519\text{ s}$

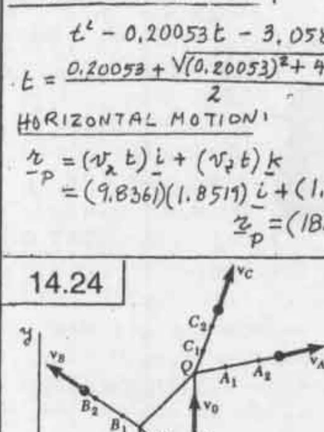
HORIZONTAL MOTION:

$\underline{r}_p = (v_x t)\underline{i} + (v_z t)\underline{k}$

$= (9.8361)(1.8519)\underline{i} + (1.3115)(1.8519)\underline{k}$

$\underline{r}_p = (18.22\text{ m})\underline{i} + (2.43\text{ m})\underline{k}$ ◀

14.24



GIVEN:
BEFORE COLLISIONS ALPHA PARTICLE A MOVED WITH $\underline{u}_0 = -(480\text{ m/s})\underline{i} + 600\underline{j} - 640\underline{k}$, NUCLEI B AND C MOVED WITH $\underline{v}_0 = (480\text{ m/s})\underline{j}$.
AFTER COLLISIONS, THEY MOVED ALONG PATHS WHERE
$A_1(240, 220, 160)$, $A_2(320, 300, 200)$
$B_1(107, 200, 170)$, $B_2(74, 270, 160)$
$C_1(200, 212, 130)$, $C_2(200, 260, 115)$

(DIMENSIONS IN mm)

FIND: SPEED OF EACH PARTICLE AFTER COLLISIONS.

MASS OF OXYGEN NUCLEUS $= m$, MASS OF α PARTICLE $= \frac{1}{4}m$

BEFORE COLLISIONS:

α PARTICLE: $\underline{u}_0 = -480\underline{i} + 600\underline{j} - 640\underline{k}$

NUCLEI B AND C: $\underline{v}_0 = 480\underline{j}$

AFTER COLLISIONS:

$\underline{v}_A = v_A \dfrac{\overline{A_1A_2}}{A_1A_2} = \dfrac{80\underline{i} + 80\underline{j} + 40\underline{k}}{120} v_A = (0.6667\underline{i} + 0.6667\underline{j} + 0.3333\underline{k})v_A$

$\underline{v}_B = v_B \dfrac{\overline{B_1B_2}}{B_1B_2} = \dfrac{-33\underline{i} + 70\underline{j} - 10\underline{k}}{78.03} v_B = (-0.4229\underline{i} + 0.8971\underline{j} - 0.12816\underline{k})v_B$

$\underline{v}_C = v_C \dfrac{\overline{C_1C_2}}{C_1C_2} = \dfrac{48\underline{j} - 15\underline{k}}{50.29} v_C = (0.9545\underline{j} - 0.2983\underline{k})$

CONSERVATION OF MOMENTUM:

$\frac{1}{4}m\underline{u}_0 + 2m\,\underline{v}_0 = \frac{1}{4}m\,\underline{v}_A + m\,\underline{v}_B + m\,\underline{v}_C$

$-120\underline{i} + 150\underline{j} - 160\underline{k} + 960\underline{j} = (0.1667\underline{i} + 0.1667\underline{j} + 0.08333\underline{k})v_A$
$(-0.4229\underline{i} + 0.8971\underline{j} - 0.12816\underline{k})v_B + (0.9545\underline{j} - 0.2983\underline{k})v_C$

EQUATING THE COEFFICIENTS OF THE UNIT VECTORS:

$0.1667\, v_A - 0.4229\, v_B = -120 \qquad (1)$

$0.1667\, v_A + 0.8971\, v_B + 0.9545\, v_C = 1110 \qquad (2)$

$0.08333\, v_A - 0.12816\, v_B - 0.2983\, v_C = -160 \qquad (3)$

MULTIPLY (2) BY 0.2983, (3) BY 0.9545 AND ADD:

$0.12926\, v_A + 0.14528\, v_B = 178.39 \qquad (4)$

MULTIPLY (1) BY 0.14528, (4) BY 0.4229 AND ADD:

$0.7888\, v_A = 58.01 \qquad v_A = 735.4$ m/s

$v_A = 735$ m/s ◀

FROM (1):

$0.1667(735.4) - 0.4229\, v_B = -120 \qquad v_B = 573.6$ m/s

$v_B = 574$ m/s ◀

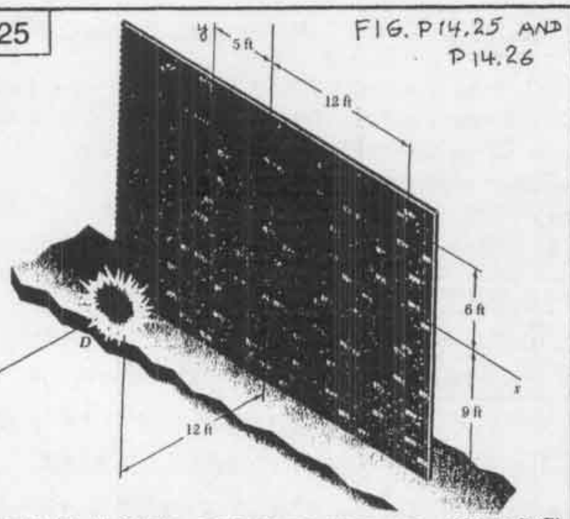
FROM (3):

$0.08333(735.4) - 0.12816(573.6) - 0.2983\, v_C = -160$

$v_C = 495.4$ m/s $\qquad v_C = 495$ m/s ◀

14.25

FIG. P14.25 AND P14.26



$\underline{\text{GIVEN}}$: 12-lb SHELL EXPLODES AT D INTO FRAGMENTS, A(5 lb), B(4 lb), AND C(3 lb), WHICH HIT WALL AS SHOWN. VELOCITY OF SHELL WAS $\underline{v}_0 = (40 \text{ ft/s})\underline{i} - (30 \text{ ft/s})\underline{j} - (1200 \text{ ft/s})\underline{k}$.

$\underline{\text{FIND}}$: SPEED OF EACH FRAGMENT.

$\underline{\text{CONSERVATION OF MOMENTUM}}$:

$$(12/g)\underline{v}_0 = (5/g)\underline{v}_A + (4/g)\underline{v}_B + (3/g)\underline{v}_C$$

$$12(40\underline{i} - 30\underline{j} - 1200\underline{k}) = 5\left(-\tfrac{5}{13}\underline{i} - \tfrac{12}{13}\underline{k}\right)v_A + 4\left(\tfrac{2}{3}\underline{i} + \tfrac{1}{3}\underline{j} - \tfrac{2}{3}\underline{k}\right)v_B + 3\left(-\tfrac{3}{5}\underline{j} - \tfrac{4}{5}\underline{k}\right)v_C$$

EQUATE COEFFICIENTS OF UNIT VECTORS:

$$(\underline{i})\ -\tfrac{25}{13}v_A + \tfrac{8}{3}v_B = 480 \quad (1)$$

$$(\underline{j})\ \tfrac{4}{3}v_B - \tfrac{9}{5}v_C = -360 \quad (2)$$

$$(\underline{k})\ -\tfrac{60}{13}v_A - \tfrac{8}{3}v_B - \tfrac{12}{5}v_C = -14{,}400 \quad (3)$$

SOLVING THESE EQUATIONS SIMULTANEOUSLY:

$v_A = 1677.64,\quad v_B = 1389.84,\quad v_C = 1229.51$

$v_A = 1678$ ft/s; $v_B = 1390$ ft/s; $v_C = 1230$ ft/s ◀

14.26

SEE FIGURE AT TOP OF PAGE

$\underline{\text{GIVEN}}$: 12-lb SHELL EXPLODES AT D INTO FRAGMENTS A(4 lb), B(3 lb), AND C(5 lb), WHICH HIT WALL AS SHOWN. VELOCITY OF SHELL WAS $\underline{v}_0 = (40 \text{ ft/s})\underline{i} - (30 \text{ ft/s})\underline{j} - (1200 \text{ ft/s})\underline{k}$.

$\underline{\text{FIND}}$: SPEED OF EACH FRAGMENT.

$\underline{\text{CONSERVATION OF MOMENTUM}}$:

$$(12/g)\underline{v}_0 = (4/g)\underline{v}_A + (3/g)\underline{v}_B + (5/g)\underline{v}_C$$

$$12(40\underline{i} - 30\underline{j} - 1200\underline{k}) = 4\left(-\tfrac{5}{13}\underline{i} - \tfrac{12}{13}\underline{k}\right)v_A + 3\left(\tfrac{2}{3}\underline{i} + \tfrac{1}{3}\underline{j} - \tfrac{2}{3}\underline{k}\right)v_B + 5\left(-\tfrac{3}{5}\underline{j} - \tfrac{4}{5}\right)\underline{k}$$

EQUATE COEFFICIENTS OF UNIT VECTORS:

$$(\underline{i})\ -\tfrac{20}{13}v_A + 2v_B = 480 \quad (1)$$

$$(\underline{j})\ v_B - 3v_C = -360 \quad (2)$$

$$(\underline{k})\ -\tfrac{48}{13}v_A - 2v_B - 4v_C = -14{,}400 \quad (3)$$

SOLVING THESE EQUATIONS SIMULTANEOUSLY:

$v_A = 2097.05,\quad v_B = 1853.11,\quad v_C = 737.705$

$v_A = 2097$ ft/s; $v_B = 1853$ ft/s; $v_C = 738$ ft/s ◀

14.27

$\underline{\text{DERIVE}}$ $\underline{H}_O = \bar{\underline{r}} \times m\bar{\underline{v}} + \underline{H}_G$, WHERE

$$\underline{H}_O = \sum(\underline{r}_i \times m_i \underline{v}_i) \quad (14.7)$$

$$\underline{H}_G = \sum(\underline{r}'_i \times m_i \underline{v}'_i) \quad (14.24)$$

AND m = TOTAL MASS OF SYSTEM, $\bar{\underline{r}}$ = POSITION VECTOR OF G; $\bar{\underline{v}}$ = VELOCITY OF G.

MAKING $\underline{r}_i = \bar{\underline{r}} + \underline{r}'_i$ IN EQ. (14.7):

$$\underline{H}_O = \sum(\bar{\underline{r}} + \underline{r}'_i) \times m_i \underline{v}_i$$

$$= \bar{\underline{r}} \times \sum m_i \underline{v}_i + \sum \underline{r}'_i \times m_i \underline{v}_i$$

BUT $\sum m_i \underline{v}_i = m\bar{\underline{v}}$

AND, BY (14.24): $\sum \underline{r}'_i \times m_i \underline{v}_i = \underline{H}_G$

THEREFORE:

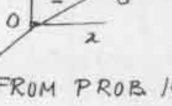
$$\underline{H}_O = \bar{\underline{r}} \times m\bar{\underline{v}} + \underline{H}_G \quad \text{(Q.E.D.)}$$ ◀

14.28

$\underline{\text{DERIVE}}$ $\sum \underline{M}_G = \dot{\underline{H}}_G \quad (14.23)$

DIRECTLY FROM $\sum \underline{M}_O = \dot{\underline{H}}_O \quad (14.11)$

BY USING EQUATION DERIVED IN PROB. 14.27.



WE REDUCE THE FORCES TO THE VECTORS SHOWN. IT FOLLOWS THAT

$$\sum \underline{M}_O = \bar{\underline{r}} \times \sum \underline{F} + \sum \underline{M}_G \quad (1)$$

FROM PROB. 14.27: $\underline{H}_O = \bar{\underline{r}} \times m\bar{\underline{v}} + \underline{H}_G$

DIFFERENTIATE: $\dot{\underline{H}}_O = \dot{\bar{\underline{r}}} \times m\bar{\underline{v}} + \bar{\underline{r}} \times m\dot{\bar{\underline{v}}} + \dot{\underline{H}}_G$

$$= \bar{\underline{v}} \times m\bar{\underline{v}} + \bar{\underline{r}} \times m\bar{\underline{a}} + \dot{H}_G$$

BUT $\bar{\underline{v}} \times m\bar{\underline{v}} = 0$ AND $m\bar{\underline{a}} = \sum \underline{F}$. THUS

$$\dot{\underline{H}}_O = \bar{\underline{r}} \times \sum \underline{F} + \dot{H}_G \quad (2)$$

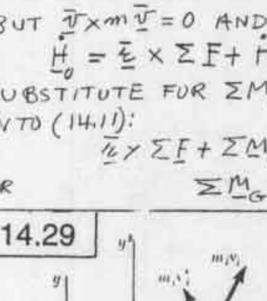
SUBSTITUTE FOR $\sum M_O$ FROM (1) AND $\dot{\underline{H}}_O$ FROM (2) INTO (14.11):

$$\bar{\underline{r}} \times \sum \underline{F} + \sum \underline{M}_G = \bar{\underline{r}} \times \sum \underline{F} + \dot{\underline{H}}_G$$

OR

$$\sum \underline{M}_G = \dot{\underline{H}}_G \quad \text{(Q.E.D.)}$$ ◀

14.29



$\underline{\text{GIVEN}}$:

NEWTONIAN FRAME $Oxyz$ AND FRAME $Ax'y'z'$ IN TRANSLATION W/R TO $Oxyz$.

LET $\underline{H}'_A = \sum \underline{r}'_i \times m_i \underline{v}'_i \quad (1)$

AND $\underline{H}_A = \sum \underline{r}'_i \times m_i \underline{v}_i \quad (2)$

WHERE $\underline{v}'_i$ AND $\underline{v}_i$ DENOTE VELOCITIES W/R $Ax'y'z'$ AND $Oxyz$, RESPECTIVELY

$\underline{\text{SHOW}}$ THAT $\underline{H}_A = \underline{H}'_A$ AT GIVEN INSTANT IF, AND ONLY IF, ONE OF THE FOLLOWING CONDITIONS IS SATISFIED AT THAT INSTANT:

(a) $\underline{v}_A = 0$ WITH RESPECT TO $Oxyz$,

(b) A COINCIDES WITH MASS CENTER G OF SYSTEM OF PARTICLES,

(c) $\underline{v}_A$ IS DIRECTED ALONG AG.

(CONTINUED)

14.29 continued

WE RECALL:

$$\underline{H}'_A = \Sigma \underline{r}'_i \times m_i \underline{v}'_i \qquad (1)$$

$$\underline{H}_A = \Sigma \underline{r}'_i \times m_i \underline{v}_i \qquad (2)$$

LET $\underline{v}_i = \underline{v}_A + \underline{v}'_i$ IN EQ. (2):

$$\underline{H}_A = \Sigma \underline{r}'_i \times m_i(\underline{v}_A + \underline{v}'_i) = (\Sigma m_i \underline{r}'_i) \times \underline{v}_A + \Sigma \underline{r}'_i \times m_i \underline{v}'_i$$

BUT, BY (14.12): $\Sigma m_i \underline{r}'_i = m\bar{\underline{r}}' = m\overrightarrow{AG}$

RECALLING EQ. (1), WE WRITE

$$\underline{H}_A = m\overrightarrow{AG} \times \underline{v}_A + \underline{H}'_A$$

THIS EQUATION REDUCES TO $\underline{H}_A = H'_A$ IF

(a) $\underline{v}_A = 0$, (b) $A \equiv G$, (c) $\underline{v}_A \parallel \overrightarrow{AG}$ (Q.E.D)

14.30

GIVEN: FRAME $Ax'y'z'$ IN TRANSLATION WITH RESPECT TO NEWTONIAN FRAME $Oxyz$.

LET $\underline{H}'_A = \Sigma \underline{r}'_i \times m_i \underline{v}'_i \qquad (1)$

WHERE $\underline{r}'_i$ AND $\underline{v}'_i$ ARE DEFINED W/R FRAME $Ax'y'z'$

AND LET $\Sigma \underline{M}_A$ BE THE SUM OF THE MOMENTS OF THE EXTERNAL FORCES ABOUT A.

SHOW THAT THE RELATION $\Sigma \underline{M}_A = \dot{\underline{H}}_A$ IS VALID IF, AND ONLY IF, ONE OF THE FOLLOWING CONDITIONS IS SATISFIED:

(a) $Ax'y'z'$ IS A NEWTONIAN FRAME OF REFERENCE,

(b) A COINCIDES WITH MASS CENTER G OF SYSTEM OF PARTICLES,

(c) $\underline{a}_A$ IS DIRECTED ALONG AG

DIFFERENTIATE EQ. (1):

$$\dot{\underline{H}}'_A = \Sigma \dot{\underline{r}}'_i \times m_i \underline{v}'_i + \Sigma \underline{r}'_i \times m_i \dot{\underline{v}}'_i$$

$$= \Sigma \underline{v}'_i \times m_i \underline{v}'_i + \Sigma \underline{r}'_i \times m_i \underline{a}'_i$$

BUT $\underline{v}'_i \times \underline{v}'_i = 0$ AND $\underline{a}'_i = \underline{a}_i - \underline{a}_A$

THUS: $\dot{\underline{H}}'_A = \Sigma(\underline{r}'_i \times m_i \underline{a}_i) - (\Sigma m_i \underline{r}'_i) \times \underline{a}_A$

BUT, BY (14.12): $\Sigma m_i \underline{r}'_i = m\bar{\underline{r}}' = m\overrightarrow{AG}$

AND, SINCE $\underline{a}_i$ IS ACCELERATION W/R NEWTONIAN FRAME, WE HAVE, BY EQ. (14.5),

$$\Sigma(\underline{r}'_i \times m_i \underline{a}_i) = \Sigma(\underline{r}'_i \times \underline{F}_i) = \Sigma \underline{M}_A$$

THEREFORE:

$$\dot{\underline{H}}_A = \Sigma \underline{M}_A - m\overrightarrow{AG} \times \underline{a}_A$$

THIS EQUATION REDUCES TO $\dot{\underline{H}}_A = \Sigma \underline{M}_A$ IF

(a) $\underline{a}_A = 0$; FRAME $Ax'y'z'$ IS IN UNIFORM TRANSLATION W/R NEWTONIAN FRAME $Oxyz$ AND IS ITSELF A NEWTONIAN FRAME,

(b) $\overrightarrow{AG} = 0$; A COINCIDES WITH G,

(c) $\overrightarrow{AG} \times \underline{a}_A = 0$; $\underline{a}_A$ IS DIRECTED ALONG AG

(Q.E.D)

14.31

GIVEN: REFERRING TO PROB. 14.1, ASSUME THAT

(1) 15-kg SUITCASE FIRST TOSSED WITH $v = 3$ m/s →

(2) 20-kg SUITCASE THEN TOSSED WITH $v = 2$ m/s →

(3) 25-kg CARRIER INITIALLY AT REST.

FIND: ENERGY LOST AS

(a) FIRST SUITCASE HITS CARRIER

(b) SECOND SUITCASE HITS CARRIER

(a) BEFORE FIRST SUITCASE HITS CARRIER:

$$T_0 = \tfrac{1}{2} m v_0^2 = \tfrac{1}{2}(15\text{ kg})(3\text{ m/s})^2 = 67.50\text{ J}$$

FIRST IMPACT: CONSERVATION OF MOMENTUM

$$(15\text{ kg})(3\text{ m/s}) = (25+15)v_1 \qquad v_1 = 1.125\text{ m/s}$$

$$T_1 = \tfrac{1}{2}(25\text{ kg} + 15\text{ kg})(1.125\text{ m/s})^2 = 25.313\text{ J}$$

$$\text{EN. LOST} = T_0 - T_1 = 67.50\text{ J} - 25.313\text{ J} = 42.2\text{ J}$$

(b) JUST BEFORE SECOND SUITCASE HITS:

$$T'_1 = T_1 + \tfrac{1}{2}(20\text{ kg})(2\text{ m/s})^2 = 25.313\text{ J} + 40\text{ J} = 65.313\text{ J}$$

SECOND IMPACT: CONSERVATION OF MOMENTUM

$$(25\text{ kg} + 15\text{ kg})(1.125\text{ m/s}) + (20\text{ kg})(2\text{ m/s}) = (60\text{ kg})v_2$$

$$v_2 = 1.4167\text{ m/s}$$

$$T_2 = \tfrac{1}{2}(60\text{ kg})(1.4167\text{ m/s})^2 = 60.208\text{ J}$$

$$\text{EN. LOST} = T'_1 - T_2 = 65.313\text{ J} - 60.208\text{ J} = 5.10\text{ J}$$

14.32

GIVEN: COLLISIONS DESCRIBED IN PROB. 14.5. WE RECALL THAT INITIAL VELOCITY OF CAR A WAS $v_0 = 1.920$ m/s

AFTER A HITS B: $(v_B)_1 = 1.680$ m/s

AFTER B HITS C: $(v_B)_2 = 0.210$ m/s

AFTER A AGAIN HITS B: $(v_B)_3 = 0.23625$ m/s

MASS OF EACH CAR = 1500 kg

FIND: ENERGY LOST AFTER ALL COLLISIONS HAVE TAKEN PLACE.

FROM SOLUTION OF PROB. 14.5 WE HAVE THE FOLLOWING FINAL VELOCITIES:

$$v_A = 0.21375\text{ m/s}, \quad v_B = 0.23625\text{ m/s}, \quad v_C = 1.470\text{ m/s}$$

INITIAL ENERGY:

$$T_0 = \tfrac{1}{2} m v_0^2 = \tfrac{1}{2}(1500\text{ kg})(1.920\text{ m/s})^2 = 2764.8\text{ J}$$

FINAL ENERGY:

$$T_f = \tfrac{1}{2} m v_A^2 + \tfrac{1}{2} m v_B^2 + \tfrac{1}{2} m v_C^2 = \tfrac{1}{2} m (v_A^2 + v_B^2 + v_C^2)$$

$$= \tfrac{1}{2}(1500\text{ kg})[(0.21375\text{ m/s})^2 + (0.23625\text{ m/s})^2 + (1.470\text{ m/s})^2]$$

$$= 1696.8\text{ J}$$

ENERGY LOST

$$= T_0 - T_f = 2764.8\text{ J} - 1696.8\text{ J} = 1068\text{ J}$$

14.33 GIVEN:
180-lb MAN AND 120-lb WOMAN OF PROB. 14.3 JUMP FROM SAME END OF 300-lb BOAT WITH VELOCITY OF 16 ft/s WITH RESPECT TO BOAT.
FIND:
WORK DONE BY WOMAN AND BY MAN IF WOMAN DIVES FIRST.

TOTAL K.E. AFTER WOMAN DIVES
FROM PART a OF SOLUTION OF PROB. 14.3:
VEL. OF BOAT $= (v_B)_1 = 3.20$ ft/s
THUS, VEL. OF WOMAN $= (v_W)_1 = 16 - 3.20 = 12.8$ ft/s

$$K.E. = T_1 = \frac{1}{2} m_W (v_W)_1^2 + \frac{1}{2}(m_B + m_M)(v_B)_1^2$$

$$T_1 = \frac{1}{2}\frac{120}{32.2}(12.8)^2 + \frac{1}{2}\frac{480}{32.2}(3.20)^2 = 381.61 \text{ ft}\cdot\text{lb}$$

WORK OF WOMAN $= T_1 = 382$ ft·lb ◀

TOTAL K.E. AFTER MAN DIVE
FROM ANSWER TO PART a OF PROB. 14.3:
VEL. OF BOAT $= (v_B)_2 = 9.20$ ft/s
THUS, VEL. OF MAN $= (v_M)_2 = 16 - 9.20 = 6.80$ ft/s

$$K.E. = T_2 = \frac{1}{2} m_W (v_W)_1^2 + \frac{1}{2} m_M (v_M)_2^2 + \frac{1}{2} m_B (v_B)_2^2$$

$$= \frac{1}{2}\frac{120}{32.2}(12.8)^2 + \frac{1}{2}\frac{180}{32.2}(6.80)^2 + \frac{1}{2}\frac{300}{32.2}(9.20)^2$$

$$T_2 = 828.82 \text{ ft}\cdot\text{lb}$$

WORK OF MAN $= T_2 - T_1 = 828.82 - 381.61 = 447$ ft·lb ◀

14.34 GIVEN:
BULLET OF PROB. 14.7 FIRED WITH $v_0 = 1500$ ft/s THROUGH 6-lb BLOCK A BECOMES EMBEDDED IN 4.95-lb BLOCK B. BLOCKS MOVE WITH $v_A = 5$ ft/s AND $v_B = 9$ ft/s.
FIND:
ENERGY LOST AS BULLET
(a) PASSES THROUGH BLOCK A
(b) BECOMES EMBEDDED IN BLOCK B

FROM ANSWERS TO PROB. 14.7:
WEIGHT OF BULLET $= W = 0.800$ oz $= 0.0500$ lb
VEL. OF BULLET BETWEEN BLOCKS $= v_1 = 900$ ft/s

(a) ENERGY LOST AS BULLET PASSES THROUGH A

$$\text{INITIAL K.E.} = T_0 = \frac{1}{2}\frac{W}{g}v_0^2 = \frac{1}{2}\frac{0.0500 \text{ lb}}{32.2 \text{ ft/s}^2}(1500 \text{ ft/s})^2$$

$$T_0 = 1746.89 \text{ ft}\cdot\text{lb}$$

K.E. OF SYSTEM AFTER BULLET PASSES THROUGH A:

$$= T_1 = \frac{1}{2}\frac{W}{g}v_1^2 + \frac{1}{2}\frac{W_A}{g}v_A^2 = \frac{1}{2}\frac{0.0500}{32.2}(900)^2 + \frac{1}{2}\frac{6}{32.2}(5)^2$$

$$T_1 = 628.88 + 2.33 = 631.21 \text{ ft}\cdot\text{lb}$$

EN. LOST $= T_0 - T_1 = 1746.89 - 631.21 = 1116$ ft·lb ◀

(b) ENERGY LOST AS BULLET BECOMES EMBEDDED IN B

$$\text{FINAL K.E.} = T_2 = \frac{1}{2}\frac{W_A}{g}v_A^2 + \frac{1}{2}\frac{(W_B + w)}{g}v_B^2$$

$$T_2 = \frac{1}{2}\frac{6}{32.2}(5)^2 + \frac{1}{2}\frac{4.95 + 0.05}{32.2}(9)^2 = 8.618 \text{ ft}\cdot\text{lb}$$

EN. LOST $= T_1 - T_2 = 631.21 - 8.618 = 623$ ft·lb ◀

14.35 GIVEN: AUTOMOBILE A, OF MASS m_A, COLLIDES WITH AUTOMOBILE B OF MASS m_B.
WE ASSUME PLASTIC IMPACT AND THAT ENERGY ABSORBED BY EACH AUTOMOBILE EQUALS ITS K.E. WITH RESPECT TO MOVING FRAME ATTACHED TO MASS CENTER OF SYSTEM.

(a) SHOW THAT $E_A/E_B = m_B/m_A$, WHERE E_A AND E_B ARE ENERGIES ABSORBED BY A AND B.
(b) FIND E_A AND E_B IF $m_A = 1600$ kg, $m_B = 900$ kg, $v_A = 90$ km/h, $v_B = 60$ km/h.

BEFORE COLLISION: VELOCITY $\bar{v}$ OF MASS CENTER G:

$$(m_A + m_B)\bar{v} = m_A v_A + m_B v_B \qquad \bar{v} = \frac{m_A v_A - m_B v_B}{m_A + m_B}$$

MOTION OF AUTOS RELATIVE TO G:

$$v_{A/G} = v_A - \bar{v} = v_A - \frac{m_A v_A - m_B v_B}{m_A + m_B} = \frac{m_A v_A + m_B v_A - m_A v_A + m_B v_B}{m_A + m_B}$$

$$v_{A/G} = \frac{m_B}{m_A + m_B}(v_A + v_B) \qquad \text{similarly: } v_{B/G} = -\frac{m_A}{m_A + m_B}(v_A + v_B)$$

$$T_{A/G} = \frac{1}{2} m_A v_{A/G}^2 = \frac{1}{2}\frac{m_A m_B^2}{(m_A + m_B)^2}(v_A + v_B)^2 \qquad (1)$$

$$T_{B/G} = \frac{1}{2} m_B v_{B/G}^2 = \frac{1}{2}\frac{m_A^2 m_B}{(m_A + m_B)^2}(v_A + v_B)^2 \qquad (2)$$

AFTER COLLISION.
SINCE THERE IS NO EXTERNAL FORCE, G KEEPS MOVING WITH VELOCITY $\bar{v}$.
SINCE IMPACT IS PLASTIC: $v_A' = v_B' = \bar{v}$
AND $v_{A/G}' = v_{B/G}' = 0$. THUS: $T_{A/G}' = T_{B/G}' = 0$
IT FOLLOWS THAT $E_A = T_{A/G}$ AND $E_B = T_{B/G}$
(a) DIVIDING (1) BY (2):

$$\frac{E_A}{E_B} = \frac{T_{A/G}}{T_{B/G}} = \frac{m_B}{m_A} \qquad \text{(Q.E.D)}$$ ◀

(b) SUBSTITUTING IN (1) AND (2) THE GIVEN DATA,
$m_A = 1600$ kg, $m_B = 900$ kg, $v_A = 90$ km/h $= 25$ m/s, $v_B = 60$ km/h $= 16.67$ m/s
WE FIND $E_A = 180.0$ kJ, $E_B = 320$ kJ ◀

14.36 GIVEN: CAR COLLISION OF PROB. 14.35
DEFINE: SEVERITY OF A COLLISION $= E/E_0$
WHERE E = ENERGY ABSORBED BY CAR IN COLLISION, AND E_0 = EN. ABSORBED BY SAME CAR IN A TEST WHERE IT HITS AN IMMOVABLE WALL WITH VELOC. v_0
SHOW THAT COLLISION OF PROB. 14.35 IS $(m_A/m_B)^2$ TIMES MORE SEVERE FOR CAR B THAN FOR CAR A.

ENERGIES ABSORBED IN TESTS OF A AND B:

$$(E_A)_0 = \frac{1}{2} m_A v_0^2 \qquad (E_B)_0 = \frac{1}{2} m_B v_0^2 \qquad (3)$$

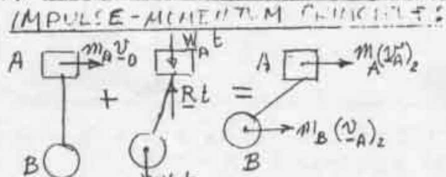
SEVERITY OF COLLISION FOR CAR A $= E_A/(E_A)_0$
SEVERITY OF COLLISION FOR CAR B $= E_B/(E_B)_0$
RECALLING EQS. (3) AND FROM PROB. 14.35 THAT $E_A/E_B = m_B/m_A$, WE HAVE

$$\frac{\text{SEVERITY OF COLL. FOR B}}{\text{SEVERITY OF COLL. FOR A}} = \frac{E_B (E_A)_0}{E_A (E_B)_0} = \frac{m_A}{m_B}\,\frac{\frac{1}{2} m_A v_0^2}{\frac{1}{2} m_B v_0^2} = \left(\frac{m_A}{m_B}\right)^2 \qquad \text{(Q.E.D.)}$$ ◀

14.37 SOLVE SAMPLE PROB. 14.4, ASSUMING THAT CART A IS GIVEN VELOCITY $\underline{v}_0 \rightarrow$ AND THAT BALL B IS AT REST.

(a) VELOCITY OF B AT MAXIMUM ELEVATION

IMPULSE-MOMENTUM PRINCIPLE:



WE NOTE THAT WHEN B REACHES MAXIMUM HEIGHT $(\underline{v}_B)_2 = (\underline{v}_A)_2$

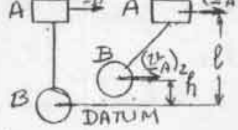
$\Sigma m \underline{v}_1 + \Sigma \text{Ext Imp}_{1\to 2} = \Sigma m \underline{v}_2$

$\xrightarrow{+} x$ COMP.: $m_A v_0 = (m_A + m_B)(v_A)_2$

$$(\underline{v}_B)_2 = (\underline{v}_A)_2 = \frac{m_A}{m_A + m_B} v_0 \rightarrow (1)$$

(b) MAXIMUM HEIGHT REACHED BY B

CONSERVATION OF ENERGY:



$V_1 = m_A g \ell \quad T_1 = \frac{1}{2} m_A v_0^2$

$V_2 = m_A g \ell + m_B g h$

$T_2 = \frac{1}{2}(m_A + m_B)(v_A)_2^2$

$T_1 + V_1 = T_2 + V_2$:

$\frac{1}{2} m_A v_0^2 + \cancel{m_A g \ell} = \frac{1}{2}(m_A + m_B)(v_A)_2^2 + \cancel{m_A g \ell} + m_B g h$

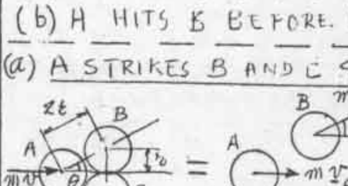
SUBSTITUTING FOR $(v_A)_2$ FROM (1):

$\frac{1}{2} m_A v_0^2 = \frac{1}{2} \frac{m_A^2}{m_A + m_B} v_0^2 + m_B g h$

$h = \frac{v_0^2}{g} \frac{m_A^2 + m_A m_B - m_A^2}{(m_A + m_B) m_B}$, $\quad h = \frac{m_A}{m_A + m_B} \frac{v_0^2}{2g}$

(SAME ANSWER AS FOR PART b OF SP 14.4)

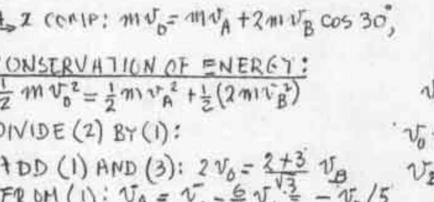
14.38



GIVEN: BALL A HITS WITH $\underline{v}_0$ BALLS B AND C WHICH ARE AT REST. ASSUME CONSERVATION OF ENERGY.

FIND: FINAL VELOCITY OF EACH BALL, IF (a) A STRIKES B AND C SIMULTANEOUSLY, (b) A HITS B BEFORE IT HITS C

(a) A STRIKES B AND C SIMULTANEOUSLY



$\sin\theta = \frac{r}{2r} = 0.5$

$\theta = 30°$

CONSERVATION OF MOMENTUM

$+\uparrow y$ COMP:

$0 = m v_B \sin 30° - m v_C \sin 30°$

$v_C = v_B$

$\xrightarrow{+} x$ COMP: $m v_0 = m v_A + 2 m v_B \cos 30°$, $\quad v_0 - v_A = v_B \sqrt{3} \quad (1)$

CONSERVATION OF ENERGY:

$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2}(2 m v_B^2) \qquad v_0^2 - v_A^2 = 2 v_B^2 \quad (2)$

DIVIDE (2) BY (1): $\qquad v_0 + v_A = \frac{2}{\sqrt{3}} v_B \quad (3)$

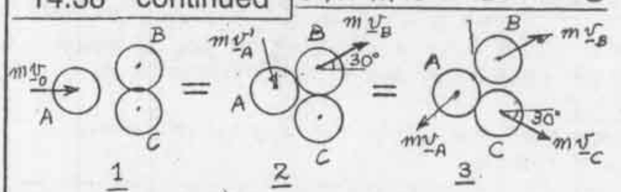
ADD (1) AND (3): $2 v_0 = \frac{2+3}{\sqrt{3}} v_B \qquad v_B = \frac{2}{5}\sqrt{3}\, v_0$

FROM (1): $v_A = v_0 - \frac{6}{5} v_0 = -v_0/5$

$v_A = 0.200 v_0 \leftarrow$; $\underline{v}_B = 0.693 v_0 \measuredangle 30°$; $\underline{v}_C = 0.693 v_0 \measuredangle 30°$

(CONTINUED)

14.38 continued (b) A HITS B BEFORE C



CONS. OF MOMENTUM FROM 1 TO 2:

$\xrightarrow{+} x$ COMP.: $m v_0 = m (v'_A)_x + m v_B \cos 30° \qquad (v'_A)_x = v_0 - v_B \cos 30° \quad (4)$

$+\uparrow y$ COMP.: $0 = m (v'_A)_y + m v_B \sin 30° \qquad (v'_A)_y = v_B \sin 30° \quad (5)$

SQUARE BOTH MEMBERS OF (4) AND (5) AND ADD:

$$v_A'^2 = v_0^2 - 2 v_0 v_B \cos 30° + v_B^2 \quad (6)$$

CONS. OF ENERGY FROM 1 TO 2:

$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A'^2 + \frac{1}{2} m v_B^2 \qquad v_A'^2 = v_0^2 - v_B^2$

CARRYING INTO (6) AND SOLVING FOR v_B: $\quad v_B = v_0 \cos 30° \quad (7)$

CONS. OF MOMENTUM FROM 1 TO 3:

$\xrightarrow{+} x$ COMP.: $m v_0 = m (v_A)_x + m v_B \cos 30° + m v_C \cos 30°$

$+\uparrow y$ COMP.: $0 = m (v_A)_y + m v_B \sin 30° - m v_C \sin 30°$

SUBSTITUTE FOR v_B FROM (7) AND SOLVE FOR $(v_A)_x$ AND $(v_A)_y$

$(v_A)_x = v_0 \sin^2 30° - v_C \cos 30° \quad (8)$

$(v_A)_y = -v_0 \sin 30° \cos 30° + v_C \sin 30° \quad (9)$

SQUARING AND ADDING: $v_A^2 = 0.25 v_0^2 - 0.866 v_0 v_C + v_C^2 \quad (10)$

CONS. OF ENERGY FROM 1 TO 3:

$\frac{1}{2} m v_1^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2 \qquad v_0^2 = v_A^2 + v_B^2 + v_C^2 \quad (11)$

SUBSTITUTE FOR v_A AND v_B FROM (10) AND (7) INTO (11):

$\cancel{v_0^2} = 0.\cancel{25} v_0^2 - 0.866 v_0 v_C + v_C^2 + 0.\cancel{75} v_0^2 + v_C^2, \quad v_C = 0.433 v_0$

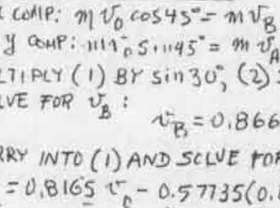
CARRYING INTO (8) AND (9):

$(v_A)_x = 0.25 v_0 - 0.433 \cos 30° = -0.1250 v_0$

$(v_A)_y = -0.433 v_0 + 0.433 v_0 \sin 30° = -0.2165 v_0$

THUS: $\underline{v}_A = 0.250 v_0 \measuredangle 60°$; $\underline{v}_B = 0.866 v_0 \measuredangle 30°$; $\underline{v}_C = 0.433 v_0 \measuredangle 30°$

14.39



GIVEN: A HITS B WITH $v_0 = 15$ ft/s. ASSUME CONS. OF ENERGY

FIND: MAGNITUDES OF $\underline{v}_A$, $\underline{v}_B$, AND $\underline{v}_C$.

CONS. OF MOMENTUM:

$\xrightarrow{+} x$ COMP: $m v_0 \cos 45° = m v_B \sin 30° + m v_C \cos 30° \quad (1)$

$+\uparrow y$ COMP: $m v_0 \sin 45° = m v_A - m v_B \cos 30° + m v_C \sin 30° \quad (2)$

MULTIPLY (1) BY $\sin 30°$, (2) BY $\cos 30°$, SUBTRACT, AND SOLVE FOR v_B:

$$v_B = 0.8660 v_A - 0.2588 v_0 \quad (3)$$

CARRY INTO (1) AND SOLVE FOR v_C:

$v_C = 0.8165 v_0 - 0.57735(0.8660 v_A - 0.2588 v_0)$

$v_C = -0.5 v_A + 0.9659 v_0 \quad (4)$

CONS. OF ENERGY:

$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2, \quad v_0^2 = v_A^2 + v_B^2 + v_C^2$

SUBSTITUTE FOR v_B AND v_C FROM (3) AND (4):

$v_0^2 = v_A^2 + (0.8660 v_A - 0.2588 v_0)^2 + (-0.5 v_A + 0.9659 v_0)^2$

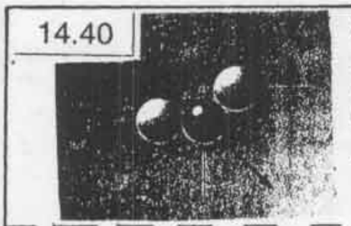
$2 v_A^2 - 1.4141 v_0 v_A = 0 \qquad v_A = 0.7071 v_0$

FROM (3) AND (4): $v_B = 0.3536 v_0$, $v_C = 0.6124 v_0$

GIVEN DATA: $v_0 = 15$ ft/s. THEREFORE:

$v_A = 10.61$ ft/s; $v_B = 5.30$ ft/s; $v_C = 9.19$ ft/s

14.40



GIVEN:
A HITS B WITH $v_0 = 15$ ft/s
ASSUME CONSERVATION OF ENERGY.
FIND:
MAGNITUDES OF $\underline{v}_A$, $\underline{v}_B$, AND $\underline{v}_C$.

CONS. OF MOMENTUM:

$\xrightarrow{+}$ x COMP.: $m v_0 \cos 30^\circ = m v_B \sin 45^\circ + m v_C \cos 45^\circ$ (1)

$+\uparrow$ y COMP.: $m v_0 \sin 30^\circ = m v_A - m v_B \cos 45^\circ + m v_C \sin 45^\circ$ (2)

SUBTRACT (2) FROM (1) AND DIVIDE BY m:

$0.3660\, v_0 = -v_A + 1.4142\, v_B \qquad v_B = 0.7071\, v_A + 0.2588\, v_0$ (3)

ADD (1) AND (2) AND DIVIDE BY m:

$1.3660\, v_0 = v_A + 1.4142\, v_C \qquad v_C = -0.7071\, v_A + 0.9659\, v_0$ (4)

CONS. OF ENERGY:

$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2 \qquad v_0^2 = v_A^2 + v_B^2 + v_C^2$

SUBSTITUTE FOR v_B AND v_C FROM (3) AND (4):

$v_0^2 = v_A^2 + (0.7071 v_A + 0.2588 v_0)^2 + (-0.7071 v_A + 0.9659 v_0)^2$

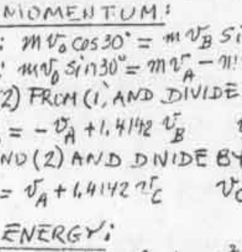
$2 v_A^2 - v_0 v_A = 0 \qquad v_A = 0.5 v_0$

FROM (3) AND (4): $v_B = 0.6124\, v_0$, $v_C = 0.6124\, v_0$

GIVEN DATA: $v_0 = 15$ ft/s. THEREFORE:

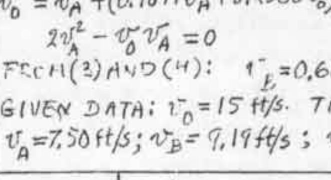
$v_A = 7.50$ ft/s; $v_B = 9.19$ ft/s; $v_C = 9.19$ ft/s ◀

14.41



GIVEN: $v_0 = 8$ m/s.
POTENTIAL ENERGY OF SPRING = 120 J.
CORD CUT WHEN $\theta = 30^\circ$.
FIND:
$\underline{v}_A$ AND $\underline{v}_B$ AFTER CORD IS CUT.

CONS. OF LINEAR MOM. W/R FRAME $Gx'y'z'$



θ COMPONENTS:

$0 = m_A v'_A - m_B v'_B$

$v'_B = \frac{m_A}{m_B} v'_A = \frac{2.5}{1.5} v'_A$

$v'_B = \frac{5}{3} v'_A$ (1)

CONS. OF ENERGY W/R FRAME $Gx'y'z'$:

$120\,\text{J} = \frac{1}{2}(2.5) v_A'^2 + \frac{1}{2}(1.5) v_B'^2 \qquad 5 v_A'^2 + 3 v_B'^2 = 480$ (2)

SUBSTITUTE FOR v'_B FROM (1) INTO (2):

$5 v_A'^2 + 3\left(\frac{5}{3} v'_A\right)^2 = 480 \qquad v_A'^2 = 36 \qquad \underline{v}'_A = 6$ m/s $\measuredangle\theta$

FROM (1): $v'_B = \frac{5}{3}(6\text{ m/s}) \qquad \underline{v}'_B = 10$ m/s $\measuredangle\theta$

WITH RESPECT TO FIXED FRAME $Oxyz$:

$\underline{v}_A = \underline{v}_0 + \underline{v}'_A = 8\text{ m/s} \rightarrow + 6\text{ m/s} \measuredangle\theta$ (3)

$\underline{v}_B = \underline{v}_0 + \underline{v}'_B = 8\text{ m/s} \rightarrow + 10\text{ m/s} \measuredangle\theta$ (4)

FOR $\theta = 30^\circ$: EQ. (3):

$\xrightarrow{+}$ x COMP.: $(v_A)_x = 8 - 6\cos 30^\circ = 2.804$ m/s

$+\uparrow$ y COMP.: $(v_A)_y = 6 \sin 30^\circ = 3$ m/s

$\underline{v}_A = 4.11$ m/s $\measuredangle 46.9^\circ$ ◀

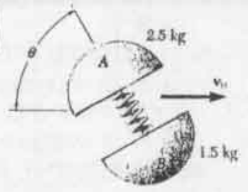
EQ. (4):

$\xrightarrow{+}$ x COMP.: $(v_B)_x = 8 + 10\cos 30^\circ = 16.660$ m/s

$+\uparrow$ y COMP.: $(v_B)_y = -10 \sin 30^\circ = -5$ m/s

$\underline{v}_B = 17.39$ m/s $\measuredangle 16.7^\circ$ ◀

14.42



GIVEN: $v_0 = 8$ m/s.
POTENTIAL ENERGY OF SPRING = 120 J.
CORD CUT WHEN $\theta = 120^\circ$.
FIND:
$\underline{v}_A$ AND $\underline{v}_B$ AFTER THE CORD IS CUT.

SEE SOLUTION OF PROB. 14.41 FOR DERIVATION OF EQS. (3) AND (4). WITH $\theta = 120^\circ$, WE HAVE

$\underline{v}_A = \underline{v}_0 + \underline{v}'_A = 8\text{ m/s} + 6\text{ m/s} \measuredangle 60^\circ$ (3')

$\underline{v}_B = \underline{v}_0 + \underline{v}'_B = 8\text{ m/s} + 10\text{ m/s} \measuredangle 60^\circ$ (4')

EQ. (3'):

$\xrightarrow{+}$ x COMP: $(v_A)_x = 8 + 6\cos 60^\circ = 8 + 3 = 11$ m/s

$+\uparrow$ y COMP: $(v_A)_y = 6 \sin 60^\circ = 5.196$ m/s

$\underline{v}_A = 12.17$ m/s $\measuredangle 25.3^\circ$ ◀

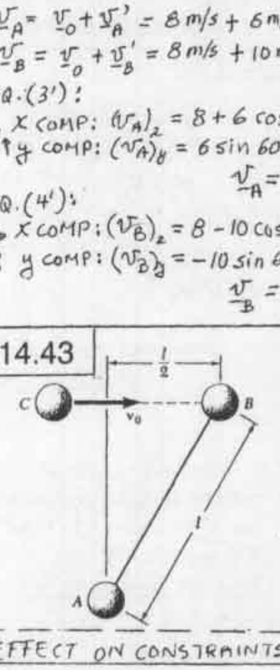
EQ. (4'):

$\xrightarrow{+}$ x COMP: $(v_B)_x = 8 - 10\cos 60^\circ = 8 - 5 = 3$ m/s

$+\uparrow$ y COMP: $(v_B)_y = -10 \sin 60^\circ = -8.660$ m/s

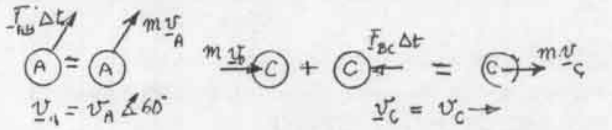
$\underline{v}_B = 9.17$ m/s $\measuredangle 70.9^\circ$ ◀

14.43



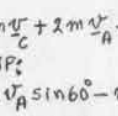
GIVEN:
THREE SPHERES, EACH OF MASS m.
A AND B ARE CONNECTED BY TAUT, INEXTENSIBLE CORD.
C STRIKES B AS SHOWN.
ASSUME CONS. OF ENERGY.
FIND:
VELOCITY OF EACH SPHERE AFTER IMPACT.

EFFECT ON CONSTRAINTS ON FINAL VELOCITIES



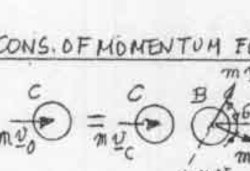
$\underline{v}_A = v_A \measuredangle 60^\circ \qquad \underline{v}_C = v_C \rightarrow$

BECAUSE CORD AB IS INEXTENSIBLE, COMPONENT OF $\underline{v}_B$ ALONG AB MUST BE EQUAL TO $\underline{v}_A$.



$\underline{v}_B = v_A \measuredangle 60^\circ + v_{B/A} \measuredangle 30^\circ$ (1)

CONS. OF MOMENTUM FOR SYSTEM:



$m\underline{v}_0 = m\underline{v}_C + 2m\underline{v}_A + m\underline{v}_{B/A}$

$+\uparrow$ y COMP:

$0 = 2m v_A \sin 60^\circ - m v_{B/A} \sin 30^\circ$

$v_{B/A} = 2\sqrt{3}\, v_A$ (2)

$\xrightarrow{+}$ x COMP:

$m v_0 = m v_C + 2m v_A \cos 60^\circ + m v_{B/A} \cos 30^\circ$

DIVIDING BY m AND SUBSTITUTING FOR $v_{B/A}$ FROM (2):

$v_0 = v_C + v_A + \frac{\sqrt{3}}{2}(2\sqrt{3} v_A) \qquad v_C = v_0 - 4 v_A$ (3)

CONS. OF ENERGY:

$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2, \qquad v_0^2 = 2 v_A^2 + v_{B/A}^2 + v_C^2$ (4)

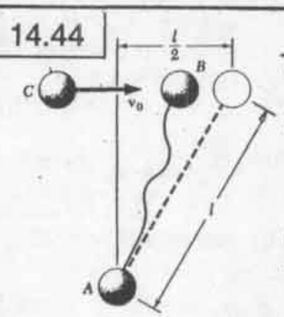
SUBSTITUTE FOR $v_{B/A}$ AND v_C FROM (2) AND (3) INTO (4):

$\cancel{v_0^2} = 2 v_A^2 + 12 v_A^2 + \cancel{v_0^2} - 8 v_0 v_A + 16 v_A^2 \qquad \underline{v}_A = \frac{4}{15} v_0 \measuredangle 60^\circ$ ◀

FROM (3): $v_C = v_0 - \frac{16}{15} v_0 = -\frac{1}{15} v_0 \qquad \underline{v}_C = \frac{1}{15} v_0 \leftarrow$ ◀

FROM (1) AND (2): $\underline{v}_B = \frac{4}{15} v_0 \measuredangle 60^\circ + \frac{8\sqrt{3}}{15} v_0 \measuredangle 30^\circ$, $\underline{v}_B = 0.961 v_0 \measuredangle 13.9^\circ$ ◀

14.44

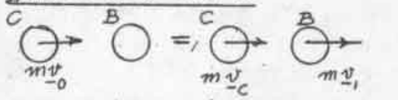


GIVEN:
THREE SPHERES, EACH OF MASS m. A AND B ARE CONNECTED BY INEXTENSIBLE CORD WHICH IS SLACK. C STRIKES B AS SHOWN WITH PERFECTLY ELASTIC IMPACT.

FIND:
(a) VELOCITY OF EACH SPHERE AFTER CORD BECOMES TAUT.
(b) FRACTION OF INITIAL K.E. LOST WHEN CORD BECOMES TAUT.

(a) DETERMINATION OF VELOCITIES

IMPACT OF C AND B



CONS. OF MOMENTUM:
$m\underline{v}_0 = m\underline{v}_C + m\underline{v}_1$
$v_C + v_1 = v_0$ (1)

CONS. OF ENERGY (PERFECTLY ELASTIC IMPACT):

$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_C^2 + \frac{1}{2} m v_1^2 \qquad v_C^2 + v_1^2 = v_0^2$ (2)

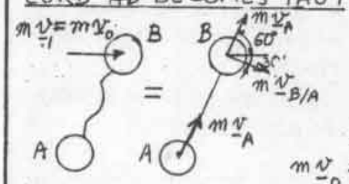
SQUARE (1): $v_C^2 + 2 v_C v_1 + v_1^2 = v_0^2$

SUBTRACT (2): $2 v_C v_1 = 0$

$v_1 = 0$ CORRESPONDS TO INITIAL CONDITIONS AND SHOULD BE ELIMINATED. THEREFORE $\underline{v}_C = 0$ ◀

FROM (1): $v_1 = v_0$

CORD AB BECOMES TAUT



BECAUSE CORD IS INEXTENSIBLE, COMPONENT OF $\underline{v}_B$ ALONG AB MUST BE EQUAL TO $\underline{v}_A$.

CONS. OF MOMENTUM:
$m\underline{v}_0 = 2m\underline{v}_A + m\underline{v}_{B/A}$

$+\uparrow y$ COMP: $0 = 2m v_A \sin 60° - m v_{B/A} \sin 30°$

$v_{B/A} = 2\sqrt{3}\, v_A$ (3)

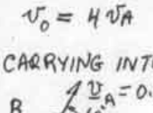
$\xrightarrow{+} x$ COMP: $m v_0 = 2m v_A \cos 60° + m v_{B/A} \cos 30°$

DIVIDING BY m AND SUBSTITUTING FOR $v_{B/A}$ FROM (3):

$v_0 = 2 v_A (0.5) + (2\sqrt{3}\, v_A)(\sqrt{3}/2)$

$v_0 = 4 v_A \qquad v_A = 0.250 v_0 \qquad \underline{v}_A = 0.250 v_0 \measuredangle 60°$ ◀

CARRYING INTO (3): $v_{B/A} = 2\sqrt{3}(0.250 v_0) = 0.866 v_0$



THUS: $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$
$= 0.250 v_0 \measuredangle 60° + 0.866 v_0 \searrow 30°$

$\underline{v}_B = (0.250 v_0 \cos 60° + 0.866 v_0 \cos 30°)\,\underline{i} + (0.250 v_0 \sin 60° - 0.866 v_0 \sin 30°)\,\underline{j}$

$\underline{v}_B = 0.875 v_0 \underline{i} - 0.2165 \underline{j}$

$\underline{v}_B = 0.90139 v_0 \searrow 13.90° \qquad v_B = 0.901 v_0 \searrow 13.9°$ ◀

(b) FRACTION OF K.E. LOST

$T_0 = \frac{1}{2} m v_0^2$

$T_{FINAL} = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_C^2$
$= \frac{1}{2} m (0.250 v_0)^2 + \frac{1}{2}(0.90139 v_0)^2 + \frac{1}{2} m (0)$
$= \frac{1}{2} m (0.875) v_0^2$

K.E. LOST $= T_0 - T_{FINAL} = \frac{1}{2} m (1 - 0.875) v_0^2 = \frac{1}{2}\,\frac{1}{8}\, m v_0^2$

FRACTION OF K.E. LOST $= \frac{1}{8}$ ◀

14.45

GIVEN:
360-kg SPACE VEHICLE WITH $\underline{v}_0 = (450\text{ m/s})\underline{k}$. AS IT PASSES THROUGH O, EXPLOSIVE CHARGES SEPARATE IT INTO 3 PARTS: A(60 kg), B(120 kg), AND C(180 kg). SHORTLY AFTER, THE POSITIONS OF THE 3 PARTS ARE A(72m, 72m, 648m), B(180m, 396m, 972m), C(−144m, −288m, 576m). VELOCITY OF B IS $\underline{v}_B = (150\text{ m/s})\underline{i} + (330\text{ m/s})\underline{j} + (660\text{ m/s})\underline{k}$. x- COMP OF VELOCITY OF C IS $(v_C)_x = -120$ m/s.

FIND: VELOCITY OF A.

CONSERVATION OF ANGULAR MOMENTUM ABOUT O

SINCE VEHICLE PASSES THROUGH O, $H_0 = 0$, OR

$$\underline{H}_0 = \underline{r}_A \times m_A \underline{v}_A + \underline{r}_B \times m_B \underline{v}_B + \underline{r}_C \times m_C \underline{v}_C = 0$$

USING DETERMINANT FORM:

$$\underline{H}_0 = 60\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 72 & 72 & 648 \\ (v_A)_x & (v_A)_y & (v_A)_z \end{vmatrix} + 120\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 180 & 396 & 972 \\ 150 & 330 & 660 \end{vmatrix} + 180\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -144 & -288 & 576 \\ -120 & (v_C)_y & (v_C)_z \end{vmatrix} = 0$$

EQUATING TO ZERO THE COEFF. OF $\underline{i}, \underline{j}, \underline{k}$, AND DIVIDING BY 60:

($\underline{i}$) $72(v_A)_z - 648(v_A)_y - 118.8\times10^3 - 864(v_C)_z - 1728(v_C)_y = 0$

($\underline{j}$) $648(v_A)_x - 72(v_A)_z + 54.0\times10^3 - 207.36\times10^3 + 432(v_C)_z = 0$

($\underline{k}$) $72(v_A)_y - 72(v_A)_x + 0 - 432(v_C)_y - 103.68\times10^3 = 0$

OR, AFTER REDUCTIONS:

$(v_A)_z - 9(v_A)_y - 12(v_C)_z - 24(v_C)_y = 1650$ (1)

$-(v_A)_z + 9(v_A)_x + 6(v_C)_z = 2130$ (2)

$(v_A)_y - (v_A)_x - 6(v_C)_y = 1440$ (3)

CONSERVATION OF LINEAR MOMENTUM

$m\underline{v}_0 = m_A \underline{v}_A + m_B \underline{v}_B + m_C \underline{v}_C$

$360(450\underline{k}) = 60[(v_A)_x\underline{i} + (v_A)_y\underline{j} + (v_A)_z\underline{k}] + 120[150\underline{i} + 330\underline{j} + 660\underline{k}] + 180[-120\underline{i} + (v_C)_y\underline{j} + (v_C)_z\underline{k}]$

EQUATING THE COEFF. OF THE UNIT VECTORS AND DIVIDING BY 60:

($\underline{i}$) $(v_A)_x + 300 - 360 = 0 \qquad (v_A)_x = 60$ m/s (4)

($\underline{j}$) $(v_A)_y + 660 + 3(v_C)_y = 0 \qquad (v_A)_y = -660 - 3(v_C)_y$ (5)

($\underline{k}$) $(v_A)_z + 1320 + 3(v_C)_z = 2700 \qquad (v_A)_z = 1380 - 3(v_C)_z$ (6)

SUBSTITUTING FROM (4), (5), (6) INTO (2) AND (3):

$-1380 + 3(v_C)_z + 9(60) + 6(v_C)_z = 2130 \qquad (v_C)_z = 330$ m/s

$-660 - 3(v_C)_y - 60 - 6(v_C)_y = 1440 \qquad (v_C)_y = -240$ m/s

SUBSTITUTING FOR $(v_C)_y$ AND $(v_C)_z$ INTO (5) AND (6):

$(v_A)_y = -660 - 3(-240) = 60$ m/s

$(v_A)_z = 1380 - 3(330) = 390$ m/s

RECALLING FROM (4) THAT $(v_A)_x = 60$ m/s, WE HAVE

$$\underline{v}_A = (60.0\text{ m/s})\,\underline{i} + (60.0\text{ m/s})\,\underline{j} + (390\text{ m/s})\,\underline{k}$$ ◀

CHECK

SINCE EQ. (1) WAS NOT USED IN OUR SOLUTION, WE CAN USE IT TO CHECK THE ANSWER. SUBSTITUTING THE VALUES OBTAINED FOR $(v_A)_z$, $(v_A)_y$, $(v_C)_z$, AND $(v_C)_y$ INTO THE LEFT-HAND MEMBER OF EQ. (1), WE OBTAIN

$390 - 9(60) - 12(330) - 24(-240) =$

$390 - 540 - 3960 + 5760 = 1650$ O.K.

14.46

GIVEN:

IN SCATTERING EXPERIMENT OF PROB. 14.24 IT IS KNOWN THAT PARTICE A IS PROJECTED FROM $A_0(260, -20, 340)$ AND COLLIDES WITH C AT $Q(200, 180, 140)$.

FIND:

COORDINATES OF B_0 WHERE PATH OF B INTERSECTS ZX PLANE.

CONS. OF ANGULAR MOMENTUM ABOUT Q:

SINCE PATHS OF A AFTER COLLISIONS AND OF C BEFORE AND AFTER COLLISION PASS THROUGH Q THE CORRESPONDING ANG. MOMENTA ARE ZERO (FIG. P14.24). CONS. OF ANGULAR MOMENTUM OF ALL PARTICLES ABOUT Q IS EXPRESSED AS

$$\overrightarrow{QA_0} \times \frac{m}{4}\underline{u}_0 + \overrightarrow{QB_0} \times m\underline{v}_0 = \overrightarrow{QB_1} \times m\underline{v}_B \qquad (1)$$

WHERE $\overrightarrow{QA_0} = \underline{r}_{A_0} - \underline{r}_Q = (260\underline{i} - 20\underline{j} + 340\underline{k}) - (200\underline{i} + 180\underline{j} + 140\underline{k})$

$= 60\underline{i} - 200\underline{j} + 200\underline{k}$

$\overrightarrow{QB_0} = (\Delta x)\underline{i} + (\Delta y)\underline{j} + (\Delta z)\underline{k}$

$\overrightarrow{QB_1} = \underline{r}_{B_1} - \underline{r}_Q = (107\underline{i} + 200\underline{j} + 170\underline{k}) - (200\underline{i} + 180\underline{j} + 140\underline{k})$

$= -93\underline{i} + 20\underline{j} + 30\underline{k}$

$\underline{u}_0 = -480\underline{i} + 600\underline{j} - 640\underline{k}$ / $\underline{v}_0 = 480\underline{j}$

AND, FROM SOLUTION OF PROB. 14.24:

$\underline{v}_B = v_B \underline{\lambda}_B = 573.6(-0.4229\underline{i} + 0.8971\underline{j} - 0.12816\underline{k})$

$= -242.6\underline{i} + 514.6\underline{j} - 73.51\underline{k}$

SUBSTITUTING INTO (1) AND USING DETERMINANTS:

$$\frac{m}{4}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 60 & -200 & 200 \\ -480 & 600 & -640 \end{vmatrix} + m\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \Delta x & \Delta y & \Delta z \\ 0 & 480 & 0 \end{vmatrix} = m\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -93 & 20 & 30 \\ -242.6 & 514.6 & -73.51 \end{vmatrix}$$

EQUATING THE COEFFICIENTS OF $\underline{i}$ AND $\underline{k}$S

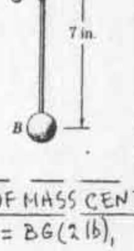
($\underline{i}$) $\frac{1}{4}(8000) - 480\Delta z = -16908$ $\qquad \Delta z = 39.39$ mm

($\underline{k}$) $\frac{1}{4}(-60000) + 480\Delta x = -43006$ $\qquad \Delta x = -58.35$ mm

$x_{B_0} = x_Q + \Delta x = 200 - 58.4$ $\qquad x_{B_0} = 141.6$ mm ◄

$z_{B_0} = z_Q + \Delta z = 140 + 39.4$ $\qquad z_{B_0} = 179.4$ mm ◄

14.47



GIVEN:

5-lb SPHERE A AND 2-lb SPHERE B CONNECTED BY RIGID ROD REST ON HORIZONTAL, FRICTIONLESS SURFACE WHEN A IS GIVEN VELOCITY $\underline{v}_0 = (10.5 \text{ ft/s})\underline{i}$.

FIND:

(a) LINEAR MOM. AND ANG. MOM. $\underline{H}_G$

(b) $\underline{v}_A$ AND $\underline{v}_B$ AFTER 180° ROTATION.

POSITION OF MASS CENTER $\qquad AG + BG = 7$ in.

$AG(5\text{ lb}) = BG(2\text{ lb})$, $\quad BG = 2.5AG$, $\quad 3.5AG = 7$, $AG = 2$ in.

(a) LINEAR AND ANG. MOMENTUM

$L = m_A \underline{v}_0 = \frac{5\text{ lb}}{32.2\text{ ft/s}^2}(10.5\text{ ft/s})\underline{i} = (1.6304\text{ lb}\cdot\text{s})\underline{i}$

$\underline{L} = (1.630\text{ lb}\cdot\text{s})\underline{i}$ ◄

$\underline{H}_G = \overrightarrow{GA} \times m_A\underline{v}_0 = (2\text{ in.})\underline{j} \times (1.6304\text{ lb}\cdot\text{s})\underline{i}$

$= -(3.2669\text{ in.}\cdot\text{lb}\cdot\text{s})\underline{k}$

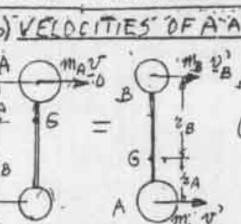
$= -(0.27174\text{ ft}\cdot\text{lb}\cdot\text{s})\underline{k}$

$\underline{H}_G = -(0.272\text{ ft}\cdot\text{lb}\cdot\text{s})\underline{k}$ ◄

(CONTINUED)

14.47 continued

(b) VELOCITIES OF A AND B AFTER 180° ROTATION



CONS. OF LINEAR MOMENTUM:

$m_A v_0 = m_A v'_A + m_B v'_B$

$(5/g)(10.5) = (5/g)v'_A + (2/g)v'_B$

$5v'_A + 2v'_B = 52.5 \qquad (1)$

CONS. OF ANG. MOM. ABOUT G:

$+\circlearrowleft\ r_A m_A v_0 = -r_A m_A v' + r_B m_B v'_B$

$(2\text{ in.})(5/g)(10.5) = -(2\text{ in.})(5/g)v'_A + (5\text{ in.})(2/g)v'_B$

MULTIPLY BY g AND DIVIDE BY 2: $\qquad -5v'_A + 5v'_B = 52.5 \qquad (2)$

ADD (1) AND (2): $7v'_B = 105$ $\qquad v'_B = +15.00$ ft/s.

FROM (1): $5v'_A + 2(15) = 52.5$ $\qquad v'_A = +4.50$ ft/s

$\underline{v}'_A = (4.50\text{ ft/s})\underline{i}$; $\underline{v}'_B = (15.00\text{ ft/s})\underline{i}$ ◄

14.48

GIVEN:

5-lb SPHERE A AND 2-lb SPHERE B CONNECTED BY RIGID ROD REST ON HORIZONTAL, FRICTIONLESS SURFACE WHEN B IS GIVEN VELOCITY $\underline{v}_0 = (10.5\text{ ft/s})\underline{i}$

FIND:

(a) LINEAR MOM. AND ANG. MOM. $\underline{H}_G$.

(b) $\underline{v}_A$ AND $\underline{v}_B$ AFTER 180° ROTATION.

POSITION OF MASS CENTER $\quad AG + BG = 7$ in.

$AG(5\text{ lb}) = BG(2\text{ lb})$, $\quad BG = 2.5AG$, $\quad 3.5AG = 7$, $\quad AG = 2$ in, $BG = 5$ in.

(a) LINEAR AND ANG. MOMENTUM

$L = m_B\underline{v}_0 = \frac{2\text{ lb}}{32.2\text{ ft/s}^2}(10.5\text{ ft/s})\underline{i} = (0.6522\text{ lb}\cdot\text{s})\underline{i}$

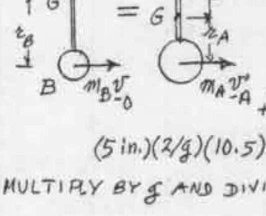
$\underline{L} = (0.652\text{ lb}\cdot\text{s})\underline{i}$ ◄

$\underline{H}_G = \overrightarrow{GB} \times m_B\underline{v}_0 = -(5\text{ in.})\underline{j} \times (0.6522\text{ lb}\cdot\text{s})\underline{i}$

$= +(3.261\text{ in.}\cdot\text{lb}\cdot\text{s})\underline{k}$

$\underline{H}_G = +(0.272\text{ ft}\cdot\text{lb}\cdot\text{s})\underline{k}$ ◄

(b) VELOCITIES OF A AND B AFTER 180° ROTATION.



CONS. OF LINEAR MOMENTUM:

$m_B v_0 = m_A v'_A + m_B v'_B$

$(2/g)(10.5) = (5/g)v'_A + (2/g)v'_B$

$5v'_A + 2v'_B = 21 \qquad (1)$

CONS. OF ANG. MOM. ABOUT G:

$+\circlearrowleft\ r_B m_B v_0 = r_A m_A v'_A - r_B m_B v'_B$

$(5\text{ in.})(2/g)(10.5) = (2\text{ in.})(5/g)v'_A - (5\text{ in.})(2/g)v'_B$

MULTIPLY BY g AND DIVIDE BY 2:

$5v'_A - 5v'_B = 52.5 \qquad (2)$

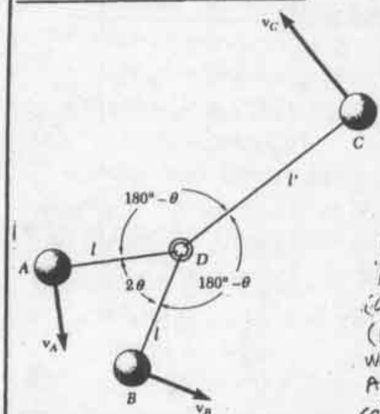
SUBTRACT (2) FROM (1):

$7v'_B = -31.5$ $\qquad v'_B = -4.50$ ft/s

FROM (1): $5v'_A + 2(-4.50) = 21$ $\qquad v'_A = +6.00$ ft/s

$\underline{v}'_A = (6.00\text{ ft/s})\underline{i}$; $\underline{v}'_B = -(4.50\text{ ft/s})\underline{i}$ ◄

14.49 and 14.50



GIVEN: THREE IDENTICAL SPHERES CONNECTED TO RING D AT THEIR MASS CENTER SLIDE ON HORIZONTAL, FRICTIONLESS SURFACE ($l' = 2\,l\cos\theta$).

$v_A = v_B = v_0$ WHEN CORD CD BREAKS.

FIND AFTER CORDS AD AND BD BECOME TAUT

(a) SPEED OF RING D

(b) RELATIVE SPEED AT WHICH A AND B ROTATE ABOUT D

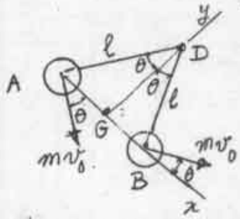
(C) PERCENT ENERGY OF ORIGINAL SYSTEM LOST WHEN AD AND BD BECOME TAUT.

PROB. 14.49: ASSUME $\theta = 30°$.

PROB. 14.50: ASSUME $\theta = 45°$.

WE CONSIDER THE FOLLOWING TWO POSTIONS OF THE SPHERES A AND B AND THE RING D.

POSITION 1: IMMEDIATELY AFTER CORD CD BREAKS



LINEAR MOMENTUM:

$$\underline{L} = (2 m v_0 \cos\theta)\,\underline{i} \quad (1)$$

ANGULAR MOMENTUM ABOUT G:

$$H_G = 2(l\sin\theta)(m v_0 \sin\theta)\underline{k}$$

$$H_G = (2\,l\,m v_0 \sin^2\theta)\underline{k} \quad (2)$$

POSITION 2: AFTER CORDS AD AND BD BECOME TAUT

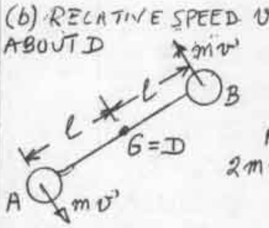
(a) SPEED OF MASS CENTER (NOW LOCATED AT D).

RECALLING (1):

$$L = (2m)\bar{\underline{v}} = (2 m v_0 \cos\theta)\underline{i} \qquad \bar{\underline{v}} = (v_0\cos\theta)\underline{i}$$

$$v_D = \bar{v} = v_0\cos\theta \quad (3)$$

(b) RELATIVE SPEED v' AT WHICH A AND B ROTATE ABOUT D



ANG. MOMENTUM ABOUT G:

$$H_G = (2 m v')l$$

RECALLING (2):

$$2 m v' l = 2 l m v_0 \sin^2\theta$$

$$v' = v_0 \sin^2\theta \quad (4)$$

(C) ENERGY LOST:

CONSIDERING SYSTEM OF 3 SPHERES:

INITIALLY, $v_C = (l'/l)v_A = (2\cos\theta)v_0$, THEREFORE

$$T_0 = \tfrac{1}{2} m v_A^2 + \tfrac{1}{2} m v_B^2 + \tfrac{1}{2} m v_C^2 = m v_0^2(1 + 2\cos^2\theta)$$

$$T_f = \tfrac{1}{2}(2m)v_D^2 + 2\left(\tfrac{1}{2} m v'^2\right) + \tfrac{1}{2} m v_C^2$$

$$= m\left[v_0^2\cos^2\theta + m(v_0\sin^2\theta)^2 + 2 v_0^2 \cos^2\theta\right]$$

$$= m v_0^2(3\cos^2\theta + \sin^4 A)$$

$$\%\text{LOSS} = 100\frac{T_0 - T_f}{T_0} = 100\frac{1 + 2\cos^2\theta - 3\cos^2\theta - \sin^4\theta}{1 + 2\cos^2\theta} = 100\frac{\sin^2\theta\cos^2\theta}{1 + 2\cos^2\theta} \quad (5)$$

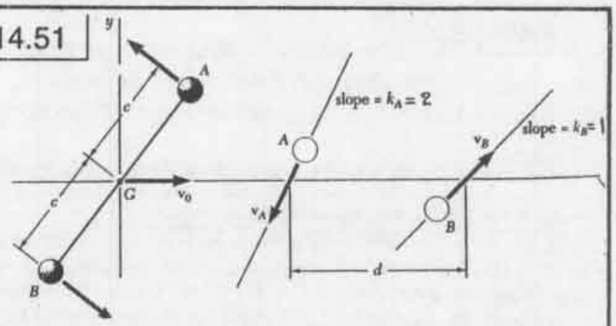
PROB. 14.49: MAKING $\theta = 30°$ IN EQS. (3), (4), AND (5):

(a) $0.866\,v_0$. (b) $0.250\,v_0$. (C) 7.50%

PROB. 14.50: MAKING $\theta = 45°$ IN EQS. (3), (4), AND (5):

(a) $0.707\,v_0$. (b) $0.500\,v_0$. (c) 12.50%

14.51



GIVEN: TWO SMALL IDENTICAL SPHERES A AND B, CONNECTED BY A CORD SLIDE ON A HORIZONTAL, FRICTIONLESS SURFACE. INITIALLY THEY ROTATE WITH $\dot{\theta} = 8$ rad/s ABOUT G, AND G HAS VELOCITY $\underline{v} = v_0\,\underline{i}$.

AFTER CORD BREAKS, SPHERES MOVE ALONG PATHS WITH $k_A = 2$, $k_B = 1$, AND $d = 625$ mm.

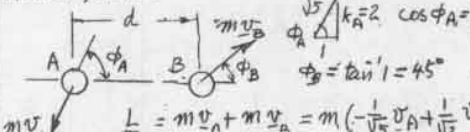
FIND:

(a) SPEEDS v_0, v_A, AND v_B, (b) length 2c of cord

CONSERVATION OF LINEAR MOMENTUM

BEFORE BREAK: $\underline{L}_0 = (2m)\underline{\bar{v}} \qquad \underline{L}_0 = 2 m v_0\,\underline{i}$

AFTER BREAK:



$\cos\phi_A = \frac{1}{\sqrt{5}}, \ \sin\phi_A = \frac{2}{\sqrt{5}}$

$\phi_B = \tan^{-1} 1 = 45° \qquad \cos\phi_B = \cos\phi_A = \frac{1}{\sqrt{2}}$

$$\underline{L} = m\underline{v}_A + m\underline{v}_B = m\left(-\tfrac{1}{\sqrt{5}}v_A + \tfrac{1}{\sqrt{2}}v_B\right)\underline{i} + m\left(-\tfrac{2}{\sqrt{5}}v_A + \tfrac{1}{\sqrt{2}}v_B\right)\underline{j}$$

SETTING $\underline{L} = \underline{L}_0$ AND EQUATING COEFF. OF UNIT VECTORS:

$(\underline{i})\quad -\frac{1}{\sqrt{5}}v_A + \frac{1}{\sqrt{2}}v_B = 2v_0 \quad (1)$

$(\underline{j})\quad -\frac{2}{\sqrt{5}}v_A + \frac{1}{\sqrt{2}}v_B = 0 \quad (2)$

SUBTRACTING (2) FROM (1): $\frac{1}{\sqrt{5}}v_A = 2v_0 \qquad v_A = 2\sqrt{5}\,v_0 \quad (3)$

SUBSTITUTING FOR v_A INTO (2): $v_B = \frac{2\sqrt{2}}{\sqrt{5}}(2\sqrt{5}\,v_0) = 4\sqrt{2}\,v_0 \quad (4)$

CONSERVATION OF ANGULAR MOMENTUM

BEFORE BREAK: $(H_G)_0 = 2mc^2\dot{\theta} = 2mc^2(8\text{ rad/s}) = 16\,mc^2$

AFTER BREAK: $H_G = H_A = m(v_B)_y d = m\frac{1}{\sqrt{2}}(4\sqrt{2}\,v_0)(0.625\,m) = 2.5\,m v_0$

SETTING $H_G = (H_G)_0$: $2.5\,m v_0 = 16\,m c^2 \qquad v_0 = 6.40\,c^2 \quad (5)$

CONSERVATION OF ENERGY

BEFORE BREAK: $T_0 = \frac{1}{2}(2m)v_0^2 + \frac{1}{2}(2m)(c\dot{\theta})^2 = m(v_0^2 + c^2\dot{\theta}^2)$

LETTING $\dot{\theta} = 8$ rad/s AND USING (5): $T_0 = m(40.96c^4 + 64c^2)$

AFTER BREAK: $T = \frac{1}{2} m v_A^2 + \frac{1}{2} m v_B^2$

RECALLING (3), (4), AND (5):

$$T = \tfrac{1}{2} m(20v_0^2 + 32v_0^2) = 26 m v_0^2 = 1064.96\,m c^4$$

SETTING $T = T_0$: $1064.96\,c^4 = 40.96\,c^4 + 64c^2$

$1024c^2 = 64 \qquad c^2 = 0.0625 \qquad c = 0.250$ m

FROM (5): $v_0 = 6.40(0.0625) = 0.400$ m/s

FROM (3): $v_A = 2\sqrt{5}(0.4) = 1.789$ m/s

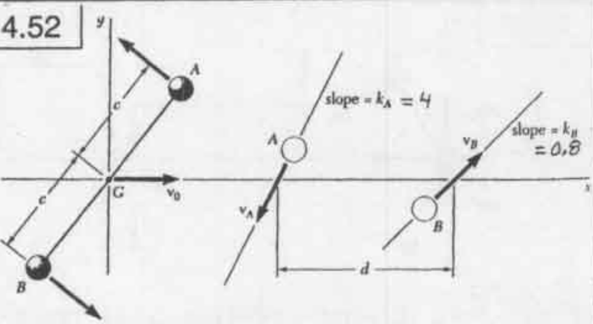
FROM (4): $v_B = 4\sqrt{2}(0.4) = 2.26$ m/s

ANSWERS:

(a) $v_0 = 0.400$ m/s; $v_A = 1.789$ m/s; $v_B = 2.26$ m/s

(b) LENGTH OF CORD = 2c = 500 mm

14.52



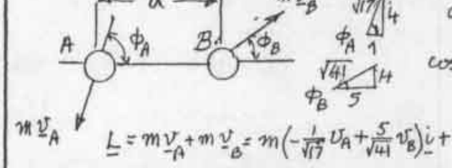
GIVEN: TWO SMALL IDENTICAL SPHERES A AND B, CONNECTED BY A CORD OF LENGTH $2c = 600$ mm SLIDE ON A HORIZONTAL, FRICTIONLESS SURFACE. INITIALLY THEY ROTATE WITH $\dot{\theta} = 12$ rad/s ABOUT G, AND G MOVES WITH $\underline{v}_0 = v_0 \underline{i}$. AFTER CORD BREAKS, SPHERES MOVE ALONG PATHS WITH $k_A = 4$ AND $k_B = 0.8$.

FIND:
SPEEDS v_0, v_A, AND v_B, (b) DISTANCE d

CONSERVATION OF LINEAR MOMENTUM

BEFORE BREAK: $\underline{L}_0 = (2m)\underline{v}_0$ $\quad \underline{L}_0 = 2m v_0 \underline{i}$

AFTER BREAK:



$\cos\phi_A = \frac{1}{\sqrt{17}}, \sin\phi_A = \frac{4}{\sqrt{17}}$

$\cos\phi_B = \frac{5}{\sqrt{41}}, \sin\phi_B = \frac{4}{\sqrt{41}}$

$\underline{L} = m\underline{v}_A + m\underline{v}_B = m\left(-\frac{1}{\sqrt{17}}v_A + \frac{5}{\sqrt{41}}v_B\right)\underline{i} + m\left(-\frac{4}{\sqrt{17}}v_A + \frac{4}{\sqrt{41}}v_B\right)\underline{j}$

SETTING $\underline{L} = \underline{L}_0$ AND EQUATING COEFF. OF UNIT VECTORS

(i) $-\frac{1}{\sqrt{17}}v_A + \frac{5}{\sqrt{41}}v_B = 2v_0$ $\quad$ (1)

(j) $-\frac{4}{\sqrt{17}}v_A + \frac{4}{\sqrt{41}}v_B = 0,$ $\quad v_B = \frac{\sqrt{41}}{\sqrt{17}}v_A$ $\quad$ (2)

SUBSTITUTE FOR v_B INTO (1):

$-\frac{1}{\sqrt{17}}v_A + \frac{5}{\sqrt{17}}v_A = 2v_0$ $\quad v_A = \frac{\sqrt{17}}{2}v_0$ $\quad$ (3)

FROM (2): $v_B = \frac{\sqrt{41}}{\sqrt{17}}\frac{\sqrt{17}}{2}v_0$ $\quad v_B = \frac{\sqrt{41}}{2}v_0$ $\quad$ (4)

CONSERVATION OF ANGULAR MOMENTUM

BEFORE BREAK: $(H_G)_0 = 2mc^2\dot{\theta} = 2m(0.3\text{ m})^2(12\text{ rad/s}) = m(2.16)$

AFTER BREAK: $H_G = H_A = m(v_B)_y d = md\frac{4}{\sqrt{41}}\frac{\sqrt{41}}{2}v_0 = 2mdv_0$

SETTING $H_G = (H_G)_0$: $2mdv_0 = m(2.16)$ $\quad v_0 d = 1.08$ $\quad$ (5)

CONSERVATION OF ENERGY

BEFORE BREAK: $T_0 = \frac{1}{2}(2m)v_0^2 + \frac{1}{2}(2m)(c\dot{\theta})^2$
$= mv_0^2 + m(0.3\times 12)^2 = m(v_0^2 + 12.96)$

AFTER BREAK: $T = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2$

RECALLING (3) AND (4): $T = \frac{1}{2}mv_0^2\left(\frac{17}{4} + \frac{41}{4}\right) = 7.25\,mv_0^2$

SETTING $T = T_0$: $7.25v_0^2 = v_0^2 + 12.96$ $\quad v_0 = 1.440$ m/s

FROM (3): $v_A = \frac{\sqrt{17}}{2}(1.440) = 2.969$ m/s

FROM (4): $v_B = \frac{\sqrt{41}}{2}(1.440) = 4.610$ m/s

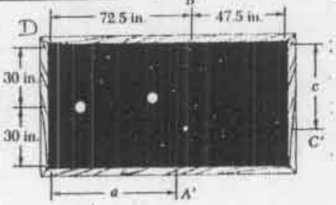
FROM (5): $d = \frac{1.08}{1.440} = 0.750$ m

ANSWERS:

(a) $v_0 = 1.440$ m/s; $v_A = 2.97$ m/s; $v_B = 4.61$ m/s

(b) DISTANCE $d = 0.750$ m $= 750$ mm

14.53



GIVEN:
BALL A HITS BALL B WITH $\underline{v}_0 = (12\text{ ft/s})\underline{i}$, THEN C, THEN SIDE OF TABLE AT A' (WHERE $a = 66$ in.) WITH $\underline{v}_A = -(5.76\text{ ft/s})\underline{j}$

FIND:
(a) VELOCITIES OF B AND C
(b) DISTANCE c WHERE BALL C HITS SIDE

CONSERVATION OF LINEAR MOMENTUM

$mv_0\underline{i} = -mv_A\underline{j} + m(v_B)_x\underline{i} + m(v_B)_y\underline{j} + mv_C\underline{i}$

EQUATING COEFF. OF UNIT VECTORS:

(i) $mv_0 = m(v_B)_x + mv_C$ $\quad (v_B)_x + v_C = v_0 = 12$ ft/s $\quad$ (1)

(j) $0 = -mv_A + (v_B)_y$ $\quad (v_B)_y = v_A = 5.76$ ft/s $\quad$ (2)

CONSERVATION OF ANG. MOMENTUM ABOUT CORNER D

$+\circlearrowleft$ $(30\text{ in.})v_0 = -(66\text{ in.})v_A + (72.5\text{ in.})(v_B)_y + cv_C$

$30(12) = -66(5.76) + (72.5)(5.76) + cv_C$

$cv_C = 322.56$ $\quad$ (3)

CONSERVATION OF ENERGY

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left[(v_B)_x^2 + (v_B)_y^2\right] + \frac{1}{2}mv_C^2$

DIVIDING BY m, MULTIPLYING BY 2, AND SUBSTITUTING FOR v_0, v_A, $(v_B)_y$ THEIR VALUES AND $(v_B)_x = 12 - v_C$ FROM (1):

$(12)^2 = (5.76)^2 + (12 - v_C)^2 + (5.76)^2 + v_C^2$

DIVIDING BY 2: $v_C^2 - 12v_C + (5.76)^2 = 0,$ $\quad v_C = 6 \pm 1.68$

WITH $v_C = 6 - 1.68 = 4.32$, EQ. (3) YIELDS $c = 74.7$ (IMPOSSIBLE)

THEREFORE: $v_C = 6 + 1.68 = 7.68$ $\quad \underline{v}_C = 7.68$ ft/s $\rightarrow$

FROM (1): $(v_B)_x = 12 - 7.68 = 4.32$

$\underline{v}_B = (4.32\text{ ft/s})\underline{i} + (5.76\text{ ft/s})$ OR $\underline{v}_B = 7.20$ ft/s $\angle 53.1°$

FROM (3): $c(7.68) = 322.56$ $\quad c = 42.0$ in.

14.54

(SEE FIGURE OF PROB. 14.53)

GIVEN: BALL A HITS B WITH $\underline{v} = (15\text{ ft/s})\underline{i}$, THEN C; BALL C HITS SIDE AT $c = 48$ in. WITH $\underline{v}_C = (9.6\text{ ft/s})\underline{i}$

FIND: (a) $\underline{v}_A$ AND $\underline{v}_B$, (b) DISTANCE a.

CONSERVATION OF LINEAR MOMENTUM

$mv_0\underline{i} = -mv_A\underline{j} + m(v_B)_x\underline{i} + m(v_B)_y\underline{j} + mv_C\underline{i}$

(i) $mv_0 = m(v_B)_x + mv_C$ $\quad (v_B)_x = 15 - 9.6 = 5.40$ ft/s $\quad$ (1)

(j) $0 = -mv_A + (v_B)_y$ $\quad (v_B)_y = v_A$ $\quad$ (2)

CONSERVATION OF ANGULAR MOMENTUM ABOUT CORNER D

$+\circlearrowleft$ $(30\text{ in.})v_0 = -av_A + (72.5\text{ in.})(v_B)_y + cv_C$

SUBSTITUTING GIVEN DATA AND USING EQ. (2):

$30(15\text{ ft/s}) = -av_A + 72.5v_A + 48(9.6\text{ ft/s})$

$(a - 72.5)v_A = 10.8$ $\quad$ (3)

CONSERVATION OF ENERGY

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left[(v_B)_x^2 + (v_B)_y^2\right] + \frac{1}{2}mv_C^2$

DIVIDING BY m, MULTIPLYING BY 2, AND SUBSTITUTING:

$(15)^2 = v_A^2 + (5.40)^2 + v_A^2 + (9.6)^2$

$v_A^2 = 51.84$ $\quad v_A = 7.20$ ft/s $\quad \underline{v}_A = 7.20$ ft/s $\downarrow$

FROM (1) AND (2):

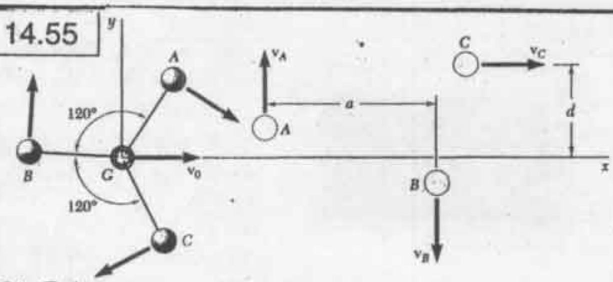
$\underline{v}_B = (v_B)_x\underline{i} + (v_B)_y\underline{j} = (5.40\text{ ft/s})\underline{i} + (7.20\text{ ft/s})\underline{j}$

OR $\underline{v}_B = 9.00$ ft/s $\angle 53.1°$

FROM (3): $(a - 72.5)7.20 = 10.8$

$a = 72.5 + 1.5$ $\quad a = 74.0$ in.

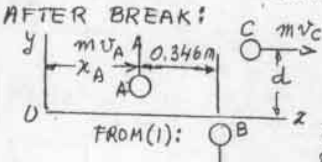
14.55



GIVEN:
THREE SMALL IDENTICAL SPHERES CONNECTED BY 200-mm-LONG STRINGS TO RING G SLIDE ON A HORIZONTAL, FRICTIONLESS SURFACE.
INITIALLY, SPHERES ROTATE ABOUT G WITH 0.8 m/s RELATIVE VELOCITY AND RING MOVES WITH $\underline{v}_0 = (0.4\,m/s)\underline{i}$. SUDDENLY RING BREAKS AND SPHERES MOVE FREELY AS SHOWN WITH $a = 346$ mm.
FIND:
(a) VELOCITY OF EACH SPHERE, (b) DISTANCE d.

CONSERVATION OF LINEAR MOMENTUM
BEFORE BREAK: $\underline{L}_0 = (3m)\bar{\underline{v}} = 3m(0.4\underline{i}) = m(1.2\,m/s)\underline{i}$
AFTER BREAK: $\underline{L} = m v_A \underline{j} - m v_B \underline{j} + m v_C \underline{i}$
$\underline{L} = \underline{L}_0$: $m v_C \underline{i} + m(v_A - v_B)\underline{j} = m(1.2\,m/s)\underline{i}$
THEREFORE: $v_A = v_B$ (1)
$v_C = 1.200$ m/s $\quad \underline{v}_C = 1.200\,m/s \rightarrow$ (2)

CONSERVATION OF ANGULAR MOMENTUM
BEFORE BREAK: $+\circlearrowleft (H_0)_0 = 3m\ell v' = 3m(0.2m)(0.8\,m/s)$
$= 0.480\,m$
AFTER BREAK:



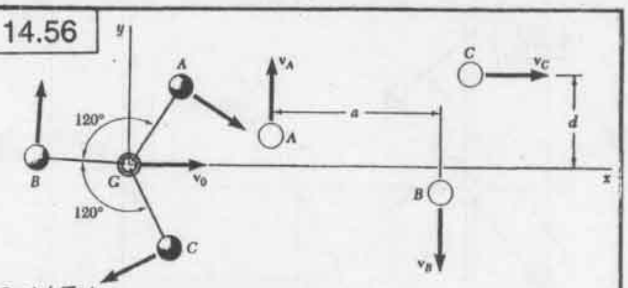
$+\circlearrowleft\ H_0 = -m\cancel{v_A} x_A + m v_A(\cancel{x_A} + 0.346) + m v_C d$
$H_0 = (H_0)_0$:
$0.346\,m v_A + m v_C d = 0.480\,m$
RECALLING (2):
$0.346 v_A + 1.200 d = 0.480$
$d = 0.400 - 0.28833 v_A$ (3)

CONSERVATION OF ENERGY
BEFORE BREAK:
$$T_0 = \tfrac{1}{2}(3m)\bar{v}^2 + 3\left(\tfrac{1}{2} m v'^2\right) = \tfrac{3}{2} m(v_0^2 + v'^2) = \tfrac{3}{2}\left[(0.4)^2 + (0.8)^2\right]m = 1.200\,m$$
AFTER BREAK:
$$T = \tfrac{1}{2} m v_A^2 + \tfrac{1}{2} m v_B^2 + \tfrac{1}{2} m v_C^2$$
$T = T_0$: SUBSTITUTING FOR v_B FROM (1) AND v_C FROM (2):
$$\tfrac{1}{2}\left[v_A^2 + v_A^2 + (1.200)^2\right] = 1.200$$
$v_A^2 = 0.480 \qquad v_A = v_B = 0.69282$ m/s

(a) VELOCITIES:
$\underline{v}_A = 0.693$ m/s ↑; $\underline{v}_B = 0.693$ m/s ↓; $\underline{v}_C = 1.200$ m/s →

(b) DISTANCE d:
FROM (3): $d = 0.400 - 0.28833(0.69282) = 0.20024$ m
$d = 200$ mm

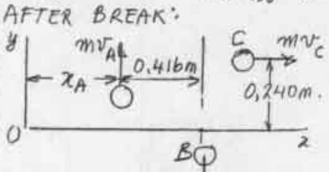
14.56



GIVEN:
THREE SMALL IDENTICAL SPHERES CONNECTED BY STRINGS OF LENGTH ℓ TO RING G SLIDE ON A HORIZONTAL, FRICTIONLESS SURFACE.
INITIALLY, SPHERES ROTATE ABOUT G AND RING MOVES AS SHOWN. SUDDENLY RING BREAKS AND SPHERE MOVE FREELY IN xy PLANE. WE KNOW THAT $\underline{v}_A = (1.039\,m/s)\underline{j}$, $\underline{v}_C = (1.800\,m/s)\underline{i}$, $a = 416$ mm, $d = 240$ mm.
FIND:
(a) VEL. $\underline{v}_0$ OF RING, (b) LENGTH ℓ OF STRINGS
(c) RATE IN rad/s AT WHICH SPHERES WERE ROTATING

CONSERVATION OF LINEAR MOMENTUM
$(3m)\bar{\underline{v}} = m\underline{v}_A + m\underline{v}_B + m\underline{v}_C$
$3m v_0 \underline{i} = m(1.039\,m/s)\underline{j} - m v_B \underline{j} + m(1.800\,m/s)\underline{i}$
EQUATING COEFF. OF UNIT VECTORS:
($\underline{i}$) $3v_0 = 1.800$ m/s $\qquad$ (a) $\underline{v}_0 = 0.600$ m/s →
($\underline{j}$) $0 = 1.039\,m/s - v_B \qquad v_B = 1.039$ m/s (1)

CONSERVATION OF ANGULAR MOMENTUM
BEFORE BREAK: $+\circlearrowleft (H_0)_0 = 3m\ell^2\dot{\theta}$
AFTER BREAK:



$+\circlearrowleft\ H_0 = -m\cancel{v_A} x_A + m v_A(\cancel{x_A} + 0.416) + m v_C(0.240)$
$= m(1.039)(0.416) + m(1.800)(0.240)$
$= m(0.864224)$

$(H_0)_0 = H_0$: $3m\ell^2\dot{\theta} = m(0.864224) \qquad \ell^2\dot{\theta} = 0.28807$ (2)

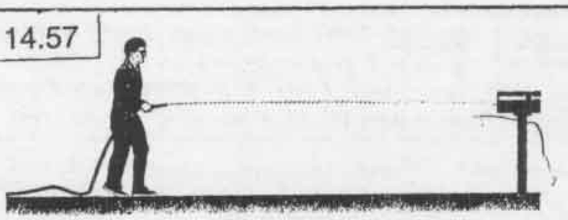
CONSERVATION OF ENERGY
BEFORE BREAK:
$$T_0 = \tfrac{1}{2}(3m)\bar{v}^2 + 3\left(\tfrac{1}{2} m v'^2\right) = \tfrac{3}{2} m v_0^2 + \tfrac{3}{2} m(\ell\dot{\theta})^2$$
$$= \tfrac{3}{2} m(0.600)^2 + \tfrac{3}{2} m\ell^2\dot{\theta}^2$$
AFTER BREAK:
$$T = \tfrac{1}{2} m v_A^2 + \tfrac{1}{2} m v_B^2 + \tfrac{1}{2} m v_C^2$$
$$= \tfrac{1}{2} m\left[(1.039)^2 + (1.039)^2 + (1.800)^2\right] = \tfrac{1}{2} m(5.399)$$
$T = T_0$: $\tfrac{1}{2} m(5.399) = \tfrac{3}{2} m(0.600)^2 + \tfrac{3}{2} m\ell^2\dot{\theta}^2$
$\ell^2\dot{\theta}^2 = 1.4397$ (3)

DIVIDING (3) BY (2): $\dot{\theta} = \dfrac{1.4397}{0.28807} = 4.9976$

(b) FROM (2): $\ell^2 = \dfrac{0.28807}{4.9976} \qquad \ell = 0.2401$ m
$\ell = 240$ mm

(c) RATE OF ROTATION $= \dot{\theta} = 5.00$ rad/s

14.57



GIVEN:
VEL. OF STREAM = 25 m/s $A = 300\ mm^2$
FIND: FORCE EXERTED BY STREAM ON MAILBOX.

$(\Delta m)v_0 \rightarrow \square + \square \leftarrow P\Delta t = 0$ $\xrightarrow{+}$ x COMP.: $(\Delta m)v_0 - P\Delta t = 0$

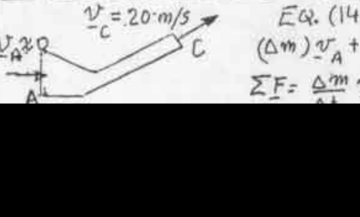
$$P = \frac{\Delta m}{\Delta t} v_0 = (\rho A v_0) v_0 = \rho A v_0^2$$

$$P = (1000\ kg/m^3)(300 \times 10^{-6}\ m^2)(25\ m/s)^2$$

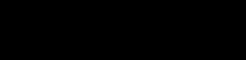
$$P = 187.5\ N$$

NOTE: FORCE $\underline{P}$ SHOWN ON SKETCH IS FORCE APPLIED BY MAILBOX ON STREAM. FORCE EXERTED BY STREAM ON MAILBOX IS 187.5 N →

14.58



GIVEN:
TREE LIMBS ARE FED INTO SHREDDER AT RATE OF 5 kg/s AND CHIPS ARE SPEWED WITH $v_C = 20$ m/s.
FIND: HORIZ. COMP. OF FORCE EXERTED ON HITCH AT D.



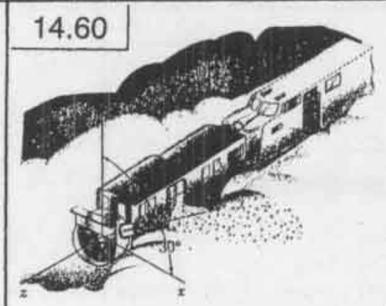
EQ. (14.38):

$$(\Delta m)\underline{v}_A + \Sigma \underline{F}\Delta t = (\Delta m)\underline{v}_C$$

$$\Sigma \underline{F} = \frac{\Delta m}{\Delta t}\underline{v}_C = (5\ kg/s)(20\ m/s \measuredangle 25°$$

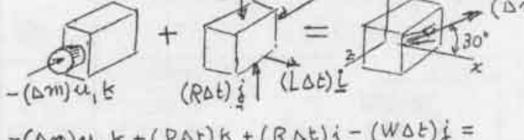
[redacted]

14.60



GIVEN:
ENGINE PROPELS PLOW AT SPEED OF 12 mi/h. PLOW PROJECTS 180 TONS OF SNOW PER MINUTE WITH VELOCITY OF 40 ft/s W/R TO CAR.
FIND:
(a) FORCE EXERTED BY ENGINE ON CAR
(b) LATERAL FORCE EXERTED BY TRACK.

WE MEASURE ALL VELOCITIES W/R PLOW CAR AND APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE PLOW CAR, THE SNOW IT CONTAINS, AND THE SNOW ENTERING IN THE TIME INTERVAL Δt.



$$-(\Delta m)u_1\underline{k} + (P\Delta t)\underline{k} + (R\Delta t)\underline{j} - (W\Delta t)\underline{j} = (\Delta m)u_2(\cos 30°\,\underline{i} + \sin 30°\,\underline{j})$$

EQUATING THE COEFF. OF THE UNIT VECTORS:

(k) $-(\Delta m)u_1 + P\Delta t = 0$ $P = \frac{\Delta m}{\Delta t}u_1$ (1)

(i) $L\Delta t = (\Delta m)u_2\cos 30°$ $L = \frac{\Delta m}{\Delta t}u_2\cos 30°$ (2)

WITH GIVEN DATA:

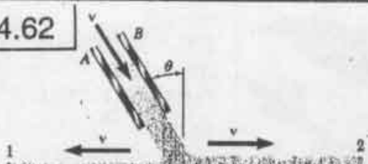
$u_1 = 12$ mi/h = 17.60 ft/s, $u_2 = 40$ ft/s

$$\frac{\Delta m}{\Delta t} = (180\ tons/min)\left(\frac{1\ min}{60\ s}\right)\left(\frac{2000\ lb}{1\ ton}\right)\left(\frac{1}{32.2\ ft/s^2}\right) = 186.34\ lb\cdot s/ft$$

(a) EQ.(1): $P = (186.34\ lb\cdot s/ft)(17.60\ ft/s)$ $\underline{P} = (3280\ lb)\underline{k}$

[redacted]

14.62



GIVEN:
$v = 40\ m/s$, $\theta = 30°$
TOTAL FORCE EXERTED BY STREAM ON PLATE = 500N ↓.
FIND: Q_1 AND Q_2 OF RESULTING STREAMS.

WE NOTE THAT $Q = Q_1 + Q_2$ (1)
SEE SOLUTION OF PROB. 14.61 FOR DERIVATION OF
$Q\sin\theta = Q_2 - Q_1$ (2) $P = \rho Q v \cos\theta$ (3)

FROM (3): $Q = \dfrac{P}{\rho v \cos\theta} = \dfrac{500\ N}{(1\ kg/L)(40\ m/s)\cos 30°} = 14.434\ L/s$
$= 866.03\ L/min$

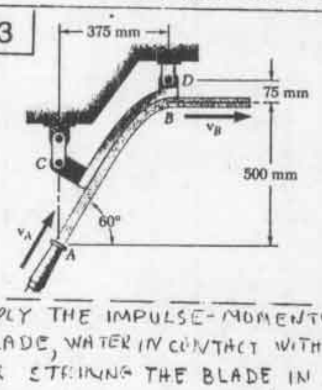
ADDING (1) AND (2): $Q(1+\sin\theta) = 2Q_2$

$Q_2 = \dfrac{1+\sin\theta}{2}Q = \dfrac{1+\sin 30°}{2}(866.03\ L/min) = 649.52\ L/min$

FROM (1): $Q_1 = Q - Q_2 = 866.03 - 649.52 = 216.51\ L/min$

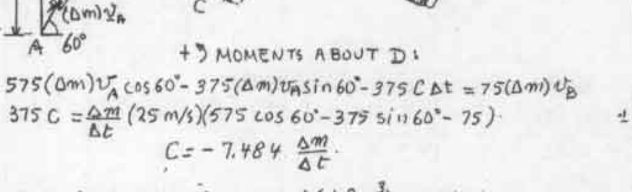
$Q_1 = 217\ L/min$; $Q_2 = 650\ L/min$ ◄

14.63



GIVEN:
WATER DISCHARGED AT RATE $Q = 1.2\ m^3/min$ WITH $v_A = v_B = 25\ m/s$
FIND:
COMPONENTS OF REACTIONS AT C AND D.
(NEGLECT WEIGHT OF VANE).

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE BLADE, WATER IN CONTACT WITH THE BLADE, AND WATER STRIKING THE BLADE IN INTERVAL Δt.



+↻ MOMENTS ABOUT D:

$575(\Delta m)v_A\cos 60° - 375(\Delta m)v_A\sin 60° - 375\,C\Delta t = 75(\Delta m)v_B$

$375\,C = \dfrac{\Delta m}{\Delta t}(25\ m/s)(575\cos 60° - 375\sin 60° - 75)$

$C = -7.484\dfrac{\Delta m}{\Delta t}$

BUT $\dfrac{\Delta m}{\Delta t} = \rho Q = (1000\ kg/m^3)\left(\dfrac{1.2\ m^3}{60\ s}\right) = 20\ kg/s$

THUS: $C = -7.484(20) = -149.68\ N$

$C_x = 0$, $C_y = 149.7\ N\downarrow$ ◄

$\xrightarrow{+}$ x COMP.: $(\Delta m)v_A\cos 60° + D_x\Delta t = (\Delta m)v_B$

$D_x = \dfrac{\Delta m}{\Delta t}(25\ m/s)(1-\cos 60°) = (20\ kg/s)(25\ m/s)(1-\cos 60°)$

$D_x = 250\ N \rightarrow$ ◄

+↑ y COMP.: $(\Delta m)v_A\sin 60° + C\Delta t + D_y\Delta t = 0$

$D_y = -\dfrac{\Delta m}{\Delta t}(25\ m/s)\sin 60° - (-149.7\ N)$

$= -(20\ kg/s)(25\ m/s)\sin 60° + 149.7\ N$

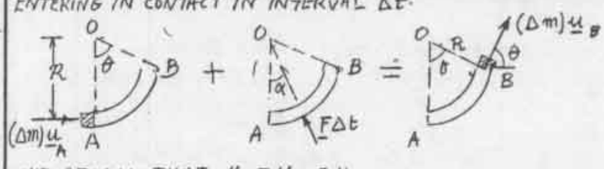
$= -433.0\ N + 149.7\ N = -283.3\ N$

$D_y = 283\ N\downarrow$ ◄

14.64

ASSUME THAT BLADE AB OF SAMPLE PROB. 14.7 IS IN THE SHAPE OF AN ARC OF CIRCLE.
SHOW THAT RESULTANT FORCE $\underline{F}$ EXERTED BY THE BLADE ON THE STREAM IS APPLIED AT MIDPOINT C OF ARC AB.

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE PORTION OF STREAM IN CONTACT WITH THE BLADE AND ENTERING IN CONTACT IN INTERVAL Δt.



WE RECALL THAT $u_A = u_B = u$

+↻ MOMENTS ABOUT O: $R(\Delta m)u + \text{MOM. OF } F = R(\Delta m)u$

THUS: MOM. OF $\underline{F}$ ABOUT $O = 0$; LINE OF ACTION OF $\underline{F}$ PASSES THROUGH O.

$\xrightarrow{+}$ x COMP.: $(\Delta m)u - (F\Delta t)\sin\alpha = (\Delta m)u\cos\theta$

OR: $F(\Delta t)\sin\alpha = (\Delta m)u(1-\cos\theta)$ (1)

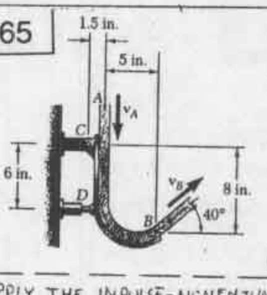
+↑ y COMP.: $0 + F(\Delta t)\cos\alpha = (\Delta m)u\sin\theta$ (2)

DIVIDE (1) BY (2):

$\tan\alpha = \dfrac{1-\cos\theta}{\sin\theta} = \dfrac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \tan\dfrac{\theta}{2}$ $\alpha = \dfrac{\theta}{2}$

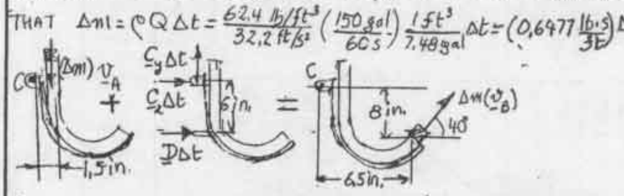
THUS: LINE OF ACTION OF $\underline{F}$ BISSECTS ∡AOB,
$\underline{F}$ IS APPLIED AT MIDPOINT C OF ARC AB ◄
(Q.E.D.)

14.65



GIVEN:
STREAM OF WATER WITH
$Q = 150\ gal/min$
$v_A = v_B = 60\ ft/s$
REACTION AT D HORIZONTAL.
FIND:
COMPONENTS OF REACTIONS AT C AND D
(NEGLECT WEIGHT OF VANE.)

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE VANE, THE WATER IN CONTACT WITH IT, AND THE MASS Δm OF WATER ENTERING AND LEAVING THE SYSTEM IN THE INTERVAL Δt. WE NOTE THAT $\Delta m = \rho Q\Delta t = \dfrac{62.4\ lb/ft^3}{32.2\ ft/s^2}\left(\dfrac{150\ gal}{60\ s}\right)\dfrac{1\ ft^3}{7.48\ gal}\Delta t = \left(0.6477\ \dfrac{lb\cdot s}{ft}\right)\Delta t$



↻ Mom. about C: $-(\Delta m)v_A(1.5\ in.) + D\Delta t(6\ in.) =$
$= (\Delta m)v_B\cos 40°(8\ in.) + (\Delta m)v_B\sin 40°(6.5\ in.)$

$D\Delta t(6\ in.) = (0.6477\ \frac{lb\cdot s}{ft})\Delta t(60\ ft/s)(11.806\ in.)$ $D = 76.47\ lb$

$\xrightarrow{+}$ x COMP.: $C_x\Delta t + D\Delta t = (\Delta m)v_B\cos 40°$

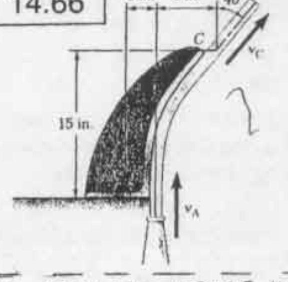
$C_x = (0.6477\ \frac{lb\cdot s}{ft})(60\ ft/s)\cos 40° - 76.47\ lb = -46.71$

+↑ y COMP.: $-(\Delta m)v_A + C_y\Delta t = (\Delta m)v_B\sin 40°$

$C_y = (0.6477\ \frac{lb\cdot s}{ft})(60\ ft/s)(\sin 40° + 1) = 63.8\ lb$

$C_x = 46.7\ lb \leftarrow$, $C_y = 63.8\ lb\uparrow$ ◄
$D_x = 76.5\ lb \rightarrow$, $D_y = 0$ ◄

14.66

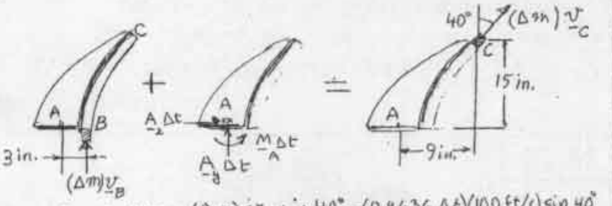


GIVEN:
STREAM OF WATER WITH $Q = 200$ gal/min
AND $v_B = v_C = 100$ ft/s

FIND:
FORCE-COUPLE SYSTEM APPLIED TO VANE AT A. (NEGLECT WEIGHT OF VANE)

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE VANE, THE WATER IN CONTACT WITH IT AND THE MASS Δm OF WATER ENTERING AND LEAVING THE SYSTEM IN Δt.
WE NOTE THAT

$$\Delta m = \rho Q \Delta t = \frac{62.4\,lb/ft^3}{32.2\,ft/s^2}\left(\frac{200\,gal}{60\,s}\right)\frac{1\,ft^3}{7.48\,gal}\,\Delta t = (0.8636\,\tfrac{lb\cdot s}{ft})\Delta t$$



$\xrightarrow{+}$ x COMP.: $A_x \Delta t = (\Delta m) v_C \sin 40° = (0.8636\,\Delta t)(100\,ft/s)\sin 40°$

$A_x = 55.51$ lb $\qquad \underline{A}_x = 55.5$ lb $\rightarrow$

$+\uparrow$ y COMP.: $(\Delta m) v_B + A_y \Delta t = (\Delta m) v_C \cos 40°$

$A_y = (0.8636)(100\,ft/s)(\cos 40° - 1) = -20.2$ lb, $\underline{A}_y = 20.2$ lb$\downarrow$

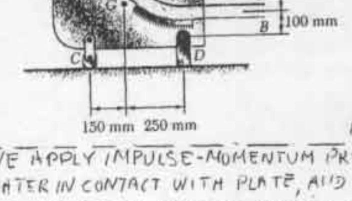
$+\circlearrowleft$ MOMENTS ABOUT A:

$(\Delta m) v_B (3\,in.) + M_A \Delta t = -(\Delta m) v_C \sin 40° (15\,in.) + (\Delta m) v_C \cos 40° (9\,in.)$

$M_A = (0.8636\,lb\cdot s/ft)(100\,ft/s)[-(15\,in.)\sin 40° + (9\,in.)\cos 40° - 3\,in.]$

$= -496.3$ lb·in. $\qquad \underline{M}_A = 496$ lb·in. $\circlearrowright$

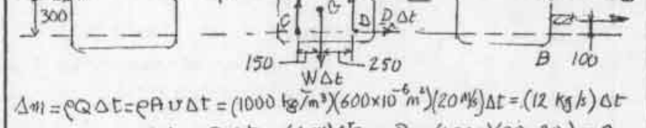
14.67



GIVEN:
STREAM OF WATER WITH CROSS-SECTION $A = 600\,mm^2$ AND $v_A = v_B = 20$ m/s. COMBINED MASS OF PLATE AND VANES IS 5 kg.

FIND:
REACTIONS AT C AND D.

WE APPLY IMPULSE-MOMENTUM PRINCIPLE TO PLATE, VANES WATER IN CONTACT WITH PLATE, AND MASS Δm OF WATER ENTERING AND LEAVING SYSTEM IN INTERVAL Δt.



$\Delta m = \rho Q \Delta t = \rho A v \Delta t = (1000\,kg/m^3)(600\times10^{-6}\,m^2)(20\,m/s)\Delta t = (12\,kg/s)\Delta t$

$\xrightarrow{+}$ x COMP: $(\Delta m) v_A + D_x \Delta t = (\Delta m) v_B$, $D_x = (1.200)(20-20) = 0$

$+\circlearrowleft$ MOM. ABOUT D: $(\Delta m) v_A (300) + C\Delta t(400) - W\Delta t(250) = (\Delta m) v_B (100)$

$400C = (12\,kg/s)(20\,m/s)(100-300) + (5\times9.81\,N)(250) = -35{,}738$

$C = -89.344$ N $\qquad \underline{C} = 89.3$ N$\downarrow$

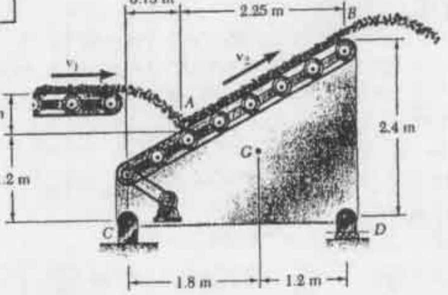
$+\uparrow$ y COMP: $(-89.344 - 5\times9.81 + D_y)\Delta t = 0$

$D_y = +138.39$

RECALLING THAT $D_x = 0$.

$\underline{D} = 138.4$ N$\uparrow$

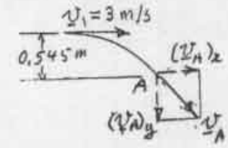
14.68



GIVEN: COAL DISCHARGED FROM FIRST TO SECOND CONVEYOR BELT AT RATE OF 120 kg/s WITH $v_1 = 3$ m/s AND $v_2 = 4.25$ m/s. MASS OF SECOND BELT ASSEMBLY AND COAL IT SUPPORTS IS 472 kg.
FIND: COMPONENTS OF REACTIONS AT C AND D.

MASS OF COAL ENTERING AND LEAVING SYSTEM IN Δt:

$$\Delta m = (120\,kg/s)\Delta t \qquad (1)$$



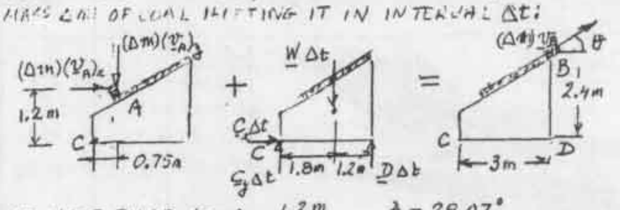
VELOCITY v_A WITH WHICH COAL HITS SECOND BELT:

$(v_A)_x = v_1 = 3$ m/s$\rightarrow \qquad (2)$

$(v_A)_y = \sqrt{2gh} = \sqrt{2(9.81)(0.545)}$

$(v_A)_y = 3.27$ m/s$\downarrow \qquad (3)$

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE SECOND BELT ASSEMBLY, THE COAL IT SUPPORTS, AND THE MASS Δm OF COAL HITTING IT IN INTERVAL Δt:



WE NOTE THAT $\tan\theta = \frac{1.2\,m}{2.25\,m}$ $\qquad \theta = 28.07°$

$+\circlearrowleft$ MOM. ABOUT C:

$(\Delta m)(v_A)_x(1.2\,m) + (\Delta m)(v_A)_y(0.75\,m) + (W\Delta t)(1.8\,m) - (D\Delta t)(3\,m)$
$= (\Delta m)(v_B\cos\theta)(2.4\,m) - (\Delta m)(v_B\sin\theta)(3\,m)$

$D(3\,m) = (472\,kg)(9.81\,m/s^2)(1.8\,m)$
$+ (120\,kg/s)[(3\,m/s)(1.2\,m) + (3.27\,m/s)(0.75\,m)]$
$-(120\,kg/s)(4.25\,m/s)[(2.4\,m)\cos 28.07° - (3\,m)\sin 28.07°]$

$D(3\,m) = 8334.6$ N·m $+ 726.30$ N·m $- 360.08$ N·m $= 8700$ N·m

$D = 2900$ N $\qquad \underline{D}_x = 0,\ \underline{D}_y = 2900$ N$\uparrow$

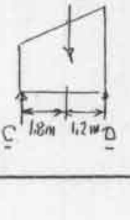
$\xrightarrow{+}$ x COMP.: $(\Delta m)(v_A)_x + C_x\Delta t = (\Delta m) v_B\cos\theta$

$C_x = (120\,kg/s)(4.25\,m/s)\cos 28.07° - (120\,kg/s)(3\,m/s)$
$= 90.0$ N $\qquad \underline{C}_x = 90.0$ N$\rightarrow$

$+\uparrow$ y COMP.: $-(\Delta m)(v_A)_y + C_y\Delta t + D\Delta t - W\Delta t = (\Delta m) v_B \sin\theta$

$C_y = -2900\,N + (472\,kg)(9.81\,m/s^2) + (120\,kg/s)(3.27 + 4.25\sin 28.07°)\,m/s$
$= 2362.7$ N $\qquad \underline{C}_y = 2360$ N$\uparrow$

NOTE: WHEN BELT IS AT REST:



$+\circlearrowleft \Sigma M_C = 0$: $D(3\,m) - W(1.8\,m) = 0$

$3D - (472\times9.81)(1.8) = 0$

$D = 2778$ N $\qquad D = 2780$ N$\uparrow$

$C = 472\times9.81 - 2778 \qquad C = 1852$ N$\uparrow$

14.69

GIVEN:
PLANE CRUISES AT 900 km/h.
SCOOPS AIR AT RATE OF 90 kg/s AND DISCHARGES IT AT 660 m/s RELATIVE TO PLANE.

FIND: TOTAL DRAG DUE TO AIR FRICTION.

WE APPLY EQ. (14.39): $\Sigma \underline{F} = \frac{dm}{dt}(\underline{v}_B - \underline{v}_A)$
WITH RESPECT TO PLANE.
WE HAVE: $\Sigma \underline{F} = \underline{D}$ = TOTAL DRAG,
$v_B = 660\ m/s,\quad v_A = 900\ km/h = 900\ \frac{1000}{3600}\frac{m}{s} = 250\ m/s$

EQ. (14.39): $D = (90\ kg/s)(660\ m/s - 250\ m/s)$

$D = 36.9\ kN$ ◀

14.70

GIVEN:
PLANE IN LEVEL FLIGHT AT 570 mi/h.
DRAG DUE TO AIR FRICTION = 7500 lb
EXHAUST VEL. = 1800 ft/s RELATIVE TO PLANE

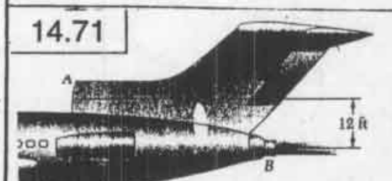
FIND: RATE IN lb/s AT WHICH AIR PASSES THRU ENGINE

WE APPLY EQ. (14.39): $\Sigma \underline{F} = \frac{dm}{dt}(\underline{v}_B - \underline{v}_A)$
WITH RESPECT TO PLANE.
WE HAVE ΣF = DRAG = 7500 lb
$v_B = 1800\ ft/s,\quad v_A = 570\ mi/h = 836\ ft/s$

EQ. (14.39): $7500\ lb = \frac{dm}{dt}(1800\ ft/s - 836\ ft/s)$

$\frac{dm}{dt} = (7.780\ \frac{lb \cdot s}{ft})(32.2\ ft/s^2) = 251\ lb/s$ ◀

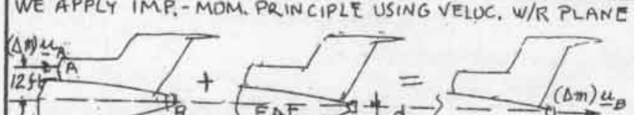
14.71



GIVEN:
ENGINE SCOOPS IN AIR AT A AT RATE OF 200 lb/s AND DISCHARGES IT AT B AT 2000 ft/s W/R PLANE

FIND:
THRUST OF ENGINE WHEN AIRPLANE SPEED IS (a) 300 mi/h, (b) 600 mi/h.

WE APPLY IMP.-MOM. PRINCIPLE USING VELOC. W/R PLANE



(F IS OPPOSITE TO PROPULSIVE FORCE)

$\xrightarrow{+}$ x COMP.: $(\Delta m)u_A + F\Delta t = (\Delta m)u_B$

$F = \frac{\Delta m}{\Delta t}(u_B - u_A) = \frac{200\ lb/s}{32.2\ ft/s^2}(2000\ ft/s - v)$ (1)

$+\circlearrowleft$ MOM. ABOUT B: $-(\Delta m)u_A(12\ ft) + (F\Delta t)d = 0$

$Fd = \frac{\Delta m}{\Delta t}(12\ ft)u_A = \frac{200\ lb/s}{32.2\ ft/s^2}(12\ ft)v$ (2)

(a) $v = 300\ mi/h = 440\ ft/s$

EQ. (1): $F = \frac{200}{32.2}(2000 - 440) = 9{,}689\ lb$

EQ. (2): $Fd = \frac{200}{32.2}(12)(440) = 32{,}795\ lb \cdot ft$

DIVIDE (2) BY (1): $d = 3.38\ ft$

ANSWER: 9690 lb, 3.38 ft BELOW B ◀

(b) $v = 600\ mi/h = 880\ ft/s$

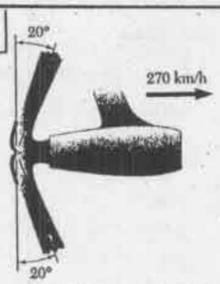
EQ. (1): $F = \frac{200}{32.2}(2000 - 880) = 6{,}956\ lb$

EQ. (2): $Fd = \frac{200}{32.2}(12)(880) = 65{,}590\ lb \cdot ft$

DIVIDE (2) BY (1): $d = 9.43\ ft$

ANSWER: 6960 lb, 9.43 ft BELOW B ◀

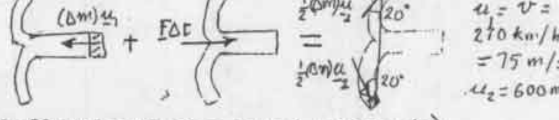
14.72



GIVEN:
IN REVERSE THRUST, ENGINE SCOOPS AIR AT RATE OF 120 kg/s AND DISCHARGES IT AS SHOWN WITH VELOCITY OF 600 m/s RELATIVE TO ENGINE.

FIND: REVERSE THRUST WHEN PLANE SPEED IS 270 km/h.

WE APPLY IMPULSE-MOMENTUM PRINCIPLE W/R PLANE



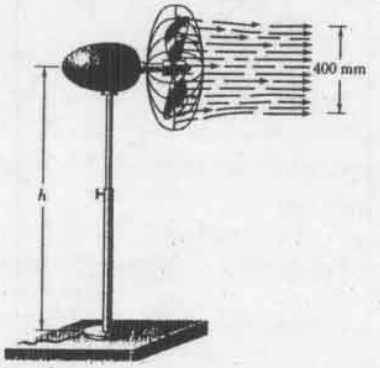
($\underline{F}$ IS OPPOSITE TO REVERSE THRUST OF ENGINE)

$\xrightarrow{+}$ x COMP.: $-(\Delta m)u_1 + F\Delta t = 2\left[\frac{1}{2}(\Delta m)u_2 \sin 20°\right]$

$F = \frac{\Delta m}{\Delta t}(u_1 + u_2 \sin 20°) = (120\ kg/s)(75 + 600 \sin 20°)\ m/s$

$F = 33.6\ kN$ ◀

14.73



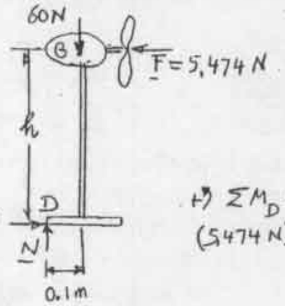
GIVEN: FLOOR FAN DELIVERS AIR WITH SPEED OF 6 m/s. IT IS SUPPORTED BY A 200-mm-DIAMETER CIRCULAR BASE AND ITS TOTAL WEIGHT IS 60 N.

FIND: MAX. HEIGHT h IF FAN IS NOT TO TIP OVER. (USE $\rho = 1.21\ kg/m^3$ FOR AIR AND ASSUME $v_A = 0$.)

THRUST:
FROM EQ. (14.39): $F = \frac{dm}{dt}(v_B - v_A) = \rho Q(v - 0) = \rho\ \ldots^2$

$F = (1.21\ kg/m^3)\frac{\pi}{4}(0.400\ m)^2(6\ m/s)^2 = 5.474\ N$

FREE BODY: FAN



FORCE EXERTED ON ... BY AIR STREAM IS EQUAL AND OPPOSITE TO THRUST.

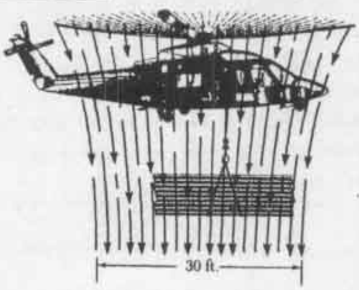
WHEN FAN IS ABOUT TO TIP OVER, NORMAL FORCE $\underline{N}$ IS APPLIED AT D.

$+\circlearrowleft \Sigma M_D = 0$:

$(5.474\ N)h - (60\ N)(0.1\ m) = 0$

$h = 1.096\ m$ ◀

14.74



GIVEN:
MAX. DOWNWARD AIR SPEED PRODUCED BY HELICOPTER IS 80 ft/s.
WEIGHT OF HELICOPTER AND CREW IS 3500 lb.

FIND:
MAX. LOAD THAT HELICOPTER CAN LIFT WHILE HOVERING.
(ASSUME $\gamma = 0.076\ \text{lb/ft}^3$ FOR AIR.)

WE USE EQ. (14.39) TO DETERMINE THE THRUST $\underline{F}$:

$$F = \frac{dm}{dt}(v_B - v_A) = \rho Q(v - 0) = \rho A v^2 = \frac{\gamma}{g} A v^2$$

$$F = \frac{0.076\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\,\frac{\pi}{4}(30\ \text{ft})^2(80\ \text{ft/s})^2 = 10{,}678\ \text{lb}$$

THE LIFT PROVIDED BY THE BLADE IS EQUAL AND OPPOSITE, THAT IS 10,678 lb $\uparrow$. WE WRITE

$$+\!\uparrow \Sigma F_y = 0: \quad 10{,}678\ \text{lb} - W - 3500\ \text{lb} = 0$$

$$W = 7178\ \text{lb} \qquad\qquad W = 7180\ \text{lb}$$

14.75

GIVEN:
AIRLINER CRUISES AT 600 mi/h WITH EACH OF ITS THREE ENGINES DISCHARGING AIR AT 2000 ft/s RELATIVE TO PLANE.

FIND:
SPEED OF PLANE AFTER IT HAS LOST THE USE OF
(a) ONE ENGINE, (b) TWO ENGINES
(ASSUME THAT DRAG IS PROPORTIONAL TO v^2.)

WE USE EQ. (14.39) TO DETERMINE THE TOTAL THRUST OF THE ENGINES:

$$F = \frac{dm}{dt}(v_B - v_A) \qquad \text{WHERE } v_B = 2000\ \text{ft/s}$$
$$v = \text{SPEED OF PLANE}$$

THUS: $F = \frac{dm}{dt}(2000 - v)$

THE DRAG IS $D = kv^2$

EQUATING THRUST AND DRAG: $\frac{dm}{dt}(2000 - v) = kv^2 \qquad (1)$

WITH THREE ENGINES, $v = 600\ \text{mi/h} = 880\ \text{ft/s}$

SUBSTITUTING IN EQ. (1):

$$\left(\frac{dm}{dt}\right)_3 (2000 - 880) = k(880)^2$$

$$\left(\frac{dm}{dt}\right)_3 = 691.43\,k$$

(a) WITH TWO ENGINES:

$$\left(\frac{dm}{dt}\right)_2 = \frac{2}{3}\left(\frac{dm}{dt}\right)_3 = \frac{2}{3}(691.43\,k) = 460.95\,k$$

SUBSTITUTING IN EQ. (1):

$$460.95\,k(2000 - v) = kv^2$$

$$v^2 + 460.95\,v - 921.9\times 10^3 = 0$$

$$v = \frac{-460.95 + \sqrt{(460.95)^2 + 4(921.9\times 10^3)}}{2} = 756.96\ \text{ft/s}$$

$$v = 516\ \text{mi/h}$$

(b) WITH ONE ENGINE:

$$\left(\frac{dm}{dt}\right)_1 = \frac{1}{3}\left(\frac{dm}{dt}\right)_3 = \frac{1}{3}(691.43\,k) = 230.48\,k$$

SUBSTITUTING IN EQ. (1):

$$230.48\,k(2000 - v) = kv^2$$

$$v^2 + 230.48\,v - 460.95\times 10^3 = 0$$

$$v = \frac{-230.48 + \sqrt{(230.48)^2 + 4(460.95\times 10^3)}}{2} = 573.41\ \text{ft/s}$$

$$v = 391\ \text{mi/h}$$

14.76

GIVEN:
16-Mg PLANE MAINTAINS $v = 774$ km/h WITH $\alpha = 18°$.
IT SCOOPS AIR AT RATE OF 300 kg/s AND DISCHARGES IT AT 665 m/s RELATIVE TO PLANE.

FIND: (a) INITIAL ACCELERATION IF PILOT CHANGES TO HORIZONTAL FLIGHT WITH SAME ENGINE SETTING
(b) MAX. HORIZONTAL SPEED THAT WILL BE ATTAINED.
(ASSUME THAT DRAG IS PROPORTIONAL TO v^2.)

DETERMINATION OF THRUST
SINCE AIRPLANE IS ACCELERATED IN HORIZONTAL FLIGHT, WE USE A REFERENCE FRAME AT REST WITH RESPECT TO THE ATMOSPHERE WHEN USING EQ. (14.39) TO DETERMINE THE THRUST $\underline{F}$ (CF. FOOTNOTE, PAGE 860).

$$F = \frac{dm}{dt}(v_B - v_A)$$

WHERE $v_A = 0$, $v_B = v_{DISCH.} - v_{PLANE}$

$$= 665\ \text{m/s} - 774\ \text{km/h}\left(\frac{1000\ \text{m}}{3600\ \text{s}}\right)$$
$$= 665\ \text{m/s} - 215\ \text{m/s} = 450\ \text{m/s}$$

$$F = (300\ \text{kg/s})(450\ \text{m/s} - 0) = 135.0\ \text{kN}$$

AIRPLANE CLIMBING (NO ACCELERATION)

$$\Sigma F \measuredangle 18° = 0$$

$$135.0\ \text{kN} - D - W\sin 18° = 0$$

$$D = 135.0\ \text{kN} - (16\ \text{Mg})(9.81\ \text{m/s}^2)\sin 18°$$
$$= 135.0\ \text{kN} - 48.50\ \text{kN} = 86.50\ \text{kN}$$

(a) AT START OF HORIZONTAL FLIGHT
THRUST AND DRAG ARE STILL THE SAME

$$\Sigma F = ma \qquad F - D = ma$$

$$(135.0 - 86.5)\times 10^3\ \text{N} = (16\times 10^3\ \text{kg})a$$

$$a = 3.03\ \text{m/s}^2$$

(b) AT MAX. SPEED IN HORIZONTAL FLIGHT
WE HAVE $a = 0$

$$F_m - D_m = 0 \qquad (1)$$

WHERE

$$F_m = \frac{dm}{dt}(u - v_m) = (300\ \text{kg/s})(665\ \text{m/s} - v_m) \qquad (2)$$

ON THE OTHER HAND

$$D_m = k v_m^2 \qquad (3)$$

BUT, INITIALLY, WE HAD $D = 86.50$ kN AND $v = 774\ \text{km/h} = 215\ \text{m/s}$ AND, THEREFORE

$$D = kv^2$$

$$86.50\times 10^3\ \text{N} = k(215\ \text{m/s})^2 \qquad (4)$$

DIVIDING (3) AND (4) MEMBER BY MEMBER:

$$\frac{D_m}{86.50\times 10^3} = \frac{v_m^2}{(215)^2} \qquad D_m = 1.8713\,v_m^2 \qquad (5)$$

SUBSTITUTING FOR F_m FROM (2) AND FOR D_m FROM (5) INTO (1):

$$300(665 - v_m) - 1.8713\,v_m^2 = 0$$

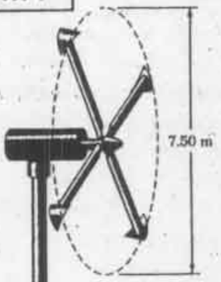
$$v_m^2 + 160.32\,v_m - 106.61\times 10^3 = 0$$

$$v_m = \frac{-160.32 + \sqrt{(160.32)^2 + 4(106.61\times 10^3)}}{2} = 256.05\ \text{m/s}$$

$$= (256.05\ \text{m/s})\,\frac{3600\ \text{s}}{1000\ \text{m}} = 921.78\ \text{km/h}$$

$$v_m = 922\ \text{km/h}$$

14.77



GIVEN:
WIND TURBINE-GENERATOR'S OUTPUT-POWER RATING IS 5 kW FOR 30 km/h WIND SPEED.
FIND FOR THAT WIND SPEED
(a) KINETIC ENERGY OF AIR PARTICLES ENTERING CIRCLE PER SECOND.
(b) EFFICIENCY OF THIS ENERGY-CONVERSION SYSTEM.
(ASSUME $\rho = 1.21\ kg/m^3$ FOR AIR)

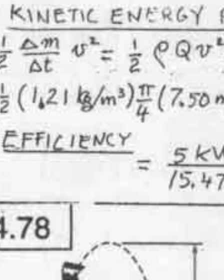
(a) KINETIC ENERGY PER SECOND

$= \frac{1}{2}\frac{\Delta m}{\Delta t} v^2 = \frac{1}{2}\rho Q v^2 = \frac{1}{2}\rho(Av)v^2 = \frac{1}{2}\rho A v^3$

$= \frac{1}{2}(1.21\ kg/m^3)\frac{\pi}{4}(7.50\ m)^2\left(\frac{30\times10^3\ m}{3.6\times10^3\ s}\right)^3 = 15.47\ kJ/s$

(b) EFFICIENCY $= \frac{5\ kW}{15.47\ kJ/s} = 0.323$

14.78



GIVEN:
WIND TURBINE GENERATOR PRODUCES 28 kW OF ELECTRIC POWER WITH AN EFFICIENCY OF 0.35 AS AN ENERGY-CONVERSION SYSTEM
FIND:
(a) KINETIC ENERGY OF AIR PARTICLES ENTERING CIRCLE PER SECOND
(b) WIND SPEED
(ASSUME $\rho = 1.21\ kg/m^3$ FOR AIR.)

(a) KINETIC ENERGY PER SECOND

$= \text{INPUT POWER} = \frac{\text{OUTPUT POWER}}{\text{EFFICIENCY}} = \frac{28\ kW}{0.35} = 80\ kJ/s$

(b) WIND SPEED

K.E. PER SECON $= \frac{1}{2}\frac{\Delta m}{\Delta t}v^2 = \frac{1}{2}\rho Q v^2 = \frac{1}{2}\rho(Av)v^2 = \frac{1}{2}\rho A v^3$

THEREFORE: $80\ kJ/s = \frac{1}{2}(1.21\ kg/m^3)\frac{\pi}{4}(7.50\ m)^2 v^3$

$v^3 = 2993.1 \quad v = 14.411\ m/s \qquad v = 51.9\ km/h$

14.79

GIVEN:
PLANE CRUISING IN LEVEL FLIGHT AT 600 mi/h SCOOPS IN AIR AT RATE OF 200 lb/s AND DISCHARGES IT AT 2200 ft/s RELATIVE TO PLANE.
FIND: (a) POWER USED TO PROPEL PLANE, (b) TOTAL ENGINE POWER (c) EFFICIENCY OF PLANE

(a) FROM EQ. (14.39):

THRUST $= F = \frac{dm}{dt}(v_B - v_A)$, WHERE $v_B = 2200\ ft/s$. $v_A = 600\ mi/h = 880\ ft/s$

$F = \frac{200\ lb/s}{32.2\ ft/s^2}(2200 - 880)\ ft/s = 8{,}198.8\ lb$

PROPULSIVE POWER $= Fv = (8{,}198.8\ lb)(880\ ft/s)$
$= 7.2149\times10^6\ ft\cdot lb/s = 13{,}120\ hp$

(b) POWER LOST IN EXHAUST $= \frac{1}{2}\frac{dm}{dt}v_{exh}^2 = \frac{1}{2}\frac{200}{32.2}(2200-880)^2$
$= 5.4112\times10^6\ ft\cdot lb/s = 9{,}838\ hp$

TOTAL POWER $= 13{,}120\ hp + 9{,}838\ hp = 22{,}960\ hp$

(c) EFFICIENCY $= \frac{13{,}120\ hp}{22{,}960\ hp} = 0.571$

14.80

GIVEN:
PROPELLER OF SMALL PLANE HAS 6-ft-DIAMETER SLIPSTREAM AND PRODUCES 800-lb THRUST WHEN PLANE IS AT REST ON GROUND.
FIND: (a) SPEED OF THE AIR IN THE SLIPSTREAM, (b) VOLUME OF AIR PASSING THROUGH PROPELLER PER SECOND. (c) KINETIC ENERGY IMPARTED TO THE AIR PER SECOND (ASSUME $\gamma = 0.076\ lb/ft^3$ FOR AIR.)

(a) SPEED v OF AIR

APPLY EQ. (14.39), ASSUMING AIR ENTERS SLIPSTREAM WITH ZERO VELOCITY:

THRUST $= F = \frac{dm}{dt}v = \rho Q v = \frac{\gamma}{g}(Av)v = \frac{\gamma}{g}Av^2$

$800\ lb = \frac{0.076\ lb/ft^3}{32.2\ ft/s^2}\frac{\pi}{4}(6\ ft)^2 v^2$

$800\ lb = (0.066734\ lb\cdot s^2/ft^2)v^2; \quad v^2 = 11{,}988\ ft^2/s^2$

$v = 109.49\ ft/s \qquad v = 109.5\ ft/s$

(b) VOLUME OF AIR PER SECOND

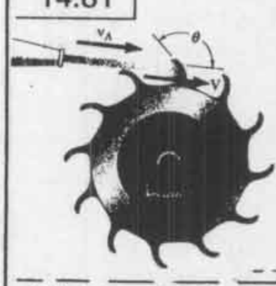
$Q = Av = \frac{\pi}{4}(6\ ft)^2(109.49\ ft/s) \qquad Q = 3100\ ft^3/s$

(c) KINETIC ENERGY IMPARTED TO AIR PER SECOND

$\frac{\Delta T}{\Delta t} = \frac{1}{2}\frac{dm}{dt}v^2 = \frac{1}{2}(\rho A v)v^2 = \frac{1}{2}\left(\frac{\gamma}{g}Av^2\right)v = \frac{1}{2}Fv$

$= \frac{1}{2}(800\ lb)(109.49\ ft/s) = 43{,}800\ ft\cdot lb/s$

14.81



GIVEN: PELTON-WHEEL TURBINE. RATE AT WHICH WATER IS DEFLECTED BY BLADES EQUALS RATE AT WHICH WATER ISSUES FROM NOZZLE: $\Delta m/\Delta t = \rho A v_A$.
FIND: (a) VELOCITY V OF BLADES FOR MAXIMUM POWER.
(b) MAXIMUM POWER.
(c) MECHANICAL EFFICIENCY.
(USE NOTATION OF SP 14.7)

IMPULSE-MOMENTUM PRINCIPLE

AS IN SAMPLE PROB. 14.7:

$\xrightarrow{+}x$ COMP: $(\Delta m)u - F_x\Delta t = (\Delta m)u\cos\theta$

BUT NOW $\Delta m = A\rho v_A \Delta t \qquad u = v_A - V$

THUS: $F_x = A\rho v_A(v_A - V)(1-\cos\theta)$

OUTPUT POWER $= F_x V = A\rho v_A(v_A - V)(1-\cos\theta)V \quad (1)$

OR: OUTPUT POWER $= A\rho v_A(v_A V - V^2)(1-\cos\theta)$

(a) FOR MAX. POWER: $d(\text{POWER})/dV = 0$:

$A\rho v_A(v_A - 2V)(1-\cos\theta) = 0 \qquad V = \frac{1}{2}v_A$

(b) MAX. POWER: MAKE $V = \frac{1}{2}v_A$ IN EQ. (1):

MAX. POWER $= A\rho v_A(v_A - \frac{1}{2}v_A)(1-\cos\theta)\frac{1}{2}v_A = \frac{1}{4}A\rho(1-\cos\theta)v_A^3$

(c) EFFICIENCY

INPUT POWER $= \frac{1}{2}\frac{\Delta m}{\Delta t}v_A^2 = \frac{1}{2}(\rho Q)v_A^2 = \frac{1}{2}(\rho A v_A)v_A^2$
$= \frac{1}{2}A\rho v_A^3 \quad (2)$

DIVIDE (1) BY (2):

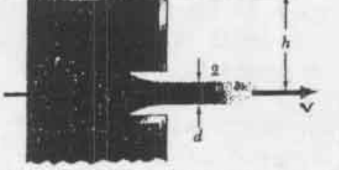
$\eta = \frac{\text{OUTPUT POWER}}{\text{INPUT POWER}} = \frac{A\rho v_A(v_A - V)(1-\cos\theta)V}{\frac{1}{2}A\rho v_A^3}$

$\eta = 2\frac{V}{v_A}\left(1 - \frac{V}{v_A}\right)(1-\cos\theta)$

NOTE: MAXIMUM EFFICIENCY IS OBTAINED WHEN $V = \frac{1}{2}v_A$ AND $\theta = 180°$:

$\eta_{MAX} = 2(\frac{1}{2})(\frac{1}{2})(2) = 1$

14.82



GIVEN:
CIRCULAR REENTRANT ORIFICE (BORDA'S MOUTHPIECE)
$v_1 = 0,\ v_2 = v = \sqrt{2gh}$
SHOW THAT:
$d = D/\sqrt{2}$

WE APPLY IMPULSE-MOMENTUM PRINCIPLE TO SECTION OF WATER INDICATED BY DASHED LINE AND TO MASS OF WATER Δm ENTERING AND LEAVING IN Δt.

$\xrightarrow{+}$ x COMP.: $0 + P\Delta t = (\Delta m)v = (\rho Q\Delta t)v = (\rho A_2 v\Delta t)v$

THUS: $P = \rho A_2 v^2 = \rho \frac{\pi}{4} d^2 v^2 \qquad (1)$

BUT, RECALLING THAT THE PRESSURE AT A DEPTH h IS $p = \rho g h$, WE HAVE

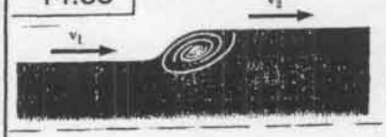
$$P = pA_1 = \rho g h A_1 = \rho g h \frac{\pi}{4} D^2$$

SUBSTITUTING THIS EXPRESSION IN (1) AND THE EXPRESSION GIVEN FOR v:

$$\rho g h \frac{\pi}{4} D^2 = \rho \frac{\pi}{4} d^2 (2gh)$$

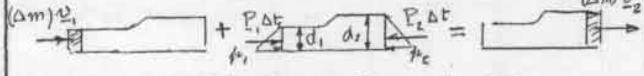
$D^2 = 2d^2 \qquad d = \frac{D}{\sqrt{2}} \qquad$ (Q.E.D.)

14.83



GIVEN:
HYDRAULIC JUMP.
CHANNEL WIDTH = b.
EXPRESS RATE OF FLOW Q IN TERMS OF b, d_1, d_2

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE WATER SECTION SHOWN AND TO THE MASS OF WATER Δm ENTERING AND LEAVING IN INTERVAL Δt.



$\xrightarrow{+}$ x COMP.: $(\Delta m)v_1 + P_1\Delta t - P_2\Delta t = (\Delta m)v_2$

$$(\rho Q\Delta t)v_1 + P_1\Delta t - P_2\Delta t = (\rho Q\Delta t)v_2$$

$$\rho Q(v_1 - v_2) = P_2 - P_1 \qquad (1)$$

BUT $Q = A_1 v_1 = b d_1 v_1 \qquad v_1 = Q/bd_1 \qquad (2)$

AND $Q = A_2 v_2 = b d_2 v_2 \qquad v_2 = Q/bd_2 \qquad (3)$

ALSO: $P_1 = \frac{1}{2}p_1 A_1 = \frac{1}{2}(\gamma d_1)(b d_1) = \frac{1}{2}\gamma b d_1^2 \qquad (4)$

SIMILARLY: $P_2 = \frac{1}{2}\gamma b d_2^2 \qquad (5)$

SUBSTITUTE FROM (2), (3), (4), (5) INTO (1):

$$\rho Q\left(\frac{Q}{bd_1} - \frac{Q}{bd_2}\right) = \frac{1}{2}\gamma b(d_2^2 - d_1^2)$$

$$\rho Q^2 \frac{d_2 - d_1}{bd_1d_2} = \frac{1}{2}\gamma b(d_2+d_1)(d_2-d_1)$$

DIVIDING THROUGH BY $d_2 - d_1$ AND RECALLING THAT $\gamma = \rho g$:

$$\frac{Q^2}{bd_1d_2} = \frac{1}{2} g b(d_1+d_2)$$

$$Q = b\sqrt{\tfrac{1}{2} g d_1 d_2 (d_1+d_2)}$$

* 14.84

GIVEN: FOR CHANNEL OF PROB. 14.83:
$b = 12$ ft, $d_1 = 4$ ft, $d_2 = 5$ ft
FIND: RATE OF FLOW.

SEE SOLUTION OF PROB. 14.83 FOR DERIVATION OF

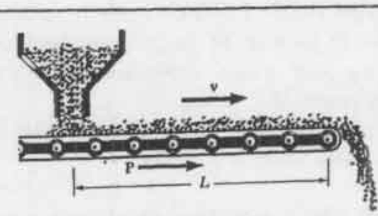
$$Q = b\sqrt{\tfrac{1}{2} g d_1 d_2 (d_1+d_2)}$$

SUBSTITUTING THE GIVEN DATA:

$$Q = (12\text{ ft})\sqrt{\tfrac{1}{2}(32.2\text{ ft/s}^2)(4\text{ ft})(5\text{ ft})(9\text{ ft})}$$

$$Q = 646\text{ ft}^3/\text{s}$$

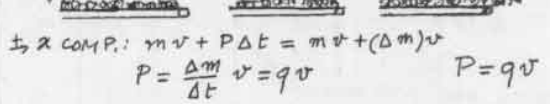
14.85



GIVEN:
GRAVEL FALLS ON CONVEYOR BELT WITH NO VELOCITY AND AT THE CONSTANT RATE $q = dm/dt$

(a) FIND MAGNITUDE OF FORCE $\underline{P}$ REQUIRED TO MAINTAIN A CONSTANT BELT SPEED.
(b) SHOW THAT K.E. REQUIRED BY GRAVEL IN GIVEN TIME INTERVAL IS HALF THE WORK DONE BY $\underline{P}$. WHAT HAPPENS TO THE OTHER HALF OF WORK OF $\underline{P}$?

(a) WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE GRAVEL ON THE BELT AND TO THE MASS Δm OF GRAVEL HITTING AND LEAVING BELT IN INTERVAL Δt.



$\xrightarrow{+}$ x COMP.: $mv + P\Delta t = mv + (\Delta m)v$

$$P = \frac{\Delta m}{\Delta t} v = qv \qquad P = qv$$

(b) KINETIC ENERGY ACQUIRED PER UNIT TIME:

$$\frac{\Delta T}{\Delta t} = \frac{1}{2}\frac{\Delta m}{\Delta t}v^2 = \frac{1}{2}qv^2 \qquad (1)$$

WORK DONE PER UNIT TIME:

$$\frac{\Delta U}{\Delta t} = \frac{P\Delta x}{\Delta t} = Pv$$

RECALLING THE RESULT OF PART a:

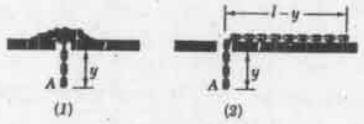
$$\frac{\Delta U}{\Delta t} = (qv)v = qv^2 \qquad (2)$$

COMPARING EQS. (1) AND (2), WE CONCLUDE THAT

$$\frac{\Delta T}{\Delta t} = \frac{1}{2}\frac{\Delta U}{\Delta t} \qquad \text{(Q.E.D.)}$$

THE OTHER HALF OF THE WORK OF $\underline{P}$ IS DISSIPATED INTO HEAT BY FRICTION AS THE GRAVEL SLIPS ON THE BELT BEFORE REACHING THE SPEED v.

14.86



GIVEN: CHAIN OF LENGTH ℓ AND MASS m FALLS THROUGH SMALL HOLE IN PLATE. CHAIN IS AT REST WHEN y IS VERY SMALL.

FIND IN EACH CASE SHOWN:

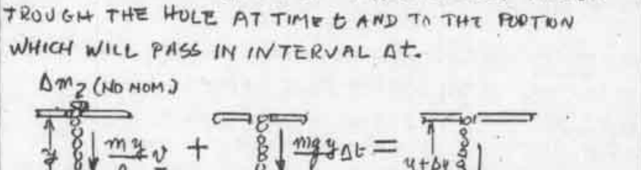
(a) ACCELERATION OF FIRST LINK A AS FUNCTION OF y.

(b) VELOCITY OF CHAIN AS LAST LINK PASSES THRU HOLE.

IN CASE 1, ASSUME THAT EACH LINK IS AT REST UNTIL IT FALLS THRU HOLE

IN CASE 2, ASSUME THAT ALL LINKS HAVE THE SAME SPEED AT ANY GIVEN INSTANT.

CASE 1. WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE PORTION OF CHAIN WHICH HAS ALREADY PASSED TROUGH THE HOLE AT TIME t AND TO THE PORTION WHICH WILL PASS IN INTERVAL Δt.



$$+\downarrow \cancel{\frac{my}{\ell}v} + \frac{mgy}{\ell}\Delta t = \frac{m}{\ell}(\cancel{yv} + y\Delta v + v\Delta y + \Delta y\Delta v)$$

DIVIDE BY Δt AND LET $\Delta t \to 0$: $gy = y\frac{dv}{dt} + v\frac{dy}{dt}$

$$= \frac{d}{dt}(yv)$$

MULTIPLY BOTH SIDES BY $yv\,dt$ AND NOTE THAT $v\,dt = dy$:

$$gy^2 dy = yv\, d(yv)$$

SET $yv = u$ AND INTEGRATE:

$$\int_0^y gy^2 dy = \int_0^{yv} u\,du$$

$$\frac{1}{3}gy^3 = \frac{1}{2}(yv)^2 \qquad v^2 = \frac{2}{3}gy \qquad (1)$$

(a) DIFFERENTIATE (1) WITH RESPECT TO t:

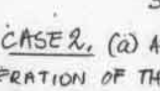
$$2v\frac{dv}{dt} = \frac{2}{3}g\frac{dy}{dt} \quad \text{OR} \quad 2va = \frac{2}{3}gv$$

$$a = \frac{1}{3}g$$

(b) AS LAST LINK PASSES THROUGH HOLE, $y = \ell$ AND EQ (1) YIELDS $v^2 = \frac{2}{3}g\ell$

$$v = \sqrt{\frac{2}{3}g\ell}$$

CASE 2. (a) AT time t, THE FORCE CAUSING THE ACCELERATION OF THE ENTIRE CHAIN IS THE WEIGHT OF THE LENGTH y OF CHAIN WHICH HAS PASSED THROUGH:



$$\Sigma F = ma:$$

$$mg\left(\frac{y}{\ell}\right) = ma \qquad a = \frac{gy}{\ell}$$

(b) SETTING $a = v\frac{dv}{dy}$, WE HAVE

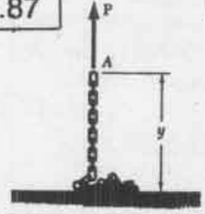
$$v\,dv = \frac{g}{\ell}y\,dy$$

INTEGRATING IN v FROM 0 TO v AND IN y FROM 0 TO ℓ:

$$\frac{1}{2}v^2 = \frac{1}{2}\frac{g}{\ell}\frac{\ell^2}{2}$$

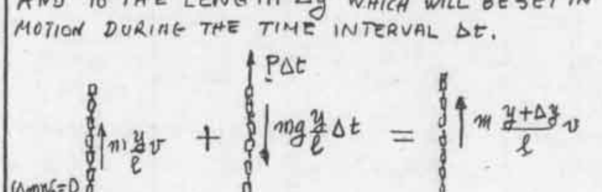
$$v = \sqrt{g\ell}$$

14.87



GIVEN:
CHAIN OF LENGTH ℓ AND MASS m IS LYING IN A PILE ON FLOOR. IT IS RAISED AT A CONSTANT v.
FIND FOR ANY y:
(a) MAGNITUDE OF FORCE P,
(b) REACTION OF FLOOR.

(a) WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE LENGTH y OF CHAIN WHICH IS OFF THE FLOOR AND TO THE LENGTH Δy WHICH WILL BE SET IN MOTION DURING THE TIME INTERVAL Δt.



$+\uparrow y$ COMP.: $m\frac{y}{\ell}v + P\Delta t - mg\frac{y}{\ell}\Delta t = m\cdot\frac{y+\Delta y}{\ell}v$

$$P\Delta t = \frac{m}{\ell}(gy\Delta t - \cancel{yv} + \cancel{yv} + v\Delta y)$$

DIVIDING BY Δt:

$$P = \frac{m}{\ell}\left(gy + v\frac{\Delta y}{\Delta t}\right)$$

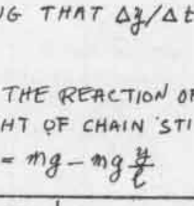
NOTING THAT $\Delta y/\Delta t = v$,

$$P = \frac{m}{\ell}(gy + v^2)$$

(b) THE REACTION OF THE FLOOR IS EQUAL TO THE WEIGHT OF CHAIN STILL ON THE FLOOR:

$$R = mg - mg\frac{y}{\ell} \qquad R = mg\left(1 - \frac{y}{\ell}\right)$$

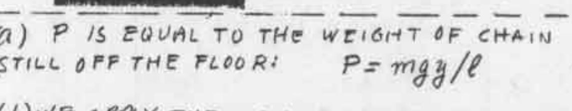
14.88



GIVEN:
CHAIN OF LENGTH ℓ AND MASS m IS LOWERED INTO A PILE ON THE FLOOR AT CONSTANT v
FIND FOR ANY y:
(a) MAGNITUDE OF FORCE P.
(b) REACTION OF THE FLOOR.

(a) P IS EQUAL TO THE WEIGHT OF CHAIN STILL OFF THE FLOOR: $P = mgy/\ell$

(b) WE APPLY THE IMPULSE MOMENTUM PRINCIPLE TO THE LENGTH $\ell - y$ OF CHAIN ON THE FLOOR AND TO THE LENGTH Δy WHICH HITS THE FLOOR IN Δt:



$+\uparrow y$ COMP.: $-m\frac{\Delta y}{\ell}v - \frac{mg}{\ell}(\ell - y + \Delta y)\Delta t + R\Delta t = 0$

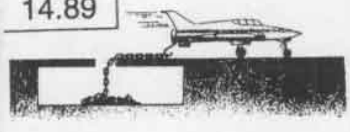
SOLVING FOR R:

$$R = \frac{m}{\ell}\left[g(\ell - y + \Delta y) + v\frac{\Delta y}{\Delta t}\right]$$

BUT $\frac{\Delta y}{\Delta t} = v$ AND $\Delta y \to 0$ WHEN $\Delta t \to 0$.

THEREFORE: $R = \frac{m}{\ell}\left[g(\ell - y) + v^2\right]$

14.89

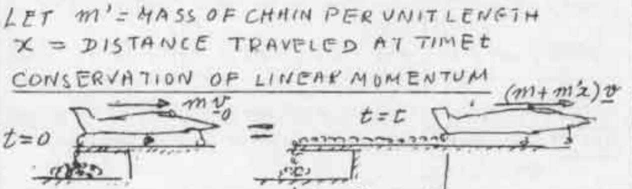


GIVEN:
AS PLANE OF MASS m LANDS WITH v_0 ON CARRIER, ITS TAIL HOOKS INTO END OF CHAIN OF LENGTH ℓ.

FIND: (a) MASS OF CHAIN REQUIRED TO REDUCE PLANE SPEED TO βv_0 (WHERE $\beta < 1$), (b) MAX. FORCE EXERTED BY CHAIN ON PLANE.

LET m' = MASS OF CHAIN PER UNIT LENGTH
x = DISTANCE TRAVELED AT TIME t

CONSERVATION OF LINEAR MOMENTUM



$\xrightarrow{+} x$ COMP.: $\quad m v_0 = (m+m'x)v \qquad (1)$

(a) WE WANT $v = \beta v_0$ FOR $x = \ell$. SUBSTITUTE:

$$m v_0 = (m + m'\ell)\beta v_0$$
$$m v_0(1-\beta) = m'\ell\beta v_0$$

MASS OF CHAIN $= m'\ell = \dfrac{1-\beta}{\beta}m \qquad (2)$ ◀

(b) SOLVE EQ. (1) FOR v:

$$v = \frac{m v_0}{(m+m'x)} \qquad (3)$$

$$a = \frac{dv}{dt} = -\frac{m v_0}{(m+m'x)^2} m' \frac{dx}{dt} = -\frac{m m' v_0 v}{(m+m'x)^2}$$

OR, RECALLING (3): $\quad a = -\dfrac{m^2 m' v_0^2}{(m+m'x)^3} \qquad (4)$

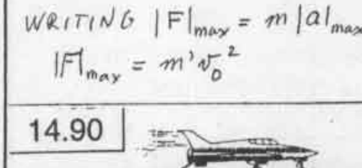
DECELERATION IS MAXIMUM FOR $x = 0$. WE HAVE

$$(-a)_{max} = \frac{m^2 m' v_0^2}{m^3} = \frac{m'}{m} v_0^2 \qquad (5)$$

WRITING $|F|_{max} = m|a|_{max}$ AND RECALLING (2):

$|F|_{max} = m' v_0^2 \qquad |F|_{max} = \dfrac{1-\beta}{\beta}\dfrac{m v_0^2}{\ell}$ ◀

14.90



GIVEN:
AS 6000-kg PLANE LANDS AT 180 km/h ON CARRIER, ITS TAIL HOOKS INTO END OF 80-m-LONG CHAIN OF MASS OF 50 kg/m.

FIND: (a) MAX. DECELERATION OF PLANE, (b) VELOCITY WHEN ENTIRE CHAIN IS PULLED OUT.

SEE SOLUTION OF PROB. 14.89 FOR DERIVATION OF EQS. (3) AND (5).

(a) FROM EQ. (5):

$$\text{MAX. DECEL.} = (-a)_{max} = \frac{m'}{m} v_0^2 = \frac{50\ kg/m}{6000\ kg}\left(\frac{180}{3.6}\ m/s\right)^2$$

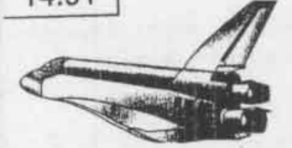
MAX. DECEL. $= 20.8\ m/s^2$ ◀

(b) FROM EQ. (3), FOR $x = \ell = 80$ m:

$$v_{max} = \frac{m v_0}{(m+m'\ell)} = \frac{(6000\ kg)(180\ km/h)}{6000\ kg + (50\ kg/m)(80 m)}$$

$v_{max} = 108.0$ km/h ◀

14.91



GIVEN:
EACH OF THE THREE ENGINES OF SPACE SHUTTLE BURNS PROPELLANT AT RATE OF 340 kg/s AND EJECTS IT WITH A RELATIVE VELOCITY OF 3750 m/s

FIND: TOTAL THRUST PROVIDED BY THE THREE ENGINES

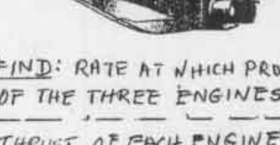
FROM EQ. (14.44) FOR EACH ENGINE

$$P = \frac{dm}{dt} u = (340\ kg/s)(3750\ m/s) = 1.275 \times 10^6\ N$$

FOR THE 3 ENGINES:

TOTAL THRUST $= 3(1.275 \times 10^6 N) = 3.83$ MN ◀

14.92



GIVEN:
THE THREE ENGINES OF SPACE SHUTTLE PROVIDE A TOTAL THRUST OF 6 MN. PROPELLANT IS EJECTED WITH A RELATIVE VEL. OF 3750 m/s.

FIND: RATE AT WHICH PROPELLANT IS BURNED BY EACH OF THE THREE ENGINES.

THRUST OF EACH ENGINE: $P = \frac{1}{3}(6 MN) = 2 \times 10^6 N$

EQ. (14.44): $P = \dfrac{dm}{dt} u$

$2 \times 10^6 N = \dfrac{dm}{dt}(3750\ m/s) \qquad \dfrac{dm}{dt} = \dfrac{2 \times 10^6 N}{3750\ m/s} = 533\ kg/s$ ◀

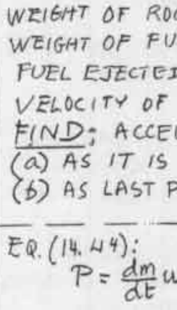
14.93

GIVEN:
ROCKET FIRED VERTICALLY FROM GROUND
WEIGHT OF ROCKET (INCLUDING FUEL) = 2400 lb
WEIGHT OF FUEL = 2000 lb
FUEL EJECTED AT RATE OF 25 lb/s WITH RELATIVE VELOCITY OF 12,000 ft/s.

FIND: ACCELERATION OF ROCKET
(a) AS IT IS FIRED.
(b) AS LAST PARTICLE OF FUEL IS BEING CONSUMED

EQ. (14.44):

$$P = \frac{dm}{dt} u = \frac{25\ lb/s}{g}(12{,}000\ ft/s) = \frac{300 \times 10^3}{g}$$



$+\uparrow \Sigma \underline{F} = m\underline{a}$:

$$P - W = ma$$
$$a = \frac{P}{m} - \frac{W}{m} = \frac{(300 \times 10^3)/g}{W/g} - g$$
$$a = \frac{300 \times 10^3}{W} - g \qquad (1)$$

(a) AS ROCKET IS FIRED:

$W = 2400$ lb

FROM (1): $\quad a = \dfrac{300 \times 10^3}{2400} - 32.2 = 125.0 - 32.2 = 92.8$

$\underline{a} = 92.8\ ft/s^2 \uparrow$ ◀

(b) AS LAST PARTICLE OF FUEL IS BEING CONSUMED:

$W = 2400 - 2000 = 400$ lb

FROM (1): $\quad a = \dfrac{300 \times 10^3}{400} - 32.2 = 750 - 32.2 = 717.8$

$\underline{a} = 718\ ft/s^2 \uparrow$ ◀

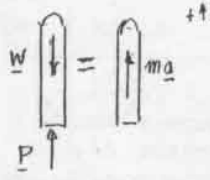
14.94

GIVEN:

ROCKET FIRED VERTICALLY FROM GROUND.
WEIGHT OF ROCKET (INCLUDING FUEL) = 3000 lb.
WEIGHT OF FUEL = 2500 lb.
FUEL CONSUMED AT RATE OF 30 lb/s.
ACCELERATION INCREASES BY 750 ft/s² FROM TIME ROCKET IS FIRED TO TIME WHEN LAST PARTICLE OF FUEL IS CONSUMED.

FIND:

RELATIVE VELOCITY WITH WHICH IS EJECTED.



$+\uparrow \Sigma F = ma$:

$$P - W = ma$$

$$a = \frac{P}{m} - \frac{W}{m} = \frac{P}{W/g} - g$$

$$a = \frac{P}{W}g - g \qquad (1)$$

AS ROCKET IS FIRED, EQ. (1) YIELD.

$$a_0 = \frac{Pg}{3000\,lb} - g \qquad (2)$$

WHEN LAST PARTICLE IS FIRED:

$$a_0 + 750\,ft/s^2 = \frac{Pg}{500\,lb} - g \qquad (3)$$

SUBTRACT (2) FROM (3): $750 = Pg\left(\frac{1}{500} - \frac{1}{3000}\right)$

$$750 = (1.6667 \times 10^{-3})Pg$$

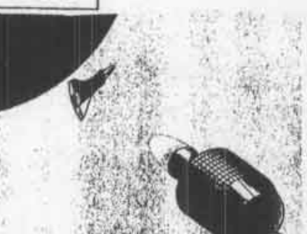
$$P = \frac{450 \times 10^3}{g}$$

BUT, FROM EQ. (14.44):

$$P = \frac{dm}{dt}u: \quad \frac{450 \times 10^3}{g} = \frac{30\,lb/s}{g}u$$

$$u = 15{,}000\ ft/s$$

14.95



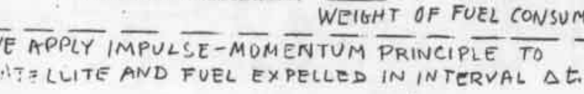
GIVEN:

ENGINE OF COMMUNICATION SATELLITE IS FIRED TO INCREASE ITS VELOCITY BY 8000 ft/s.
WEIGHT OF SATELLITE (INCLUDING FUEL) = 10,000 lb.
FUEL EJECTED WITH RELATIVE VEL. OF 13,750 ft/s

FIND:

WEIGHT OF FUEL CONSUMED

WE APPLY IMPULSE-MOMENTUM PRINCIPLE TO SATELLITE AND FUEL EXPELLED IN INTERVAL Δt.



$\xrightarrow{+}$ $mv = (m - \Delta m)(v + \Delta v) - \Delta m(u - v - \Delta v)$

$\cancel{mv} = \cancel{mv} - \cancel{v\Delta m} + m\Delta v - u\Delta m + \cancel{v\Delta m}$ + Second-order terms

$$m\Delta v = u\Delta m$$

BUT $\Delta m = q\Delta t$ AND $m = m_0 - qt$

THUS $(m_0 - qt)\Delta v = uq\,\Delta t$

AS $\Delta t \to 0$: $\frac{dv}{dt} = \frac{uq}{m_0 - qt}$

$$v = \int_0^t \frac{uq\,dt}{m_0 - qt} = -u\left[\ln(m_0 - qt)\right]_0^t$$

$$v = u \ln \frac{m_0}{m_0 - qt} \qquad (1)$$

(CONTINUED)

14.95 continued

EXPRESSING EQ. (1) IN EXPONENTAL FORM:

$$\frac{m_0}{m_0 - qt} = e^{v/u} \qquad (2)$$

SETTING $m_0 = (10{,}000\,lb)/g$, $v = 8000\,ft/s$, $u = 13{,}750\,ft/s$, AND EXPRESSING q IN lb/s, WE HAVE

$$\frac{10{,}000/g}{(10{,}000 - qt)/g} = e^{\frac{8000}{13{,}750}} = e^{0.58182} = 1.7893$$

$$10{,}000 - qt = \frac{10{,}000}{1.7893} = 5{,}588.8 \qquad qt = 4{,}411.2\ lb$$

WEIGHT OF FUEL EXPANDED $= qt = 4410\ lb$

14.96

GIVEN: COMMUNICATION SATELLITE OF PROB. 14.95.

FIND: INCREASE IN VELOCITY AFTER 2500 lb HAS BEEN CONSUMED.

SEE SOLUTION OF PROB 14.95 FOR DERIVATION OF EQ. (1).

$$v = u \ln \frac{m_0}{m_0 - qt} \qquad (1)$$

FROM DATA OF PROBS. 14.95 AND 14.96:

$u = 13{,}750\,ft/s$, $m_0 = (10{,}000\,lb)/g$, $qt = (2{,}500\,lb)/g$

SUBSTITUTE IN (1):

$$v = (13{,}750\,ft/s)\ln\frac{10{,}000/g}{7{,}500/g} = (13{,}750\,ft/s)\ln(1.3333)$$

$$v = 3960\ ft/s$$

14.97

GIVEN:

A 540-kg SPACECRAFT IS MOUNTED ON TOP OF ROCKET OF MASS OF 19 Mg, INCLUDING 17.8 Mg OF FUEL.
FUEL IS CONSUMED AT THE RATE OF 225 kg/s AND EJECTED WITH A RELATIVE VELOCITY OF 3600 m/s.

FIND:

MAXIMUM SPEED OF SPACECRAFT IF ROCKET IS FIRED VERTICALLY FROM THE GROUND.

SEE SAMPLE PROB. 14.8 FOR DERIVATION OF

$$v = u \ln \frac{m_0}{m_0 - qt} - gt \qquad (1)$$

DATA:

$u = 3600\,m/s$, $q = 225\,kg/s$, $m_{fuel} = 17\,800\,kg$

$m_0 = 19\,000\,kg + 540\,kg = 19\,540\,kg$

WE HAVE $m_{fuel} = qt$, $17800\,kg = (225\,kg/s)\,t$

$$t = \frac{17800\,kg}{225\,kg/s} = 79.111\,s$$

MAX. VELOCITY IS REACHED WHEN ALL FUEL HAS BEEN CONSUME, THAT IS, WHEN $qt = m_{fuel}$. EQ. (1) YIELDS

$$v_m = u \ln \frac{m_0}{m_0 - m_{fuel}} - gt$$

$$= (3600\,m/s)\ln\frac{19540}{19540 - 17800} - (9.81\,m/s^2)(79.111\,s)$$

$$= (3600\,m/s)\ln 11.230 - 776.1\,m/s = 7930.8\,m/s$$

$$v_m = 7930\ m/s$$

14.98

GIVEN:
A 540-kg SPACECRAFT IS MOUNTED ON A TWO-STAGE ROCKET.
EACH STAGE HAS A MASS OF 9.5 Mg, INCLUDING 8.9 Mg OF FUEL.
FUEL IS CONSUMED AT A RATE OF 225 kg/s AND EJECTED WITH A RELATIVE VELOCITY OF 3600 m/s.
AS STAGE A EXPELS ITS LAST PARTICLE OF FUEL, ITS CASING IS JETTISONED.
FIND:
(a) SPEED OF ROCKET AT THAT INSTANT.
(b) MAXIMUM SPEED OF SPACECRAFT

SEE SAMPLE PROB. 14.8 FOR DERIVATION OF

$$v = u \ln \frac{m_0}{m_0 - qt} - gt \qquad (1)$$

(a) FIRST STAGE

$u = 3600$ m/s, $q = 225$ kg/s, MASS OF FUEL $= m_f = 8900$ kg

$m_0 = 2(9500\text{ kg}) + 540\text{ kg} = 19540$ kg

WE HAVE $m_f = qt_1 \quad t_1 = \frac{m_f}{q} = \frac{8900\text{ kg}}{225\text{ kg/s}} \quad t_1 = 39.556$ s

SUBSTITUTE INTO (1):

$$v_1 = (3600\text{ m/s}) \ln \frac{19540}{19540 - 8900} - (9.81\text{ m/s}^2)(39.556\text{ s})$$

$$= (3600\text{ m/s}) \ln 1.8365 - 388.04\text{ m/s} = 1800.3\text{ m/s}$$

$$v_1 = 1800\text{ m/s}$$

(b) SECOND STAGE

$u = 3600$ m/s, $q = 225$ kg/s, MASS OF FUEL $= m_f = 8900$ kg

$m_1 = 9500\text{ kg} + 540\text{ kg} = 10040$ kg

$m_f = qt_2 \quad t_2 = \frac{m_f}{q} = \frac{8900\text{ kg}}{225\text{ kg/s}} \quad t_2 = 39.556$ s

REPLACING v BY $v_2 - v_1$ AND m_0 BY m_1 IN EQ. (1):

$$v_2 - v_1 = u \ln \frac{m_1}{m_1 - qt_2} - gt_2$$

$$= (3600\text{ m/s}) \ln \frac{10040}{10040 - 8900} - (9.81\text{ m/s}^2)(39.556\text{ s})$$

$$= (3600\text{ m/s}) \ln 8.8070 - 388.04\text{ m/s} = 7444\text{ m/s}$$

$$v_2 = v_1 + 7444 = 1800 + 7444 = 9244, \quad v_2 = 9240\text{ m/s}$$

14.99

GIVEN: SPACECRAFT OF PROB. 14.97.
FIND: ALTITUDE REACHED WHEN ALL THE FUEL OF THE LAUNCHING ROCKET IS CONSUMED.

WE RECALL DATA FROM PROB. 14.97 AND EQ(1). SETTING $v = dy/dt$, WE HAVE

$$dy = \left(u \ln \frac{m_0}{m_0 - qt} - gt\right) dt \qquad (1')$$

$$\int_0^h dy = \int_0^t \left(u \ln \frac{m_0}{m_0 - qt} - gt\right) dt = u \int_0^t \ln \frac{m_0}{m_0 - qt}\, dt - \frac{1}{2} g t^2$$

SETTING $\frac{m_0 - qt}{m_0} = z$, WE FIND THAT

$$u \int_0^t \ln \frac{m_0}{m_0 - qt}\, dt = u \int_1^z (-\ln z)\left(-\frac{m_0}{q} dz\right) = u \frac{m_0}{q}\left[z \ln z - z\right]_1^z$$

THUS: $h = \int_0^h dy = \frac{u m_0}{q}(z \ln z - z - 0 + 1) - \frac{1}{2} g t^2$

$$h = u\left[t + \frac{m_0 - qt}{q} \ln \frac{m_0 - qt}{m_0}\right] - \frac{1}{2} g t^2 \qquad (2)$$

GIVEN DATA: $u = 3600$ m/s, $q = 225$ kg/s, $m_0 = 19540$ kg

$t = 79.111$ s, $qt = m_f = 17800$ kg, $m_0 - qt = 19540 - 17800 = 1740$ kg

$$h = 3600\left[79.111 + \frac{1740}{225} \ln \frac{1740}{19540}\right] - \frac{1}{2}(9.81)(79.111)^2$$

$$= 3600(79.111 - 18.704) - 30698 = 186\,770\text{ m}$$

[NOTE THAT g WAS ASSUMED CONSTANT] $\quad h = 186.8$ km

14.100

GIVEN: SPACECRAFT AND TWO-STAGE LAUNCHING ROCKET OF PROB. 14.98.
FIND ALTITUDE AT WHICH
(a) STAGE A IS RELEASED.
(b) FUEL OF BOTH STAGES HAS BEEN CONSUMED.

SEE SOLUTIONS OF SAMPLE PROB. 14.8 AND PROB. 14.99 FOR DERIVATION OF EQ. (2):

$$h = u\left[t + \frac{m_0 - qt}{q} \ln \frac{m_0 - qt}{m_0}\right] - \frac{1}{2} g t^2 \qquad (2)$$

(a) FIRST STAGE

FROM PROB 14.98 WE HAVE

$u = 3600$ m/s, $q = 225$ kg/s, $m_0 = 19\,540$ kg, $t_1 = 39.556$ s

$qt_1 = m_f = 8900$ kg, $m_0 - qt_1 = 19540 - 8900 = 10\,640$ kg

EQ. (2) YIELDS

$$h_1 = (3600)\left[39.556 + \frac{10640}{225} \ln \frac{10\,640}{19540}\right] - \frac{1}{2}(9.81)(39.556)^2$$

$$= (3600)(39.556 - 28.744) - 7675 = 31\,248\text{ m}$$

$$h_1 = 31.2\text{ km}$$

(b) SECOND STAGE

USING AGAIN EQ. (2) AND ADDING h_1 AND $v_1 t_2$ TO IT,

$$h_2 = h_1 + v_1 t_2 + u\left[t_2 + \frac{m_1 - qt_2}{q} \ln \frac{m_1 - qt_2}{m_1}\right] - \frac{1}{2} g t_2^2 \qquad (3)$$

FROM PROB. 14.98, WE HAVE

$v_1 = 1800.3$ m/s, $u = 3600$ m/s, $q = 225$ kg/s, $t_2 = 39.556$ s

$m_1 = 10\,040$ kg, $qt_2 = m_f = 8900$ kg, $m_1 - qt_2 = 1140$ kg

EQ. (3) YIELDS

$$h_2 = 31248 + (1800.3)(39.556) + 3600\left[39.556 + \frac{1140}{225} \ln \frac{1140}{10040}\right] - \frac{1}{2}(9.81)(39.556)^2$$

$$h_2 = 31248 + 71\,213 + 3600(39.556 - 11.023) - 7675$$

$$= 197\,500\text{ m} \qquad h_2 = 197.5\text{ km}$$

14.101

GIVEN:
COMMUNICATION SATELLITE OF PROB. 14.95
FUEL CONSUMED AT RATE OF 37.5 lb/s.
FIND: DISTANCE FROM SATELLITE TO SHUTTLE AT $t = 60$ s.

SEE SOLUTION OF PROB. 14.95 FOR DERIVATION OF

$$v = u \ln \frac{m_0}{m_0 - qt} \qquad (1)$$

SETTING $v = dx/dt$, WE HAVE

$$dx = \left(u \ln \frac{m_0}{m_0 - qt}\right) dt \qquad (1')$$

$$x = \int_0^t \left(u \ln \frac{m_0}{m_0 - qt}\right) dt = -u \int_0^t \ln \frac{m_0 - qt}{m_0}\, dt$$

SETTING $\frac{m_0 - qt}{m_0} = z$ WE HAVE $dt = -\frac{m_0}{q} dz$ AND

$$x = \frac{m_0 u}{q} \int_1^z \ln z\, dz = \frac{m_0 u}{q}\left[z \ln z - z\right]_1^z = \frac{m_0 u}{q}(z \ln z - z + 1)$$

$$= \frac{m_0 u}{q}\left(\frac{m_0 - qt}{m_0} \ln \frac{m_0 - qt}{m_0} - 1 + \frac{qt}{m_0} + 1\right)$$

$$x = u\left(t + \frac{m_0 - qt}{q} \ln \frac{m_0 - qt}{m_0}\right) \qquad (2)$$

GIVEN DATA: $q = (37.5\text{ lb/s})/g$, $t = 60$ s,

AND FROM PROB. 14.95:

$u = 13{,}750$ ft/s, $m_0 = (10{,}000\text{ lb})/g$

THUS: $m_0 - qt = (10{,}000/g) - (37.5/g)(60) = (7750\text{ lb})/g$

AFTER SUBSTITUTION, EQ. (2) YIELDS

$$x = (13{,}750\text{ ft/s})\left(60 + \frac{7750}{37.5} \ln \frac{7750}{10{,}000}\right)\text{s}$$

$$= (13{,}750)(60 - 52.678) = 100{,}680\text{ ft}$$

$$= (100{,}680\text{ ft})\ \frac{1\text{ mi}}{5280\text{ ft}} = 19.068\text{ mi}$$

$$x = 19.07\text{ mi}$$

14.102 GIVEN:

ROCKET OF PROB. 14.93.

FIND: (a) ALTITUDE AT WHICH ALL FUEL IS CONSUMED,
(b) VELOCITY OF ROCKET AT THAT TIME.

SEE SAMPLE PROB. 14.8 FOR DERIVATION OF

$$v = u \ln \frac{m_0}{m_0 - qt} - gt \qquad (1)$$

AND SOLUTION OF PROB. 14.99 FOR DERIVATION OF

$$h = u\left[t + \frac{m_0 - qt}{q} \ln \frac{m_0 - qt}{m_0}\right] - \frac{1}{2} g t^2 \qquad (2)$$

FROM STATEMENT OF PROB. 14.93, WE RECALL

$u = 12{,}000$ ft/s, $m_0 = (2400\text{ lb})/g$, $qt = m_f = (2000\text{ lb})/g$

$q = (25\text{ lb/s})/g \qquad t = \frac{m_f}{q} = \frac{2000\text{ lb}}{25\text{ lb/s}} = 80\text{ s}$

(a) ALTITUDE AT WHICH ALL FUEL IS CONSUMED

SUBSTITUTING DATA IN EQ (2):

$$h = (12{,}000\text{ ft/s})\left[80 + \frac{2400-2000}{25} \ln \frac{2400-2000}{2400}\right]\text{s} - $$
$$-\frac{1}{2}(32.2\text{ ft/s}^2)(80\text{ s})^2$$

$$h = (12{,}000)(80 - 28.668) - 103{,}040 = 512{,}944\text{ ft}$$

$$h = \frac{512{,}944}{5280} = 97.148\text{ mi} \qquad h = 97.1\text{ mi}$$

(b) VELOCITY OF ROCKET AT THAT TIME

SUBSTITUTING DATA IN EQ. (1):

$$v = (12{,}000\text{ ft/s}) \ln \frac{2400}{2400-2000} - (32.2\text{ ft/s}^2)(80\text{ s})$$
$$= 12{,}000 \ln 6 - 2576 = 18{,}925\text{ ft/s}$$
$$v = 18{,}930\text{ ft/s}$$

14.103 GIVEN: JET AIRPLANE WITH

v = SPEED OF AIRPLANE

u = RELATIVE SPEED OF EXPELLED GASES

SHOW THAT MECHANICAL EFFICIENCY IS $\eta = \frac{2v}{u+v}$

EXPLAIN WHY $\eta = 1$ WHEN $u = v$.

THRUST $\underline{P}$ IS OBTAINED FROM EQ. (14.39):

$\Sigma \underline{F} = \frac{dm}{dt}(\underline{v}_B - \underline{v}_A)$ WHERE $v_A = v$ = AIRPLANE SPEED

$v_B = u$ = EXHAUST VEL. REL. TO PLANE

THUS: $F = \frac{dm}{dt}(u - v)$

USEFUL POWER $= Fv = \frac{dm}{dt}(u - v)v$

WASTED POWER = K.E. IMPARTED PER SECOND TO EXHAUST GASES WHOSE ABSOLUTE VEL. IS $u - v$.

$$= \frac{1}{2}\frac{dm}{dt}(u-v)^2$$

TOTAL POWER = USEFUL POWER + WASTED POWER

$$= \frac{dm}{dt}\left[(u-v)v + \frac{1}{2}(u-v)^2\right] = \frac{dm}{dt}\left(\cancel{uv} - v^2 + \tfrac{1}{2}u^2 + \tfrac{1}{2}v^2 - \cancel{uv}\right)$$
$$= \frac{1}{2}\frac{dm}{dt}(u^2 - v^2) = \frac{1}{2}\frac{dm}{dt}(u+v)(u-v)$$

$$\text{EFFICIENCY} = \eta = \frac{\text{USEFUL POWER}}{\text{TOTAL POWER}} = \frac{(u-v)v}{\frac{1}{2}(u+v)(u-v)}$$

$$\eta = \frac{2v}{u+v} \qquad \text{(Q.E.D.)}$$

WHEN $u = v$, THE ABSOLUTE VELOCITY $u - v$ OF THE EXPELLED GASES IS ZERO. THUS, NO ENERGY IS IMPARTED TO THE EXPELLED GASES AND NO POWER IS WASTED.

14.104 GIVEN:

ROCKET WITH SPEED v, EXPELLING FUEL WITH RELATIVE SPEED u.

SHOW THAT MECHANICAL EFFICIENCY IS $\eta = 2uv/(u^2+v^2)$.

EXPLAIN WHY $\eta = 1$ WHEN $u = v$.

WE RECALL EQ. (14.44) FOR THRUST $\underline{P}$ OF ROCKET:

$$P = \frac{dm}{dt}u$$

USEFUL POWER $= Pv = \frac{dm}{dt}uv$

WASTED POWER = K.E. ENERGY IMPARTED PER SECOND TO EXPELLED FUEL WHOSE ABSOLUTE VELOCITY IS $u - v$.

$$= \frac{1}{2}\frac{dm}{dt}(u-v)^2$$

TOTAL POWER = USEFUL POWER + WASTED POWER

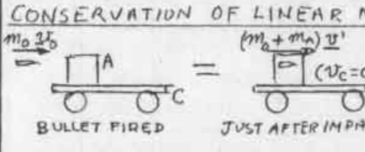
$$= \frac{dm}{dt}uv + \frac{1}{2}\frac{dm}{dt}(u-v)^2$$
$$= \frac{1}{2}\frac{dm}{dt}(2uv + u^2 + v^2 - 2uv)$$
$$= \frac{1}{2}\frac{dm}{dt}(u^2 + v^2)$$

$$\text{EFFICIENCY} = \eta = \frac{\text{USEFUL POWER}}{\text{TOTAL POWER}} = \frac{\frac{dm}{dt}uv}{\frac{1}{2}\frac{dm}{dt}(u^2+v^2)}$$

$$\eta = \frac{2uv}{u^2+v^2} \qquad \text{(Q.E.D.)}$$

WHEN $u = v$, THE ABSOLUTE VELOCITY $u - v$ OF THE EXPELLED FUEL IS ZERO. THUS, NO ENERGY IS IMPARTED TO THE EXPELLED FUEL AND NO POWER IS WASTED.

14.105



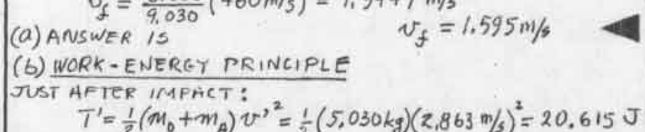
GIVEN:

30-g BULLET FIRED WITH $v_0 = 480$ m/s INTO 5-kg BLOCK A, WHICH RESTS ON 4-kg CART C.

$\mu_k = 0.50$ BETWEEN BLOCK A AND CART C.

FIND (a) FINAL VELOCITY v_f OF CART AND BLOCK,
(b) FINAL POSITION OF BLOCK ON CART.

CONSERVATION OF LINEAR MOMENTUM



$$m_0 v_0 = (m_0 + m_A)v' = (m_0 + m_A + m_C)v_f$$

$$(0.030\text{ kg})(480\text{ m/s}) = (5.030\text{ kg})v' = (9.030\text{ kg})v_f$$

$$v' = \frac{0.030}{5.030}(480\text{ m/s}) = 2.863\text{ m/s}$$

$$v_f = \frac{0.030}{9.030}(480\text{ m/s}) = 1.5947\text{ m/s}$$

(a) ANSWER IS $\qquad v_f = 1.595$ m/s

(b) WORK-ENERGY PRINCIPLE

JUST AFTER IMPACT:

$$T' = \frac{1}{2}(m_0 + m_A)v'^2 = \frac{1}{2}(5.030\text{ kg})(2.863\text{ m/s})^2 = 20.615\text{ J}$$

FINAL KINETIC ENERGY:

$$T_f = \frac{1}{2}(m_0 + m_A + m_C)v_f^2 = \frac{1}{2}(9.030\text{ kg})(1.5947\text{ m/s})^2 = 11.482\text{ J}$$

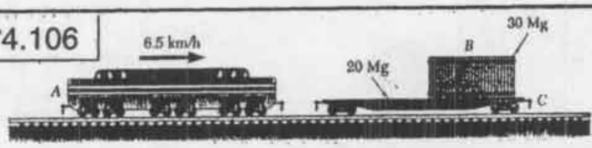
WORK OF FRICTION FORCE:

$$F = \mu_k N = \mu_k(m_0 + m_A)g = 0.50(5.030)(9.81) = 24.672\text{ N}$$

$$\text{WORK} = U = -Fx = -24.672x$$

$$T' + U = T_f: \; 20.615 - 24.672x = 11.482 \qquad x = 0.370\text{ m}$$

14.106

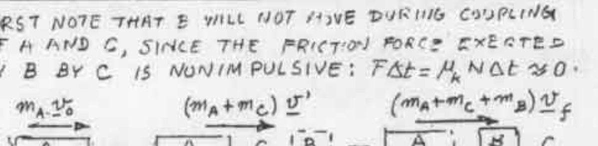


GIVEN: 80-Mg ENGINE A WITH $v_0 = 6.5$ km/h STRIKES 20-Mg FLATCAR C WHICH IS AT REST AND CARRIES 30-Mg LOAD B. A AND C ARE COUPLED UPON IMPACT. B CAN SLIDE ON C WITH $\mu_k = 0.25$.

FIND VELOCITY OF CAR C
(a) IMMEDIATELY AFTER IMPACT
(b) AFTER B HAS SLID TO A STOP RELATIVE TO C.

CONSERVATION OF LINEAR MOMENTUM

FIRST NOTE THAT B WILL NOT MOVE DURING COUPLING OF A AND C, SINCE THE FRICTION FORCE EXERTED ON B BY C IS NONIMPULSIVE: $F\Delta t = \mu_k N \Delta t \approx 0$.



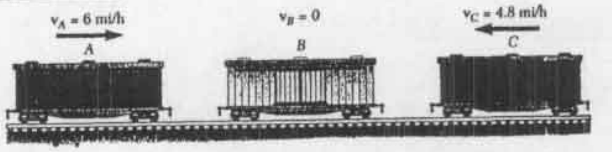
$$m_A v_0 = (m_A + m_C)v' = (m_A + m_C + m_B) v_f$$

$$(80\text{ Mg})(6.5\text{ km/h}) = (100\text{ Mg})v' = (130\text{ Mg})v_f$$

(a) $v' = \frac{80}{100}(6.5\text{ km/h})$ $\qquad v' = 5.20$ km/h ◀

(b) $v_f = \frac{80}{130}(6.5\text{ km/h})$ $\qquad v_f = 4.00$ km/h ◀

14.107



GIVEN: THREE IDENTICAL CARS WITH VELOCITIES SHOWN. CAR B IS FIRST HIT BY CAR A.

FIND FINAL VELOCITY OF EACH CAR IF
(a) ALL CARS GET AUTOMATICALLY COUPLED,
(b) A AND B GET COUPLED, BUT B AND C BOUNCE OFF EACH OTHER WITH $e = 1$ (i.e. NO ENERGY LOSS).

(a) ALL CARS AUTOMATICALLY COUPLED

CONSERVATION OF LINEAR MOMENTUM:

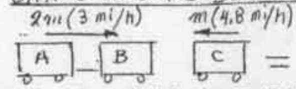
$$m_A v_A + m_B v_B + m_C v_C = (m_A + m_B + m_C) v_f$$

$$m(6\text{ mi/h}) + 0 - m(4.8\text{ mi/h}) = (3m) v_f$$

$$v_f = \frac{6 - 4.8}{3} = +0.4 \qquad \underline{v}_f = 0.400 \text{ mi/h} \rightarrow$$ ◀

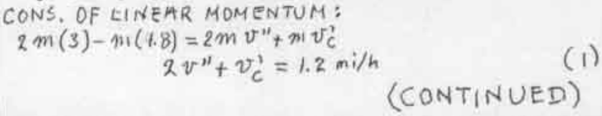
(b) CARS A AND B ONLY GET COUPLED

CONSERVATION OF LINEAR MOMENTUM FOR A AND B:



$$m(6\text{ mi/h}) = (2m)v'$$
$$v' = 3\text{ mi/h} \rightarrow$$

CAR C HITS AND BOUNCES OFF CARS A AND B



CONS. OF LINEAR MOMENTUM:

$$2m(3) - m(4.8) = 2m v'' + m v_C'$$

$$2v'' + v_C' = 1.2\text{ mi/h} \qquad (1)$$

(CONTINUED)

14.107 continued

CONSERVATION OF ENERGY ($e = 1$):
RELATIVE VELOCITY AFTER AND BEFORE IMPACT ARE EQUAL:

$$v_C' - v'' = (3 + 4.8)\text{ mi/h} \qquad (2)$$

SUBTRACTING (2) FROM (1):

$$3v'' = 1.2 - 7.8 \qquad v'' = -2.20\text{ mi/h}$$

THUS: $\qquad \underline{v}_A'' = \underline{v}_B'' = 2.20$ mi/h ← ◀

SUBSTITUTING $v'' = -2.20$ mi/h IN (1):

$$2(-2.20\text{ mi/h}) + v_C' = 1.2\text{ mi/h}$$

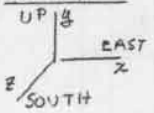
$$v_C' = +5.60\text{ mi/h} \qquad \underline{v}_C' = 5.60\text{ mi/h} \rightarrow$$ ◀

14.108

GIVEN:
9000-lb HELICOPTER A IS TRAVELING DUE EAST AT 75 mi/h AT ALTITUDE OF 2500 ft WHEN IT IS HIT BY 12,000-lb HELICOPTER B. THEIR ENTANGLED WRECKAGE FALLS TO THE GROUND IN 12 s AT POINT LOCATED 1500 ft EAST AND 384 ft SOUTH OF POINT OF IMPACT.

FIND VELOCITY COMPONENTS OF HELICOPTER B JUST BEFORE COLLISION. (NEGLECT AIR RESISTANCE.)

VELOCITY OF WRECKAGE IMMEDIATELY AFTER COLLISION



$$\underline{v}' = v_x' \underline{i} + v_y' \underline{j} + v_z' \underline{k}$$

BUT:

$$x = v_x' t \qquad v_x' = \frac{x}{t} = \frac{1500\text{ ft}}{12\text{ s}} = 125\text{ ft/s}$$

$$z = v_z' t \qquad v_z' = \frac{z}{t} = \frac{384\text{ ft}}{12\text{ s}} = 32\text{ ft/s}$$

$$-h = v_y' t - \frac{1}{2} g t^2 \qquad v_y' = -\frac{h}{t} + \frac{1}{2} g t$$

$$= -\frac{2500\text{ ft}}{12\text{ s}} + \frac{1}{2}(32.2\text{ ft/s}^2)(12\text{ s})$$

$$v_y' = -15.133\text{ ft/s}$$

THUS: $\quad \underline{v}' = (125\text{ ft/s})\underline{i} - (15.133\text{ ft/s})\underline{j} + (32\text{ ft/s})\underline{k}$

IMPACT: CONSERVATION OF LINEAR MOMENTUM

$$m_A \underline{v}_A + m_B \underline{v}_B = (m_A + m_B)\underline{v}'$$

AFTER SUBSTITUTING DATA AND EXPRESSION FOUND FOR $\underline{v}'$, AND NOTING THAT $v_A = 75\text{ mi/h} = 110\text{ ft/s}$,

$$\frac{9000\text{ lb}}{g}(110\text{ ft/s})\underline{i} + \frac{12{,}000\text{ lb}}{g}\underline{v}_B$$

$$= \frac{21{,}000\text{ lb}}{g}\left[(125\text{ ft/s})\underline{i} - (15.133\text{ ft/s})\underline{j} + (32\text{ ft/s})\underline{k}\right]$$

SOLVING FOR $\underline{v}_B$:

$$\underline{v}_B = 1.75\left[(125\text{ ft/s})\underline{i} - (15.133\text{ ft/s})\underline{j} + (32\text{ ft/s})\underline{k}\right] - (82.5\text{ ft/s})\underline{i}$$

IT FOLLOWS THAT

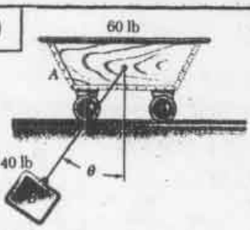
$$(v_B)_x = 1.75(125\text{ ft/s}) - 82.5\text{ ft/s} = 136.25\text{ ft/s}$$
$$= 92.90\text{ mi/h}$$

$$(v_B)_y = -1.75(15.133\text{ ft/s}) = -26.48\text{ ft/s}$$

$$(v_B)_z = 1.75(32\text{ ft/s}) = 56.0\text{ ft/s} = 38.18\text{ mi/h}$$

ANSWER:
92.9 mi/h EAST, 38.2 mi/h SOUTH, 26.5 ft/s DOWN ◀

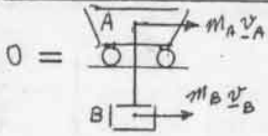
14.109



GIVEN: BLOCK B IS SUSPENDED FROM 6-ft CORD ATTACHED TO CART A. SYSTEM IS RELEASED FROM REST WHEN $\theta = 35°$.

FIND: VELOCITIES OF A AND B WHEN $\theta = 0$.

CONSERVATION OF LINEAR MOMENTUM



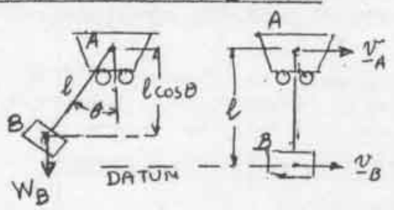
$0 = m_A \underline{v}_A + m_B \underline{v}_B$

$\xrightarrow{+}$ x COMP:

$m_A v_A + m_B v_B = 0$

$v_A = -\dfrac{m_B}{m_A} v_B \qquad (1)$

CONSERVATION OF ENERGY



INITIALLY: $T_0 = 0 \qquad V_0 = W_B l(1-\cos\theta)$

$= m_B g l(1-\cos\theta)$

AS B PASSES UNDER A:

$T = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \qquad V = 0$

$T_0 + V_0 = T + V$:

$m_B g l(1-\cos\theta) = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$

$m_A v_A^2 + m_B v_B^2 = 2 m_B g l(1-\cos\theta)$

SUBSTITUTING FOR v_A FROM (1):

$m_A\left(\dfrac{m_B}{m_A}\right)^2 v_B^2 + m_B v_B^2 = 2 m_B g l(1-\cos\theta)$

$\dfrac{m_B}{m_A}(m_A + m_B) v_B^2 = 2 m_B g l(1-\cos\theta)$

$\dfrac{m_A + m_B}{m_A} v_B^2 = 2 g l(1-\cos\theta)$

$v_B = \sqrt{\dfrac{2 m_A}{m_A + m_B} g l(1-\cos\theta)}$

GIVEN DATA:

$\dfrac{m_A}{m_A + m_B} = \dfrac{W_A}{W_A + W_B} = \dfrac{60\text{ lb}}{60\text{ lb} + 40\text{ lb}} = 0.6$

$l = 6$ ft, $\theta = 35°$

$v_B = \sqrt{2(0.6)(32.2\text{ ft/s}^2)(6\text{ ft})(1-\cos 35°)} = 6.4752$ ft/s

CARRYING THIS VALUE INTO (1):

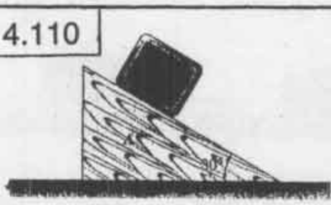
$v_A = -\dfrac{m_B}{m_A} v_B = -\dfrac{W_B}{W_A} v_B = -\dfrac{40\text{ lb}}{60\text{ lb}}(6.4752\text{ ft/s})$

$= -4.3168$ ft/s

ANSWER:

$\underline{v}_A = 4.32$ ft/s ←; $\underline{v}_B = 6.48$ ft/s →

14.110

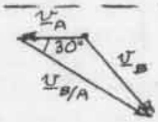


GIVEN: 9-kg BLOCK B STARTS FROM REST AND SLIDES DOWN 15-kg WEDGE A.

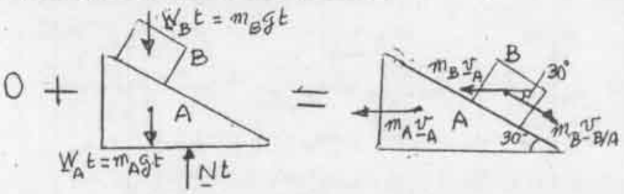
FIND: (a) VELOCITY OF B RELATIVE TO A AFTER IT HAS SLID 0.6 m

(b) CORRESPONDING VELOCITY OF WEDGE A. (NEGLECT FRICTION.)

WE RESOLVE $\underline{v}_B$ INTO ITS COMPONENTS: $\underline{v}_A$ ← AND $\underline{v}_{B/A}$ ⦪ 30°



IMPULSE-MOMENTUM PRINCIPLE



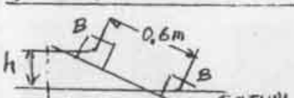
$\Sigma m\underline{v}_0 + \Sigma \underline{F} t = \Sigma m\underline{v}$

$\xrightarrow{+}$ x COMP.: $0 + 0 = m_B v_{B/A}\cos 30° - m_A v_A - m_B v_A$

$v_A = \dfrac{m_B \cos 30°}{m_A + m_B} v_{B/A} = \dfrac{(9\text{ kg})\cos 30°}{15\text{ kg} + 9\text{ kg}} v_{B/A}$

$v_A = 0.32476\, v_{B/A} \qquad (1)$

CONSERVATION OF ENERGY



$T_0 = 0$

$V_0 = m_B g h$

$= (9\text{ kg})(9.81\text{ m/s}^2)(0.6\text{ m})\sin 30°$

$= 26.487$ J

$T = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 \qquad V = 0$

REFERRING TO VELOCITY TRIANGLE SHOWN ABOVE AND USING THE LAW OF COSINES:

$T = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B (v_A^2 + v_{B/A}^2 - 2 v_A v_{B/A}\cos 30°)$

RECALLING (1) AND SUBSTITUTING THE GIVEN DATA

$T = \frac{1}{2}(15)(0.32476)^2 v_{B/A}^2 +$

$+ \frac{1}{2}(9)\left[(0.32476)^2 + 1 - 2(0.32476)\cos 30°\right] v_{B/A}^2$

$= 0.79102\, v_{B/A}^2 + 2.44336\, v_{B/A}^2 = 3.2344\, v_{B/A}^2$

$T + V = T_0 + V_0$:

$3.2344\, v_{B/A}^2 = 26.487$ J

$v_{B/A} = 2.8617$ m/s

(a) $\underline{v}_{B/A} = 2.86$ m/s ⦪ 30°

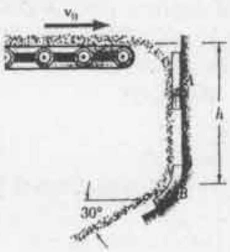
(b) FROM EQ. (1):

$v_A = 0.32476(2.8617\text{ m/s})$

$= 0.92936$ m/s

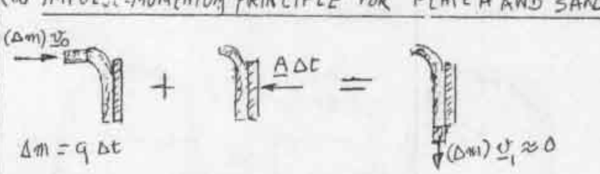
$\underline{v}_A = 0.929$ m/s ←

14.111



GIVEN:
MASS q OF SAND DISCHARGED PER UNIT TIME FROM CONVEYOR BELT AND DEFLECTED BY PLATE AT A SO THAT IT FALLS IN A VERTICAL STREAM UNTIL IT IS DEFLECTED BY PLATE AT B.
FIND FORCE REQUIRED TO HOLD
(a) PLATE A,
(b) PLATE B.
(NEGLECT FRICTION BETWEEN SAND AND PLATES.)

(a) IMPULSE-MOMENTUM PRINCIPLE FOR PLATE A AND SAND



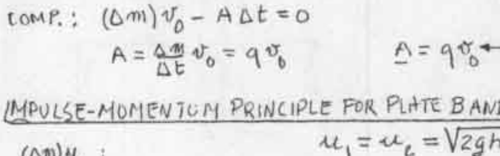
$\Delta m = q\,\Delta t$

$\xrightarrow{+}$ x COMP.: $(\Delta m)v_0 - A\Delta t = 0$

$A = \frac{\Delta m}{\Delta t}v_0 = qv_0$ $\quad\quad \underline{A} = qv_0 \leftarrow$

(b) IMPULSE-MOMENTUM PRINCIPLE FOR PLATE B AND SAND

$u_1 = u_2 = \sqrt{2gh}$



$\Delta m = q\,\Delta t$

$\xrightarrow{+}$ x COMP.: $0 - B_x\Delta t = -(\Delta m)u_2\cos 30°$

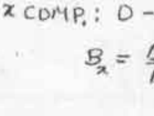
$B_x = \frac{\Delta m}{\Delta t}u_2\cos 30° = q\sqrt{2gh}\,\frac{\sqrt{3}}{2}$

$B_x = \frac{1}{2}q\sqrt{6gh}$

$+\downarrow$ y COMP.: $(\Delta m)u_1 - B_y\Delta t = (\Delta m)u_2\sin 30°$

$B_y = \frac{\Delta m}{\Delta t}(u_1 - u_2\sin 30°) = q\sqrt{2gh}\left(1-\frac{1}{2}\right)$

$B_y = \frac{1}{2}q\sqrt{2gh}$

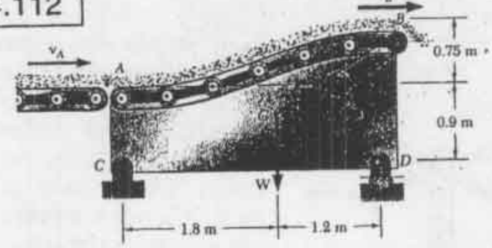


$B^2 = B_x^2 + B_y^2$
$= \left(\frac{q}{2}\right)^2(6gh + 2gh)$
$= 2q^2gh$
$B = q\sqrt{2gh}$

$\tan\theta = \frac{B_y}{B_x} = \frac{\sqrt{2gh}}{\sqrt{6gh}} = \frac{1}{\sqrt{3}}, \quad \theta = 30°$

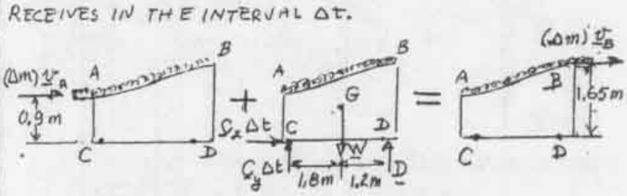
$\underline{B} = q\sqrt{2gh}$ ⦪ $30°$

14.112



GIVEN:
SAND RECEIVED AT A AND DISCHARGED AT B AT A RATE OF 100 kg/s AND WITH $v_A = v_B = 4.5$ m/s.
COMBINED WEIGHT OF COMPONENT AND SAND IT SUPPORTS IS $W = 4$ kN.
FIND:
REACTIONS AT C AND D.

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE COMPONENT, THE SAND IT SUPPORTS AND THE SAND IT RECEIVES IN THE INTERVAL Δt.



$\xrightarrow{+}$ x COMP.: $(\Delta m)v_A + C_x\Delta t = (\Delta m)v_B$

$C_x = \frac{\Delta m}{\Delta t}(v_B - v_A) = (100\text{ kg/s})(4.5\text{ m/s} - 4.5\text{ m/s}) = 0$

$+\circlearrowleft$ MOMENTS ABOUT C:

$-(\Delta m)v_A(0.9\text{m}) - (W\Delta t)(1.8\text{m}) + (D\Delta t)(3\text{m}) = -(\Delta m)v_B(1.65\text{m})$

$3D = 1.8W + \frac{\Delta m}{\Delta t}(0.9v_A) - \frac{\Delta m}{\Delta t}(1.65\,v_B)$

$= 1.8(4000\text{N}) + 0.9(100\text{ kg/s})(4.5\text{ m/s}) - 1.65(100\text{ kg/s})(4.5\text{ m/s})$

$= 6862.5$ N

$D = 2287.5$ N $\quad\quad \underline{D} = 2.29$ kN $\uparrow$

$+\uparrow$ y COMP.:

$C_y\Delta t - W\Delta t + D\Delta t = 0$

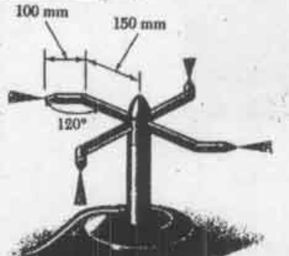
$C_y = W - D = 4000\text{ N} - 2287.5\text{ N} = 1712.5$ N

RECALLING THAT $C_x = 0$: $\quad\quad \underline{C} = 1.712$ kN $\uparrow$

NOTE. IF COMPONENT WAS STOPPED AND THE SAND WAS NOT MOVING, WE WOULD HAVE

$\underline{C} = 1.600$ kN $\uparrow$, $\quad \underline{D} = 2.40$ kN $\uparrow$

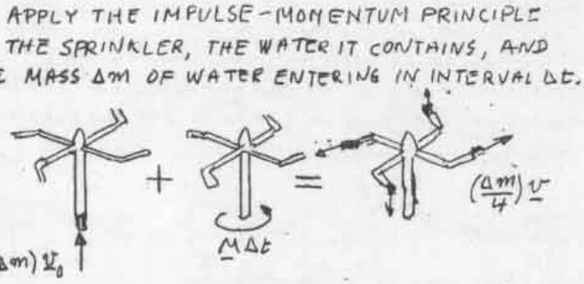
14.113



GIVEN:
EACH OF THE FOUR ROTATING ARMS OF SPRINKLER CONSISTS OF TWO STRAIGHT PORTIONS OF PIPE FORMING 120° ANGLE. EACH ARM DISCHARGES WATER AT THE RATE OF 20 L/min WITH RELATIVE VELOCITY OF 18 m/s. FRICTION IS EQUIVALENT TO COUPLE $M = 0.375$ N.m.

FIND:
CONSTANT RATE AT WHICH SPRINKLER ROTATES.

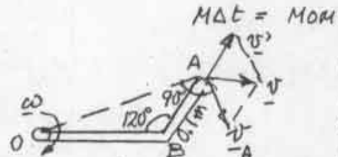
WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE SPRINKLER, THE WATER IT CONTAINS, AND THE MASS Δm OF WATER ENTERING IN INTERVAL Δt.



EQUATING MOMENTS ABOUT AXIS OF ROTATION ↺:

$$0 + M\Delta t = 4\,[\text{MOMENT OF } (\tfrac{\Delta m}{4})v]$$

$$M\Delta t = \text{MOMENT OF } (\Delta m)\,v \qquad (1)$$



THE VELOCITY v OF THE WATER LEAVING AN ARM IS THE RESULTANT OF THE VELOCITY v' RELATIVE TO THE ARM AND OF THE VELOCITY v_A OF NOZZLE

$$v = v' + v_A$$

WHERE $v' = 18$ m/s AND $v_A = (OA)\omega$

BUT APPLYING THE LAW OF COSINES TO TRIANGLE OAB:

$$(OA)^2 = (OB)^2 + (BA)^2 - 2(OB)(BA)\cos 120°$$
$$= (0.15m)^2 + (0.10m)^2 - 2(0.15m)(0.10m)\cos 120°$$
$$(OA)^2 = 0.0475\ m^2$$

THEREFORE:

+↺ MOM. OF v ABOUT O = MOM. OF v' + MOM. OF v_A

$$= (0.15m)\,v'\cos 30° - (OA)(OA)\,\omega$$
$$= (0.15m)(18\ m/s)\cos 30° - (OA)^2\omega$$
$$= 2.3383\ m^2/s - (0.0475\ m^2)\,\omega$$

SUBSTITUTING INTO EQ.(1) AND RECALLING THAT $M = 0.375$ N.m:

$$(0.375\ N.m)\Delta t = (\Delta m)[2.3383\ m^2/s - (0.0475\ m^2)\omega]$$

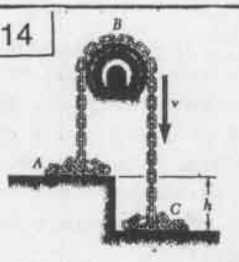
DIVIDING BY Δt, AND NOTING THAT

$$\frac{\Delta m}{\Delta t} = \rho Q = (1\ kg/L)(80\ L/min)\,\frac{1\,min}{60\,s} = \frac{4}{3}\ kg/s$$

WE HAVE

$$0.375\ N.m = (\tfrac{4}{3}\ kg/s)[2.3383\ m^2/s - (0.0475\ m^2)\,\omega]$$
$$2.3383\ m^2/s - (0.0475\ m^2)\,\omega = 0.28125\ m^2/s$$
$$\omega = 43.306\ rad/s \qquad \omega = 414\ rpm$$

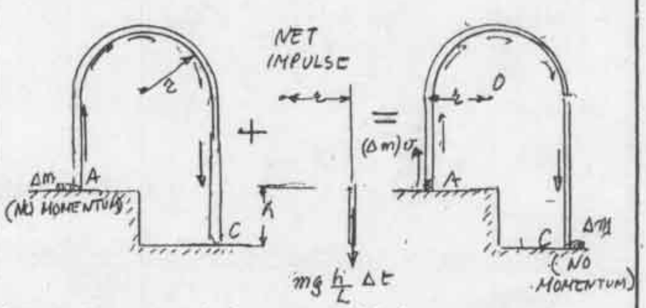
14.114



GIVEN:
WHEN GIVEN AN INITIAL SPEED v, THE CHAIN KEEPS MOVING OVER THE PULLEY.

FIND:
HEIGHT h.
(NEGLECT FRICTION.)

WE APPLY THE IMPULSE-MOMENTUM PRINCIPLE TO THE PORTION OF CHAIN OF MASS m AND LENGTH L IN MOTION A TIME t AND TO THE ELEMENT OF LENGTH Δx AND MASS $\Delta m = \frac{m}{L}\Delta x$ WHICH WILL BE SET IN MOTION IN THE TIME INTERVAL Δt.



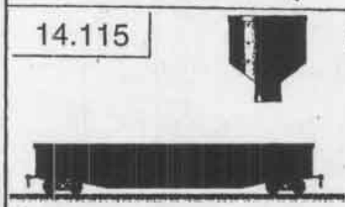
WE NOTE THAT THE ELEMENT AT A ACQUIRES A LINEAR MOMENTUM $(\Delta m)v$ WHICH IS ADDED TO THE SYSTEM, WHILE THE MOMENTUM OF THE ELEMENT AT C IS LOST TO THE SYSTEM.

EQUATING MOMENTS ABOUT O:

$$+↻\quad 0 + (mg\tfrac{h}{L}\Delta t)r = (\Delta m)vr$$
$$= (\tfrac{m}{L}\Delta x)vr$$

$$h = \frac{\Delta x}{\Delta t}\frac{v}{g} = v\frac{v}{g} \qquad h = \frac{v^2}{g}$$

14.115

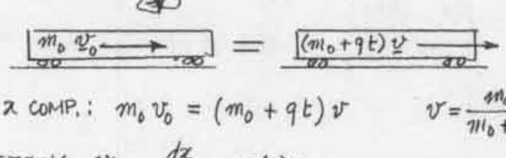


GIVEN:
RAILROAD CAR OF MASS m_0 AND LENGTH L APPROACHES CHUTE AT SPEED v_0 TO BE LOADED WITH SAND AT RATE $dm/dt = q$.

FIND: (a) MASS OF CAR AND LOAD AFTER CAR HAS PASSED.
(b) SPEED OF CAR AT THAT TIME.

CONSERVATION OF MOMENTUM IN HORIZONTAL DIRECTION

WE CONSIDER THE CAR AND THE MASS OF SAND qt WHICH FALLS INTO THE CAR IN THE TIME t.



$$\pm\ x\ \text{COMP.}:\quad m_0 v_0 = (m_0 + qt)\,v \qquad v = \frac{m_0 v_0}{m_0 + qt} \qquad (1)$$

LETTING $v = \frac{dx}{dt}$ IN (1):

$$dx = \frac{m_0 v_0\,dt}{m_0 + qt} \qquad x = m_0 v_0 \int_0^t \frac{dt}{m_0 + qt}$$

(CONTINUED

14.115 continued

$$x = \frac{m_0 v_0}{q}\left[\ln(m_0+qt)\right]_0^t = \frac{m_0 v_0}{q}\ln\frac{m_0+qt}{m_0} \qquad (2)$$

USING THE EXPONENTIAL FORM: $m_0+qt = m_0 e^{qx/m_0 v_0}$

WHERE m_0+qt REPRESENTS THE MASS AT TIME t, AFTER THE CAR HAS MOVED THROUGH x.

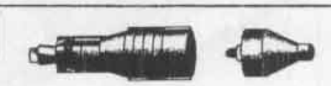
(a) MAKING $x=L$ IN (2), WE OBTAIN THE FINAL MASS:

$$m_f = m_0+qt_f = m_0 e^{qL/m_0 v_0}$$ ◀

(b) MAKING $t=t_f$ IN (1), WE OBTAIN THE FINAL SPEED:

$$v = \frac{m_0 v_0}{m_0+qt_f} = \frac{m_0}{m_f}v_0 = v_0 e^{-qL/m_0 v_0}$$ ◀

14.116



GIVEN:

SPACE VEHICLE DESCRIBING CIRCULAR ORBIT ABOUT THE EARTH AT SPEED OF 15,000 mi/h RELEASES AT ITS FRONT END A CAPSULE WITH A GROSS WEIGHT OF 1200 lb, INCLUDING 800 lb OF FUEL, WHICH IS CONSUMED AT THE RATE OF 40 lb/s AND EJECTED WITH RELATIVE VELOCITY OF 9000 ft/s.

FIND:

(a) TANGENTIAL ACCELERATION OF CAPSULE AS IT IS FIRED,

(b) MAX. SPEED ATTAINED BY THE CAPSULE.

FROM EQ. (14.44):

$$\text{THRUST} = P = \frac{dm}{dt}u = \frac{40\text{ lb/s}}{32.2\text{ ft/s}^2}(9000\text{ ft/s}) = \frac{360\times10^3}{32.2}\text{ lb}$$

(a) $P = m_0 a_t$: $\frac{360\times10^3}{32.2}\text{ lb} = \frac{1200\text{ lb}}{32.2\text{ ft/s}^2}a_t$

$$a_t = \frac{360\times10^3}{1.2\times10^3}\text{ ft/s}^2 \qquad a_t = 300\text{ ft/s}^2$$ ◀

(b) MAX. SPEED OF CAPSULE RELATIVE TO SPACE VEHICLE IS OBTAINED FROM EXPRESSION DERIVED IN PROB. 14.95, OR FROM EXPRESSION OBTAINED IN SAMPLE PROB. 14.8 BY OMITTING THE TERM DUE TO GRAVITY.

$$v_{C/V} = u\ln\frac{m_0}{m_0-qt}$$

WHERE $u = (9000\text{ ft/s})$

$$m_0 = \frac{1200\text{ lb}}{g}, \quad m_0-qt = \frac{1200\text{ lb}-800\text{ lb}}{g} = \frac{400\text{ lb}}{g}.$$

$$\frac{m_0}{m_0-qt} = \frac{1200}{400} = 3$$

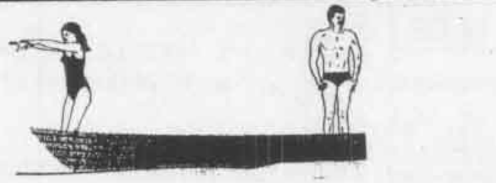
THUS:

$$v_{C/V} = (9000\text{ ft/s})\ln 3 = (9000\text{ ft/s})(1.0986) = 9887.5\text{ ft/s} = 6741\text{ mi/h}$$

$$v_C = v_V + v_{C/V} = 15{,}000\text{ mi/h} + 6741\text{ mi/h} = 21{,}741.5\text{ mi/h}$$

$$v_C = 21{,}700\text{ mi/h}$$ ◀

14.C1



GIVEN:

WOMAN OF WEIGHT W_w STANDS READY TO DIVE WITH VELOCITY v_w RELATIVE TO BOAT OF WEIGHT W_b.
MAN OF WEIGHT W_m READY TO DIVE FROM OTHER END OF BOAT WITH RELATIVE VELOCITY v_m.

FIND:

VELOCITY OF BOAT AFTER BOTH SWIMMERS HAVE DIVED IF (a) WOMAN DIVES FIRST, (b) MAN DIVES FIRST

USE $W_w = 120$ lb, $W_m = 180$ lb, $W_b = 300$ lb, AND

(PROB. 14.4): $v_w = v_m = 16$ ft/s

(i) $v_w = 14$ ft/s, $v_m = 18$ ft/s,

(ii) $v_w = 18$ ft/s, $v_m = 14$ ft/s

ANALYSIS

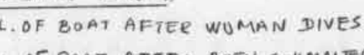
(a) WOMAN DIVES FIRST:

v_b' = VEL. OF BOAT AFTER WOMAN DIVES

v_b = VEL. OF BOAT AFTER BOTH SWIMMERS HAVE DIVED

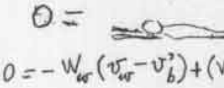
CONSERVATION OF MOMENTUM:

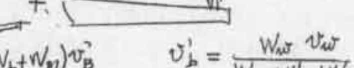
$$0 = \overleftarrow{W_w(v_w - v_b')} + \overrightarrow{(W_b+W_m)v_b'}$$



$$0 = -W_w(v_w-v_b') + (W_b+W_m)v_b' \qquad v_b' = \frac{W_w v_w}{W_w+W_m+W_b} \quad (1)$$

$$\overrightarrow{(W_b+W_m)v_b'} = \overrightarrow{W_b v_b} + \overrightarrow{W_m(v_m+v_b)}$$





$$(W_b+W_m)v_b' = W_b v_b + W_m(v_m+v_b) \qquad v_b = v_b' - \frac{W_m v_m}{W_m+W_b}$$

SUBSTITUTING FOR v_b' FROM (1):

$$\overset{+}{\rightarrow} \quad v_b = \frac{W_w v_w}{W_w+W_m+W_b} - \frac{W_m v_m}{W_m+W_b} \qquad (2)$$

(b) MAN DIVES FIRST:

INTERCHANGE SUB w AND SUB m IN (2) AND CHANGE ALL SIGNS

$$\overset{+}{\rightarrow} \quad v_b = -\frac{W_m v_m}{W_m+W_w+W_b} + \frac{W_w v_w}{W_w+W_b} \qquad (3)$$

OUTLINE OF PROGRAM

INPUT W_w, W_m, W_b, v_w, v_m, AND EQS. (2) AND (3).

PROGRAM OUTPUT

PROB. 14.4

```
(a) Woman dives first
Velocity of boat = -2.800
(b) Man dives first
Velocity of boat = -0.229
```

(i)

```
(a) Woman dives first
Velocity of boat = -3.950
(b) Man dives first
Velocity of boat = -1.400
```

(ii)

```
(a) Woman dives first
Velocity of boat = -1.650
(b) Man dives first
Velocity of boat =  0.943
```

14.C2

GIVEN:

SYSTEM OF n PARTICLES A_i OF MASS m_i, COORDINATES x_i, y_i, z_i, WITH VELOCITIES OF COMPONENTS $(v_x)_i, (v_y)_i, (v_z)_i$.

FIND:

COMPONENTS OF ANGULAR MOMENTUM OF SYSTEM ABOUT ORIGIN O. USE PROGRAM TO SOLVE PROBS. 14.9 AND 14.13.

ANALYSIS

$$\underline{H}_O = \sum_{i=1}^{n} \underline{r}_i \times m_i \underline{v}_i = \sum_{i=1}^{n} m_i \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ x_i & y_i & z_i \\ (v_x)_i & (v_y)_i & (v_z)_i \end{vmatrix}$$

$$H_x = \sum_{i=1}^{n} m_i \left[y_i (v_z)_i - z_i (v_y)_i \right] \quad (1)$$

$$H_y = \sum_{i=1}^{n} m_i \left[z_i (v_x)_i - x_i (v_z)_i \right] \quad (2)$$

$$H_z = \sum_{i=1}^{n} m_i \left[x_i (v_y)_i - y_i (v_x)_i \right] \quad (3)$$

OUTLINE OF PROGRAM

ENTER PROBLEM NUMBER AND SYSTEM OF UNITS USED

IF SI UNITS, ENTER FOR $i = 1$ TO $i = n$:

m_i (kg); x_i, y_i, z_i (m); $(v_x)_i, (v_y)_i, (v_z)_i$ (m/s)

IF U.S. CUSTOMARY UNITS, ENTER FOR $i = 1$ TO $i = n$:

W_i (lb); x_i, y_i, z_i (ft); $(v_x)_i, (v_y)_i, (v_z)_i$ (ft/s)

AND COMPUTE $m_i = W_i / 32.2$

COMPUTE THE SUMS (1), (2), AND (3).

PRINT PROBLEM NUMBER

PRINT VALUES OBTAINED FOR H_x, H_y, H_z.

IF SI UNITS, RESULTS ARE EXPRESSED IN kg·m²/s.

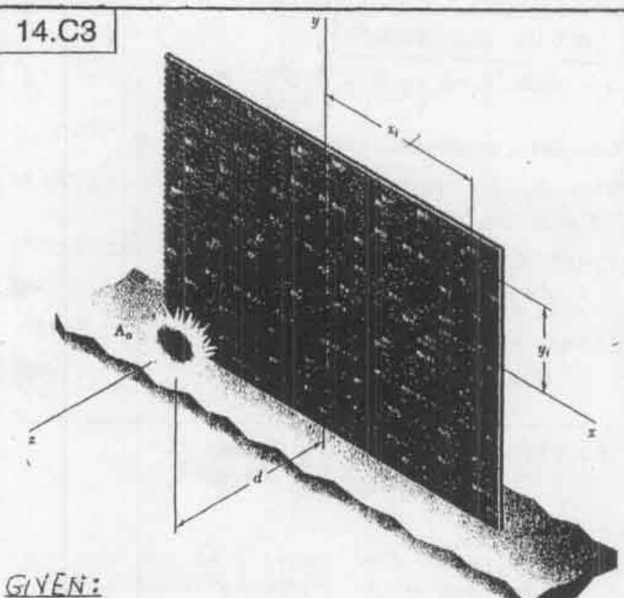
IF U.S. CUSTOMARY UNITS, RESULTS ARE EXPRESSED IN ft·lb·s.

PROGRAM OUTPUT

```
Problem 14.09
Hx = -31.2 kg*m^2/s
Hy = -64.8 kg*m^2/s
Hz =  48.0 kg*m^2/s
```

```
Problem 14.13
Hx =  0.000 ft*lb*s
Hy = -0.720 ft*lb*s
Hz =  1.440 ft*lb*s
```

14.C3



GIVEN:

SHELL MOVING WITH VELOCITY OF COMPONENTS v_x, v_y, v_z EXPLODES IN THREE FRAGMENTS OF WEIGHTS W_1, W_2, W_3 AT POINT A_0 AT DISTANCE d FROM WALL. FRAGMENTS HIT THE WALL AT POINTS A_i ($i = 1, 2, 3$) OF COORDINATES x_i AND y_i.

FIND: SPEED OF EACH FRAGMENT AFTER EXPLOSION. USE PROGRAM TO SOLVE (a) PROB. 14.25, (b) PROB. 14.26.

ANALYSIS

DETERMINE DIRECTION COSINES OF PATH A_0A_i ($i=1,2,3$)

FIRST COMPUTE $\ell_i = \sqrt{x_i^2 + y_i^2 + d^2}$ (1)

THEN $(\lambda_x)_i = x_i/\ell_i$, $(\lambda_y)_i = y_i/\ell_i$, $(\lambda_z)_i = -d/\ell_i$ (2)

CONSERVATION OF LINEAR MOMENTUM:

$$\frac{1}{g}(W_1+W_2+W_3)(v_x \underline{i} + v_y \underline{j} + v_z \underline{k}) = \frac{W_1}{g} v_1 \underline{\lambda}_1 + \frac{W_2}{g} v_2 \underline{\lambda}_2 + \frac{W_3}{g} v_3 \underline{\lambda}_3$$

x-COMP: $W_1(\lambda_x)_1 v_1 + W_2(\lambda_x)_2 v_2 + W_3(\lambda_x)_3 v_3 = (W_1+W_2+W_3) v_x$ (3)

y-COMP: $W_1(\lambda_y)_1 v_1 + W_2(\lambda_y)_2 v_2 + W_3(\lambda_y)_3 v_3 = (W_1+W_2+W_3) v_y$ (4)

z-COMP: $W_1(\lambda_z)_1 v_1 + W_2(\lambda_z)_2 v_2 + W_3(\lambda_z)_3 v_3 = (W_1+W_2+W_3) v_z$ (5)

THESE 3 EQS. ARE SOLVED SIMULTANEOUSLY FOR v_1, v_2, v_3.

OUTLINE OF PROGRAM

ENTER PROBLEM NUMBER

ENTER VALUES OF v_x, v_y, v_z, AND d

ENTER VALUES OF W_i, x_i, y_i FOR $i = 1, 2, 3$

COMPUTE DIRECTION COSINES FROM EQS. (1) AND (2)

COMPUTE COEFF. IN EQS. (3), (4), (5) AND SOLVE FOR v_1, v_2, v_3 BY COMPUTING

$$D = \begin{vmatrix} W_1(\lambda_x)_1 & W_2(\lambda_x)_2 & W_3(\lambda_x)_3 \\ W_1(\lambda_y)_1 & W_2(\lambda_y)_2 & W_3(\lambda_y)_3 \\ W_1(\lambda_z)_1 & W_2(\lambda_z)_2 & W_3(\lambda_z)_3 \end{vmatrix}, \quad D_1 = \begin{vmatrix} (\Sigma W)v_x & W_2(\lambda_x)_2 & W_3(\lambda_x)_3 \\ (\Sigma W)v_y & W_2(\lambda_y)_2 & W_3(\lambda_y)_3 \\ (\Sigma W)v_z & W_2(\lambda_z)_2 & W_3(\lambda_z)_3 \end{vmatrix}, \text{ etc.}$$

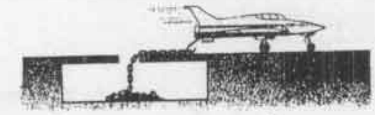
AND $v_1 = D_1/D$, $v_2 = D_2/D$, $v_3 = D_3/D$

PROGRAM OUTPUT

```
(a) Problem 14.25
    VA = 1678 ft/s
    VB = 1390 ft/s
    VC = 1230 ft/s
```

```
(b) Problem 14.26
    VA = 2097 ft/s
    VB = 1853 ft/s
    VC =  738 ft/s
```

14.C4

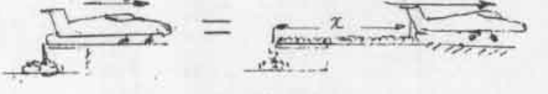


GIVEN: AS 6000-kg PLANE LANDS ON CARRIER AT 180 Km/h, ITS TAIL HOOKS INTO END OF 80-m LONG CHAIN OF MASS PER UNIT LENGTH $m' = 50$ kg/m LYING BELOW DECK.

TABULATE, USING 5-m INCREMENTS, THE DISTANCE TRAVELED BY THE PLANE AND THE CORRESPONDING VALUES OF THE TIME, THE VELOCITY, AND THE ACCELERATION OF THE PLANE.
(ASSUME NO OTHER RETARDING FORCE.)

ANALYSIS

CONSERVATION OF LINEAR MOMENTUM:



$$\overset{+}{\rightarrow} \quad m v_0 = (m + m'x) v \qquad (1)$$

LETTING $v = dx/dt$:

$$m v_0 \, dt = (m + m'x) dx$$

$$t = \int_0^x \frac{m + m'x}{m v_0} dx = \left[\frac{(m+m'x)^2}{2 m m' v_0}\right]_0^x = \frac{(m+m'x)^2 - m^2}{2 m m' v_0} \qquad (2)$$

SOLVING (1) FOR v: $\quad v = \dfrac{m v_0}{m + m'x} \qquad (3)$

DIFFERENTIATING (1) WITH RESPECT TO t:

$$0 = m' \frac{dx}{dt} v + (m + m'x) \frac{dv}{dt}$$

NOTING THAT $dx/dt = v$ AND $dv/dt = a$:

$$0 = m' v^2 + (m + m'x) a \qquad a = -\frac{m' v^2}{m + m'x} \qquad (4)$$

OUTLINE OF PROGRAM

ENTER $m = 6000$ kg, $m' = 50$ kg/m, $v_0 = 180$ km/h $= 50$ m/s
FOR $x = 0$ TO $x = 80$ m AND USING 5-m INCREMENTS
CALCULATE t, v, AND a FROM EQS. (2), (3), (4) AND TABULATE

PROGRAM OUTPUT

Distance (m)	Time (s)	Velocity (km/h)	Acceleration (m/s^2)
0.000	0.000	180.000	-20.833
5.000	0.102	172.800	-18.432
10.000	0.208	166.154	-16.386
15.000	0.319	160.000	-14.632
20.000	0.433	154.286	-13.120
25.000	0.552	148.966	-11.809
30.000	0.675	144.000	-10.667
35.000	0.802	139.355	-9.667
40.000	0.933	135.000	-8.789
45.000	1.069	130.909	-8.014
50.000	1.208	127.059	-7.327
55.000	1.352	123.429	-6.717
60.000	1.500	120.000	-6.173
65.000	1.652	116.757	-5.686
70.000	1.808	113.684	-5.249
75.000	1.969	110.769	-4.855
80.000	2.133	108.000	-4.500

14.C5

GIVEN:
A 16-Mg PLANE MAINTAINS A CONSTANT SPEED OF 774 km/h WHILE CLIMBING AT AN ANGLE $\alpha = 18°$.
PLANE SCOOPS IN AIR AT RATE OF 300 kg/s AND DISCHARGES IT AT A RELATIVE SPEED OF 665 m/s.
PILOT THEN CHANGES ANGLE OF CLIMB α WHILE MAINTAINING THE SAME ENGINE SETTING.
FIND FOR VALUES OF α FROM 0 TO 20° USING 1° INCREMENTS:
(a) INITIAL ACCELERATION OF PLANE,
(b) MAXIMUM SPEED THAT WILL BE ATTAINED.
(ASSUME DRAG TO BE PROPORTIONAL TO v^2.)

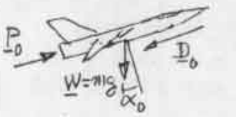
ANALYSIS

FROM EQ. (14.39): THRUST $= P = \dfrac{dm}{dt}(u - v)$

DENOTING RATE dm/dt by R:

$$P = R(u - v) \qquad (1)$$

WHILE CLIMBING AT v_0 AND α_0:

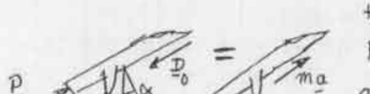


$$P_0 = R(u - v_0) \qquad (2)$$

SINCE PLANE IS IN EQUILIBRIUM:

$$D_0 = P_0 - mg \sin \alpha_0 \qquad (3)$$

(a) PLANE CLIMBING AT ANGLE α AND SPEED v_0:



$$+\!\measuredangle\alpha \quad \Sigma F_x = ma$$

$$P_0 - D_0 - mg \sin \alpha = ma$$

$$a = (P_0 - D_0 - mg \sin\alpha)/m \qquad (4)$$

(b) MAX. SPEED WHILE CLIMBING AT ANGLE α:
ACCEL. WILL THEN BE ZERO AND PLANE IN EQUILIBRIUM.

$$\measuredangle\alpha \; \Sigma F_x = 0: \; P - D - mg \sin\alpha = 0 \qquad (5)$$

BUT, SINCE $D \sim v^2$, WE HAVE

$$D = D_0 (v/v_0)^2$$

SUBSTITUTING FOR D INTO (5) AND FOR P FROM (1):

$$R(u - v) - (D_0/v_0^2) v^2 - mg \sin\alpha = 0$$

$$v^2 + \frac{R v_0^2}{D_0} v + (v_0^2/D_0)(mg \sin\alpha - Ru) = 0$$

SET $\quad B = R v_0^2/D_0, \qquad C = (v_0^2/D_0)(mg \sin\alpha - Ru) \qquad (6)$

MAX. SPEED:

$$v_{max} = v = \tfrac{1}{2}\left(-B + \sqrt{B^2 - 4C}\right) \qquad (7)$$

OUTLINE OF PROGRAM

ENTER $m = 16 \times 10^3$ kg, $v_0 = 774$ km/h $= 215$ m/s, $\alpha_0 = 18°$,
$R = 300$ kg/s, $u = 665$ m/s, $g = 9.81$ m/s^2.
USE EQS. (2) AND (3) TO CALCULATE P_0 AND D_0.
CALCULATE $B = R v_0^2 / D_0$.
FOR α FROM 0 TO 20°, WITH 1° INCREMENTS,
(a) USE EQ. (4) TO CALCULATE a
(b) USE EQS. (6) AND (7) TO CALCULATE v_{max}

(CONTINUED)

14.C5 continued

PROGRAM OUTPUT

alpha degrees	acceleration m/s^2	max v km/h
0.000	3.031	921.796
1.000	2.860	913.933
2.000	2.689	906.020
3.000	2.518	898.060
4.000	2.347	890.053
5.000	2.176	882.002
6.000	2.006	873.907
7.000	1.836	865.770
8.000	1.666	857.594
9.000	1.497	849.378
10.000	1.328	841.126
11.000	1.160	832.839
12.000	0.992	824.518
13.000	0.825	816.166
14.000	0.658	807.785
15.000	0.492	799.375
16.000	0.327	790.940
17.000	0.163	782.481
18.000	0.000	774.000
19.000	-0.162	765.499
20.000	-0.324	756.981

14.C6

GIVEN:
ROCKET OF WEIGHT 2400 lb, INCLUDING 2000 lb OF FUEL, IS FIRED VERTICALLY FROM GROUND. IT CONSUMES FUEL AT RATE OF 25 lb/s AND EJECTS IT WITH RELATIVE VELOCITY OF 12,000 ft/s.

FIND FROM TIME OF IGNITION TO TIME WHEN LAST PARTICLE OF FUEL IS CONSUMED, AND AT 4-s TIME INTERVALS:

(a) ACCELERATION a OF ROCKET IN ft/s^2,

(b) ITS VELOCITY v IN ft/s,

(c) ITS ELEVATION h ABOVE GROUND IN MILES.

ANALYSIS

WE RECALL FROM SAMPLE PROB. 14.8 THAT

$$v = u \ln \frac{m_0}{m_0 - qt} - gt \qquad (1)$$

WHERE v = VELOCITY OF ROCKET

m_0 = INITIAL WEIGHT OF ROCKET AND FUEL

q = RATE AT WHICH FUEL IS CONSUMED

u = RELATIVE VELOCITY AT WHICH FUEL IS EJECTED

LETTING $v = dy/dt$ AND INTEGRATING y FROM 0 TO h:

$$h = \int_0^h dy = u \int_0^t \ln \frac{m_0}{m_0 - qt} dt - \frac{1}{2} g t^2 \qquad (2)$$

TO CALCULATE THE INTEGRAL, WE SET $\frac{m_0 - qt}{m_0} = z$

AND OBTAIN $dt = -\frac{m_0}{q} dz$. THEREFORE:

$$\int_0^t \ln \frac{m_0}{m_0 - qt} dt = \int_1^z (-\ln z)\left(-\frac{m_0}{q} dz\right)$$

$$= \frac{m_0}{q} \int_1^z \ln z \, dz = \frac{m_0}{q}\left[z \ln z - z\right]_1^z = \frac{m_0}{q}(z \ln z - z + 1)$$

THUS, EQ.(2) YIELDS

$$h = \frac{m_0 u}{q}\left(\frac{m_0 - qt}{m_0} \ln \frac{m_0 - qt}{m_0} - 1 + \frac{qt}{m_0} + 1\right) - \frac{1}{2} g t^2$$

$$h = u\left(t + \frac{m_0 - qt}{q} \ln \frac{m_0 - qt}{m_0}\right) - \frac{1}{2} g t^2 \qquad (3)$$

REWRITING EQ.(1) AS

$$v = u \ln m_0 - u \ln(m_0 - qt) - gt$$

AND DIFFERENTIATING WITH RESPECT TO t,

$$a = \frac{dv}{dt} = -u \frac{-q}{m_0 - qt} - g \qquad a = \frac{uq}{m_0 - qt} - g \qquad (4)$$

(CONTINUED)

14.C6 continued

OUTLINE OF PROGRAM

ENTER $g = 32.2\ ft/s^2$, $m_0 = 2400/g$, $m_f = 2000/g$, $q = 25/g$, $u = 12{,}000\ ft/s$

COMPUTE FINAL TIME $= t_f = m_f/q = 2000/25 = 80$ s

FOR t FROM 0 TO 80 s AT 4-s INTERVALS, COMPUTE

(a) ACCELERATION a FROM EQ.(4)

(b) VELOCITY v FROM EQ.(1)

(c) ELEVATION h FROM EQ.(3), DIVIDING RESULT BY 5280 TO OBTAIN h IN MILES.

PROGRAM OUTPUT

t s	a ft/s^2	v 10^3 ft/s	h mi
0.000	92.800	0.000	0.000
4.000	98.235	0.382	0.143
8.000	104.164	0.787	0.584
12.000	110.657	1.216	1.341
16.000	117.800	1.673	2.434
20.000	125.695	2.159	3.883
24.000	134.467	2.679	5.714
28.000	144.271	3.236	7.952
32.000	155.300	3.835	10.628
36.000	167.800	4.481	13.775
40.000	182.086	5.180	17.431
44.000	198.569	5.940	21.639
48.000	217.800	6.772	26.449
52.000	240.527	7.688	31.921
56.000	267.800	8.702	38.122
60.000	301.133	9.838	45.137
64.000	342.800	11.123	53.066
68.000	396.371	12.596	62.037
72.000	467.800	14.317	72.213
76.000	567.800	16.376	83.814
80.000	717.800	18.925	97.148

15.1

GIVEN: $\theta = 1.5t^3 - 4.5t^2 + 10$

FIND: θ, ω, AND α WHEN (a) $t=0$, (b) $t=4\,s$.

$$\omega = d\theta/dt = 4.5t^2 - 9t$$
$$\alpha = d\omega/dt = 9t - 9$$

(a) $t=0$: $\theta = 10\ rad$; $\omega = 0$; $\alpha = -9\ rad/s^2$

(b) $t=4s$: $\theta = 1.5(4)^3 - 4.5(4)^2 + 10$, $\quad \theta = 34\ rad$

$\omega = 4.5(4)^2 - 9(4)$, $\quad \omega = 36\ rad/s$

$\alpha = 9(4) - 9$; $\quad \alpha = 27\ rad/s^2$

15.2

GIVEN: $\theta = 1.5t^3 - 4.5t^2 + 10$

FIND: t, θ, AND α WHEN $\omega = 0$

$$\omega = d\theta/dt = 4.5t^2 - 9t$$
$$\alpha = d\omega/dt = 9t - 9$$

FOR $\omega = 0$: $4.5t^2 - 9t = 0$; $\quad t = 0$ AND $t = 2$.

$t=0$: $\theta = 10\ rad$, $\alpha = -9\ rad/s^2$

$t = 2s$: $\theta = 1.5(2)^3 - 4.5(2)^2 + 10$, $\quad \theta = 4\ rad$

$\alpha = 9(2) - 9$, $\quad \alpha = 9\ rad/s^2$

15.3

GIVEN: $\theta = \theta_0(1 - e^{-t/4})$ WITH $\theta_0 = 0.40\ rad$

FIND: θ, ω, AND α WHEN (a) $t=0$, (b) $t=3s$, (c) $t=\infty$

$$\theta = 0.40\,(1 - e^{-t/4})$$
$$\omega = \frac{d\theta}{dt} = \frac{1}{4}(0.40)e^{-t/4} = 0.10\,e^{-t/4}$$
$$\alpha = \frac{d\omega}{dt} = -\frac{1}{4}(0.10)e^{-t/4} = -0.025e^{-t/4}$$

(a) $t=0$: $\theta = 0.40(1 - e^0)$, $\quad \theta = 0$

$\omega = 0.10\,e^0$, $\quad \omega = 0.1\ rad/s$

$\alpha = -0.025e^0$, $\quad \alpha = -0.025\ rad/s^2$

(b) $t = 3s$: $\theta = 0.40(1 - e^{-3/4})$
$= 0.40(1 - 0.4724)$, $\quad \theta = 0.211\ rad$

$\omega = 0.10\,e^{-3/4}$
$= 0.10\,(0.4724)$, $\quad \omega = 0.0472\ rad/s$

$\alpha = -0.025\,e^{-3/4}$
$= -0.025\,(0.4724)$, $\quad \alpha = -0.01181\ rad/s^2$

(c) $t = \infty$: $\theta = 0.40(1 - e^{-\infty})$
$= 0.40\,(1 - 0)$, $\quad \theta = 0.4\ rad$

$\omega = 0.10e^{-\infty}$, $\quad \omega = 0$

$\alpha = -0.025e^{-\infty}$, $\quad \alpha = 0$

15.4

GIVEN: $\theta = \theta_0 \sin\left(\frac{\pi t}{T}\right) - 0.5\theta_0 \sin\left(\frac{2\pi t}{T}\right)$
WHERE $\theta_0 = 6\ rad$, $T = 4s$.

FIND: θ, ω, AND α WHEN (a) $t = 0$, (b) $t = 2s$.

$$\omega = \frac{d\theta}{dt} = \theta_0\frac{\pi}{T}\cos\left(\frac{\pi t}{T}\right) - 0.5\theta_0\frac{2\pi}{T}\cos\left(\frac{2\pi t}{T}\right)$$
$$\alpha = \frac{d\omega}{dt} = -\theta_0\left(\frac{\pi}{T}\right)^2\sin\left(\frac{\pi t}{T}\right) + 0.5\theta_0\left(\frac{2\pi}{T}\right)^2\sin\left(\frac{2\pi t}{T}\right)$$

(a) $t=0$: $\theta = 0$

$\omega = 6\,\frac{\pi}{4} - 0.5(6)\frac{2\pi}{4}$, $\quad \omega = 0$

$\alpha = 0$

(b) $t = 2s$:

$\theta = 6\sin\left(\frac{2\pi}{4}\right) - 0.5(6)\sin\left(\frac{4\pi}{4}\right) = 6 - 0$, $\quad \theta = 6\ rad$

$\omega = 6\left(\frac{\pi}{4}\right)\cos\left(\frac{2\pi}{4}\right) - 0.5(6)\,\frac{2\pi}{4}\cos\left(\frac{4\pi}{4}\right)$
$= 6\,\frac{\pi}{4}(0) - 0.5(6)\,\frac{2\pi}{4}(-1) = \frac{6\pi}{4}$

$\omega = 4.71\ rad/s$

$\alpha = -6\left(\frac{\pi}{4}\right)^2\sin\left(\frac{2\pi}{4}\right) + 0.5(6)\left(\frac{2\pi}{4}\right)^2\sin\left(\frac{4\pi}{4}\right)$
$= -6\left(\frac{\pi}{4}\right)^2(1) + 3\left(\frac{2\pi}{4}\right)^2(0) = -\frac{3}{8}\pi^2$

$\alpha = -3.70\ rad/s^2$

15.5

GIVEN: $\theta = \theta_0 \sin\left(\frac{\pi t}{T}\right) - 0.5\,\theta_0 \sin\left(\frac{2\pi t}{T}\right)$
WHERE $\theta_0 = 6\ rad$, $T = 4s$

FIND: θ, ω, AND α WHEN $t = 1s$

$$\omega = \frac{d\theta}{dt} = \theta_0\frac{\pi}{T}\cos\left(\frac{\pi t}{T}\right) - 0.5\theta_0\,\frac{2\pi}{T}\cos\left(\frac{2\pi t}{T}\right)$$
$$\alpha = \frac{d\omega}{dt} = -\theta_0\left(\frac{\pi}{T}\right)^2\sin\left(\frac{\pi t}{T}\right) + 0.5\theta_0\left(\frac{2\pi}{T}\right)^2\sin\left(\frac{2\pi t}{T}\right)$$

$t = 1s$: $\theta = 6\sin\left(\frac{\pi}{4}\right) - 0.5(6)\sin\left(\frac{2\pi}{4}\right)$
$= 6\,\frac{\sqrt{2}}{2} - 0.5(6)(1)$, $\quad \theta = 1.243\ rad$

$\omega = 6\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{4}\right) - 0.5(6)\left(\frac{2\pi}{4}\right)\cos\left(\frac{2\pi}{4}\right)$
$= 6\left(\frac{\pi}{4}\right)\frac{\sqrt{2}}{2} - 0.5(6)\left(\frac{\pi}{2}\right)(0)$, $\quad \omega = 3.33\ rad/s$

$\alpha = -6\left(\frac{\pi}{4}\right)^2\sin\left(\frac{\pi}{4}\right) + 0.5(6)\left(\frac{2\pi}{4}\right)^2\sin\left(\frac{2\pi}{4}\right)$
$= -6\left(\frac{\pi}{4}\right)^2\frac{\sqrt{2}}{2} + 0.5(6)\left(\frac{2\pi}{4}\right)^2(1)$, $\quad \alpha = 4.79\ rad/s^2$

15.6

GIVEN: $t = 0$, $\omega = 0$
$t = 6s$, $\omega_1 = 3300\ rpm = 110\pi\ rad/s$.
THEN COASTS TO REST IN 80s.

FIND: NUMBER OF REVOLUTIONS
(a) TO REACH SPEED OF 3300 rpm.
(b) TO COAST TO REST.

UNIFORMLY ACCELERATED MOTION: $\omega_0 = 0$, $t = 6s$.

(a) $\omega = \omega_0 + \alpha t$; $\quad 110\pi = 0 + \alpha(6)$, $\quad \alpha = \frac{110}{6}\pi\ rad/s^2$

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\left(\frac{110}{6}\pi\right)(6s)^2 = 330\pi\ rad$

$\theta = (330\pi\ rad)\,\frac{1\ rev}{2\pi\ rad}$, $\quad \theta = 165\ rev$

(b) $\omega_1 = 110\pi\ rad/s$, $\quad \omega_2 = 0$ WHEN $t = 80s$

$\omega_2 = \omega_1 + \alpha t$; $\quad 0 = 110\pi + \alpha(80s)$, $\quad \alpha = -\frac{110}{80}\pi\ rad/s^2$

$\theta = \omega_1 t + \frac{1}{2}\alpha t^2 = (110\pi)(80) - \frac{1}{2}\left(\frac{110}{80}\pi\right)(80s)^2$
$= 8800\pi - 4400\pi = 4400\pi\ rad$

$\theta = (4400\pi)\,\frac{1\ rev}{2\pi\ rad}$, $\quad \theta = 2200\ rev$

15.7 GIVEN: ROTOR COASTS TO REST IN 4 min. FROM RATED SPEED OF $\omega_0 = 6900$ rpm.

FOR UNIFORMLY ACCELERATED MOTION,

FIND: (a) ANG. ACCEL. α. (b) NUMBER OF REVOLUTIONS

$\omega_0 = 6900\,\text{rpm}\left(\frac{2\pi}{60}\right) = 722.57\ \text{rad/s},\ t = 4\,\text{min} = 240\,\text{s}$

(a) $\omega = \omega_0 + \alpha t;\quad 0 = 722.57 + \alpha(240)$

$\alpha = -3.0107\ \text{rad/s},\quad \alpha = -3.01\ \text{rad/s}^2$ ◄

(b) $\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (722.57)(240) + \frac{1}{2}(-3.0107)(240)^2$

$\theta = 173{,}416 - 86{,}708 = 86{,}708\ \text{rad}$

$\theta = 86{,}708\ \text{rad}\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right),\quad \theta = 13{,}800\ \text{rev}$ ◄

15.8 GIVEN: $\alpha = -k\theta$.

FIND: (a) VALUE OF k FOR WHICH $\omega = 8$ rad/s WHEN $\theta = 0$ AND $\theta = 4$ rad/s WHEN $\omega = 0$.

(b) ANGULAR VELOCITY WHEN $\theta = 3$ rad.

$\alpha = -k\theta \qquad \omega\frac{d\omega}{d\theta} = -k\theta \qquad \omega\, d\omega = -k\theta\, d\theta$

(a) $\displaystyle\int_{8\,\text{rad/s}}^{0} \omega\, d\omega = -\int_0^{4\,\text{rad}} k\theta\, d\theta\ ;\quad \left|\tfrac{1}{2}\omega^2\right|_8^0 = -\left|\tfrac{1}{2}k\theta^2\right|_0^4$

$\frac{1}{2}(0 - 8^2) = -\frac{1}{2}k(4^2 - 0) \qquad k = 4\,\text{s}^{-2}$ ◄

(b) $\displaystyle\int_{8\,\text{rad/s}}^{\omega} \omega\, d\omega = -\int_0^{3\,\text{rad}} k\theta\, d\theta\ ;\quad \left|\tfrac{1}{2}\omega^2\right|_0^{\omega} = -\left|\tfrac{1}{2}(4\text{s}^{-1})\theta^2\right|_0^3$

$\frac{1}{2}(\omega^2 - 8^2) = -\frac{1}{2}(4)(3^2 - 0)$

$\omega^2 - 64 = -36;\quad \omega^2 = 64 - 36 = 28;\quad \omega = 5.29\ \frac{\text{rad}}{\text{s}}$ ◄

15.9 GIVEN: $\alpha = -0.25\,\omega$; WHEN $t = 0$, $\omega_0 = 20$ rad/s

FIND: (a) REVOLUTIONS BEFORE $\omega = 0$.

(b) TIME WHEN $\omega = 0$.

(c) TIME WHEN $\omega = 0.01\,\omega_0$.

$\alpha = -0.25\omega\ ;\quad \omega\frac{d\omega}{d\theta} = -0.25\omega;\quad d\omega = -0.25\, d\theta$

(a) $\displaystyle\int_{20\,\text{rad/s}}^{0} d\omega = -0.25\int_0^{\theta} d\theta\ ;\quad (0 - 20) = -0.25\theta$

$\theta = 80\ \text{rad}$

$\theta = (80\,\text{rad})\frac{\text{rev}}{2\pi\,\text{rad}} \qquad \theta = 12.73\ \text{rev}$ ◄

(b)

$\alpha = -0.25\omega\ ;\quad \frac{d\omega}{dt} = -0.25\,\omega\ ;\quad \frac{d\omega}{\omega} = -0.25\,dt$

$\displaystyle\int_{20\,\text{rad/s}}^{\omega} \frac{d\omega}{\omega} = -0.25\int_0^t dt \qquad \left|\ln\omega\right|_{20}^{\omega} = -0.25t$

$t = -\frac{1}{0.25}(\ln\omega - \ln 20) = 4(\ln 20 - \ln\omega)$

$t = 4\ln\frac{20}{\omega} \qquad (1)$

FOR $\omega = 0 \qquad t = 4\ln\frac{20}{0} = 4\ln\infty$

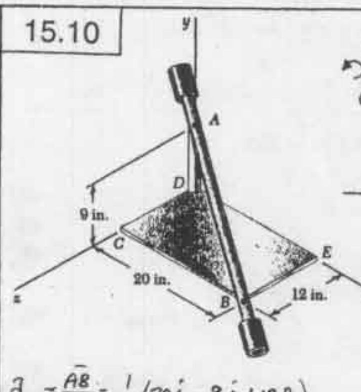
$t = \infty$ ◄

(c) FOR $\omega = 0.01\,\omega_0 = 0.01(20) = 0.2$ rad

USE EQ(1): $t = 4\ln\left(\frac{20}{0.2}\right) = 4\ln 100 = 4(4.605)$

$t = 18.42\ \text{s}$ ◄

15.10



GIVEN: $\omega_{AB} = 7.5$ rad/s ↺ AS VIEWED FROM B.

$\alpha_{AB} = 0$.

FIND: $\underline{v}_E$ AND $\underline{a}_E$

$AB^2 = 20^2 + 9^2 + 12^2$

$AB = 25$ in.

$\lambda_{AB} = \frac{\overline{AB}}{AB}$

$\underline{\lambda}_{AB} = \frac{\overline{AB}}{AB} = \frac{1}{25}(20\underline{i} - 9\underline{j} + 12\underline{k})$

$\underline{\omega} = \omega_{AB}\underline{\lambda}_{AB} = (7.5\ \text{rad/s})\frac{1}{25}(20\underline{i} - 9\underline{j} + 12\underline{k})$

$\underline{\omega} = (6\ \text{rad/s})\underline{i} - (2.7\ \text{rad/s})\underline{j} + (3.6\ \text{rad/s})\underline{k}$

$\underline{r}_{E/B} = -(12\ \text{in.})\underline{k}$

$\underline{v}_E = \underline{\omega}\times\underline{r}_{E/B} = (6\underline{i} - 2.7\underline{j} + 3.6\underline{k})\times(-12\underline{k})$

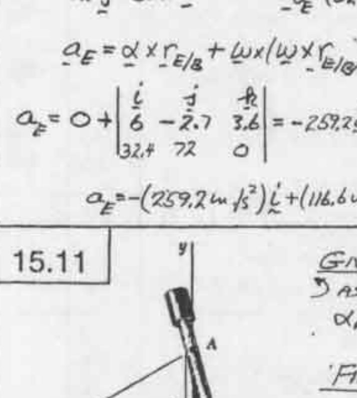
$= 72\underline{j} + 32.4\underline{i} \qquad \underline{v}_E = (32.4\ \text{in./s})\underline{i} + (72\ \text{in./s})\underline{j}$ ◄

$\underline{a}_E = \underline{\alpha}\times\underline{r}_{E/B} + \underline{\omega}\times(\underline{\omega}\times\underline{r}_{E/B}) = \underline{\alpha}\times\underline{r}_{E/B} + \underline{\omega}\times\underline{v}_E$

$\underline{a}_E = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 6 & -2.7 & 3.6 \\ 32.4 & 72 & 0 \end{vmatrix} = -259.2\underline{i} + 116.6\underline{j} + (432 + 87.4)\underline{k}$

$\underline{a}_E = -(259.2\ \text{in./s}^2)\underline{i} + (116.6\ \text{in./s}^2)\underline{j} + (519\ \text{in./s}^2)\underline{k}$ ◄

15.11



GIVEN: $\omega_{AB} = 7.5$ rad/s ↺ AS VIEWED FROM B,

$\alpha_{AB} = -30\ \text{rad/s}^2$

FIND: $\underline{v}_C$ AND $\underline{a}_C$

$AB^2 = 20^2 + 9^2 + 12^2$

$AB = 25$ in.

$\underline{\lambda}_{AB} = \frac{\overline{AB}}{AB}$

$\underline{r}_{C/B} = -(20\ \text{in.})\underline{i}$

$\underline{\lambda}_{AB} = \frac{\overline{AB}}{AB} = \frac{1}{25}(20\underline{i} - 9\underline{j} + 12\underline{k})$

$\underline{\omega} = \omega_{AB}\underline{\lambda}_{AB} = (7.5\ \text{rad/s})\frac{1}{25}(20\underline{i} - 9\underline{j} + 12\underline{k})$

$\underline{\omega} = (6\ \text{rad/s})\underline{i} - (2.7\ \text{rad/s})\underline{j} + (3.6\ \text{rad/s})\underline{k}$

$\underline{\alpha} = \alpha_{AB}\underline{\lambda}_{AB} = (-30\ \text{rad/s}^2)\frac{1}{25}(20\underline{i} - 9\underline{j} + 12\underline{k})$

$\underline{\alpha} = -(24\ \text{rad/s}^2)\underline{i} + (10.8\ \text{rad/s}^2)\underline{j} - (14.4\ \text{rad/s}^2)\underline{k}$

$\underline{v}_C = \underline{\omega}\times\underline{r}_{C/B} = (6\underline{i} - 2.7\underline{j} + 3.6\underline{k})\times(-20\underline{i})$

$= -54\underline{k} - 72\underline{j} \qquad \underline{v}_C = -(72\ \text{in./s})\underline{j} - (54\ \text{in./s})\underline{k}$ ◄

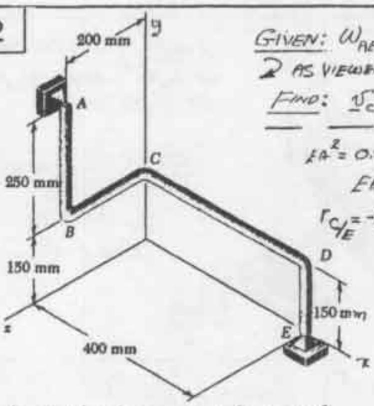
$\underline{a}_C = \underline{\alpha}\times\underline{r}_{C/B} + \underline{\omega}\times(\underline{\omega}\times\underline{r}_{C/B}) = \underline{\alpha}\times\underline{r}_{C/B} + \underline{\omega}\times\underline{v}_C$

$\underline{a}_C = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -24 & 10.8 & -14.4 \\ -20 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 6 & -2.7 & 3.6 \\ 0 & -72 & -54 \end{vmatrix}$

$\underline{a}_C = 288\underline{j} + 216\underline{k} + (157.2 + 259.2)\underline{i} + 324\underline{j} - 432\underline{k}$

$\underline{a}_C = (405\ \text{in./s}^2)\underline{i} + (612\ \text{in./s}^2)\underline{j} - (216\ \text{in./s}^2)\underline{k}$ ◄

15.12



GIVEN: $\omega_{AE} = 9$ rad/s, $\alpha_{AE} = 0$ ↻ AS VIEWED FROM E.

FIND: $\underline{v}_C$ AND $\underline{a}_C$

$EA^2 = 0.4^2 + 0.4^2 + 0.2^2$

$EA = 0.6$ m

$r_{C/E} = -(0.4\,m)\underline{i} + (0.15\,m)\underline{j}$

$\overline{EA} = -(0.4\,m)\underline{i} + (0.4\,m)\underline{j} + (0.2\,m)\underline{k}$

$\underline{\lambda}_{EA} = \frac{\overline{EA}}{EA} = \frac{1}{0.6}(-0.4\underline{i} + 0.4\underline{j} + 0.2\underline{k}) = \frac{1}{3}(-2\underline{i} + 2\underline{j} + \underline{k})$

$\underline{\omega} = \omega_{AE}\underline{\lambda}_{EA} = (9\,rad/s)\frac{1}{3}(-2\underline{i} + 2\underline{j} + \underline{k})$

$\underline{\omega} = -(6\,rad/s)\underline{i} + (6\,rad/s)\underline{j} + (3\,rad/s)\underline{k}$

$$\underline{v}_C = \underline{\omega} \times r_{C/E} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -6 & 6 & 3 \\ -0.4 & 0.15 & 0 \end{vmatrix} = -0.45\underline{i} - 1.2\underline{j} + (-0.9 + 2.4)\underline{k}$$

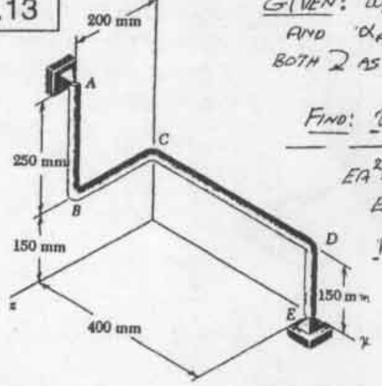
$$\underline{v}_C = -(0.45\,m/s)\underline{i} - (1.2\,m/s)\underline{j} + (1.5\,m/s)\underline{k} \blacktriangleleft$$

$\underline{a}_C = \underline{\alpha}_{AC} \times \underline{r}_{C/E} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{C/E}) = \underline{\alpha}_{AC} \times \underline{r}_{C/E} + \underline{\omega} \times \underline{v}_C$

$$\underline{a}_C = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -6 & 6 & 3 \\ -0.45 & -1.2 & 1.5 \end{vmatrix} = (9 + 3.6)\underline{i} + (-1.35 + 9)\underline{j} + (7.2 + 2.7)\underline{k}$$

$$\underline{a}_C = (12.60\,m/s^2)\underline{i} + (7.65\,m/s^2)\underline{j} + (9.90\,m/s^2)\underline{k} \blacktriangleleft$$

15.13



GIVEN: $\omega_{AE} = 9$ rad/s AND $\alpha_{AE} = 45$ rad/s^2, BOTH ↻ AS VIEWED FROM E.

FIND: $\underline{v}_B$ AND $\underline{a}_B$

$EA^2 = 0.4^2 + 0.4^2 + 0.2^2$

$EA = 0.6$ m

$\underline{r}_{B/A} = -(0.25\,m)\underline{j}$

$\overline{EA} = -(0.4\,m)\underline{i} + (0.4\,m)\underline{j} + (0.2\,m)\underline{k}$

$\underline{\lambda}_{EA} = EA/EA = (-0.4\underline{i} + 0.4\underline{j} + 0.2\underline{k})/0.6 = \frac{1}{3}(-2\underline{i} + 2\underline{j} + \underline{k})$

$\underline{\omega} = \omega_{AE}\underline{\lambda}_{EA} = (9\,rad/s)\frac{1}{3}(-2\underline{i} + 2\underline{j} + \underline{k})$

$\underline{\omega} = -(6\,rad/s)\underline{i} + (6\,rad/s)\underline{j} + (3\,rad/s)\underline{k}$

$\underline{v}_B = \underline{\omega} \times r_{C/E} = (-6\underline{i} + 6\underline{j} + 3\underline{k}) \times (-0.25)\underline{j}$

$= 1.5\underline{k} + 0.75\underline{i}$; $\underline{v}_B = (0.75\,m/s)\underline{i} + (1.5\,m/s)\underline{k} \blacktriangleleft$

$\underline{\alpha} = \alpha_{AE}\underline{\lambda}_{EA} = (45\,rad/s^2)\frac{1}{3}(-2\underline{i} + 2\underline{j} + \underline{k})$

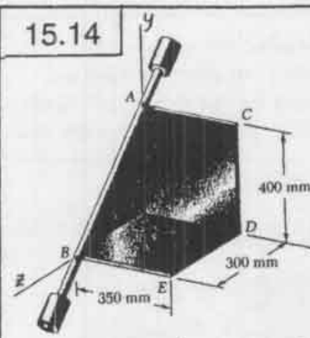
$\underline{\alpha} = -(30\,rad/s^2)\underline{i} + (30\,rad/s^2)\underline{j} + (15\,rad/s^2)\underline{k}$

$\underline{a}_B = \underline{\alpha} \times \underline{r}_{B/A} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{B/A}) = \underline{\alpha} \times \underline{r}_{B/A} + \underline{\omega} \times \underline{v}_B$

$$\underline{a}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -30 & 30 & 15 \\ 0 & -0.25 & 0 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -6 & 6 & 3 \\ 0.75 & 0 & 1.5 \end{vmatrix} = 3.75\underline{i} + 7.5\underline{k} + 9\underline{i} + (2.25 + 9)\underline{j} - 4.5\underline{k}$$

$$\underline{a}_B = (12.75\,m/s^2)\underline{i} + (11.25\,m/s^2)\underline{j} + (3\,m/s^2)\underline{k} \blacktriangleleft$$

15.14



GIVEN: $\omega_{AB} = 5$ rad/s, $\alpha_{AB} = 0$ AND VELOCITY OF E IS DOWNWARD

FIND: $\underline{v}_D$ AND $\underline{a}_D$

SINCE $\underline{v}_E$ IS DOWNWARD, ω_{AB} IS ↺ WHEN VIEWED FROM A.

$AB^2 = 0.4^2 + 0.3^2$: $AB = 0.5$ m

$\underline{r}_{D/B} = (0.35\,m)\underline{i} - (0.3\,m)\underline{k}$

$\overline{BA} = (0.4\,m)\underline{j} - (0.3\,m)\underline{k}$

$\underline{\lambda}_{BA} = \frac{\overline{BA}}{BA} = \frac{1}{0.5}(0.4\underline{j} - 0.3\underline{k}) = 0.8\underline{j} - 0.6\underline{k}$

$\underline{\omega} = \omega_{AB}\underline{\lambda}_{BA} = (5\,rad/s)(0.8\underline{j} - 0.6\underline{k}) = (4\,rad/s)\underline{j} - (3\,rad/s)\underline{k}$

$$\underline{v}_D = \underline{\omega} \times \underline{r}_{D/B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & -3 \\ 0.35 & 0 & -0.3 \end{vmatrix} = -1.2\underline{i} - 1.05\underline{j} - 1.4\underline{k}$$

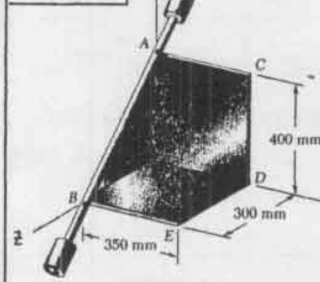
$$\underline{v}_D = -(1.2\,m/s)\underline{i} - (1.05\,m/s)\underline{j} - (1.4\,m/s)\underline{k} \blacktriangleleft$$

$\underline{a}_D = \underline{\alpha} \times \underline{r}_{D/B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{D/B}) = \underline{\alpha} \times \underline{r}_{D/B} + \underline{\omega} \times \underline{v}_D$

$$\underline{a}_D = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & -3 \\ -1.2 & -1.05 & -1.4 \end{vmatrix} = (-5.6 - 3.15)\underline{i} + 3.6\underline{j} + 4.8\underline{k}$$

$$\underline{a}_D = -(8.75\,m/s^2)\underline{i} + (3.6\,m/s^2)\underline{j} + (4.8\,m/s^2)\underline{k} \blacktriangleleft$$

15.15



GIVEN: $\omega_{AB} = 5$ rad/s, $\alpha_{AB} = -20$ rad/s AND VELOCITY OF E IS DOWNWARD.

FIND: $\underline{v}_D$ AND $\underline{a}_D$.

SINCE $\underline{v}_E$ IS DOWNWARD, ω_{AB} IS ↺ WHEN VIEWED FROM A.

$AB^2 = 0.4^2 + 0.3^2$; $AB = 0.5$ m

$\underline{r}_{D/B} = (0.35\,m)\underline{i} - (0.3\,m)\underline{k}$

$\overline{BA} = (0.4\,m)\underline{j} - (0.3\,m)\underline{k}$

$\underline{\lambda}_{BA} = \frac{\overline{BA}}{BA} = \frac{1}{0.5}(0.4\underline{j} - 0.3\underline{k}) = 0.8\underline{j} - 0.6\underline{k}$

$\underline{\omega} = \omega_{AB}\underline{\lambda}_{BA} = (5\,rad/s)(0.8\underline{j} - 0.6\underline{k}) = (4\,rad/s)\underline{j} - (3\,rad/s)\underline{k}$

$\underline{\alpha} = \alpha_{AB}\underline{\lambda}_{BA} = (-20\,rad/s^2)(0.8\underline{j} - 0.6\underline{k}) = (-16\,rad/s^2)\underline{j} + (12\,rad/s^2)\underline{k}$

$$\underline{v}_D = \underline{\omega} \times \underline{r}_{D/B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & -3 \\ 0.35 & 0 & -0.3 \end{vmatrix} = -1.2\underline{i} - 1.05\underline{j} - 1.4\underline{k}$$

$$\underline{v}_D = -(1.2\,m/s)\underline{i} - (1.05\,m/s)\underline{j} - (1.4\,m/s)\underline{k} \blacktriangleleft$$

$\underline{a}_D = \underline{\alpha} \times \underline{r}_{D/B} + \underline{\omega} \times (\underline{\omega} \times \underline{r}_{D/B}) = \underline{\alpha} \times \underline{r}_{D/B} + \underline{\omega} \times \underline{v}_D$

$$\underline{a}_D = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & -16 & 12 \\ 0.35 & 0 & -0.3 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & -3 \\ -1.2 & -1.05 & -1.4 \end{vmatrix}$$

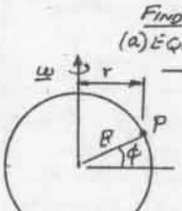
$= 4.8\underline{i} + 4.2\underline{j} + 5.6\underline{k} + (-5.6 - 3.15)\underline{i} + 3.6\underline{j} + 4.8\underline{k}$

$$\underline{a}_D = -(3.95\,m/s^2)\underline{i} + (7.8\,m/s^2)\underline{j} + (10.40\,m/s^2)\underline{k} \blacktriangleleft$$

15.16

GIVEN: EARTH ROTATES 2π radians IN 23 h 56 m. RADIUS OF EARTH = 3960 mi

FIND: VELOCITY AND ACCELERATION OF POINT AT (a) EQUATOR, (b) PHILA, LATITUDE 40°, (c) NORTH POLE



$$24h\ 56m = 23.933\ h$$

$$\omega = \frac{2\pi \text{ rad}}{(23.933h)\left(\frac{3600s}{h}\right)} = 72.925\times10^{-6} \text{ rad/s}$$

$$R = (3960 mi)\left(\frac{5280 ft}{mi}\right) = 20.91\times10^{6} ft$$

r = RADIUS OF PATH = $R\cos\phi$

(a) EQUATOR: LATITUDE = $\phi = 0$

$v = r\omega = R(\cos 0)\omega = (20.91\times10^6 ft)(1)(72.925\times10^{-6} \text{ rad/s})$

$v = 1525$ ft/s ◀

$a = r\omega^2 = R(\cos 0)\omega^2 = (20.91\times10^6 ft)(1)(72.925\times10^{-6} \text{ rad/s})^2$

$a = 0.1112$ ft/s² ◀

(b) PHILADELPHIA: LATITUDE = $\phi = 40°$

$v = r\omega = R(\cos 40°)\omega = (20.91\times10^6 ft)(\cos 40°)(72.925\times10^{-6} \text{ rad/s})$

$v = 1168$ ft/s ◀

$a = r\omega^2 = R(\cos 40°)\omega^2 = (20.91\times10^6 ft)(\cos 40°)(72.925\times10^{-6} \text{ rad/s})^2$

$a = 0.0852$ ft/s² ◀

(c) NORTH POLE: LATITUDE = $\phi = 0$

$r = R\cos 0 = 0$ $\quad v = a = 0$ ◀

15.17

GIVEN: ONE YEAR = 365.24 DAYS AND RADIUS OF ORBIT OF EARTH = 93×10^6 mi.

FIND: FOR THE EARTH, v AND a.

$$\omega = \frac{2\pi \text{ rad}}{(365.24 \text{ DAYS})\left(\frac{24h}{DAY}\right)\left(\frac{3600s}{h}\right)} = 199.11\times10^{-9} \text{ rad/s}$$

$v = r\omega = (93\times10^6 mi)\left(\frac{5280 ft}{mi}\right)(199.11\times10^{-9} \text{ rad/s})$

$v = 97{,}770$ ft/s $\quad v = 66{,}700$ mi/h ◀

$a = r\omega^2 = (93\times10^6)(5280)(199.11\times10^{-9} \text{ rad/s})^2$

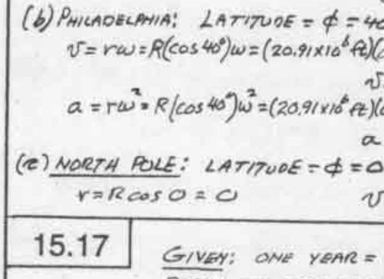
$a = 19.47\times10^{-3}$ ft/s² ◀

15.18

GIVEN: $r = 200$ mm

$\alpha = 0.3$ rad/s² (CONSTANT)

AT $t = 0$, $\omega_0 = 0$



FIND: a_B WHEN (a) $t = 0$, (b) $t = 2$ s, (c) $t = 4$ s.

UNIFORMLY ACCELERATED MOTION

$\omega = \omega_0 + \alpha t = 0 + \alpha t \quad \omega = \alpha t$

$a_t = r\alpha \quad a_n = r\omega^2 = r\alpha^2 t^2$

$a_B^2 = a_t^2 + a_n^2 = r^2\alpha^2 + r^2\alpha^4 t^4 = r^2\alpha^2(1+\alpha^2 t^4)$

$a_B = r\alpha(1+\alpha^2t^4)^{1/2}$

$r = 0.2$ m, $\alpha = 0.3$ rad/s²

$a_B = (0.2)(0.3)(1+(0.3)^2t^4)^{1/2} = 0.06(1+0.09t^4)^{1/2}$

(a) $t=0$: $a_B = 0.06(1+0)$ $\quad a_B = 0.06$ m/s² ◀

(b) $t=2s$: $a_B = 0.06(1+0.09\times2^4)^{1/2}$ $\quad a_B = 0.0937$ m/s² ◀

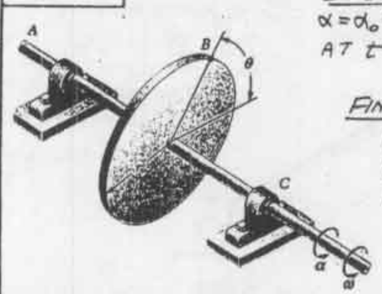
(c) $t=4s$: $a_B = 0.06(1+0.09\times4^4)^{1/2}$ $\quad a_B = 0.294$ m/s² ◀

15.19

GIVEN: $r = 600$ mm

$\alpha = \alpha_0 e^{-t}$ WHERE $\alpha_0 = 10$ rad/s².

AT $t=0$, $\omega = 0$.



FIND: a_B WHEN (a) $t = 0$, (b) $t = 0.5$ s, (c) $t = \infty$.

$\alpha = \frac{d\omega}{dt} = \alpha_0 e^{-t}; \quad \int_0^\omega d\omega = \int_0^t \alpha_0 e^{-t}dt$

$\omega = \alpha_0 \left| -e^{-t}\right|_0^t \quad \omega = \alpha_0(1-e^{-t})$

$a_t = r\alpha = r\alpha_0 e^{-t} = (0.6 m)(10 \text{ rad/s}^2)e^{-t} = 6e^{-t}$ ◁

$a_n = r\omega^2 = r\alpha_0^2(1-e^{-t})^2 = (0.6)(10)^2(1-e^{-t})^2 = 60(1-e^{-t})^2$ ◁

(a) $t=0$: $a_t = 6e^0 = 6$ m/s² $\quad a_n = 60(1-e^0)^2 = 0$

$a_B^2 = a_t^2 + a_n^2 = 6^2 \quad a_B = 6$ m/s² ◀

(b) $t = 0.5s$:

$a_t = 6e^{-0.5} = 6(0.6065) = 3.639$ m/s²

$a_n = 60(1-e^{-0.5})^2 = 60(1-0.6065)^2 = 9.289$ m/s²

$a_B^2 = a_t^2 + a_n^2 = (3.639)^2 + (9.289)^2 \quad a_B = 9.98$ m/s² ◀

(c) $t = \infty$:

$a_t = 6e^{-\infty} = 0 \quad a_n = 60(1-e^{-\infty})^2 = 60$ m/s²

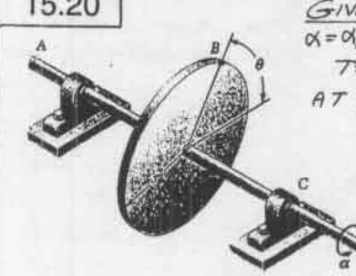
$a_B^2 = a_t^2 + a_n^2 = 0 + 60^2 \quad a_B = 60$ m/s² ◀

15.20

GIVEN: $r = 250$ mm

$\alpha = \alpha_0\cos(\pi t/T)$ WHERE $T = 1.5$ s AND $\alpha_0 = 10$ rad/s²

AT $t = 0$, $\omega = 0$



FIND: a_B WHEN (a) $t = 0$, (b) $t = 0.5$ s, (c) $t = 0.75$ s.

$\alpha = \frac{d\omega}{dt} = \alpha_0\cos\left(\frac{\pi t}{T}\right): \quad \int_0^\omega d\omega = \int_0^t \alpha_0\cos\left(\frac{\pi t}{T}\right)dt$

$\omega = \alpha_0\frac{T}{\pi}\left|\sin\left(\frac{\pi t}{T}\right)\right|_0^t \quad \omega = \alpha_0\frac{T}{\pi}\sin\left(\frac{\pi t}{T}\right)$

$a_t = r\alpha = r\alpha_0\cos(\pi t/T) = (0.25m)(10 \text{ rad/s}^2)\cos\left(\frac{\pi t}{1.5}\right) = 2.5\cos\left(\frac{\pi t}{1.5}\right)$ ◁

$a_n = r\omega^2 = r\alpha_0^2\frac{T^2}{\pi^2}\sin^2\frac{\pi t}{T} = (0.25)(10^2)\frac{1.5^2}{\pi^2}\sin^2\left(\frac{\pi t}{1.5}\right) = 5.70\sin^2\left(\frac{\pi t}{1.5}\right)$ ◁

(a) $t=0$: $a_t = 2.5\cos(0) = 2.5$ m/s²

$a_n = 5.70\sin(0) = 0$

$a_B^2 = a_t^2 + a_n^2 = 2.5^2 + 0 \quad a_B = 2.5$ m/s² ◀

(b) $t = 0.5s$:

$a_t = 2.5\cos\left(\frac{0.5\pi}{1.5}\right) = 2.5\cos\frac{\pi}{3} = 1.25$ m/s²

$a_n = 5.70\sin^2\left(\frac{0.5\pi}{1.5}\right) = 5.70\sin^2\frac{\pi}{3} = 4.275$ m/s²

$a_B^2 = a_t^2 + a_n^2 = 1.25^2 + 4.275^2 \quad a_B = 4.45$ m/s² ◀

(c) $t = 0.75s$:

$a_t = 2.5\cos\left(\frac{0.75\pi}{1.5}\right) = 2.5\cos\frac{\pi}{2} = 0$

$a_n = 5.70\sin\left(\frac{0.75\pi}{1.5}\right) = 5.70\sin\frac{\pi}{2} = 5.70$ m/s²

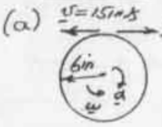
$a_B^2 = a_t^2 + a_n^2 = 0 + 5.70^2 \quad a_B = 5.70$ m/s² ◀

15.21

GIVEN: $\underline{v}_A = 15\ in./s$, $\underline{a}_A = 9\ in./s^2$

FIND: (a) ω AND $\underline{\alpha}$ OF PULLEY. (b) $\underline{a}_B$

(a)



$v = r\omega$

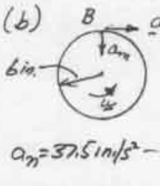
$15\ in./s = (6\ in.)\omega$; $\omega =$

$\omega = 2.5\ rad/s$

$a = r\alpha$; $9\ in./s^2 = (6\ in.)\alpha$

$\underline{\alpha} = 1.5\ rad/s^2$

(b)



$a_n = r\omega^2 = (6\ in.)(2.5\ rad/s)^2$

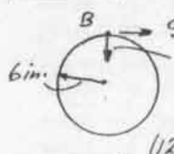
$\underline{a}_n = 37.5\ in/s^2 \downarrow$

$\underline{a}_B = 38.6\ in/s^2 \measuredangle 76.5°$

15.22

GIVEN: $\underline{\omega} = 4\ rad/s$

FIND: $\underline{\alpha}$ FOR WHICH $a_B = 120\ in./s^2$



$\underline{a}_t = r\alpha = (6\ in.)\alpha$

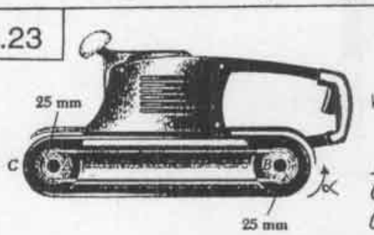
$\underline{a}_n = r\omega^2 = (6\ in.)(4\ rad/s)^2 = 96\ in./s^2 \downarrow$

$a_B^2 = a_t^2 + a_n^2$

$(120\ in/s^2)^2 = (6\alpha)^2 + (96\ in./s^2)^2$

$\alpha^2 = 144$, $\alpha = \pm 12$, $\underline{\alpha} = 12\ rad/s^2$

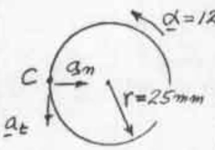
15.23



GIVEN: $\underline{\alpha} = 120\ rad/s^2$

WHEN $t=0$, $\omega = 0$

FIND: a_C WHEN (a) $t = 0.5\ s$, (b) $t = 2\ s$.



$a_t = r\alpha = (0.025\ m)(120\ rad/s^2)$

$\underline{a}_t = 3\ m/s^2 \downarrow$

(a) $\underline{t = 0.5\ s}$:

$\omega = \alpha t = (120\ rad/s^2)(0.5\ s) = 60\ rad/s$

$a_n = r\omega^2 = (0.025\ m)(60\ rad/s)^2$

$a_n = 90\ m/s^2 \rightarrow$

$a_B^2 = a_t^2 + a_n^2 = 3^2 + 90^2$ $\quad a_B = 90.05\ m/s^2$

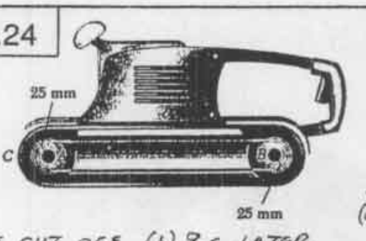
(b) $\underline{t = 2\ s}$: $\omega = \alpha t = (120\ rad/s^2)(2\ s) = 240\ rad/s$

$a_n = r\omega^2 = (0.025\ m)(240\ rad/s)^2$

$a_n = 1440\ m/s^2$

$a_B^2 = a_t^2 + a_n^2 = 3^2 + 1440^2$ $\quad a_B = 1440\ m/s^2$

15.24



GIVEN: RATED SPEED OF DRUMS IS 2400 rpm. SANDER COASTS TO REST IN 10 s.

FIND: v_C AND a_C (a) BEFORE POWER IS CUT OFF, (b) 9 s LATER.

$\omega_0 = 2400\ rpm = 251.3\ rad/s$ $\quad r = 0.025\ m$

(a) $v_C = r\omega = (0.025\ m)(251.3\ rad/s)$; $v_C = 6.28\ m/s$

$a_C = r\omega^2 = (0.025\ m)(251.3\ rad/s)^2$; $a_C = 1579\ m/s^2$

(b) WHEN $t = 10\ s$, $\omega = 0$.

$\omega = \omega_0 + \alpha t$; $0 = 251.3\ rad/s + \alpha(10\ s)$; $\alpha = -25.13\ rad/s^2$

WHEN $t = 9\ s$:

$\omega = \omega_0 + \alpha t$; $\omega_9 = 251.3\ rad/s - (25.13\ rad/s^2)(9\ s) = 25.13\ rad/s$

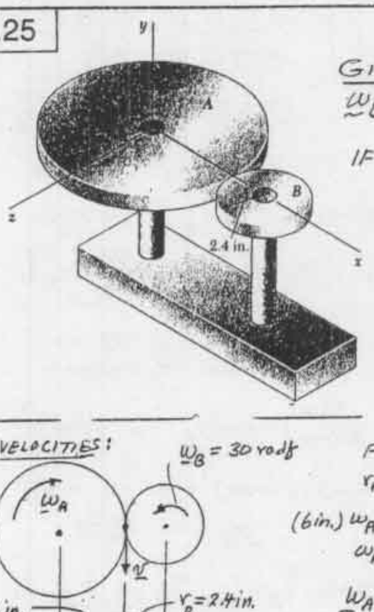
$v_C = r\omega_9 = (0.025\ m)(25.13\ rad/s)$; $v_9 = 0.628\ m/s$

$(a_C)_t = r\alpha = (0.025\ m)(-25.13\ rad/s^2)$; $(a_C)_t = 0.628\ m/s^2$

$(a_C)_n = r\omega_9^2 = (0.025\ m)(25.13\ rad/s)^2$; $(a_C)_n = 15.79\ m/s^2$

$a_C^2 = (a_C)_t^2 + (a_C)_n^2 = (0.628\ m/s^2)^2 + (15.79\ m/s^2)^2$ $\quad a_C = 15.80\ m/s^2$

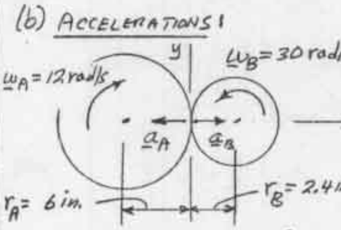
15.25



GIVEN: $\underline{\omega}_B = (30\ rad/s)\mathbf{j}$

IF NO SLIPPING OCCURS, FIND: (a) $\underline{\omega}_A$ (b) ACCELERATIONS OF POINTS IN CONTACT.

(a) VELOCITIES:



FOR NO SLIPPING.

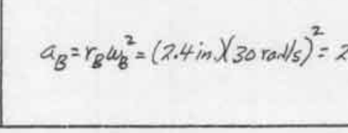
$r_A\omega_A = r_B\omega_B$

$(6\ in.)\omega_A = (2.4\ in.)(30\ rad/s)$

$\omega_A = 12\ rad/s$

$\underline{\omega}_A = -(12\ rad/s)\mathbf{j}$

(b) ACCELERATIONS:



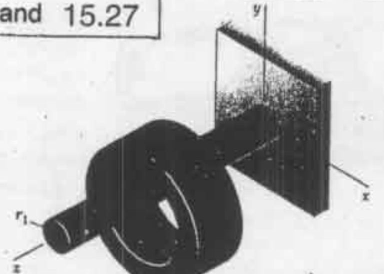
$a_A = r_A\omega_A^2 = (6\ in.)(12\ rad/s)^2 = 864\ in./s^2 = 72\ ft/s^2$

$\underline{a}_A = -(72\ ft/s^2)\mathbf{i}$

$a_B = r_B\omega_B^2 = (2.4\ in.)(30\ rad/s)^2 = 2160\ in./s^2 = 180\ ft/s^2$

$\underline{a}_B = (180\ ft/s^2)\mathbf{i}$

15.26 and 15.27



GIVEN: CONSTANT ANG. VELOCITY OF SHAFT : $\underline{\omega}_A = \omega_A \underline{k}$

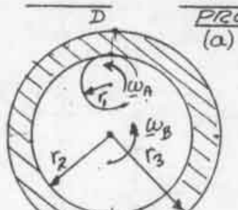
FIND: (a) ANG. VELOCITY OF RING $\underline{\omega}_B$

(b) ACCELERATIONS OF POINTS SHAFT AND RING WHICH ARE IN CONTACT.

PROB. 15.26: IN TERMS OF ω_A, r_1, r_2, AND r_3.

PROB. 15.27: WHEN $\omega_A = 25$ rad/s, $r_1 = 12$ mm, $r_2 = 30$ mm, AND $r_3 = 40$ mm

ALSO, FIND ACCEL. OF POINT ON OUTSIDE OF B.



PROB. 15.26

(a) AT POINT OF CONTACT

$r_1 \omega_A = r_2 \omega_B \qquad \underline{\omega}_B = \frac{r_1}{r_2} \omega_A \circlearrowleft$ ◀

(b) ACCEL OF POINTS OF CONTACT

SHAFT A: $\underline{a}_A = r_1 \omega_A^2 \downarrow$ ◀

RING B: $a_B = r_2 \omega_B^2 = r_2 \left(\frac{r_1}{r_2}\omega_A\right)^2$

$\underline{a}_B = \frac{r_1^2}{r_2} \omega_A^2 \downarrow$ ◀

ACCEL. OF POINT D ON OUTSIDE OF RING

$a_D = r_3 \omega_B^2 = r_3 \left(\frac{r_1}{r_2}\omega_A\right)^2; \quad a_D = r_3 \left(\frac{r_1}{r_2}\right)^2 \omega_A^2 \downarrow$

PROB. 15.27 $\omega_A = 25$ rad/s, $r_1 = 12$ mm, $r_2 = 30$ mm, $r_3 = 40$ mm

(a) $\omega_B = \frac{r_1}{r_2}\omega_A = \frac{12\text{ mm}}{30\text{ mm}}(25\text{ rad/s}); \quad \underline{\omega}_B = 10\text{ rad/s} \circlearrowleft$ ◀

(b) $a_A = r_1 \omega_A^2 = (12\text{ mm})(25\text{ rad/s})^2 = 7.5\times10^3\text{ mm/s}^2$

$\underline{a}_A = 7.5\text{ m/s}^2 \downarrow$ ◀

$a_B = \frac{r_1^2}{r_2}\omega_A^2 = \frac{(12\text{ mm})^2}{(30\text{ mm})}(25\text{ rad/s})^2 = 3\times10^3\text{ mm/s}^2$

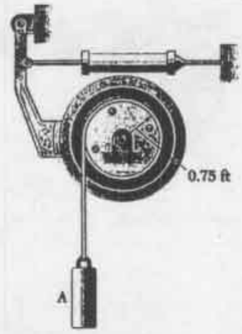
$\underline{a}_B = 3\text{ m/s}^2 \downarrow$ ◀

$a_D = r_3\left(\frac{r_1}{r_2}\right)^2 \omega_A^2 = (40\text{ mm})\left(\frac{12\text{ mm}}{30\text{ mm}}\right)^2 (25\text{ rad/s})^2$

$a_D = 4\times10^3\text{ mm/s}^2$

$\underline{a}_D = 4\text{ m/s}^2 \downarrow$ ◀

15.28



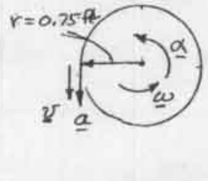
GIVEN: WHEN $t = 0$, $\underline{v}_A = 9$ ft/s $\downarrow$ BRAKE IS APPLIED AND BLOCK COMES TO REST AFTER MOVING 18 ft $\downarrow$. ASSUMING UNIFORM MOTION, FIND: (a) α OF DRUM (b) TIME TO COME TO REST

BLOCK A: $v^2 - v_0^2 = 2as$

$0 - (9\text{ ft/s})^2 = 2a(18\text{ ft})$

$a = -2.25\text{ ft/s}^2; \quad \underline{a} = 2.25\text{ ft/s}^2 \uparrow$

DRUM:



$v_A = r\omega_0$

$9\text{ ft/s} = (0.75\text{ ft})\omega$

$\omega_0 = 12\text{ rad/s}$

$a = r\alpha$

$-(2.25\text{ ft/s}^2) = (0.75\text{ ft})\alpha$

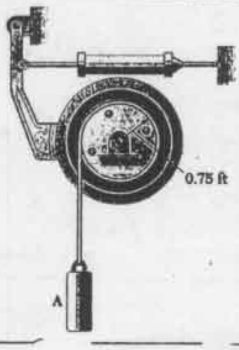
$\alpha = -3\text{ rad/s}^2; \quad \alpha = 3\text{ rad/s}^2 \circlearrowright$ ◀

UNIFORM MOTION: $\omega = 0$ WHEN $t = t_1$

$\omega = \omega_0 + \alpha t: \quad 0 = (12\text{ rad/s}) - (3\text{ rad/s}^2)t_1$

$t_1 = 4\text{ s}$ ◀

15.29



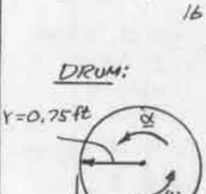
GIVEN: WHEN $t = 0$, $v = 0$. WHEN $t = 5$, BLOCK HAS MOVED 16 ft $\downarrow$ ASSUMING UNIFORM MOTION, FIND: (a) α OF DRUM (b) ω OF DRUM WHEN $t = 4$ s

BLOCK A: $s = v_0 t + \frac{1}{2}at^2$

$16\text{ ft} = 0 + \frac{1}{2}a(5\text{ s})^2$

$a = +1.28\text{ ft/s}^2 \qquad \underline{a} = 1.28\text{ ft/s}^2 \downarrow$

DRUM:



$a = r\alpha$

$(1.28\text{ ft/s}^2) = (0.75\text{ ft})\alpha$

$\alpha = 1.707\text{ rad/s}^2$

$\underline{\alpha} = 1.707\text{ rad/s}^2 \circlearrowleft$ ◀

UNIFORM MOTION $\omega_0 = 0$ WHEN $t = 0$

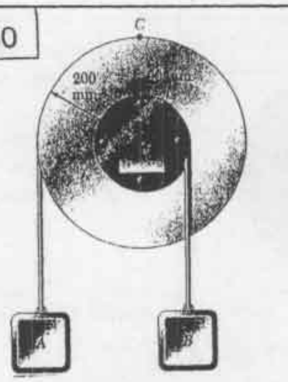
$\omega = \omega_0 + \alpha t$

WHEN $t = 4$ s: $\omega = 0 + (1.707\text{ rad/s}^2)(4\text{ s})$

$\omega = 6.83\text{ rad/s}$

$\underline{\omega} = 6.83\text{ rad/s} \circlearrowleft$ ◀

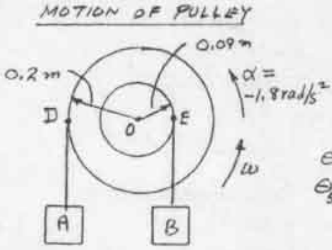
15.30



GIVEN: FOR PULLEY, $\omega_0 = 0.8$ rad/s ↻, $\alpha = 1.8$ rad/s² ↺.

FIND: WHEN $t = 5$ s, THE VELOCITY AND POSITION OF
(a) BLOCK A.
(b) BLOCK B.

MOTION OF PULLEY



UNIF. ACCEL. MOTION

$\omega = \omega_0 + \alpha t$

$\omega_5 = +(0.8 \text{ rad/s}) - (1.8 \text{ rad/s}^2)(5 \text{ s})$

$\omega_5 = -8.2$ rad/s

$\omega_5 = 8.2$ rad/s ↺ ◁

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta_5 = 0 + (0.8 \text{ rad/s})(5 \text{ s}) - \frac{1}{2}(1.8 \text{ rad/s}^2)(5 \text{ s})^2$

$\theta_5 = -18.5$ rad

$\theta_5 = 18.5$ rad ↺ ◁

BLOCK A:

$v_A = v_D = r\omega_5 = (0.2 \text{ m})(8.2 \text{ rad/s}) = 1.64$ m/s; $v_A = 1.640$ m/s ↑ ◀

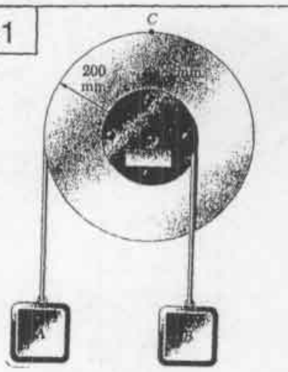
$s_A = r\theta_5 = (0.2 \text{ m})(18.5 \text{ rad}) = 3.70$ m; $s_A = 3.70$ m ↑ ◀

BLOCK B:

$v_B = v_E = r\omega_5 = (0.09 \text{ m})(8.2 \text{ rad/s}) = 0.738$ m/s; $v_B = 0.738$ m/s ↓ ◀

$s_B = r\theta_5 = (0.09 \text{ m})(18.5 \text{ rad}) = 1.665$ m; $s_B = 1.665$ m ↓ ◀

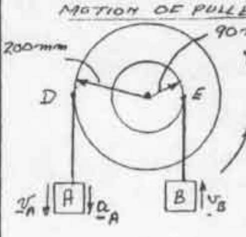
15.31



GIVEN: BLOCK A. $v_0 = 120$ mm/s ↓, $a_A = 75$ mm/s² ↓

FIND:
(a) REVOLUTIONS OF PULLEY IN 6 s.
(b) WHEN $t = 6$ s THE VELOCITY AND POSITION OF BLOCK B.
(c) a_C WHEN $t = 0$

MOTION OF PULLEY



UNIF. ACCEL. MOTION

$v_0 = r\omega_0$; $120 \text{ mm/s} = (200 \text{ mm})\omega_0$

$\omega_0 = 0.6$ rad/s ↻

$a_A = r\alpha$, $75 \text{ mm/s} = (200 \text{ mm})\alpha$

$\alpha = 0.375$ rad/s² ↻

(a) PULLEY: WHEN $t = 6$ s

(a) $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\theta = 0 + (0.6 \text{ rad/s})(6 \text{ s}) + \frac{1}{2}(0.375 \text{ rad/s}^2)(6 \text{ s})^2$

$\theta = 10.35$ rad

$\theta = 10.35 \text{ rad}\left(\frac{\text{rev}}{2\pi \text{ rad}}\right)$: $\theta = 1.647$ rev ◀

$\omega_6 = \omega_0 + \alpha t = 0.6 \text{ rad/s} + (0.375 \text{ rad/s}^2)(6 \text{ s})$; $\omega_6 = 2.85$ rad/s ↻

BLOCK B: WHEN $t = 6$ s

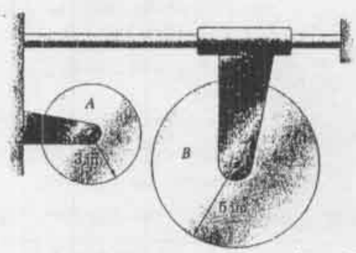
$v_B = r\omega_6 = (90 \text{ mm})(2.85 \text{ rad/s})$ $v_B = 256$ mm/s ↑ ◀

$s_B = r\theta = (90 \text{ mm})(10.35 \text{ rad})$ $s_B = 932$ mm ↑ ◀

POINT C, WHEN $t = 0$

$\left.\begin{array}{l} a_t = r\alpha = (200 \text{ mm})(0.375 \text{ rad/s}^2) = 75 \text{ mm/s}^2 \leftarrow \\ a_n = r\omega^2 = (200 \text{ mm})(0.6 \text{ rad/s})^2 = 72 \text{ mm/s}^2 \downarrow \end{array}\right\}$ $a_C = 104.0$ mm/s² ↙ 43.8° ◀

15.32



GIVEN: WHEN $t = 0$, $(\omega_A)_0 = 450$ rpm ↺, $(\omega_B)_0 = 0$

AFTER SLIPPAGE, WHEN $t = 6$ s, $\omega_A = 140$ rpm ↺

FIND: DURING SLIPPAGE α_A AND α_B

DISK A: $(\omega_A)_0 = 450 \text{ rpm} = 47.124$ rad/s ↺

WHEN $t = 6$: $\omega_A = 140 \text{ rpm} = 14.661$ rad/s ↺

$\omega_A = (\omega_A)_0 + \alpha_A t$

$14.661 \text{ rad/s} = 47.124 \text{ rad/s} + \alpha_A(6 \text{ s})$

$\alpha_A = -5.41$ rad/s $\qquad \alpha_A = 5.41$ rad/s² ↻ ◀

DISK B: $\omega_0 = 0$

WHEN $t = 6$ s: (END OF SLIPPAGE)

$+$ $r_A\omega_A = r_B\omega_B$: $(3 \text{ in.})(14.661 \text{ rad/s}) = (5 \text{ in.})(\omega_B)$

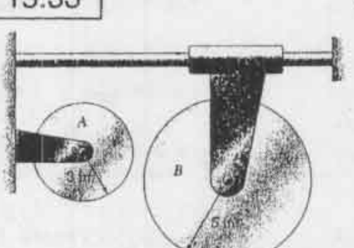
$\omega_B = 8.796$ rad/s ↻

$\omega_B = (\omega_B)_0 + \alpha_B t$

$8.796 \text{ rad/s} = 0 + \alpha_B(6 \text{ s})$

$\alpha_B = 1.466$ rad/s² $\qquad \alpha_B = 1.466$ rad/s² ↻ ◀

15.33



GIVEN:

DISK A: $(\omega_A)_0 = 500$ rpm ↺ WILL COAST TO REST IN 60 s

DISK B: $(\omega_B)_0 = 0$, $\alpha_B = 2.5$ rad/s² ↻

FIND:
(a) WHEN DISKS CAN BE BROUGHT TOGETHER WITH NO SLIPPAGE
(b) FINAL ω_A AND ω_B.

DISK A: $(\omega_A)_0 = 500 \text{ rpm} = 52.36$ rad/s ↺

DISK A WILL COAST TO REST IN 60 s

$\omega_A = (\omega_A)_0 + \alpha_A t$; $0 = 52.36 \text{ rad/s} + \alpha_A(60 \text{ s})$

$\alpha_A = -0.87266$ rad/s²

AT TIME t:

$\omega_A = (\omega_A)_0 + \alpha_A t$; $\qquad \omega_A = 52.36 - 0.87266t \qquad (1)$

DISK B: $\alpha_B = 2.5$ rad/s² $\qquad (\omega_B)_0 = 0$

AT TIME t: $\omega_B = (\omega_B)_0 + \alpha_B t$; $\qquad \omega_B = 2.5t \qquad (2)$

(a) BRING DISKS TOGETHER WHEN: $r_A\omega_A = r_B\omega_B$

$(3 \text{ in.})(52.36 - 0.87266t) = (5 \text{ in.})(2.5t)$

$157.08 - 2.618t = 12.5t$

$157.08 = 15.118t$

$t = 10.39$ s ◀

(b) WHEN CONTACT IS MADE ($t = 10.39$ s)

EQ. (1): $\omega_A = 52.36 - 0.87266(10.39)$

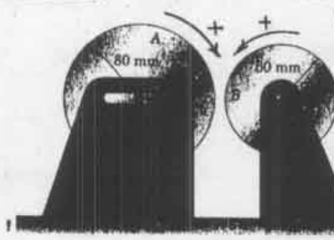
$\omega_A = 43.29$ rad/s $\qquad \omega_A = 413$ rpm ↺ ◀

EQ. (2): $\omega_B = 2.5(10.39)$

$\omega_B = 25.975$ rad/s $\qquad \omega_B = 248$ rpm ↻ ◀

15.34

GIVEN: DISK A: $(\omega_A)_0 = 500$ rpm ↻ WILL COAST TO REST IN 60S.
DISK B: $(\omega_B)_0 = 0$
$\alpha_B = 2.5$ rad/s² ↺
FIND:
(a) WHEN DISKS CAN BE BROUGHT TOGETHER WITH NO SLIPPAGE
(b) FINAL ω_A AND ω_B



DISK A: $(\omega_A)_0 = 500$ rpm $= 52.36$ rad/s ↻

DISK A WILL COAST TO REST IN 60S

$\omega_A = (\omega_A)_0 + \alpha_A t; \quad 0 = 52.36 + \alpha_A(60\text{ s})$

$\alpha_A = -0.87266$ rad/s²

AT TIME t:

$\omega_A = (\omega_A)_0 + \alpha_A t: \qquad \omega_A = 52.36 - 0.87266\,t \qquad (1)$

DISK B: $\alpha_B = 2.5$ rad/s² $\quad (\omega_B)_0 = 0$

AT TIME t: $\omega_B = (\omega_B)_0 + \alpha_B t; \quad \omega_B = 2.5t \qquad (2)$

(a) BRING DISKS TOGETHER WHEN: $r_A\omega_A = r_B\omega_B$

$(80\text{ mm})(52.36 - 0.87266t) = (60\text{ mm})(2.5t)$

$4188.8 - 69.813t = 150t$

$4188.8 = 219.813\,t$

$t = 19.056$ s $\qquad t = 19.06$ s ◀

(b) CONTACT IS MADE:

EQ.(1): $\omega_A = 52.36 - 0.87266(19.056)$

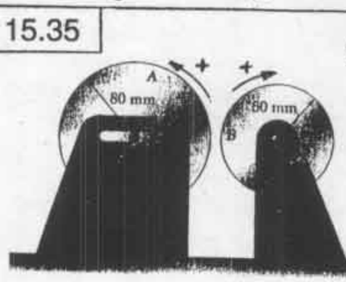
$\omega_A = 35.73$ rad/s $\qquad \omega_A = 341$ rpm ↻ ◀

EQ.(2) $\omega_B = 2.5(19.056)$

$\omega_B = 47.64$ rad/s $\qquad \omega_B = 455$ rpm ↺ ◀

15.35

GIVEN:
$(\omega_A)_0 = (\omega_B)_0 = 240$ rpm ↺
DISKS ARE BROUGHT TOGETHER, AFTER 8S OF SLIPPAGE,
$\omega_A = 60$ rpm ↺
FIND: (a) α_A AND α_B.
(b) TIME AT WHICH $\omega_B = 0$.



(a) DISK A: $(\omega_A)_0 = 240$ rpm $= 25.133$ rad/s ↺

WHEN $t = 8$s, $\omega_A = 60$ rpm $= 6.283$ rad/s ↺

$\omega_A = (\omega_A)_0 + \alpha_A t; \quad 6.283\text{ rad/s} = 25.133\text{ rad/s} + \alpha_A(8\text{s})$

$\alpha_A = -2.356$ rad/s²

$\alpha_A = 2.36$ rad/s² ↻ ◀

DISK B: $(\omega_B)_0 = 240$ rpm $= 25.133$ rad/s ↺

WHEN $t = 8$s: (SLIPPAGE STOPS)

$r_A\omega_A = r_B\omega_B$

$(80\text{ mm})(6.283\text{ rad/s}) = (60\text{ mm})\,\omega_B$

$\omega_B = 8.378$ rad/s $\qquad \omega_B = 8.38$ rad/s ↻

FOR +↻: $\omega_B = (\omega_B)_0 + \alpha_B t$

$8.378\text{ rad/s} = -25.133\text{ rad/s} + \alpha_B(8\text{ s})$

$\alpha_B = 4.188$ rad/s² $\qquad \alpha_B = 4.19$ rad/s² ↻ ◀

(b) TIME WHEN $\omega_B = 0$

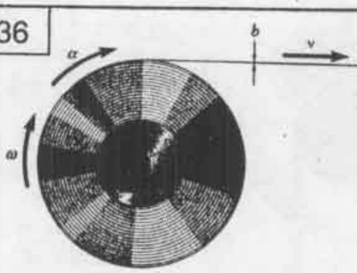
FOR +↻: $\omega_B = (\omega_B)_0 + \alpha_B t$

$0 = -25.133\text{ rad/s} + (4.188\text{ rad/s}^2)\,t$

$t = 6.00$ s $\qquad t = 6.00$ s ◀

15.36

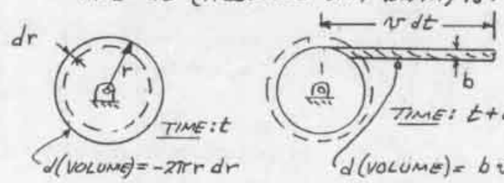
GIVEN: PAPER MOVES AT CONSTANT SPEED v.
DERIVE AN EXPRESSION FOR α OF ROLL.



ANG. VELOCITY IS $\omega = v/r$. SINCE v IS CONSTANT,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{v}{r}\right) = v\,\frac{d}{dt}\left(\frac{1}{r}\right)\frac{dr}{dt} = -\frac{v}{r^2}\frac{dr}{dt} \qquad (1)$$

WE NOTE THAT THE VOLUME OF PAPER UNROLLED IN TIME dt (ASSUMING UNIT WIDTH) IS:



$d(\text{VOLUME}) = -2\pi r\,dr \qquad d(\text{VOLUME}) = b\,v\,dt$

THUS: $-2\pi r\,dr = b\,v\,dt$

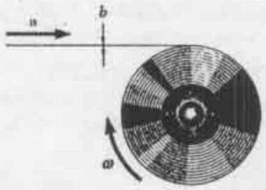
$$\frac{dr}{dt} = -\frac{bv}{2\pi r} \qquad (2)$$

SUBSTITUTE FOR dr/dt FROM (2) INTO (1),

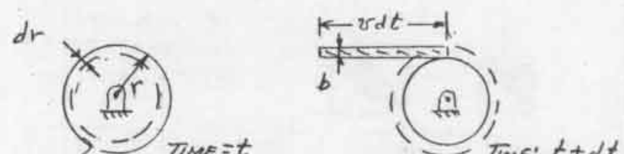
$$\alpha = -\frac{v}{r^2}\left(-\frac{bv}{2\pi r}\right) \qquad \alpha = \frac{bv^2}{2\pi r^3} ↻ ◀$$

15.37

GIVEN:
$\omega = \omega_0$ ↻
$\alpha = 0$
FIND: ACCEL. a OF TAPE



$v = r\omega_0 \qquad \frac{v}{r} = \omega_0 \qquad (1)$



$d(\text{VOLUME}) = 2\pi r\,dr \qquad d(\text{VOLUME}) = b\,v\,dt$

THUS: $2\pi r\,dr = b\,v\,dt$

$$\frac{dr}{dt} = \frac{bv}{2\pi r} \qquad (2)$$

$v = r\omega_0; \quad a = \frac{dv}{dt} = \omega_0\frac{dr}{dt}$

SUBSTITUTE $\frac{dr}{dt}$ FROM (2):

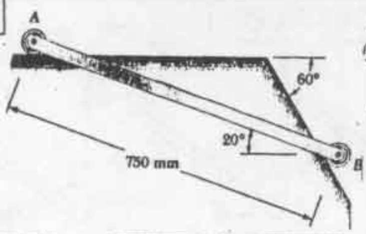
$$a = \omega_0\,\frac{bv}{2\pi r} = \frac{\omega_0 b}{2\pi}\left(\frac{v}{r}\right)$$

SUBSTITUTE v/r FROM (1):

$$a = \frac{\omega_0 b}{2\pi}\,\omega_0 \qquad a = \frac{b\omega_0^2}{2\pi} ◀$$

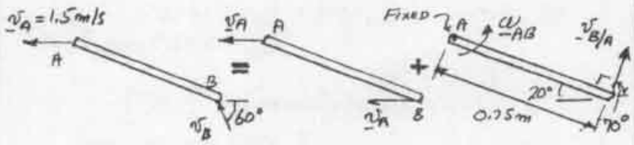
NOTE: a IS INDEPENDENT OF THE RADIUS r.

15.38



GIVEN: $\underline{v}_A = 1.5\ m/s \leftarrow$

FIND: (a) $\underline{\omega}_{AB}$ (b) $\underline{v}_B$



$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$[v_B \measuredangle 60°] = [1.5\ m/s \leftarrow] + [v_{B/A} \measuredangle 70°]$

LAW OF SINES

$$\frac{v_B}{\sin 70°} = \frac{v_{B/A}}{\sin 60°} = \frac{1.5\ m/s}{\sin 50°}$$

$\underline{v}_B = 1.840\ m/s \measuredangle 60°$ ◀

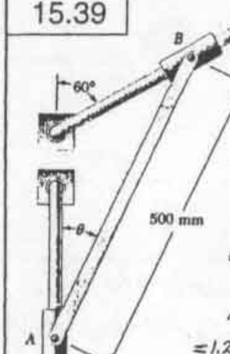
$\underline{v}_{B/A} = 1.696\ m/s \measuredangle 70°$

$v_{B/A} = (AB)\,\omega_{AB}$

$1.696\ m/s = (0.75\ m)\,\omega_{AB}$

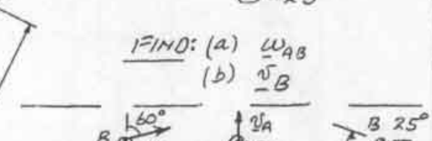
$\omega_{AB} = 2.261\ rad/s$ $\qquad \underline{\omega}_{AB} = 2.26\ rad/s \circlearrowleft$ ◀

15.39



GIVEN: $\underline{v}_A = 1.2\ m/s \uparrow$, $\theta = 25°$

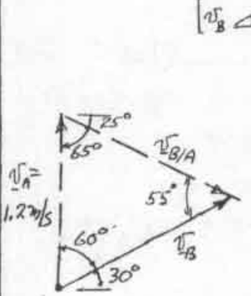
FIND: (a) $\underline{\omega}_{AB}$ (b) $\underline{v}_B$



$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$[v_B \measuredangle 30°] = [1.2\ m/s \uparrow] + [v_{B/A} \measuredangle 25°]$

LAW OF SINES



$$\frac{v_B}{\sin 65°} = \frac{v_{B/A}}{\sin 60°} = \frac{1.2\ m/s}{\sin 55°}$$

$\underline{v}_B = 1.328\ m/s \measuredangle 30°$ ◀

$\underline{v}_{B/A} = 1.269\ m/s \measuredangle 65°$

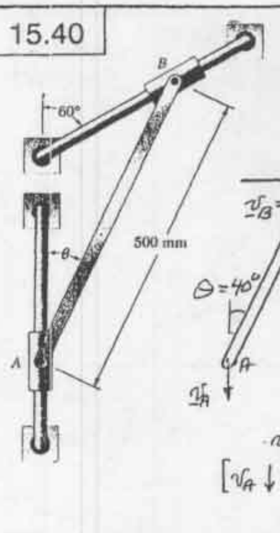
$v_{B/A} = (AB)\,\omega_{AB}$

$1.269\ m/s = (0.5\ m)\,\omega_{AB}$

$\omega_{AB} = 2.538\ rad/s$

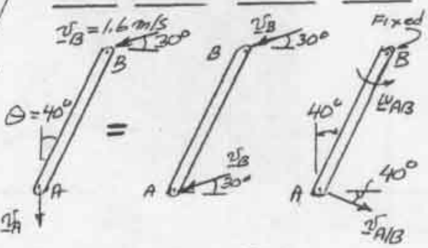
$\underline{\omega}_{AB} = 2.54\ rad/s \circlearrowright$ ◀

15.40



GIVEN: $\underline{v}_B = 1.6\ m/s \swarrow$, $\theta = 40°$

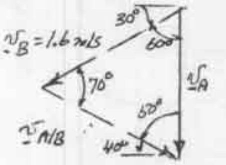
FIND: (a) ω_{AB} (b) $\underline{v}_A$



$v_A = v_B + v_{A/B}$

$[v_A \downarrow] = [1.6\ m/s \measuredangle 30°] + [v_{A/B} \measuredangle]$

LAW OF SINES



$$\frac{v_A}{\sin 70°} = \frac{v_{A/B}}{\sin 60°} = \frac{1.6\ m/s}{\sin 50°}$$

$\underline{v}_A = 1.963\ m/s \downarrow$ ◀

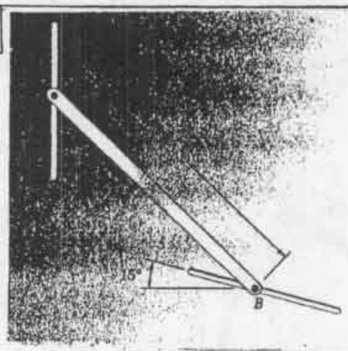
$\underline{v}_{A/B} = 1.809\ m/s \measuredangle 40°$

$v_{A/B} = (AB)\,\omega_{AB}$ $\qquad AB = 0.5\ m$

$1.809\ m/s = (0.5\ m)\,\omega_{AB}$

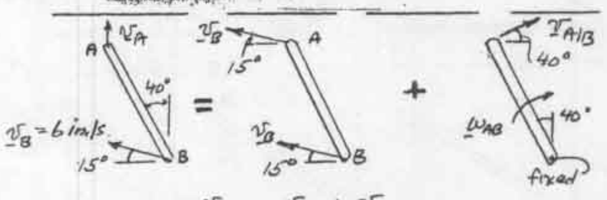
$\omega_{AB} = 3.618\ rad/s$ $\qquad \underline{\omega}_{AB} = 3.62\ rad/s \circlearrowleft$ ◀

15.41



GIVEN: $\underline{v}_B = 6\ in./s \leftarrow$, $\theta = 40°$

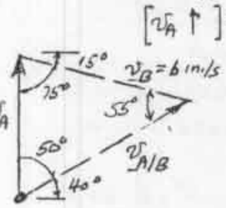
FIND: (a) $\underline{\omega}_{AB}$ (b) $\underline{v}_A$



$v_A = v_B + v_{A/B}$

$[v_A \uparrow] = [6\ in./s \measuredangle 15°] + [v_{A/B} \measuredangle 40°]$

LAW OF SINES



$$\frac{v_A}{\sin 55°} = \frac{v_{A/B}}{\sin 75°} = \frac{6\ in./s}{\sin 50°}$$

$\underline{v}_A = 6.42\ in./s \uparrow$ ◀

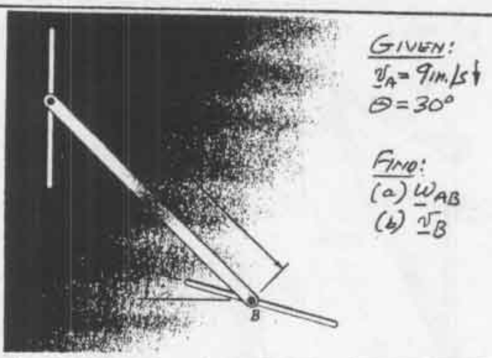
$\underline{v}_{A/B} = 7.566\ in./s \measuredangle 40°$

$v_{A/B} = (AB)\,\omega_{AB}$ $\qquad AB = 20\ in.$

$7.566\ in./s = (20\ in.)\,\omega_{AB}$

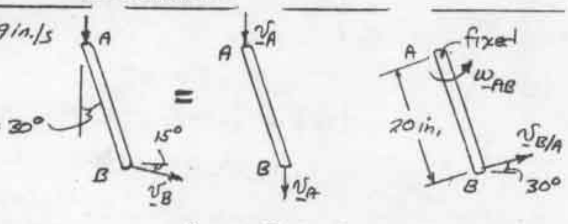
$\omega_{AB} = 0.3783\ rad/s$ $\qquad \underline{\omega}_{AB} = 0.378\ rad/s \circlearrowright$ ◀

15.42



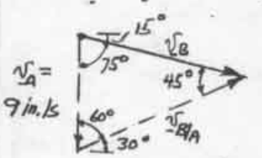
GIVEN: $v_A = 9\text{ in./s}\downarrow$, $\theta = 30^\circ$

FIND: (a) ω_{AB} (b) v_B



$$v_B = v_A + v_{B/A}$$

$$\left[v_B \measuredangle 15^\circ\right] = \left[9\text{ in./s}\downarrow\right] + \left[v_{B/A} \measuredangle 30^\circ\right]$$



LAW OF SINES

$$\frac{v_B}{\sin 60^\circ} = \frac{v_{B/A}}{\sin 75^\circ} = \frac{9\text{ in./s}}{\sin 45^\circ}$$

$v_B = 11.02\text{ in./s} \measuredangle 15^\circ$ ◀

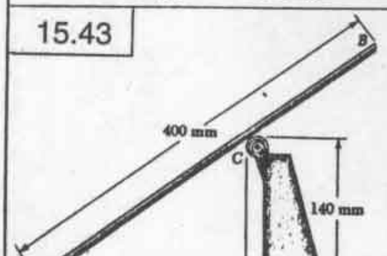
$v_{B/A} = 12.294\text{ in/s} \measuredangle 30^\circ$

$v_{B/A} = (AB)\,\omega_{AB}$

$12.294\text{ in/s} = (20\text{ in.})\,\omega_{AB}$

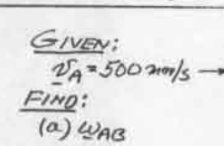
$\omega_{AB} = 0.6147\text{ rad/s}$ $\qquad \omega_{AB} = 0.615\text{ rad/s}\circlearrowleft$ ◀

15.43



GIVEN: $v_A = 500\text{ mm/s}\rightarrow$

FIND: (a) ω_{AB} (b) v_B

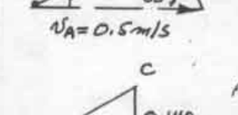


$$\theta = \tan^{-1}\frac{140}{200} = 35.0^\circ$$

RIGHT TRIANGLE

$v_{C/A} = (0.5\text{ m/s})\sin 35.0^\circ$

$= 0.2868\text{ m/s}$



$AC^2 = (0.140)^2 + (0.200)^2$; $AC = 0.2441\text{ m}$

$v_{C/A} = (AC)\,\omega_{ACB}$

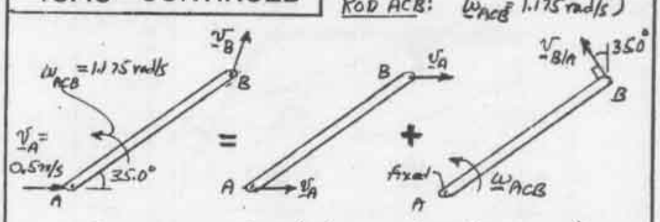
$0.2868\text{ m/s} = (0.2441\text{ m})\,\omega_{ACB}$

$\omega_{ACB} = 1.1747\text{ rad/s}$ $\qquad \omega_{ACB} = 1.175\text{ rad/s}\circlearrowleft$ ◀

(CONTINUED)

15.43 CONTINUED

ROD ACB: ω_{ACB} $1.175\text{ rad/s}\circlearrowleft$

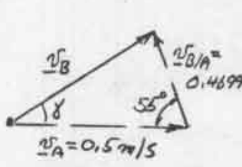


$AB = 0.4\text{ m}$ $\qquad v_{B/A} = (AB)\,\omega_{ACB} = (0.4\text{ m})(1.175\text{ rad/s})$

$v_{B/A} = 0.4699\text{ m/s} \measuredangle 55^\circ$

$$v_B = v_A + v_{B/A}$$

$$= \left[0.5\text{ m/s}\rightarrow\right] + \left[0.4699\text{ m/s} \measuredangle 55^\circ\right]$$



LAW OF COSINES

$$v_B^2 = v_A^2 + v_{B/A}^2 - 2v_A v_{B/A}\cos 55^\circ$$

$$= (0.5)^2 + (0.4699)^2 - 2(0.5)(0.4699)\cos 55^\circ$$

$v_B = 0.449\text{ m/s}$

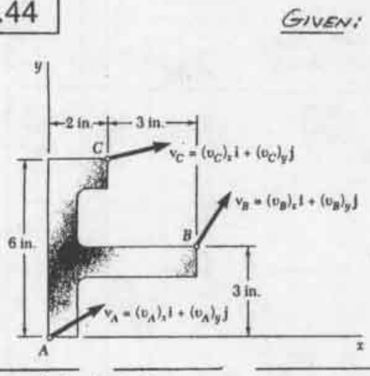
LAW OF SINES $\quad \dfrac{0.4699\text{ m/s}}{\sin\gamma} = \dfrac{0.449\text{ m/s}}{\sin 55^\circ}$; $\gamma = 59.1^\circ$

$v_B = 0.449\text{ m/s} \measuredangle 59.1^\circ$ ◀

15.44

GIVEN: $(v_A)_x = 4\text{ in/s}$, $(v_B)_y = -3\text{ in./s}$, $(v_C)_x = 16\text{ in./s}$

FIND: (a) ω of plate (b) v_A



$v_A = (4\text{ in/s})\,i + (v_A)_y\, j$ $\qquad v_C = (16\text{ in./s})\,i + (v_C)_y\, j$

$\omega = \omega k$; $r_{C/A} = (2\text{ in.})\,i + (6\text{ in.})\,j$; $r_{B/A} = (5\text{ in./s})\,i + (3\text{ in/s})\,j$

$$v_C = v_A + v_{C/A} = v_A + \omega \times r_{C/A} = v_A + \omega k \times (2i + 6j)$$

$$16i + (v_C)_y j = 4i + (v_A)_y j + 2\omega j - 6\omega i$$

COEFFICIENTS OF i: $16 = 4 - 6\omega$; $\omega = -2$

$\omega = -(2\text{ rad/s})\,k$ ◀

$$v_B = v_A + v_{B/A} = v_A + \omega \times r_{B/A} = v_A + \omega k \times (5i + 3j)$$

$$(v_B)_x i - 3j = 4i + (v_A)_y j + 5\omega j - 3\omega i$$

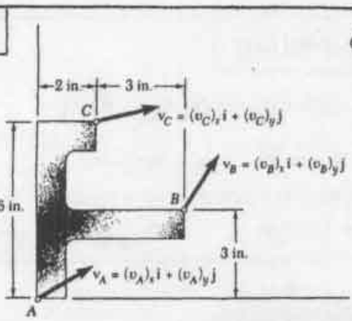
COEFFICIENTS OF j: $-3 = (v_A)_y + 5\omega$

$-3 = (v_A)_y + 5(-2)$; $(v_A)_y = 7\text{ in./s}$

$v_A = (v_A)_x i + (v_A)_y j$

$v_A = (4\text{ in./s})\,i + (7\text{ in./s})\,j$ ◀

15.45



GIVEN:
$(v_A)_x = 4 \text{ in./s}$
$(v_B)_y = -3 \text{ in/s}$
$(v_C)_x = 16 \text{ in./s}$

FIND:
LOCUS OF POINTS OF PLATE WITH $v = 8 \text{ in/s}$

FROM THE ANSWER OF PROB. 15.44, WE HAVE

$\underline{\omega} = -(2 \text{ rad/s})\underline{k} \qquad \underline{v}_A = (4 \text{ in./s})\underline{i} + (7 \text{ in./s})\underline{j}$

LET $\underline{P} = x\underline{i} + y\underline{j}$ BE AN ARBITRARY POINT

THUS: $\underline{r}_{P/A} = x\underline{i} + y\underline{j}$

$$\underline{v}_P = \underline{v}_A + \underline{v}_{P/A} = \underline{v}_A + \underline{\omega} \times \underline{r}_{P/A} = 4\underline{i} + 7\underline{j} + (-2\underline{k}) \times (x\underline{i} + y\underline{j})$$

$$v_A = (4+2y)\underline{i} + (7-2x)\underline{j}$$

$$(v_A)_x = 4 + 2y \qquad (v_A)_y = 7 - 2x$$

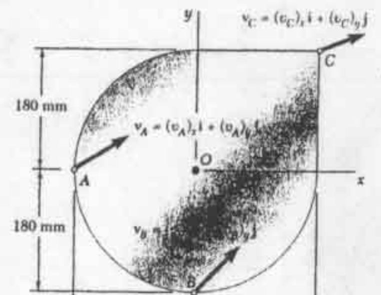
$$v_A^2 = (v_A)_x^2 + (v_A)_y^2 \qquad \text{WE SEEK } v_A = 8 \text{ in./s}$$

$$8^2 = (4+2y)^2 + (7-2x)^2$$

SIMPLIFY: $(x - 3.5)^2 + (y+2)^2 = 4^2$ ◀

NOTE: LOCUS IS A CIRCLE OF RADIUS 4 in. WITH CENTER AT $x = 3.5 \text{ in.}, \ y = -2 \text{ in.}$

15.46



GIVEN:
$(v_A)_x = 120 \text{ mm/s}$
$(v_B)_y = 300 \text{ mm/s}$
$(v_C)_y = -60 \text{ mm/s}$

FIND:
(a) $\underline{\omega}$
(b) $\underline{v}_A$

$\underline{r}_{C/B} = (180 \text{ mm})\underline{i} + (360 \text{ mm})\underline{j}$
$\underline{\omega} = \omega \underline{k}$

$$\underline{v}_B = (v_B)_x \underline{i} + (300 \text{ mm/s})\underline{j} \ ; \ \underline{v}_C = (v_C)_x \underline{i} - (60 \text{ mm/s})\underline{j}$$

(a)

$$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B}$$

$$(v_C)_x \underline{i} - (60 \text{ mm/s})\underline{j} = (v_B)_x \underline{i} + (300 \text{ mm/s})\underline{j} + \underline{\omega} \times \underline{r}_{C/B}$$

$$(v_C)_x \underline{i} - 60\underline{j} = (v_B)_x \underline{i} + 300\underline{j} + \omega \underline{k} \times (180\underline{i} + 360\underline{j})$$

$$(v_C)_x \underline{i} - 60\underline{j} = (v_B)_x \underline{i} + 300\underline{j} + 180\omega\underline{j} - 360\omega\underline{i}$$

COEFFICIENTS OF $\underline{j}$: $-60 = 300 + 180\omega$

$$\omega = -2 \text{ rad/s} \qquad \underline{\omega} = 2 \text{ rad/s} \circlearrowright$$ ◀

(b) VELOCITY OF A:

$$\underline{r}_{A/B} = -(180 \text{ mm})\underline{i} + (180 \text{ mm})\underline{j}$$

$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$$

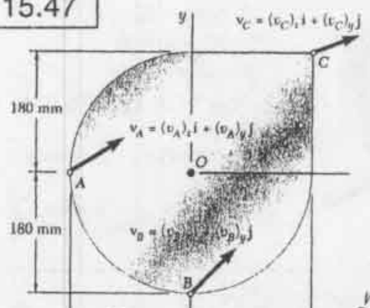
$$120\underline{i} + (v_A)_y \underline{j} = (v_B)_x \underline{i} + 300\underline{j} + (-2\underline{k}) \times (-180\underline{i} + 180\underline{j})$$

$$120\underline{i} + (v_A)_y \underline{j} = (v_B)_x \underline{i} + 300\underline{j} + 360\underline{j} + 360\underline{i}$$

COEFFICIENTS OF $\underline{j}$: $(v_A)_y = 300 + 360 = 660 \text{ mm/s}$

$$\underline{v}_A = (120 \text{ mm/s})\underline{i} + (660 \text{ mm/s})\underline{j}$$ ◀

15.47



GIVEN:
$(v_A)_x = 120 \text{ mm/s}$
$(v_B)_y = 300 \text{ mm/s}$
$(v_C)_y = -60 \text{ mm/s}$

FIND:
(a) $\underline{v}_B$
(b) POINT OF ZERO VELOCITY

$\underline{r}_{B/A} = (180 \text{ mm/s})\underline{i} - (180 \text{ mm/s})\underline{j}$

FROM THE ANSWER OF PROB. 15.46, WE HAVE

$\underline{\omega} = -(2 \text{ rad/s})\underline{k} \ ; \ \underline{v}_A = (120 \text{ mm/s})\underline{i} + (660 \text{ mm/s})\underline{j}$

(a) VELOCITY OF B:

$$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A} = \underline{v}_A + \underline{\omega} \times \underline{r}_{B/A}$$

$$= 120\underline{i} + 660\underline{j} - 2\underline{k} \times (180\underline{i} - 180\underline{j})$$

$$= 120\underline{i} + 660\underline{j} - 360\underline{j} - 360\underline{i}$$

$$\underline{v}_B = -(240 \text{ mm/s})\underline{i} + (300 \text{ mm/s})\underline{j}$$ ◀

(b) POINT WITH $v = 0$:

LET $\underline{P} = x\underline{i} + y\underline{j}$ BE AN ARBITRARY PONT

THUS: $\underline{r}_{P/A} = (180 + x)\underline{i} + y\underline{j}$

$$\underline{v}_P = \underline{v}_A + \underline{v}_{P/A} = \underline{v}_A + \underline{\omega} \times \underline{r}_{P/A}$$

$$v_P = 120\underline{i} + 660\underline{j} + (-2\underline{k}) \times [(180+x)\underline{i} + y\underline{j}]$$

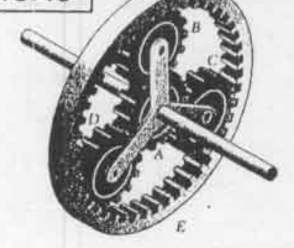
$$v_P = 120\underline{i} + 660\underline{j} - (360 + 2x)\underline{j} + 2y\underline{i}$$

$$v_P = (120 + 2y)\underline{i} + (300 - 2x)\underline{j}$$

FOR $v_P = 0 \quad 120 + 2y = 0$ and $300 - 2x = 0$

$v = 0$ AT: $\quad y = -60 \text{ mm}, \ x = 150 \text{ mm}$ ◀

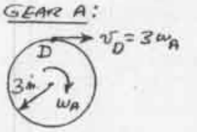
15.48



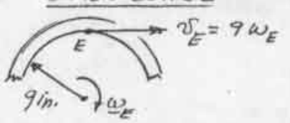
GIVEN
$r_A = r_B = r_C = 3 \text{ in.} ; \ r_E = 9 \text{ in.}$
$\underline{\omega}_A = 150 \text{ rpm} \circlearrowright$
$\underline{\omega}_E = 120 \text{ rpm} \circlearrowright$

FIND: (a) $\underline{\omega}_B$
(b) $\underline{\omega}_S = \underline{\omega}_{SPIDER}$

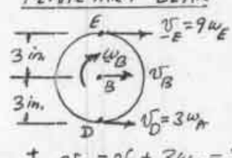
GEAR A:



OUTER GEAR E



PLANETARY GEAR



$$v_E = v_D + 6\omega_B$$

$$\overset{+}{\rightarrow} \ 9\omega_E = 3\omega_A + 6\omega_B$$

$$9(120 \text{ rpm}) = 3(150 \text{ rpm}) + 6\omega_B$$

$$\omega_B = 105 \text{ rpm} \qquad \underline{\omega}_B = 105 \text{ rpm} \circlearrowright$$ ◀

$$\overset{+}{\rightarrow} \ v_B = v_D + 3\omega_B = 3\omega_A + 3\omega_B = 3(\omega_A + \omega_B)$$

SPIDER:

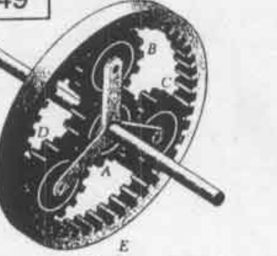
$$v_B = 6\omega_S$$

$$3(\omega_A + \omega_B) = 6\omega_S$$

$$\omega_S = \tfrac{1}{2}(\omega_A + \omega_B) = \tfrac{1}{2}(150 \text{ rpm} + 105 \text{ rpm})$$

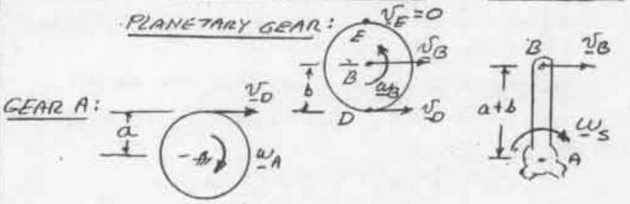
$$\omega_S = 127.5 \text{ rpm} \qquad \underline{\omega}_S = 127.5 \text{ rpm} \circlearrowright$$ ◀

15.49



GIVEN: $r_A = a$
$r_B = r_C = r_D = b$
$r_E = a + 2b$
$\underline{\omega}_A = \omega_A \circlearrowright$, $\omega_E = 0$

FIND:
(a) RATIO b/a FOR WHICH: $\underline{\omega}_S = \omega_{SPIDER} = \frac{1}{5}\omega_A \circlearrowright$
(b) $\underline{\omega}_S$



GEAR A: $\underline{v}_D = a\omega_A \rightarrow$

PLANETARY GEAR: $\underline{v}_D = \underline{v}_E + \underline{v}_{D/E}$

$$a\omega_A = 0 + (2b)\omega_B: \quad \omega_B = \frac{a}{2b}\omega_A \circlearrowleft \quad (1)$$

$$v_B = v_E + v_{B/E} = 0 + b\,\omega_B = b\left(\frac{a}{2b}\omega_A\right)$$

$$\underline{v}_B = \frac{1}{2}a\omega_A \rightarrow$$

SPIDER: $v_B = (a+b)\omega_S$

$$\frac{1}{2}a\omega_A = (a+b)\omega_S \qquad \underline{\omega}_S = \frac{a}{2(a+b)}\omega_A \circlearrowright \quad (2)$$

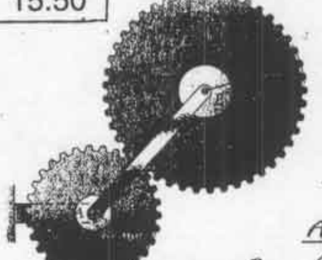
SUBSTITUTE DATA: $\omega_S = \frac{1}{5}\omega_A$ INTO (2)

$$\frac{1}{5}\omega_A = \frac{a}{2(a+b)}\omega_A; \quad 2a + 2b = 5a$$

$$2b = 3a \qquad \frac{b}{a} = 1.5 \blacktriangleleft$$

EQ(1): $\omega_B = \frac{1}{2}\left(\frac{a}{b}\right)\omega_A = \frac{1}{2}\left(\frac{1}{1.5}\right)\omega_A \qquad \underline{\omega}_B = \frac{1}{3}\omega_A \circlearrowleft \blacktriangleleft$

15.50



GIVEN:
$\underline{\omega}_A = 120 \text{ rpm} \circlearrowright$
$\underline{\omega}_{AB} = 90 \text{ rpm} \circlearrowright$

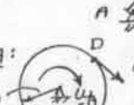
FIND: $\underline{\omega}_B$

ARM AB:

$AB = (60+90)\text{ mm} = 150 \text{ mm}$

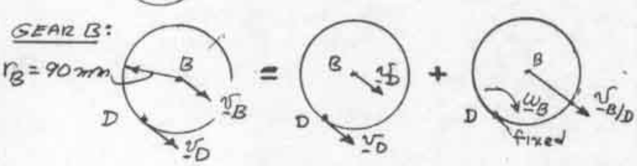
$\underline{v}_B = (AB)\omega_{AB} = 150\,\omega_{AB}$

GEAR A:



$\underline{v}_D = r_A\omega_A = 60\,\omega_A$

GEAR B:



$+\circlearrowright \quad \underline{v}_B = \underline{v}_D + \underline{v}_{B/D}; \quad 150\,\omega_{AB} = 60\,\omega_A + 90\,\omega_B \qquad (1)$

(CONTINUED)

15.50 CONTINUED

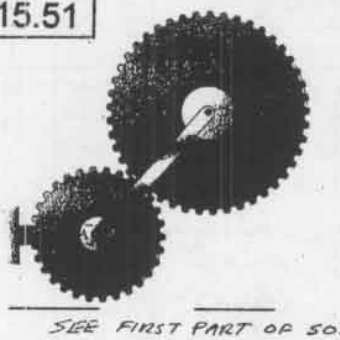
EQ(1): $+\circlearrowright \quad 150\,\omega_{AB} = 60\,\omega_A + 90\,\omega_B$

DATA: $\underline{\omega}_A = 120 \text{ rpm} \circlearrowright \quad \underline{\omega}_{AB} = 90 \text{ rpm} \circlearrowright$

$$150(90 \text{ rpm}) = 60(120 \text{ rpm}) + 90\,\omega_B$$

$$\omega_B = +70 \text{ rpm} \qquad \underline{\omega}_B = 70 \text{ rpm} \circlearrowright \blacktriangleleft$$

15.51



GIVEN:
$\omega_{AB} = 42 \text{ rpm} \circlearrowright$

FIND:
(a) $\underline{\omega}_A$ FOR WHICH $\underline{\omega}_B$ IS $20 \text{ rpm} \circlearrowleft$
(b) ω_A FOR WHICH $\underline{\omega}_B = 0$ (CURVILINEAR TRANSLATION)

SEE FIRST PART OF SOLUTION OF PROB 15.50 FOR DERIVATION OF

$$+\circlearrowright \quad 150\,\omega_{AB} = 60\,\omega_A + 90\,\omega_B \qquad (1)$$

(a) FOR $\underline{\omega}_B = 20 \text{ rpm} \circlearrowleft$, $\quad \omega_B = -20 \text{ rpm}$

EQ(1): $+\circlearrowright \quad 150(42 \text{ rpm}) = 60\,\omega_A + 90(-20 \text{ rpm})$

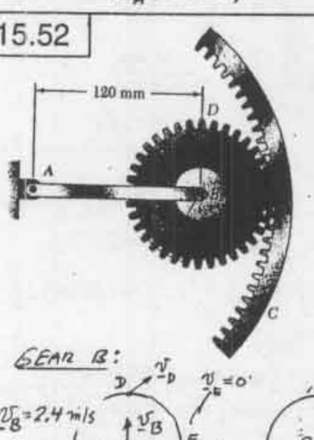
$$\omega_A = +135 \text{ rpm} \qquad \underline{\omega}_A = 135 \text{ rpm} \circlearrowright \blacktriangleleft$$

(b) FOR $\omega_B = 0$:

EQ(1): $+\circlearrowright \quad 150(42 \text{ rpm}) = 60\,\omega_A + 0$

$$\omega_A = +105 \text{ rpm} \qquad \underline{\omega}_A = 105 \text{ rpm} \circlearrowright \blacktriangleleft$$

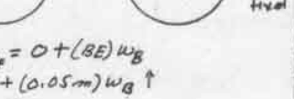
15.52



GIVEN: $\omega_{AB} = 20 \text{ rad/s} \circlearrowleft$

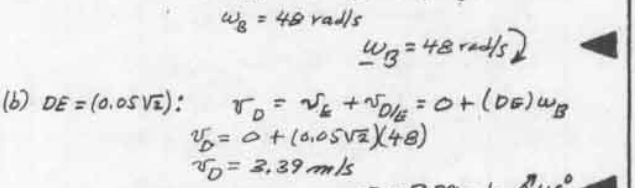
FIND:
(a) $\underline{\omega}_B$
(b) $\underline{v}_D$

ARM AB



$\underline{v}_B = (120 \text{ mm})(20 \text{ r/s}) = 2.4 \text{ m/s} \uparrow$

GEAR B:



(a) $BE = 0.05\text{ m}$: $\quad v_B = v_E + v_{B/E} = 0 + (BE)\,\omega_B$

$$2.4 \text{ m/s}\uparrow = 0 + (0.05 \text{ m})\,\omega_B \uparrow$$

$$\omega_B = 48 \text{ rad/s}$$

$$\underline{\omega}_B = 48 \text{ rad/s} \circlearrowright \blacktriangleleft$$

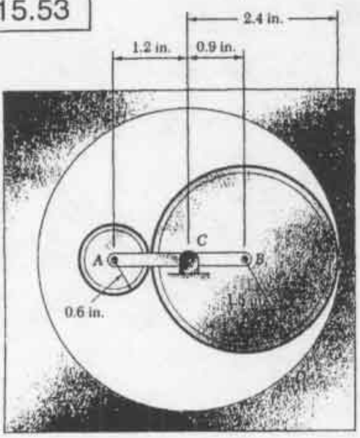
(b) $DE = (0.05\sqrt{2})$: $\quad v_D = v_E + v_{D/E} = 0 + (DE)\,\omega_B$

$$v_D = 0 + (0.05\sqrt{2})(48)$$

$$v_D = 3.39 \text{ m/s}$$

$$\underline{v}_D = 3.39 \text{ m/s} \angle 45^\circ \blacktriangleleft$$

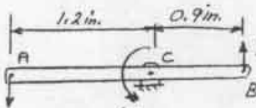
15.53



GIVEN: $\underline{\omega}_{ACB} = 40 \text{ rad/s} \circlearrowleft$

FIND:
(a) $\underline{\omega}_A$
(b) $\underline{\omega}_B$

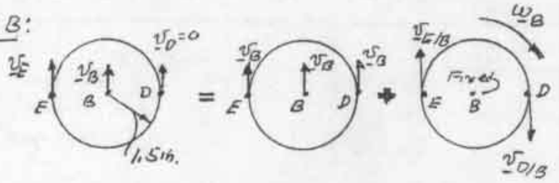
ARM ACB:



$\underline{v}_B = (0.9 \text{ in.})\,\omega_{ACB}$

$\underline{v}_A = (1.2 \text{ in.})\,\omega_{ACB}$ $\qquad \underline{\omega}_{ACB}$

DISK B:



DISK ROLLS ON D: $\quad \underline{v}_D = \underline{v}_B + \underline{v}_{D/B} = \underline{v}_B + (BD)\,\omega_B$

$+\uparrow \quad 0 = (0.9 \text{ in.})\,\omega_{ACB} - (1.5 \text{ in.})\,\omega_B$

$\omega_B = 0.6\,\omega_{ACB} = 0.6(40 \text{ rad/s})$

$\underline{\omega}_B = 24 \text{ rad/s} \circlearrowright$ ◀

POINT OF CONTACT E OF THE DISKS:

$\underline{v}_E = \underline{v}_B + \underline{v}_{E/B} = v_B + (EB)\,\omega_B$

$+\uparrow \quad \underline{v}_E = (0.9 \text{ in})\,\omega_{ACB} + (1.5 \text{ in.})(0.6\,\omega_{ACB})$

$v_E = (0.9 \text{ in} + 0.9 \text{ in.})\,\omega_{ACB} = (1.8 \text{ in.})\,\omega_{ACB}$

DISK A

$\underline{v}_E = v_A + v_{E/A} = \underline{v}_A + (AE)\,\omega_A$

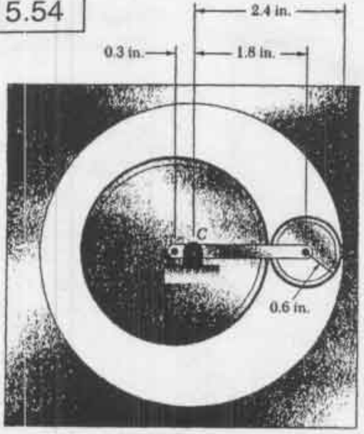
$+\uparrow \quad (1.8 \text{ in.})\,\omega_{ACB} = -(1.2 \text{ in.})\,\omega_{ACB} + (0.6 \text{ in.})\,\omega_A$

$\omega_A = \dfrac{1.8 + 1.2}{0.6}\,\omega_{ACB} = 5\,\omega_{ACB}$

$\omega_A = 5(40 \text{ rad/s}) = 200 \text{ rad/s}$

$\underline{\omega}_A = 200 \text{ rad/s} \circlearrowleft$ ◀

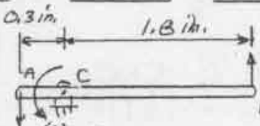
15.54



GIVEN: $\underline{\omega}_{ACB} = 40 \text{ rad/s} \circlearrowleft$

FIND:
(a) $\underline{\omega}_A$
(b) $\underline{\omega}_B$

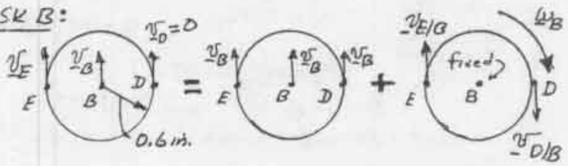
ARM ACB:



$\underline{v}_B = (1.8 \text{ in.})\,\omega_{ACB}$

$\underline{v}_A = (0.3 \text{ in.})\,\omega_{ACB}$ $\qquad \underline{\omega}_{ACB}$

DISK B:



DISK ROLLS ON D: $\quad \underline{v}_D = \underline{v}_B + \underline{v}_{D/B} = v_B + (BD)\,\omega_B$

$+\uparrow \quad 0 = (1.8 \text{ in.})\,\omega_{ACB} - (0.6 \text{ in.})\,\omega_B$

$\omega_B = 3\,\omega_{ACB} = 3(40 \text{ rad/s})$

$\underline{\omega}_B = 120 \text{ rad/s} \circlearrowright$ ◀

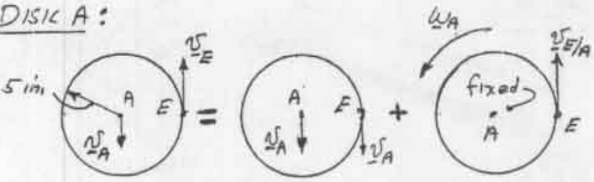
POINT OF CONTACT E OF THE DISKS:

$\underline{v}_E = \underline{v}_B + \underline{v}_{E/B} = v_B + (EB)\,\omega_B$

$+\uparrow \quad v_E = (1.8 \text{ in.})\,\omega_{ACB} + (0.6 \text{ in.})(3\,\omega_{ACB})$

$v_E = (1.8 \text{ in.} + 1.8 \text{ in.})\,\omega_{ACB} = (3.6 \text{ in.})\,\omega_{ACB}$

DISK A:



$v_E = v_A + v_{E/A} = v_A + (AE)\,\omega_A$

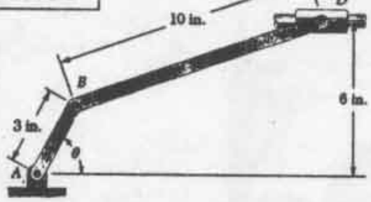
$+\uparrow \quad (3.6 \text{ in.})\,\omega_{ACB} = -(0.3 \text{ in.})\,\omega_{ACB} + (1.5 \text{ in.})\,\omega_A$

$\omega_A = \dfrac{3.6 + 0.3}{1.5}\,\omega_{ACB} = 2.6\,\omega_{ACB}$

$\omega_A = 2.6(40 \text{ rad/s}) = 104 \text{ rad/s}$

$\underline{\omega}_A = 104 \text{ rad/s} \circlearrowleft$ ◀

15.55



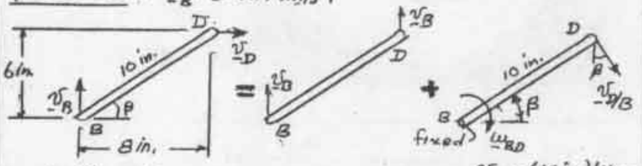
GIVEN: $\underline{\omega}_{AB} = 160\text{ rpm}$ ↻

FIND: $\underline{\omega}_{BD}$ AND $\underline{v}_D$ WHEN
(a) $\theta = 0$
(b) $\theta = 90°$

CRANK AB:
$\omega_{AB} = 160\text{ rpm} = 16.755\text{ rad/s}$

$v_B = (3\text{in})\omega_{AB} = 3(16.755) = 50.27\text{ in./s}$

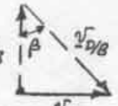
(a) $\theta = 0$: $v_B = 50.27\text{ in./s}$ ↑



$\beta = \sin^{-1}\frac{6}{10} = 36.9°$

$v_{D/B} = (10\text{ in.})\omega_{BD}$

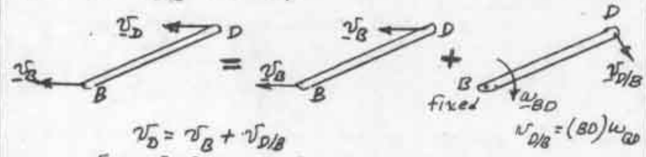
$$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$$
$$[v_D \rightarrow] = [v_B \uparrow] + [10\omega_{BD}]$$



$v_D = v_B \tan\beta = (50.27\text{ in./s})\tan 36.9$
$v_D = 37.7\text{ in/s}$ $\quad \underline{v}_D = 37.7\text{ in/s} \rightarrow$

$v_B = v_{D/B}\cos\beta$
$50.27\text{ in./s} = (10\omega_{BD})\cos 36.9°$
$\omega_{BD} = 6.28\text{ rad/s} = 60\text{ rpm}$ $\quad \underline{\omega}_{BD} = 60\text{ rpm}$ ↻

(b) $\theta = 90°$: $\underline{v}_B = 50.27\text{ in./s} \leftarrow$



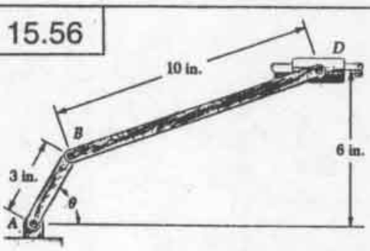
$v_{D/B} = (BD)\omega_{BD}$

$$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$$
$$[v_D \leftarrow] = [v_B \leftarrow] + [(BD)\omega_{BD} \searrow]$$

+↓ YIELDS $(BD)\omega_{BD} = 0$ $\quad \omega_{BD} = 0$

$\overset{+}{\leftarrow}$ $v_D = v_D + 0 = 50.27\text{ in./s}$ $\quad \underline{v}_D = 50.3\text{ in./s} \leftarrow$

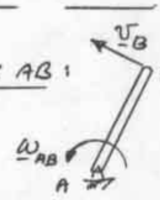
15.56



GIVEN:
$\underline{\omega}_{AB} = 160\text{ rpm}$ ↻
$\theta = 60°$

FIND:
$\underline{\omega}_{BD}$ AND $\underline{v}_D$

CRANK AB:



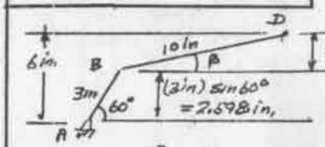
$\omega_{AB} = 160\text{ rpm} = 16.755\text{ rad/s}$
$AB = 3\text{ in.}$

$v_B = (AB)\,\omega_{AB}$
$= (3\text{ in.})(16.755\text{ rad/s})$
$\underline{v}_B = 50.27\text{ in./s} \measuredangle 30°$

(CONTINUED)

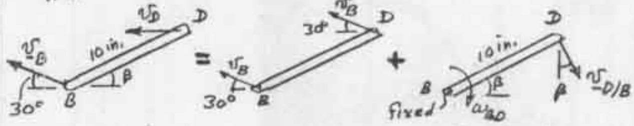
15.56 CONTINUED

ROD BD:

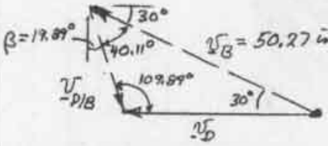


$6\text{ in.} - 2.598\text{ in.} = 3.402\text{ in.}$

$\beta = \sin^{-1}\frac{3.402}{10} = 19.89°$



$v_{D/B} = (10\text{ in.})\,\omega_{BD}$



$v_B = 50.27\text{ in./s}$

LAW OF SINES

$$\frac{v_D}{\sin 40.11°} = \frac{v_{D/B}}{\sin 30°} = \frac{50.27\text{ in./s}}{\sin 109.89°}$$

$v_D = 34.44\text{ in/s}$ $\quad \underline{v}_D = 34.4\text{ in./s} \leftarrow$

$v_{D/B} = (10\text{ in.})\,\omega_{BD} = 26.73\text{ in/s}$ $\quad \omega_{BD} = 2.67\text{ rad/s}$ ↻

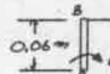
15.57

GIVEN:
$l = 160\text{ mm}$
$b = 60\text{ mm}$
$\underline{\omega}_{AB} = 1000\text{ rpm}$ ↻

FIND: v_P AND ω_{BD}
WHEN (a) $\theta = 0$
(b) $\theta = 90°$

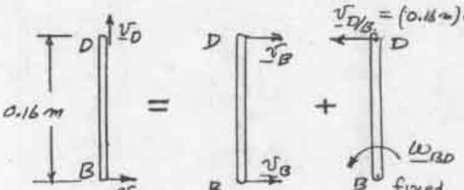
CRANK AB: $\underline{\omega}_{AB} = 1000\text{ rpm} = 104.72\text{ rad/s}$ ↻

(a) $\theta = 0$:



$v_B = (0.06\text{ m})(104.72\text{ rad/s})$
$\underline{v}_B = 6.283\text{ m/s} \rightarrow$

$\underline{v}_{D/B} = (0.16\text{ m})\,\omega_{BD}$



$$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$$
$$\underline{v}_D \uparrow = [6.283\text{ m/s} \rightarrow] + [\underline{v}_{D/B} \leftarrow]$$

+↑ $v_D = 0$; $v_P = v_D$; $\quad \underline{v}_P = 0$

$\overset{+}{\rightarrow}$ $0 = 6.283\text{ m/s} - v_{D/B}$

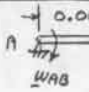
$\underline{v}_{D/B} = 6.283\text{ m/s} \leftarrow$

$(0.16\text{ m})\,\omega_{BD} = 6.283\text{ m/s}$; $\quad \underline{\omega}_{BD} = 39.3\text{ rad/s}$ ↺

(CONTINUED)

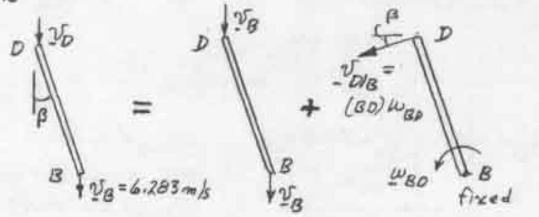
15.57 CONTINUED

(b) $\theta = 90°$:



$v_B = (0.06\,m)(104.72\,rad/s)$

$v_B = 6.283\,m/s \downarrow$



$v_D = v_B + v_{D/B}$

$v_D \downarrow = [6.283\,m/s \downarrow] + [v_{D/B} \angle \beta]$

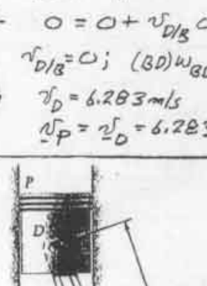
$\xleftarrow{+}$ $0 = 0 + v_{D/B}\cos\beta$

$v_{D/B} = 0$; $(BD)\omega_{BD} = 0$; $\quad \omega_{BD} = 0$ ◀

$+\downarrow$ $v_D = 6.283\,m/s$

$v_P = v_D = 6.283\,m/s \downarrow$; $v_P = 6.28\,m/s \downarrow$ ◀

15.58



GIVEN:

$l = 160\,mm$

$b = 60\,mm$

$\theta = 60°$

$\omega_{AB} = 1000\,rpm$ ↻

FIND: v_P AND ω_{BD}

CRANK AB: $\quad \omega_{AB} = 1000\,rpm = 104.72\,\frac{rad}{s}$

$v_B = (0.06\,m)(104.72\,rad/s)$

$v_B = 6.283\,m/s \angle 60°$

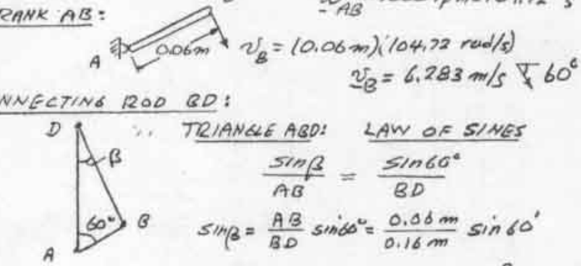
CONNECTING ROD BD:

TRIANGLE ABD: LAW OF SINES

$$\frac{\sin\beta}{AB} = \frac{\sin 60°}{BD}$$

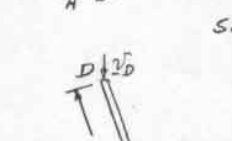
$$\sin\beta = \frac{AB}{BD}\sin 60° = \frac{0.06\,m}{0.16\,m}\sin 60°$$

$\sin\beta = 0.3248 \qquad \beta = 18.95°$



$v_D = v_B + v_{D/B}$

$v_D \downarrow = [6.283\,m/s \angle 60°] + [v_{D/B} \angle \beta]$



LAW OF SINES

$$\frac{v_D}{\sin 78.95°} = \frac{v_{D/B}}{\sin 30°} = \frac{6.283\,m/s}{\sin 71.05°}$$

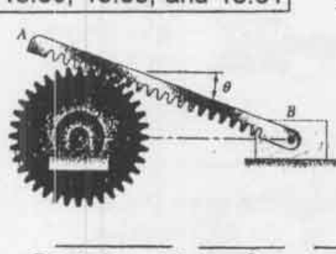
$v_D = 6.52\,m/s \qquad v_P = 6.52\,m/s \downarrow$ ◀

$v_{D/B} = (BD)\omega_{BD} = 3.322\,m/s$

$(0.16\,m)\omega_{BD} = 3.322\,m/s$

$\omega_{BD} = 20.8\,rad/s$ ↺ ◀

15.59, 15.60, and 15.61



PROB. 15.59: GIVEN: ω_D ↻

FIND: v_B + ω_{AB} IN TERMS OF r, θ, AND ω_D

PROB. 15.60: GIVEN: $\theta = 20°$

$r = 75\,mm$, $\omega_D = 15\,rpm$ ↺

FIND: v_B AND ω_{AB}

PROB. 15.61: GIVEN: $\theta = 25°$

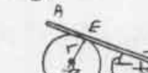
$r = 60\,mm$, $v_B = 200\,mm \rightarrow$

FIND: ω_D AND ω_{AB}

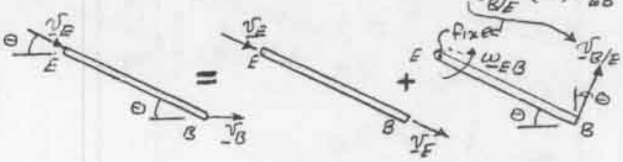
GEAR D: TOOTH E IS IN CONTACT WITH RACK AB

$v_E = r\,\omega_D \angle \theta$

RACK AB:

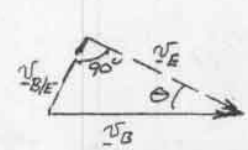


$$EB = \frac{r}{\tan\theta}$$



$v_B = v_E + v_{B/E}$

$v_B \rightarrow = [v_E \angle \theta] + [v_{B/E} \angle \theta]$



FORCE TRIANGLE

$$v_B = \frac{v_E}{\cos\theta} = \frac{r\omega_D}{\cos\theta} \rightarrow$$ ◀

$v_{B/E} = v_E \tan\theta$

$$(EB)\omega_{EB} = r\omega_D\tan\theta; \quad \omega_{EB} = \frac{r\omega_D\tan\theta}{EB} = \frac{r\omega_D\tan\theta}{r/\tan\theta}$$

$\omega_{EB} = \omega_D\tan^2\theta$ ↺ ◀

PROBLEM 15.60

GIVEN: $r = 75\,mm$, $\omega_D = 15\,rpm$ ↺; $\theta = 20°$

FIND: (a) v_B, (b) ω_{EB}

$\omega_D = 15\,rpm = 1.571\,rad/s$ ↺

(a) $$v_B = \frac{r\,\omega_D}{\cos\theta} = \frac{(75\,mm)(1.571\,rad/s)}{\cos 20°} = 125.39\,mm/s$$

$v_B = 125.4\,mm/s \leftarrow$ ◀

(b) $\omega_{EB} = \omega_D\tan^2\theta = (1.571\,rad/s)\tan^2 20° = 0.2081\,rad/s$

$\omega_{EB} = 0.208\,rad/s$ ↻ ◀

PROBLEM 15.61

GIVEN: $r = 60\,mm$, $v_B = 200\,mm/s \rightarrow$, $\theta = 25°$

FIND: (a) ω_D, (b) ω_{EB}

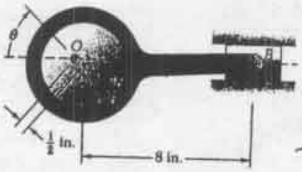
(a) $$v_B = \frac{r\,\omega_D}{\cos\theta}; \quad 200\,mm/s = \frac{(60\,mm)\,\omega_D}{\cos 25}$$

$\omega_D = 3.02\,rad/s$ ↻ ◀

(b) $\omega_{EB} = \omega_D\tan^2\theta = (3.02\,rad/s)\tan^2 25°$

$\omega_{EB} = 0.657\,rad/s$ ↺ ◀

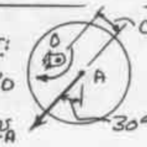
15.62



GIVEN: $\theta = 30°$
$\omega_O = 900$ rpm
$OA = 0.5$ in.

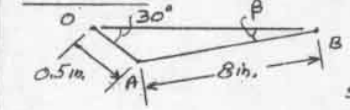
FIND: v_B

SHAFT:



$\omega_O = 900 \text{ rpm} = 94.248 \text{ rad/s}$
$v_A = (0.5 \text{ in.})(94.248 \text{ rad/s})$
$v_A = 47.124 \text{ in./s} \ \angle 60°$

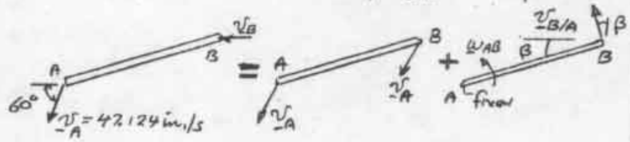
ECCENTRIC:



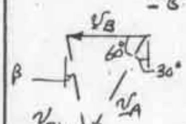
LAW OF SINES

$$\frac{\sin\beta}{0.5 \text{ in.}} = \frac{\sin 30°}{8 \text{ in.}}$$

$\sin\beta = \frac{0.5}{8.0}\sin 30°; \quad \beta = 1.79°$



$$v_B = v_A + v_{A/B}$$



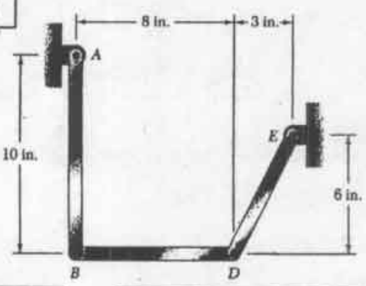
LAW OF SINES

$$\frac{v_B}{\sin(30° + \beta)} = \frac{v_A}{\sin(90° - \beta)}$$

$$v_B = \frac{\sin(30° + 1.79°)}{\sin(90° - 1.79°)}(47.124 \text{ in./s})$$

$$v_B = 24.8 \text{ in./s} \leftarrow$$

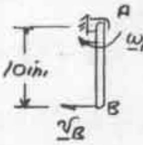
15.63



GIVEN:
$\omega_{AB} = 4$ rad/s

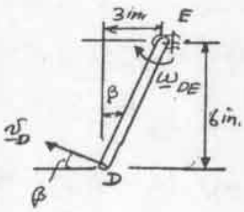
FIND:
ω_{BD}
ω_{DE}

BAR AB:



$v_B = (10 \text{ in.})(4 \text{ rad/s})$
$v_B = 40 \text{ in./s} \leftarrow$

BAR DE:



$\tan\beta = \frac{3 \text{ in.}}{6 \text{ in.}}; \quad \beta = 26.57°$

$DE = \frac{6 \text{ in.}}{\cos\beta} = 6.708 \text{ in.}$

$v_D = (DE)\,\omega_{DE}$
$= (6.708 \text{ in.})\,\omega_{DE}$

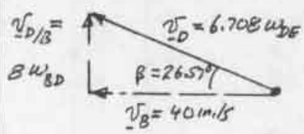
$v_D = 6.708\,\omega_{DE} \ \angle 26.57°$

(CONTINUED)

15.63 CONTINUED

BAR BD

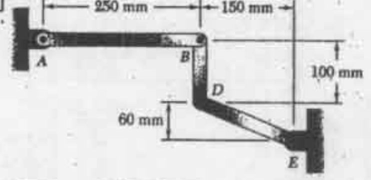
$$v_D = v_B + v_{D/B}$$
$$[v_D \angle \beta] = [v_B \leftarrow] + [v_{D/B} \uparrow]$$



$v_{D/B} = v_B \tan\beta$
$8\omega_{BD} = 40 \tan 26.57°$
$\omega_{BD} = 2.5$ rad/s

$v_B = v_D \cos\beta$
$40 = 6.708\,\omega_{DE} \cos 26.57°$
$\omega_{DE} = 6.67$ rad/s

15.64



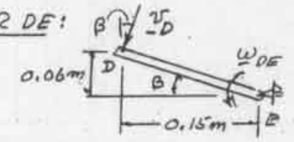
GIVEN:
$\omega_{AB} = 4$ rad/s

FIND:
ω_{BD}
ω_{DE}

BAR AB:

$v_B = (0.25 \text{ m})(4 \text{ rad/s}) = 1 \text{ m/s}$

BAR DE:

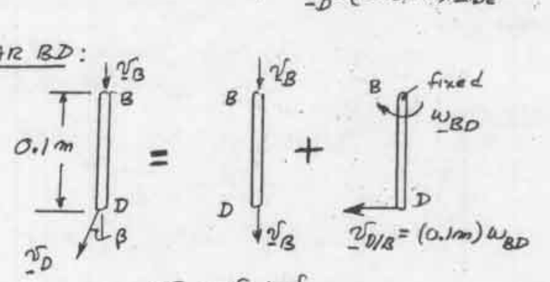


$\beta = \tan^{-1}\frac{0.06 \text{ m}}{0.15 \text{ m}} = 21.8°$

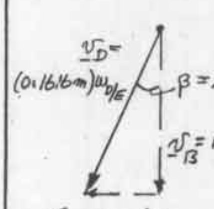
$DE = \frac{0.15 \text{ m}}{\cos\beta} = 0.1616 \text{ m}$

$v_D = (0.1616 \text{ m})\,\omega_{DE}$

BAR BD:



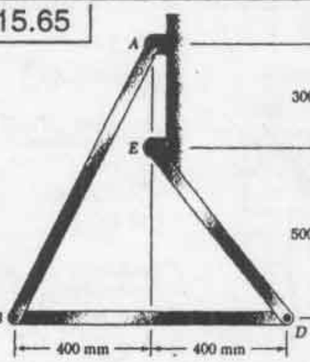
$$v_D = v_B + v_{D/B}$$
$$[v_D \angle \beta] = [v_B \downarrow] + [v_{D/B} \leftarrow]$$



$v_{D/B} = v_B \tan\beta$
$(0.1 \text{ m})\,\omega_{BD} = (1 \text{ m/s}) \tan 21.8°$
$\omega_{BD} = 4$ rad/s

$v_D = v_B / \cos\beta$
$(0.1616 \text{ m})\,\omega_{DE} = (1 \text{ m/s})/\cos 21.8°$
$\omega_{DE} = 6.67$ rad/s

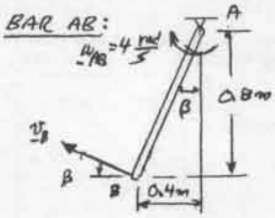
15.65



GIVEN: $\underline{\omega}_{AB} = 4$ rad/s ↻

FIND: $\underline{\omega}_{BD}$, $\underline{\omega}_{DE}$

BAR AB:



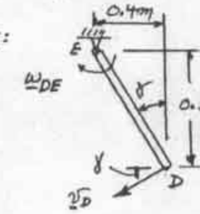
$\beta = \tan^{-1}\frac{0.4}{0.8} = 26.56°$

$AB = \frac{0.8}{\cos\beta} = 0.8944\text{ m}$

$v_B = (AB)\omega_{AB}$
$= (0.8944\text{ m})(4\text{ m/s})$

$\underline{v}_B = 3.578\text{ m/s} \;\searrow 26.56°$

BAR DE:



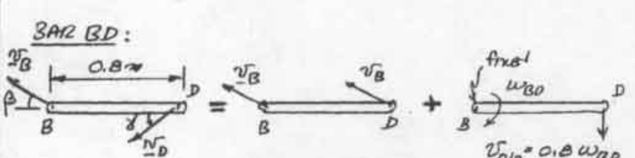
$\gamma = \tan^{-1}\frac{0.4}{0.5} = 38.66°$

$DE = \frac{0.5}{\cos\gamma} = 0.6403\text{ m}$

$v_D = (DE)\omega_{DE}$

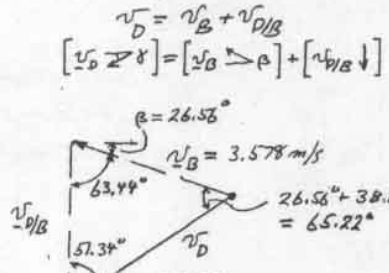
$\underline{v}_D = (0.6403\text{ m})\omega_{DE} \;\swarrow 38.66°$

BAR BD:



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[\underline{v}_D \swarrow \gamma] = [\underline{v}_B \searrow \beta] + [v_{D/B} \downarrow]$



LAW OF SINES

$$\frac{v_D}{\sin 63.44°} = \frac{v_{D/B}}{\sin 65.22} = \frac{3.578\text{ m/s}}{\sin 51.34°}$$

$v_D = 4.099\text{ m/s}$

$(0.6403\text{ m})\omega_{DE} = 4.099\text{ m/s}$

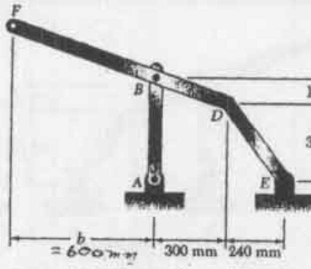
$\underline{\omega}_{DE} = 6.4\text{ rad/s}$ ↻ ◀

$v_{D/B} = 4.160\text{ m/s}$

$(0.8\text{ m})v_{BD} = 4.16\text{ m/s}$

$\underline{\omega}_{BD} = 5.2\text{ rad/s}$ ↻ ◀

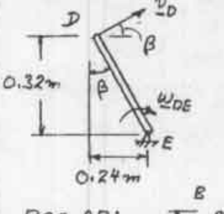
15.66



GIVEN: $\omega_{DE} = 15$ rad/s ↻, $b = 600$ mm

FIND: (a) $\underline{\omega}_{FBD}$ (b) $\underline{v}_F$

BAR DE:



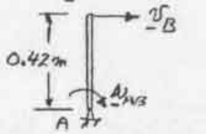
$\beta = \tan^{-1}\frac{0.24\text{ m}}{0.32\text{ m}} = 36.87°$

$DE = \frac{0.32\text{ m}}{\cos\beta} = 0.4\text{ m}$

$\underline{v}_D = (DE)\omega_{DE} = (0.4\text{ m})(15\text{ rad/s})$

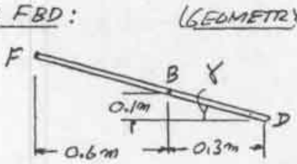
$\underline{v}_D = 6\text{ m/s} \;\angle 36.87°$

BAR AB:



$\underline{v}_B \rightarrow$

BAR FBD: (GEOMETRY)

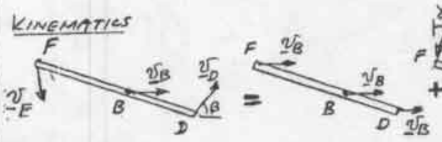


$\gamma = \tan^{-1}\frac{0.1}{0.3} = 18.43°$

$BD = \frac{0.3}{\cos\gamma} = 0.316\text{ m}$

$BF = 2(BD) = 0.632\text{ m}$

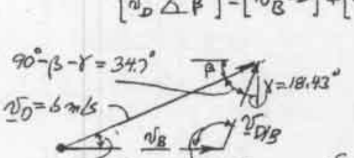
KINEMATICS



$v_{D/B} = (BD)\omega_{FBD} = (0.316)\omega_{FBD}$

(a) $\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \angle \beta] = [v_B \rightarrow] + [v_{D/B} \nearrow \gamma]$



LAW OF SINES

$$\frac{v_B}{\sin 34.7°} = \frac{v_{D/B}}{\sin 36.87°} = \frac{6\text{ m/s}}{\sin 108.43°}$$

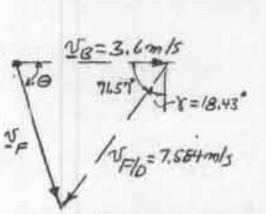
$v_B = 3.6\text{ m/s}$

$v_{D/B} = (0.316\text{ m})\omega_{FBD} = 3.795\text{ m/s}$

$\underline{\omega}_{FBD} = 12\text{ rad/s}$ ↺ ◀

(b) $\underline{v}_F = \underline{v}_B + \underline{v}_{F/B}$

$\underline{v}_F = [\underline{v}_B \rightarrow] + [\underline{v}_{F/B} \swarrow \gamma]$



$v_{F/B} = (BF)\omega_{FBD}$
$= (0.632\text{ m})(12\text{ rad/s})$

$v_{F/B} = 7.584\text{ m/s}$

LAW OF COSINES

$v_F^2 = (3.6)^2 + (7.584)^2 - 2(3.6)(7.584)\cos 71.57°$

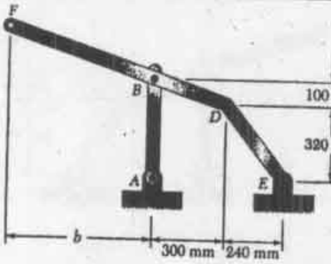
$v_F = 7.295\text{ m/s}$

LAW OF SINES

$$\frac{v_{F/D}}{\sin\theta} = \frac{v_F}{\sin 71.57};\quad \frac{7.584\text{ m/s}}{\sin\theta} = \frac{7.295\text{ m/s}}{\sin 71.57°};\quad \theta = 80.5°$$

$\underline{v}_F = 7.30\text{ m/s} \;\searrow 80.5°$ ◀

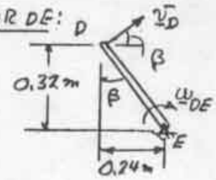
15.67



GIVEN: $\underline{\omega}_{DE} = 15 \frac{rad}{s}$ ↻

FIND: (a) DISTANCE b FOR $\underline{v}_F$ TO BE VERTICAL
(b) $\underline{v}_F$

BAR DE:



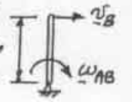
$\beta = \tan^{-1}\frac{0.24}{0.32} = 36.87°$

$DE = 0.32\,m/\cos\beta = 0.4\,m$

$v_D = (DE)\omega_{DE} = (0.4\,m)(15\,rad/s)$

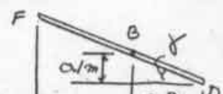
$\underline{v}_D = 6\,m/s \measuredangle 36.87°$

BAR AB:



$\underline{v}_B \rightarrow$

BAR FBD:

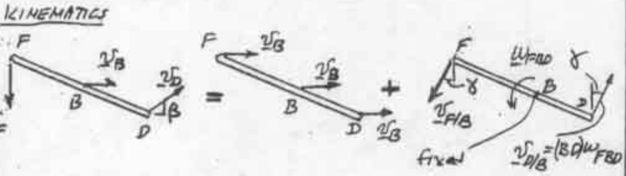


(GEOMETRY) $\gamma = \tan^{-1}\frac{0.1}{0.3} = 18.43°$

$BD = 0.3\,m/\cos\beta = 0.316\,m$

$BF = b/\cos\beta = 1.0541\,b \quad (1)$

KINEMATICS

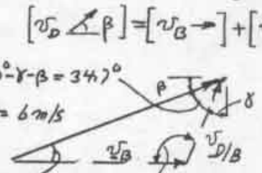


$\underline{v}_{D/B} = (BD)\omega_{FBD}$

$\underline{v}_{D/B} = 0.316\,\omega_{FBD}$

$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$

$[v_D \measuredangle \beta] = [v_B \rightarrow] + [v_{D/B} \measuredangle \gamma]$



$90° - \gamma - \beta = 34.7°$

$\underline{v}_D = 6\,m/s$

LAW OF SINES

$$\frac{v_B}{\sin 34.7} = \frac{v_{D/B}}{\sin 36.87°} = \frac{6\,m/s}{\sin 108.43}$$

$\underline{v}_B = 3.6\,m/s \rightarrow$

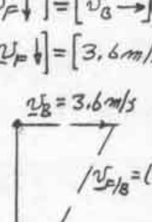
$v_{D/B} = (0.316)\omega_{FBD} = 3.795\,m/s$

$\underline{\omega}_{FBD} = 12\,rad/s$ ↺

(a) $\underline{v}_F = \underline{v}_B + \underline{v}_{F/B}$ $\qquad v_{F/B} = (BF)\omega_{FBD}$

$[v_F \downarrow] = [v_B \rightarrow] + [v_{F/B} \measuredangle \gamma]$

$[v_F \downarrow] = [3.6\,m/s \rightarrow] + [(BF)\omega_{FBD} \measuredangle 18.43°]$



RIGHT TRIANGLE

$$v_F = \frac{3.6\,m/s}{\tan 18.43°}$$

$\underline{v}_F = 10.80\,m/s \downarrow$ ◀

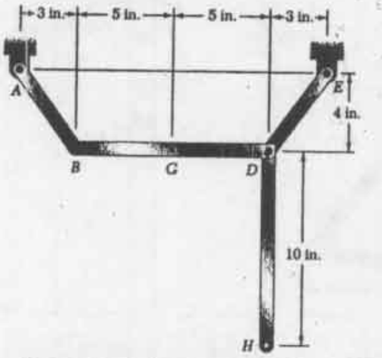
$$v_{F/B} = \frac{3.6\,m/s}{\sin 18.43°} = 11.387\,m/s$$

$v_{F/B} = (BF)\omega_{FBD}$; $11.387\,m/s = (BF)(12\,rad/s)$

$BF = 0.9489\,m$

EQ(1): $BF = 1.0541\,b$; $0.9489 = 1.0541\,b$; $b = 0.900\,m$ ◀

15.68 and 15.69



GIVEN: $\underline{\omega}_{AB} = 20\,rad/s$ ↺

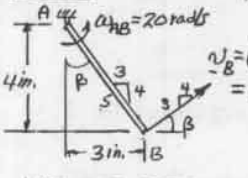
PROBLEM 15.68

FIND: (a) $\underline{\omega}_{BDH}$
(b) $\underline{v}_G$

PROBLEM 15.69

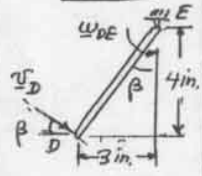
(a) $\underline{\omega}_{BDH}$
(b) $\underline{v}_H$

BAR AB: $AB = 5\,in.$ $\quad \beta = \tan^{-1}\frac{3}{4} = 36.87°$

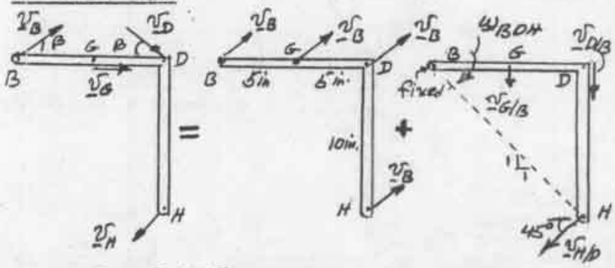


$\underline{v}_B = (5\,in.)(20\,rad/s) = 100\,in./s$

BAR DE:

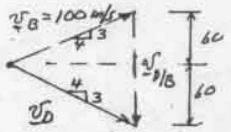


MEMBER BDH



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$ $\qquad \underline{v}_{D/B} = (10\,in.)\omega_{BDH} \downarrow$

$[v_D \measuredangle^{3}_{4}] = [v_B \measuredangle^{3}_{4}] + [(10)\omega_{BHD} \downarrow]$



$v_{D/B} = 60 + 60 = 120\,in./s$

$(10\,in.)\omega_{BDH} = 120\,in./s$

$\underline{\omega}_{BDH} = 12\,rad/s$ ↻ ◀

PROBLEM 15.68

$\underline{v}_G = \underline{v}_B + \underline{v}_{G/B}$

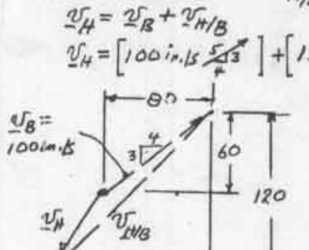
$\underline{v}_G = [v_B \measuredangle^{3}_{4}] + [v_{G/B} \downarrow]$

$\overset{+}{\rightarrow}$ $v_G = (100\,in./s)\frac{4}{5} + 0$ $\qquad \underline{v}_G = 80\,in./s \rightarrow$ ◀

PROBLEM 15.69 $\qquad v_{H/B} = (BH)\omega_{BDH} = 10\sqrt{2}(12) = 120\sqrt{2}\,in./s$

$\underline{v}_H = \underline{v}_B + \underline{v}_{H/B}$

$\underline{v}_H = [100\,in./s \measuredangle^{3}_{4}] + [120\sqrt{2}\,in./s \measuredangle^{1}_{1}]$



$(v_H)_x = 120 - 80 = 40$

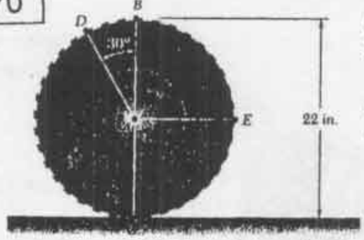
$(v_H)_y = 120 - 60 = 60$

$\theta = \tan^{-1}\frac{60}{40} = 56.3°$

$v_H = (40^2 + 60^2)^{1/2} = 72.11\,in./s$

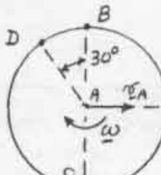
$\underline{v}_H = 72.11\,in./s \measuredangle 56.3°$ ◀

15.70



GIVEN: $\underline{v}_A = 48 \text{ mi/h} \rightarrow$

FIND: VELOCITIES OF POINTS B, C, D, AND E



$\underline{v}_A = 48 \frac{mi}{h} = 70.4 \text{ ft/s} \rightarrow$

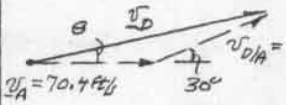
$\omega = \frac{v_A}{r} = \frac{70.4 \text{ ft/s}}{r}$

$r = 11 \text{ in}$

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A} = \left[70.4 \rightarrow\right] + \left[r\left(\frac{70.4}{r}\right) \rightarrow\right] = 140.8 \text{ ft/s} \rightarrow$ ◀

$\underline{v}_C = \underline{v}_A + \underline{v}_{C/A} = \left[70.4 \rightarrow\right] + \left[r\left(\frac{70.4}{r}\right) \leftarrow\right] = 0$ ◀

$\underline{v}_D = \underline{v}_A + \underline{v}_{D/A} = \left[70.4 \rightarrow\right] + \left[r\left(\frac{70.4}{r}\right) \measuredangle 30°\right]$



$2\theta = 30° \quad \theta = 15°$

$v_D = 2 v_A \cos 15°$

$= 2(70.4)\cos 15$

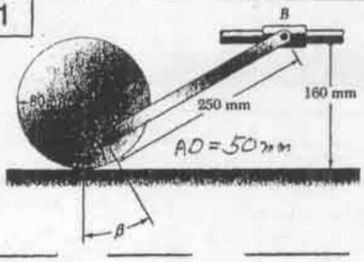
$\underline{v}_D = 136.0 \text{ ft/s} \measuredangle 15°$ ◀

$v_E = v_A + v_{E/A} = \left[70.4 \rightarrow\right] + \left[r\left(\frac{70.4}{r}\right) \downarrow\right]$

$= 70.4\sqrt{2} = 99.6 \text{ ft/s}$

$\underline{v}_E = 99.6 \text{ ft/s} \searrow 45°$ ◀

15.71



GIVEN: $\underline{v}_D = 900 \text{ mm/s} \leftarrow$

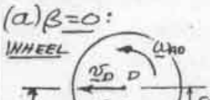
FIND: $\underline{v}_B$ AND ω_{AB} WHEN

(a) $\beta = 0$

(b) $\beta = 90°$

(a) $\underline{\beta = 0}$:

WHEEL



$v_C = 0, \quad \omega_{AD} = \frac{v_D}{CD} = \frac{900 \text{ mm/s}}{80 \text{ mm}} = 11.25 \frac{rad}{s}$

$\underline{v}_A = \underline{v}_D + \underline{v}_{A/D}$

$\underline{v}_A = \left[900 \text{ mm/s} \leftarrow\right] + \left[(50 \text{ mm})(11.25 \tfrac{rad}{s}) \rightarrow\right]$

$\overset{+}{\leftarrow} \; v_A = 900 - 562.5 = 337.5 \text{ mm/s} \leftarrow$

ROD AB: $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$\left[v_B \leftrightarrow\right] = \left[337.5 \text{ mm/s} \leftarrow\right] + \left[(AB)\omega_{AB} \overset{\theta}{\nwarrow}\right]$

$+\uparrow \quad 0 = (AB)\omega_{AB}\cos\theta \qquad \underline{\omega}_{AB} = 0$ ◀

$\overset{+}{\leftarrow} \; \underline{v}_B = 337.5 \text{ mm/s} \leftarrow$

$\underline{v}_B = 338 \text{ mm/s} \leftarrow$ ◀

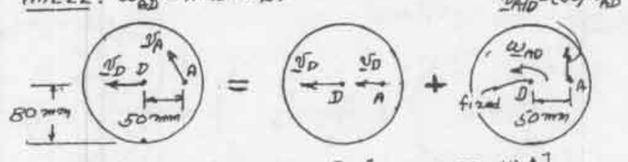
(CONTINUED)

15.71 CONTINUED

(b) $\underline{\beta = 90°}$

WHEEL: $\omega_{AD} = 11.25 \text{ rad/s}$

$v_{A/D} = (50)\omega_{AD}$

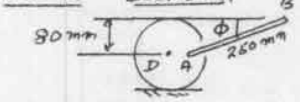


$\underline{v}_A = \underline{v}_D + \underline{v}_{A/D} = \left[900 \text{ mm/s} \leftarrow\right] + \left[(50 \text{ mm})(11.25 \text{ rad/s}) \uparrow\right]$

$= \left[900 \text{ mm/s} \leftarrow\right] + \left[562.5 \text{ mm/s} \uparrow\right]$

$\underline{v}_A = 1061 \text{ mm/s} \measuredangle 32.0°$

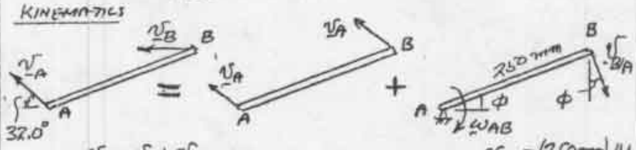
ROD AB GEOMETRY



$\phi = \sin^{-1} \frac{80 \text{ mm}}{250 \text{ mm}}$

$\phi = 18.66°$

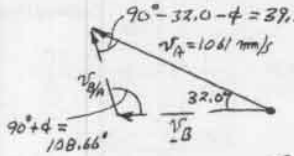
KINEMATICS



$v_{B/A} = (250 \text{ mm})\,\omega_{AB}$

$\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$\left[v_B \leftrightarrow\right] = \left[1061 \text{ mm/s} \measuredangle 32.0°\right] + \left[250\, \omega_{AB} \measuredangle \phi\right]$



LAW OF SINES

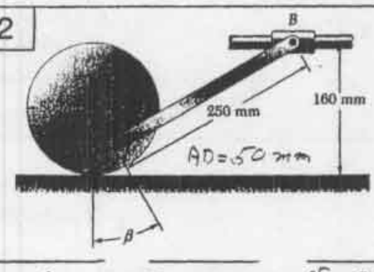
$\frac{v_B}{\sin 39.37} = \frac{v_{B/A}}{\sin 32.0°} = \frac{1061 \text{ mm/s}}{\sin 108.66°}$

$\underline{v}_B = 710 \text{ mm/s} \leftarrow$ ◀

$v_{B/A} = (250 \text{ mm})\,\omega_{AB} = 593.4 \text{ mm/s}$

$\underline{\omega}_{AB} = 2.37 \text{ rad/s}$ ◀

15.72



GIVEN: $\underline{v}_D = 900 \text{ mm/s} \leftarrow$

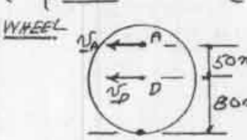
FIND: $\underline{v}_B$ AND $\underline{\omega}_{AB}$ WHEN

(a) $\beta = 180°$

(b) $\beta = 270°$

(a) $\underline{\beta = 180°}$ $\quad v_C = 0 \quad \omega_{AD} = \frac{v_D}{CD} = \frac{900 \text{ mm/s}}{80 \text{ mm}} = 11.25 \frac{rad}{s}$

WHEEL



$\underline{v}_A = \underline{v}_D + \underline{v}_{A/D}$

$v_A = \left[900 \text{ mm/s} \leftarrow\right] + \left[(50 \text{ mm})(11.25 \tfrac{rad}{s}) \leftarrow\right]$

$\overset{+}{\leftarrow} \; v_A = 900 + 562.5 = 1461.5 \text{ mm/s} \leftarrow$

ROD AB: $\underline{v}_B = \underline{v}_A + \underline{v}_{B/A}$

$\left[v_B \rightarrow\right] = \left[1461.5 \text{ mm/s} \leftarrow\right] + \left[(AB)\omega_{AB} \overset{\theta}{\nwarrow}\right]$

$+\uparrow \quad 0 = (AB)\,\omega_{AB}\cos\theta \qquad \underline{\omega}_{AB} = 0$ ◀

$\overset{+}{\leftarrow} \; v_B = 1461.5 \text{ mm/s} \leftarrow$

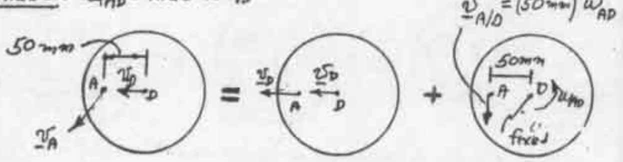
$\underline{v}_B = 1.463 \text{ m/s} \leftarrow$ ◀

(CONTINUED)

15.72 CONTINUED

(b) $\beta = 270°$:

WHEEL: $\omega_{AD} = 11.25$ rad/s



$v_A = v_D + v_{A/D} = [900 \text{ mm/s} \leftarrow] + [(50 \text{ mm})(11.25 \text{ rad/s}) \downarrow]$

$= [900 \text{ mm/s} \leftarrow] + [562.5 \text{ mm/s} \downarrow]$

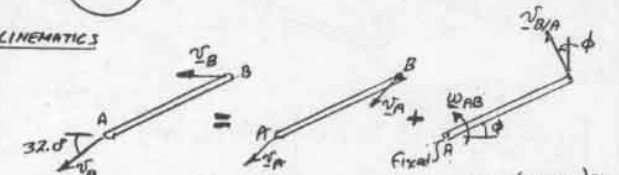
$v_D = 1061 \text{ mm/s} \measuredangle 32.0°$

ROD AB: GEOMETRY

$\phi = \sin^{-1}\frac{80 \text{ mm}}{250 \text{ mm}}$

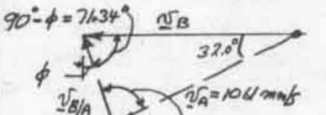
$\phi = 18.66°$

KINEMATICS



$v_B = v_A + v_{B/A}$

$[v_B \longleftrightarrow] = [1061 \text{ mm/s} \measuredangle 32.0°] + [250\,\omega_{AB} \measuredangle \phi]$



LAW OF SINES

$\frac{v_B}{\sin 76.66°} = \frac{v_{B/A}}{\sin 32.0°} = \frac{1061 \text{ mm/s}}{\sin 71.34°}$

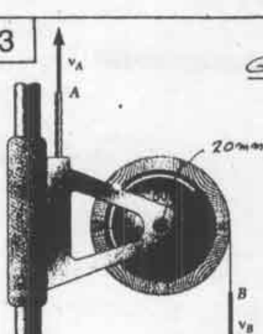
$v_B = 1090$ mm/s

$v_B = 1.090 \text{ m/s} \leftarrow$

$v_{B/A} = (250 \text{ mm})\,\omega_{AB} = 593.4$ mm/s

$\omega_{AB} = 2.37 \text{ rad/s} \circlearrowright$

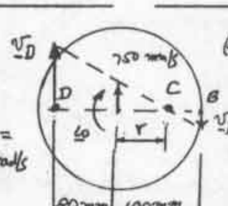
15.73



GIVEN: $v_A = 750 \text{ mm/s} \uparrow$
$\omega = 15 \text{ rad/s} \circlearrowright$

FIND:
(a) INST. CTR. OF ROTATION
(b) v_B AND v_D

$v_0 = v_A = 750 \text{ mm/s} \uparrow$



(a) INST. CTR. LOCATED ON LINE DOB

AT $r = \frac{750 \text{ mm}}{15 \text{ rad/s}}$; $r = 50$ mm

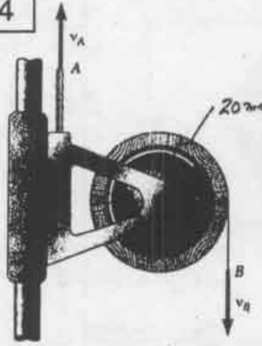
(b) $v_B = (CB)\omega$
$(100 \text{ mm} - 50 \text{ mm})(15 \text{ rad/s})$

$v_B = 750 \text{ mm/s} \downarrow$

$v_D = (DC)\omega$
$= (80 \text{ mm} + 50 \text{ mm})(15 \text{ rad/s})$
$= 1950$ mm/s

$v_D = 1.950 \text{ m/s} \uparrow$

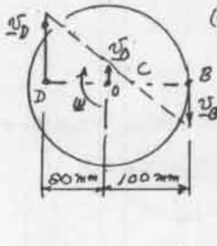
15.74



GIVEN: $v_A = 100 \text{ mm/s} \uparrow$
$v_B = 300 \text{ mm/s} \downarrow$

FIND:
(a) INST. CTR. OF ROTATION
(b) v_D

$v_0 = v_A = 100$ mm/s



(a) SINCE v_0 AND v_B ARE PARALLEL, INST. CTR. C IS LOCATED AT INTERSECTION OF BC AND LINE JOINING END POINTS OF v_0 AND v_B

SIMILAR TRIANGLES

$\frac{OC}{v_0} = \frac{BC}{v_B} = \frac{OC + BC}{v_0 + v_B}$

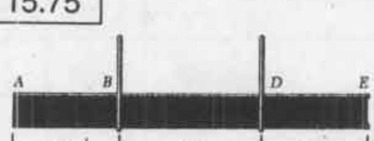
$OC = \frac{v_0}{v_0 + v_B}(OC + BC)$

$OC = \frac{100 \text{ mm/s}}{(100 + 300) \text{ mm/s}}(100 \text{ mm}) = 25 \text{ mm}$

(b) $\frac{v_D}{(DO) + (OC)} = \frac{v_0}{(OC)}$; $\frac{v_D}{(80 + 25) \text{ mm}} = \frac{100 \text{ mm/s}}{25 \text{ mm}}$

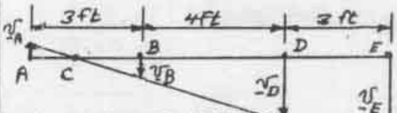
$v_D = 420 \text{ mm/s} \uparrow$

15.75



GIVEN:
$v_D = 24 \text{ in./s} \downarrow$
$v_E = 36 \text{ in./s} \downarrow$

FIND:
(a) INST. CTR. OF ROTATION
(b) v_A



(a) SINCE v_D AND v_E ARE PARALLEL, THE INST. CTR. C IS LOCATED AT INTERSECTION OF AE AND LINE JOINING END POINTS OF v_D AND v_E

(a) SIMILAR TRIANGLES

$\frac{v_E}{CE} = \frac{v_D}{CD} = \frac{v_E - v_D}{CE - CD}$

BUT: $CE - CD = 3$ ft

$\frac{v_D}{CD} = \frac{v_E - v_D}{CE - CD}$; $\frac{24 \text{ in./s}}{CD} = \frac{(36 - 24) \text{ in./s}}{3 \text{ ft}}$; $CD = 6$ ft

$AC = AD - CD = 7 \text{ ft} - 6 \text{ ft} = 1$ ft

INST. CTR. IS 1 ft TO RIGHT OF A

(b) $\frac{v_A}{AC} = \frac{v_D}{CD}$

$\frac{v_A}{1 \text{ ft}} = \frac{24 \text{ in./s}}{6 \text{ ft}}$; $v_A = 4 \text{ in./s} \uparrow$

15.76

GIVEN:

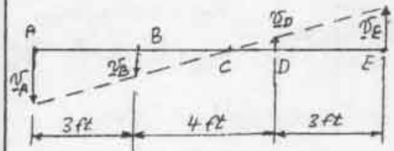
$\underline{v}_A = 13$ in./s ↓

$\underline{v}_E = 7$ in./s ↑

FIND:

(a) INST. CTR. OF ROTATION

(b) $\underline{v}_D$



(a) SINCE v_A AND v_E ARE PARALLEL, THE INST. CTR. C IS LOCATED AT INTERSECTION OF AE AND LINE JOINING END POINTS OF v_A AND v_E

(a) SIMILAR TRIANGLES

$$\frac{AC}{v_A} = \frac{CE}{v_E} = \frac{AC+CE}{v_A+v_E}$$

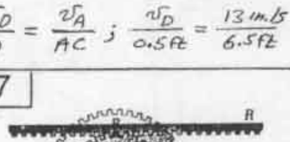
$$AC = \frac{v_A}{v_A+v_E}(AC+CE) = \frac{13\text{ in./s}}{(13+7)\text{ in./s}}(10\text{ ft}) = 6.5\text{ ft}$$

$$CD = AD - AC = 7\text{ ft} - 6.5\text{ ft} = 0.5\text{ ft}$$

INST. CTR. IS 0.5 ft TO LEFT OF D ◀

(b) $\dfrac{v_D}{CD} = \dfrac{v_A}{AC}$; $\dfrac{v_D}{0.5\text{ ft}} = \dfrac{13\text{ in./s}}{6.5\text{ ft}}$; $\underline{v}_D = 1$ in./s ↑ ◀

15.77



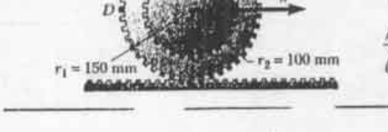
GIVEN:

$\underline{v}_A = 1.2$ m/s →

VELOCITY OF LOWER RACK IS $\underline{v}_C = 0.6$ m/s ←

FIND: (a) $\underline{\omega}$

(b) $\underline{v}_R$ AND $\underline{v}_D$



SINCE $\underline{v}_A$ AND $\underline{v}_C$ ARE PARALLEL THE INST. CTR. OF ROTATION IS AT THE INTERSECTION OF BC AND THE LINE JOINING THE END POINTS OF v_A AND v_C

(a) ANGULAR VELOCITY $\quad v_A = (AI)\omega$

$$1.2\text{ m/s} = (0.15\text{ m})\,\omega$$

$$\underline{\omega} = 12\text{ rad/s}$$ ◀

(b) UPPER RACK $\quad v_R = v_B = (BI)\omega$

$$v_R = (0.2\text{ m})(12\text{ rad/s})$$

$$\underline{v}_R = 2.4\text{ m/s} \rightarrow$$ ◀

VELOCITY OF POINT D: $\beta = \tan^{-1}\dfrac{0.15\text{ m}}{0.1\text{ m}} = 56.3°$

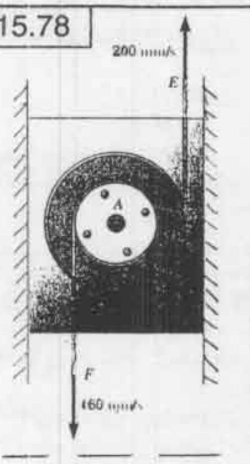
$$DI = \frac{DA}{\cos\beta} = \frac{0.15\text{ m}}{\cos 56.3°} = 0.1803\text{ m}$$

$$v_D = (DI)\omega$$

$$v_D = (0.1803\text{ m})(12\text{ rad/s})$$

$$\underline{v}_D = 2.16\text{ m/s} \measuredangle 56.3°$$ ◀

15.78



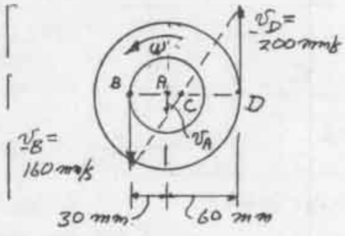
GIVEN: INNER RADIUS = 30 mm

OUTER RADIUS = 60 mm

FIND: (a) INST. CTR. OF ROTATION

(b) $\underline{v}_{BLOCK} = \underline{v}_A$

(c) LENGTH OF CORD WRAPPED OR UNWRAPPED PER SECOND ON EACH PULLEY.



(a) SINCE v_B AND v_D ARE PARALLEL, INST. CTR. IS LOCATED AT THE INTERSECTION OF BD AND LINE JOINING END POINTS OF $\underline{v}_B + \underline{v}_D$

$\dfrac{BC}{160} = \dfrac{CD}{200} = \dfrac{BC+CD}{160+200}$; BUT $BC+CD = 90$ mm

$\dfrac{BC}{160} = \dfrac{90\text{ mm}}{360}$; $BC = 40$ mm ; $AC = BC - AB = 40\text{ mm} - 30\text{ mm} = 10$ mm

INST. CTR. C IS 10 mm TO RIGHT OF A ◀

(b) $v_{BLOCK} = v_A$ $\quad \omega = \dfrac{v_B}{BC} = \dfrac{160\text{ mm/s}}{40\text{ mm}}$ $\quad \underline{\omega} = 4$ rad/s

$v_A = (AC)\omega = (10\text{ mm})(4\text{ rad/s}) = 40$ mm/s $\quad \underline{v}_{BLOCK} = 40$ mm/s ↓ ◀

(c) OUTER PULLEY:

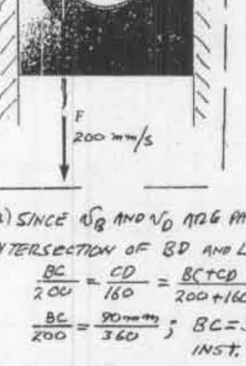
SINCE v_D ↑ AND v_A ↓, CORD IS UNWRAPPED AT RATE $(v_A + v_D)$/s.

$v_A + v_D = 40 + 200 = 240$ mm/s ; 240 mm, UNWRAPPED/s ◀

INNER PULLEY: v_B ↓ > v_A ↓, CORD IS UNWRAPPED AT RATE $(v_B - v_A)$/s

$v_B - v_A = 160 - 40 = 120$ mm/s ; 120 mm, UNWRAPPED/s ◀

15.79



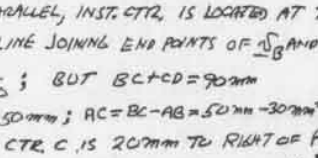
GIVEN: INNER PULLEY = 30 mm

OUTER PULLEY = 60 mm

FIND: (a) INST. CTR. OF ROTATION

(b) $\underline{v}_{BLOCK} = \underline{v}_A$

(c) LENGTH OF CORD WRAPPED OR UNWRAPPED PER SECOND ON EACH PULLEY



(a) SINCE v_B AND v_D ARE PARALLEL, INST. CTR. IS LOCATED AT THE INTERSECTION OF BD AND LINE JOINING END POINTS OF $\underline{v}_B$ AND $\underline{v}_D$.

$\dfrac{BC}{200} = \dfrac{CD}{160} = \dfrac{BC+CD}{200+160}$; BUT $BC+CD = 90$ mm

$\dfrac{BC}{200} = \dfrac{90\text{ mm}}{360}$; $BC = 50$ mm ; $AC = BC - AB = 50\text{ mm} - 30\text{ mm} = 20$ mm

INST. CTR. C IS 20 mm TO RIGHT OF A ◀

(b) $v_{BLOCK} = v_A$: $\omega = v_B/BC = (200\text{ mm/s})/50\text{ mm}$; $\underline{\omega} = 4$ rad/s

$v_A = (AC)\omega = (20\text{ mm})(4\text{ rad/s}) = 80$ mm/s ; $\underline{v}_{BLOCK} = 80$ mm/s ↓ ◀

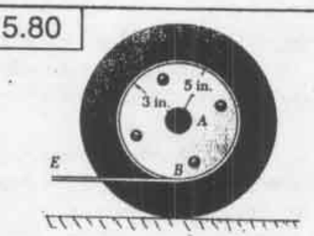
(c) OUTER PULLEY: v_D ↑ AND v_A ↓, CORD IS UNWRAPPED AT $(v_D + v_A)$/s

$v_D + v_A = 160 + 80 = 240$ mm/s ; 240 mm, UNWRAPPED/s ◀

INNER PULLEY: v_B ↓ > $\underline{v}_A$ ↓, CORD IS UNWRAPPED AT $(v_B - v_A)$/s

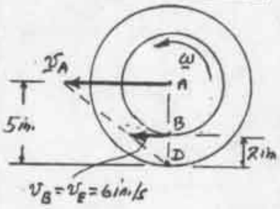
$v_B - v_A = 200 - 80 = 120$ mm/s ; 120 mm, UNWRAPPED/s ◀

15.80



GIVEN: $\underline{v}_E = 6\,in./s \leftarrow$

FIND: (a) $\underline{\omega}$
(b) $\underline{v}_A$
(c) CORD WOUND OR UNWOUNDED PER SECOND



SINCE DRUM ROLLS WITHOUT SLIDING, INST. CTR. OF ROTATION IS AT D.

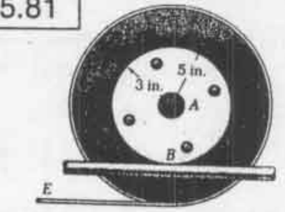
(a) $v_E = (BC)\omega$
$6in/s = (2in)\omega$; $\underline{\omega = 3\,rad/s}$ ↶

(b) $v_A = (AC)\omega$
$v_A = (5in)(3rad/s)$; $\underline{v_A = 15\,in./s} \leftarrow$

(c) SINCE $v_A > v_E$, DRUM GAINS ON CORD AND CORD IS WOUND ON DRUM AT RATE $(v_A - v_E) = (15in/s) - (6in/s) = 9in/s$

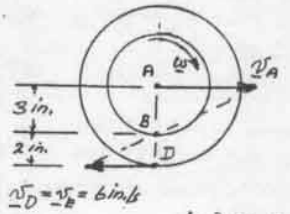
CORD WOUND PER SECOND = 9 in.

15.81



GIVEN: $\underline{v}_E = 6\,in./s \leftarrow$

FIND: (a) ω
(b) $\underline{v}_A$
(c) CORD WOUND OR UNWOUNDED PER SECOND



SINCE DRUM ROLLS WITHOUT SLIDING, INST. CTR. OF ROTATION IS AT B.

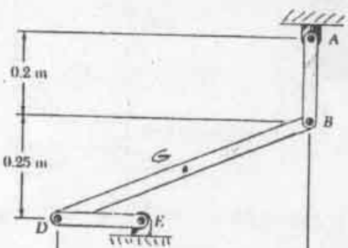
(a) $v_D = (BD)\omega$
$6in/s = (2in)\omega$; $\underline{\omega = 3\,rad/s}$ ↷

(b) $v_A = (AB)\omega$
$v_A = (3in.)(3rad/s)$
$\underline{v_A} = 9\,in./s \rightarrow$

(c) SINCE $v_E \leftarrow$ AND $v_A \rightarrow$, CORD MOVES TO LEFT AND DRUM MOVES TO THE RIGHT, CORD IS UNWOUND FROM DRUM AT RATE $(v_A + v_E) = (9+6) = 15\,in./s$

CORD UNWOUND PER SECOND = 15 in.

15.82



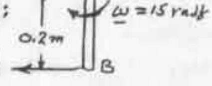
GIVEN: $\underline{\omega}_{AB} = 15\,rad/s$ ↷

FIND: (a) ω_{BD}
(b) $\underline{v}_G$ WHERE G IS MIDPOINT OF BD

ROD AB:

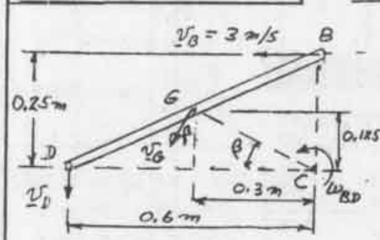
$v_B = (AB)\omega = (0.2m)(15\,rad/s)$
$\underline{v}_B = 3\,m/s \leftarrow$

ROD DE:



(CONTINUED)

15.82 CONTINUED



ROD BD:
DRAW LINES ⊥ TO v_B AND v_E TO LOCATE INST. CTR. C

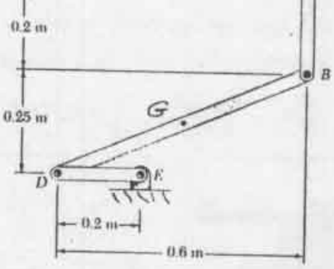
(a) $v_B = (BC)\omega_{BD}$
$3\,m/s = (0.25m)\,\omega_{BD}$
$\underline{\omega}_{BD} = 12\,rad/s$ ↶

(b) $\beta = \tan \frac{0.125m}{0.3m} = 22.6°$; $CG = \frac{0.3m}{\cos\beta} = 0.325\,m$

$v_G = (CG)\omega_{BD} = (0.325m)(12\,rad/s) = 3.90\,m/s$

$\underline{v}_G = 3.90\,m/s \measuredangle 22.6°$; $\underline{v}_G = 3.90\,m/s \measuredangle 67.4°$

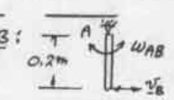
15.83



GIVEN: $\underline{v}_D = 2.4\,m/s \uparrow$

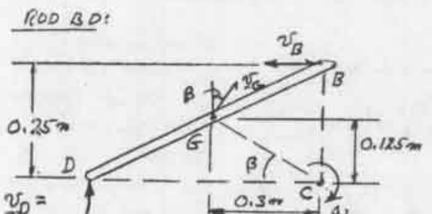
FIND: (a) $\underline{\omega}_{AB}$
(b) v_G WHERE G IS MIDPOINT OF BD

ROD AB:



$v_B = (AB)\,\omega_{AB}$
$\underline{v}_B = (0.2m)\omega_{AB} \leftrightarrow$ (1)

ROD BD:



DRAW LINES ⊥ TO $\underline{v}_B$ AND $\underline{v}_D$ TO LOCATE INST. CTR. OF ROTATION C.

(a) $v_D = (CD)\omega_{BD}$
$2.4\,m/s = (0.6m)\,\omega_{BD}$ $\underline{\omega}_{BD} = 4\,rad/s$ ↷

$v_B = (BC)\omega_{BD} = (0.25m)(4\,rad/s)$
$\underline{v}_B = 1\,rad/s \rightarrow$

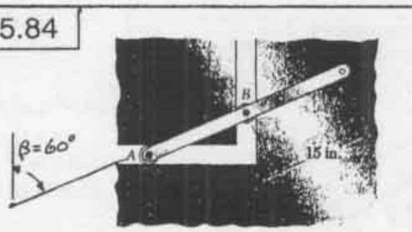
EQ(1): $v_B = (0.2m)\,\omega_{AB}$
$1\,m/s = (0.2m)\,\omega_{AB}$ $\underline{\omega}_{AB} = 5\,rad/s$ ↶

(b) $\beta = \tan^{-1}\frac{0.125m}{0.3m} = 22.6°$
$CG = \frac{0.3m}{\cos\beta} = 0.325\,m$

$v_G = (CG)\,\omega_{BD} = (0.325m)(4\,rad/s) = 1.300\,m/s$

$\underline{v}_G = 1.300\,m/s \measuredangle 22.6°$ $\underline{v}_G = 1.300\,m/s \measuredangle 67.4°$

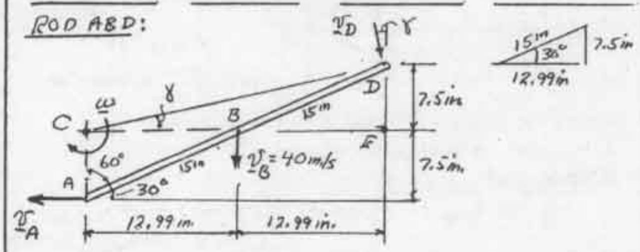
15.84



GIVEN:
$AB = BD = 15$ in.
$\underline{v}_B = 40$ in./s ↓
$\beta = 60°$

FIND: (a) ω
(b) $\underline{v}_D$

ROD ABD:



WE LOCATE INST. CTR. C BY DRAWING LINES ⊥ $\underline{v}_A$ AND $\underline{v}_D$

(a) ANGULAR VELOCITY

$v_B = (BC)\omega$

$40 \text{ in./s} = (12.99 \text{ in.})\omega$

$\omega = 3.079$ rad/s

$\underline{\omega} = 3.08$ rad/s ↻ ◀

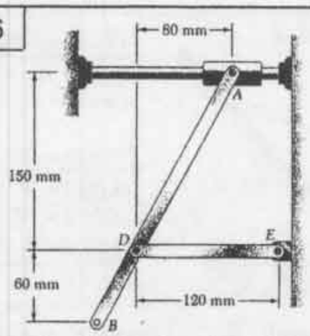
(b) VELOCITY OF D:

IN ΔCDE: $\gamma = \tan^{-1}\frac{7.5}{25.98} = 16.1°$; $CD = \frac{25.98}{\cos\gamma} = 27.04$ in.

$v_D = (CD)\omega = (27.04 \text{ in.})(3.079 \text{ rad/s}) = 83.3$ in./s

$\underline{v}_D = 83.3$ in./s ∡ 16.1°; $\underline{v}_D = 83.3$ in./s ∡ 73.9° ◀

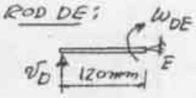
15.86



GIVEN:
$\underline{\omega}_{DE} = 2.4$ rad/s ↻

FIND:
(a) $\underline{v}_A$
(b) $\underline{v}_B$

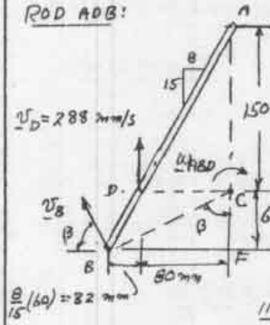
ROD DE:



$v_D = (120 \text{ mm})(2.4 \text{ rad/s})$

$\underline{v}_D = 288$ mm/s ↑

ROD ADB:



LOCATE INST. CTR. AT INTERSECTION OF LINES DRAWN ⊥ TO $\underline{v}_A$ AND $\underline{v}_D$

$v_D = (CD)\omega_{ABD}$

$288 \text{ mm/s} = (80 \text{ mm})\omega_{ABD}$; $\omega_{ABD} = 3.6 \frac{rad}{s}$ ↻

(a) VELOCITY OF A:

$v_A = (AC)\omega_{ABD} = (150 \text{ mm})(3.6 \text{ rad/s})$

$\underline{v}_A = 540$ mm/s → ◀

(b) VELOCITY OF B:

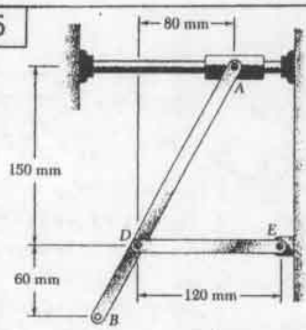
IN ΔBCF: $\beta = \tan^{-1}\frac{32+80}{60} = 61.82°$

$BC = \frac{32+80}{\sin\beta} = 127.06$ mm

$v_B = (BC)\omega_{ABD} = (127.06 \text{ mm})(3.6 \text{ rad/s}) = 457$ mm/s

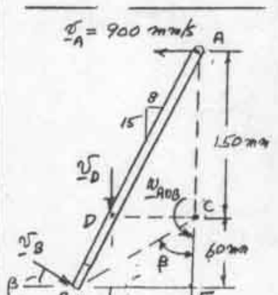
$\underline{v}_B = 457$ mm/s ∡ 61.8° ◀

15.85

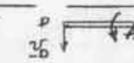


GIVEN:
$\underline{v}_A = 900$ mm/s ←

FIND: (a) $\underline{\omega}_{ADB}$
(b) $\underline{v}_B$



ROD DE



ROD ADB: LOCATE INST. CTR. C AT INTERSECTION OF LINES DRAWN ⊥ $\underline{v}_A$ AND v_D

(a) ANGULAR VELOCITY ω_{ABD}

$v_A = (CA)\,\omega_{ABD}$

$900 \text{ mm/s} = (150 \text{ mm})\,\omega_{ABD}$

$\underline{\omega}_{ABD} = 6$ rad/s ↻ ◀

(b) VELOCITY OF D

IN ΔBCF: $\beta = \tan^{-1}\frac{32+80}{60} = 61.82°$

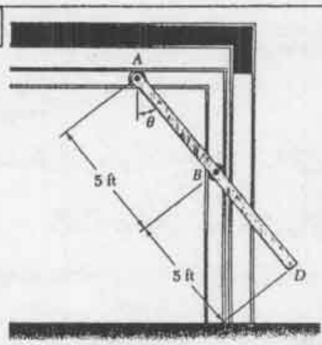
$BC = \frac{32+80}{\sin\beta} = 127.06$ mm

$v_B = (BC)(\omega_{ABD})$

$= (127.06 \text{ mm})(6 \text{ rad/s}) = 762.4$ mm/s

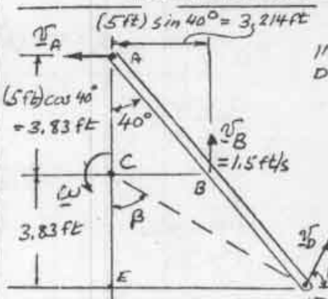
$\underline{v}_D = 762$ mm/s ∡ 61.8° ◀

15.87



GIVEN:
$\theta = 40°$
$\underline{v}_B = 1.5$ ft/s ↑

FIND:
(a) $\underline{\omega}$
(b) $\underline{v}_D$



LOCATE INST. CTR. AT INTERSECTION OF LINES DRAWN ⊥ TO $\underline{v}_A$ AND $\underline{v}_B$.

(a) ANGULAR VELOCITY

$v_B = (BC)\omega$

$1.5 \text{ ft/s} = (3.214 \text{ ft})\omega$

$\omega = 0.4667$ rad/s

$\underline{\omega} = 0.467$ rad/s ↺ ◀

(b) VELOCITY OF D:

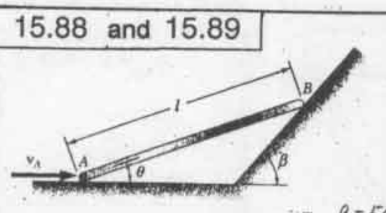
IN ΔCDE: $\beta = \tan^{-1}\frac{6.427}{3.83} = 59.2°$

$CD = \frac{6.427}{\sin\beta} = 7.482$ ft

$v_D = (CD)\omega = (7.482 \text{ ft})(0.4667 \text{ rad/s}) = 3.49$ ft/s

$\underline{v}_D = 3.49$ ft/s ∡ 59.2° ◀

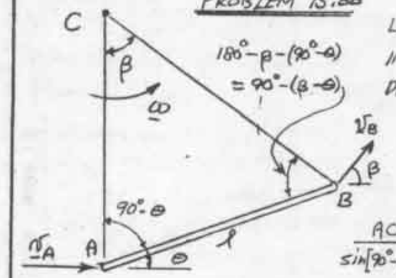
15.88 and 15.89



PROBLEM 15.88
DERIVE AN EXPRESSION FOR (a) ω, (b) v_B.

PROBLEM 15.89
GIVEN: $\theta = 20°$, $\beta = 50°$, $l = 0.6\,m$, $v_A = 3\,m/s$
FIND: (a) ω, (b) v_B

PROBLEM 15.88



LOCATE INST. CTR. AT INTERSECTION OF LINES DRAWN ⊥ TO v_A AND v_B

LAW OF SINES

$$\frac{AC}{\sin[90°-(\beta-\theta)]} = \frac{BC}{\sin(90°-\theta)} = \frac{l}{\sin\beta}$$

$$\frac{AC}{\cos(\beta-\theta)} = \frac{BC}{\cos\theta} = \frac{l}{\sin\beta}$$

$$AC = l\,\frac{\cos(\beta-\theta)}{\sin\beta} \qquad BC = l\,\frac{\cos\theta}{\sin\beta}$$

(a) ANGULAR VELOCITY: $v_A = (AC)\omega = l\,\frac{\cos(\beta-\theta)}{\sin\beta}\,\omega$

$$\omega = \frac{v_A}{l}\cdot\frac{\sin\beta}{\cos(\beta-\theta)}$$ ◄

(b) VELOCITY OF B:

$$v_B = (BC)\omega = l\,\frac{\cos\theta}{\sin\beta}\left[\frac{v_A}{l}\cdot\frac{\sin\beta}{\cos(\beta-\theta)}\right]$$

$$v_B = v_A\,\frac{\cos\theta}{\cos(\beta-\theta)}$$ ◄

PROBLEM 15.89: DATA, $\theta = 20°$, $\beta = 50°$, $l = 0.6\,m$, $v_A = 3\,m/s$

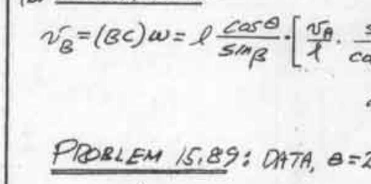
(a) $\omega = \frac{v_A}{l}\,\frac{\sin\beta}{\cos(\beta-\theta)} = \frac{3\,m/s}{0.6\,m}\cdot\frac{\sin 50°}{\cos(50°-20°)}$

$\omega = 4.423$ rad/s $\qquad \omega = 4.42$ rad/s ↻ ◄

(b) $v_B = v_A\,\frac{\cos\theta}{\cos(\beta-\theta)} = (3\,m/s)\,\frac{\cos 20°}{\cos(50°-20°)}$

$v_B = 3.2552$ m/s $\qquad v_B = 3.26$ m/s ∠50° ◄

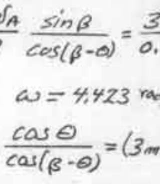
15.90



GIVEN: $v_B = 400$ mm/s ↑

FIND:
(a) ω_{ABD}
(b) v_A

CRANK DE



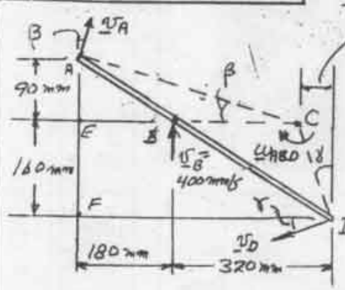
$\gamma = \tan^{-1}\frac{125}{300} = 22.62°$

$v_D = v_D$ ∠ 22.62°

(CONTINUED)

15.90 CONTINUED

ARM ABD:



$(160\,mm)\tan\gamma = 160\tan 22.62° = 66.7$ mm

$CE = DF - 66.7$ mm

$CE = 500 - 66.7 = 433.3$ mm

$\tan\beta = \frac{AE}{CE} = \frac{90\,mm}{433.3\,mm}$

$\beta = 11.734°$

$AC = \frac{90\,mm}{\sin\beta} = 442.5$ mm

INST. CTR. C IS LOCATED AT INTERSECTION OF LINES DRAWN ⊥ TO v_B AND v_D.

(a) ANGULAR VELOCITY:

$v_B = (BC)\omega_{ABD}$; $\quad 400\,mm/s = [320\,mm - 66.7\,mm]\,\omega_{ABD}$

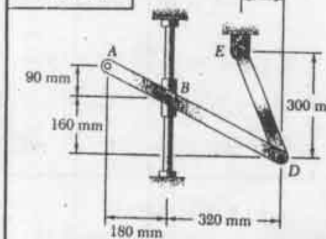
$\omega_{ABD} = 1.579$ rad/s ↺ ◄

(b) VELOCITY OF A:

$v_A = (AC)\,\omega_{ABD} = (442.5\,mm)(1.579\,rad/s) = 699$ mm/s

$v_A = 699$ mm/s ∡11.7° $\qquad v_A = 699$ mm/s ∠78.3° ◄

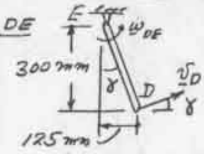
15.91



GIVEN: $\omega_{DE} = 1.2$ rad/s ↺

FIND:
(a) ω_{ABD}
(b) v_A

CRANK DE



$\gamma = \tan^{-1}\frac{125}{300} = 22.62$

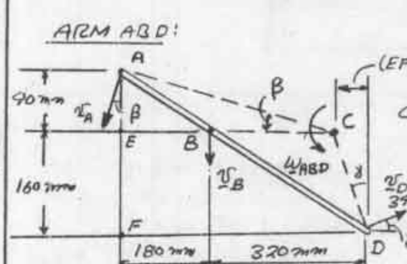
$DE = \frac{300}{\cos\gamma} = 325$ mm

$v_D = (DE)\,\omega_{DE}$

$v_D = (325\,mm)(1.2\,rad/s)$

$v_D = 390$ mm/s ∠22.6°

ARM ABD:



$(EF)\tan\gamma = (160)\tan 22.6° = 66.7$ mm

$CD = \frac{160\,mm}{\cos\gamma} = 173.3$ mm

INST. CTR. C IS LOCATED AT INTERSECTION OF LINES DRAWN ⊥ TO v_B AND v_D

(a) ANGULAR VELOCITY:

$v_D = (CD)\,\omega_{ABD}$; $\quad 390\,mm/s = (173.3\,mm)\,\omega_{ABD}$

$\omega_{ABD} = 2.25$ rad/s ; $\quad \omega_{ABD} = 2.25$ rad/s ↺ ◄

(b) VELOCITY OF A:

$CE = DF - 66.7\,mm = 500\,mm - 66.7\,mm = 433.3\,mm$

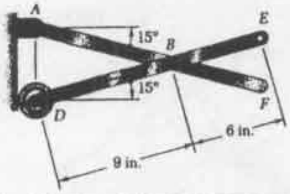
$\beta = \tan^{-1}\frac{AE}{CE} = \tan^{-1}\frac{90\,mm}{433.3\,mm} = 11.734°$

$AC = (CE)/\cos\beta = (433.3\,mm)/\cos 11.734° = 442.5$ mm

$v_A = (AC)\,\omega_{ABD} = (442.5\,mm)(2.25\,rad/s) = 996$ mm/s

$v_A = 996$ mm/s ∡11.7° $\qquad v_A = 996$ mm/s ∠78.3° ◄

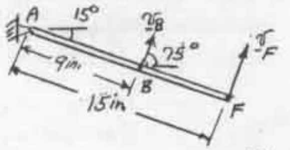
15.92



GIVEN:
$\underline{v}_D = 10 \text{ in./s} \uparrow$

FIND: (a) $\underline{v}_E$
(b) $\underline{v}_F$

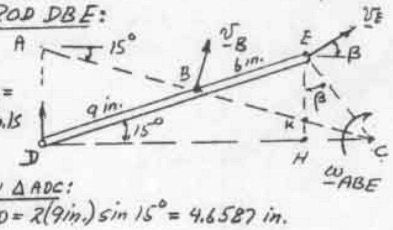
ROD ABF:



$v_B = v_B \angle 75°$

$\underline{v}_F = \frac{15}{9}\underline{v}_B \quad (1)$

ROD DBE:



INST. CTR. C IS LOCATED AT INTERSECTION OF LINES DRAWN ⊥ TO $\underline{v}_B$ AND $\underline{v}_D$

IN Δ ADC:
$AD = 2(9 \text{ in.}) \sin 15° = 4.6587 \text{ in.}$
$CD = AD/\tan 15° = 17.387 \text{ in.}$

IN Δ DEH:
$EH = (DE)\sin 15° = (15 \text{ in.})\sin 15° = 3.8823 \text{ in.}$
$DH = (DE)\cos 15° = (15 \text{ in.})\cos 15° = 14.489 \text{ in.}$

IN Δ CEH:
$HC = CD - DH = 17.387 \text{ in.} - 14.489 \text{ in.} = 2.898 \text{ in.}$

$\beta = \tan^{-1}\frac{HC}{EH} = \tan^{-1}\frac{2.898 \text{ in.}}{3.8823 \text{ in.}} = 36.74°$

$EC = (EH)/\cos\beta = (3.8823 \text{ in.})/\cos\beta = 4.844 \text{ in.}$

(CHECK) $CK = (HC)/\sin 15° = 3.000 \text{ in.}$ OK.

$BC = AB = 9 \text{ in.}$

ANGULAR VELOCITY

$v_D = (CD)\omega_{ABE}$

$10 \text{ in./s} = (17.387 \text{ in.})\omega_{ABE}$; $\omega_{ABE} = 0.5751 \text{ rad/s}$

(a) VELOCITY OF E:

$v_E = (EC)\omega_{ABE} = (4.844 \text{ in.})(0.5751 \text{ rad/s}) = 2.79 \text{ in./s}$

$\underline{v}_E = 2.79 \text{ in./s} \angle 36.7°$ ◀

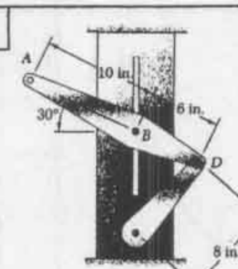
(b) VELOCITY OF F;

$v_B = (BC)\omega_{ABE} = (9 \text{ in.})(0.5751 \text{ rad/s}) = 5.176 \text{ in./s}$

EQ(1): $v_F = \frac{15}{9}v_B = \frac{15}{9}(5.176 \text{ in./s}) = 8.63 \text{ in./s}$

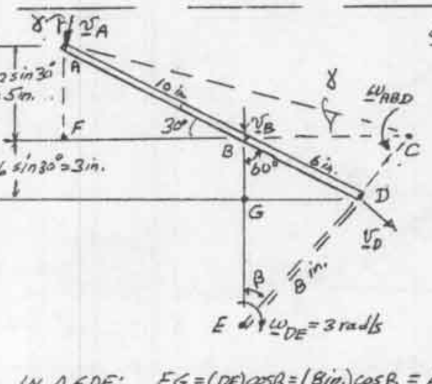
$\underline{v}_F = 8.63 \text{ in./s} \angle 75°$ ◀

15.93



GIVEN:
$\underline{\omega}_{DE} = 3 \text{ rad/s}$

FIND:
(a) $\underline{\omega}_{ABD}$
(b) $\underline{v}_A$



GEOMETRY
IN Δ BDE

$\frac{\sin\beta}{6 \text{ in.}} = \frac{\sin 60°}{8 \text{ in.}}$

$\sin\beta = \frac{3}{4}\sin 60°$

$\sin\beta = 0.6495$

$\beta = 40.505°$

IN Δ GDE: $EG = (DE)\cos\beta = (8 \text{ in.})\cos\beta = 6.083 \text{ in.}$

IN Δ BCE: $BC = (BE)\tan\beta = [BG + EG]\tan\beta$
$= (3 \text{ in.} + 6.083 \text{ in.})\tan\beta = 7.759 \text{ in.}$
$EC = (BE)/\cos\beta = (3 \text{ in.} + 6.083 \text{ in.})/\cos\beta = 11.946 \text{ in.}$

$FB = (AB)\cos 30° = (10 \text{ in.})\cos 30° = 8.660 \text{ in.}$

$FC = FB + BC = 8.660 \text{ in.} + 7.759 \text{ in.} = 16.419 \text{ in.}$

IN Δ AFC: $\gamma = \tan^{-1}\frac{AF}{FC} = \tan^{-1}\frac{5 \text{ in.}}{16.419 \text{ in.}} = 16.937°$

$AC = \frac{FC}{\cos\gamma} = \frac{16.419 \text{ in.}}{\cos\gamma} = 17.163 \text{ in.}$

ARM DE: $v_D = (DE)\omega_{DE} = (8 \text{ in.})(3 \text{ rad/s})$
$\underline{v}_D = 24 \text{ in./s} \measuredangle \beta$

MEMBER ABD: THE INST. CTR. C IS LOCATED AT INTERSECTION OF LINES DRAWN ⊥ $\underline{v}_B$ AND $\underline{v}_D$.

$CD = EC - ED = 11.946 \text{ in.} - 8 \text{ in.} = 3.946 \text{ in.}$

(a) ANGULAR VELOCITY ω_{ABD}:

$v_D = (CD)\omega_{ABD}$
$24 \text{ in./s} = (3.946 \text{ in.})\omega_{ABD}$
$\omega_{ABD} = 6.082 \text{ rad/s}$ $\quad \underline{\omega}_{ABD} = 6.08 \text{ rad/s}$ ◀

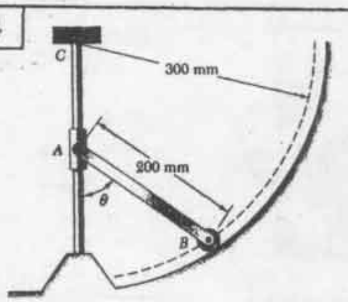
(b) VELOCITY OF A:

$v_A = (AC)\omega_{ABD} = (17.163 \text{ in.})(6.082 \text{ rad/s})$
$v_A = 104.4 \text{ in./s}$

$\underline{v}_A = 104.4 \text{ in./s} \measuredangle \gamma = 104.4 \text{ in./s} \measuredangle 16.9°$

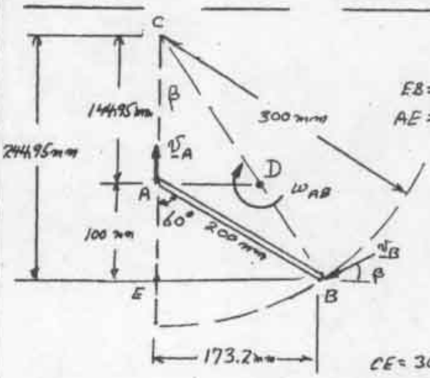
$\underline{v}_A = 104.4 \text{ in./s} \measuredangle 73.1°$ ◀

15.94



GIVEN:
$\theta = 60°$
$v_A = 250$ mm/s

FIND:
(a) ω_{AB}
(b) v_B



IN ΔABE:
$EB = 200 \sin 60° = 173.2$ mm
$AE = 200 \cos 60° = 100$ mm

IN ΔBCE:
$\beta = \sin^{-1} \frac{173.2 \text{ mm}}{300 \text{ mm}}$

$\beta = 35.264°$

$CE = 300 \cos\beta = 244.95$ mm
$AC = CE - 100 = 144.95$ mm

SIMILAR TRIANGLES: ΔCAD AND CEB

$$\frac{CD}{CB} = \frac{AD}{EB} = \frac{CA}{CE}$$

$$\frac{CD}{300 \text{ mm}} = \frac{AD}{173.2 \text{ mm}} = \frac{144.95 \text{ mm}}{244.95 \text{ mm}}$$

$CD = 177.53$ mm $\qquad AD = 102.49$ mm

$BD = CB - CD = 300 \text{ mm} - 177.53 \text{ mm} = 122.47$ mm

THE INST. CTR. IS LOCATED AT POINT D WHICH IS THE POINT OF INTERSECTION OF LINES DRAWN $\perp$ TO v_A AND v_B

(a) ANGULAR VELOCITY ω_{AB}:

$v_A = (AD)\,\omega_{AB}$

$250 \text{ mm/s} = (102.49 \text{ mm})\,\omega_{AB}$

$\omega_{AB} = 2.439$ rad/s $\qquad \omega_{AB} = 2.44$ rad/s ↻ ◀

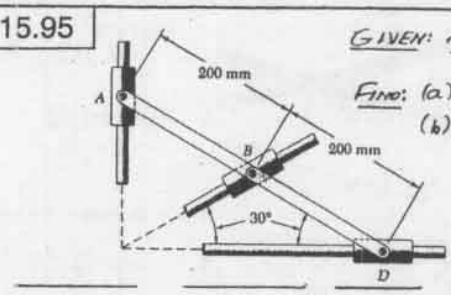
(b) VELOCITY OF B:

$v_B = (BD)\,\omega_{AB} = (122.47 \text{ mm})(2.439 \text{ rad/s})$

$v_B = 298.7$ mm/s

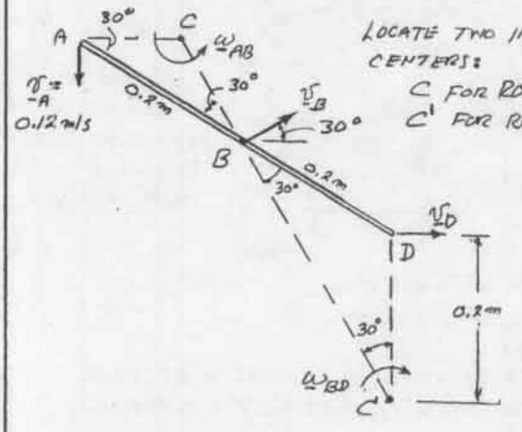
$v_B = 299 \text{ mm/s} \angle 35.3°$ ◀

15.95



GIVEN: $v_A = 120$ mm/s ↓

FIND: (a) ω_{AB}, ω_{BD}
(b) v_D



LOCATE TWO INSTANTANEOUS CENTERS:
C FOR ROD AB AND
C' FOR ROD BD.

ISOSCELES ΔACB:
$AC = BC = (0.1 \text{ m})/\cos 30° = 0.11547$ m

ISOSCELES $\Delta BC'D$
$BD = DC' = 0.2$ m
$BC' = 2(BD)\cos 30° = 2(0.2 \text{ m})\cos 30° = 0.3464$ m

(a) ROD AB:

$v_A = (AC)\,\omega_{AB}$

$0.120 \text{ m/s} = (0.11547 \text{ m})\,\omega_{AB}$

$\omega_{AB} = 1.0392$ rad/s

$\omega_{AB} = 1.039$ rad/s ↺ ◀

$v_B = (BC)\,\omega_{AB}$

SINCE $AC = BC$, $v_B = v_A$

$v_B = 0.12 \text{ m/s} \angle$

ROD BD:

$v_B = (BC')\,\omega_{BD}$

$0.12 \text{ m/s} = (0.3464 \text{ m})\,\omega_{BD}$

$\omega_{BD} = 0.3464$ rad/s

$\omega_{BD} = 0.346$ rad/s ↻

(b) VELOCITY OF D:

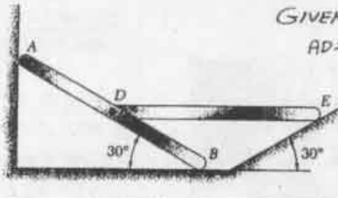
$v_D = (DC')\,\omega_{BD}$

$= (0.2 \text{ m})(0.3464 \text{ rad/s})$

$v_D = 0.06928$ m/s

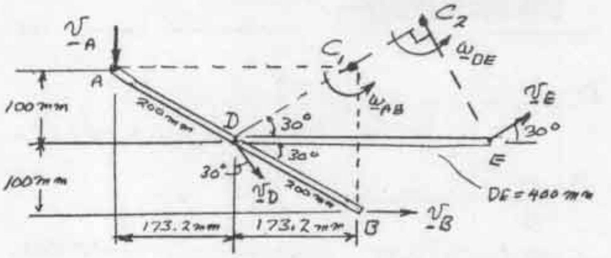
$v_D = 69.3$ mm/s → ◀

15.96



GIVEN: $v_A = 240$ mm/s ↓
$AD = DE = 400$ mm

FIND:
(a) ω_{DE}
(b) v_E



WE LOCATE TWO INST. CTRS. AT INTERSECTIONS OF LINES DRAWN AS FOLLOWS:

C_1: FOR ROD AB, DRAW LINES $\perp$ TO $v_A + v_B$

C_2: FOR ROD DE, DRAW LINES $\perp$ TO $v_D + v_E$

GEOMETRY: $AC_1 = (400\text{ mm})\cos 30° = 346.4$ mm

$BC_1 = (400\text{ mm})\sin 30° = 200$ mm

$DC_1 = AD = 200$ mm

$DC_2 = (DE)\cos 30° = (400\text{ mm})\cos 30° = 346.4$ mm

$EC_2 = (DE)\sin 30° = (400\text{ mm})\sin 30° = 200$ mm

ROD AB:

$v_A = (AC_1)\omega_{AB}$; $240\text{ mm/s} = (346.4\text{ mm})\,\omega_{AB}$

$\omega_{AB} = 0.69284$ rad/s ↻

$v_D = (DC_1)\,\omega_{AB} = (200\text{ mm})(0.69284\text{ rad/s})$

$v_D = 138.57$ rad/s ⦪ 30°

ROD DE:

$v_D = (DC_2)\,\omega_{DE}$

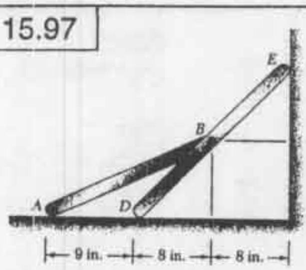
$138.57\text{ mm/s} = (346.4\text{ mm})\,\omega_{DE}$

(a) $\omega_{DE} = 0.400$ rad/s $\qquad \omega_{DE} = 0.4$ rad/s ↻ ◀

(b) $v_E = (EC_2)\,\omega_{DE} = (200\text{ mm})(0.400\text{ rad/s})$

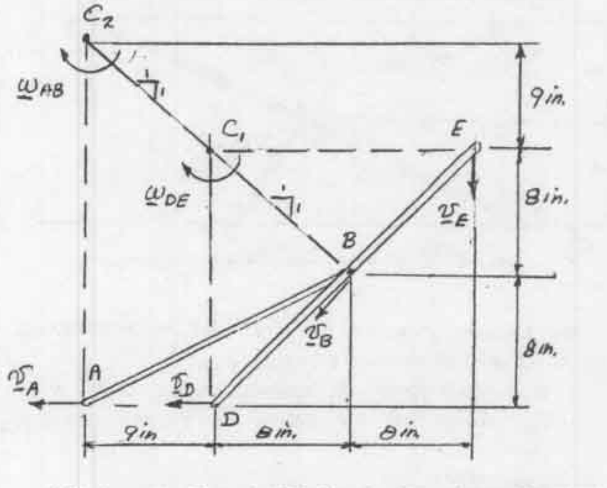
$v_E = 80$ mm/s $\qquad v_E = 80$ mm/s ∠ 30° ◀

15.97



GIVEN:
$v_D = 40$ in./s ←

FIND:
(a) ω_{AB} AND ω_{DE}
(b) v_A



WE LOCATE TWO INST. CTRS. AT INTERSECTIONS OF LINES DRAWN AS FOLLOWS:

C_1: FOR ROD DE, DRAW LINES $\perp$ TO v_D AND v_E

C_2: FOR ROD AB, DRAW LINES $\perp$ TO v_A AND v_B

GEOMETRY: $BC_1 = (8\text{ in.})\sqrt{2} = 8\sqrt{2}$ in.

$DC_1 = 16$ in.

$BC_2 = (9\text{ in} + 8\text{ in})\sqrt{2} = 17\sqrt{2}$ in.

$AC_2 = 25$ in.

(a) ROD DE: $v_D = (DC_1)\,\omega_{DE}$

$40\text{ in./s} = (16\text{ in.})\,\omega_{DE}$

$\omega_{DE} = 2.5$ rad/s $\qquad \omega_{DE} = 2.5$ rad/s ↻ ◀

$v_B = (BC_1)\,\omega_{DE}$

$= (8\sqrt{2}\text{ in.})(2.5\text{ rad/s})$

$v_B = 20\sqrt{2}$ in./s ⦫ 45°

ROD AB: $v_B = (BC_2)\,\omega_{AB}$

$20\sqrt{2}\text{ in./s} = (17\sqrt{2}\text{ in.})\,\omega_{AB}$

$\omega_{AB} = \frac{20}{17}$ rad/s $= 1.1765$ rad/s

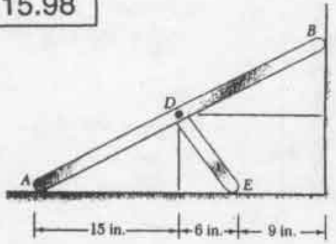
$\omega_{AB} = 1.176$ rad/s ↻ ◀

(b) $v_A = (AC_2)\,\omega_{AB}$

$= (25\text{ in.})(1.1765\text{ rad/s})$

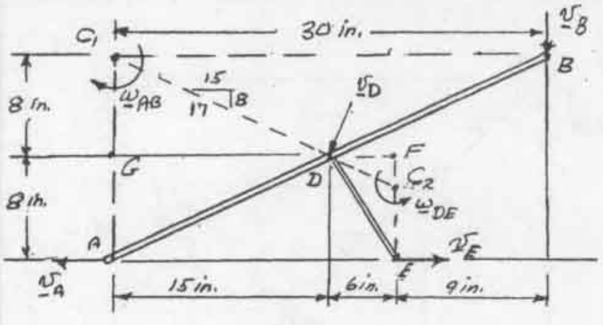
$v_A = 29.41$ in./s $\qquad v_A = 29.4$ in./s ← ◀

15.98



GIVEN: $\underline{v}_B = 60\text{ in./s}\downarrow$

FIND: (a) $\underline{\omega}_{AB}$ AND $\underline{\omega}_{DE}$ (b) $\underline{v}_E$



WE LOCATE TWO INST. CTRS. AT INTERSECTIONS OF LINES DRAWN AS FOLLOWS:

C_1: FOR ROD AB DRAW LINES $\perp$ TO $\underline{v}_A$ AND $\underline{v}_B$

C_2: FOR ROD DE DRAW LINES $\perp$ TO $\underline{v}_D$ AND $\underline{v}_E$

GEOMETRY:

$$DC_1 = (8^2+15^2)^{1/2} = 17\text{ in.}$$

SINCE ΔC_1DG AND ΔDFC_2 ARE SIMILAR,

$$\frac{C_2F}{8\text{ in.}} = \frac{C_2D}{17\text{ in.}} = \frac{6\text{ in.}}{15\text{ in.}}$$

$$C_2F = 3.2\text{ in.} \qquad C_2D = 6.8\text{ in.}$$

$$EC_2 = 8\text{ in.} - C_2F = 8 - 3.2 = 4.8\text{ in.}$$

(a) ROD AB: $v_B = (BC_1)\omega_{AB}$

$60\text{ in./s} = (30\text{ in.})\,\omega_{AB}$

$\omega_{AB} = 2\text{ rad/s}$ $\qquad \underline{\omega}_{AB} = 2\text{ rad/s}$ ◄

$v_D = (DC_1)\omega_{AB}$

$v_D = (17\text{ in.})(2\text{ rad/s}) = 34\text{ in./s}$

ROD DE: $v_D = (DC_2)\,\omega_{DE}$

$34\text{ in./s} = (6.8\text{ in.})\,\omega_{DE}$

$\omega_{DE} = 5\text{ rad/s}$ $\qquad \underline{\omega}_{DE} = 5\text{ rad/s}$ ◄

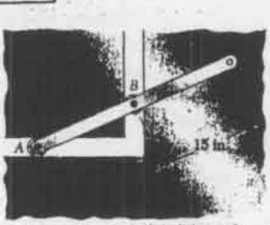
(b) $v_E = (EC_2)\,\omega_{DE}$

$v_E = (4.8\text{ in.})(5\text{ rad/s})$

$v_E = 24\text{ in./s}$

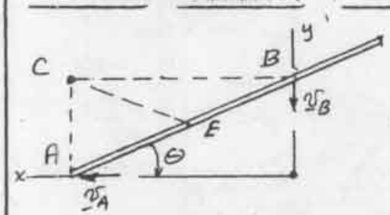
$\underline{v}_E = 24\text{ in./s} \rightarrow$ ◄

15.99



GIVEN: $AB = BD = 15\text{ in.}$

DESCRIBE THE SPACE CENTRODE AND BODY CENTRODE OF ROD ABD.



LET: $AB = l = 15\text{ in.}$

SPACE CENTRODE: COORDINATES OF INST. CTR.

$$x = l\cos\theta \qquad y = l\sin\theta$$

$$x^2+y^2 = l^2(\cos^2\theta+\sin^2\theta)$$

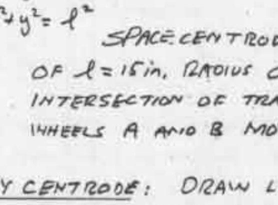
$$x^2+y^2 = l^2$$

SPACE CENTRODE IS A QUARTER CIRCLE OF $l = 15$ in. RADIUS CENTERED AT INTERSECTION OF TRACKS IN WHICH WHEELS A AND B MOVE ◄

BODY CENTRODE: DRAW LINE CE WHICH CONNECTS INST. CTR. C AND POINT E LOCATED MIDWAY BETWEEN A AND B.

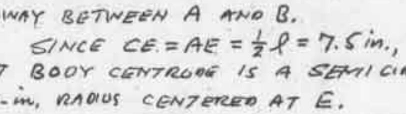
SINCE $CE = AE = \frac{1}{2}l = 7.5\text{ in.}$, WE NOTE THAT BODY CENTRODE IS A SEMICIRCLE OF 7.5-in. RADIUS CENTERED AT E. ◄

15.100



GIVEN: GEAR ROLLS ON STATIONARY LOWER RACK.

DESCRIBE THE SPACE CENTRODE AND BODY CENTRODE OF THE GEAR.



SINCE GEAR ROLLS ON LOWER RACK, THE INST. CTR. IS ALWAYS AT POINT OF CONTACT BETWEEN GEAR AND LOWER RACK.

SPACE CENTRODE: LOWER RACK ◄

BODY CENTRODE: CIRCUMFERENCE OF GEAR ◄

15.101

(PROB 15.62)

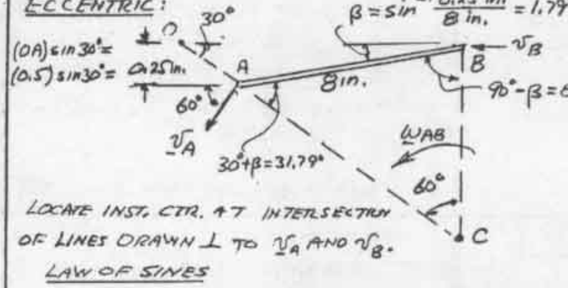
GIVEN: $\theta = 30°$
$\omega_0 = 900\ \text{rpm}$
$OA = 0.5\ \text{in.}$

FIND: $\underline{v}_B$

SHAFT:

$\omega_0 = 900\ \text{rpm} = 94.248\ \text{rad/s}$
$v_A = (0.5\ \text{in.})(94.248\ \text{rad/s})$
$\underline{v}_A = 47.124\ \text{in./s} \ \angle 60°$

ECCENTRIC:



$\beta = \sin^{-1}\dfrac{0.25\ \text{in.}}{8\ \text{in.}} = 1.79°$

LOCATE INST. CTR. AT INTERSECTION OF LINES DRAWN ⊥ TO $\underline{v}_A$ AND v_B.

LAW OF SINES

$$\frac{AC}{\sin 88.21°} = \frac{BC}{\sin 31.79°} = \frac{8\ \text{in.}}{\sin 60°}; \quad AC = 9.233\ \text{in.} \quad BC = 4.866\ \text{in.}$$

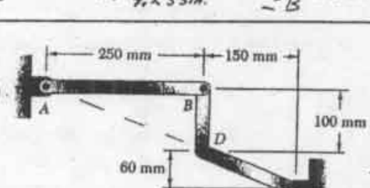
$v_A = (AC)\omega_{AB} \qquad \omega_{AB} = v_A/(AC)$

$v_B = (BC)\omega_{AB} = (BC)\, v_A/(AC) = v_A \dfrac{BC}{AC}$

$v_B = (47.124\ \text{in./s})\dfrac{4.866\ \text{in}}{9.233\ \text{in.}}; \quad \underline{v}_B = 24.8\ \text{in/s} \leftarrow$

15.102

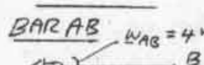
(PROB. 15.64)



GIVEN: $\underline{\omega}_{AB} = 4\ \frac{\text{rad}}{\text{s}}$

FIND: $\underline{\omega}_{BD}$ AND $\underline{\omega}_{DE}$

BAR AB



$\underline{v}_B = (AB)\omega_{AB} = (0.25\ \text{m})(4\ \text{rad/s}) = 1\ \text{m/s}$

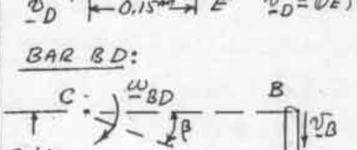
BAR DE:

$\beta = \tan^{-1}\dfrac{0.06\ \text{m}}{0.15\ \text{m}} = 21.8°$

$DE = \dfrac{0.15\ \text{m}}{\cos\beta}$

$\underline{v}_D = (DE)\omega_{DE} = \dfrac{0.15\ \text{m}}{\cos\beta}\omega_{DE} \angle\beta \qquad (1)$

BAR BD:



LOCATE INST. CTR. C AT INTERSECTION OF LINES DRAWN ⊥ TO $\underline{v}_B$ + $\underline{v}_D$.

$BC = \dfrac{0.1\ \text{m}}{\tan\beta} = \dfrac{0.1\ \text{m}}{\tan 21.8°}$

$BC = 0.25\ \text{m}$

$DC = \dfrac{0.25\ \text{m}}{\cos\beta}$

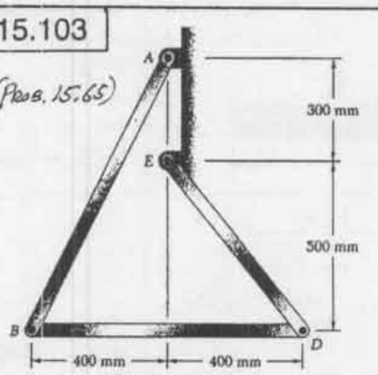
$v_B = (BC)\omega_{BD}$
$1\ \text{m/s} = (0.25\ \text{m})\,\omega_{BD} \qquad \underline{\omega}_{BD} = 4\ \text{rad/s}$

$v_D = (DC)\omega_{BD} = \dfrac{0.25\ \text{m}}{\cos\beta}(4\ \text{rad/s})$

EQ(1) $v_D = \dfrac{0.15\ \text{m}}{\cos\beta}\omega_{DE}; \quad \dfrac{1\ \text{m/s}}{\cos\beta} = \dfrac{0.15\ \text{m}}{\cos\beta}\omega_{DE}; \quad \underline{\omega}_{DE} = 6.67\ \frac{\text{rad}}{\text{s}}$

15.103

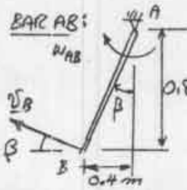
(PROB. 15.65)



GIVEN: $\omega_{AB} = 4\ \text{rad/s}$

FIND: $\underline{\omega}_{BD}$, $\underline{\omega}_{DE}$

BAR AB:



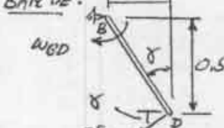
$\beta = \tan^{-1}\dfrac{0.4\ \text{m}}{0.8\ \text{m}} = 26.56°$

$AB = \dfrac{0.8\ \text{m}}{\cos\beta} = 0.8944\ \text{m}$

$v_B = (AB)\omega_{AB} = (0.8944\ \text{m})(4\ \text{rad/s})$

$\underline{v}_B = 3.578\ \text{m/s} \ \angle 26.56°$

BAR DE:



$\gamma = \tan^{-1}\dfrac{0.4\ \text{m}}{0.5\ \text{m}} = 38.66°$

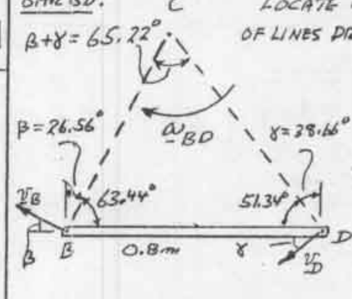
$DE = \dfrac{0.5\ \text{m}}{\cos\gamma} = 0.6403\ \text{m}$

$v_D = (DE)\omega_{DE}$

$\underline{v}_D = (0.6403\ \text{m})\omega_{DE} \ \angle 38.66°$

BAR BD:

LOCATE INST. CTR. AT INTERSECTION OF LINES DRAWN ⊥ TO $\underline{v}_B$ AND $\underline{v}_D$



LAW OF SINES

$$\frac{BC}{\sin 51.34°} = \frac{CD}{\sin 63.44°} = \frac{0.8\ \text{m}}{\sin 65.22°}$$

$BC = 0.688\ \text{m}$
$CD = 0.7881\ \text{m}$

$v_B = (BC)\omega_{BD}$

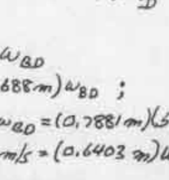
$3.578\ \text{m/s} = (0.688\ \text{m})\,\omega_{BD}; \qquad \underline{\omega}_{BD} = 5.2\ \text{rad/s}$

$v_D = (CD)\,\omega_{BD} = (0.7881\ \text{m})(5.2\ \text{m/s}) = 4.098\ \text{m/s}$

EQ(1) $4.098\ \text{m/s} = (0.6403\ \text{m})\,\omega_{DE}; \qquad \underline{\omega}_{DE} = 6.4\ \text{rad/s}$

15.104

(PROB. 15.70)

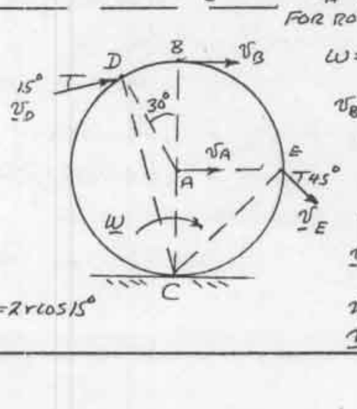


GIVEN: $v_A = 48\ \text{mi/h} \rightarrow$
$r = 11\ \text{in.}$

FIND: $\underline{v}_B$, $\underline{v}_C$, $\underline{v}_D$, AND $\underline{v}_E$

$v_A = 48\ \text{mi/h} = 70.4\ \text{ft/s}$

FOR ROLLING INST. CTR. AT C



$\omega = \dfrac{v_A}{r} = \dfrac{70.4\ \text{ft/s}}{r}$

$v_B = (BC)\omega = (2r)\dfrac{70.4}{r}$

$\underline{v}_B = 140.8\ \text{ft/s} \rightarrow$

$\underline{v}_C = 0$

$CD = 2r\cos 15°$

$v_D = (CD)\omega = (2r\cos 15°)\dfrac{70.4}{r}$

$\underline{v}_D = 136.0\ \text{ft/s} \ \angle 15°$

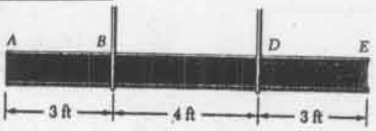
$CE = \sqrt{2}\, r$

$v_E = (CE)\omega = (\sqrt{2}\, r)\dfrac{70.4}{r}$

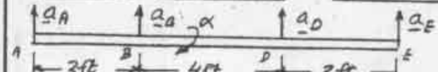
$\underline{v}_E = 99.6\ \text{ft/s} \ \angle 45°$

$CD = 2r\cos 15°$

15.105



GIVEN:
$\underline{a}_B = 5\ \mathrm{ft/s^2} \uparrow$
$\underline{a}_D = 3\ \mathrm{ft/s^2} \uparrow$
FIND: (a) $\underline{\alpha}$
(b) $\underline{a}_A$ AND $\underline{a}_E$



(a) $\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = a_B \uparrow + (BD)\alpha \downarrow$

$3\ \mathrm{ft/s^2}\uparrow = 5\ \mathrm{ft/s^2}\uparrow + (4\ \mathrm{ft})\alpha\downarrow$

$2\ \mathrm{ft/s^2}\downarrow = (4\ \mathrm{ft})\alpha\downarrow$ $\qquad \alpha = 0.5\ \mathrm{rad/s^2}$ ◀

(b) $\underline{a}_A = \underline{a}_B + \underline{a}_{A/B} = \underline{a}_B + (AB)\alpha$

$\underline{a}_A = 5\ \mathrm{ft/s^2}\uparrow + (3\ \mathrm{ft})(0.5\ \mathrm{rad/s^2})\uparrow$

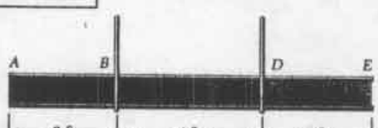
$\underline{a}_A = 6.5\ \mathrm{ft/s^2}\uparrow$ $\qquad \underline{a}_A = 6.5\ \mathrm{ft/s^2}\uparrow$ ◀

$\underline{a}_E = \underline{a}_D + \underline{a}_{E/D} = a_D + (DE)\alpha$

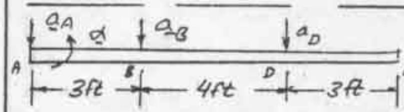
$\underline{a}_E = 3\ \mathrm{ft/s^2}\uparrow + (3\ \mathrm{ft})(0.5\ \mathrm{rad/s^2})\downarrow$

$\underline{a}_E = 1.5\ \mathrm{ft/s^2}\uparrow$ $\qquad \underline{a}_E = 1.5\ \mathrm{ft/s^2}\uparrow$ ◀

15.106



GIVEN:
$\underline{a}_A = 4\ \mathrm{ft/s^2}\downarrow$
$\underline{\alpha} = 1.2\ \mathrm{rad/s^2}$
FIND:
(a) $\underline{a}_B$
(b) $\underline{a}_D$



(a) $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = a_A\downarrow + (AB)\alpha\uparrow$

$\underline{a}_B = 4\ \mathrm{ft/s^2}\downarrow + (3\ \mathrm{ft})(1.2\ \mathrm{rad/s^2})\uparrow$

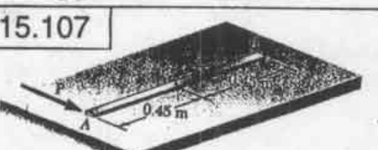
$\underline{a}_B = 4\ \mathrm{ft/s^2}\downarrow + 3.6\ \mathrm{ft/s^2}\uparrow$ $\qquad \underline{a}_B = 0.4\ \mathrm{ft/s^2}\downarrow$ ◀

$\underline{a}_D = a_A + a_{D/A} = a_A\downarrow + (AD)\alpha\uparrow$

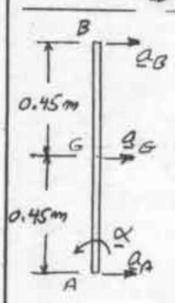
$\underline{a}_D = 4\ \mathrm{ft/s^2}\downarrow + (7\ \mathrm{ft})(1.2\ \mathrm{rad/s^2})\uparrow$

$\underline{a}_D = 4\ \mathrm{ft/s^2}\downarrow + 8.4\ \mathrm{ft/s^2}\uparrow$ $\qquad \underline{a}_D = 4.4\ \mathrm{ft/s^2}\uparrow$ ◀

15.107



GIVEN: $\underline{a}_A = 3.6\ \mathrm{m/s^2}\rightarrow$
$\underline{\alpha} = 6\ \mathrm{rad/s^2}$
FIND: (a) $\underline{a}_G$
(b) $\underline{a}_B$



$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A} = a_A \rightarrow + (AG)\alpha \leftarrow$

$\underline{a}_G = 3.6\ \mathrm{m/s^2}\rightarrow + (0.45\ \mathrm{m})(6\ \mathrm{rad/s^2})\leftarrow$

$\underline{a}_G = 3.6\ \mathrm{m/s^2}\rightarrow + 2.7\ \mathrm{m/s^2}\leftarrow$

$\underline{a}_G = 0.9\ \mathrm{m/s^2}\rightarrow$ ◀

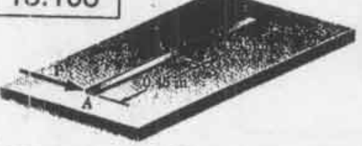
$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = a_A\rightarrow + (AB)\alpha\leftarrow$

$\underline{a}_B = 3.6\ \mathrm{m/s^2}\rightarrow + (0.9\ \mathrm{m})(6\ \mathrm{rad/s^2})\leftarrow$

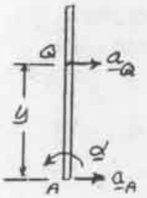
$\underline{a}_B = 3.6\ \mathrm{m/s^2}\rightarrow + 5.4\ \mathrm{m/s^2}\leftarrow$

$\underline{a}_B = 1.8\ \mathrm{m/s^2}\leftarrow$ ◀

15.108



GIVEN: $\underline{a}_A = 3.6\ \mathrm{m/s^2}\rightarrow$
$\underline{\alpha} = 6\ \mathrm{rad/s^2}$
FIND: POINT OF ROD FOR
(a) $\underline{a} = 0$
(b) $\underline{a} = 2.4\ \mathrm{m/s^2}\rightarrow$



(a) FOR $\underline{a}_Q = 0$

$\underline{a}_Q = \underline{a}_A + \underline{a}_{Q/A} = \underline{a}_A\rightarrow + (AQ)\alpha\leftarrow$

$0 = 3.6\ \mathrm{m/s^2}\rightarrow + (y)(6\ \mathrm{rad/s^2})\leftarrow$

$y = \dfrac{3.6\ \mathrm{m/s^2}}{6\ \mathrm{rad/s}} = 0.6\ \mathrm{m}$

$\underline{a} = 0$ AT 0.6 m FROM A ◀

(b) FOR $\underline{a}_Q = 2.4\ \mathrm{m/s^2}\rightarrow$

$\underline{a}_Q = \underline{a}_A + \underline{a}_{Q/A} = a_A\rightarrow + (AQ)\alpha\leftarrow$

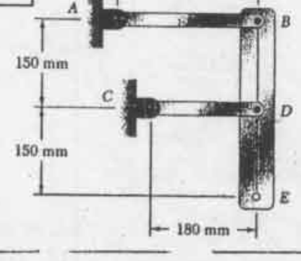
$2.4\ \mathrm{m/s^2}\rightarrow = 3.6\ \mathrm{m/s^2}\rightarrow + (y)(6\ \mathrm{rad/s^2})\leftarrow$

$1.2\ \mathrm{m/s^2}\leftarrow = (y)(6\ \mathrm{rad/s^2})\leftarrow$

$y = 0.2\ \mathrm{m}$

$\underline{a} = 2.4\ \mathrm{m/s^2}\rightarrow$ AT 0.2 m FROM A ◀

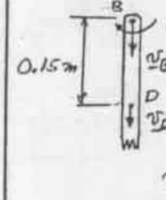
15.109



GIVEN:
$\omega_{AB} = 3\ \mathrm{rad/s}$
$\underline{\alpha}_{AB} = 0$

FIND:
(a) $\underline{a}_D$
(b) $\underline{a}_E$

VELOCITY $\quad v_B = (AB)\omega_{AB} = (0.24\ \mathrm{m})(3\ \mathrm{rad/s}) = 0.72\ \mathrm{m/s}\downarrow$



$v_D = v_B + v_{D/B} = v_B\downarrow + (BD)\omega_{BD} \leftrightarrow$

$\underline{v}_D\downarrow = 0.72\ \mathrm{m/s}\downarrow + (0.15\ \mathrm{m})\omega_{BD} \longleftrightarrow$

$\underline{\omega}_{BD} = 0$

$\underline{v}_D = 0.72\ \mathrm{m/s}\downarrow$

$v_D = (CD)\omega_{CD}$: $\quad 0.72\ \mathrm{m/s}\downarrow = (0.18\ \mathrm{m})\omega_{CD}\downarrow$

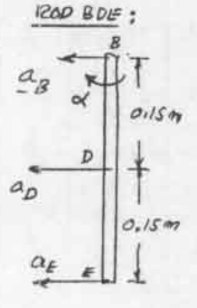
$\underline{\omega}_{CD} = 4\ \mathrm{rad/s}$

ACCELERATIONS

ROD AB: $\underline{a}_B = (AB)\omega_{AB}^2 = (0.24\ \mathrm{m})(3\ \mathrm{rad/s})^2 = 2.16\ \mathrm{m/s^2}\leftarrow$

ROD CD: $\underline{a}_D = (CD)\omega_{CD}^2 = (0.18\ \mathrm{m})(4\ \mathrm{rad/s})^2 = 2.88\ \mathrm{m/s^2}\leftarrow$ ◀

ROD BDE:



$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = a_B\leftarrow + (BD)\alpha\leftarrow$

$2.88\ \mathrm{m/s^2}\leftarrow = 2.16\ \mathrm{m/s^2}\leftarrow + (0.15\ \mathrm{m})\alpha\leftarrow$

$0.72\ \mathrm{m/s^2}\leftarrow = (0.15\ \mathrm{m})\alpha\leftarrow$

$\underline{\alpha} = 4.8\ \mathrm{rad/s^2}$

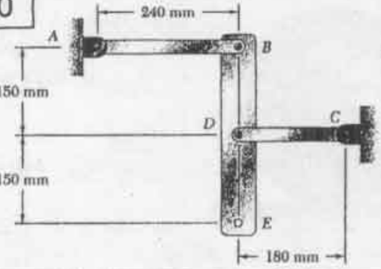
$\underline{a}_E = \underline{a}_D + \underline{a}_{E/D} = a_D\leftarrow + (DE)\alpha\leftarrow$

$\underline{a}_E = 2.88\ \mathrm{m/s^2}\leftarrow + (0.15\ \mathrm{m})(4.8\ \mathrm{rad/s^2})\leftarrow$

$\underline{a}_E = 2.88\ \mathrm{m/s^2}\leftarrow + 0.72\ \mathrm{m/s^2}\leftarrow$

$\underline{a}_E = 3.6\ \mathrm{m/s^2}\leftarrow$ ◀

15.110

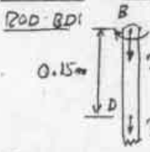


GIVEN: $\underline{\omega}_{AB} = 3\ \text{rad/s}$ ↻, $\alpha_{AB} = 0$

FIND: (a) $\underline{a}_D$ (b) a_E

VELOCITY: ROD AB $\quad v_B = (AB)\omega_{AB} = (0.24\,m)(3\ \text{rad/s}) = 0.72\ m/s$

ROD BD:



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B} = v_B\downarrow + (BD)\omega_{BD} \leftarrow$

$v_D\downarrow = 0.72\ m/s\downarrow + (BD)\omega_{BD} \rightarrow$

$\underline{\omega}_{BD} = 0$

$\underline{v}_D = 0.72\ m/s\downarrow$

ROD DC: $v_D = (DC)\omega_{DC}$; $0.72\ m/s^2\downarrow = (0.18\,m)\,\omega_{DC}$

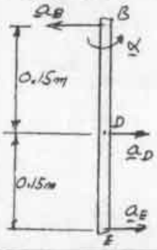
$\underline{\omega}_{DC} = 4\ \text{rad/s}$ ↺

ACCELERATION:

ROD AB: $a_B = (AB)\omega_{AB}^2 = (0.24\,m)(3\ \text{rad/s})^2 = 2.16\ m/s^2 \leftarrow$

ROD DC: $\underline{a}_D = (DC)\omega_{DC}^2 = (0.18\,m)(4\ \text{rad/s})^2 = 2.88\ m/s^2 \rightarrow$ ◀

ROD BDE:



$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = a_B \leftarrow + (BD)\alpha \rightarrow$

$2.88\ m/s^2 \rightarrow = 2.16\ m/s^2 \leftarrow + (0.15\,m)\alpha \rightarrow$

$5.04\ m/s^2 \rightarrow = (0.15\,m)\alpha \rightarrow$

$\alpha = 33.6\ \text{rad/s}^2$ ↺

$\underline{a}_E = \underline{a}_D + \underline{a}_{E/D} = a_D \rightarrow + (DE)\alpha \rightarrow$

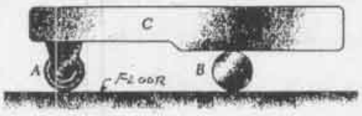
$\underline{a}_E = 2.88\ m/s^2 \rightarrow + (0.15\,m)(33.6\ \text{rad/s}^2) \rightarrow$

$\underline{a}_E = 2.88\ m/s^2 \rightarrow + 5.04\ m/s^2 \rightarrow$

$\underline{a}_E = 7.92\ m/s^2 \rightarrow$ ◀

15.112

CASTER AND CYLINDER EACH OF 50-mm DIAM.



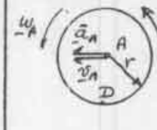
GIVEN: $\underline{v}_C = 1.5\ m/s \leftarrow$, $\underline{a}_C = 2.4\ m/s^2 \leftarrow$

FIND: (a) $\underline{\alpha}_A$ AND $\underline{\alpha}_B$ (b) $\bar{\underline{a}}_A$ AND $\bar{\underline{a}}_B$

ROLLING OCCURS AT ALL SURFACES OF CONTACT

INST. CENTERS AT POINTS OF CONTACT WITH FLOOR

CASTER: $r = 0.025\ m$



$\bar{a}_A = a_C = 2.4\ m/s^2 \leftarrow$ ◀

$(a_D)_x = 0$ (ROLLING WITH NO SLIDING)

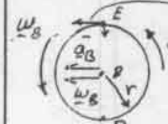
$\bar{\underline{a}}_A = \underline{a}_D + \underline{a}_{A/D}$

$\bar{a}_A \leftarrow = (a_D)_x \leftarrow + (a_D)_y\uparrow + r\alpha_A \leftarrow + r\omega_A^2\downarrow$

$\overset{+}{\leftarrow}\ \bar{a}_A = 0 + r\alpha_A$

$2.4\ m/s^2 \leftarrow = (0.025\,m)$; $\quad \underline{\alpha}_A = 96\ \text{rad/s}^2$ ↻ ◀

CYLINDER: $r = 0.025\,m$



$(a_E)_x = a_C = 2.4\ m/s^2 \leftarrow$

$(a_D)_x = 0$

$\underline{a}_E = \underline{a}_D + \underline{a}_{E/D}$

$(a_E)_x \leftarrow + (a_E)_y\downarrow = (a_D)_x \leftarrow + (a_D)_y\downarrow + 2r\alpha_B \leftarrow + 2r\omega_B^2\downarrow$

$\overset{+}{\leftarrow}\ (a_E)_x = (a_D)_x + 2r\alpha_B$

$2.4\ m/s^2 \leftarrow = 0 + 2(0.025\,m)\alpha_B$

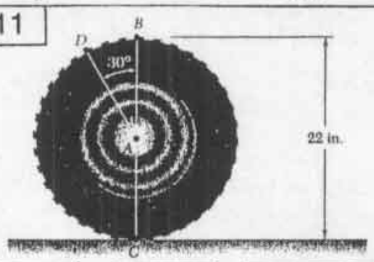
$\underline{\alpha}_B = 48\ \text{rad/s}^2$ ↻ ◀

$\bar{a}_B \leftarrow = (a_D)_x \leftarrow + (a_D)_y\uparrow + r\alpha \leftarrow + r\omega^2\downarrow$

$\overset{+}{\leftarrow}\ a_B = 0 + r\alpha_B$

$\bar{a}_B = (0.025\,m)(48\ \text{rad/s}^2)$; $\quad \bar{\underline{a}}_B = 1.2\ m/s^2 \leftarrow$ ◀

15.111



GIVEN: $\underline{v}_A = 48\ mi/h \rightarrow$, $\underline{a}_A = 0$

FIND: (a) $\underline{a}_B$ (b) $\underline{a}_C$ (c) $\underline{a}_D$

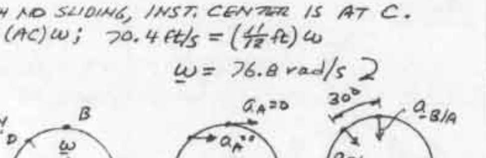
$\underline{v}_A = 48\ \frac{mi}{h} \cdot \frac{h}{3600\,s} \cdot \frac{5280\,ft}{mi} = 70.4\ ft/s \rightarrow$

ROLLING WITH NO SLIDING, INST. CENTER IS AT C.

$\therefore\ v_A = (AC)\omega$; $\quad 70.4\ ft/s = (\frac{11}{12}\,ft)\,\omega$

$\underline{\omega} = 76.8\ \text{rad/s}$ ↻

ACCELERATION



PLANE MOTION = TRANS. WITH A + ROTATION ABOUT A

$\underline{a}_{B/A} = \underline{a}_{C/A} = \underline{a}_{D/A} = r\omega^2 = (\frac{11}{12}\,ft)(76.8\ \text{rad/s})^2 = 5407\ ft/s^2$

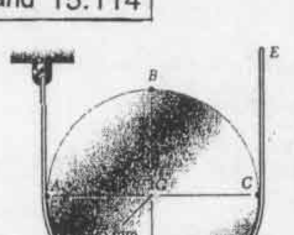
(a) $\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = 0 + 5407\ ft/s^2\downarrow \qquad \underline{a}_B = 5410\ ft/s^2\downarrow$ ◀

(b) $\underline{a}_C = \underline{a}_A + \underline{a}_{C/A} = 0 + 5407\ ft/s^2\uparrow \qquad \underline{a}_C = 5410\ ft/s^2\uparrow$ ◀

(c) $\underline{a}_D = \underline{a}_A + \underline{a}_{D/A} = 0 + 5407\ ft/s^2 \measuredangle 30°$

$\underline{a}_D = 5410\ ft/s^2 \measuredangle 60°$ ◀

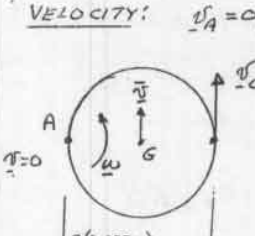
15.113 and 15.114



GIVEN: $\underline{v}_E = 300\ mm/s\uparrow$, $\underline{a}_E = 480\ mm/s^2\uparrow$

FIND: (a) $\underline{a}_A$ (b) $\underline{a}_B$

VELOCITY: $v_A = 0$, THUS INST. CENTER IS AT A.



$\underline{v}_C = \underline{v}_E = 0.3\ m/s\uparrow$

$v_C = (0.15\,m)\,\omega$

$0.3\ m/s = (0.15\,m)\,\omega$

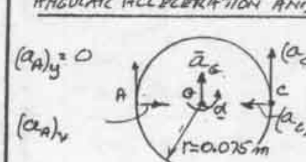
$\underline{\omega} = 2\ \text{rad/s}$ ↻

(CONTINUED)

15.113 and 15.114 CONTINUED

$\underline{\omega} = 2\,\text{rad/s}\curvearrowleft$

ANGULAR ACCELERATION AND $\bar{a}_G$ $\quad (a_A)_y = 0$; a



$$\underline{a}_C = \underline{a}_A + \underline{a}_{C/A}$$

$$(a_C)_x \leftarrow + (a_C)_y \uparrow = (a_A)_x \rightarrow + (a_C)_y \uparrow + 2r\alpha \uparrow$$

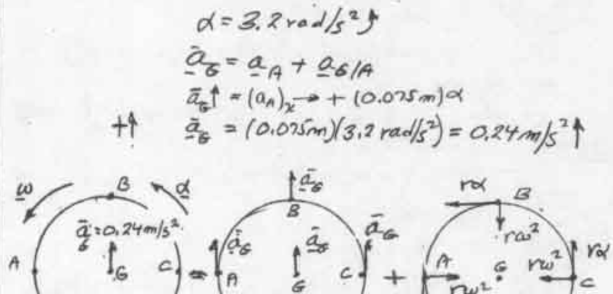
$$+\uparrow \quad 0 = 0.48\,\text{m/s}^2 + 2(0.075\,\text{m})\alpha$$

$$\alpha = 3.2\,\text{rad/s}^2 \curvearrowright$$

$$\bar{\underline{a}}_G = \underline{a}_A + \underline{a}_{G/A}$$

$$\bar{a}_G \uparrow = (a_A)_x \rightarrow + (0.075\,\text{m})\alpha$$

$$+\uparrow \quad \bar{a}_G = (0.075\,\text{m})(3.2\,\text{rad/s}^2) = 0.24\,\text{m/s}^2 \uparrow$$



PLANE MOTION = TRANS. WITH G + ROTATION ABOUT G

$$r\alpha = (0.075\,\text{m})(3.2\,\text{rad/s}^2) = 0.24\,\text{m/s}^2$$

$$r\omega^2 = (0.075\,\text{m})(2\,\text{rad/s})^2 = 0.3\,\text{m/s}^2$$

FOR EACH POINT:

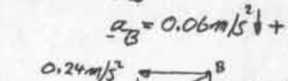
$$\underline{a} = \bar{\underline{a}}_G + r\alpha + r\omega^2$$

POINT A: $\underline{a}_A = 0.24\,\text{m/s}^2 \uparrow + 0.24\,\text{m/s}^2 \downarrow + 0.3\,\text{m/s}^2 \rightarrow$

$$\underline{a}_A = 0.3\,\text{m/s}^2 \rightarrow \quad \triangleleft$$

POINT B: $\underline{a}_B = 0.24\,\text{m/s}^2 \uparrow + 0.24\,\text{m/s}^2 \leftarrow + 0.3\,\text{m/s}^2 \downarrow$

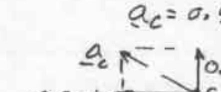
$$\underline{a}_B = 0.06\,\text{m/s}^2 \downarrow + 0.24\,\text{m/s}^2 \leftarrow$$



$\underline{a}_B = 0.247\,\text{m/s}^2 \measuredangle 14.0^\circ \quad \triangleleft$

POINT C: $\underline{a}_C = 0.24\,\text{m/s}^2 \uparrow + 0.24\,\text{m/s}^2 \uparrow + 0.3\,\text{m/s}^2 \leftarrow$

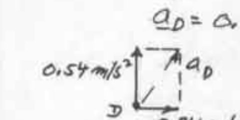
$$\underline{a}_C = 0.48\,\text{m/s}^2 \uparrow + 0.3\,\text{m/s}^2 \leftarrow$$



$\underline{a}_C = 0.566\,\text{m/s}^2 \measuredangle 58.0^\circ \quad \triangleleft$

POINT D: $\underline{a}_D = 0.24\,\text{m/s}^2 \uparrow + 0.24\,\text{m/s}^2 \rightarrow + 0.3\,\text{m/s}^2 \uparrow$

$$\underline{a}_D = 0.54\,\text{m/s}^2 \uparrow + 0.24\,\text{m/s}^2 \rightarrow$$



$a_D = 0.591\,\text{m/s}^2 \measuredangle 66^\circ \quad \triangleleft$

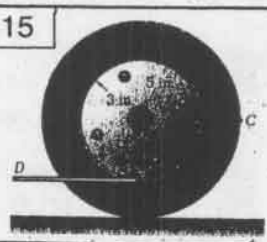
PROBLEM 15.113 $\quad \underline{a}_A = 300\,\text{mm/s}^2 \rightarrow$ ◀

$\underline{a}_B = 247\,\text{mm/s}^2 \measuredangle 14.0^\circ$ ◀

PROBLEM 15.114 $\quad \underline{a}_C = 566\,\text{mm/s}^2 \measuredangle 58.0^\circ$ ◀

$\underline{a}_D = 591\,\text{mm/s}^2 \measuredangle 66.0^\circ$ ◀

15.115



GIVEN:
$\underline{v}_D = 8\,\text{in./s} \leftarrow$
$\underline{a}_D = 30\,\text{in./s}^2 \leftarrow$

FIND:
$\underline{a}_A$, $\underline{a}_B$, AND $\underline{a}_C$

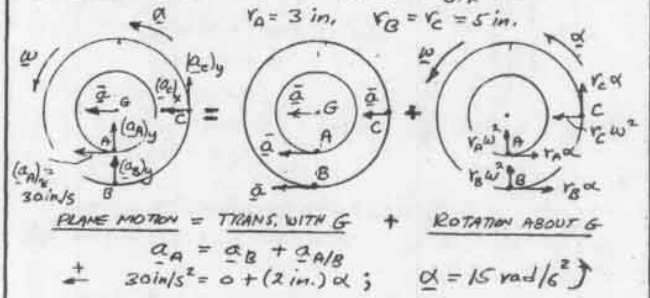
VELOCITY: INST. CENTER AT B.

$$v_D = (2\,\text{in.})\omega \,; \quad 8\,\text{in./s} = (2\,\text{in.})\omega$$

$$\underline{\omega} = 4\,\text{rad/s} \curvearrowright$$

ACCELERATION: FOR NO SLIDING: $(a_B)_x = 0$

$$r_A = 3\,\text{in.} \quad r_B = r_C = 5\,\text{in.}$$



PLANE MOTION = TRANS. WITH G + ROTATION ABOUT G

$$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$$

$$\overset{+}{\leftarrow} \quad 30\,\text{in/s}^2 = 0 + (2\,\text{in.})\alpha \,; \quad \underline{\alpha} = 15\,\text{rad/s}^2 \curvearrowright$$

$$\bar{\underline{a}}_G = \underline{a}_B + \underline{a}_{G/B}$$

$$\overset{+}{\leftarrow} \quad \bar{\underline{a}} = 0 + (5\,\text{in.})(15\,\text{rad/s}^2); \quad \bar{\underline{a}} = 75\,\text{in/s}^2 \leftarrow$$

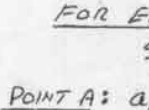
FOR EACH POINT:

$$\underline{a} = \bar{\underline{a}}_G + r\alpha + r\omega^2$$

POINT A: $\underline{a}_A = 75\,\text{in/s}^2 \leftarrow + (3\,\text{in.})(15\,\text{rad/s}^2) \rightarrow + (3\,\text{in.})(4\,\text{rad/s})^2 \uparrow$

$$= 75\,\text{in./s}^2 \leftarrow + 45\,\text{in./s}^2 \rightarrow + 48\,\text{in./s}^2 \uparrow$$

$$\underline{a}_A = 30\,\text{in./s}^2 \leftarrow + 48\,\text{in./s}^2 \uparrow$$



$\underline{a}_A = 56.6\,\text{in./s}^2 \measuredangle 58.0^\circ$ ◀

POINT B: $\underline{a}_B = 75\,\text{in/s}^2 \leftarrow + (5\,\text{in.})(15\,\text{rad/s}^2) \rightarrow + (5\,\text{in.})(4\,\text{rad/s})^2 \uparrow$

$$= 75\,\text{in./s}^2 \leftarrow + 75\,\text{in./s}^2 \rightarrow + 80\,\text{in/s}^2 \uparrow$$

$$\underline{a}_B = 80\,\text{in./s}^2$$

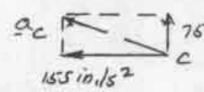
$$\underline{a}_B = 80\,\text{in./s}^2 \uparrow$$ ◀

POINT C:

$$\underline{a}_C = 75\,\text{in/s}^2 \leftarrow + (5\,\text{in.})(15\,\text{rad/s}^2) \uparrow + (5\,\text{in.})(4\,\text{rad/s})^2 \leftarrow$$

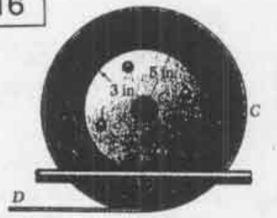
$$= 75\,\text{in./s}^2 \leftarrow + 75\,\text{in./s}^2 \uparrow + 80\,\text{in./s}^2 \leftarrow$$

$$\underline{a}_C = 155\,\text{in./s}^2 \leftarrow + 75\,\text{in./s}^2 \uparrow$$



$\underline{a}_C = 172.2\,\text{in./s}^2 \measuredangle 25.8^\circ$ ◀

15.116



GIVEN:
$\underline{v}_D = 8\,in./s \leftarrow$
$\underline{a}_D = 30\,in./s^2 \leftarrow$

FIND:
$\underline{a}_A$, $\underline{a}_B$ AND $\underline{a}_C$

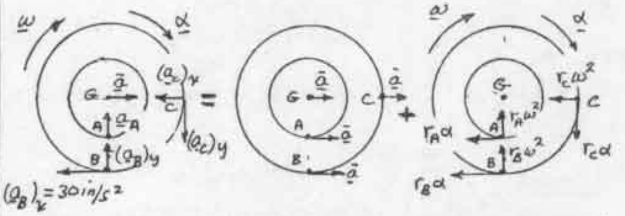
VELOCITY: INST. CENTER AT A

$v_D = (2\,in.)\omega$; $8\,in./s = (2\,in.)\omega$

$\underline{\omega} = 4\,rad/s$ ↻

ACCELERATION: FOR NO SLIDING: $(a_A)_x = 0$

$r_A = 3\,in.$ $r_B = r_C = 5\,in.$



PLANE MOTION = TRANS. WITH G + ROTATION ABOUT G

$$\underline{a}_B = \underline{a}_A + a_{B/A}$$

$\xleftarrow{+}$ $30\,in./s^2 \leftarrow = 0 + (2\,in.)\alpha$; $\underline{\alpha} = 15\,rad/s^2$ ↻

$$\bar{\underline{a}}_G = \underline{a}_A + \underline{a}_{G/A}$$

$\xleftarrow{+}$ $\bar{\underline{a}} = 0 + (3\,in.)(15\,rad/s^2)$; $\bar{\underline{a}} = 45\,in/s^2 \rightarrow$

FOR EACH POINT

$$a = \bar{a}_G + r\alpha + r\omega^2$$

POINT A: $\underline{a}_A = 45\,in./s^2 \rightarrow + (3\,in.)(15\,rad/s^2) \leftarrow + (3\,in.)(4\,rad/s)^2 \uparrow$

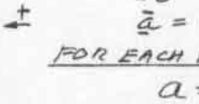
$= 45\,in/s^2 \rightarrow + 45\,in./s^2 \leftarrow + 48\,in./s^2 \uparrow$

$\underline{a}_A = 48\,in./s^2 \uparrow$ ◀

POINT B: $\underline{a}_B = 45\,in./s^2 \rightarrow + (5\,in.)(15\,rad/s^2) \leftarrow + (5\,in.)(4\,rad/s)^2 \uparrow$

$= 45\,in/s^2 \rightarrow + 75\,in./s^2 \leftarrow + 80\,in./s^2 \uparrow$

$\underline{a}_B = 30\,in./s^2 \leftarrow + 80\,in./s^2 \uparrow$

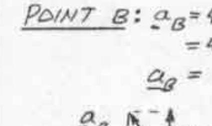


$\underline{a}_B = 85.4\,in/s^2 \measuredangle 69.4°$ ◀

POINT C: $\underline{a}_C = 45\,in/s^2 \rightarrow + (5\,in.)(15\,rad/s^2) \downarrow + (5\,in.)(4\,rad/s)^2 \leftarrow$

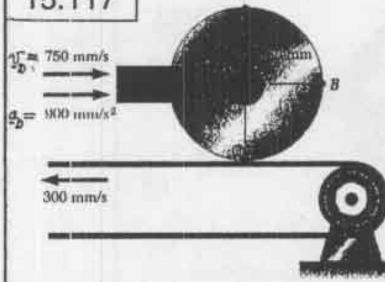
$= 45\,in/s^2 \rightarrow + 75\,in/s^2 \downarrow + 80\,in/s^2 \leftarrow$

$= 35\,in./s^2 \leftarrow + 75\,in./s^2 \downarrow$



$\underline{a}_C = 82.8\,in./s^2 \measuredangle 65.0°$ ◀

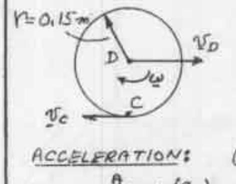
15.117



GIVEN:
$\underline{v}_{BELT} = 300\,mm/s \leftarrow$
$\underline{a}_{BELT} = 0$

FIND:
$\underline{a}_A$, $\underline{a}_B$, AND $\underline{a}_C$

VELOCITY: $v_C = v_{BELT} = 0.3\,m/s \leftarrow$; $\underline{v}_D = 0.75\,m/s \rightarrow$



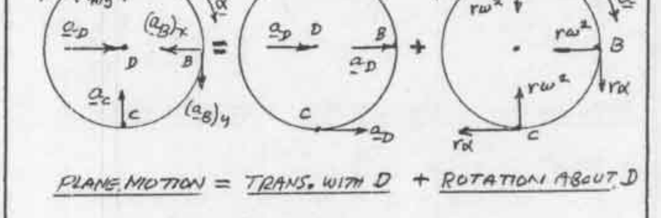
$v_D = v_C + v_{D/C} = v_C + r\omega$

$0.75\,m/s \rightarrow = 0.3\,m/s \leftarrow + r\omega \rightarrow$

$1.05\,m/s \rightarrow = (0.15\,m)\,\omega$

$\underline{\omega} = 7\,rad/s$ ↻

ACCELERATION: $(a_C)_x = a_{BELT} = 0$



PLANE MOTION = TRANS. WITH D + ROTATION ABOUT D

$$\underline{a}_D = \underline{a}_C + \underline{a}_{D/C}$$
$$= \underline{a}_C + r\alpha$$

$0.9\,m/s^2 \rightarrow = a_C \uparrow + (0.15\,m)\alpha \rightarrow$

$\underline{\alpha} = 6\,rad/s^2$ ↻

$r\alpha = (0.15\,m)(6\,rad/s^2) = 0.9\,m/s^2$

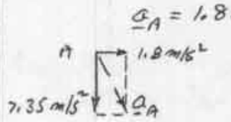
$r\omega^2 = (0.15\,m)(7\,rad/s)^2 = 7.35\,m/s^2$

FOR EACH POINT

$$\underline{a} = \underline{a}_D + r\alpha + r\omega^2$$

POINT A: $\underline{a}_A = 0.9\,m/s^2 \rightarrow + 0.9\,m/s^2 \rightarrow + 7.35\,m/s^2 \downarrow$

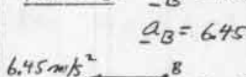
$\underline{a}_A = 1.8\,m/s^2 \rightarrow + 7.35\,m/s^2 \downarrow$



$\underline{a}_A = 7.57\,m/s^2 \measuredangle 76.2°$ ◀

POINT B: $\underline{a}_B = 0.9\,m/s^2 \rightarrow + 0.9\,m/s^2 \downarrow + 7.35\,m/s^2 \leftarrow$

$\underline{a}_B = 6.45\,m/s^2 \leftarrow + 0.9\,m/s^2 \downarrow$

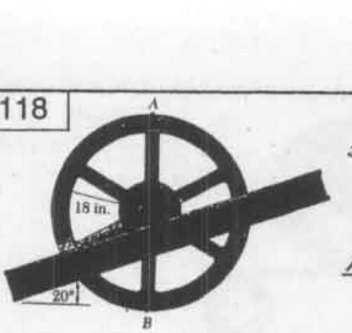


$\underline{a}_B = 6.51\,m/s^2 \measuredangle 7.9°$ ◀

POINT C: $\underline{a}_C = 0.9\,m/s^2 \rightarrow + 0.9\,m/s^2 \leftarrow + 7.35\,m/s^2 \uparrow$

$\underline{a}_C = 7.35\,m/s^2 \uparrow$ ◀

15.118



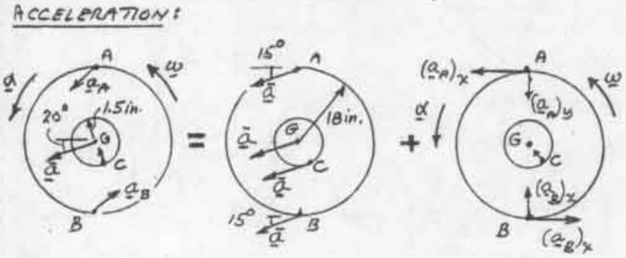
GIVEN:
SHAFT: $r = 1.5$ in.
$\bar{v} = 1.2$ in./s ↙
$\bar{a} = 0.5$ in./s² ↙

FIND: (a) a_A
(b) a_B

VELOCITY: SHAFT, ROLLING, NO SLIDING
INST. CENTER AT C

$\bar{v} = r\omega$
1.2 in./s $= (1.5$ in.$)\,\omega$
$\omega = 0.8$ rad/s ↻

ACCELERATION:



PLANE MOTION = TRANS. WITH G + ROTATION ABOUT G

$+\nearrow 15°$: $a_G = a_C + a_{G/C}$
$\bar{a} = 0 + r\alpha$
0.5 in./s² $= 0 + (1.5$ in.$)\alpha$; $\alpha = \frac{1}{3}$ rad/s² ↻

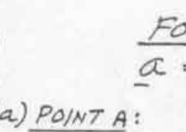
FOR EACH POINT $r_A = r_B = 18$ in.

$$a = \bar{a} + r\alpha + r\omega^2$$

(a) POINT A:

$$a_A = (0.5 \text{ in./s}^2)\nearrow 20° + (18 \text{ in.})(\tfrac{1}{3}\text{ rad/s}^2) \leftarrow + (18\text{ in.})(0.8\text{ rad/s})^2 \downarrow$$
$$= 0.470 \text{ in./s}^2 \leftarrow + 0.171 \text{ in./s}^2 \downarrow + 6 \text{ in./s}^2 \leftarrow + 11.52 \text{ in./s}^2 \downarrow$$

$$a_A = 6.47 \text{ in./s}^2 \leftarrow + 11.69 \text{ in./s}^2 \downarrow$$

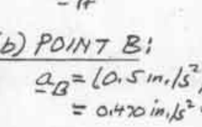


$a_A = 13.36$ in./s² $\measuredangle 61.0°$ ◀

(b) POINT B:

$$a_B = (0.5\text{ in./s}^2)\nearrow 20° + (18\text{ in.})(\tfrac{1}{3}\text{ rad/s}^2) \rightarrow + (18\text{ in.})(0.8\text{ rad/s})^2 \uparrow$$
$$= 0.470 \text{ in./s}^2 \leftarrow + 0.171 \text{ in./s}^2 \downarrow + 6 \text{ in./s}^2 \rightarrow + 11.52 \text{ in./s}^2 \uparrow$$

$$a_B = 5.53 \text{ in./s}^2 \rightarrow + 11.35 \text{ in./s}^2 \uparrow$$



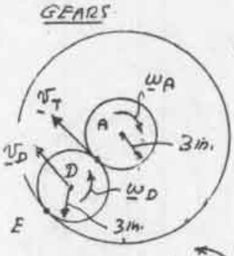
$a_B = 12.62$ in./s² $\measuredangle 64.0°$ ◀

15.119

GIVEN:
$r_A = r_B = r_C = 3$ in.; $r_E = 9$ in.
$\omega_A = 150$ rpm ↻, $\alpha_A = 0$
$\omega_E = 0$
FIND: MAGNITUDE OF ACCELERATION OF TOOTH OF GEAR D IN CONTACT WITH (a) GEAR A, (b) GEAR E.

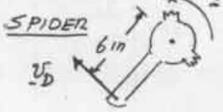
VELOCITY: T = TOOTH OF GEAR D IN CONTACT WITH GEAR A

GEARS



$v_T = r\omega_A = (3$ in.$)\,\omega_A$
SINCE $v_E = 0$, E IS INST. CENTER OF GEAR D
$v_T = 2r\,\omega_D$
$(3 \text{ in.})\omega_A = 2(3 \text{ in.})\omega_D$
$\omega_D = \frac{1}{2}\omega_A$
$v_D = r\omega_D = (3 \text{ in.})\frac{1}{2}\omega_A = (1.5 \text{ in.})\omega_A$

SPIDER

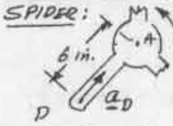


$v_D = (6 \text{ in.})\,\omega_S$
$(1.5 \text{ in.})\omega_A = (6 \text{ in.})\omega_S$
$\omega_S = \frac{1}{4}\omega_A$

$\omega_A = 150$ rpm $= 15.708$ rad/s ↻
$\omega_D = \frac{1}{2}\omega_A = 7.854$ rad/s ↻
$\omega_S = \frac{1}{4}\omega_A = 3.927$ rad/s ↻

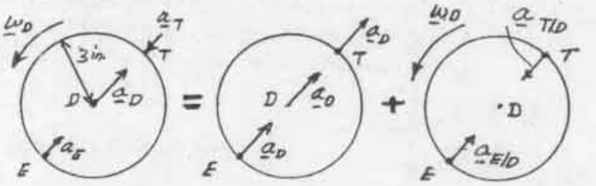
ACCELERATION

SPIDER:



$a_D = (AD)\omega_S^2 = (6 \text{ in.})(3.927 \text{ rad/s})^2$
$a_D = 92.53$ in./s² ↗

GEAR D:



PLANE MOTION = TRANS. WITH D + ROTATION ABOUT D

(a) TOOTH T IN CONTACT WITH GEAR A

$$a_T = a_D + a_{T/D} = a_D + (DT)\omega_D^2$$
$$= 92.53 \text{ in./s}^2 \nearrow + (3 \text{ in.})(7.854 \text{ rad/s})^2 \swarrow$$
$$= 92.53 \text{ in./s}^2 \nearrow + 185.06 \text{ in./s}^2 \swarrow$$
$$a_T = 92.53 \text{ in./s}^2 \swarrow$$

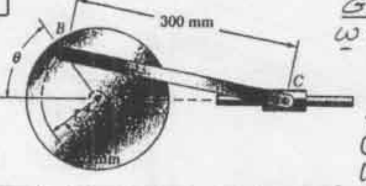
$a_T = 92.5$ in./s² ◀

(b) TOOTH E IN CONTACT WITH GEAR E

$$a_E = a_D + a_{E/D} = a_D + (ED)\omega_D^2$$
$$= 92.53 \text{ in./s}^2 \nearrow + (3 \text{ in.})(7.854 \text{ rad/s})^2 \nearrow$$
$$= 92.53 \text{ in./s}^2 \nearrow + 185.06 \text{ in./s}^2 \nearrow$$
$$a_E = 277.6 \text{ in./s}^2 \nearrow$$

$a_E = 278$ in./s² ◀

15.120



GIVEN: $\omega = 360$ rpm ↻, $\alpha = 0$

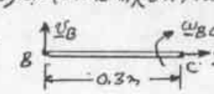
FIND: a_C WHEN, (a) $\theta = 0$ (b) $\theta = 180°$

DISK: $\omega = 360$ rpm $= 37.7$ rad/s ↻

$v_B = (AB)\omega = (0.075\,m)(37.7\,rad/s)$ ↑ $\quad v_B = 2.8275$ m/s

$a_B = (AB)\omega^2 = (0.075\,m)(37.7\,rad/s)^2$; $\quad a_B = 106.59$ m/s²

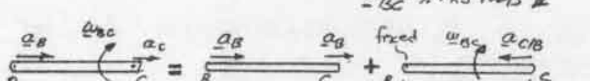
(a) $\theta = 0$:



$v_B = (BC)\omega_{BC}$

$2.8275\,m/s = (0.3\,m)\omega_{BC}$

$\omega_{BC} = 9.425$ rad/s ↻



PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

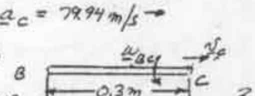
$a_C = a_B + a_{C/B} = a_B + (BC)\omega_{BC}^2$

$= 106.59\,m/s^2 \rightarrow + (0.3\,m)(9.425\,rad/s)^2 \leftarrow$

$= 106.59\,m/s^2 \rightarrow + 26.65\,m/s^2 \leftarrow$

$a_C = 79.94$ m/s → $\qquad a_C = 79.9$ m/s² → ◀

(b) $\theta = 180°$:



$v_B = (BC)\omega_{BC}$

$2.8275\,m/s = (0.3\,m)\omega_{BC}$

$\omega_{BC} = 9.425$ rad/s ↺

PLANE MOTION = TRANS WITH B + ROTATION ABOUT B

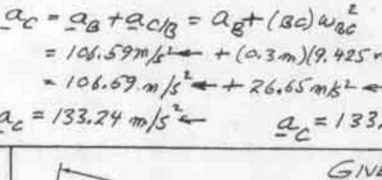
$a_C = a_B + a_{C/B} = a_B + (BC)\omega_{BC}^2$

$= 106.59\,m/s^2 \leftarrow + (0.3\,m)(9.425\,rad/s)^2 \leftarrow$

$= 106.59\,m/s^2 \leftarrow + 26.65\,m/s^2 \leftarrow$

$a_C = 133.24$ m/s² ← $\qquad a_C = 133.2$ m/s² ← ◀

15.121



GIVEN: $\omega = 360$ rpm ↻, $\alpha = 0$, $\theta = 90°$

FIND: a_C

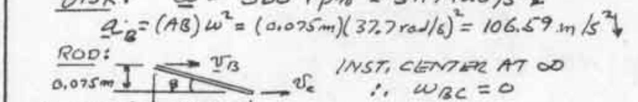
DISK: $\omega = 360$ rpm $= 37.7$ rad/s ↻

$a_B = (AB)\omega^2 = (0.075\,m)(37.7\,rad/s)^2 = 106.59$ m/s² ↓

ROD:

$\beta = 14.477°$

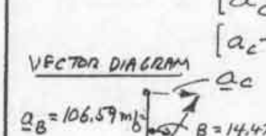
INST. CENTER AT ∞ ∴ $\omega_{BC} = 0$



$[a_C \rightarrow] = [a_B \downarrow] + [a_{C/B} \measuredangle \beta]$

$[a_C \rightarrow] = [106.59\,m/s^2 \downarrow] + [a_{C/B} \measuredangle 14.477°]$

VECTOR DIAGRAM

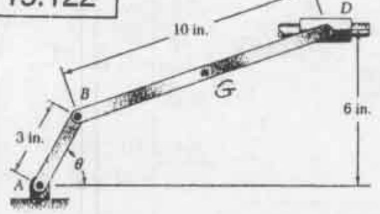


$a_C = (106.59\,m/s^2)\tan 14.477°$

$a_C = 27.52$ m/s²

$a_C = 27.5$ m/s² → ◀

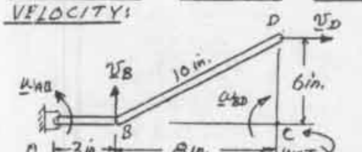
15.122



GIVEN: $\theta = 0$, $\omega_{AB} = 16$ rad/s ↺, $\alpha_{AB} = 0$

FIND: (a) a_D (b) a_G

VELOCITY:



$v_B = (AB)\omega_{AB} = (3\,in.)(16\,rad/s)$

$v_B = 48$ in./s ↑

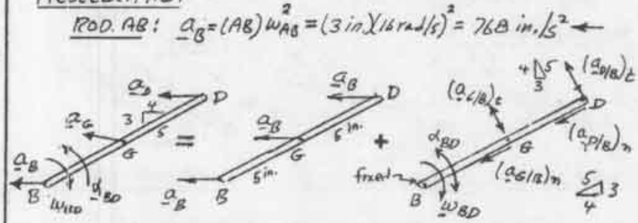
$v_B = (BC)\omega_{BD}$

$48\,in./s = (8\,in.)\omega_{BD}$

$\omega_{BD} = 6$ rad/s ↻

ACCELERATION

ROD AB: $a_B = (AB)\omega_{AB}^2 = (3\,in.)(16\,rad/s)^2 = 768\,in./s^2 \leftarrow$



PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

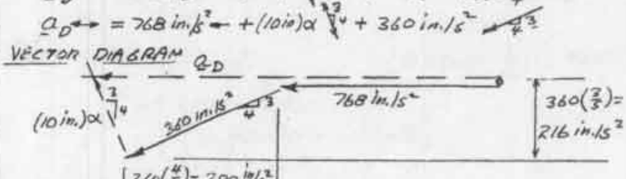
(a) $a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$

$a_D \leftrightarrow = a_B \leftarrow + (BD)\alpha \measuredangle^3_4 + (BD)\omega^2 \measuredangle^3_4$

$a_D \leftrightarrow = 768\,in/s^2 \leftarrow + (10\,in)\alpha \measuredangle^3_4 + (10\,in.)(6\,rad/s)^2 \measuredangle^3_4$

$a_D \leftrightarrow = 768\,in./s^2 \leftarrow + (10\,in)\alpha \measuredangle^3_4 + 360\,in./s^2 \measuredangle^3_4$

VECTOR DIAGRAM



$(10\,in)\alpha\frac{4}{5} = 216\,in/s^2$; $\alpha = 27$ rad/s² ↺

$a_D = 768 + 288 + \frac{3}{5}(10\,in.)\alpha$

$= 768 + 288 + \frac{3}{5}(10)(27) = 768 + 288 + 162 = 1218$ in/s²

$a_D = 1218$ in./s² ← ◀

(b) $a_G = a_B + a_{G/B} = a_B + (a_{G/B})_t + (a_{G/B})_n$

$a_G = a_B \leftarrow + (BG)\alpha \measuredangle^3_4 + (BG)\omega^2 \measuredangle^3_4$

$a_G = 768\,in./s^2 \leftarrow + (5\,in.)(27\,rad/s) \measuredangle^3_4 + (5\,in.)(6\,rad/s)^2 \measuredangle^3_4$

$a_G = 768\,in./s^2 \leftarrow + 135\,in/s^2 \measuredangle^3_4 + 180\,in./s^2 \measuredangle^3_4$

+↑ COMPONENTS: $(a_G)_y = 135(\frac{4}{5}) - 180(\frac{3}{5})$

$= 108\,in/s^2 - 108\,in./s^2$

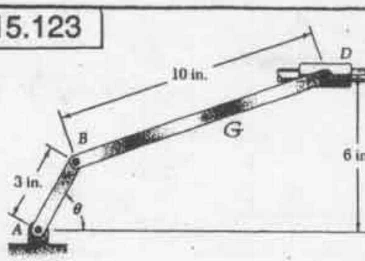
$(a_G)_y = 0$

$\stackrel{+}{\leftarrow}$ COMPONENTS: $(a_G)_x = 768 + 135(\frac{3}{5}) + 180(\frac{4}{5})$

$= 768 + 81 + 144 = 993$ in./s²

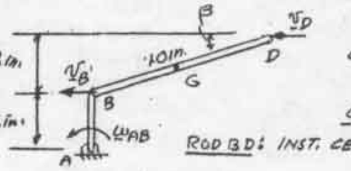
$a_G = 993$ in./s² ← ◀

15.123



GIVEN:
$\theta = 90°$
$\underline{\omega}_{AB} = 16 \text{ rad/s} \circlearrowright$
$\underline{\alpha}_{AB} = 0$

FIND: (a) $\underline{a}_D$
(b) $\underline{a}_G$

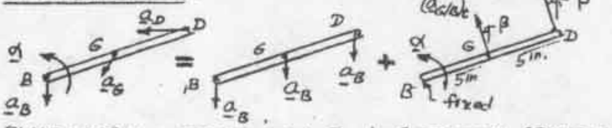


ROD AB:
$a_B = (AB)\omega_{AB}^2$
$= (3 \text{ in.})(16 \text{ rad/s})^2$
$\underline{a}_B = 768 \text{ in./s}^2 \downarrow$

ROD BD: INST. CENTER IS AT ∞; $\underline{\omega}_{BD} = 0$

$\sin\beta = (3 \text{ in.})/(10 \text{ in.}) = 0.3$; $\beta = 17.46°$

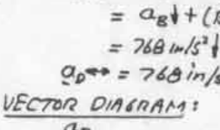
ACCELERATION:



PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

(a) $\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = \underline{a}_B + (\underline{a}_{D/B})_t + (\underline{a}_{D/B})_n$
$= a_B \downarrow + (BD)\alpha \;\nwarrow\beta + (BD)\omega_{BD}^2 \swarrow\beta$
$= 768 \text{ in/s}^2 \downarrow + (10 \text{ in})\alpha \;\nwarrow\beta + (10 \text{ in.})(0)^2 \swarrow\beta$
$\underline{a}_D \leftrightarrow = 768 \text{ in/s} \downarrow + (10 \text{ in})\alpha \;\nwarrow\beta$

VECTOR DIAGRAM:



$\underline{a}_D = (768 \text{ in/s}^2)\tan 17.46°$
$= 241.52 \text{ in./s}^2$

$\underline{a}_D = 242 \text{ in./s}^2 \leftarrow$ ◀

$(10 \text{ in.})\alpha = (768 \text{ in./s}^2)/\cos 17.46°$
$(10 \text{ in.})\alpha = 805.08 \text{ in./s}^2$
$\underline{\alpha} = 80.5 \text{ rad/s}^2 \circlearrowright$

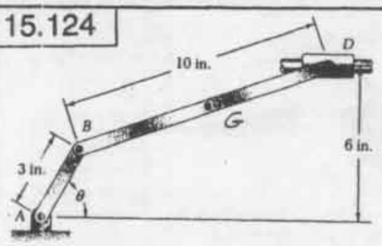
(b) $\underline{a}_G = \underline{a}_B + \underline{a}_{G/B} = \underline{a}_B + (\underline{a}_{G/B})_t + (\underline{a}_{G/B})_n$
$= \underline{a}_B \downarrow + (BG)\alpha \;\nwarrow\beta + (BG)\omega_{BD}^2 \swarrow\beta$
$= 768 \text{ in/s}^2 \downarrow + (5 \text{ in})(80.5 \text{ rad/s}^2) \;\nwarrow\beta + (BG)(0)^2$
$\underline{a}_G = 768 \text{ in/s}^2 \downarrow + 402.5 \text{ in/s}^2 \;\nwarrow 17.46°$

$\overset{+}{\leftarrow}$ COMPONENTS: $(a_G)_x = (402.5 \text{ in/s}^2)\sin 17.46°$
$(a_G)_x = 120.77 \text{ in./s}^2 \leftarrow$

$+\downarrow$ COMPONENTS: $(a_G)_y = 768 \text{ in/s}^2 - (402.5 \text{ in/s}^2)\cos 17.46°$
$= 768 \text{ in/s}^2 - 384 \text{ in/s}^2$
$(a_G)_y = 384 \text{ in/s}^2 \downarrow$

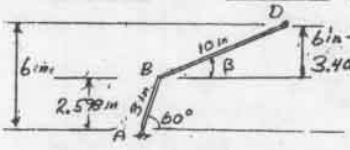
$\underline{a}_G = 403 \text{ in/s}^2 \measuredangle 72.5°$ ◀

15.124



GIVEN:
$\theta = 60°$
$\underline{\omega}_{AB} = 16 \text{ rad/s} \circlearrowright$
$\underline{\alpha}_{AB} = 0$

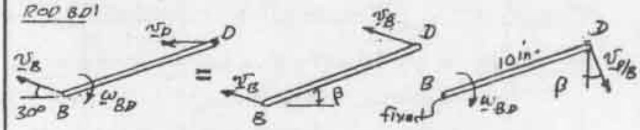
FIND: $\underline{a}_D$



$\beta = \sin^{-1}\dfrac{3.402 \text{ in}}{10 \text{ in}}$
$\beta = 19.89°$

VELOCITY: $\underline{v}_B = (AB)\omega_{AB} = (3 \text{ in.})(16 \text{ rad/s}) = 48 \text{ in./s} \measuredangle 30°$

ROD BD:

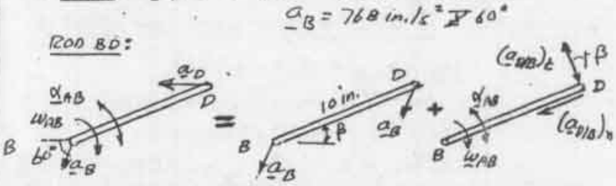


PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B
$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B} = \underline{v}_B + (BD)\omega_{BD} \measuredangle \beta$
$\underline{v}_D \leftrightarrow = 48 \text{ in./s} \measuredangle 30° + (10 \text{ in.})\omega_{BD} \measuredangle 19.89°$

$+\uparrow$ COMPONENTS: $(48 \text{ in./s})\sin 30° - (10 \text{ in.})\omega_{BD}\cos 19.89°$
$\underline{\omega}_{BD} = \dfrac{(48 \text{ in./s})\sin 30°}{(10 \text{ in.})\cos 19.890} = 2.552 \text{ rad/s} \circlearrowright$

ACCELERATION:
ROD AB: $\underline{a}_B = (AB)\omega_{AB}^2 \measuredangle 60° = (3 \text{ in.})(16 \text{ rad/s})^2 \measuredangle 60°$
$\underline{a}_B = 768 \text{ in./s}^2 \measuredangle 60°$

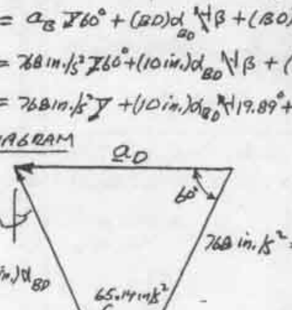
ROD BD:



PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

$\underline{a}_D = \underline{a}_B + \underline{a}_{B/D} = \underline{a}_B + (\underline{a}_{D/B})_t + (\underline{a}_{D/B})_n$
$a_D \leftrightarrow = a_B \measuredangle 60° + (BD)\alpha_{BD} \;\nwarrow\beta + (BD)\omega_{BD}^2 \measuredangle \beta$
$= 768 \text{ in./s}^2 \measuredangle 60° + (10 \text{ in.})\alpha_{BD} \;\nwarrow\beta + (10 \text{ in.})(2.552 \text{ rad/s})^2 \measuredangle \beta$
$a_D \leftrightarrow = 768 \text{ in./s}^2 \measuredangle + (10 \text{ in.})\alpha_{BD} \;\nwarrow 19.89° + 65.14 \text{ in./s}^2 \measuredangle 19.89°$

VECTOR DIAGRAM

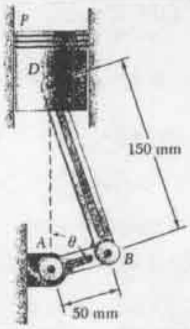


y COMPONENTS
$+\downarrow$ $768\sin 60° + 65.14\sin 19.89° - 10\alpha_{BD}\cos 19.89° = 0$
$\alpha_{BD} = 73.09 \text{ rad/s}^2$

x COMPONENTS:
$\overset{+}{\leftarrow}$ $a_D = 768\cos 60° + 65.14\cos 19.89° + (10)(73.09)\sin 19.89°$
$a_D = 693.9 \text{ in./s}^2$

$\underline{a}_D = 694 \text{ in./s}^2 \leftarrow$ ◀

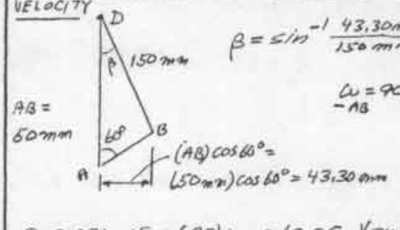
15.125



GIVEN:
$\theta = 60°$
$\underline{\omega}_{AB} = 900$ rpm ↻
$\underline{\alpha}_{AB} = 0$

FIND:
$\underline{a}_P$

VELOCITY

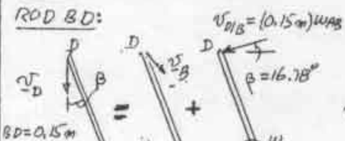


$\beta = \sin^{-1}\frac{43.30\text{ mm}}{150\text{ mm}} = 16.78°$

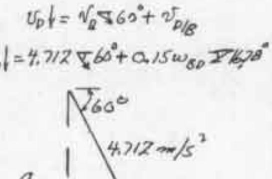
$\underline{\omega}_{AB} = 900$ rpm $= 94.248$ rad/s ↻

ROD AB: $\underline{v}_B = (AB)\omega_{AB} = (0.05\text{ m})(94.248\text{ rad/s}) = 4.712$ m/s $\measuredangle 60°$

ROD BD:



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$
$v_D\downarrow = v_B \measuredangle 60° + v_{D/B}$
$v_D\downarrow = 4.712 \measuredangle 60° + 0.15\,\omega_{BD} \measuredangle 16.78°$



VECTOR DIAGRAM

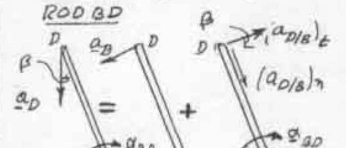
x COMPONENTS:
$\xrightarrow{+}$ $(4.712\text{ m/s})\cos 60° - (0.15\text{ m})\omega_{BD}\cos 16.78° = 0$

$\underline{\omega}_{BD} = 16.41$ rad/s ↺

ACCELERATION:

ROD AB $\underline{a}_B = (AB)\omega_{AB}^2 = (0.05\text{ m})(94.248\text{ rad/s})^2 = 444.1$ m/s² $\measuredangle 30°$

ROD BD



$(\underline{a}_{D/B})_t = (0.15\text{ m})\alpha_{BD} \measuredangle \beta$

$(a_{D/B})_n = (0.15\text{ m})\omega_{BD}^2 = (0.15\text{ m})(16.41\text{ rad/s})^2$

$(\underline{a}_{D/B})_n = 40.4$ m/s² $\measuredangle \beta$

$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = \underline{a}_B + (\underline{a}_{D/B})_t + (\underline{a}_{D/B})_n$

$\underline{a}_D\downarrow = (444.1\text{ m/s}^2)\measuredangle 30° + (0.15\text{ m})\alpha_{BD}\measuredangle 16.78° + (40.4\text{ m/s}^2)\measuredangle 16.78°$

x COMPONENTS:
$\xleftarrow{+}$: $444.1\cos 30° - 0.15\alpha_{BD}\cos 16.78° - 40.4\sin 16.78° = 0$

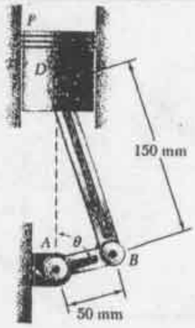
$\underline{\alpha}_{BD} = 2597$ rad/s² ↺

y COMPONENTS:
$+\downarrow$ $a_D = 444.1\sin 30° - (0.15)(2597)\sin 16.78 + 40.4\cos 16.78°$

$a_D = 148.3$ m/s²

$\underline{a}_P = \underline{a}_D$ $\qquad \underline{a}_P = 148.3$ m/s² ↓ ◀

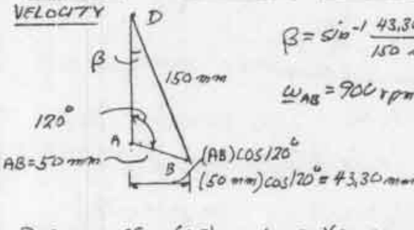
15.126



GIVEN:
$\theta = 120°$
$\underline{\omega}_{AB} = 900$ rpm ↻
$\underline{\alpha}_{AB} = 0$

FIND:
a_P

VELOCITY

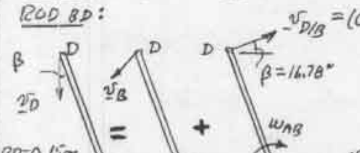


$\beta = \sin^{-1}\frac{43.30\text{ mm}}{150\text{ mm}} = 16.78°$

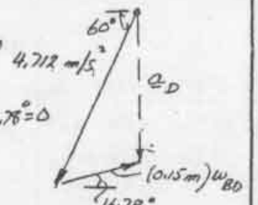
$\underline{\omega}_{AB} = 900$ rpm $= 94.248$ rad/s ↻

ROD AB: $\underline{v}_B = (AB)\omega_{AB} = (0.05\text{ m})(94.248\text{ rad/s}) = 4.712$ m/s $\measuredangle 60°$

ROD BD:



$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B}$
$v_D\downarrow = v_B \measuredangle 60° + v_{D/B} \measuredangle 16.78°$
$\underline{v}_D\downarrow = 4.712 \measuredangle 60° + 0.15\,\omega_{BD} \measuredangle 16.78°$



VECTOR DIAGRAM

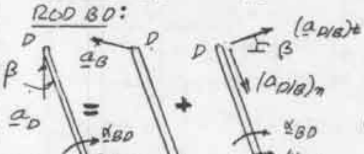
x COMPONENTS:
$\xrightarrow{+}$ $-(4.712\text{ m/s}^2)\cos 60° + (0.15\text{ m})\omega_{BD}\cos 16.78° = 0$

$\underline{\omega}_{BD} = 16.41$ rad/s ↻

ACCELERATION:

ROD AB: $\underline{a}_B = (AB)\omega_{AB}^2 = (0.05\text{ m})(94.248\text{ rad/s})^2 = 444.1$ m/s² $\measuredangle 30°$

ROD BD:



$(\underline{a}_{D/B})_t = (0.15\text{ m})\alpha_{BD} \measuredangle \beta$

$(a_{D/B})_n = (0.15\text{ m})\omega_{BD}^2 = (0.15\text{ m})(16.41\text{ rad/s})^2$

$(\underline{a}_{D/B})_n = 40.4$ m/s² $\measuredangle \beta$

$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = \underline{a}_B + (\underline{a}_{D/B})_t + (\underline{a}_{D/B})_n$

$\underline{a}_D\uparrow = (444.1\text{ m/s}^2)\measuredangle 30° + (0.15\text{ m})\alpha_{BD}\measuredangle 16.78° + (40.4\text{ m/s}^2)\measuredangle 16.78°$

x COMPONENTS:
$\xleftarrow{+}$: $444.1\cos 30° - (0.15\text{ m})\alpha_{BD}\cos 16.78° - 40.4\sin 16.78° = 0$

$\underline{\alpha}_{BD} = 2597$ rad/s² ↻

y COMPONENTS:
$+\uparrow$ $a_D = 444.1\sin 30° + (0.15)(2597)\sin 16.78 - 40.4\cos 16.78°$

$a_D = 296$ m/s²

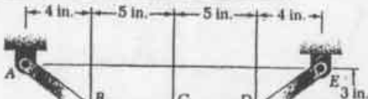
$\underline{a}_P = \underline{a}_D$ $\qquad \underline{a}_P = 296$ m/s² ↑ ◀

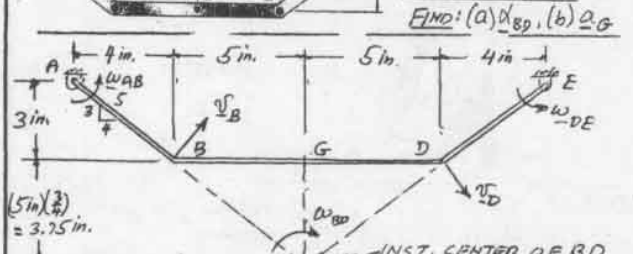
15.127 and 15.128

GIVEN: $\alpha_{AB}=0$, $\omega_{AB}=15$ rad/s ↻

PROBLEM 15.127 FIND: (a) α_{DE}, (b) a_D

PROBLEM 15.128 FIND: (a) α_{BD}, (b) a_G





$BC=(3.75^2+5^2)^{1/2}=6.25$ in. $v_B=(AB)\omega_{AB}=(5\text{ in})\omega_{AB}$

$\omega_{BD}=\frac{v_B}{BC}=\frac{5\text{ in}}{6.25\text{ in}}\omega_{AB}=\frac{5\text{ in}}{6.25\text{ in}}(15\text{ rad/s})$; $\omega_{BD}=12$ rad/s ↺

$v_D=v_B$, THUS, $\omega_{DE}=\omega_{AB}$; $\omega_{DE}=15$ rad/s ↻

ACCELERATION: ROD AB: $\alpha_{AB}=0$

$a_B=(AB)\omega_{AB}^2=(5\text{ in.})(15\text{ rad/s}^2)=1125\text{ in./s}^2$ ↖ (4, 3)

ROD DE:

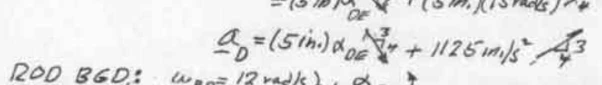
$DE=5$ in

$a_D=(a_D)_t$ ↘ (3, 4) $+(a_D)_n$ ↗ (4, 3)

$=(5\text{ in})\alpha_{DE}$ ↘ $+(5\text{ in.})(15\text{ rad/s})^2$ ↗

$a_D=(5\text{ in.})\alpha_{DE}$ ↘ (3, 4) $+1125\text{ in./s}^2$ ↗ (4, 3)

ROD BGD: $\omega_{BD}=12$ rad/s ↺, α_{BD} ↻



PLANE MOTION = TRANS WITH B + ROTATION ABOUT B

$a_D=a_B+a_{D/B}=a_B+(a_{D/B})_t+(a_{D/B})_n$

$a_D=a_B+(BD)\alpha_{BD}\uparrow+(BD)\omega_{BD}^2\leftarrow$

$a_D=a_B+(10\text{ in.})\alpha_{BD}\uparrow+(10\text{ in.})(12\text{ rad/s})^2\leftarrow$

$(5\text{ in.})\alpha_{DE}$ ↘ (3, 4) $+1125\text{ in/s}^2$ ↗ (4, 3) $=1125\text{ in./s}^2$ ↖ (4, 3) $+(10\text{ in.})\alpha_{BD}\uparrow$
$+1440\text{ in./s}^2\leftarrow$ (1)

x COMPONENTS:

$\xrightarrow{+}$: $(5\text{ in.})\alpha_{DE}\left(\frac{3}{5}\right)+(1125\text{ in/s}^2)\left(\frac{4}{5}\right)=-(1125\text{ in/s}^2)\left(\frac{4}{5}\right)-1440\text{ in/s}^2$

$3\alpha_{DE}+900=-900-1440$

$\alpha_{DE}=-1080\text{ rad/s}^2$ ↻

WE ASSUMED α_{DE} ↻, MINUS SIGN SHOWS α_{DE} IS ↺

$\alpha_{DE}=1080\text{ rad/s}^2$ ↺ ◀

y COMPONENTS: (USING $\alpha_{DE}=-1080\text{ rad/s}^2$)

$+\uparrow$: $(5\text{ in})(-1080\text{ rad/s}^2)\left(\frac{4}{5}\right)-(1125\text{ in/s}^2)\left(\frac{3}{5}\right)=-(1125\text{ in/s}^2)\left(\frac{3}{5}\right)$
$-(10\text{ in.})\alpha_{BD}$

$-4320\text{ in/s}^2-675\text{ in/s}^2=-675\text{ in/s}^2-(10\text{ in.})\alpha_{BD}$

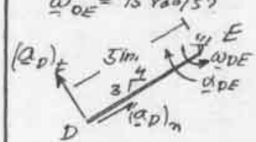
$\alpha_{BD}=+432\text{ rad/s}^2$

WE ASSUMED α_{BD} ↺, THUS $\alpha_{BD}=432\text{ rad/s}^2$ ↺ ◀

(CONTINUED)

15.127 and 15.128 CONTINUED

ACCELERATION OF D: WE KNOW $\alpha_{DE}=1080\text{ rad/s}^2$ ↺ AND $\omega_{DE}=15$ rad/s ↻



$a_D=(a_D)_t$ ↖ (3, 4) $+(a_D)_n$ ↗ (4, 3)

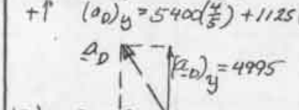
$=(DE)\alpha_{DE}$ ↖ $+(DE)\omega_{DE}^2$ ↗

$a_D=(5)(1080)$ ↖ $+(5)(15)^2$ ↗

$a_D=5400\text{ in/s}^2$ ↖ (3, 4) $+1125\text{ in/s}^2$ ↗ (4, 3)

$\xleftarrow{+}$ $(a_D)_x=5400\left(\frac{3}{5}\right)-1125\left(\frac{4}{5}\right)=+2340\text{ in/s}^2$

$+\uparrow$ $(a_D)_y=5400\left(\frac{4}{5}\right)+1125\left(\frac{3}{5}\right)=+4995\text{ in/s}^2$



$a_D=5520\text{ in/s}^2$ ↖ 64.9° ◀

ACCELERATION OF G: WE AGAIN USE THE FREE-BODY EQUATION.

$a_G=a_B+a_{G/B}=a_B+(a_{G/B})_t+(a_{G/B})_n$

$a_G=a_B+(BG)\alpha_{BD}\uparrow+(BD)\omega_{BD}^2\leftarrow$

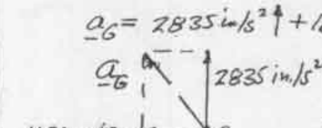
RECALL WE FOUND $\alpha_{BD}=+432\text{ rad/s}^2$ AND USE THIS VALUE HERE TOGETHER WITH $\omega_{BD}=12$ rad/s AND

$a_B=1125\text{ in/s}^2$ ↖ (4, 3)

$a_G=1125\text{ in/s}^2$ ↖ (4, 3) $+(5\text{ in.})(432\text{ rad/s}^2)\uparrow+(5\text{ in})(12\text{ rad/s})^2\leftarrow$

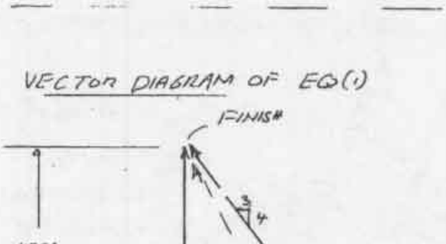
$=[900\text{ in/s}^2\leftarrow+675\text{ in/s}^2\uparrow]+2160\text{ in/s}^2\uparrow+720\text{ in/s}^2\leftarrow$

$a_G=2835\text{ in/s}^2\uparrow+1620\text{ in/s}^2\leftarrow$



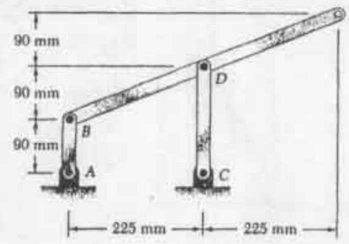
$a_G=3270\text{ in/s}^2$ ↖ 60.3° ◀

VECTOR DIAGRAM OF EQ(1)



DIMENSIONS in: in./s²

15.129 and 15.130

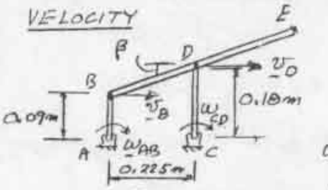


GIVEN: $\omega_{AB} = 6\ rad/s$ ↻, $\alpha_{AB} = 0$

PROBLEM 15.129: FIND: a_D

PROBLEM 15.130: FIND: (a) α_{BDE} (b) a_E

VELOCITY



INST. CENTER OF BDE AT ∞. $\omega_{BDE} = 0$

$v_B = v_D$

$(AB)\omega_{AB} = (CD)\omega_{CD}$

$\omega_{CD} = \frac{(AB)}{(CD)}\omega_{AB} = \frac{0.09m}{0.18m}\omega_{AB}$

$\omega_{CD} = \frac{1}{2}\omega_{AB} = 3\ rad/s$ ↻

$\beta = \tan^{-1}\frac{0.09m}{0.225m} = 21.8°$

ACCELERATION $\alpha_{AB} = 0$

ROD AB: $a_B = (AB)\omega_{AB}^2 \downarrow = (0.09m)(6\ rad/s)^2 = 3.24\ m/s^2 \downarrow$

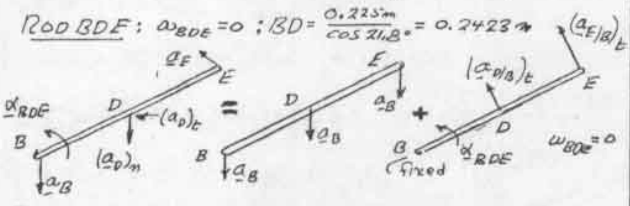
ROD CD: $(a_D)_t = (0.18m)\alpha_{CD}$

$(a_D)_n = (0.18m)(3\ rad/s)^2$

$(a_D)_n = 1.62\ m/s^2$

$a_D = (0.18m)\alpha_{CD} \leftarrow + 1.62\ m/s^2 \downarrow$

ROD BDE: $\omega_{BDE} = 0$; $BD = \frac{0.225m}{\cos 21.8°} = 0.2423m$



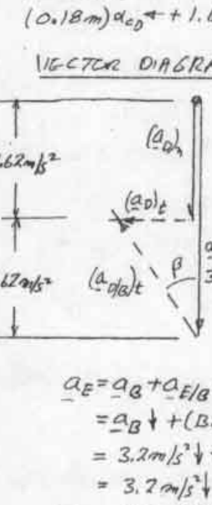
PLANE MOTION = TRANS WITH B + ROTATION ABOUT B

$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t$

$(0.18m)\alpha_{CD} \leftarrow + 1.62\ m/s^2 \downarrow = 3.24\ m/s^2 \downarrow + (BD)\alpha_{BDE} \nwarrow \beta$

$(0.18m)\alpha_{CD} \leftarrow + 1.62\ m/s^2 \downarrow = 3.24\ m/s^2 \downarrow + (0.2423m)\alpha_{BDE} \nwarrow \beta$

VECTOR DIAGRAM



$(a_{D/B})_t = 0.2423\,\alpha_{BDE} = \frac{1.62\ m/s^2}{\cos\beta}$

$\alpha_{BDE} = 7.2\ rad/s^2$ ↺ ◀

$(a_D)_t = (1.62\ m/s^2)\tan\beta = 0.648\ m/s^2 \leftarrow$

$a_D = 0.648\ m/s^2 \leftarrow + 1.62\ m/s^2 \downarrow$

$a_D = 1.745\ m/s^2 \measuredangle 68.2°$ ◀

$BE = 2\,\frac{0.225}{\cos\beta} = 0.4847m$

$a_E = a_B + a_{E/B} = a_B + (a_{E/B})_t$

$= a_B \downarrow + (BE)\alpha_{BDE} \nwarrow \beta$

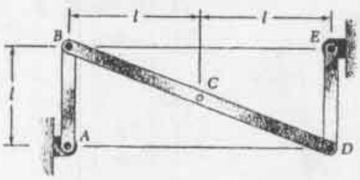
$= 3.2\ m/s^2 \downarrow + (0.4847m)(7.2\ rad/s^2) \nwarrow \beta$

$= 3.2\ m/s^2 \downarrow + 3.4896\ m/s^2 \nwarrow 21.8°$

$a_E = 3.2\ m/s^2 \downarrow + 1.296\ m/s^2 \leftarrow + 3.2\ m/s^2 \uparrow$

$a_E = 1.296\ m/s^2 \leftarrow$ ◀

15.131 and 15.132

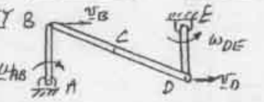


GIVEN: $\omega_{AB} = \omega_0$ ↻, $\alpha_{AB} = 0$

PROBLEM 15.131: FIND: (a) α_{DE} (b) a_D

PROBLEM 15.132: FIND: a_C KNOWING $\omega_0 = 8\ rad/s$ AND $l = 0.3m$

VELOCITY



INST. CENTER OF BD IS AT ∞

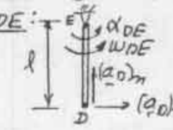
THUS, $\omega_{BD} = 0$

$v_D = v_B$; $(AB)\omega_{AB} = (DE)\omega_{DE}$; $\omega_{DE} = \omega_{AB} = \omega_0$

ACCELERATION $\alpha_{AB} = 0$

ROD AB: $a_B = (AB)\omega_{AB}^2 \downarrow = l\omega_0^2 \downarrow$

ARM DE:

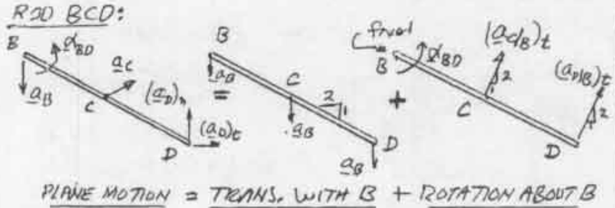


$a_D = (a_D)_t \rightarrow + (a_D)_n \uparrow$

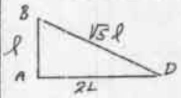
$= (DE)\alpha_{DE} \rightarrow + (DE)\omega_{DE}^2 \uparrow$

$a_D = l\alpha_{DE} \rightarrow + l\omega_0^2 \uparrow$

ROD BCD:



PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

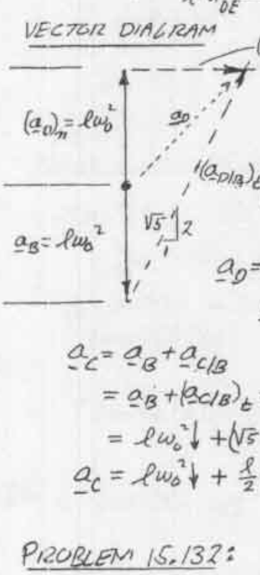


$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t$

$l\alpha_{DE} \rightarrow + l\omega_0^2 \uparrow = l\omega_0^2 \downarrow + (BD)\alpha_{BD} \measuredangle$

$l\alpha_{DE} \rightarrow + l\omega_0^2 \uparrow + l\omega_0^2 \downarrow + \sqrt{5}\,l\,\alpha_{BD} \measuredangle$

VECTOR DIAGRAM



$(a_D)_t = l\alpha_{DE} = \frac{1}{2}(2l\omega_0^2) = l\omega_0^2$

$\alpha_{DE} = \omega_0^2$ ↺ ◀

$(a_{D/B})_t = \sqrt{5}\,l\,\alpha_{BD} = \frac{\sqrt{5}}{2}(2l\omega_0^2) = \sqrt{5}\,l\omega_0^2$

$\alpha_{BD} = \omega_0^2$ ↺

$a_D = (a_D)_t + (a_D)_n = l\omega_0^2 \rightarrow + l\omega_0^2 \uparrow$

$a_D = \sqrt{2}\,l\omega_0^2 \measuredangle 45°$ ◀

$a_C = a_B + a_{C/B}$

$= a_B + (a_{C/B})_t = a_B \downarrow + (BC)\alpha_{BD} \measuredangle$

$= l\omega_0^2 \downarrow + \left(\sqrt{5}\,\frac{l}{2}\right)\omega_0^2 \measuredangle$

$a_C = l\omega_0^2 \downarrow + \frac{l}{2}\omega_0^2 \rightarrow + l\omega_0^2 \uparrow$

$a_C = \frac{l}{2}\omega_0^2 \uparrow$

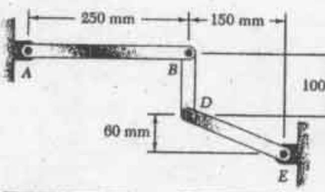
PROBLEM 15.132:

FOR $\omega_0 = 8\ rad/s$ and $l = 0.3m$, WE HAVE

$a_C = \frac{l}{2}\omega_0^2 = \left(\frac{0.3m}{2}\right)(8\ rad/s)^2 = 9.6\ m/s^2$

$a_C = 9.6\ m/s^2 \rightarrow$ ◀

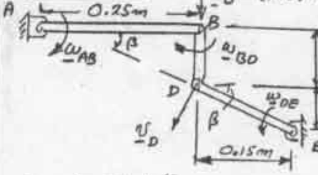
15.133



GIVEN:
$\omega_{AB} = 4$ rad/s ↻
$\alpha_{AB} = 0$

FIND:
(a) α_{BD}
(b) α_{DE}

VELOCITY: $v_B = (AB)\omega_{AB}$



INST. CENTER OF BD IS AT A, THUS
$\omega_{BD} = \omega_{AB}$
$\omega_{BD} = 4$ rad/s ↻

$AD = (0.25\text{m})/\cos\beta \qquad DE = (0.15\text{m})/\cos\beta$

$v_D = (AD)\,\omega_{AB}$

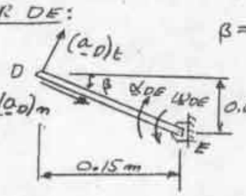
$$\omega_{DE} = \frac{v_D}{(DE)} = \frac{(AD)}{(DE)}\omega_{AB} = \frac{(0.25/\cos\beta)}{(0.15/\cos\beta)}(4\text{ rad/s}) = 6.667\ \frac{\text{rad}}{\text{s}}\ ↺$$

ACCELERATION:

BAR AB: $a_B = (AB)\omega_{AB}^2 = (0.25\text{ m})(4\text{ rad/s})^2 = 4\text{ m/s}^2 \leftarrow$

BAR DE:

$\beta = \tan^{-1}\frac{0.06}{0.15} = 21.8°$



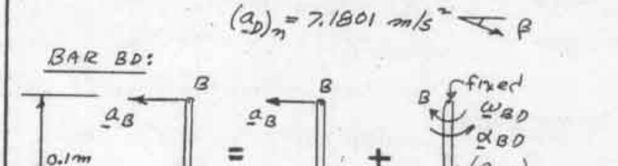
$$DE = \frac{0.15\text{ m}}{\cos 21.8°} = 0.16155\text{ m}$$

$(a_D)_t = (DE)\alpha_{DE}$ ∠β

$(a_D)_n = (DE)\omega_{DE}^2 = (0.16155\text{ m})(6.6667\text{ rad/s})^2$

$(a_D)_n = 7.1801\text{ m/s}^2$ ∠β

BAR BD:



PLANE MOTION = TRANS WITH B + ROTATION ABOUT B

$$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$$

$(a_D)_t$ ∠β $+ (a_D)_n$ ∠β $= a_B \leftarrow + (BD)\alpha_{BD} \rightarrow + (BD)\omega_{BD}^2 \uparrow$

$(0.16155\text{m})\alpha_{DE}$ ∠β $+ 7.1803\text{ m/s}^2$ ∠β $= 4\text{ m/s}^2 \leftarrow + (0.1\text{m})\alpha_{BD} \rightarrow$
$+ (0.1\text{m})(4\text{ rad/s})^2 \uparrow$

y COMPONENTS: $\beta = 21.8°$

$+\uparrow\ (0.16155\text{m})\alpha_{DE}\cos\beta - (7.1801\text{ m/s}^2)\sin\beta = 1.6\text{ m/s}^2$

$0.15\,\alpha_{DE} - 2.6665 = 1.6$

$\alpha_{DE} = 28.445\text{ rad/s}^2$

$\alpha_{DE} = 28.4\text{ rad/s}^2$ ↻ ◀

x COMPONENTS:

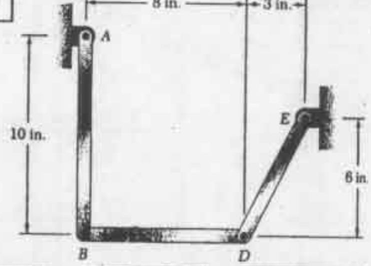
$\xrightarrow{+}\ (0.16155\text{m})(28.445\text{ rad/s})\sin\beta + (7.1801\text{ m/s}^2)\cos\beta$
$= -4\text{ m/s}^2 + (0.1\text{m})\alpha_{BD}$

$1.7066 + 6.6667 = -4 + 0.1\,\alpha_{BD}$

$\alpha_{BD} = 123.73\text{ rad/s}^2$

$\alpha_{BD} = 123.7\text{ rad/s}^2$ ↺ ◀

15.134

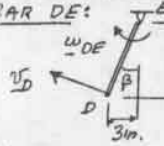


GIVEN:
$\omega_{AB} = 4$ rad/s ↻
$\alpha_{AB} = 0$

FIND:
(a) α_{BD}
(b) α_{DE}

VELOCITY: BAR AB: $v_B = (10\text{ in.})(4\text{ rad/s}) = 40\text{ in./s} \leftarrow$

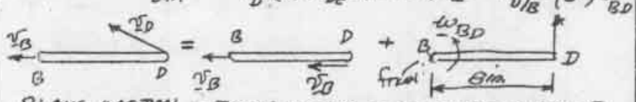
BAR DE:



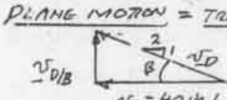
$\beta = \tan^{-1}\frac{3}{6} = 26.56°$

$DE = \frac{6}{\cos\beta} = 6.708\text{ in}$

$v_D = (DE)\omega_{DE} = (6.708\text{ in})\omega_{DE} \qquad v_{D/B} = (8\text{ in})\omega_{BD}$



PLANE MOTION = TRANS WITH B + ROTATION ABOUT B



$v_D = v_B + v_{D/B}$

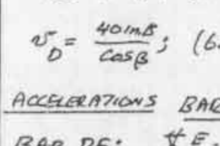
v_D ∠ $= 40\text{ in/s} \leftarrow + v_{D/B} \uparrow$

$v_{D/B} = \frac{1}{2}v_B$; $(8\text{ in.})\,\omega_{BD} = \frac{1}{2}(40\text{ in/s})$; $\omega_{BD} = 2.5\text{ rad/s}$ ↺

$v_D = \frac{40\text{ in/s}}{\cos\beta}$; $(6.708\text{ in.})\omega_{DE} = \frac{40\text{ in/s}}{\cos\beta}$; $\omega_{DE} = 6.667\text{ rad/s}$ ↻

ACCELERATIONS BAR AB: $a_B = (AB)\omega_{AB}^2 = (10\text{ in})\left(4\ \frac{\text{rad}}{\text{s}}\right)^2 = 160\text{ in/s}^2 \uparrow$

BAR DE:

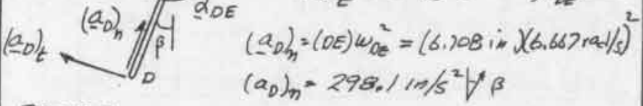


$(a_D)_t = (DE)\alpha_{DE} = (6.708\text{ in})\alpha_{DE}$ ∠β

$(a_D)_n = (DE)\omega_{DE}^2 = (6.708\text{ in})(6.667\text{ rad/s})^2$

$(a_D)_n = 298.1\text{ in/s}^2$ ∠β

BAR BD:



PLANE MOTION = TRANS WITH B + ROTATION ABOUT B

$$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$$

$(a_D)_t$ ∠β $+ (a_D)_n$ ∠β $= a_B \uparrow + (8\text{ in})\alpha_{BD} \uparrow + (8\text{ in.})(2.5\text{ rad/s})^2 \leftarrow$

$(6.708\text{ in})\alpha_{DE}$ ∠β $+ 298.1\text{ in/s}^2$ ∠β $= 160\text{ in./s}^2 \uparrow + (8\text{ in.})\alpha_{BD} \uparrow$
$+ 50\text{ in./s}^2 \leftarrow$

x COMPONENTS: $\beta = 26.56°$

$\xrightarrow{+}\ (6.708\text{ in.})\alpha_{DE}\cos\beta - (298.1\text{ in./s}^2)\sin\beta = +50\text{ m/s}^2$

$6.000\,\alpha_{DE} - 133.31 = -50$

$\alpha_{DE} = 30.55\text{ rad/s}^2$

$\alpha_{DE} = 30.6\text{ rad/s}^2$ ↻ ◀

y COMPONENTS:

$+\uparrow\ (6.708\text{in.})\alpha_{DE}\sin\beta + (298.1\text{ in./s}^2)\cos\beta = 160\text{ in/s}^2 + (8\text{ in.})\alpha_{BD}$

$(6.708)(30.55)\sin\beta + 298.1\cos\beta = 160 + 8\alpha_{BD}$

$\alpha_{BD} = 24.8\text{ rad/s}^2$

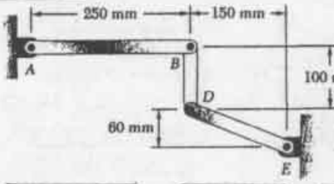
$\alpha_{BD} = 24.8\text{ rad/s}^2$ ↺ ◀

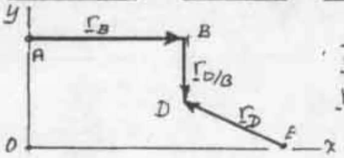
15.135

GIVEN: $\omega_{AB} = 4$ rad/s, $\alpha_{AB} = 0$

FIND: (a) α_{BD}, (b) α_{DE}

[USE FULL VECTOR APPROACH AS DONE IN SAMPLE PROB 15.8]





$r_B = (0.25\text{ m})\,i$

$r_D = -(0.15\text{ m})\,i + (0.06\text{ m})\,j$

$r_{D/B} = -(0.1)\,j$

$\omega_{AB} = -(4\text{ rad/s})\,k \qquad \omega_{BD} = \omega_{BD}\,k \qquad \omega_{DE} = \omega_{DE}\,k$

VELOCITY:

$$v_D = v_B + v_{D/B}$$

$$\omega_{DE}\,k \times r_D = \omega_{AB}\,k \times r_B + \omega_{BD}\,k \times r_{D/B}$$

$$\omega_{DE}\,k \times (-0.15\,i + 0.06\,j) = -4k \times 0.25\,i + \omega_{BD}\,k \times -0.1\,j$$

$$-0.15\,\omega_{DE}\,j - 0.06\,\omega_{DE}\,i = -j + 0.1\,\omega_{BD}\,i$$

COEFFICIENTS OF j: $-0.15\,\omega_{DE} = -1 \qquad \omega_{DE} = 6.667$ rad/s

COEFFICIENTS OF i: $-0.06\,\omega_{DE} = 0.1\,\omega_{BD}$

$-0.06(6.667) = 0.1\,\omega_{BD} \qquad \omega_{BD} = 4$ rad/s

ACCELERATION:

$\alpha_{AB} = 0 \qquad \alpha_{BD} = \alpha_{BD}\,k \qquad \alpha_{DE} = \alpha_{DE}\,k$

$$a_D = a_B + a_{D/B} \qquad (1)$$

$a_D = \alpha_{DE}\,k \times r_D - \omega_{DE}^2\,r_D$

$= \alpha_{DE}\,k \times (-0.15\,i + 0.06\,j) - (6.667)^2(-0.15\,i + 0.06\,j)$

$a_D = -0.15\,\alpha_{DE}\,j - 0.06\,\alpha_{DE}\,i + 6.667\,i - 2.667\,j$ ◁

$a_B = \alpha_{AB}\,k \times r_B - \omega_{AB}^2\,r_B \qquad$ NOTE: $\alpha_{AB} = 0$

$= 0 - (4)^2(+0.25\,i)$

$a_B = -4\,i$ ◁

$a_{D/B} = \alpha_{BD}\,k \times r_{D/B} - \omega_{BD}^2\,r_{D/B}$

$= \alpha_{BD}\,k \times (-0.1\,j) - (4)^2(-0.1\,j)$

$a_{D/B} = -0.1\,\alpha_{BD}\,i + 1.6\,j$ ◁

SUBSTITUTE FOR a_D, a_B, $a_{D/B}$ IN EQ. (1),

$$a_D = a_B + a_{D/B}$$

$$-0.15\,\alpha_{DE}\,j - 0.06\,\alpha_{DE}\,i + 6.667\,i - 2.667\,j = -4\,i - 0.1\,\alpha_{BD}\,i + 1.6\,j$$

COEFFICIENTS OF j:

$-0.15\,\alpha_{DE} - 2.667 = 1.6$

$\alpha_{DE} = -28.44$ rad/s $\qquad \alpha_{DE} = 28.4$ rad/s ◀

COEFFICIENTS OF i:

$-0.06\,\alpha_{DE} + 6.667 = -4 - 0.1\,\alpha_{BD}$

$-(0.06)(-28.44) + 6.667 = -4 - 0.1\,\alpha_{BD}$

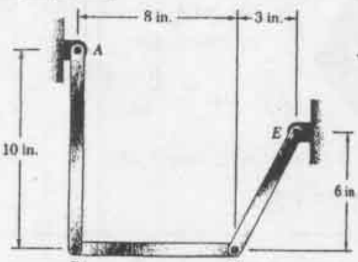
$\alpha_{BD} = 127.73$ rad/s$^2 \qquad \alpha_{BD} = 123.7$ rad/s^2 ◀

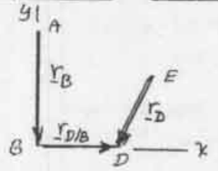
15.136

GIVEN: $\omega_{AB} = 4$ rad/s, $\alpha_{AB} = 0$

FIND: (a) α_{BD}, (b) α_{DE}

[USE FULL VECTOR APPROACH AS DONE IN SAMPLE PROB. 15.8]





$r_B = -(10\text{ in.})\,j$

$r_D = -(3\text{ in.})\,i - (6\text{ in.})\,j$

$r_{D/B} = (8\text{ in.})\,i$

$\omega_{AB} = -(4\text{ rad/s})\,k \qquad \omega_{BD} = \omega_{BD}\,k \qquad \omega_{DE} = \omega_{DE}\,k$

VELOCITY: $\quad v_D = v_B + v_{D/B}$

$$\omega_{DE}\,k \times r_D = \omega_{AB}\,k \times r_B + \omega_{BD}\,k \times r_{D/B}$$

$$\omega_{DE}\,k \times (-3\,i - 6\,j) = -4k \times (-10\,j) + \omega_{BD}\,k \times 8\,i$$

$$-3\omega_{DE}\,j + 6\omega_{DE}\,i = -40\,i + 8\,\omega_{BD}\,j$$

COEFFICIENTS OF i: $6\omega_{DE} = -40 \qquad \omega_{DE} = -6.667$ rad/s

COEFFICIENTS OF j: $-3\omega_{DE} = 8\,\omega_{BD}$

$-3(-6.667) = 8\omega_{BD} \qquad \omega_{BD} = 2.5$ rad/s

ACCELERATION:

$\alpha_{AB} = 0, \quad \alpha_{BD} = \alpha_{BD}\,k \qquad \alpha_{DE} = \alpha_{DE}\,k$

$$a_D = a_B + a_{D/B} \qquad (1)$$

$a_D = \alpha_{DE}\,k \times r_D - \omega_{DE}^2\,r_D$

$= \alpha_{DE}\,k \times (-3\,i - 6\,j) - (-6.667)^2(-3\,i - 6\,j)$

$= -3\alpha_{DE}\,j + 6\alpha_{DE}\,i + 133.3\,i + 266.7\,j$ ◁

$a_B = \alpha_{AB}\,k \times r_B - \omega_{AB}^2\,r_B \qquad$ NOTE: $\alpha_{AB} = 0$.

$= 0 - (4)^2(-10\,j)$

$a_B = 160\,j$ ◁

$a_{D/B} = \alpha_{BD}\,k \times r_{D/B} - \omega_{BD}^2\,r_{D/B}$

$= \alpha_{BD}\,k \times 8\,i - (2.5)^2(8\,i)$

$a_{D/B} = 8\alpha_{BD}\,j - 50\,i$ ◁

EQ(1): $\quad a_D = a_B + a_{D/B}$

$$-3\alpha_{DE}\,j + 6\alpha_{DE}\,i + 133.3\,i + 266.7\,j = 160\,j + 8\alpha_{BD}\,j - 50\,i$$

COEFFICIENTS OF i:

$+6\alpha_{DE} + 133.3 = -50$

$\alpha_{DE} = -30.55$ rad/s^2

$\alpha_{DE} = 30.6$ rad/s^2 ◀

COEFFICIENTS OF j

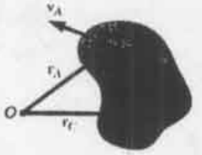
$-3\,\alpha_{DE} + 266.7 = 160 + 8\alpha_{BD}$

$-3(-30.55) + 266.7 = 160 + 8\alpha_{BD}$

$\alpha_{BD} = 24.8$ rad/s^2

$\alpha_{BD} = 24.8$ rad/s^2 ◀

15.137



INSTANTANEOUS CENTER OF ROTATION AT C

(a) SHOW THAT

$$\underline{r}_C = \underline{r}_A + \frac{\underline{\omega} \times \underline{v}_A}{\omega^2}$$

(b) SHOW THAT $a_C = 0$, IF, AND ONLY IF,

$$\underline{a}_A = \frac{\alpha}{\omega} \underline{v}_A + \underline{\omega} \times \underline{v}_A$$

$$\underline{v}_A = \underline{v}_C + \underline{v}_{A/C} = \underline{v}_C + \underline{\omega} \times (\underline{r}_{A/C})$$

$$\underline{v}_A = \underline{v}_C + \underline{\omega} \times (\underline{r}_A - \underline{r}_C)$$

BUT $v_C = 0$: $\quad \underline{v}_A = \underline{\omega} \times (\underline{r}_A - \underline{r}_C)$

CROSS MULTIPLY EACH MEMBER BY $\underline{\omega}$

$$\underline{\omega} \times \underline{v}_A = \underline{\omega} \times [\underline{\omega} \times (\underline{r}_A - \underline{r}_C)]$$

SINCE $\underline{\omega} \perp$ TO PLANE CONTAINING $(\underline{r}_A - \underline{r}_C)$, CROSS MULTIPLYING TWICE BY $\underline{\omega}$ IS EQUIVALENT TO MULTIPLYING $(r_A - r_C)$ BY ω^2 AND ROTATING IT THROUGH 180°. THUS,

$$\underline{\omega} \times \underline{v}_A = -\omega^2 (\underline{r}_A - \underline{r}_C)$$

SOLVING FOR r_C: $\quad \underline{r}_C = \underline{r}_A + \dfrac{\underline{\omega} \times \underline{v}_A}{\omega^2} \quad$ (Q.E.D.)

(b) SINCE WE WANT $a_C = 0$, WE SHALL WRITE

$$a_C = a_A + a_{C/A} = 0 \qquad (1)$$

USING EQ. 15.11, page 891

$$\underline{a}_{C/A} = \underline{\alpha} \times r_{C/A} - \omega^2 r_{C/A} \qquad (2)$$

FROM PART a: $\quad r_{C/A} = r_C - \underline{r}_A = \dfrac{\underline{\omega} \times \underline{v}_A}{\omega^2}$

EQ(2): $\quad \underline{a}_{C/A} = \alpha \times \dfrac{\underline{\omega} \times \underline{v}_A}{\omega^2} - (\underline{\omega} \times \underline{v}_A)$

BUT $\underline{\alpha} = \alpha \underline{k}$ AND $\underline{\omega} = \omega \underline{k}$, AND SINCE $\underline{k} \perp \underline{v}_A$

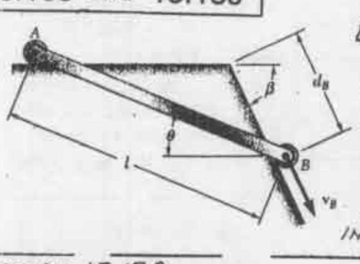
$$\underline{a}_{C/A} = \frac{\alpha}{\omega}\left[\underline{k} \times (\underline{k} \times \underline{v}_A)\right] - \underline{\omega} \times \underline{v}_A$$

$$= -\frac{\alpha}{\omega} \underline{v}_A - \underline{\omega} \times \underline{v}_A$$

SUBSTITUTING INTO (1) AND SOLVING FOR a_A, WE HAVE FOR $a_C = 0$

$$\underline{a}_A = \frac{\alpha}{\omega} \underline{v}_A + \underline{\omega} \times \underline{v}_A \qquad \text{(Q.E.D)}$$

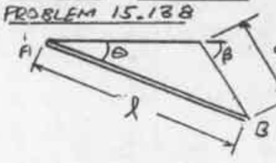
*15.138 and 15.139



PROBLEM 15.138
EXPRESS ω OF ROD IN TERMS OF v_B, θ, l, AND β

PROBLEM 15.139
GIVEN: $a_B = 0$, EXPRESS α OF ROD IN TERMS OF v_B, θ, l, + β

PROBLEM 15.138



LAW OF SINES

$$\frac{d_B}{\sin\theta} = \frac{l}{\sin\beta}$$

$$d_B = \frac{l}{\sin\beta} \sin\theta$$

$$v_B = \frac{d}{dt}(d_B) = \frac{l}{\sin\beta} \cos\theta \frac{d\theta}{dt} = \frac{l}{\sin\beta} \cos\theta \; \omega$$

$$\omega = \frac{v_B \sin\beta}{l \cos\theta} \quad ◀$$

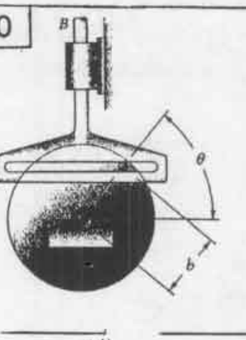
PROBLEM 15.139 $\quad$ NOTE THAT $a_B = \dfrac{dv_B}{dt} = 0$.

$$\alpha = \frac{d\omega}{dt} = \frac{v_B \sin\beta}{l} \cdot \frac{\sin\theta}{\cos^2\theta} \cdot \frac{d\theta}{dt}$$

$$\alpha = \frac{v_B \sin\beta \sin\theta}{l \cos^2\theta} \cdot \frac{v_B \sin\beta}{l \cos\theta}$$

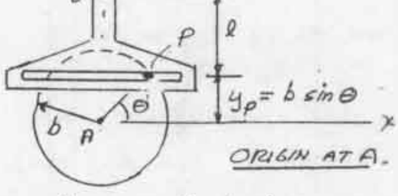
$$\alpha = \left[\frac{v_B \sin\beta}{l}\right]^2 \frac{\sin\theta}{\cos^3\theta} \quad ◀$$

* 15.140



GIVEN: FOR DISK, $\underline{\omega}$ AND $\underline{\alpha}$ ARE ↺

DERIVE EXPRESSIONS FOR v_B AND a_B



ORIGIN AT A.

$$y_B = l + y_P = l + b\sin\theta$$

$$v_B = \dot{y}_B = b\cos\theta\, \dot{\theta} = b\cos\theta\, \omega$$

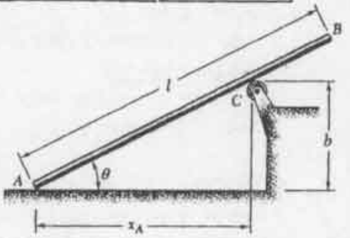
$$v_B = b\omega\cos\theta \quad ◀$$

$$a_B = \ddot{y}_B = \frac{d}{dt} v_B = \frac{d}{dt}(b\cos\theta\, \dot{\theta})$$

$$a_B = -b\sin\theta\, \dot{\theta}^2 + b\cos\theta\, \ddot{\theta}$$

$$a_B = b\alpha\cos\theta - b\omega^2\sin\theta \quad ◀$$

* 15.141 and 15.142



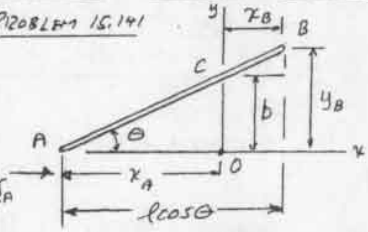
GIVEN: $a_A = 0$, $v_A = v_A \rightarrow$

DERIVE EXPRESSIONS,

PROBLEM 15.141: α AND ω

PROBLEM 15.142: $(v_B)_x$ AND $(v_B)_y$

PROBLEM 15.141



$x_A = \frac{b}{\tan\theta}$ $\qquad \dot{x}_A = -v_A$

$$x_A = \frac{b}{\tan\theta} = b\,\frac{\cos\theta}{\sin\theta}; \qquad \dot{x}_A = -v_A$$

$$\dot{x}_A = -v_A = b\,\frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta}\,\dot{\theta} - \frac{b}{\sin^2\theta}\,\omega$$

$$v_A = \frac{b}{\sin^2\theta}\,\omega \qquad \omega = \frac{v_A}{b}\sin^2\theta$$

$$\alpha = \dot{\omega} = \frac{v_A}{b}\,2\sin\theta\cos\theta\,\dot{\theta} + \frac{\sin^2\theta}{b}\,\dot{v}_A$$

BUT $\dot{v}_A = a_A = 0$; $\quad \alpha = \frac{v_A}{b}\,2\sin\theta\cos\theta\left[\frac{v_A}{b}\sin^2\theta\right]$

$$\alpha = 2\left(\frac{v_A}{b}\right)^2 \sin^3\theta\cos\theta$$

PROBLEM 15.142

$$x_B = l\cos\theta - x_A$$

$$\dot{x}_B = -l\sin\theta\,\dot{\theta} - \dot{x}_A$$

$$(v_B)_x = -l\sin\theta\,\omega + \frac{b}{\sin^2\theta}\,\omega$$

$$(v_B)_x = \left(-l\sin\theta + \frac{b}{\sin^2\theta}\right)\left(\frac{v_A}{b}\sin^2\theta\right)$$

$$(v_B)_x = v_A\left(1 - \frac{l}{b}\sin^3\theta\right)$$

$y_B = l\sin\theta$; $\quad \dot{y}_B = l\cos\theta\,\dot{\theta}$

$$\dot{y}_B = (v_B)_y = l\cos\theta\left(\frac{v_A}{b}\sin^2\theta\right)$$

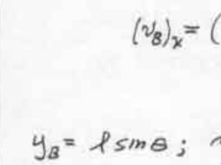
$$(v_B)_y = v_A\,\frac{l}{b}\cos\theta\sin^2\theta$$

* 15.143

GIVEN: $v_A = v \rightarrow$, $a_A = 0$

AT $t=0$, P IS ON GROUND AT O.

FIND: $\dot{x}_P$ AND $\dot{y}_P$ AT ANY TIME t



$x_P = x_A - r\sin\theta = r\theta - r\sin\theta$

$y_P = y_A - r\cos\theta = r - r\cos\theta$

$$\dot{x}_P = r(\dot{\theta} + \cos\theta\,\dot{\theta}) = r\dot{\theta}(1 - \cos\theta)$$

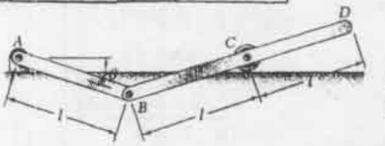
$$\dot{y}_P = r(\sin\theta\,\dot{\theta}) = r\dot{\theta}\sin\theta$$

ROLLING MOTION: $\theta = \frac{v}{r}t$; $\dot{\theta} = \frac{v}{r}$; $r\dot{\theta} = v$

$v_x = \dot{x}_P \qquad v_x = v\left(1 - \cos\frac{v}{r}t\right)$

$v_y = \dot{y}_P \qquad v_y = v\sin\frac{v}{r}t$

* 15.144 and 15.145

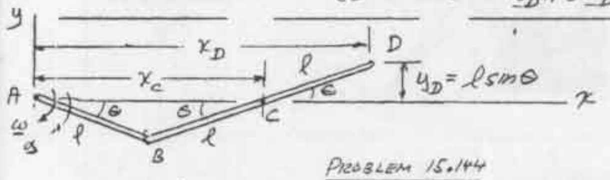


GIVEN: $\omega_{AB} = \omega$ ↻, $\alpha_{AB} = \alpha$ ↻

DERIVE EXPRESSIONS,

PROBLEM 15.144: v_A AND a_C

PROBLEM 15.145: COMPONENTS OF v_D AND a_D



PROBLEM 15.144

$$x_C = 2l\cos\theta$$

$$\dot{x}_C = -2l\sin\theta\,\dot{\theta}$$

$$v_C = -2l\omega\sin\theta$$

$$\ddot{x}_C = -2l\sin\theta\,\ddot{\theta} - 2l\cos\theta\,\dot{\theta}^2$$

$$a_C = -2l\alpha\sin\theta - 2l\omega^2\cos\theta$$

PROBLEM 15.145

$$x_D = 3l\cos\theta$$

$$\dot{x}_D = -3l\sin\theta\,\dot{\theta}$$

$$(v_D)_x = -3l\omega\sin\theta$$

$$\ddot{x}_D = -3l\sin\theta\,\ddot{\theta} - 3l\cos\theta\,\dot{\theta}^2$$

$$(a_D)_x = -3l\alpha\sin\theta - 3l\omega^2\cos\theta$$

$$y_D = l\sin\theta$$

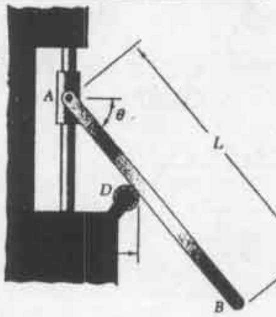
$$\dot{y}_D = l\cos\theta\,\dot{\theta}$$

$$(v_D)_y = l\omega\cos\theta$$

$$\ddot{y}_D = l\cos\theta\,\ddot{\theta} - l\sin\theta\,\dot{\theta}^2$$

$$(a_D)_y = l\alpha\cos\theta - l\omega^2\sin\theta$$

* 15.146 and 15.147



GIVEN: $\underline{v}_A = v_A \uparrow$

$\underline{a}_A = 0$

DERIVE EXPRESSIONS,

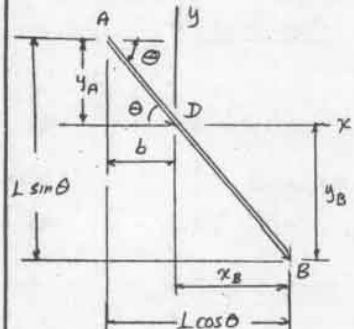
PROBLEM 15.146

(a) ω_{AB}

(b) COMPONENTS OF v_B

PROBLEM 15.147

$\underline{\alpha}_{AB}$



POSITIVE θ IS ↻

PROBLEM 15.146 $y_A = b\tan\theta$

$$v_A = \dot{y}_A = b\frac{1}{\cos^2\theta}\dot{\theta} = \frac{b\omega}{\cos^2\theta}$$

$$\omega = \frac{v_A}{b}\cos^2\theta \quad ◀$$

$$x_B = L\cos\theta - b$$

$$\dot{x}_B = -L\sin\theta\,\dot{\theta} = -L\omega\sin\theta$$

$$= -L\left(\frac{v_A}{b}\cos^2\theta\right)\sin\theta$$

$$\xrightarrow{+} \quad (v_B)_x = \dot{x}_B = -v_A\frac{L}{b}\sin\theta\cos^2\theta \quad ◀$$

$$y_B = L\sin\theta - y_A = L\sin\theta - b\tan\theta$$

$$\dot{y}_B = L\cos\theta\,\dot{\theta} - b\frac{1}{\cos^2\theta}\dot{\theta}$$

$$= \left(L\cos\theta - \frac{b}{\cos^2\theta}\right)\left(\frac{v_A}{b}\cos^2\theta\right)$$

$$+\uparrow \quad (v_B)_y = \dot{y}_B = v_A\left(\frac{L}{b}\cos^3\theta - 1\right) \quad ◀$$

PROBLEM 15.147

RECALL THAT $a_A = \dot{v}_A = 0$ and

$$\omega = \dot{\theta} = \frac{v_A}{b}\cos^2\theta$$

$$\alpha = \dot{\omega} = \frac{v_A}{b}(-2\cos\theta\sin\theta)\,\dot{\theta}$$

$$\alpha = -2\frac{v_A}{b}\cos\theta\sin\theta\left(\frac{v_A}{b}\cos^2\theta\right)$$

$$\alpha = -2\left(\frac{v_A}{b}\right)^2\sin\theta\cos^3\theta \quad ◀$$

NOTE: SINCE POSITIVE θ IS ↻, THE DIRECTION OF $\underline{\alpha}$ IS ↺.

$$\underline{\alpha} = 2\left(\frac{v_A}{b}\right)^2\sin\theta\cos^3\theta \;↺$$

* 15.148 and 15.149

GIVEN: POSITION SHOWN IS WHEN $t=0$

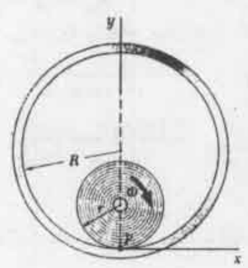
ω = CONSTANT ($\alpha = 0$)

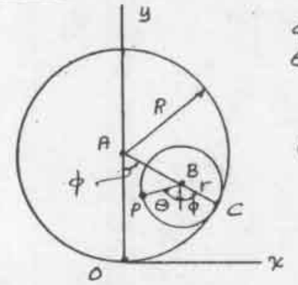
PROBLEM 15.148

DERIVE EXPRESSIONS FOR $(v_P)_x$ AND $(v_P)_y$

PROBLEM 15.149

WHEN $r = R/2$ SHOW THAT PATH OF P IS y AXIS AND DERIVE EXPRESSIONS FOR v_P AND a_P





$\phi = \angle OAB$

θ = ANGLE BP FORMS WITH THE VERTICAL

$$\theta = \omega t; \quad \dot{\theta} = \omega \qquad (1)$$

$$v_B = (AB)\dot{\phi}$$

$$v_B = (R-r)\dot{\phi}$$

SINCE C IS INSTANTANEOUS CENTER, $v_B = r\omega$

EQUATING THE TWO EXPRESSIONS OBTAINED FOR v_B

$$(R-r)\dot{\phi} = r\omega \qquad \dot{\phi} = \frac{r}{R-r}\omega \qquad (2)$$

$$x_P = (R-r)\sin\phi - r\sin\theta$$

$$y_P = R - (R-r)\cos\phi - r\cos\theta$$

DIFFERENTIATING AND USING (1) AND (2):

$$\dot{x}_P = (R-r)\cos\phi\,\dot{\phi} - r\cos\theta\,\dot{\theta}$$

$$\dot{y}_P = (R-r)\sin\phi\,\dot{\phi} + r\sin\theta\,\dot{\theta}$$

$$\dot{x}_P = (R-r)\cos\phi\left(\frac{r}{R-r}\right)\omega - r\cos\theta\,\omega$$

$$\dot{y}_P = (R-r)\sin\phi\left(\frac{r}{R-r}\right)\omega + r\sin\theta\,\omega$$

$$\dot{x}_P = r\omega(\cos\phi - \cos\theta)$$

$$\dot{y}_P = r\omega(\sin\phi + \sin\theta)$$

$$(v_P)_x = \dot{x}_P = r\omega\left[\cos\frac{r\omega t}{R-r} - \cos\omega t\right] \quad ◀$$

$$(v_P)_y = \dot{y}_P = r\omega\left[\sin\frac{r\omega t}{R-r} + \sin\omega t\right] \quad ◀$$

PROBLEM 15.149   FOR $r = R/2$

$$\dot{x}_P = r\omega(\cos\omega t - \cos\omega t) = 0$$

THUS P MOVES ALONG THE y AXIS ◀

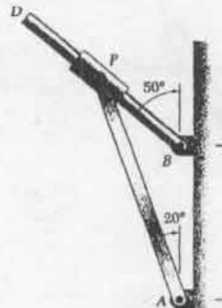
$$v = \dot{y}_P = r\omega(\sin\omega t + \sin\omega t)$$

$$v = 2r\omega\sin\omega t \qquad \underline{v} = (R\omega\sin\omega t)\,\underline{j} \quad ◀$$

$$a = \frac{dv}{dt} = 2r\omega(\omega\cos\omega t) \qquad [\text{RECALL } \omega = \text{CONSTANT}]$$

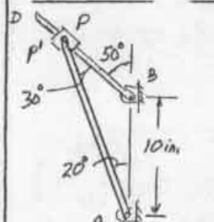
$$\underline{a} = (R\omega^2\cos\omega t)\,\underline{j} \quad ◀$$

15.150



GIVEN:
$b = 10$ in,
$\omega_B = 5$ rad/s ↻

FIND:
(a) $\underline{\omega}_{AP}$
(b) $\underline{v}_{P/BP}$



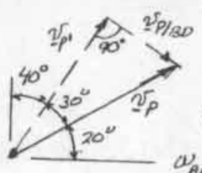
GEOMETRY: LAW OF SINES

$$\frac{AP}{\sin 50^\circ} = \frac{BP}{\sin 20^\circ} = \frac{10 \text{ in.}}{\sin 30^\circ}$$

$AP = 15.32$ in.
$BP = 6.84$ in.

NOTE: POINT P′ OF BP COINCIDES WITH P

VELOCITIES: $v_{P'} = (BP)\omega_B = (6.84 \text{ in.})(5 \text{ rad/s}) = 34.20$ in/s $\angle 40^\circ$



$$v_P = v_{P'} + v_{P/BD}$$

$$[v_P \angle 20^\circ] = [34.2 \measuredangle 40^\circ] + [v_{P/BD} \measuredangle 40^\circ]$$

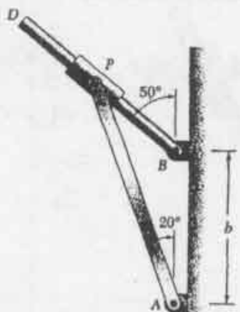
(a) $v_P = \frac{v_{P'}}{\cos 30^\circ} = \frac{34.2}{\cos 30^\circ} = 39.49$ in/s

$\omega_{AP} = \frac{v_P}{AP} = \frac{39.49 \text{ in/s}}{15.32 \text{ in}}$ $\quad \underline{\omega}_{AP} = 2.58$ rad/s ↻ ◀

(b) $v_{P/BD} = v_{P'} \tan 30^\circ = (34.2 \text{ in/s}) \tan 30^\circ$

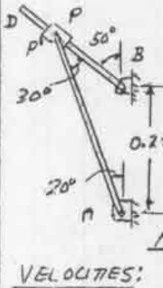
$\underline{v}_{P/BD} = 19.75$ in/s $\measuredangle 40^\circ$ ◀

15.152



GIVEN:
$b = 200$ mm
$\underline{v}_{P/BD} = 300$ mm/s $\measuredangle 50^\circ$

FIND:
$\underline{\omega}_{AP}$
$\underline{\omega}_{BD}$



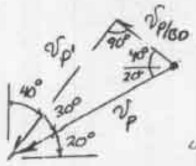
GEOMETRY LAW OF SINES

$$\frac{AP}{\sin 30^\circ} = \frac{BP}{\sin 20^\circ} = \frac{0.2 \text{ m}}{\sin 30^\circ}$$

$AP = 0.3064$ m $\quad BP = 0.1368$ m

NOTE: POINT P′ OF BP COINCIDES WITH P.

VELOCITIES: $v_{P'} = (BP)\omega_{BD} = (0.1368 \text{ m})\,\omega_{BD} \measuredangle 40^\circ$



$$v_P = v_{P'} + v_{P/BD}$$

$$[v_P \measuredangle 20^\circ] = [v_{P'} \measuredangle 40^\circ] + [v_{P/BD} \measuredangle 40^\circ]$$

$v_{P'} = \frac{v_{P/BD}}{\tan 30} = \frac{0.3 \text{ m/s}}{\tan 30^\circ} = 0.5196$ m/s

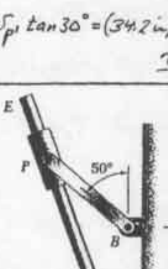
$\omega_{BD} = \frac{v_{P'}}{BP} = \frac{0.5196 \text{ m/s}}{0.1368 \text{ m}} = 3.80$ rad/s

$\underline{\omega}_{BD} = 3.80$ rad/s ↻ ◀

$v_P = \frac{v_{P/BD}}{\sin 30^\circ} = \frac{0.3 \text{ m/s}}{\sin 30^\circ} = 0.6$ m/s

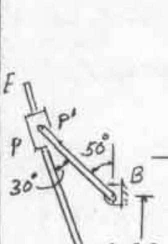
$\omega_{AP} = \frac{v_P}{AP} = \frac{0.6 \text{ m/s}}{0.3064 \text{ m}}$ $\quad \underline{\omega}_{AP} = 1.958$ rad/s ↺ ◀

15.151



GIVEN:
$b = 200$ mm
$\underline{\omega}_B = 9$ rad/s ↻

FIND:
(a) $\underline{\omega}_{AP}$
(b) $\underline{v}_{P/AE}$



GEOMETRY: LAW OF SINES

$$\frac{AP}{\sin 50^\circ} = \frac{BP}{\sin 20^\circ} = \frac{0.2 \text{ m}}{\sin 30^\circ}$$

$AP = 0.3064$ m $\quad BP = 0.1368$ m

NOTE: POINT P′ OF AE COINCIDES WITH P

VELOCITIES: $v_P = (BP)\omega_B = (0.1368 \text{ m})(9 \text{ rad/s}) = 1.231$ m/s $\measuredangle 40^\circ$

$$v_P = v_{P'} + v_{P/AE}$$

$$[1.231 \measuredangle 40^\circ] = [v_{P'} \measuredangle 20^\circ] + [v_{P/AE} \measuredangle 20^\circ]$$

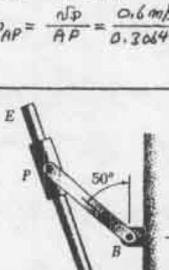
(a) $v_{P'} = v_P \cos 30^\circ = (1.231 \text{ m/s}) \cos 30^\circ = 1.066$ m/s

$\omega_{AE} = \frac{v_{P'}}{AP} = \frac{1.066 \text{ m/s}}{0.3064 \text{ m}}$; $\quad \underline{\omega}_{AE} = 3.48 \frac{\text{rad}}{\text{s}}$ ↻ ◀

(b) $v_{P/AE} = v_P \sin 30^\circ = (1.231 \text{ m/s}) \sin 30^\circ$

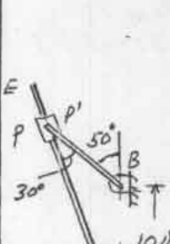
$\underline{v}_{P/AE} = 0.616$ m/s $\measuredangle 70^\circ$ ◀

15.153



GIVEN:
$b = 10$ in.
$\underline{v}_{P/AE} = 15$ in./s $\measuredangle 20^\circ$

FIND:
$\underline{\omega}_{AP}$
$\underline{\omega}_{BP}$



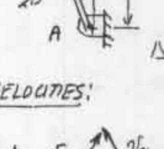
GEOMETRY: LAW OF SINES

$$\frac{AP}{\sin 50^\circ} = \frac{BP}{\sin 20^\circ} = \frac{10 \text{ in}}{\sin 30^\circ}$$

$AP = 15.32$ in. $\quad BP = 6.84$ in.

NOTE: POINT P′ OF AE COINCIDES WITH P

VELOCITIES:



$$v_P = v_{P'} + v_{P/AE}$$

$$[v_P \measuredangle 40^\circ] = [v_{P'} \angle 20^\circ] + [v_{P/AE} \measuredangle 20^\circ]$$

$v_{P'} = \frac{v_{P/AE}}{\tan 30^\circ} = \frac{15 \text{ in/s}}{\tan 30^\circ} = 25.98$ in/s

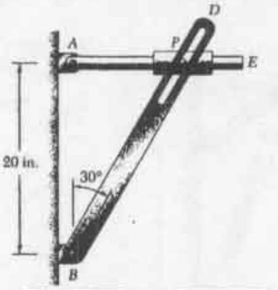
$\omega_{AE} = \frac{v_{P'}}{AP} = \frac{25.98 \text{ in/s}}{15.32 \text{ in.}} = 1.696$ rad/s

$\underline{\omega}_{AE} = 1.696$ rad/s ↻ ◀

$v_P = \frac{v_{P/AE}}{\sin 30^\circ} = \frac{15 \text{ in/s}}{\sin 30^\circ} = 30$ in/s

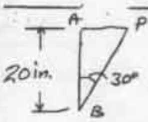
$\omega_{BP} = \frac{v_P}{BP} = \frac{30 \text{ in/s}}{6.84 \text{ in}} = 4.39 \frac{\text{rad}}{\text{s}}$ $\quad \underline{\omega}_{BP} = 4.39$ rad/s ↻ ◀

15.154



GIVEN: $\omega_{AE} = 4$ rad/s ↻, $\omega_{BD} = 1.5$ rad/s ↻

FIND: $\underline{v}_P$

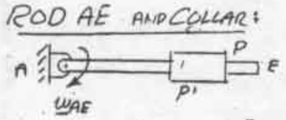


GEOMETRY:

$AP = (20 \text{ in}) \tan 30° = 11.547$ in

$BP = (20 \text{ in})/\cos 30° = 23.094$ in.

ROD AE AND COLLAR:



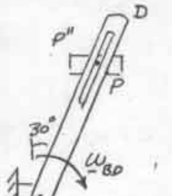
$\underline{v}_P = \underline{v}_{P'} + \underline{v}_{P/AE}$

$\underline{v}_P = [(AP)\omega_{AE} \downarrow] + [v_{P/AE} \rightarrow]$

$\underline{v}_P = [(11.547 \text{ in.})(4 \text{ rad/s}) \downarrow] + [v_{P/AE} \rightarrow]$

$\underline{v}_P = [46.188 \text{ in./s} \downarrow] + [v_{P/AE} \rightarrow]$ (1)

ROD BD AND COLLAR:



$\underline{v}_P = \underline{v}_{P''} + \underline{v}_{P/BD}$ (2)

$\underline{v}_P = [(BP)\omega_{BD} \measuredangle 30°] + [v_{P/BD} \measuredangle 30°]$

$\underline{v}_P = [(23.094 \text{ in.})(1.5 \text{ rad/s}) \measuredangle 30°] + [v_{P/BD} \measuredangle 30°]$

$\underline{v}_P = [34.64 \text{ in/s} \measuredangle 30°] + [v_{P/BD} \measuredangle 30°]$ (3)

$v_{P''} = (BP)\omega_{BD} = (23.094 \text{ in.})(1.5 \text{ rad/s})$; $\underline{v}_{P''} = 34.64 \text{ in/s} \measuredangle 30°$

EQUATE EQS. (1) AND (3):

$[46.188 \text{ in/s} \downarrow] + [v_{P/AE} \rightarrow] = [34.64 \text{ in/s} \measuredangle 30°] + [v_{P/BD} \measuredangle 30°]$

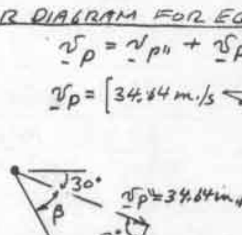
$+\downarrow$ y COMPONENTS: $46.188 = 34.64 \sin 30° + v_{P/BD} \cos 30°$

$v_{P/BD} = +33.33$ in./s $\quad \underline{v}_{P/BD} = 33.33 \text{ in./s} \measuredangle 30°$

VECTOR DIAGRAM FOR EQ (2):

$\underline{v}_P = \underline{v}_{P''} + \underline{v}_{P/BD}$

$\underline{v}_P = [34.64 \text{ in./s} \measuredangle 30°] + [33.33 \text{ in./s} \measuredangle 30°]$



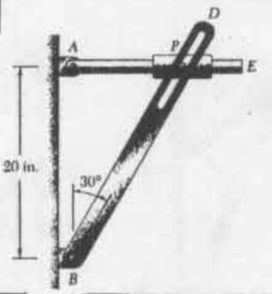
$\beta = \tan^{-1} \dfrac{33.33 \text{ in./s}}{34.64 \text{ in./s}}$

$\beta = 43.9°$

$v_P = 48.07$ in/s

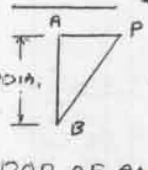
$\underline{v}_P = 48.07 \text{ in/s} \measuredangle 73.9°$ ◀

15.155



GIVEN: $\omega_{AE} = 3.5$ rad/s ↻, $\omega_{BD} = 2.4$ rad/s ↻

FIND: $\underline{v}_P$

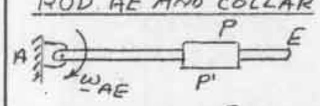


GEOMETRY:

$AP = (20 \text{ in.}) \tan 30° = 11.547$ in.

$BP = (20 \text{ in.})/\cos 30° = 23.094$ in.

ROD AE AND COLLAR:



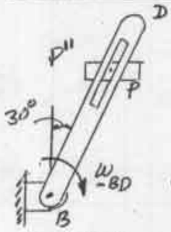
$\underline{v}_P = \underline{v}_{P'} + \underline{v}_{P/AE}$

$\underline{v}_P = [(AP)\omega_{AE} \downarrow] + [v_{P/AE} \rightarrow]$

$\underline{v}_P = [(11.547 \text{ in.})(3.5 \text{ rad/s}) \downarrow] + [v_{P/AE} \rightarrow]$

$\underline{v}_P = [40.415 \text{ in./s} \downarrow] + [v_{P/AE} \rightarrow]$ (1)

ROD BD AND COLLAR:



$\underline{v}_P = \underline{v}_{P''} + \underline{v}_{P/BD}$ (2)

$\underline{v}_P = [(BP)\omega_{BD} \measuredangle 30°] + [v_{P/BD} \measuredangle 30°]$

$\underline{v}_P = [(23.094 \text{ in.})(2.4 \text{ rad/s}) \measuredangle 30°] + [v_{P/BD} \measuredangle 30°]$

$\underline{v}_P = [55.426 \text{ in./s} \measuredangle 30°] + [v_{P/BD} \measuredangle 30°]$ (3)

$v_{P''} = (BP)\omega_{BD} = (23.094 \text{ in.})(2.4 \text{ rad/s})$; $\underline{v}_{P''} = 55.426 \text{ in/s} \measuredangle 30°$

EQUATE EQS (1) AND (3):

$[40.415 \text{ in./s} \downarrow] + [v_{P/AE} \rightarrow] = [55.426 \text{ in/s} \measuredangle 30°] + [v_{P/BD} \measuredangle 30°]$

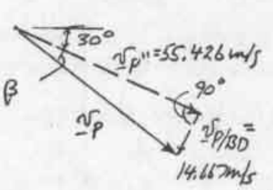
$+\downarrow$ y COMPONENTS: $40.415 = 55.426 \sin 30° + v_{P/BD} \cos 30°$

$v_{P/BD} = +14.667$ in/s $\quad \underline{v}_{P/BD} = 14.667 \text{ in/s} \measuredangle 30°$

VECTOR DIAGRAM FOR EQ (2):

$\underline{v}_P = \underline{v}_{P''} + \underline{v}_{P/BD}$

$\underline{v}_P = [55.426 \text{ in/s} \measuredangle 30°] + [14.667 \text{ in/s} \measuredangle 30°]$



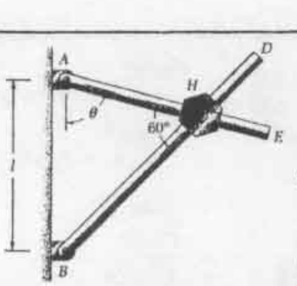
$\beta = \tan^{-1} \dfrac{14.667 \text{ in/s}}{55.426 \text{ in/s}}$

$\beta = 14.82°$

$v_P = 57.3$ in/s

$\underline{v}_P = 57.3 \text{ in/s} \measuredangle 44.8°$ ◀

15.156



GIVEN: $\underline{\omega}_{AE} = \omega$ ↻

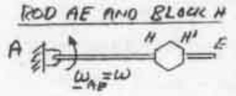
FIND: $\underline{v}_{H/AE}$ AND $\underline{v}_{H/BD}$ WHEN (a) $\theta = 90°$ (b) $\theta = 60°$

ANGLE BETWEEN RODS IS CONSTANT, $\therefore \underline{\omega}_{AE} = \underline{\omega}_{BD} = \underline{\omega}$ ↻

(a) $\theta = 90°$: GEOMETRY

$AH = l/\sqrt{3}$; $BD = 2l/\sqrt{3}$

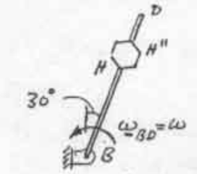
ROD AE AND BLOCK H



$\underline{v}_H = \underline{v}_{H'} + \underline{v}_{H/AE} = (AH)\omega \uparrow + v_{H/AE} \leftrightarrow$

$\underline{v}_H = \frac{l}{\sqrt{3}}\omega \uparrow + v_{H/AE} \leftrightarrow$ (1)

ROD BD AND BLOCK H



$\underline{v}_H = v_{H''} + v_{H/BD}$

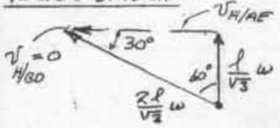
$\underline{v}_H = (BD)\omega \measuredangle 30° + v_{H/BD} \measuredangle 30°$

$\underline{v}_H = \frac{2l}{\sqrt{3}}\omega \measuredangle 30° + v_{H/BD} \measuredangle 30°$ (2)

EQUATE RIGHT-HAND MEMBERS OF EQS. (1) + (2):

$$\frac{l}{\sqrt{3}}\omega \uparrow + v_{H/AE} \leftrightarrow = \frac{2l}{\sqrt{3}}\omega \measuredangle 30° + v_{H/BD} \measuredangle 30°$$

VECTOR DIAGRAM:

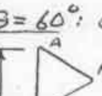


$v_{H/AE} = \frac{l}{\sqrt{3}}\omega \tan 60° = \frac{l}{\sqrt{3}}\omega(\sqrt{3})$

$v_{H/AE} = l\omega \leftarrow$ ◀

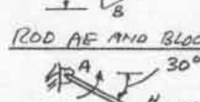
$v_{H/BD} = 0$ ◀

(b) $\theta = 60°$: GEOMETRY:



EQUILATERAL TRIANGLE $AH = BH = l$

ROD AE AND BLOCK H:

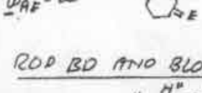


$\underline{v}_H = \underline{v}_{H'} + \underline{v}_{H/AE}$

$= [(AH)\omega \measuredangle 60°] + [v_{H/AE} \measuredangle 30°]$

$\underline{v}_H = [l\omega \measuredangle 60°] + [v_{H/AE} \measuredangle 30°]$ (1)

ROD BD AND BLOCK H:



$v_H = v_{H''} + v_{H/BD}$

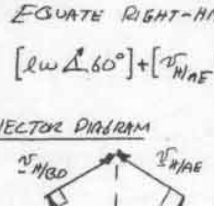
$= [(BH)\omega \measuredangle 60°] + [v_{H/BD} \measuredangle 30°]$

$v_H = [l\omega \measuredangle 60°] + [v_{H/BD} \measuredangle 30°]$ (2)

EQUATE RIGHT-HAND MEMBERS OF EQS. (1) AND (2):

$$[l\omega \measuredangle 60°] + [v_{H/AE} \measuredangle 30°] = [l\omega \measuredangle 60°] + [v_{H/BD} \measuredangle 30°]$$

VECTOR DIAGRAM

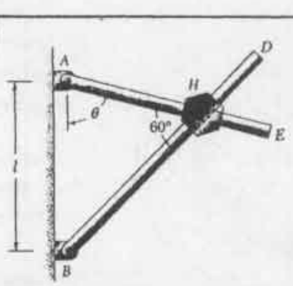


$v_{H/BD} = v_{H/AE} = l\omega \tan 30° = \frac{l\omega}{\sqrt{3}}$

$\underline{v}_{H/AE} = \frac{l\omega}{\sqrt{3}} \measuredangle 30°$ ◀

$\underline{v}_{H/BD} = \frac{l\omega}{\sqrt{3}} \measuredangle 30°$ ◀

15.157

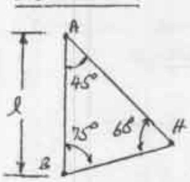


GIVEN: $\omega_{AE} = \omega$ ↻, $\theta = 45°$

FIND: $\underline{v}_{H/AE}$, $\underline{v}_{H/BD}$

ANGLE BETWEEN RODS IS CONSTANT, $\therefore \omega_{AE} = \omega_{BD} = \omega$ ↻

GEOMETRY:



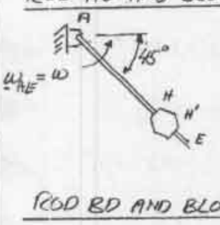
LAW OF SINES

$$\frac{AH}{\sin 75°} = \frac{BH}{\sin 45°} = \frac{l}{\sin 60°}$$

$AH = 1.115\, l$

$BH = 0.8165\, l$

ROD AE AND BLOCK H:

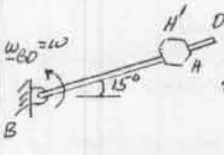


$\underline{v}_H = \underline{v}_{H'} + \underline{v}_{H/AE}$

$= [(AH)\omega \measuredangle 45°] + [v_{H/AE} \measuredangle 45°]$

$\underline{v}_H = [1.115\, l\omega \measuredangle 45°] + [v_{H/AE} \measuredangle 45°]$ (1)

ROD BD AND BLOCK H:



$\underline{v}_H = \underline{v}_{H''} + \underline{v}_{H/BD}$

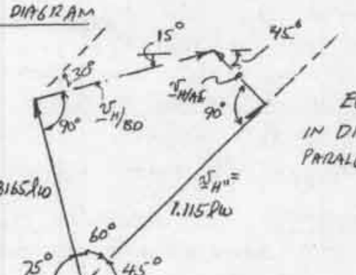
$= [(BH)\omega \measuredangle 75°] + [v_{H/BD} \measuredangle 15°]$

$\underline{v}_H = [0.8165\, l\omega \measuredangle 75°] + [v_{H/BD} \measuredangle 15°]$ (2)

EQUATE RIGHT-HAND MEMBERS OF EQS. (1) AND (2):

$$[1.115\, l\omega \measuredangle 45°] + [v_{H/AE} \measuredangle 45°] = [0.8165\, l\omega \measuredangle 75°] + [v_{H/BD} \measuredangle 15°]$$

VECTOR DIAGRAM



EQUATE COMPONENTS IN DIRECTION PARALLEL TO $v_{H''} \measuredangle 45°$

EQUATE COMPONENTS IN DIRECTION PARALLEL TO $v_{H''} \measuredangle 60°$

$$(0.8165\, l\omega)\cos 60° + v_{H/BD}\cos 30° = 1.115\, l\omega$$

$v_{H/BD} = +0.816\, l\omega$ $\underline{v}_{H/BD} = 0.816\, l\omega \measuredangle 15°$ ◀

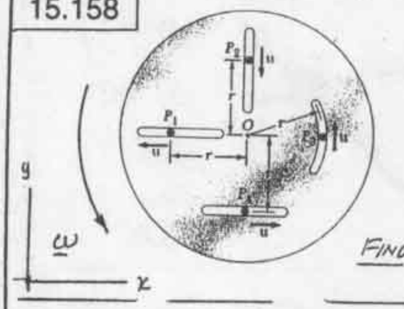
EQUATE COMPONENTS IN DIRECTION PERPENDICULAR TO $v_{H''} \measuredangle 45°$

$(0.8165\, l\omega)\sin 60° - v_{H/BD}\sin 30° = v_{H/AE}$

$(0.8165\, l\omega)\sin 60° - (0.816\, l\omega)\sin 30° = v_{H/AE}$

$v_{H/AE} = +0.299\, l\omega$ $\underline{v}_{H/AE} = 0.299\, l\omega \measuredangle 45°$ ◀

15.158



GIVEN: CONSTANT ANGULAR VELOCITY = $\underline{\omega}$ ↺

CONSTANT SPEED OF PINS RELATIVE TO PLATE = u

FIND: ACCELERATION OF EACH PIN.

FOR EACH PIN: $\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/\mathcal{F}} + \underline{a}_c$

ACCELERATION OF COINCIDING POINT P':

FOR EACH PIN: $\underline{a}_{P'} = r\omega$ TOWARD CENTER $\underline{O}$

ACCELERATION OF PIN WITH RESPECT TO PLATE:

FOR P_1, P_2, AND P_4: $a_{P/\mathcal{F}} = 0$

FOR P_4: $a_{P/\mathcal{F}} = u^2/r$ TOWARD CENTER O

CORIOLIS ACCELERATION FOR EACH PIN $\underline{a}_c = 2u\omega$, WITH $\underline{a}_c$ IN A DIRECTION OBTAINED BY ROTATING $\underline{u}$ THROUGH 90° IN THE SENSE OF $\underline{\omega}$.

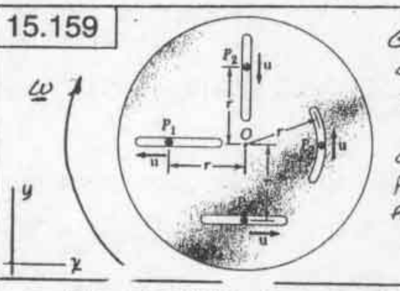
$\underline{a}_1 = [r\omega^2 \rightarrow] + [2u\omega \downarrow]$; $\underline{a}_1 = r\omega^2 \underline{i} - 2u\omega \underline{j}$ ◀

$\underline{a}_2 = [r\omega^2 \downarrow] + [2u\omega \rightarrow]$; $\underline{a}_2 = 2u\omega \underline{i} - r\omega^2 \underline{j}$ ◀

$\underline{a}_3 = [r\omega^2 \leftarrow] + [\frac{u^2}{r} \leftarrow] + [2u\omega \leftarrow]$; $\underline{a}_3 = -(r\omega^2 + \frac{u^2}{r} + 2u\omega)\underline{i}$ ◀

$\underline{a}_4 = [r\omega^2 \uparrow] + [2u\omega \uparrow]$; $\underline{a}_4 = (r\omega^2 + 2u\omega)\underline{j}$ ◀

15.159



GIVEN: CONSTANT ANGULAR VELOCITY = $\underline{\omega}$ ↻

CONSTANT SPEED OF PINS RELATIVE TO PLATE = u

FOR EACH PIN: $\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/\mathcal{F}} + \underline{a}_c$

ACCELERATION OF COINCIDING POINT P':

FOR EACH PIN: $a_{P'} = r\omega^2$ TOWARD CENTER $\underline{O}$

ACCELERATION OF PIN WITH RESPECT TO PLATE:

FOR P_1, P_2, AND P_4: $a_{P/\mathcal{F}} = 0$

FOR P_4: $a_{P/\mathcal{F}} = u^2/r$ TOWARD CENTER O.

CORIOLIS ACCELERATION: FOR EACH PIN, $\underline{a}_c = 2u\omega$, WITH a_c IN A DIRECTION OBTAINED BY ROTATING $\underline{u}$ THROUGH 90° IN THE SENS OF $\underline{\omega}$.

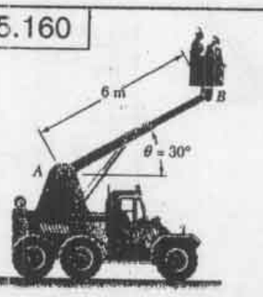
$\underline{a}_1 = [r\omega^2 \rightarrow] + [2u\omega \uparrow]$; $\underline{a}_1 = r\omega^2 \underline{i} + 2u\omega \underline{j}$ ◀

$\underline{a}_2 = [r\omega^2 \downarrow] + [2u\omega \leftarrow]$; $\underline{a}_2 = -2u\omega \underline{i} - r\omega^2 \underline{j}$ ◀

$\underline{a}_3 = [r\omega^2 \leftarrow] + [\frac{u^2}{r} \leftarrow] + [2u\omega \rightarrow]$; $\underline{a}_3 = -(r\omega^2 + \frac{u^2}{r} - 2u\omega)\underline{i}$ ◀

$\underline{a}_4 = [r\omega^2 \uparrow] + [2u\omega \downarrow]$; $\underline{a}_4 = (r\omega^2 - 2u\omega)\underline{j}$ ◀

15.160



GIVEN:

$\underline{\omega}_{AB} = 0.08$ rad/s ↻

$\underline{\alpha}_{AB} = 0$

$\underline{v}_{B/A} = 0.2$ m/s ∠ 30°

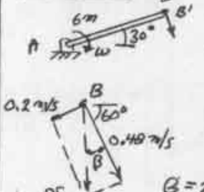
$\underline{a}_{B/A} = 0$

FIND:

(a) $\underline{v}_B$

(b) $\underline{a}_B$

(a) VELOCITY:



$v_{B/\mathcal{F}} = v_{B/A} = 0.2$ m/s ∠ 30°

$\underline{v}_B = \underline{v}_{B'} + \underline{v}_{B/\mathcal{F}}$

$\underline{v}_B = [(AB)\omega \angle 60°] + [0.2\text{ m/s} \angle 30°]$

$= [(6\text{m})(0.08\text{ m/s}) \angle 60°] + [0.2\text{ m/s} \angle 30°]$

$v_B = 0.48\text{ m/s} \angle 60 + 0.2\text{ m/s} \angle 30°$

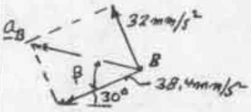
$\beta = 22.6°$ $\underline{v}_B = 0.52\text{ m/s} \angle 82.6°$ ◀

(b) ACCELERATION: $\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/\mathcal{F}} + \underline{a}_c$; $a_{B/\mathcal{F}} = a_{B/A} = 0$

$\underline{a}_{B'} = (AB)\omega^2 = (6\text{m})(0.08\text{ rad/s})^2 = 0.0384\text{ m/s}^2 = 38.4\text{ mm/s}^2 \angle 30°$

$\underline{a}_c = 2u\omega = 2(0.2\text{ m/s})(0.08\text{ rad/s}) = 0.032\text{ m/s}^2 = 32\text{ mm/s}^2 \angle 60°$

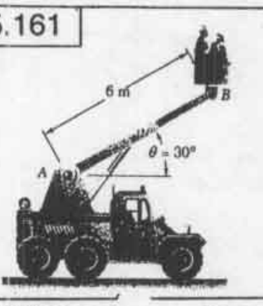
$\underline{a}_B = [38.4\text{ mm/s}^2 \angle 30°] + 0 + [32\text{ mm/s}^2 \angle 60°]$



$\beta = 39.8°$ $a_B = 50.0\text{ mm/s}^2$

$\underline{a}_B = 50.0\text{ mm/s}^2 \angle 9.8°$ ◀

15.161



GIVEN:

$\underline{\omega}_{AB} = 0.08$ rad/s ↻

$\underline{\alpha}_{AB} = 0$

$\underline{v}_{B/A} = 0.2$ m/s ∠ 30°

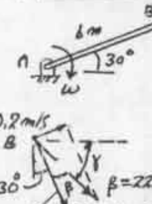
$\underline{a}_{B/A} = 0$

FIND:

(a) $\underline{v}_B$

(b) $\underline{a}_B$

(a) VELOCITY



$\underline{v}_{B/\mathcal{F}} = \underline{v}_{B/A} = 0.2$ m/s ∠ 30°

$\underline{v}_B = \underline{v}_{B'} + \underline{v}_{B/\mathcal{F}}$

$\underline{v}_B = [(AB)\omega \angle 60°] + [v_{B/\mathcal{F}} \angle 30°]$

$= [(6\text{m})(0.08\text{ m/s}) \angle 60°] + [0.2\text{ m/s} \angle 30°]$

$\underline{v}_B = 0.48\text{ m/s} \angle 60° + 0.2\text{ m/s} \angle 30°$

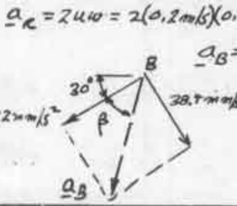
$\gamma = 90° - 30° - 22.6° = 37.4°$; $\underline{v}_B = 0.52\text{ m/s} \angle 37.4°$ ◀

(b) ACCELERATION $\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/\mathcal{F}} + \underline{a}_c$; $a_{B/\mathcal{F}} = 0$

$\underline{a}_{B'} = (AB)\omega^2 = (6\text{m})(0.08\text{ rad/s})^2 = 0.0384\text{ m/s}^2 = 38.4\text{ mm/s}^2 \angle 30°$

$\underline{a}_c = 2u\omega = 2(0.2\text{ m/s})(0.08\text{ rad/s}) = 0.032\text{ m/s}^2 = 32\text{ mm/s}^2 \angle 60°$

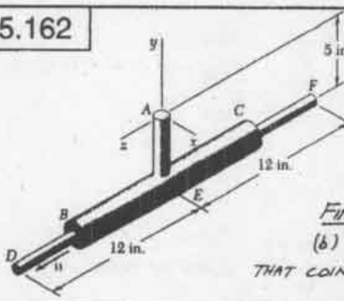
$\underline{a}_B = [38.4\text{ mm/s} \angle 30°] + 0 + [32\text{ mm/s}^2 \angle 60°]$



$\beta = 39.8°$ $a_B = 50.0\text{ mm/s}^2$

$\underline{a}_B = 50.0\text{ mm/s}^2 \angle 69.8°$ ◀

15.162



GIVEN:
$\underline{\omega}_{ABC} = \underline{\omega} = (3\,rad/s)\,\underline{i}$
$\underline{\alpha}_{ABC} = 0$
$\underline{v}_{D/BC} = (16\,in./s)\,\underline{k}$
$\underline{a}_{D/BC} = 0$

FIND: (a) $\underline{a}_D$
(b) ACCEL. OF POINT P OF DF THAT COINCIDES WITH E.

(a) POINT D: $\underline{v}_{D/\mathcal{F}} = \underline{v}_{D/BC} = (16\,in/s)\,\underline{k}$; $a_{D/\mathcal{F}} = 0$

$\overline{AD} = -(5\,in)\,\underline{j} + (12\,in.)\,\underline{k}$

$\underline{a}_{D'} = \underline{\omega}\times\underline{\omega}\times\overline{AD} = -\omega^2(AD) = -(3\,rad/s)^2 AD = +(45\,in./s^2)\,\underline{j} - (108\,in./s^2)\,\underline{k}$

$\underline{a}_c = 2\underline{\omega}\times\underline{v}_{D/\mathcal{F}} = 2[(3\,rad/s)\,\underline{i}]\times(16\,in./s)\,\underline{k} = -(96\,in./s^2)\,\underline{j}$

$$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$$
$$= [(45\,in/s)\,\underline{j} + (108\,in./s)\,\underline{k}] + 0 + [-(96\,in/s)\,\underline{j}]$$
$$\underline{a}_D = -(51\,in/s)\,\underline{j} + (108\,in./s)\,\underline{k}$$ ◀

(b) POINT P OF DF THAT COINCIDES WITH E

$\underline{v}_{P/\mathcal{F}} = \underline{v}_{D/BC} = (16\,in./s)\,\underline{k}$; $\underline{a}_{P/\mathcal{F}} = 0$

$\overline{AE} = -(5\,in.)\,\underline{j}$

$\underline{a}_P = \underline{\omega}\times\underline{\omega}\times\overline{AE} = -\omega^2 AE = -(3\,rad/s)^2 AE = (45\,in./s^2)\,\underline{j}$

$\underline{a}_c = 2\underline{\omega}\times\underline{v}_{D/\mathcal{F}} = 2[(3\,rad/s)\,\underline{i}]\times(16\,in/s)\,\underline{k} = -(96\,in./s^2)\,\underline{j}$

$$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/\mathcal{F}} + \underline{a}_c$$
$$= [(45\,in./s^2)\,\underline{j}] + 0 + [-(96\,in./s^2)\,\underline{j}]$$
$$\underline{a}_P = -(51\,in./s^2)\,\underline{j}$$ ◀

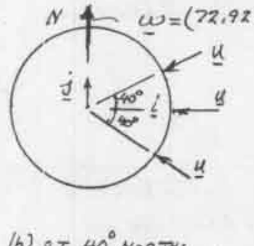
15.164

GIVEN: ELEVATOR MOVES DOWNWARD AT 40 ft/s
FIND: CORIOLIS ACCELERATION OF ELEVATOR IF IT IS LOCATED AT: (a) EQUATOR, (b) 40° NORTH, (c) 40° SOUTH.

EARTH MAKES ONE REVOLUTION IN $23\,h\,56\,m = 23.933\,h$

$$\omega = \frac{2\pi\,rad}{(23.933\,h)(3600\,s/h)} = 72.92\times10^{-6}\,rad/s \quad *$$

$\underline{\omega} = (72.92\times10^{-6}\,rad/s)\,\underline{j}$



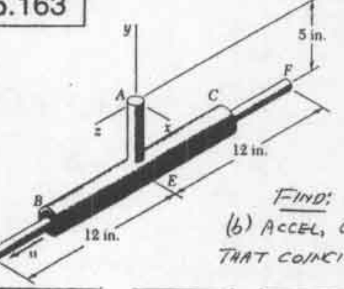
(a) AT EQUATOR:
$\underline{a}_c = 2\,\underline{\omega}\times\underline{u}$
$= 2(72.92\times10^{-6}\,rad/s)\,\underline{j}\times(-40\,ft/s)\,\underline{i}$
$\underline{a}_c = (5.83\times10^{-3}\,ft/s^2)\,\underline{k}$
$a_c = 5.83\times10^{-3}\,ft/s^2$ WEST ◀

(b) AT 40° NORTH:
$\underline{u} = 40\,ft/s\,(-\cos40°\,\underline{i} - \sin40°\,\underline{j})$
$\underline{a}_c = 2\underline{\omega}\times\underline{u} = 2(72.92\times10^{-6}\,rad/s)\,\underline{j}\times(40\,ft/s)(-\cos40°\,\underline{i} - \sin40°\,\underline{j})$
$\underline{a}_c = (4.47\,ft/s^2)\,\underline{k}$ $\qquad a_c = 4.47\,ft/s^2$ WEST ◀

(c) AT 40° SOUTH:
$\underline{u} = 40\,ft/s\,(-\cos40°\,\underline{i} + \sin40°\,\underline{j})$
$\underline{a}_c = 2\underline{\omega}\times\underline{u} = 2(72.92\times10^{-6}\,rad/s)\,\underline{j}\times(40\,ft/s)(-\cos40°\,\underline{i} + \sin40°\,\underline{j})$
$\underline{a}_c = (4.47\times10^{-3})\,ft/s^2$ $\qquad a_c = (4.47\times10^{-3})\,ft/s^2$ WEST ◀

* NOTE: EARTH ROTATES COUNTER CLOCKWISE WHEN OBSERVED FROM ABOVE THE NORTH POLE.

15.163



GIVEN:
$\underline{\omega}_{ABC} = \underline{\omega} = (3\,rad/s)\,\underline{j}$
$\underline{\alpha}_{ABC} = 0$
$\underline{v}_{D/BC} = (16\,in./s)\,\underline{k}$
$\underline{a}_{D/BC} = 0$

FIND: (a) $\underline{a}_D$
(b) ACCEL. OF POINT P OF DF THAT COINCIDES WITH E

(a) POINT D: $\underline{v}_{D/\mathcal{F}} = \underline{v}_{D/BC} = (16\,in./s)\,\underline{k}$; $\underline{a}_{D/BC} = 0$

$\overline{AD} = -(5\,in.)\,\underline{j} + (12\,in.)\,\underline{k}$

$\underline{v}_{D'} = \underline{\omega}\times\overline{AD} = (3\,rad/s)\,\underline{j}\times[-(5\,in.)\,\underline{j} + (12\,in.)\,\underline{k}] = (36\,in/s)\,\underline{i}$

$\underline{a}_{D'} = \underline{\omega}\times\underline{v}_{D'} = (3\,rad/s)\,\underline{j}\times(36\,in/s)\,\underline{i} = -(108\,in./s^2)\,\underline{k}$

$\underline{a}_c = 2\underline{\omega}\times\underline{v}_{D/\mathcal{F}} = (3\,rad/s)\,\underline{j}\times(16\,in/s)\,\underline{k} = (96\,in./s^2)\,\underline{i}$

$$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$$
$$= -(108\,in./s^2)\,\underline{k} + 0 + (96\,in./s^2)\,\underline{i}$$
$$\underline{a}_D = (96\,in./s^2)\,\underline{i} - (108\,in./s^2)\,\underline{k}$$ ◀

(b) POINT P OF DF THAT COINCIDES WITH E

$\underline{v}_{P/\mathcal{F}} = \underline{v}_{D/BC} = (16\,in./s)\,\underline{k}$; $\underline{a}_{P/\mathcal{F}} = 0$

$\overline{AE} = -(5\,in.)\,\underline{j}$; $\underline{v}_{P'} = \underline{\omega}\times\overline{AE} = (3\,rad/s)\,\underline{j}\times(-5\,in/s)\,\underline{j} = 0$

$\underline{a}_{P'} = \underline{\omega}\times\underline{v}_{E'} = 0$

$\underline{a}_c = 2\,\underline{\omega}\times\underline{v}_{E/\mathcal{F}} = 2(3\,rad/s)\,\underline{j}\times(16\,in/s)\,\underline{k} = (96\,in./s^2)\,\underline{i}$

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/\mathcal{F}} + \underline{a}_C$

$\underline{a}_P = 0 + 0 + (96\,in/s^2)\,\underline{i}$ $\qquad \underline{a}_P = (96\,in./s^2)\,\underline{i}$ ◀

15.165

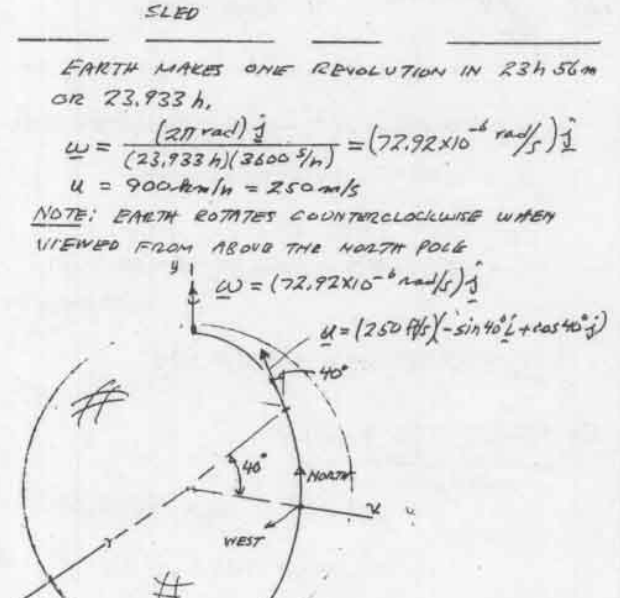
GIVEN: TEST SLED MOVING DUE NORTH AT 900 km/h. AT 40° NORTH LATITUDE.
FIND: CORIOLIS ACCELERATION OF SLED

EARTH MAKES ONE REVOLUTION IN 23 h 56 m OR 23.933 h.

$$\underline{\omega} = \frac{(2\pi\,rad)\,\underline{j}}{(23.933\,h)(3600\,s/h)} = (72.92\times10^{-6}\,rad/s)\,\underline{j}$$

$u = 900\,km/h = 250\,m/s$

NOTE: EARTH ROTATES COUNTERCLOCKWISE WHEN VIEWED FROM ABOVE THE NORTH POLE

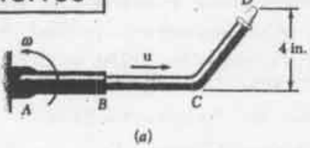


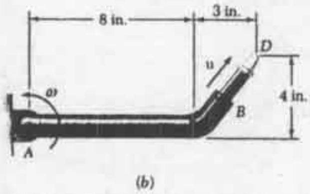
$\underline{a}_c = 2\underline{\omega}\times\underline{u} = 2(72.92\times10^{-6}\,rad/s)\,\underline{j}\times(250\,m/s)(-\sin40°\,\underline{i} + \cos40°\,\underline{j})$

$\underline{a}_C = (23.4\times10^{-3})\,m/s^2\,\underline{k} = (23.4\times10^{-3})\,m/s^2$ WEST

$a_c = (23.4\times10^{-3})\,ft/s^2$ TO LEFT OF SLED ◀

15.166

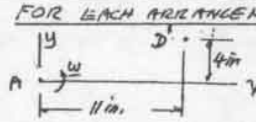




GIVEN:
$\underline{\omega}_{AB} = \underline{\omega} = 2.4\ rad/s$ ↺
$\underline{\alpha}_{AB} = 0$
$u = 10\ in./s$
$\dot{u} = 0$

FIND: $\underline{a}_D$ FOR EACH ARRANGEMENT

FOR EACH ARRANGEMENT: $\underline{\omega} = (2.4\ rad/s)\underline{k}$



$\overline{AD} = (11\ in.)\underline{i} + (4\ in.)\underline{j}$

$\underline{a}_{D'} = \underline{\omega} \times \underline{\omega} \times \overline{AD} = -\omega^2 \overline{AD}$
$= -(2.4\ rad/s)^2[(11\ in)\underline{i} + (4\ in)\underline{j}]$

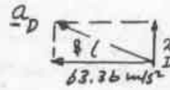
$\underline{a}_{D'} = -(63.36\ in/s^2)\underline{i} - (23.04\ in/s^2)\underline{j}$

(a) $\underline{u} = \underline{v}_{D/\mathcal{F}} = (10\ in./s)\underline{i} \qquad a_{D/\mathcal{F}} = 0$

$\underline{a}_c = 2\underline{\omega} \times \underline{v}_{D/\mathcal{F}} = 2(2.4\ rad/s)\underline{k} \times (10\ in./s)\underline{i} = (48\ in/s)\underline{j}$

$a_D = a_{D'} + a_{D/\mathcal{F}} + a_c$
$= [-(63.36\ in/s^2)\underline{i} - (23.04\ in/s^2)\underline{j}] + 0 + (48\ in/s^2)\underline{j}$

$a_D = -(63.36\ in/s^2)\underline{i} + (24.96\ in/s^2)\underline{j}$



$\beta = 21.5^\circ \qquad a_D = 68.1\ in/s^2$

$\underline{a}_D = 68.1\ in/s^2 \measuredangle 21.5^\circ$ ◀

(b) $\underline{u} = 10\ in./s^2$ (slope 3, 4, 5)

$\underline{u} = \frac{3}{5}(10)\underline{i} + \frac{4}{5}(10)\underline{j}$

$\underline{u} = \underline{v}_{D/\mathcal{F}} = (6\ in./s)\underline{i} + (8\ in./s)\underline{j}$; $\underline{a}_{D/\mathcal{F}} = 0$

$\underline{a}_c = 2\underline{\omega} \times \underline{v}_{D/\mathcal{F}} = 2(2.4\ rad/s)\underline{k} \times [(6\ in/s)\underline{i} + (8\ in/s)\underline{j}]$

$\underline{a}_c = -(38.4\ in./s^2)\underline{i} + (28.8\ in/s^2)\underline{j}$

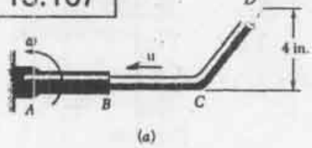
$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$
$= -(63.36\ in/s^2)\underline{i} - (23.04\ in./s^2)\underline{j} + 0 - (38.4\ in/s^2)\underline{i} + (28.8\ in/s^2)\underline{j}$

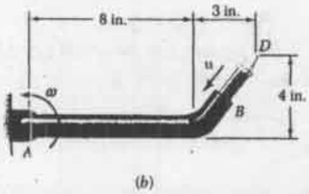
$\underline{a}_D = -(101.76\ in/s^2)\underline{i} + (5.76\ in./s^2)\underline{j}$

$\beta = 3.2^\circ \qquad a_D = 101.9\ in./s^2$

$\underline{a}_D = 101.9\ in./s^2 \measuredangle 3.2^\circ$ ◀

15.167

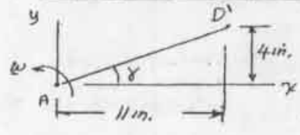




GIVEN:
$\underline{\omega}_{AB} = \underline{\omega} = 2.4\ rad/s$ ↺
$\alpha_{AB} = 0$
$\underline{v} = 10\ in/s$
$\dot{v} = 0$

FIND: $\underline{a}_D$ FOR EACH ARRANGEMENT

FOR EACH ARRANGEMENT:



$AD = (4^2 + 11^2)^{1/2} = 11.705\ in$
$\gamma = 19.98^\circ$

$\underline{a}_{D'} = (AD)\omega^2 \measuredangle 19.98^\circ$
$= (11.705\ in)(2.4\ rad/s)^2 \measuredangle 19.98^\circ$
$\underline{a}_{D'} = 67.42\ in/s^2 \measuredangle 19.98^\circ$

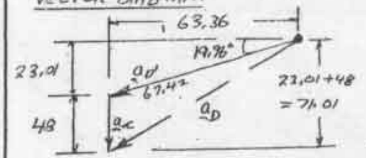
(a) $\underline{u} = v_{D/\mathcal{F}} = 10\ in./s \rightarrow$; $a_{D/\mathcal{F}} = 0$

$\underline{a}_c = 2\omega v_{D/\mathcal{F}} = 2(2.4\ rad/s)(10\ in/s) = 48\ in/s^2$

IN DIRECTION 90° ↺ FROM $v_{D/\mathcal{F}}$: $\underline{a}_c = 48\ in./s^2 \downarrow$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$
$\underline{a}_D = 67.42\ in/s^2 \measuredangle + 0 + 48\ in./s^2 \downarrow$

VECTOR DIAGRAM



$\underline{a}_D = 95.2\ in/s^2 \measuredangle 48.3^\circ$ ◀

(b) $\underline{u} = 10\ in./s$ (slope 3, 4)

$\underline{u} = \underline{v}_{D/\mathcal{F}} = 10\ in/s$ (53.13°)

$\underline{a}_{D/\mathcal{F}} = 0$

$a_c = 2\omega v_{D/\mathcal{F}} = 2(2.4\ rad/s)(10\ in./s) = 48\ in/s^2$

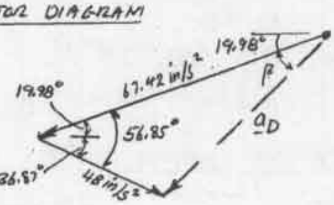
IN DIRECTION 90° ↺ FROM $v_{D/\mathcal{F}}$

$\underline{a}_c = 48\ in/s^2 \measuredangle 53.13^\circ = 48\ in/s^2 \measuredangle 36.87^\circ$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$

$\underline{a}_D = 67.42\ in/s^2 \measuredangle 19.98^\circ + 0 + 48\ in/s^2 \measuredangle 36.87^\circ$

VECTOR DIAGRAM

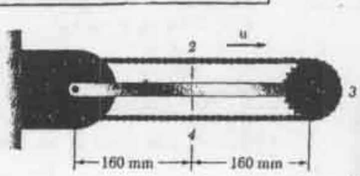


LAW OF COSINES
$a_D^2 = (67.42)^2 + (48)^2 - 2(67.42)(48)\cos 56.85^\circ$; $a_D = 57.53\ in/s^2$

LAW OF SINES
$\frac{a_D}{\sin 56.85} = \frac{48}{\sin\beta}$; $\sin\beta = \frac{48}{57.53}\sin 56.85$; $\beta = 44.3^\circ$

$a_D = 57.53 \measuredangle (19.98^\circ + 44.3^\circ)$; $\underline{a}_D = 57.5\ in/s^2 \measuredangle 64.3^\circ$ ◀

15.168 and 15.169



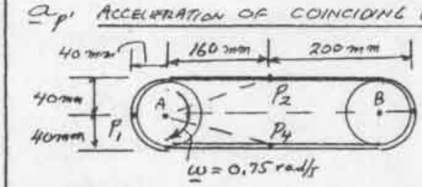
GIVEN:
$\omega_{AB} = \omega = 0.75$ rad/s
$\alpha_{AB} = 0$
$u = 80$ mm/s
$\dot{u} = 0$

FIND: ACCEL. OF LINKS
PROB. 15.168: LINKS 1 + 2
PROB. 15.169: LINKS 3 + 4

$$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/\mathcal{F}} + \underline{a}_c \qquad (1)$$

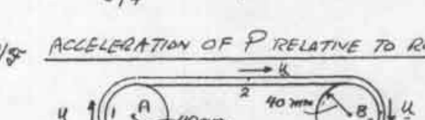
EACH TERM IS COMPUTED SEPARATELY FOR EACH LINK

$\underline{a}_{P'}$ ACCELERATION OF COINCIDING POINT P'



$a_{P_1} = (AP_1)\omega^2 = 40 \times 0.75^2 = 22.5$ mm/s² →
$a_{P_2} = (AP_2)\omega^2 = 160 \times 0.75^2 \leftarrow + 40 \times 0.75^2 \downarrow$
$a_{P_2} = 90$ mm/s² ← + 22.5 mm/s² ↓
$a_{P_3} = (AP_3)\omega^2 = (160 + 200)0.75^2 = 202.5$ mm/s² ←
$a_{P_4} = (AP_4)\omega^2 = 160 \times 0.75^2 \leftarrow + 40 \times 0.75^2 \uparrow$
$a_{P_4} = 90$ mm/s² ← + 22.5 mm/s² ↑

$\underline{a}_{P/\mathcal{F}}$ ACCELERATION OF P RELATIVE TO ROTATING FRAME



$u = 80$ mm/s

$a_{P_1/\mathcal{F}} = u^2/r = (80)^2/40 = 160$ mm/s² →
$a_{P_2/\mathcal{F}} = a_{P_4/\mathcal{F}} = 0$
$a_{P_3/\mathcal{F}} = u^2/r = (80)^2/40 = 160$ mm/s² ←

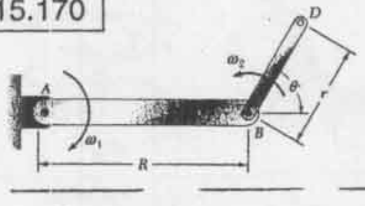
$\underline{a}_c$ CORIOLIS ACCELERATION
MAGNITUDE FOR ALL LINKS
$a_c = 2\omega u = 2(0.75 \text{ rad/s})(80 \text{ mm/s}) = 120$ mm/s²
DIRECTION ROTATE $\underline{u}$ THROUGH 90° ↻
LINK 1: $a_c = 120$ mm/s² →
LINK 2: $a_c = 120$ mm/s² ↓
LINK 3: $a_c = 120$ mm/s² ←
LINK 4: $a_c = 120$ mm/s² ↑

$$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/\mathcal{F}} + \underline{a}_c$$

PROBLEM 15.168:
LINK 1: $\underline{a}_{P_1} = (22.5$ mm/s² →$) + (160$ mm/s² →$) + (120$ mm/s² →$)$
$\underline{a}_{P_1} = 302.5$ mm/s² →
LINK 2: $\underline{a}_{P_2} = (90$ mm/s² ← + 22.5 mm/s² ↓$) + 0 + (120$ mm/s² ↓$)$
$\underline{a}_{P_2} = 90$ mm/s² ← + 142.5 mm/s² ↓ = 168.5 mm/s² ⦫ 57.7°

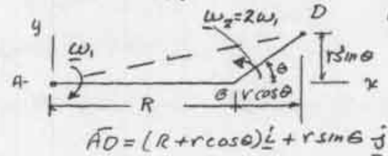
PROBLEM 15.169:
LINK 3: $\underline{a}_{P_3} = (202.5$ mm/s² ←$) + (160$ mm/s² ←$) + (120$ mm/s² ←$)$
$\underline{a}_{P_3} = 482.5$ mm/s² ←
LINK 4: $\underline{a}_{P_4} = (90$ mm/s² ← + 22.5 mm/s² ↑$) + 0 + (120$ mm/s² ↑$)$
$\underline{a}_{P_4} = 90$ mm/s² ← + 142.5 mm/s² ↑ = 168.5 mm/s² ⦪ 57.7°

15.170



GIVEN:
$\omega_2 = 2\omega_1$
SHOW THAT $\underline{a}_D$ PASSES THROUGH A AND THAT THE RESULT IS INDEPENDENT OF R, r, θ

$$a_D = a_{D'} + a_{D/\mathcal{F}} + a_c$$



$\overline{BD} = r\cos\theta\,\underline{i} + r\sin\theta\,\underline{j}$
$\underline{\omega}_1 = -\omega_1\underline{k}$
$\underline{\omega}_2 = 2\omega_1\underline{k}$

$$\overline{AD} = (R + r\cos\theta)\underline{i} + r\sin\theta\,\underline{j}$$

$$\underline{a}_{D'} = -\omega_1^2(\overline{AD}) = -(R + r\cos\theta)\omega_1^2\underline{i} - r\omega_1^2\sin\theta\,\underline{j}$$

$$\underline{v}_{P/\mathcal{F}} = \underline{\omega}_2 \times (\overline{BD}) = 2\omega_1\underline{k} \times (r\cos\theta\,\underline{i} + r\sin\theta\,\underline{j})$$
$$\underline{v}_{P/\mathcal{F}} = -2\omega_1 r\sin\theta\,\underline{i} + 2\omega_1 r\cos\theta\,\underline{j}$$

$$\underline{a}_{D/\mathcal{F}} = -\omega_2^2(\overline{BD}) = -(2\omega_1)^2(r\cos\theta\,\underline{i} + r\sin\theta\,\underline{j})$$
$$a_{D/\mathcal{F}} = -4\omega_1^2 r\cos\theta\,\underline{i} - 4\omega_1^2 r\sin\theta\,\underline{j}$$

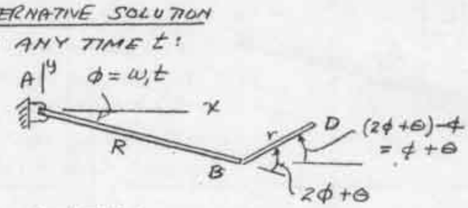
$$\underline{a}_c = 2\omega_1 \times \underline{v}_{P/\mathcal{F}} = 2(-\omega_1\underline{k}) \times (-2\omega_1 r\sin\theta\,\underline{i} + 2\omega_1 r\cos\theta\,\underline{j})$$
$$\underline{a}_c = +4\omega_1^2 r\cos\theta\,\underline{i} + 4\omega_1^2 r\sin\theta\,\underline{j}$$

$$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$$
$$= -(R + r\cos\theta)\omega_1^2\underline{i} - r\omega_1^2\sin\theta\,\underline{j} - 4\omega_1^2 r\cos\theta\,\underline{i}$$
$$- 4\omega_1^2 r\sin\theta\,\underline{j} + 4\omega_1^2 r\cos\theta\,\underline{i} + 4\omega_1^2 r\sin\theta\,\underline{j}$$

$$\underline{a}_D = -\omega_1^2\left[(R + r\cos\theta)\underline{i} - r\sin\theta\,\underline{j}\right].$$

$$\underline{a}_D = -\omega_1^2(\overline{AD}) \qquad \text{QED}$$

ALTERNATIVE SOLUTION
AT ANY TIME t:



FOR POINT D:
$$x = R\cos\phi + r\cos(\phi + \theta) = R\cos\omega_1 t + r\cos(\omega_1 t + \theta)$$
$$y = -R\sin\phi + r\sin(\phi + \theta) = -R\sin\omega_1 t + r\sin(\omega_1 t + \theta)$$

$$\dot{x} = -R\omega_1\sin\omega_1 t - r\omega_1\sin(\omega_1 t + \theta)$$
$$\dot{y} = -R\omega_1\cos\omega_1 t + r\omega_1\cos(\omega_1 t + \theta)$$

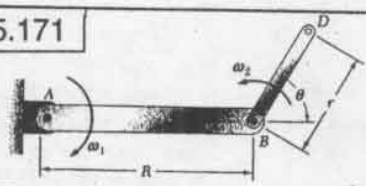
$$\ddot{x} = -R\omega_1^2\cos\omega_1 t - r\omega_1^2\cos(\omega_1 t + \theta)$$
$$\ddot{y} = +R\omega_1^2\sin\omega_1 t - r\omega_1^2\sin(\omega_1 t + \theta)$$

$$\ddot{x} = -\omega_1^2(R\cos\omega_1 t + r\cos(\omega_1 t + \theta)) = -\omega_1^2 x$$
$$\ddot{y} = -\omega_1^2(-R\sin\omega_1 t + r\sin(\omega_1 t + \theta)) = -\omega_1^2 y$$

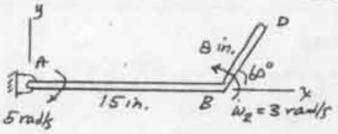
$$\therefore\ \underline{a}_D = -\omega_1^2(AD)$$

WHEN $\omega_2 = 2\omega_1$, $\underline{a}_D$ PASSES THROUGH POINT A DURING ENTIRE MOTION

15.171



GIVEN: $R = 15$ in. $r = 8$ in., $\theta = 60°$
$\underline{\omega}_1 = 5$ rad/s ↻
$\underline{\omega}_2 = 3$ rad/s ↺
FIND: $\underline{a}_D$



$\underline{\omega}_1 = -(5 \text{ rad/s})\underline{k}$

$\underline{\omega}_2 = (3 \text{ rad/s})\underline{k}$

$\underline{r}_{D/B} = (4 \text{ in.})\underline{i} + (6.928 \text{ in.})\underline{j}$

$\underline{r}_D = \overline{AD} = (15 + 8\cos 60°)\,\underline{i} + 8\sin 60°\,\underline{j} = (19 \text{ in.})\underline{i} + (6.928 \text{ in.})\underline{j}$

$\underline{a}_{D'}$ ACCELERATION OF COINCIDING POINT D'

$\underline{a}_D = -\omega_1^2 r_D = -5^2(19\underline{i} + 6.928\underline{j}) = -(475 \text{ in/s}^2)\underline{i} - (173.2 \text{ in/s}^2)\underline{j}$

MOTION OF D RELATIVE TO FRAME

$\underline{v}_{D/\mathcal{F}} = \underline{\omega}_2 \times \underline{r}_{D/B} = (3\underline{k}) \times (4\underline{i} + 6.928\underline{j}) = -(20.78 \text{ in/s})\underline{i} + (12 \text{ in/s})\underline{j}$

$a_{D/\mathcal{F}} = -\omega_2^2\, \underline{r}_{D/B} = -3^2(4\underline{i} + 6.928\underline{j}) = -(36 \text{ in/s}^2)\underline{i} - (62.35 \text{ in/s}^2)\underline{j}$

$\underline{a}_c$ CORIOLIS ACCELERATION

$\underline{a}_c = 2\,\underline{\omega}_1 \times \underline{v}_{D/\mathcal{F}} = 2(-5 \text{ rad/s})\underline{k} \times [-(20.78 \text{ in/s})\underline{i} + (12 \text{ in/s})\underline{j}]$

$\underline{a}_c = (120 \text{ in/s}^2)\underline{i} + (207.8 \text{ in/s}^2)\underline{j}$

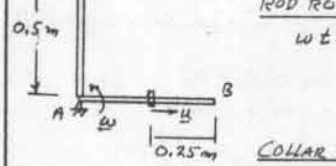
$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$

$= -(475 \text{ in/s}^2)\underline{i} - (173.2 \text{ in/s}^2)\underline{j} - (36 \text{ in/s}^2)\underline{i}$
$\quad - (62.35 \text{ in/s}^2)\underline{j} + (120 \text{ in/s}^2)\underline{i} + (207.8 \text{ in/s}^2)\underline{j}$

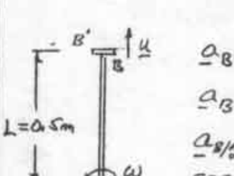
$\underline{a}_D = -(391 \text{ in/s}^2)\underline{i} - (27.75 \text{ in/s}^2)\underline{j}$

$a_D = 392 \text{ in/s}^2 \measuredangle 4.05°$ ◀

15.172



GIVEN: $\underline{\omega} = 20$ rpm ↺
$r = 250$ mm WHEN $\theta = 0$
AND COLLAR REACHES B
WHEN $\theta = 90°$
FIND: a_D JUST AS COLLAR REACHES B.



$\underline{\omega} = 20 \text{ rpm} = 2.094 \text{ rad/s}$ ↺

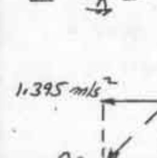
ROD ROTATES $90° = \pi/2$ radians

$\omega t = \frac{\pi}{2}$; $t = \frac{\pi}{2\omega} = \frac{\pi}{2(2.094 \text{ rad/s})}$

$t = 0.75$ s

COLLAR MOVES 0.25 m IN $t = 0.75$ s

$ut = 0.25$ m; $u = \frac{0.25 \text{ m}}{0.75 \text{ s}} = 0.333$ m/s



$\underline{a}_{B'}$ ACCEL. OF COINCIDING POINT B'

$\underline{a}_{B'} = L\omega^2 \downarrow = 0.5 \text{ m}(2.094 \text{ rad/s})^2 = 2.19 \text{ m/s}^2 \downarrow$

$\underline{a}_{B/\mathcal{F}} = 0$, SINCE u = CONSTANT

CORIOLIS ACCELERATION

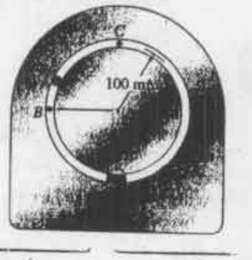
$\underline{a}_c = 2u\omega \leftarrow = 2(0.333 \text{ m/s})(2.094 \text{ rad/s})$

$\underline{a}_c = 1.395 \text{ m/s}^2 \leftarrow$

$a_B^2 = (2.19 \text{ m/s}^2)^2 + (1.395 \text{ m/s}^2)^2$

$a_B = 2.60 \text{ m/s}^2$ ◀

15.173 and 15.174

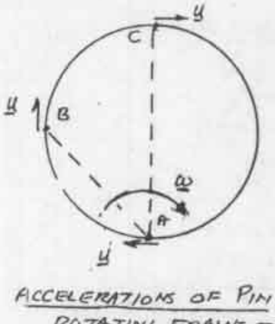


GIVEN: $\omega = 3$ rad/s ↺
$u = 90$ mm/s, $\dot{u} = 0$

PROBLEM 15.173: FOR $\alpha = 0$
FIND: $\underline{a}_P$ WHEN PIN IS AT
(a) POINT A, (b) POINT B, (c) POINT C.

PROBLEM 15.174: SOLVE SAME PROBLEM IF $\alpha = 5 \text{ rad/s}^2$ ↺ AS PIN IS AT POINTS A, B, + C.

PROBLEM 15.173: $\overline{AB} = 0.1 \text{ m}\uparrow + 0.1 \text{ m} \leftarrow$; $\overline{AC} = 0.2 \text{ m}\uparrow$



ACCELERATIONS OF COINCIDING POINTS

$\underline{a}_{A'} = 0$

$\underline{a}_{B'} = -\omega^2(\overline{AB}) = -(3)^2(\overline{AB})$
$= 0.9 \text{ m/s}^2 \rightarrow + 0.9 \text{ m/s}^2 \downarrow$

$\underline{a}_{C'} = -\omega^2(\overline{AC}) = -(3)^2(\overline{AC})$
$= 1.8 \text{ m/s}^2 \downarrow$

ACCELERATIONS OF PIN RELATIVE TO THE ROTATING FRAME $= u^2/r = (0.09 \text{ m/s})^2/(0.1 \text{ m}) = 0.081 \text{ m/s}^2$

WE HAVE:

$\underline{a}_{A/\mathcal{F}} = 0.081 \text{ m/s}^2 \uparrow$

$\underline{a}_{B/\mathcal{F}} = 0.081 \text{ m/s}^2 \rightarrow$

$\underline{a}_{C/\mathcal{F}} = 0.081 \text{ m/s}^2 \downarrow$

CORIOLIS ACCELERATIONS

POINT A: $\underline{a}_c = 2u\omega = 2(0.09 \text{ m/s})(3 \text{ rad/s}) = 0.54 \text{ m/s}^2 \uparrow$

POINT B: SAME MAGNITUDE $\quad \underline{a}_c = 0.54 \text{ m/s}^2 \rightarrow$

POINT C: " " $\quad \underline{a}_c = 0.54 \text{ m/s}^2 \downarrow$

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/\mathcal{F}} + \underline{a}_c$

POINT A: $\underline{a}_A = 0 + 0.081 \text{ m/s}^2 \uparrow + 0.54 \text{ m/s}^2 \uparrow = 0.621 \text{ m/s}^2 \uparrow$ ◀

POINT B: $\underline{a}_B = 0.9 \text{ m/s}^2 \rightarrow + 0.9 \text{ m/s}^2 \downarrow + 0.081 \text{ m/s}^2 \rightarrow + 0.54 \text{ m/s}^2 \rightarrow$
$= 1.521 \text{ m/s}^2 \rightarrow + 0.9 \text{ m/s}^2 \downarrow = 1.767 \text{ m/s}^2 \measuredangle 30.6°$ ◀

POINT C: $\underline{a}_C = 1.8 \text{ m/s}^2 \downarrow + 0.081 \text{ m/s}^2 \downarrow + 0.54 \text{ m/s}^2 \downarrow = 2.42 \text{ m/s}^2 \downarrow$ ◀

PROBLEM 15.174 WE NOW ALSO HAVE $\underline{\alpha} = 5 \text{ rad/s}^2$ ↺

THIS ADDITION CHANGES ONLY THE ACCELERATIONS OF THE COINCIDING POINT BY ADDING THE TERM $\underline{\alpha} \times \underline{r}$

AT POINT A: $r = 0$ and $\underline{\alpha} \times \underline{r} = 0$

AT POINT B: $\alpha r = \alpha(AB) = (5 \text{ rad/s}^2)AB$
$= 0.5 \text{ m/s}^2 \leftarrow + 0.5 \text{ m/s}^2 \downarrow$

AT POINT C: $\alpha r = \alpha(AC) = (5 \text{ rad/s}^2)(0.2 \text{ m}) = 1 \text{ m/s}^2 \leftarrow$

WE NOW ADD αr TO RESULTS OF PROB. 15.173

POINT A: $\underline{a}_A = 0 + 0.621 \text{ m/s}^2 \uparrow \qquad \underline{a}_A = 0.621 \text{ m/s}^2 \uparrow$ ◀

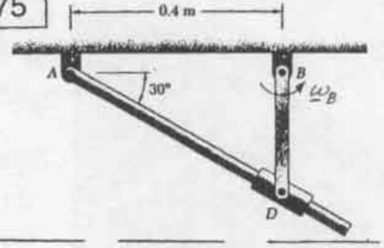
POINT B: $\underline{a}_B = 0.5 \text{ m/s}^2 \leftarrow + 0.5 \text{ m/s}^2 \downarrow + 1.521 \text{ m/s}^2 \rightarrow + 0.9 \text{ m/s}^2 \downarrow$
$= 1.021 \text{ m/s}^2 \rightarrow + 1.4 \text{ m/s}^2 \downarrow$

$\underline{a}_B = 1.733 \text{ m/s}^2 \measuredangle 53.9°$ ◀

POINT C: $\underline{a}_C = 1 \text{ m/s}^2 \leftarrow + 2.42 \text{ m/s}^2 \downarrow$

$\underline{a}_C = 2.62 \text{ m/s}^2 \measuredangle 67.6°$ ◀

15.175



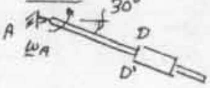
GIVEN:
$\underline{\omega}_B = 6$ rad/s ↻
$\underline{\alpha}_B = 0$

FIND:
$\underline{\omega}_A$
$\underline{\alpha}_A$

GEOMETRY: $BD = (0.4\,m)\tan 30° = 0.23094\,m$
$AD = (0.4\,m)/\cos 30° = 0.4618\,m$

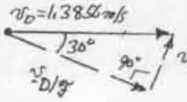
VELOCITY: ROD BD: $\underline{v}_D = (BD)\omega_B = (0.2309\,m)(6\,rad/s) = 1.3856\,m/s \rightarrow$

ROD AD



$v_{D'} = (AD)\omega_A = (0.4618\,m)\,\omega_A \measuredangle 60°$ (1)
$\underline{v}_{D/\mathcal{F}} = \searrow 30°$

VECTOR DIAGRAM:

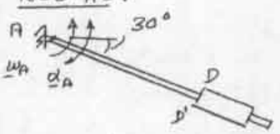


$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/\mathcal{F}}$
$[1.3856\,m/s \rightarrow] = [v_{D'} \measuredangle 60°] + [v_{D/\mathcal{F}} \searrow 30°]$
$v_{D'} = 1.3856 \sin 30° = 0.6928\,m/s$
EQ(1): $0.6928\,m/s = (0.4618\,m)\,\omega_A$
$\underline{\omega}_A = 1.5\,m/s$ ↻ ◀

$v_{D/\mathcal{F}} = (1.3856\,m/s)\cos 30° = 1.2\,m/s \searrow 30°$

ACCELERATION: ROD BD $a_D = (BD)\omega_B^2 = (0.23094\,m)(6\,rad/s)^2$
$\underline{a}_D = 8.314\,m/s^2 \uparrow$

ROD AD:

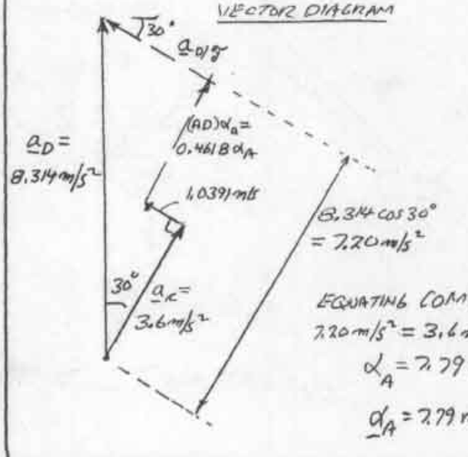


$\underline{\omega}_A = 1.5$ rad/s ↻
$\underline{v}_{D/\mathcal{F}} = 1.2\,m/s \searrow 30°$

$\underline{a}_{D'} = (AD)\omega_A^2 \searrow 30° + (AD)\alpha_A \measuredangle 60°$
$= (0.4618\,m)(1.5\,rad/s)^2 \searrow 30° + (0.4618\,m)\alpha_A \measuredangle 60°$
$\underline{a}_{D'} = 1.0391\,m/s^2 \searrow 30° + (0.4618\,m)\alpha_A \measuredangle 60°$
$\underline{a}_{D/\mathcal{F}} = \searrow 30°$
$\underline{a}_c = 2\omega_A v_{D/\mathcal{F}} = 2(1.5\,rad/s)(1.2\,m/s) = 3.6\,m/s^2 \measuredangle 60°$

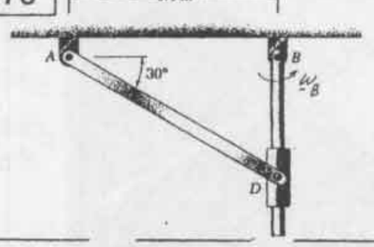
$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$
$[8.314\,m/s^2 \uparrow] = [1.0391\,m/s^2 \searrow 30° + (0.4618\,m)\alpha_A \measuredangle 60°]$
$+ [a_{D/\mathcal{F}} \searrow 30°] + [3.6\,m/s^2 \measuredangle 60°]$

VECTOR DIAGRAM



EQUATING COMPONENTS $\measuredangle 60°$
$7.20\,m/s^2 = 3.6\,m/s^2 + 0.4618\,\alpha_A$
$\alpha_A = 7.79\,rad/s^2$
$\underline{\alpha}_A = 7.79\,rad/s^2$ ↻ ◀

15.176

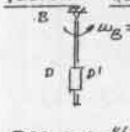


GIVEN:
$\underline{\omega}_B = 6$ rad/s ↺
$\underline{\alpha}_B = 0$

FIND:
$\underline{\omega}_A$
α_A

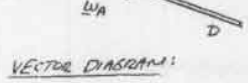
GEOMETRY: $BD = (0.4\,m)\tan 30° = 0.23094\,m$
$AD = (0.4\,m)/\cos 30° = 0.4618\,m$

VELOCITY ROD BD:



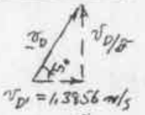
$v_{D'} = (BD)\omega_B = (0.2309\,m)(6\,rad/s)$
$\underline{v}_{D'} = 1.3856\,m/s \rightarrow$
$v_{D/\mathcal{F}} = \updownarrow$

ROD AD:



$v_D = (AD)\omega_A = (0.4618\,m)\,\omega_A \measuredangle 60°$ (1)

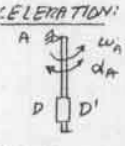
VECTOR DIAGRAM:



$v_D = v_{D'} + v_{D/\mathcal{F}}$
$[v_D \measuredangle 60°] = [1.3856\,m/s \rightarrow] + [v_{D/\mathcal{F}} \updownarrow]$
$v_D = (1.3856)/\cos 60 = 2.7712\,m/s$
EQ(1): $2.7712\,m/s = (0.4618\,m)\,\omega_A$
$\underline{\omega}_A = 6$ rad/s ↺ ◀

$v_{D/\mathcal{F}} = (1.3856\,m/s)\tan 60 = 2.4\,m/s \uparrow$

ACCELERATION: ROD BD



$\underline{a}_{D'} = (BD)\omega_A^2 \uparrow$
$\underline{a}_{D'} = (0.23094\,m)(6\,rad/s)^2 \uparrow = 8.314\,m/s^2 \uparrow$
$\underline{a}_{D/\mathcal{F}} = \updownarrow$

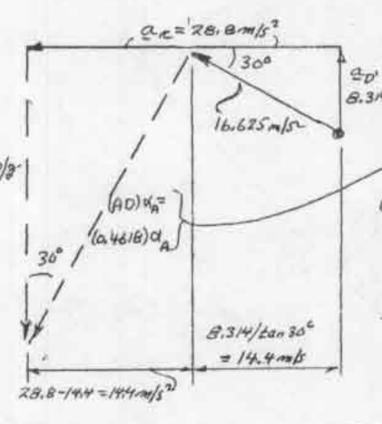
ROD AD:

$\underline{a}_D = (AD)\omega_A^2 \searrow 30° + (AD)\alpha_A \measuredangle 60°$
$= (0.4618\,m)(6\,rad/s)^2 \searrow 30° + (0.4618\,m)\alpha_A \measuredangle 60°$
$\underline{a}_D = 16.625\,m/s^2 \searrow 30° + (0.4618\,m)\alpha_A \measuredangle 60°$

$\underline{a}_c = 2\omega_A v_{D/\mathcal{F}} = 2(6\,rad/s)(2.4\,m/s) = 28.8\,m/s^2 \leftarrow$

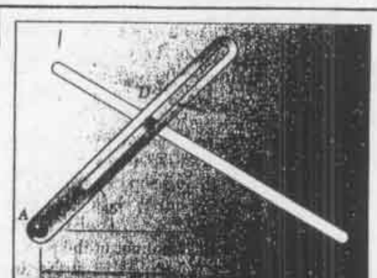
$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$
$[16.625\,m/s^2 \searrow 30° + (0.4618\,m)\alpha_A \measuredangle 60°] = [8.314\,m/s^2 \uparrow]$
$+ [a_{D/\mathcal{F}} \updownarrow] + [28.8\,m/s^2 \leftarrow]$

VECTOR DIAGRAM



$(AD)\alpha_A = \dfrac{14.4\,m/s^2}{\sin 30°}$
$(0.4618\,m)\alpha_A = 28.8\,m/s^2$
$\alpha_A = 62.4\,rad/s^2$
$\underline{\alpha}_A = 62.4\,rad/s^2$ ↻ ◀

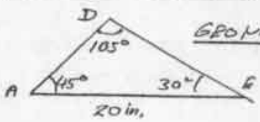
15.177



GIVEN: $\omega_A = 3$ rad/s ↺; $\alpha_A = 5$ rad/s² ↺

FIND: $\underline{a}_D$

GEOMETRY

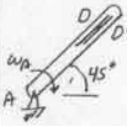


LAW OF SINES

$$\frac{AD}{\sin 30^\circ} = \frac{20\text{ in}}{\sin 105^\circ}$$

$$AD = 10.353\text{ in.}$$

VELOCITY:

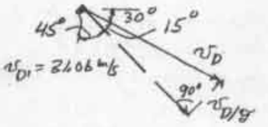


$\underline{v}_{D'} = (AD)\omega_A = (10.353\text{ in.})(3\text{ rad/s}) = 31.06\text{ in/s} \measuredangle 45^\circ$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/\mathcal{F}}$

$[v_D \measuredangle 30^\circ] = [31.06\text{ in/s} \measuredangle 45^\circ] + [v_{D/\mathcal{F}} \measuredangle 45^\circ]$

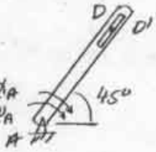
VECTOR DIAGRAM



$v_{D/\mathcal{F}} = (31.06\text{ in/s})\tan 15^\circ$

$\underline{v}_{D/\mathcal{F}} = 8.322\text{ in/s} \measuredangle 45^\circ$

ACCELERATION



$a_{D'} = (AD)\omega_A^2 \measuredangle 45^\circ + (AD)\alpha_A \measuredangle 45^\circ$
$= (10.353\text{ in.})(3\text{ rad/s})^2 \measuredangle 45^\circ$
$+ (10.353\text{ in.})(5\text{ rad/s}^2) \measuredangle 45^\circ$

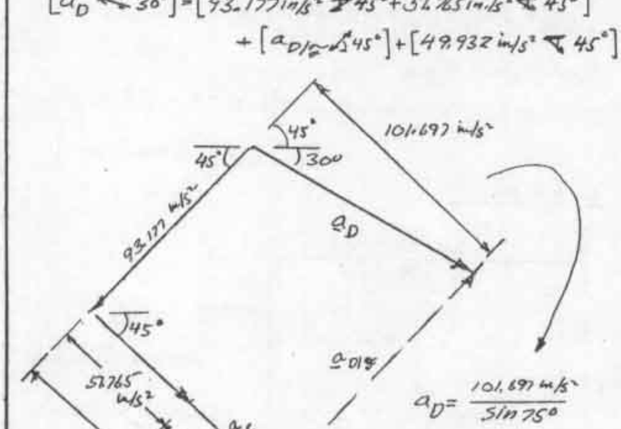
$a_{D'} = 93.177\text{ in/s}^2 \measuredangle 45^\circ + 51.765\text{ in/s}^2 \measuredangle 45^\circ$

$\underline{a}_c = 2\omega_A v_{D/\mathcal{F}} = 2(3\text{ rad/s})(8.322\text{ in/s})$

$\underline{a}_c = 49.932\text{ in/s}^2 \measuredangle 45^\circ$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$

$[a_D \measuredangle 30^\circ] = [93.177\text{ in/s}^2 \measuredangle 45^\circ + 51.765\text{ in/s}^2 \measuredangle 45^\circ]$
$+ [a_{D/\mathcal{F}} \measuredangle 45^\circ] + [49.932\text{ in/s}^2 \measuredangle 45^\circ]$

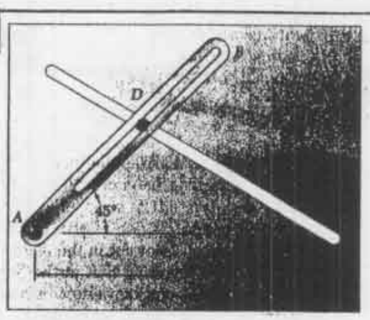


$$a_D = \frac{101.697\text{ in/s}^2}{\sin 75^\circ}$$

$a_D = 105.3\text{ in/s}^2$

$\underline{a}_D = 105.3\text{ in/s}^2 \measuredangle 30^\circ$ ◀

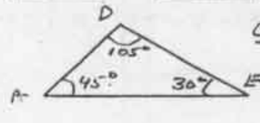
15.178



GIVEN: $\omega_A = 3$ rad/s ↻; $\alpha_A = 5$ rad/s² ↻

FIND: $\underline{a}_D$

GEOMETRY

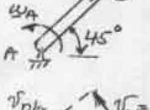


LAW OF SINES

$$\frac{AD}{\sin 30^\circ} = \frac{20\text{ in.}}{\sin 105^\circ}$$

$$AD = 10.353\text{ in}$$

VELOCITY:

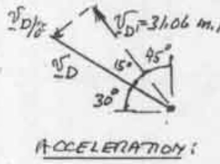


$v_{D'} = (AD)\omega_A = (10.353\text{ in})(3\text{ rad/s})^2 = 31.06\text{ in/s}^2 \measuredangle 45^\circ$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/\mathcal{F}}$

$[v_D \measuredangle 30^\circ] = [31.06\text{ in/s} \measuredangle 45^\circ] + [v_{D/\mathcal{F}} \measuredangle 45^\circ]$

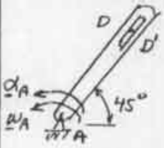
VECTOR DIAGRAM



$v_D = (31.06\text{ in/s})\tan 15^\circ$

$\underline{v}_{D/\mathcal{F}} = 8.322\text{ in/s} \measuredangle 45^\circ$

ACCELERATION:



$\underline{a}_{D'} = (AD)\omega_A^2 \measuredangle 45^\circ + (AD)\alpha_A \measuredangle 45^\circ$
$= (10.353\text{ in})(3\text{ rad/s})^2 \measuredangle 45^\circ$
$+ (10.353\text{ in})(5\text{ rad/s}^2) \measuredangle 45^\circ$

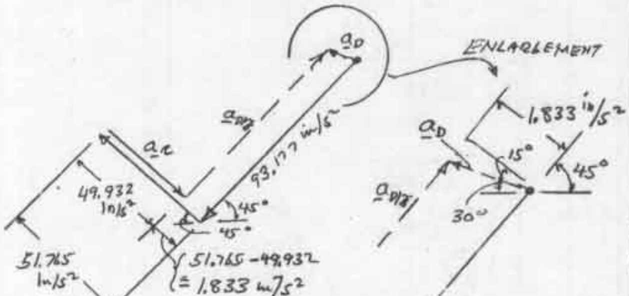
$\underline{a}_{D'} = 93.177\text{ in/s}^2 \measuredangle 45^\circ + 51.765\text{ in/s}^2 \measuredangle 45^\circ$

$\underline{a}_c = 2\omega_A v_{D/\mathcal{F}} = 2(3\text{ rad/s})(8.322\text{ in/s})$

$\underline{a}_c = 49.932\text{ in/s}^2 \measuredangle 45^\circ$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$

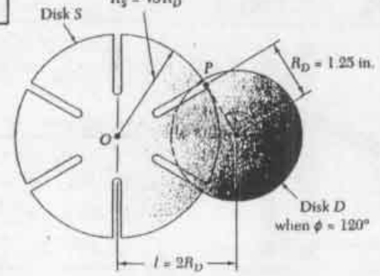
$[a_D \measuredangle 30^\circ] = [93.177\text{ in/s}^2 \measuredangle 45^\circ + 51.765\text{ in/s}^2 \measuredangle 45^\circ]$
$+ [a_{D/\mathcal{F}} \measuredangle 45^\circ] + [49.932\text{ in/s}^2 \measuredangle 45^\circ]$



$$a_D = \frac{1.833\text{ in./s}^2}{\cos 15^\circ}$$

$\underline{a}_D = 1.898\text{ in./s}^2 \measuredangle 30^\circ$ ◀

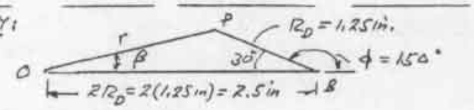
15.179



GIVEN: $\underline{\omega}_D = 8$ rad/s ↺, $\phi = 150°$

FIND: $\underline{\omega}_S$, $\underline{\alpha}_S$

GEOMETRY:



$r^2 = (1.25)^2 + (2.5)^2 - 2(1.25)(2.5)\cos 30°$ $\quad r = 1.5491$ in

$\dfrac{\sin\beta}{1.25\text{ in}} = \dfrac{\sin 30°}{1.5491\text{ in}}$ $\quad \beta = 23.79°$

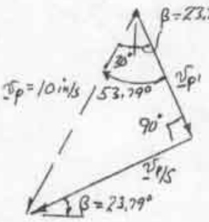
VELOCITY: $\underline{v}_P = (BP)\omega_D \measuredangle 30° = (1.25\text{ in})(8\text{ rad/s}) = 10$ in/s $\measuredangle 30°$

$\underline{v}_{P/S} = v_{P/S} \measuredangle \beta$

$\underline{v}_{P'} = v_{P'} \measuredangle \beta$, WHERE P' IS COINCIDING POINT ON S

$\underline{v}_P = \underline{v}_{P'} + \underline{v}_{P/S}$

$[10 \text{ in/s} \measuredangle 30°] = [v_{P'} \measuredangle \beta] + [v_{P/S} \measuredangle\,]$



$v_{P/S} = (10 \text{ in/s})\sin 53.79° = 8.0686$ in/s $\measuredangle 23.79°$

$v_{P'} = (10\text{ in/s})\cos 53.79° = 5.9075$ in/s $\measuredangle 23.79°$

$\omega_S = \dfrac{v_{P'}}{r} = \dfrac{5.9075\text{ in/s}}{1.5491\text{ in}} = 3.8135$ rad/s

$\underline{\omega}_S = 3.81$ rad/s ↻ ◀

ACCELERATION: $\underline{a}_P = (BP)\omega_D^2 \measuredangle 30° = (1.25\text{ in})(8\text{ rad/s})^2 = 80$ in/s² $\measuredangle 30°$

$\underline{a}_{P'} = r\omega_S^2 \measuredangle \beta + r\alpha_S \measuredangle \beta$

$= (1.5491\text{ in})(3.8135\text{ rad/s})^2 \measuredangle \beta + (1.5491\text{ in})\alpha_S \measuredangle \beta$

$\underline{a}_{P'} = 22.528$ in/s² $\measuredangle 23.79° + (1.5491\text{ in})\alpha_S \measuredangle 23.79°$

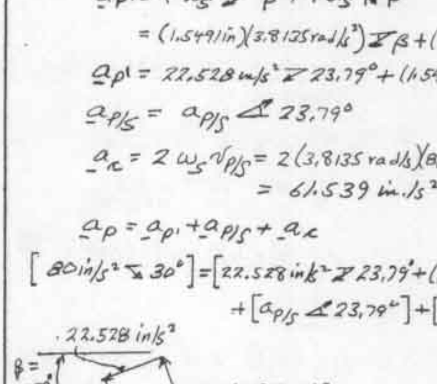
$\underline{a}_{P/S} = a_{P/S} \measuredangle 23.79°$

$\underline{a}_c = 2\omega_S v_{P/S} = 2(3.8135\text{ rad/s})(8.0686\text{ in/s})$

$= 61.539$ in/s² $\measuredangle 23.79°$

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/S} + \underline{a}_c$

$[80\text{ in/s}^2 \measuredangle 30°] = [22.528\text{ in/s}^2 \measuredangle 23.79° + (1.5491\text{ in})\alpha_S \measuredangle 23.79°]$
$+ [a_{P/S} \measuredangle 23.79°] + [61.539\text{ in/s}^2 \measuredangle 23.79°]$

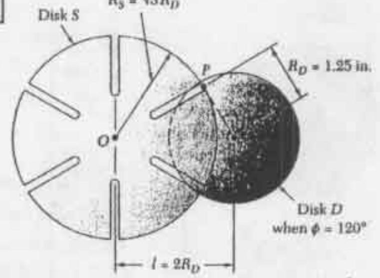


SUM COMPONENTS $\measuredangle \beta$ (THAT IS, SUM ⊥ TO SLOT)

$+\measuredangle\beta$: $(80\text{ in/s}^2)\cos 36.21° = (1.5491\text{ in})\alpha_S - (61.539\text{ in/s}^2)$

$\alpha_S = 81.39$ rad/s² $\quad \underline{\alpha}_S = 81.4$ rad/s² ↻ ◀

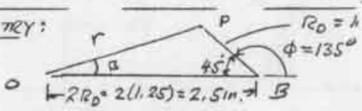
15.180



GIVEN: $\underline{\omega}_D = 8$ rad/s ↺, $\phi = 135°$

FIND: $\underline{\omega}_S$, $\underline{\alpha}_S$

GEOMETRY:



$r^2 = (1.25)^2 + (2.5)^2 - 2(1.25)(2.5)\cos 45°$ $\quad r = 1.842$ in.

$\dfrac{\sin\beta}{1.25\text{ in.}} = \dfrac{\sin 45°}{1.842\text{ in}}$ $\quad \beta = 28.68$

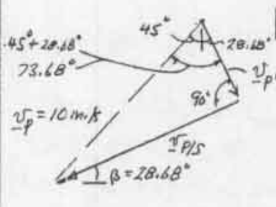
VELOCITY: $\underline{v}_P = (BP)\omega_D \measuredangle 45° = (1.25\text{ in})(8\text{ rad/s}) = 10$ in/s $\measuredangle 45°$

$\underline{v}_{P/S} = v_{P/S} \measuredangle \beta$

$v_{P'} = v_{P'} \measuredangle \beta$, WHERE P' IS COINCIDING POINT ON S

$\underline{v}_P = \underline{v}_{P'} + \underline{v}_{P/S}$

$[10\text{ in/s} \measuredangle 45°] = [v_{P'} \measuredangle \beta] + [v_{P/S} \measuredangle \beta]$



$v_{P/S} = (10)\sin 73.68° = 9.597$ in/s $\measuredangle \beta$

$v_{P'} = (10)\cos 73.68° = 2.819$ in/s $\measuredangle \beta$

$\omega_S = \dfrac{v_{P'}}{r} = \dfrac{2.819\text{ in/s}}{1.842\text{ in}} = 1.5255$ rad/s

$\underline{\omega}_S = 1.526$ rad/s ↻ ◀

ACCELERATION $\underline{a}_P = (BP)\omega_D^2 \measuredangle 45° = (1.25\text{ in.})(8\text{ rad/s})^2 = 80$ in/s² $\measuredangle 45°$

$\underline{a}_{P'} = r\omega_S^2 \measuredangle \beta + r\alpha_S \measuredangle \beta$

$= (1.842\text{ in})(1.526\text{ rad/s})^2 \measuredangle \beta + (1.842\text{ in.})\alpha_S \measuredangle \beta$

$\underline{a}_{P'} = 4.289$ in/s² $\measuredangle 28.68° + (1.842\text{ in.})\alpha_S \measuredangle \beta$

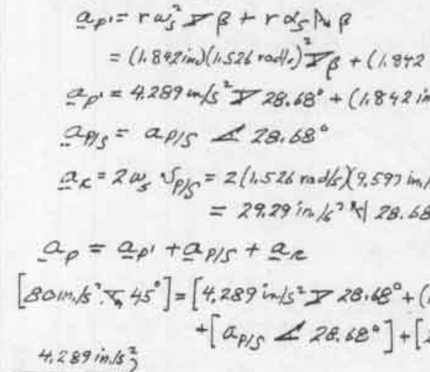
$\underline{a}_{P/S} = a_{P/S} \measuredangle 28.68°$

$\underline{a}_c = 2\omega_S v_{P/S} = 2(1.526\text{ rad/s})(9.597\text{ in/s})$

$= 29.29$ in/s² $\measuredangle 28.68°$

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/S} + \underline{a}_c$

$[80\text{ in/s}^2 \measuredangle 45°] = [4.289\text{ in/s}^2 \measuredangle 28.68° + (1.842\text{ in.})\alpha_S \measuredangle 28.68°]$
$+ [a_{P/S} \measuredangle 28.68°] + [29.29\text{ in/s}^2 \measuredangle 28.68°]$

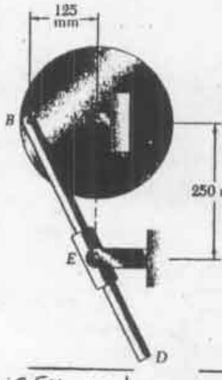


SUM COMPONENTS $\measuredangle \beta$ (THAT IS, SUM ⊥ TO SLOT)

$+\measuredangle\beta$: $(80\text{ in/s}^2)\cos 16.32° = (1.842\text{ in})\alpha_S - (29.29\text{ in/s}^2)$

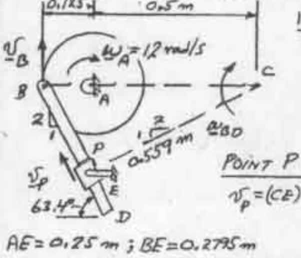
$\alpha_S = 57.58$ rad/s² $\quad \underline{\alpha}_S = 57.6$ rad/s² ↻ ◀

15.181



GIVEN: $\omega_A = 12$ rad/s ↺, $\alpha_A = 0$

FIND: (a) ω_{BD} AND α_{BD} (b) VELOCITY AND ACCELERATION OF POINT OF BD THAT COINCIDES WITH E



VELOCITY: INST. CENTER AT C

$v_B = (0.125\text{ m})(12\text{ rad/s}) = 1.5\text{ m/s} \uparrow$

$\omega_{BD} = \frac{v_B}{BC} = \frac{1.5\text{ m/s}}{0.625\text{ m}} = 2.4$ rad/s

$\omega_{BD} = 2.4$ rad/s ↺ ◀

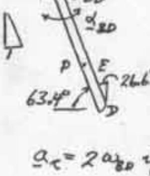
POINT P OF BD COINCIDES WITH E

$v_P = (CE)\omega_{BD} = (0.559\text{ m})(2.4\text{ rad/s})$

$v_P = 1.342$ m/s ⦪ 63.4° ◀

AE = 0.25 m ; BE = 0.2795 m

ACCELERATION: $a_B = (BA)\omega_A^2 = (0.125\text{ m})(12\text{ rad/s})^2 = 18\text{ m/s}^2 \rightarrow$



$a_P = a_B + a_{P/B}$

$= [18\text{ m/s}^2 \rightarrow] + [(BE)\omega_{BD}^2 \measuredangle 63.4° + (BD)\alpha_{BD} \measuredangle 26.6°]$

$= [18\text{ m/s}^2 \rightarrow] + [(0.2795)(2.4)^2 \measuredangle 63.4° + (0.2795)\alpha_{BD} \measuredangle 26.6°]$

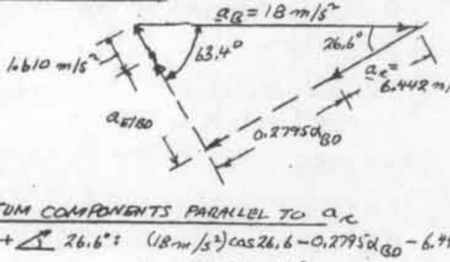
$= [18\text{ m/s}^2 \rightarrow] + [1.610\text{ m/s}^2 \measuredangle 63.4 + 0.2795\alpha_{BD} \measuredangle 26.6°]$

$a_{E/BD} = a_{E/BD} \measuredangle 63.4° = a_{E/BD} \measuredangle 26.6°$

$a_c = 2\omega_{BD} v_{E/BD}$ NOTE: $v_{E/BD} = -v_P = 1.342$ m/s ⦪ 63.4°

$a_c = 2(2.4\text{ rad/s})(1.342\text{ m/s}) = 6.442\text{ m/s}^2 \measuredangle 63.4°$

VECTOR DIAGRAM



SUM COMPONENTS PARALLEL TO a_c

$+\measuredangle 26.6°$: $(18\text{ m/s}^2)\cos 26.6 - 0.2795\alpha_{BD} - 6.442\text{ m/s}^2 = 0$

$\alpha_{BD} = 34.54$ rad/s² $\alpha_{BD} = 34.5$ rad/s² ↺ ◀

SUM COMPONENTS ⊥ TO a_c

$+ 26.6°$: $a_{E/BD} + 1.610\text{ m/s}^2 - (18\text{ m/s}^2)\sin 26.6° = 0$

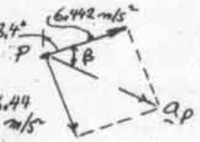
$a_{E/BD} = 6.44\text{ m/s}^2 \measuredangle 63.4°$

SINCE POINT E IS FIXED, WE NOTE THAT

$a_P = a_{P/E} = -a_{E/P}$

$= -[a_{E/BD} \measuredangle 63.4° + a_c \measuredangle 63.4°]$

$a_P = [6.45\text{ m/s}^2 \measuredangle 63.4° + 6.442\text{ m/s}^2 \measuredangle 63.4°]$



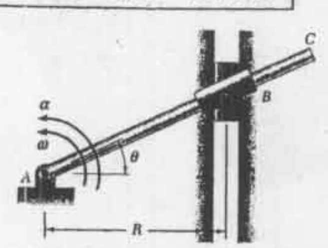
$a_P = 9.11$ m/s² $\beta = 45.0°$

63.4 + 45° = 108.4°

108.4° − 90° = 18.4°

$a_P = 9.11$ m/s² ⦪ 18.4° ◀

15.182 and 15.183



PROBLEM 15.182

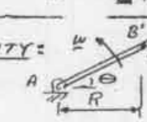
DERIVE EXPRESSIONS FOR v_B AND a_B

PROBLEM 15.183

GIVEN: R = 15 in., θ = 25°, ω = 3 rad/s ↻, α = 8 rad/s² ↻

FIND: v_B AND a_B

VELOCITY:



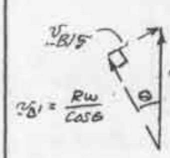
$AB = R/\cos\theta$

$v_{B'} = (AB)\omega = R\omega/\cos\theta \measuredangle\theta$

$v_B = v_{B'} + v_{B/\mathcal{F}}$

$[v_B \uparrow] = [R\omega/\cos\theta \measuredangle\theta] + [v_{B/\mathcal{F}} \measuredangle\theta]$

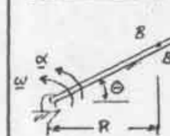
VECTOR DIAGRAM:



$v_B = \frac{v_{B'}}{\cos\theta} = \frac{R\omega}{\cos^2\theta} \uparrow$ ◀

$v_{B/\mathcal{F}} = v_{B'}\tan\theta = \frac{R\omega}{\cos\theta}\tan\theta \measuredangle\theta$

ACCELERATION:



$a_{B'} = (AB)\omega^2 \measuredangle\theta + (AB)\alpha \measuredangle\theta$

$= \frac{R\omega^2}{\cos\theta} \measuredangle\theta + \frac{R\alpha}{\cos\theta} \measuredangle\theta$

$a_{B/\mathcal{F}} = a_{B/\mathcal{F}} \measuredangle\theta$

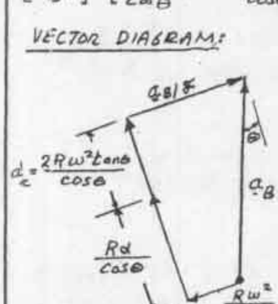
$a_c = 2\omega v_{B/\mathcal{F}} = 2\omega\frac{R\omega}{\cos\theta}\tan\theta$

$a_c = \frac{2R\omega^2}{\cos\theta}\tan\theta \measuredangle\theta$

$a_B = a_{B'} + a_{B/\mathcal{F}} + a_c$

$[a_B \uparrow] = \left[\frac{R\omega^2}{\cos\theta} \measuredangle\theta + \frac{R\alpha}{\cos\theta} \measuredangle\theta\right] + [a_{B/\mathcal{F}} \measuredangle\theta] + \left[\frac{2R\omega^2\tan\theta}{\cos\theta}\right] \measuredangle\theta$

VECTOR DIAGRAM:



SUM COMPONENTS PARALLEL TO a_c

$+\measuredangle\theta$:

$a_B\cos\theta - \frac{2R\omega^2\tan\theta}{\cos\theta} - \frac{R\alpha}{\cos\theta} = 0$

$a_B = \frac{2R\omega^2\tan\theta}{\cos^2\theta} + \frac{R\alpha}{\cos^2\theta}$

$a_B = \frac{R}{\cos^2\theta}(\alpha + 2\omega^2\tan\theta) \uparrow$ ◀

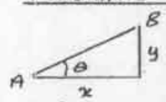
PROBLEM 15.183 R = 15 in., θ = 25°, ω = 3 rad/s ↻, α = 8 rad/s² ↻

$v_B = \frac{R\omega}{\cos^2\theta} = \frac{(15\text{ in.})(3\text{ rad/s})}{\cos^2 25} = 54.78$ in./s; $v_B = 54.8$ in./s ↑ ◀

$a_B = \frac{R}{\cos^2\theta}(\alpha + 2\omega^2\tan\theta) = \frac{15\text{ in.}}{\cos^2 25°}(8\text{ rad/s}^2 + 2(3\text{ rad/s})^2\tan 25°)$

$a_B = 18.262(8 + 8.333) = 298.7$ m/s²; $a_B = 299$ in./s² ↑ ◀

ALTERNATIVE DERIVATION USING A PARAMETER. Sec 15.9



$\omega = \dot\theta$

$\alpha = \ddot\theta$

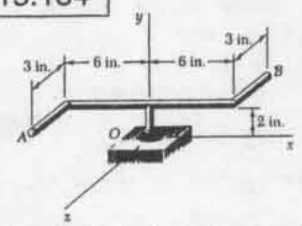
$y = R\tan\theta$

$v_B = \dot y = (R/\cos^2\theta)\dot\theta = \frac{R\omega}{\cos^2\theta}$ ◀

$a_B = \ddot y = \frac{R}{\cos^2\theta}\ddot\theta + \left(2\frac{R\sin\theta}{\cos^3\theta}\dot\theta\right)\dot\theta$

$a_B = \frac{R}{\cos^2\theta}(\alpha + 2\omega^2\tan\theta)$ ◀

15.184



GIVEN: $\omega_y = 30\ \text{rad/s}$

$\mathbf{v}_A = (100\ \text{in./s})\mathbf{i} + (6\ \text{in./s})\mathbf{j} + (v_A)_z\mathbf{k}$

FIND: (a) $\boldsymbol{\omega}$ (b) $\mathbf{v}_B$

$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k} \qquad \mathbf{r}_A = -(6\ \text{in.})\mathbf{i} + (2\ \text{in.})\mathbf{j} + (3\ \text{in.})\mathbf{k}$

$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & 30 & \omega_z \\ -6 & 2 & 3 \end{vmatrix}$$

$$\mathbf{v}_A = (90 - 2\omega_z)\mathbf{i} + (-3\omega_x - 6\omega_z)\mathbf{j} + (2\omega_x + 180)\mathbf{k}$$

$(v_A)_x = 90 - 2\omega_z = 100 \qquad \omega_z = -5\ \text{rad/s}$

$(v_A)_y = -3\omega_x - 6(-5) = 6$

$-3\omega_x + 30 = 6 \qquad \omega_x = 8\ \text{rad/s}$

(a) $\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

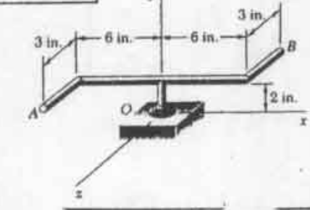
$$\boldsymbol{\omega} = (8\ \text{rad/s})\mathbf{i} + (30\ \text{rad/s})\mathbf{j} - (5\ \text{rad/s})\mathbf{k}$$

(b) $\mathbf{r}_B = (6\ \text{in.})\mathbf{i} + (2\ \text{in.})\mathbf{j} - (3\ \text{in.})\mathbf{k}$

$$\mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 30 & -5 \\ 6 & 2 & -3 \end{vmatrix} = (-90+10)\mathbf{i} + (-30+24)\mathbf{j} + (16-180)\mathbf{k}$$

$$\mathbf{v}_B = -(80\ \text{in./s})\mathbf{i} - (6\ \text{in./s})\mathbf{j} - (164\ \text{in./s})\mathbf{k}$$

15.185



GIVEN: $\omega_y = 40\ \text{rad/s}$

$\mathbf{v}_A = (100\ \text{in./s})\mathbf{i} + (6\ \text{in./s})\mathbf{j} + (v_A)_z\mathbf{k}$

FIND: (a) $\boldsymbol{\omega}$ (b) $\mathbf{v}_B$

$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k} \qquad \mathbf{r}_A = -(6\ \text{in.})\mathbf{i} + (2\ \text{in.})\mathbf{j} + (3\ \text{in.})\mathbf{k}$

$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & 40 & \omega_z \\ -6 & 2 & 3 \end{vmatrix}$$

$$\mathbf{v}_A = (120 - 2\omega_z)\mathbf{i} + (-3\omega_x - 6\omega_z)\mathbf{j} + (2\omega_x + 240)\mathbf{k}$$

$(v_A)_x = 120 - 2\omega_z = 100 \qquad \omega_z = 10$

$(v_A)_y = -3\omega_x - 6(10) = 6$

$-3\omega_x - 60 = 6 \qquad \omega_x = -22$

(a) $\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$$\boldsymbol{\omega} = -(22\ \text{rad/s})\mathbf{i} + (40\ \text{rad/s})\mathbf{j} + (10\ \text{rad/s})\mathbf{k}$$

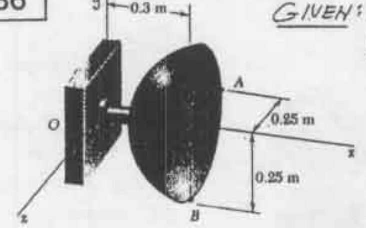
(b) $\mathbf{r}_B = (6\ \text{in.})\mathbf{i} + (2\ \text{in.})\mathbf{j} - (3\ \text{in.})\mathbf{k}$

$$\mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -22 & 40 & 10 \\ 6 & 2 & -3 \end{vmatrix}$$

$$= (-120-20)\mathbf{i} + (60-66)\mathbf{j} + (-44-240)\mathbf{k}$$

$$\mathbf{v}_B = -(140\ \text{in./s})\mathbf{i} - (6\ \text{in./s})\mathbf{j} - (284\ \text{in./s})\mathbf{k}$$

15.186



GIVEN: $(v_A)_y = 300\ \text{mm/s}$

$(v_B)_y = 180\ \text{mm/s}$

$(v_B)_z = 360\ \text{mm/s}$

FIND: (a) $\boldsymbol{\omega}$ (b) $\mathbf{v}_A$

$\mathbf{r}_A = (0.3\ \text{m})\mathbf{i} - (0.25\ \text{m})\mathbf{k}$

$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & 0 & -0.25 \end{vmatrix}$$

$(v_A)_x = -0.25\omega_y \quad (1)$

$(v_A)_y = 0.3\omega_z + 0.25\omega_x \quad (2)$

$(v_A)_z = -0.3\omega_y \quad (3)$

$\mathbf{r}_B = (0.3\ \text{m})\mathbf{i} - (0.25\ \text{m})\mathbf{j}$

$$\mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & -0.25 & 0 \end{vmatrix}$$

$(v_B)_x = 0.25\omega_z \quad (4)$

$(v_B)_y = 0.3\omega_z \quad (5)$

$(v_B)_z = -0.25\omega_x - 0.3\omega_y \quad (6)$

EQ 5: $(v_B)_y = 0.18\ \text{m/s} = (0.3\ \text{m})\omega_z\ ; \qquad \omega_z = 0.6\ \text{rad/s}$

EQ 2: $(v_A)_y = 0.3\ \text{m/s} = (0.3)(0.6) + 0.25\omega_x\ ; \qquad \omega_x = 0.48\ \text{rad/s}$

EQ 6: $(v_B)_z = 0.36\ \text{m/s} = -(0.25)(0.48) - 0.3\omega_y\ ; \qquad \omega_y = -1.6\ \text{rad/s}$

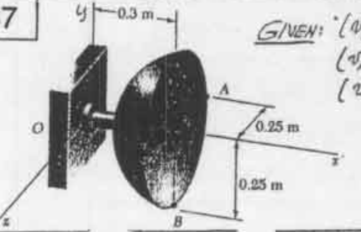
(a) $\boldsymbol{\omega} = (0.48\ \text{rad/s})\mathbf{i} - (1.6\ \text{rad/s})\mathbf{j} + (0.6\ \text{rad/s})\mathbf{k}$

(b) $\mathbf{v}_A = (v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} + (v_A)_z\mathbf{k} = -0.25(-1.6)\mathbf{i} + 0.3\mathbf{j} - (0.3)(-1.6)\mathbf{k}$

$\mathbf{v}_A = (0.4\ \text{m/s})\mathbf{i} + (0.3\ \text{m/s})\mathbf{j} + (0.48\ \text{m/s})\mathbf{k}$

$\mathbf{v}_A = (400\ \text{mm/s})\mathbf{i} + (300\ \text{mm/s})\mathbf{j} + (480\ \text{mm/s})\mathbf{k}$

15.187



GIVEN: $(v_A)_x = 100\ \text{mm/s}$

$(v_A)_y = -90\ \text{mm/s}$

$(v_B)_z = 120\ \text{mm/s}$

FIND: (a) $\boldsymbol{\omega}$ (b) $\mathbf{v}_A$

$\mathbf{r}_A = (0.3\ \text{m})\mathbf{i} - (0.25\ \text{m})\mathbf{k}$

$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & 0 & -0.25 \end{vmatrix}$$

$(v_A)_x = -0.25\omega_y \quad (1)$

$(v_A)_y = 0.3\omega_z + 0.25\omega_x \quad (2)$

$(v_A)_z = -0.3\omega_y \quad (3)$

$\mathbf{r}_B = (0.3\ \text{m})\mathbf{i} - (0.25\ \text{m})\mathbf{j}$

$$\mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & -0.25 & 0 \end{vmatrix}$$

$(v_B)_x = 0.25\omega_z \quad (4)$

$(v_B)_y = 0.3\omega_z \quad (5)$

$(v_B)_z = -0.25\omega_x - 0.3\omega_y \quad (6)$

EQ(1): $(v_A)_x = 0.1\ \text{m/s} = -0.25\omega_y\ ; \qquad \omega_y = -0.4\ \text{rad/s}$

EQ(6): $(v_B)_z = 0.12\ \text{m/s} = -0.25(\omega_x) - 0.3(-0.4); \qquad \omega_x = 0$

EQ(2): $(v_A)_y = -0.09\ \text{m/s} = 0.3\omega_z + 0.25(0)\ ; \qquad \omega_z = -0.3\ \text{rad/s}$

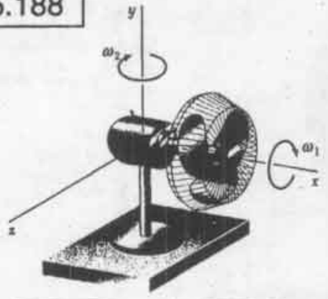
(a) $\boldsymbol{\omega} = -(0.4\ \text{rad/s})\mathbf{j} - (0.3\ \text{rad/s})\mathbf{k}$

(b) $\mathbf{v}_A = (v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} + (v_A)_z\mathbf{k} = -0.25(-0.4)\mathbf{i} + 0.3(-0.3)\mathbf{j} - 0.3(-0.4)\mathbf{k}$

$\mathbf{v}_A = (0.1\ \text{m/s})\mathbf{i} - (0.09\ \text{m/s})\mathbf{j} + (0.12\ \text{m/s})\mathbf{k}$

$\mathbf{v}_A = (100\ \text{mm/s})\mathbf{i} - (90\ \text{mm/s})\mathbf{j} + (120\ \text{mm/s})\mathbf{k}$

15.188



GIVEN: $\underline{\omega}_1 = (360 \text{ rpm})\underline{i}$

$\alpha_1 = 0$

$\underline{\omega}_2 = -(2.5 \text{ rpm})\underline{j}$

$\underline{\alpha}_2 = 0$

FIND: FOR HOUSING OF MOTOR

(a) $\underline{\omega}_H$

(b) $\underline{\alpha}_H$

$\underline{\omega}_1 = -(360 \text{ rpm})\underline{i} = -(12\pi \text{ rad/s})\underline{i}$

$\underline{\omega}_2 = -(2.5 \text{ rpm})\underline{j} = -(\pi/12 \text{ rad/s})\underline{j}$

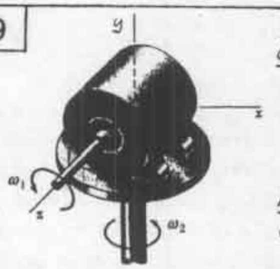
$\underline{\omega}_2$ = ROTATION OF FRAME Oxyz

$\underline{\alpha} = (\dot{\underline{\omega}}_1 + \dot{\underline{\omega}}_2) = (\dot{\omega}_1 + \dot{\omega}_2)_{Oxyz} + \underline{\omega}_2 \times (\underline{\omega}_1 + \underline{\omega}_2)$

$\underline{\alpha} = \underline{\omega}_2 \times \underline{\omega}_1 = (-\pi/12 \text{ rad/s})\underline{j} \times (-12\pi \text{ rad/s})\underline{i}$

$\underline{\alpha} = -(9.8696 \text{ rad/s}^2)\underline{k}$ $\qquad \underline{\alpha} = -(9.87 \text{ rad/s}^2)\underline{k}$ ◀

15.189



GIVEN: $\omega_1 = 1800$ rpm

$\alpha_1 = 0$

$\omega_2 = 6$ rpm

$\alpha_2 = 0$

FIND: FOR ROTOR OF MOTOR, α

$\underline{\omega}_1 = (1800 \text{ rpm})\underline{k} = (60\pi \text{ rad/s})\underline{k}$

$\underline{\omega}_2 = (6 \text{ rpm})\underline{j} = (\pi/5 \text{ rad/s})\underline{j}$

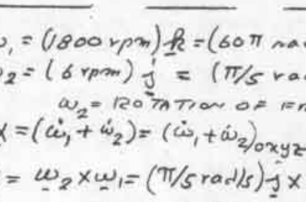
ω_2 = ROTATION OF FRAME Oxyz

$\underline{\alpha} = (\dot{\omega}_1 + \dot{\omega}_2) = (\dot{\omega}_1 + \dot{\omega}_2)_{Oxyz} + \underline{\omega}_2 \times (\underline{\omega}_1 + \underline{\omega}_2)$

$\underline{\alpha} = \underline{\omega}_2 \times \underline{\omega}_1 = (\pi/5 \text{ rad/s})\underline{j} \times (60\pi \text{ rad/s})\underline{k}$

$\underline{\alpha} = (118.44 \text{ rad/s}^2)\underline{i}$ $\qquad \underline{\alpha} = (118.4 \text{ rad/s}^2)\underline{i}$ ◀

15.190



GIVEN: $\underline{\omega}_1 = \omega_1 \underline{k}$

$\alpha_1 = 0$

$\omega_2 = \alpha_2 = 0$

FIND: FOR DISK

(a) $\underline{v}_A$

(b) $\underline{\alpha}_A$

DISK A: (IN ROTATION ABOUT O)

SINCE $\omega_y = \omega_1$ $\qquad \underline{\omega}_A = \omega_x \underline{i} + \omega_1 \underline{j} + \omega_z \underline{k}$

POINT D IS POINT OF CONTACT OF WHEEL + DISK

$\underline{r}_{D/O} = -r\underline{j} - R\underline{k}$

$$\underline{v}_D = \underline{\omega}_A \times \underline{r}_{D/O} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_1 & \omega_z \\ 0 & -r & -R \end{vmatrix}$$

$\underline{v}_D = (-R\omega_1 + r\omega_z)\underline{i} + R\omega_x\underline{j} - r\omega_x\underline{k}$

(CONTINUED)

15.190 CONTINUED

SINCE $\omega_2 = 0$, $v_D = 0$

EACH COMPONENT OF $\underline{v}_D$ IS ZERO

$(v_D)_z = r\omega_x = 0$; $\qquad \omega_x = 0$

$(v_D)_x = -R\omega_1 + r\omega_z = 0$; $\qquad \omega_z = (R/r)\omega_1$

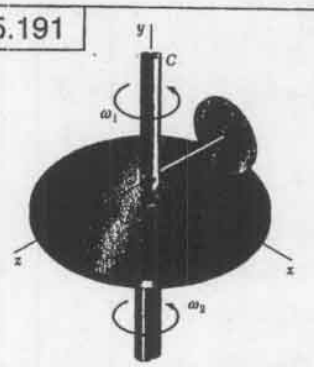
(a) $\qquad \underline{\omega}_A = \omega_1 \underline{j} + (R/r)\omega_1 \underline{k}$ ◀

(b) DISK A: ROTATES ABOUT y AXIS AT RATE ω_1

$\underline{\alpha}_A = \dfrac{d\omega_A}{dt} = \underline{\omega}_y \times \underline{\omega}_A = \omega_1\underline{j} \times (\omega_1 \underline{j} + \frac{R}{r}\omega_1 \underline{k})$

$\underline{\alpha}_A = \frac{R}{r}\omega_1^2 \underline{i}$ ◀

15.191



GIVEN: $\underline{\omega}_1 = \omega_1 \underline{k}$

$\alpha_1 = 0$

$\underline{\omega}_2 = \omega_2 \underline{k}$

$\alpha_2 = 0$

FIND: FOR DISK,

(a) $\underline{v}_A$

(b) $\underline{\alpha}_A$

DISK A: (IN ROTATION ABOUT O)

SINCE $\omega_y = \omega_1$, $\qquad \underline{\omega}_A = \omega_x \underline{i} + \omega_1 \underline{j} + \omega_z \underline{k}$

POINT D IS POINT OF CONTACT OF WHEEL AND DISK

$\underline{r}_{D/O} = -r\underline{j} - R\underline{k}$

$$\underline{v}_D = \underline{\omega}_A \times \underline{r}_{D/O} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_1 & \omega_z \\ 0 & -r & -R \end{vmatrix}$$

$\underline{v}_D = (-R\omega_1 + r\omega_z)\underline{i} + R\omega_x \underline{j} - r\omega_x \underline{k}$ $\qquad (1)$

DISK B: $\underline{\omega}_B = \omega_2 \underline{j}$

$\underline{v}_D = \underline{\omega}_B \times \underline{r}_{D/O} = \omega_2 \underline{j} \times (-r\underline{j} - R\underline{k}) = -R\omega_2 \underline{i}$ $\qquad (2)$

FROM EQS 1 AND 2:

$\underline{v}_D = \underline{v}_D$: $(-R\omega_1 + r\omega_z)\underline{i} + R\omega_x\underline{j} - r\omega_x\underline{k} = -R\omega_2\underline{i}$

COEF. OF $\underline{k}$: $-r\omega_x = 0$; $\qquad \omega_x = 0$

COEF. OF $\underline{i}$: $(-R\omega_1 + r\omega_z) = -R\omega_2$; $\qquad \omega_z = \frac{R}{r}(\omega_1 - \omega_2)$

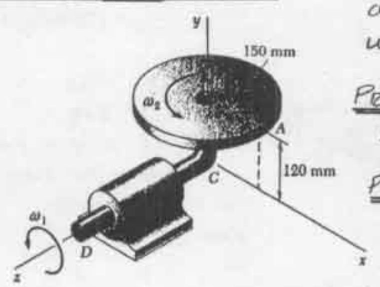
(a) $\qquad \underline{\omega}_A = \omega_1 \underline{j} + \frac{R}{r}(\omega_1 - \omega_2)\underline{k}$ ◀

(b) DISK A ROTATES ABOUT y AXIS AT RATE ω_1

$\underline{\alpha}_A = \dfrac{d\omega_A}{dt} = \underline{\omega}_y \times \omega_A = \omega_1 \underline{j} \times \left[\omega_1 \underline{j} + \frac{R}{r}(\omega_1 - \omega_2)\underline{k}\right]$

$\underline{\alpha}_A = \frac{R}{r}\omega_1(\omega_1 - \omega_2)\underline{i}$ ◀

15.192 and 15.193



GIVEN:
$\omega_1 = 5\ \text{rad/s},\ \alpha_1 = 0$
$\omega_2 = 4\ \text{rad/s},\ \alpha_2 = 0$

PROBLEM 15.192
FIND: $\alpha_{DISK} = \alpha$

PROBLEM 15.193
FIND: (a) $\underline{v}_A$
(b) $\underline{a}_A$

DISK: $\underline{\omega} = \omega_2 \underline{j} + \omega_1 \underline{k} = (4\ \text{rad/s})\underline{j} + (5\ \text{rad/s})\underline{k}$

PROBLEM 15.192:

DISK ROTATES ABOUT Z AXIS AT RATE $\omega_1 = \omega_1 \underline{k}$

$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = (5\ \text{rad/s})\underline{k} \times [(4\ \text{rad/s})\underline{j} + (5\ \text{rad/s})\underline{k}]$

$$\underline{\alpha} = -(20\ \text{rad/s}^2)\underline{i}$$

PROBLEM 15.193:

$\underline{r}_A = (0.15\ \text{mm})\underline{i} + (0.12\ \text{mm})\underline{j}$

$$\underline{v}_A = \underline{\omega} \times \underline{r}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & 5 \\ 0.15 & 0.12 & 0 \end{vmatrix} = -0.6\underline{i} + 0.75\underline{j} - 0.6\underline{k}$$

$$\underline{v}_A = -(0.6\ \text{m/s})\underline{i} + (0.75\ \text{m/s})\underline{j} - (0.6\ \text{m/s})\underline{k}$$

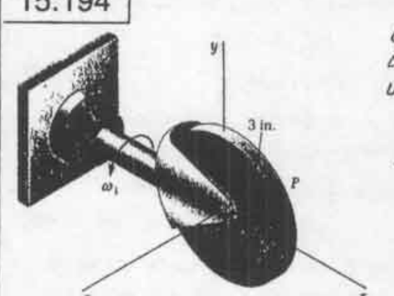
$\underline{a}_A = \underline{\alpha} \times \underline{r}_A + \underline{\omega} \times \underline{v}_A$

$$= -20\underline{i} \times (0.15\underline{i} + 0.12\underline{j}) + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 4 & 5 \\ -0.6 & 0.75 & -0.6 \end{vmatrix}$$

$\underline{a}_A = -2.4\underline{k} + (-2.4 - 3.75)\underline{i} - 3\underline{j} + 2.4\underline{k}$

$$\underline{a}_A = -(6.15\ \text{m/s}^2)\underline{i} - (3\ \text{m/s}^2)\underline{j}$$

15.194



GIVEN:
$\omega_1 = 5\ \text{rad/s},\ \alpha_1 = 0$
$\omega_2 = 4\ \text{rad/s},\ \alpha_2 = 0$

FIND:
(a) $\alpha_{DISK} = \alpha$
(b) $\underline{a}_P$ WHEN $\theta = 0$
(c) $\underline{a}_P$ WHEN $\theta = 90°$

$\underline{\omega} = \omega_1 \underline{i} + \omega_2 \underline{k} = (5\ \text{rad/s})\underline{i} + (4\ \text{rad/s})\underline{k}$

$\underline{\alpha} = \omega_1 \times \underline{\omega} = (5\ \text{rad/s})\underline{i} \times [(5\ \text{rad/s})\underline{i} + (4\ \text{rad/s})\underline{k}]$

(a) $\underline{\alpha} = -(20\ \text{rad/s}^2)\underline{j}$

(b) $\underline{\theta = 0}$: $\underline{r}_P = (3\ \text{in.})\underline{i}$

$\underline{v}_P = \underline{\omega} \times \underline{r}_P = (5\underline{i} + 4\underline{k}) \times 3\underline{i}$; $\quad \underline{v}_P = (12\ \text{in./s})\underline{j}$

$\underline{a}_P = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times \underline{v}_P$

$= -20\underline{j} \times 3\underline{i} + (5\underline{i} + 4\underline{k}) \times 12\underline{j}$

$= 60\underline{k} + 60\underline{k} - 48\underline{i} = -48\underline{i} + 120\underline{k}$

$$\underline{a}_P = -(48\ \text{in./s}^2)\underline{i} + (120\ \text{in./s}^2)\underline{k}$$

(CONTINUED)

15.194 CONTINUED

(c) $\underline{\theta = 90°}$: $\underline{r}_P = (3\ \text{in.})\underline{j}$

$\underline{v}_P = \underline{\omega} \times \underline{r}_P = (5\underline{i} + 4\underline{k}) \times 3\underline{j}$; $\quad \underline{v}_P = -(12\ \text{in./s})\underline{i} + (15\ \text{in./s})\underline{k}$

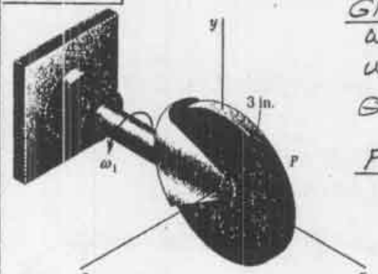
$\underline{a}_P = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times \underline{v}_P$

$= -20\underline{j} \times 3\underline{j} + (5\underline{i} + 4\underline{k}) \times (-12\underline{i} + 15\underline{k})$

$= 0 - 75\underline{j} - 48\underline{j} = -123\underline{j}$

$$\underline{a}_P = -(123\ \text{in./s}^2)\underline{j}$$

15.195



GIVEN:
$\omega_1 = 5\ \text{rad/s},\ \alpha_1 = 0$
$\omega_2 = 4\ \text{rad/s},\ \alpha_2 = 0$
$\theta = 30°$

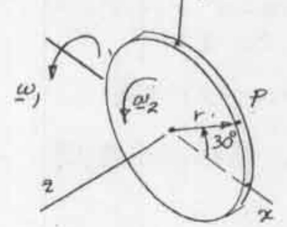
FIND: $\underline{a}_P$

$\underline{\omega} = \omega_1 \underline{i} + \omega_2 \underline{k} = (5\ \text{rad/s})\underline{i} + (4\ \text{rad/s})\underline{k}$

$\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = (5\ \text{rad/s})\underline{i} \times [(5\ \text{rad/s})\underline{i} + (4\ \text{rad/s})\underline{k}]$

$$\underline{\alpha} = -(20\ \text{rad/s}^2)\underline{j}$$

FOR $\theta = 30°$



$r = 3\ \text{in.}$

$(r_P)_x = r\cos 30° = (3\ \text{in.})\cos 30° = 2.598\ \text{in.}$

$(r_P)_y = r\sin 30° = (3\ \text{in.})\sin 30° = 1.5\ \text{in.}$

$\underline{r}_P = (2.598\ \text{in.})\underline{i} + (1.5\ \text{in.})\underline{j}$

$$\underline{v}_P = \underline{\omega} \times \underline{r}_P = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & 0 & 4 \\ 2.598 & 1.5 & 0 \end{vmatrix}$$

$= -6\underline{i} + 10.392\underline{j} + 7.5\underline{k}$

$$\underline{v}_P = -(6\ \text{in./s})\underline{i} + (10.392\ \text{in./s})\underline{j} + (7.5\ \text{in./s})\underline{k}$$

$\underline{a}_P = \underline{\alpha} \times \underline{r}_P + \underline{\omega} \times \underline{v}_P$

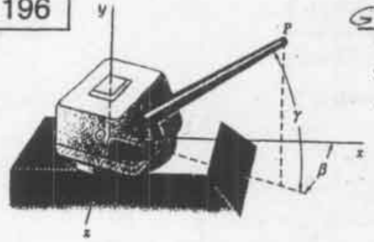
$$\underline{a}_P = -20\underline{j} \times (2.598\underline{i} + 1.5\underline{j}) + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 5 & 0 & 4 \\ -6 & 10.392 & 7.5 \end{vmatrix}$$

$= 51.96\underline{k} - 41.57\underline{i} + (-24 - 37.5)\underline{j} + 51.96\underline{k}$

$\underline{a}_P = -41.57\underline{i} - 61.5\underline{j} + 103.92\underline{k}$

$$\underline{a}_P = -(41.6\ \text{in./s}^2)\underline{i} - (61.5\ \text{in./s}^2)\underline{j} + (103.9\ \text{in./s}^2)\underline{k}$$

15.196



GIVEN: OP = 4 m

$d\beta/dt = 30°/s$

$d\gamma/dt = 10°/s$

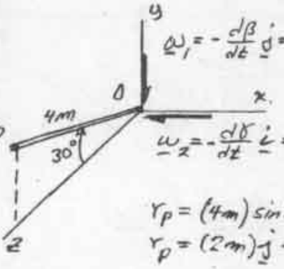
$\beta = 90°$

$\gamma = 30°$

FIND: (a) $\boldsymbol{\omega}$

(b) $\boldsymbol{\alpha}$

(c) $\mathbf{v}_P$ AND $\mathbf{a}_P$



$$\boldsymbol{\omega}_1 = -\frac{d\beta}{dt}\mathbf{j} = -(30°/s)\mathbf{j} = -\left(\frac{\pi}{6}\text{ rad/s}\right)\mathbf{j}$$

$$\boldsymbol{\omega}_2 = -\frac{d\gamma}{dt}\mathbf{i} = -(10°/s)\mathbf{j} = -\left(\frac{\pi}{18}\text{ rad/s}\right)\mathbf{i}$$

$$r_p = (4m)\sin 30°\,\mathbf{j} + (4m)\cos 30°\,\mathbf{k}$$

$$r_p = (2m)\mathbf{j} + (3.464 m)\mathbf{k}$$

(a) $\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 = -\frac{\pi}{18}\mathbf{i} - \frac{\pi}{6}\mathbf{j} = -0.17453\mathbf{i} - 0.5236\mathbf{j}$

$$\boldsymbol{\omega} = -(0.1745\text{ rad/s})\mathbf{i} - (0.524\text{ rad/s})\mathbf{j}$$

(b) NOTE TURRET ROTATES ABOUT y AXIS AT RATE $\boldsymbol{\omega}_1$

$$\boldsymbol{\alpha} = \boldsymbol{\omega}_1 \times \boldsymbol{\omega} = -\frac{\pi}{6}\mathbf{j} \times \left(-\frac{\pi}{18}\mathbf{i} - \frac{\pi}{6}\mathbf{j}\right) = -\frac{\pi^2}{(6)(18)}\mathbf{k}$$

$$\boldsymbol{\alpha} = -(0.0914\text{ rad/s}^2)\mathbf{k}$$

(c)

$$\mathbf{v}_P = \boldsymbol{\omega} \times \mathbf{r}_P = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.17453 & -0.5236 & 0 \\ 0 & 2 & 3.464 \end{vmatrix}$$

$$\mathbf{v}_P = -1.8138\mathbf{i} + 0.6046\mathbf{j} - 0.34906\mathbf{k}$$

$$\mathbf{v}_P = -(1.814 m/s)\mathbf{i} + (0.605 m/s)\mathbf{j} - (0.349 m/s)\mathbf{k}$$

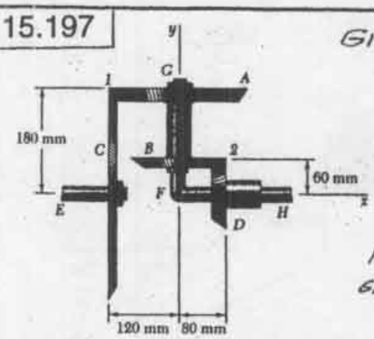
$$\mathbf{a}_P = \boldsymbol{\alpha} \times \mathbf{r}_P + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_P) = \boldsymbol{\alpha} \times \mathbf{r}_P + \boldsymbol{\omega} \times \mathbf{v}_P$$

$$\mathbf{a}_P = -0.0914\mathbf{k} \times (2\mathbf{j} + 3.464\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.17453 & -0.5236 & 0 \\ -1.814 & 0.605 & -0.349 \end{vmatrix}$$

$$\mathbf{a}_P = 0.1828\mathbf{i} + 0.1828\mathbf{i} - 0.0609\mathbf{j} + (-0.1056 - 0.9498)\mathbf{k}$$
$$= 0.3656\mathbf{i} - 0.0609\mathbf{j} - 1.0554\mathbf{k}$$

$$\mathbf{a}_P = (0.366 m/s^2)\mathbf{i} - (0.0609 m/s^2)\mathbf{j} - (1.055 m/s^2)\mathbf{k}$$

15.197



GIVEN: $\boldsymbol{\omega}_C = (15\text{ rad/s})\mathbf{i}$

$\boldsymbol{\omega}_D = (30\text{ rad/s})\mathbf{i}$

FIND: (a) $\boldsymbol{\omega}_{AB}$

(b) $\boldsymbol{\alpha}_{AB}$

(c) ACCELERATION OF TOOTH OF GEAR B IN CONTACT WITH GEAR D

$$\mathbf{r}_1 = -(0.12 m)\mathbf{i} + (0.18 m)\mathbf{j}\ ;\quad \mathbf{r}_2 = (0.08 m)\mathbf{i} + (0.06 m)\mathbf{j}$$

$$\mathbf{v}_1 = \boldsymbol{\omega}_C \times \mathbf{r}_1 = 15\mathbf{i} \times (-0.12\mathbf{i} + 0.18\mathbf{j}) = (2.7 m/s)\mathbf{k} \qquad (1)$$

$$\mathbf{v}_2 = \boldsymbol{\omega}_D \times \mathbf{r}_2 = 30\mathbf{i} \times (0.08\mathbf{i} + 0.06\mathbf{j}) = (1.8 m/s)\mathbf{k} \qquad (2)$$

MOTION OF GEARS A & B $\quad \boldsymbol{\omega}_{AB} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

VELOCITY:

$$\mathbf{v}_1 = \boldsymbol{\omega}_{AB} \times \mathbf{r}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -0.12 & 0.18 & 0 \end{vmatrix}$$

$$\mathbf{v}_1 = -0.18\omega_z\mathbf{i} - 0.12\omega_z\mathbf{j} + (0.18\omega_x + 0.12\omega_y)\mathbf{k}$$

FROM EQ 1: $\mathbf{v}_1 = (2.7 m/s)\mathbf{k}$

$$2.7\mathbf{k} = -0.18\omega_z\mathbf{i} - 0.12\omega_z\mathbf{j} + (0.18\omega_x + 0.12\omega_y)\mathbf{k}$$

COEFFICIENTS OF $\mathbf{i}$: $\quad \omega_z = 0$

COEFFICIENTS OF $\mathbf{k}$: $\quad 2.7 = 0.18\omega_x + 0.12\omega_y \qquad (3)$

$(\omega_z = 0)$

$$\mathbf{v}_2 = \boldsymbol{\omega}_{AB} \times \mathbf{r}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & 0 \\ 0.08 & 0.06 & 0 \end{vmatrix} = (0.06\omega_x - 0.08\omega_y)\mathbf{k}$$

FROM EQ 2: $\mathbf{v}_2 = (1.8 m/s)\mathbf{k} = (0.06\omega_x - 0.08\omega_y)\mathbf{k}$

COEFFICIENTS of $\mathbf{k}$: $\quad 1.8 = 0.06\omega_x - 0.08\omega_y \qquad (4)$

SOLVING SIMULTANEOUSLY EQS 1 AND 2, WE FIND

$$\omega_x = 20 \qquad \omega_y = -7.5$$

(a) $\boldsymbol{\omega}_{AB} = (20\text{ rad/s})\mathbf{i} - (7.5\text{ rad/s})\mathbf{j}$

ACCELERATION GEARS A & B ROTATE ABOUT x AXIS AT RATE $\boldsymbol{\omega}_{FH} = (\boldsymbol{\omega}_{AB})_x = (20\text{ rad/s})\mathbf{i}$

$$\boldsymbol{\alpha}_{AB} = \boldsymbol{\omega}_{FH} \times \boldsymbol{\omega}_{AB} = (20\mathbf{i}) \times (20\mathbf{i} - 7.5\mathbf{j}) = -150\mathbf{k}$$

(b) $\boldsymbol{\alpha}_{AB} = -(150\text{ rad/s}^2)\mathbf{k}$

(c) FOR TOOTH OF GEAR B IN CONTACT WITH GEAR D

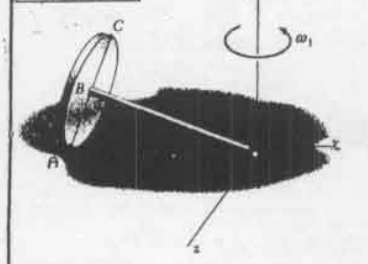
$$\mathbf{r}_2 = (0.08 m)\mathbf{i} + (0.06 m)\mathbf{j}\ ;\quad \mathbf{v}_2 = (1.8 m/s)\mathbf{k}$$

$$\mathbf{a}_2 = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_2 + \boldsymbol{\omega}_{AB} \times \mathbf{v}_2$$
$$= (-150\mathbf{k}) \times (0.08\mathbf{i} + 0.06\mathbf{j}) + (20\mathbf{i} - 7.5\mathbf{j}) \times (1.8\mathbf{k})$$
$$= -12\mathbf{j} + 9\mathbf{i} - 36\mathbf{j} - 13.5\mathbf{i}$$

$$\mathbf{a}_2 = -4.5\mathbf{i} - 48\mathbf{j}$$

$$\mathbf{a}_2 = -(4.5 m/s^2)\mathbf{i} - (48 m/s^2)\mathbf{j}$$

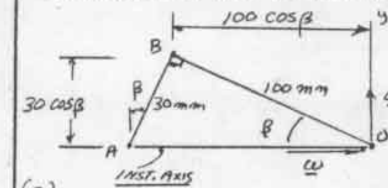
15.198



GIVEN:
$r = AB = BC = 30$ mm
$l = OB = 100$ mm
$\underline{\omega}_1 = (2.4 \text{ rad/s})\,\underline{j}$
$\alpha_1 = 0$

FIND: FOR WHEEL,
(a) $\underline{\omega}$
(b) $\underline{\alpha}$
(c) $\underline{a}_C$

FOR WHEEL-ROD UNIT: ANGULAR VELOCITY = $\underline{\omega}$
INSTANTANEOUS AXIS OF ROTATION IS THE x AXIS



(a)
$\tan\beta = \frac{30}{100}$; $\beta = 16.7^\circ$

CONSIDER MOTION ABOUT y AXIS: $v_B = (100\cos\beta)(2.4)$

CONSIDER MOTION ABOUT INST. AXIS: $v_B = (30\cos\beta)\,\omega$

$v_B = v_B$: $(100\cos\beta)(2.4) = (30\cos\beta)\,\omega$

$\omega = \frac{100}{30}(2.4)$ $\quad \underline{\omega} = (8 \text{ rad/s})\,\underline{i}$ ◀

(b) $\underline{\alpha} = \underline{\omega}_1 \times \underline{\omega} = (2.4 \text{ rad/s})\underline{j} \times (8 \text{ rad/s})\underline{i}$

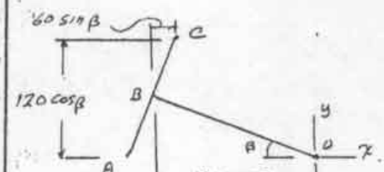
$\underline{\alpha} = -(19.2 \text{ rad/s}^2)\,\underline{k}$ ◀

(c) POINT C: $CA = 2r = 0.06$ m

$\underline{r}_{C/A} = (0.06\text{m})(-\sin\beta\,\underline{i} + \cos\beta\,\underline{j})$

$\underline{v}_C = \underline{\omega} \times \underline{r}_{C/A} = (8 \text{ rad/s})\underline{i} \times (0.06\text{m})(-\sin\beta\,\underline{i} + \cos\beta\,\underline{j})$
$= 0.48\cos\beta\,\underline{k} = 0.48\cos 16.7\,\underline{k}$

$v_C = (0.4598 \text{ m/s})\,\underline{k}$



BC = 30 mm
OB = 100 mm
$\beta = 16.7^\circ$

$\underline{r}_C = -(100\cos\beta - 30\sin\beta)\,\underline{i} + (60\cos\beta)\,\underline{j}$
$= -(95.782 - 8.621)\,\underline{i} + 57.47\,\underline{j}$

$\underline{r}_C = -(87.16 \text{ mm})\,\underline{i} + (57.47 \text{ mm})\,\underline{j}$

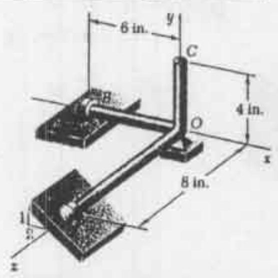
$r_C = (0.08716\text{m})\,\underline{i} + (0.05747\text{m})\,\underline{j}$

$\underline{a}_C = \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times \underline{v}_C$
$= -(19.2)\underline{k} \times (-0.08716\,\underline{i} + 0.05747\,\underline{j}) + 8\,\underline{i} \times 0.4598\,\underline{k}$
$= +1.673\,\underline{j} + 1.103\,\underline{i} - 3.678\,\underline{j}$
$= 1.103\,\underline{i} - 2.005\,\underline{j}$

$\underline{a}_C = (1.103 \text{ m/s}^2)\,\underline{i} - (2.005 \text{ m/s}^2)\,\underline{j}$ ◀

15.199 and 15.200

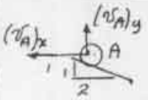
GIVEN: $\underline{v}_B = -(15 \text{ in./s})\,\underline{k}$
$(\underline{a}_B)_z = 0$



PROBLEM 15.199
FIND: (a) $\underline{\omega}$
(b) $\underline{v}_C$

PROBLEM 15.200
FIND: (a) $\underline{\alpha}$
(b) $\underline{a}_C$

PROBLEM 15.199



$(v_A)_x = -2(v_A)_y$

$\underline{r}_B = -(6 \text{ in.})\,\underline{i}$
$\underline{r}_A = (8 \text{ in.})\,\underline{k}$
$\underline{\omega} = \omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}$

$\underline{v}_B = \underline{\omega} \times \underline{r}_B = (\omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}) \times (-6\text{in.})\,\underline{i}$

$-(15 \text{ in./s})\,\underline{k} = (6\text{in.})\,\omega_y\,\underline{k} - (6\text{in.})\,\omega_z\,\underline{j}$

COEF. OF $\underline{k}$: $-15 \text{ in./s} = (6\text{in.})\,\omega_y$ $\quad \omega_y = -2.5$ rad/s

COEF. OF $\underline{j}$: $0 = (6\text{in})\,\omega_z$ $\quad \omega_z = 0$

$\underline{v}_A = \underline{\omega} \times \underline{r}_A = (\omega_x\underline{i} - 2.5\underline{j}) \times 8\underline{k}$

$\underline{v}_A = -(20 \text{ in./s})\,\underline{i} - (8 \text{ in.})\,\omega_x\,\underline{j}$ (1)

BUT $(v_A)_x = -2(v_A)_y$: $-20 \text{ in./s} = -2(-8 \text{ in.})\,\omega_x$

$\omega_x = -1.25$ rad/s

$\underline{\omega} = -(1.25 \text{ rad/s})\,\underline{i} - (2.5 \text{ rad/s})\,\underline{j}$ ◀

$\underline{r}_C = (4 \text{ in.})\,\underline{j}$

$\underline{v}_C = \underline{\omega} \times \underline{r}_C = (-1.25\,\underline{i} - 2.5\,\underline{j}) \times 4\underline{j} = -5\underline{k}$

$\underline{v}_C = -(5 \text{ in./s})\,\underline{k}$ ◀

$\underline{v}_A = \underline{\omega} \times \underline{r}_A = (-1.25\underline{i} - 2.5\underline{j}) \times 8\underline{k} = -(20 \text{ in./s})\underline{i} + (10 \text{ in./s})\underline{j}$

PROBLEM 15.200 SINCE $(a_B)_z = 0$, $\underline{a}_B = (a_B)_x\,\underline{i}$

$\underline{a}_B = \underline{\alpha} \times \underline{r}_B + \underline{\omega} \times \underline{v}_B$
$= (\alpha_x\underline{i} + \alpha_y\underline{j} + \alpha_z\underline{k}) \times 6\,\underline{i} + (-1.25\,\underline{i} - 2.5\,\underline{j}) \times (-15\,\underline{k})$
$= +6\alpha_y\,\underline{k} - 6\alpha_z\,\underline{j} - 18.75\,\underline{j} + 37.5\,\underline{i}$

COEF. OF $\underline{k}$: $0 = \alpha_y$ $\quad \alpha_y = 0$

COEF. OF $\underline{j}$: $0 = -6\alpha_z - 18.75$ $\quad \alpha_z = -3.125 \text{ rad/s}^2$

$\underline{a}_A = \underline{\alpha} \times \underline{r}_A + \underline{\omega} \times \underline{v}_A$
$= (\alpha_x\,\underline{i} - 3.125\,\underline{k}) \times 8\underline{k} + (-1.25\,\underline{i} - 2.5\,\underline{j}) \times (-20\,\underline{i} + 10\,\underline{j})$

$\underline{a}_A = -8\alpha_x\,\underline{j} - 12.5\,\underline{k} - 50\,\underline{k}$

$(a_A)_x = 0 \quad (a_A)_y = -8\alpha_x \quad (a_A)_z = -62.5$

SINCE $(v_A)_x = -2(v_A)_y$, $(a_A)_x = -2(a_A)_y$

$0 = -8\alpha_x \quad \alpha_x = 0$

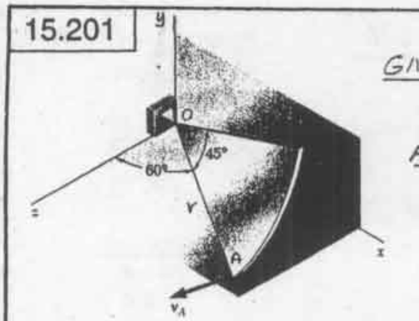
$\underline{\alpha} = -(3.125 \text{ rad/s}^2)\,\underline{k}$ ◀

$\underline{r}_C = (4 \text{ in.})\,\underline{j}$

$\underline{a}_C = \underline{\alpha} \times \underline{r}_C + \underline{\omega} \times \underline{v}_C$
$= (-3.125\,\underline{k}) \times 4\,\underline{j} + (-1.25\,\underline{i} - 2.5\,\underline{j}) \times (-5\,\underline{k})$
$= 12.5\,\underline{i} - 6.25\,\underline{j} + 12.5\,\underline{i}$

$\underline{a}_C = (25 \text{ in./s}^2)\,\underline{i} - (6.25 \text{ in./s}^2)\,\underline{j}$ ◀

15.201



GIVEN: $r = 10$ in.
$v_A = 60$ in./s

FIND: (a) α
(b) $\underline{v}_B$

FIND OB: $\overline{OA} = (10\text{in.})\sin 60°\,\underline{i} + (10\text{in.})\cos 60°\,\underline{k}$

$\underline{r}_A = \overline{OA} = (8.6603\text{ in.})\underline{i} + (5\text{ in.})\underline{k}$

$\overline{OB} = (r_B)_x\underline{i} + (r_B)_y\underline{j}$

SCALAR PRODUCT:

$\overline{OA}\cdot\overline{OB} = (OA)(OB)\sin 45°$

$(8.6603\underline{i} + 5\underline{k})\cdot((r_B)_x\underline{i} + (r_B)_y\underline{j}) = (10)(10)\sin 45°$

$8.6603(r_B)_x = 70.711 \qquad (r_B)_x = 8.165$ in.

$(r_B)_y = OB^2 - (r_B)_x^2 = 10^2 - 8.165^2\,; \quad (r_B)_y = 5.773$ in

$\underline{r}_B = (8.165\text{ in.})\underline{i} + (5.773\text{ in.})\underline{j}$

SINCE $v_A \perp OA$, v_A FORMS $30°$ ANGLE WITH Z AXIS

$\underline{v}_A = (60\text{ in/s})(-\sin 30°\,\underline{i} + \cos 60°\,\underline{j})$

$\underline{v}_A = -(30\text{ in/s})\underline{i} + (51.96\text{ in/s})\underline{k}$

PLATE OAB: $\underline{\omega} = \omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k}$

$$\underline{v}_A = \underline{\omega}\times\underline{r}_A = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 8.6603 & 0 & 5 \end{vmatrix}$$

$-30\underline{i} + 51.96\underline{k} = 5\omega_y + (8.6603\omega_z - 5\omega_x)\underline{j} - 8.6603\omega_y\underline{k}$

COEF $\underline{i}$: $-30 = 5\omega_y \quad\rightarrow\quad \omega_y = -6$ rad/s

COEF $\underline{j}$: $0 = 8.6603\omega_z - 5\omega_x \quad\rightarrow\quad \omega_z = 0.57735\omega_x$ (1)

COEF $\underline{k}$: $51.96 = -8.6603\omega_y \quad\rightarrow\quad \omega_y = -6$ rad/s

$$\underline{v}_B = \underline{\omega}\times\underline{r}_B = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 8.165 & 5.773 & 0 \end{vmatrix}$$

$\underline{v}_B = -5.773\omega_z\underline{i} + 8.165\omega_z\underline{j} + (5.773\omega_x - 8.165\omega_y)\underline{k}$

SINCE POINT B MOVES IN xy PLANE

$(v_B)_z = 0 = 5.773\omega_x - 8.165\omega_y$

$0 = 5.773\omega_x - 8.165(-6)$

$\omega_x = -8.486$ rad/s

EQ(1): $\omega_z = 0.57735(-8.486) = -4.899$ rad/s

$\underline{\omega} = -(8.49\text{ rad/s})\underline{i} - (6\text{ rad/s})\underline{j} - (4.90\text{ rad/s})\underline{k}$ ◀

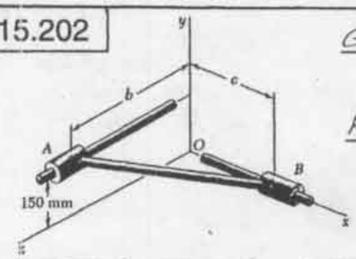
$(v_B)_x = -5.773(-4.899) = 28.3$ m/s

$(v_B)_y = 8.165(-4.899) = -40.0$ m/s

$(v_B)_z = 0$

$\underline{v}_B = (28.3\text{ in./s})\underline{i} - (40.0\text{ in./s})\underline{j}$ ◀

15.202



GIVEN: $c = 175$ mm
$\underline{v}_B = -(180\text{ mm/s})\underline{i}$
$AB = 275$ mm

FIND: $\underline{v}_A$

$c = 175$ mm ; $275^2 = 150^2 + 175^2 + b^2$; $b = 150$ mm

$\underline{v}_B = -(180\text{ mm/s})\underline{i}$; $\underline{v}_A = v_A\underline{k}$; $\underline{r}_{A/B} = -175\underline{i} + 150\underline{j} + 150\underline{k}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega}\times\underline{r}_{A/B}$

$$v_A\underline{k} = -180\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -175 & 150 & 150 \end{vmatrix}$$

$v_A\underline{k} = -180\underline{i} + (150\omega_y - 150\omega_z)\underline{i} + (-175\omega_z - 150\omega_x)\underline{j} + (150\omega_x + 175\omega_y)\underline{k}$

COEF OF $\underline{i}$: $+180 = \quad +150\omega_y - 150\omega_z$ (1)

COEF. OF $\underline{j}$: $0 = -150\omega_x \quad -175\omega_z$ (2)

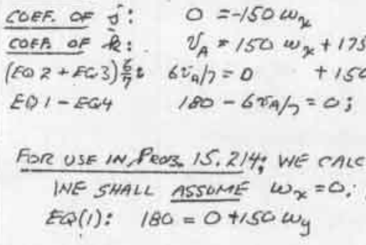
COEF OF $\underline{k}$: $v_A = 150\omega_x + 175\omega_y$ (3)

(EQ 2 + EQ 3)$\frac{6}{7}$: $6v_A/7 = 0 \quad +150\omega_y - 150\omega_z$ (4)

EQ 1 − EQ 4 $\quad 180 - 6v_A/7 = 0$; $\quad \underline{v}_A = (210\text{ mm/s})\underline{k}$ ◀

FOR USE IN PROB. 15.214; WE CALCULATE A POSSIBLE $\underline{\omega}$.
WE SHALL ASSUME $\omega_x = 0$. FROM EQ (2), WE HAVE $\omega_z = 0$.
EQ(1): $180 = 0 + 150\omega_y$ $\qquad \underline{\omega} = +(1.2\text{ rad/s})\underline{j}$

15.203



GIVEN: $c = 50$ mm
$\underline{v}_B = -(180\text{ mm/s})\underline{i}$
$AB = 275$ mm

FIND: $\underline{v}_A$

$c = 50$ mm ; $275^2 = 150^2 + 50^2 + b^2$; $b = 225$ mm

$\underline{v}_B = -(180\text{ mm/s})\underline{i}$; $\underline{v}_A = v_A\underline{k}$; $\underline{r}_{A/B} = -50\underline{i} + 150\underline{j} + 225\underline{k}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega}\times\underline{r}_{A/B}$

$$v_A\underline{k} = -180\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -50 & 150 & 225 \end{vmatrix}$$

$v_A\underline{k} = -180\underline{i} + (225\omega_y - 150\omega_z)\underline{i} + (-50\omega_z - 225\omega_x)\underline{j} + (150\omega_x + 50\omega_y)\underline{k}$

COEF OF $\underline{i}$: $+180 = \quad 225\omega_y - 150\omega_z$ (1)

COEF OF $\underline{j}$: $0 = -225\omega_x \quad -50\omega_z$ (2)

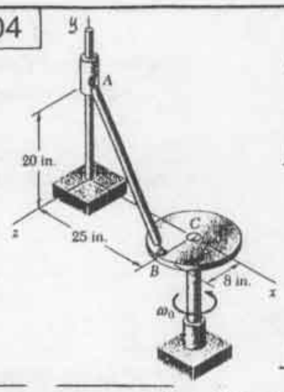
COEF OF $\underline{k}$: $v_A = 150\omega_x + 50\omega_y$ (3)

[EQ.1 + 3×EQ.2 − 4.5 EQ.3]: $\quad 180 - 4.5v_A = 0$

$\underline{v}_A = +(40\text{ mm/s})\underline{k}$ ◀

FOR USE IN PROB. 15.215:
WE SHALL CALCULATE A POSSIBLE $\underline{\omega}$; SINCE ω IS INDETERMINATE, WE CAN ASSUME A VALUE FOR ANY COMPONENT OF $\underline{\omega}$. WE ASSUME $\omega_x = 0$,
FROM EQ (2), WE FIND $\omega_z = 0$
EQ(1): $+180 = 225\omega_y$ $\qquad \underline{\omega} = (0.8\text{ rad/s})\underline{j}$

15.204



GIVEN:
$\underline{\omega}_0 = (3\text{ rad/s})\,\underline{j}$

FIND: $\underline{v}_A$

$\underline{r}_{A/B} = -(25\text{ in.})\underline{i} + (20\text{ in.})\underline{j} - (8\text{ in.})\underline{k}$

DISK: $\underline{v}_B = \underline{\omega}_0 \times \underline{r}_{B/C} = (3\text{ rad/s})\underline{j} \times (8\text{ in.})\underline{k} = (24\text{ in./s})\underline{i}$

ROD AB: $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{B/C}$

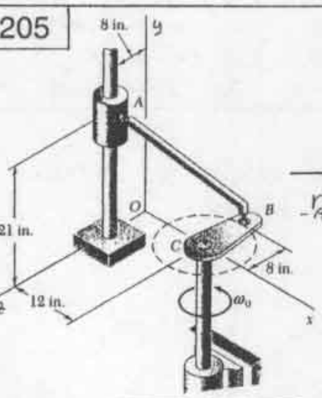
$$v_A\underline{j} = (24\text{ in./s})\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -25 & +20 & -8 \end{vmatrix}$$

$$v_A\underline{j} = 24\underline{i} + (-8\omega_y - 20\omega_z)\underline{i} + (-25\omega_z + 8\omega_x)\underline{j} + (20\omega_x + 25\omega_y)\underline{k}$$

COEF OF $\underline{i}$: $-24 = -8\omega_y - 20\omega_z$ (1)
COEF OF $\underline{j}$: $v_A = 8\omega_x - 25\omega_z$ (2)
COEF OF $\underline{k}$: $0 = 20\omega_x + 25\omega_y$ (3)

SINCE DETERMINANT OF $\omega_x, \omega_y, \omega_z$ IS ZERO, ω IS INDETERMINATE. WE CAN ASSUME ANY ONE COMPONENT
ASSUME $\omega_x = 0$, EQ.3 YIELDS $\omega_y = 0$.
EQ.1: $-24 = 0 - 20\omega_z$; $\omega_z = 1.2\text{ rad/s}$
EQ.2: $v_A = 0 - 25(1.2) = -30$; $\underline{v}_A = -(30\text{ in./s})\underline{j}$ ◀

15.205



GIVEN:
$\underline{\omega}_0 = (10\text{ rad/s})\underline{j}$

FIND: $\underline{v}_A$

$\underline{r}_{A/B} = -(12\text{ in.})\underline{i} + (21\text{ in.})\underline{j} + (16\text{ in.})\underline{k}$

CRANK BC:
$\underline{r}_{B/C} = -(8\text{ in.})\underline{k}$
$\underline{v}_B = \underline{\omega} \times \underline{r}_{B/C}$
$= (10\text{ rad/s})\underline{j} \times (-8\text{ in.})\underline{i}$
$\underline{v}_B = -(80\text{ in/s})\underline{i}$

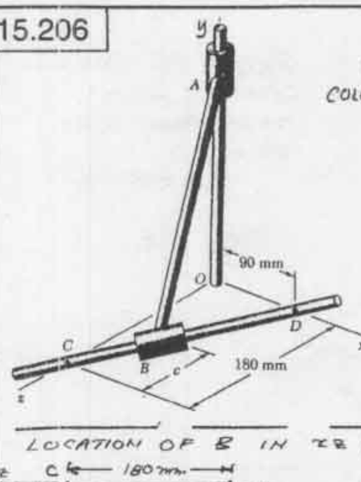
ROD AB: $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{B/C}$

$$v_A\underline{j} = -80\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -12 & 21 & 16 \end{vmatrix}$$

COEF. OF $\underline{i}$: $-80 = 16\omega_y - 21\omega_z$ (1)
COEF. OF $\underline{j}$: $v_A = -16\omega_x - 12\omega_z$ (2)
COEF. OF $\underline{k}$: $0 = 21\omega_x + 12\omega_y$ (3)

SINCE DETERMINANT OF $\omega_x, \omega_y, \omega_z$ IS ZERO, THE ANGULAR VELOCITY IS INDETERMINATE. WE CAN ASSUME VALUE OF ANY ONE COMPONENT.
ASSUME $\omega_x = 0$. EQ.2 YIELDS $\omega_y = 0$
EQ.(1): $-80 = 0 - 21\omega_z$; $\omega_z = -\frac{80}{21}\text{ rad/s}$
EQ.(2): $v_A = 0 - 12(-\frac{80}{21})$; $\underline{v}_A = (45.7\text{ in./s})\underline{j}$ ◀

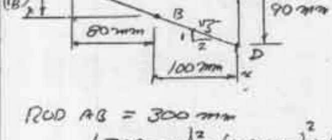
15.206



GIVEN: AB = 300 mm
COLLAR B MOVES TOWARD POINT D AT 50 mm/s
$r = 80$ mm

FIND: $\underline{v}_A$

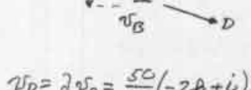
LOCATION OF B IN XZ PLANE



$(r_B)_x = \frac{1}{2}r = \frac{80\text{ mm}}{2}$

$(r_B)_x = 40$ mm

ROD AB = 300 mm
$(300\text{ mm})^2 = (100\text{ mm})^2 + (40\text{ mm})^2 + (r_A)^2$
$\underline{r}_A = (280\text{ mm})\underline{j}$



VELOCITY OF B:
$v_B = 50$ mm/s
$\underline{\lambda}_{CD} = \frac{1}{\sqrt{5}}(-2\underline{k} + \underline{i})$

$\underline{v}_B = \underline{\lambda} v_B = \frac{50}{\sqrt{5}}(-2\underline{k} + \underline{i}) = -(44.72\text{ mm/s})\underline{k} + (22.36\text{ mm/s})\underline{i}$

$\underline{r}_{A/B} = -(40\text{ mm})\underline{i} + (280\text{ mm})\underline{j} - (100\text{ mm})\underline{k}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$$v_A\underline{j} = -44.72\underline{k} + 22.36\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -40 & 280 & -100 \end{vmatrix}$$

$$v_A\underline{j} = -44.72\underline{k} + 22.36\underline{i} + (-100\omega_y - 280\omega_z)\underline{i} + (-40\omega_z + 100\omega_x)\underline{j} + (280\omega_x + 40\omega_y)\underline{k}$$

COEF. OF $\underline{i}$: $-22.36 = -100\omega_y - 280\omega_z$ (1)
COEF. OF $\underline{j}$: $v_A = 100\omega_x - 40\omega_z$ (2)
COEF. OF $\underline{k}$: $+44.72 = 280\omega_x + 40\omega_y$ (3)

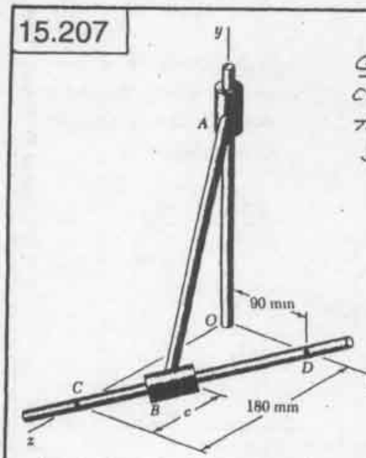
SINCE DETERMINANT OF $\omega_x, \omega_y, \omega_z$ IS ZERO, THE ANGULAR VELOCITY IS INDETERMINANT. WE CAN ASSUME VALUE OF ANY COMPONENT.
ASSUME $\omega_x = 0$
EQ.3: $44.72 = 0 + 40\omega_y$; $\omega_y = 1.118\text{ rad/s}$
EQ.1: $-22.36 = -100(1.118) - 280\omega_z$
$\omega_z = -0.3194\text{ rad/s}$

$\underline{\omega} = (1.118\text{ rad/s})\underline{j} - (0.3194\text{ rad/s})\underline{k}$

EQ.2: $v_A = 0 - 40(-0.3194) = 12.777\text{ mm/s}$

$\underline{v}_A = (12.78\text{ mm/s})\underline{j}$ ◀

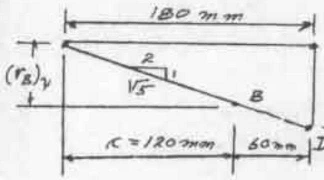
15.207



GIVEN: $AB = 300\ mm$
COLLAR B MOVES TOWARD POINT D AT $50\ mm/s$.
$c = 120\ mm$

FIND: v_A

LOCATION OF B IN xz PLANE



$(r_B)_x = \frac{c}{2} = \frac{120\ mm}{2}$

$(r_B)_y = 60\ mm$

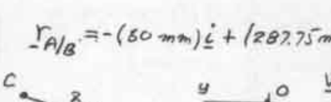
$(r_B)_z = 60\ mm$

ROD $AB = 300\ mm$

$(300\ mm)^2 = (60\ mm)^2 + (60\ mm)^2 + (r_A)^2$

$r_A = 287.75\ mm$

$\underline{r}_{A/B} = -(60\ mm)\underline{i} + (287.75\ mm)\underline{j} - (60\ mm)\underline{k}$



VELOCITY OF B:

$v_B = 50\ mm/s$

$\lambda_{CD} = \frac{1}{\sqrt{5}}(-2\underline{k} + \underline{i})$

$\underline{v}_B = v_B \underline{\lambda} = \frac{50}{\sqrt{5}}(\underline{i} - 2\underline{k}) = +(22.36\ mm/s)\underline{i} - (44.72\ mm/s)\underline{k}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$$v_A\underline{j} = 22.36\underline{i} - 44.72\underline{k} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -60 & 287.75 & -60 \end{vmatrix}$$

$$v_A\underline{j} = 22.36\underline{i} - 44.72\underline{k} + (-60\omega_y - 287.75\omega_z)\underline{i} + (-60\omega_z + 60\omega_x)\underline{j} + (287.75\omega_x + 60\omega_y)\underline{k}$$

COEF. OF $\underline{i}$: $\quad -22.36 = -60\omega_y - 287.75\omega_z \quad (1)$

COEF. OF $\underline{j}$: $\quad v_A = 60\omega_x - 60\omega_z \quad (2)$

COEF. OF $\underline{k}$: $\quad 44.72 = 287.75\omega_x + 60\omega_y \quad (3)$

SINCE DETERMINANT OF $\omega_x, \omega_y, \omega_z$ IS ZERO, THE ANGULAR VELOCITY IS INDETERMINANT. WE CAN THUS ASSUME THE VALUE OF ANY COMPONENT.

ASSUME $\omega_x = 0$:

EQ.3: $\quad 44.72 = 0 + 60\omega_y; \quad \omega_y = 0.7453\ rad/s$

EQ.1: $\quad -22.36 = -60(0.7453) - 287.75\omega_z$

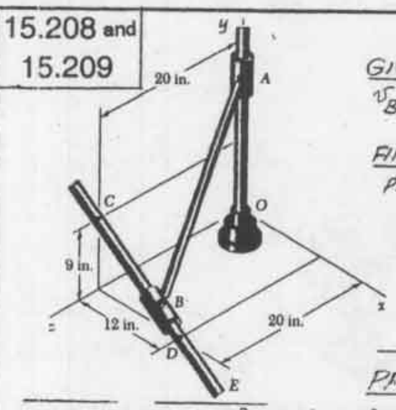
$\omega_z = -0.0777\ rad/s$

$\underline{\omega} = (0.7453\ rad/s)\underline{j} - (0.0777\ rad/s)\underline{k}$

EQ.2: $\quad v_A = 0 - 60(-0.0777) = 4.66$

$\underline{v}_A = (4.66\ mm/s)\underline{j}$ ◀

15.208 and 15.209



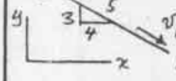
GIVEN: $AB = 25\ in.$
$v_B = 20\ in./s$ TOWARD E

FIND: v_A AS COLLAR B PASSES POINT D

PROBLEM 15.208

COLLAR AT D: $AB^2 = 25^2 = 12^2 + 20^2 + r_A^2; \quad r_A = 9\ in.$

$\underline{r}_{A/B} = -(12\ in.)\underline{i} + (9\ in.)\underline{j} - (20\ in.)\underline{k}$



$v_B = 20\ in/s \quad \underline{v}_B = v_B\underline{\lambda}_{CD} = v_B(0.8\underline{i} - 0.6\underline{j})$

$\underline{v}_B = (16\ in./s)\underline{i} - (12\ in./s)\underline{j}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$$v_A\underline{j} = 16\underline{i} - 12\underline{j} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -12 & 9 & -20 \end{vmatrix}$$

COEF. OF $\underline{i}$: $\quad -16 = -20\omega_y - 9\omega_z \quad (1)$

COEF. OF $\underline{j}$: $\quad v_A + 12 = 20\omega_x - 12\omega_z \quad (2)$

COEF. OF $\underline{k}$: $\quad 0 = 9\omega_x + 12\omega_y \quad (3)$

SINCE DETERMINANT OF $\omega_x, \omega_y, \omega_z$ IS ZERO, THE ANGULAR VELOCITY IS INDETERMINATE. WE CAN THUS ASSUME THE VALUE OF ANY COMPONENT

ASSUME $\omega_x = 0$, EQ 3. YIELDS $\omega_y = 0$

EQ.1: $\quad -16 = 0 - 9\omega_z \quad \omega_z = \frac{16}{9}\ rad/s$

EQ.2: $\quad v_A + 12 = 0 - 12(\frac{16}{9})\underline{j}; \quad v_A + 12 = -21.33$

$\underline{v}_A = -(33.3\ in./s)\underline{j}$ ◀

PROBLEM 15.209

COLLAR AT C: $AB^2 = 25^2 = 20^2 + r_A^2; \quad r_A = 15\ in.$

$\underline{r}_{A/B} = (15\ in.)\underline{j} - (20\ in.)\underline{k}$

$\underline{v}_B = (16\ in./s)\underline{i} - (12\ in./s)\underline{j}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$$v_A\underline{j} = 16\underline{i} - 12\underline{j} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 0 & 15 & -20 \end{vmatrix}$$

COEF. OF $\underline{i}$: $\quad 16 = -20\omega_y - 15\omega_z \quad (1)$

COEF. OF $\underline{j}$: $\quad v_A + 12 = 20\omega_x \quad (2)$

COEF. OF $\underline{k}$: $\quad 0 = 15\omega_x \quad (3)$

EQ.3: $\quad \omega_x = 0$

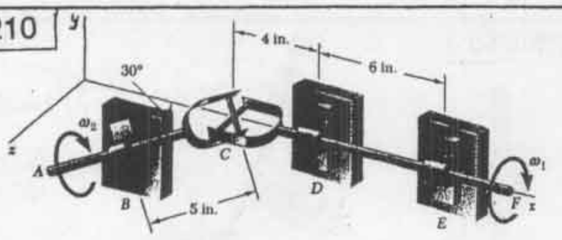
EQ.2: $\quad v_A + 12 = 0 \quad v_A = -12\ in/s$

$\underline{v}_A = -(12\ in./s)\underline{j}$ ◀

NOTE: ω IS INDETERMINATE. ANY VALUE CAN BE CHOSEN FOR EITHER ω_y OR ω_z. FOR EXAMPLE, IF $\omega_x = 0$, THEN

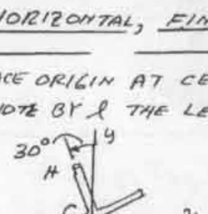
EQ.1: $\quad 16 = -20\omega_y \quad \omega_y = -0.8\ rad/s$

15.210



WHEN ARM OF CROSSPIECE ATTACHED TO SHAFT CF IS HORIZONTAL, FIND ω_2 OF SHAFT AC.

PLACE ORIGIN AT CENTER OF CROSSPIECE AND DENOTE BY ℓ THE LENGTH OF EACH ARM.



$$\mathbf{r}_G = \ell\mathbf{k}$$
$$\mathbf{r}_H = -\ell\sin 30°\,\mathbf{i} + \ell\cos 30°\,\mathbf{j}$$
$$\boldsymbol{\omega}_1 = -\omega_1\mathbf{i}$$
$$\boldsymbol{\omega}_2 = -\omega_2\cos 30°\,\mathbf{i} - \omega_2\sin 30°\,\mathbf{j}$$

SHAFT CF: $\mathbf{v}_G = \boldsymbol{\omega}_1\times\mathbf{r}_G = -\omega_1\ell\,\mathbf{i}\times\ell\mathbf{k} = \ell\omega_1\mathbf{j}$ (1)

SHAFT AC: $\mathbf{v}_H = \boldsymbol{\omega}_2\times\mathbf{r}_H$

$$= (-\omega_2\cos 30°\,\mathbf{i} - \omega_2\sin 30°\,\mathbf{j})\times(-\ell\sin 30°\,\mathbf{i} + \ell\cos 30°\,\mathbf{j})$$
$$\mathbf{v}_H = -\ell\omega_2\cos^2 30°\,\mathbf{k} - \ell\omega_2\sin^2 30°\,\mathbf{k}$$
$$\mathbf{v}_H = -\ell\omega_2(\cos^2 30° + \sin^2 30°)\mathbf{k} = -\ell\omega_2\mathbf{k} \quad (2)$$

CROSSPIECE $\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$$\mathbf{v}_G = \boldsymbol{\omega}\times\mathbf{r}_G = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k})\times\ell\mathbf{k}$$
$$\mathbf{v}_G = -\ell\omega_x\mathbf{j} + \ell\omega_y\mathbf{i} \quad (3)$$

EQ 1 = EQ 3: $\mathbf{v}_G = \mathbf{v}_G$

$$\ell\omega_1\mathbf{j} = -\ell\omega_x\mathbf{j} + \ell\omega_y\mathbf{i}$$

COEF. of $\mathbf{j}$: $\ell\omega_1 = -\ell\omega_x$ $\quad \omega_x = -\omega_1$ (4)

COEF. of $\mathbf{i}$: $\quad \omega_y = 0$ (5)

$$\mathbf{v}_H = \boldsymbol{\omega}\times\mathbf{r}_H = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -\ell\sin 30° & \ell\cos 30° & 0 \end{vmatrix}$$
$$= -\ell\omega_z\cos 30°\,\mathbf{i} - \ell\omega_z\sin 30°\,\mathbf{j} + (\ell\omega_x\cos 30° + \ell\omega_y\sin 30°)\mathbf{k}$$

SUBSTITUTE FROM EQS. 4 AND 5: $\omega_x =$ AND $\omega_y = 0$

$$\mathbf{v}_H = -\ell\omega_z\cos 30°\,\mathbf{i} - \ell\omega_z\sin 30°\,\mathbf{j} - \ell\omega_1\cos 30°\,\mathbf{k}$$

SUBSTITUTE FOR v_H FROM EQ. 2.

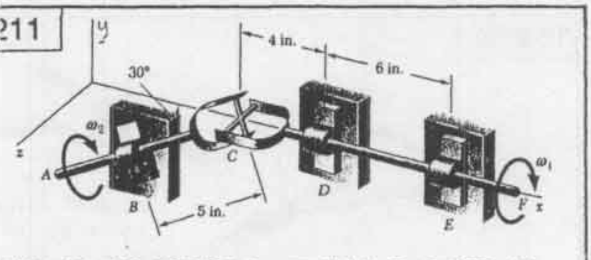
$$-\ell\omega_2\mathbf{k} = -\ell\omega_z\cos 30°\,\mathbf{i} - \ell\omega_z\sin 30°\,\mathbf{j} - \ell\omega_1\cos 30°\,\mathbf{k}$$

COEF OF $\mathbf{j}$: $0 = -\ell\omega_z\sin 30°$ $\quad \omega_z = 0$

COEF OF $\mathbf{k}$: $-\ell\omega_2 = -\ell\omega_1\cos 30°$

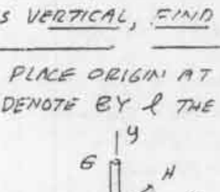
$$\omega_2 = \omega_1\cos 30°$$

15.211



WHEN ARM OF CROSSPIECE ATTACHED TO SHAFT CF IS VERTICAL, FIND ω_2 OF SHAFT AC.

PLACE ORIGIN AT CENTER OF CROSSPIECE AND DENOTE BY ℓ THE LENGTH OF EACH ARM.



$$\mathbf{r}_G = \ell\mathbf{j}$$
$$\mathbf{r}_H = -\ell\mathbf{k}$$
$$\boldsymbol{\omega}_1 = -\omega_1\mathbf{i}$$
$$\boldsymbol{\omega}_2 = -\ell\cos 30°\,\mathbf{i} - \ell\sin 30°\,\mathbf{j}$$

SHAFT CF: $\mathbf{v}_G = \boldsymbol{\omega}_1\times\mathbf{r}_G = -\omega_1\mathbf{i}\times\ell\mathbf{j} = -\ell\omega_1\mathbf{k}$ (1)

SHAFT AC: $\mathbf{v}_H = \boldsymbol{\omega}_2\times\mathbf{r}_H$

$$= (-\ell\omega_2\cos 30°\,\mathbf{i} - \ell\omega_2\sin 30°\,\mathbf{j})\times -\ell\mathbf{k}$$
$$\mathbf{v}_H = -\ell\omega_2\cos 30°\,\mathbf{j} + \ell\omega_2\sin 30°\,\mathbf{i} \quad (2)$$

CROSSPIECE: $\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$$\mathbf{v}_G = \boldsymbol{\omega}\times\mathbf{r}_G = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k})\times\ell\mathbf{j}$$
$$\mathbf{v}_G = \ell\omega_x\mathbf{k} - \ell\omega_z\mathbf{i} \quad (3)$$
$$\mathbf{v}_H = \boldsymbol{\omega}\times\mathbf{r}_H = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k})\times(-\ell\mathbf{k})$$
$$\mathbf{v}_H = \ell\omega_x\mathbf{j} - \ell\omega_y\mathbf{i} \quad (4)$$

EQ. 1 = EQ. 3: $\mathbf{v}_G = \mathbf{v}_G$

$$-\ell\omega_1\mathbf{k} = \ell\omega_x\mathbf{k} - \ell\omega_z\mathbf{i}$$

COEF. OF $\mathbf{k}$: $-\ell\omega_1 = \ell\omega_x$ $\quad \omega_x = -\omega_1$ (4)

COEF. OF $\mathbf{i}$: $\quad \omega_z = 0$ (5)

EQ 2 = EQ. 4: $\mathbf{v}_H = \mathbf{v}_H$

$$-\ell\omega_2\cos 30°\,\mathbf{j} + \ell\omega_2\sin 30°\,\mathbf{i} = \ell\omega_x\mathbf{j} - \ell\omega_y\mathbf{i}$$

COEF. OF $\mathbf{i}$: $\ell\omega_2\sin 30° = -\ell\omega_y$ $\quad \omega_y = -\omega_2\sin 30°$

COEF. OF $\mathbf{j}$: $-\ell\omega_2\cos 30° = \ell\omega_x$

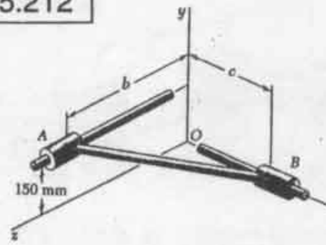
$$\omega_2 = -\frac{\omega_x}{\cos 30°}$$

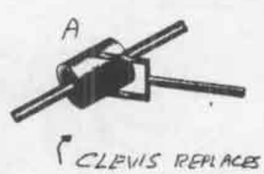
FROM EQ. 5: $\omega_x = -\omega_1$

THUS, $\omega_2 = -\dfrac{(-\omega_1)}{\cos 30°}$

$$\omega_2 = \frac{\omega_1}{\cos 30°}$$

15.212



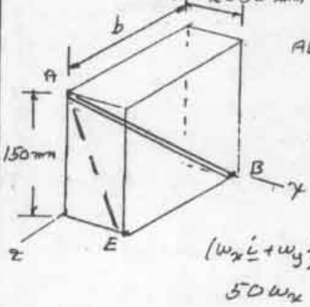


CLEVIS REPLACES JOINT AT A

$AB = 275 \text{ mm}$

GIVEN: $c = 50 \text{ mm}$, $\underline{v}_B = -(180 \text{ mm/s})\underline{i}$.

FIND: (a) ω, (b) $\underline{v}_A$



SINCE $AB = 275 \text{ mm}$

$AB^2 = 275^2 = 150^2 + 50^2 + b^2$

$b = 225 \text{ mm}$

CLEVIS PREVENTS ROTATION OF ROD ABOUT AE

$r_{E/A} = (50 \text{mm})\underline{i} - (150 \text{mm})\underline{j}$

$\omega \cdot r_{E/A} = 0$

$(\omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}) \cdot (50\underline{i} - 150\underline{j})$

$50\omega_x - 150\omega_y = 0$

$\omega_y = \frac{1}{3}\omega_x \qquad (1)$

$r_{A/B} = -(50 \text{ mm})\underline{i} + (150 \text{ mm})\underline{j} + (225 \text{ mm})\underline{k}$

$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$$v_A \underline{k} = -180\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -50 & 150 & 225 \end{vmatrix}$$

$v_A \underline{k} = -180\underline{i} + (225\omega_y - 150\omega_z)\underline{i} + (-50\omega_z - 225\omega_x)\underline{j} + (150\omega_x + 50\omega_y)\underline{k}$

COEF. OF $\underline{i}$: $\quad +180 = 225\omega_y - 150\omega_z \qquad (2)$

COEF. OF $\underline{j}$: $\quad 0 = -225\omega_x - 50\omega_z \qquad (3)$

COEF. OF $\underline{k}$: $\quad v_A = 150\omega_x + 50\omega_y \qquad (4)$

3(EQ 3): $\quad 0 = -675\omega_x - 150\omega_z \qquad (5)$

EQ 1 − EQ 5: $\quad 180 = 675\omega_x + 225\omega_y$

SUBSTITUTE $\omega_y = \frac{1}{3}\omega_x$ FROM EQ 2 INTO EQ 4:

$180 = 675\omega_x + 225(\frac{1}{3}\omega_x)$

$180 = 750\omega_x \qquad \omega_x = 0.24 \text{ rad/s}$

$\omega_y = \frac{1}{3}\omega_x = \frac{1}{3}(0.24) \qquad \omega_y = 0.08 \text{ rad/s}$

EQ 3: $\quad 0 = -225(0.24) - 50\omega_z \qquad \omega_z = -1.08 \text{ rad/s}$

(a) $\underline{\omega} = (0.24 \text{ rad/s})\underline{i} + (0.080 \text{ rad/s})\underline{j} - (1.08 \text{ rad/s})\underline{k}$ ◀

(b) EQ 4: $\quad v_A = 150(0.24) + 50(0.08)$

$= 36 + 4$

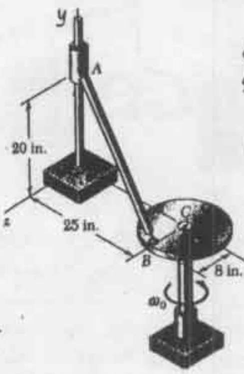
$\underline{v}_A = (40 \text{ mm/s})\underline{k}$ ◀

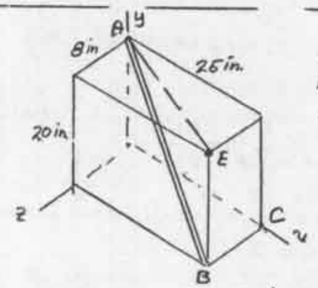
15.213

GIVEN:
$\underline{\omega}_0 = (3 \text{ rad/s})\underline{j}$

FIND: (a) $\underline{\omega}$
(b) $\underline{v}_A$

CLEVIS REPLACES JOINT AT A





CLEVIS PREVENTS ROTATION OF AB ABOUT AE.

$\underline{r}_{E/A} = (25 \text{ in.})\underline{i} + (8 \text{ in.})\underline{k}$

$\underline{\omega} \cdot r_{E/A} = 0$

$(\omega_x \underline{i} + \omega_y \underline{j} + \omega_z \underline{k}) \cdot (25\underline{i} + 8\underline{k})$

$25\omega_x + 8\omega_z = 0$

$\omega_z = -\frac{25}{8}\omega_x \qquad (1)$

$\underline{r}_{A/B} = -(25 \text{ in.})\underline{i} + (20 \text{ in.})\underline{j} - (8 \text{ in.})\underline{k}$

DISK: $\underline{v}_B = \underline{\omega}_0 \times r_{B/C} = (3 \text{ rad/s})\underline{j} \times (8 \text{ in.})\underline{k} = (24 \text{ in/s})\underline{i}$

ROD: $\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{A/B}$

$$v_A \underline{j} = 24\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ -25 & 20 & -8 \end{vmatrix}$$

COEF. OF $\underline{i}$: $\quad -24 = -8\omega_y - 20\omega_z \qquad (2)$

COEF. OF $\underline{j}$: $\quad v_A = 8\omega_x - 25\omega_z \qquad (3)$

COEF. OF $\underline{k}$: $\quad 0 = 20\omega_x + 25\omega_y \qquad (4)$

SUBSTITUTE $\omega_z = -\frac{25}{8}\omega_x$ FROM EQ.1 INTO EQ.2

$-24 = -8\omega_y - 20(-\frac{25}{8})\omega_x$

$-24 = -8\omega_y + 62.5\omega_x \qquad (5)$

FROM EQ. 4: $\quad \omega_y = -\frac{20}{25}\omega_x = -0.8\omega_x \qquad (6)$

SUBSTITUTE INTO EQ. 5:

$-24 = -8(-0.8\omega_x) + 62.5\omega_x$

$\omega_x = -0.3483 \text{ rad/s}$

EQ. 6: $\quad \omega_y = -0.8(-0.3483); \qquad \omega_y = 0.2787 \text{ rad/s}$

EQ. 1: $\quad \omega_z = -\frac{25}{8}(-0.3483); \qquad \omega_z = 1.0885 \text{ rad/s}$

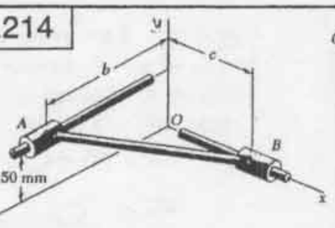
(a) $\underline{\omega} = -(0.348 \text{ rad/s})\underline{i} + (0.279 \text{ rad/s})\underline{j} + (1.089 \text{ rad/s})\underline{k}$ ◀

(b) EQ. 3: $\quad v_A = 8(-0.3483) - 25(1.0885)$

$= -2.79 - 27.21 = -30$

$\underline{v}_A = -(30 \text{ in./s})\underline{j}$ ◀

15.214



GIVEN: $c = 175\ mm$

$\underline{v}_B = -(180\ mm/s)\,\underline{i}$

$\underline{a}_B = 0$

$AB = 275\ mm$

FIND: $\underline{a}_A$

FROM SOLUTION OF PROB. 15.202 WE RECALL

$b = 150\ mm;\quad \underline{r}_{A/B} = -(175\ mm)\underline{i} + (150\ mm)\underline{j} + (150\ mm)\underline{k}$

$\underline{\omega} = +(1.2\ rad/s)\,\underline{j}$

WE NOW CALCULATE:

$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = -1.2\,\underline{j} \times (-175\underline{i} + 150\underline{j} + 150\underline{k})$

$\underline{v}_{A/B} = +(210\ mm/s)\underline{k} + (180\ mm/s)\underline{i}$

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B} = \underline{a}_B + \underline{\alpha} \times \underline{r}_{A/B} + \underline{\omega} \times \underline{v}_{A/B}$

$$a_A\underline{k} = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -175 & 150 & 150 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & +1.2 & 0 \\ 180 & 0 & 210 \end{vmatrix}$$

$$a_A\underline{k} = (150\alpha_y - 150\alpha_z)\underline{i} + (-175\alpha_z - 150\alpha_x)\underline{j} + (150\alpha_x + 175\alpha_y)\underline{k} + 252\underline{i} - 216\underline{k}$$

COEF OF $\underline{i}$: $\quad -252 = 150\alpha_y - 150\alpha_z \qquad (1)$

COEF OF $\underline{j}$: $\quad 0 = -150\alpha_x - 175\alpha_z \qquad (2)$

COEF OF $\underline{k}$: $\quad a_A + 216 = 150\alpha_x + 175\alpha_y \qquad (3)$

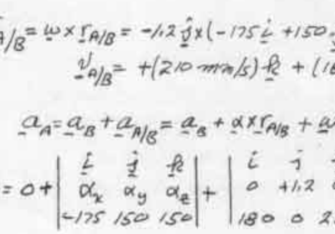
$\underline{\alpha}$ IS INDETERMINANT: ASSUME $\alpha_x = 0$, FROM EQ 2, $\alpha_z = 0$

THEN EQ. 1, YIELDS: $-252 = 150\alpha_y;\quad \alpha_y = -1.68$

EQ. 3: $\quad a_A + 216 = 0 + 175(-1.68) = -294$

$a_A = -216 - 294 = -510 \qquad \underline{a}_A = -(510\ mm/s^2)\,\underline{k}$ ◀

15.215



GIVEN: $c = 50\ mm$

$\underline{v}_B = -(180\ mm/s)\,\underline{i}$

$\underline{a}_B = 0$

$AB = 275\ mm$

FIND: $\underline{a}_A$

FROM SOLUTION OF PROB 15.203, WE RECALL:

$b = 225\ mm;\quad \underline{r}_{A/B} = -(50\ mm)\underline{i} + (150\ mm)\underline{j} + (225\ mm)\underline{k}$

$\underline{\omega} = (0.8\ rad/s)\,\underline{j}$

$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = 0.8\,\underline{j} \times (-50\underline{i} + 150\underline{j} + 225\underline{k})$

$\underline{v}_{A/B} = (40\ mm/s)\underline{k} + (180\ mm/s)\underline{i}$

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B} = \underline{a}_B + \underline{\alpha} \times \underline{r}_{A/B} + \underline{\omega} \times \underline{v}_{A/B}$

$$a_A\underline{k} = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -50 & 150 & 225 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0.8 & 0 \\ 180 & 0 & 40 \end{vmatrix}$$

$$a_A\underline{k} = (225\alpha_y - 150\alpha_z)\underline{i} + (-50\alpha_z - 225\alpha_x)\underline{j} + (150\alpha_x + 50\alpha_y)\underline{k} + 32\underline{i} - 144\underline{k}$$

(CONTINUED)

15.215 CONTINUED

COEF. OF $\underline{i}$: $\quad -32 = 225\alpha_y - 150\alpha_z \qquad (1)$

COEF. OF $\underline{j}$: $\quad 0 = -225\alpha_x - 50\alpha_z \qquad (2)$

COEF. OF $\underline{k}$: $\quad a_A + 144 = 150\alpha_x + 50\alpha_y \qquad (3)$

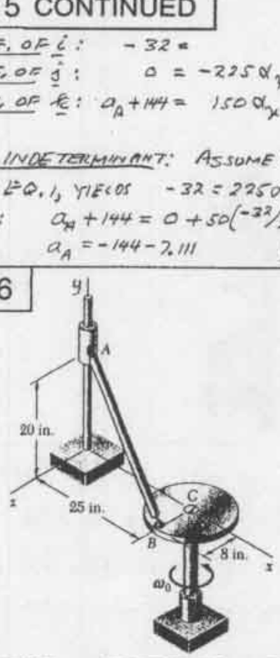
α IS INDETERMINANT: ASSUME $\alpha_x = 0$, FROM EQ 2, $\alpha_z = 0$

THEN EQ. 1, YIELDS $\quad -32 = 225\alpha_y \qquad \alpha_y = -32/225$

EQ. 3: $\quad a_A + 144 = 0 + 50(-32/225)$

$a_A = -144 - 7.111 \qquad \underline{a}_A = -(151.1\ mm/s^2)\,\underline{k}$ ◀

15.216



GIVEN:

$\underline{\omega}_0 = (3\ rad/s)\,\underline{j}$

$\underline{\alpha}_0 = 0$

FIND: $\underline{a}_A$

FROM PROB 15.204, WE RECALL: $\quad \underline{v}_B = (24\ in./s)\,\underline{i}$

$\underline{r}_{A/B} = -(25\ in.)\underline{i} + (20\ in.)\underline{j} - (8\ in.)\underline{k};\quad \underline{\omega} = (1.2\ rad/s)\,\underline{k}$

WE NOW CALCULATE: $\quad \underline{a}_B = \underline{\omega}_0 \times \underline{v}_{B} = 3\underline{j} \times 24\underline{i} = -(72\ in/s)\underline{k}$

$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = 1.2\,\underline{k} \times (-25\underline{i} + 20\underline{j} - 8\underline{k})$

$\underline{v}_{A/B} = -(30\ in./s)\,\underline{j} - (24\ in./s)\,\underline{i}$

$\underline{a}_A = \underline{a}_B + \underline{\alpha} \times \underline{r}_{A/B} + \underline{\omega} \times \underline{v}_{A/B}$

$$a_A\underline{j} = -72\underline{k} - \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -25 & 20 & -8 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & 1.2 \\ -24 & -30 & 0 \end{vmatrix}$$

$$a_A\underline{j} = -72\underline{k} + (-8\alpha_y - 20\alpha_z)\underline{i} + (-25\alpha_z + 8\alpha_x)\underline{j} + (20\alpha_x + 25\alpha_y)\underline{k} + 36\underline{i} - 28.8\underline{j}$$

COEF. OF $\underline{i}$: $\quad -36 = -8\alpha_y - 20\alpha_z \qquad (1)$

COEF. OF $\underline{j}$: $\quad a_A + 28.8 = 8\alpha_x - 25\alpha_z \qquad (2)$

COEF. OF $\underline{k}$: $\quad 72 = 20\alpha_x + 25\alpha_y \qquad (3)$

α IS INDETERMINATE: ASSUME $\alpha_y = 0$:

EQ. 3: $\quad 72 = 20\alpha_x + 0 \qquad \alpha_x = 3.6$

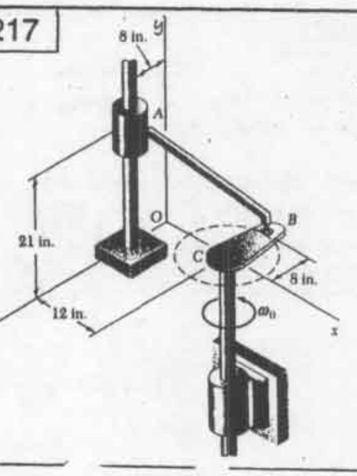
EQ. 1: $\quad -36 = 0 - 20\alpha_z \qquad \alpha_z = 1.8$

EQ. 2: $\quad a_A + 28.8 = 8(3.6) - 25(1.8)$

$a_A = -28.8 + 28.8 - 45$

$\underline{a}_A = -(45\ in/s^2)\,\underline{j}$ ◀

15.217



GIVEN:
$\underline{\omega}_0 = (10 \text{ rad/s})\underline{j}$
$\underline{\alpha}_0 = 0$

FIND: $\underline{a}_A$

FROM SOLUTION OF PROB. 15.205: $\omega = -\left(\frac{80}{21} \text{ rad/s}\right)\underline{k}$

$$\underline{r}_{A/B} = -(12 \text{ in.})\underline{i} + (21 \text{ in.})\underline{j} + (16 \text{ in.})\underline{k}$$

WE NOW CALCULATE: $\underline{v}_B = \underline{\omega}_0 \times \underline{r}_{B/C} = 10\underline{j} \times -8\underline{k} = -(80 \text{ in./s})\underline{i}$

$$\underline{a}_B = \underline{\omega}_0 \times \underline{v}_B = 10\underline{j} \times -80\underline{i} = (800 \text{ in/s}^2)\underline{k}$$

$$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = -\frac{80}{21}\underline{k} \times (-12\underline{i} + 21\underline{j} + 16\underline{k})$$

$$\underline{v}_{A/B} = (45.714 \text{ in/s})\underline{j} - (80 \text{ in./s})\underline{i}$$

$$\underline{a}_A = \underline{a}_B + \underline{\alpha} \times \underline{r}_{A/B} + \underline{\omega} \times \underline{v}_{A/B}$$

$$a_A\underline{j} = 800\underline{k} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -12 & 21 & 16 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0 & -80/21 \\ -80 & 45.714 & 0 \end{vmatrix}$$

$$a_A\underline{j} = 800\underline{k} + (16\alpha_y - 21\alpha_z)\underline{i} + (-12\alpha_z - 16\alpha_x)\underline{j} + (21\alpha_x + 12\alpha_y)\underline{k} + 174.15\underline{i} + 304.76\underline{j}$$

COEF. OF $\underline{i}$: $-174.15 = 16\alpha_y - 21\alpha_z$ (1)
COEF. OF $\underline{j}$: $a_A - 304.76 = -16\alpha_x - 12\alpha_z$ (2)
COEF. OF $\underline{k}$: $-800 = 21\alpha_x + 12\alpha_y$ (3)

α IS INDETERMINANT:
ASSUME $\alpha_y = 0$

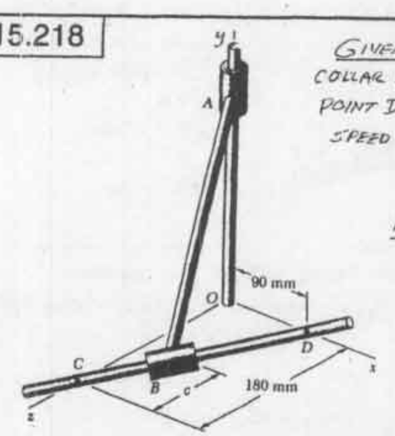
EQ. 1: $-174.15 = 0 - 21\alpha_z$ $\quad \alpha_z = 8.293$

EQ. 3: $-800 = 21\alpha_x + 0$ $\quad \alpha_x = -38.095$

EQ. 2) $a_A - 304.76 = -16(-38.095) - 12(8.293)$
$a_A - 304.76 = 609.53 - 99.50$

$$\underline{a}_A = (815 \text{ in./s}^2)\underline{j}$$

15.218



GIVEN: AB = 300 mm
COLLAR B MOVES TOWARD POINT D AT CONSTANT SPEED OF 50 mm/s
$c = 80$ mm

FIND: $\underline{a}_A$

$c = 80$ mm: FROM PROB. 15.206 WE RECALL:

$$\underline{v}_B = +(22.36 \text{ mm/s})\underline{i} - (44.72 \text{ mm/s})\underline{k}$$
$$\underline{\omega} = (1.118 \text{ rad/s})\underline{j} - (0.3194 \text{ rad/s})\underline{k}$$
$$\underline{r}_{A/B} = -(40 \text{ mm})\underline{i} + (280 \text{ mm})\underline{j} - (100 \text{ mm})\underline{k}$$

$$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1.118 & -0.3194 \\ -40 & 280 & -100 \end{vmatrix} = (-111.8 + 89.432)\underline{i} + 12.776\underline{j} + 44.72\underline{k}$$

$$\underline{v}_{A/B} = -(22.37 \text{ mm/s})\underline{i} + (12.776 \text{ mm/s})\underline{j} + (44.72 \text{ mm/s})\underline{k}$$

$$\underline{a}_A = \underline{a}_B + \underline{\alpha}_{AB} \times \underline{r}_{A/B} + \underline{\omega} \times \underline{v}_{A/B}$$

$$a_A\underline{j} = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -40 & 280 & -100 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1.118 & -0.3194 \\ -22.37 & 12.776 & 44.72 \end{vmatrix}$$

$$a_A\underline{j} = (-100\alpha_y - 280\alpha_z)\underline{i} + (-40\alpha_z + 100\alpha_x)\underline{j} + (280\alpha_x + 40\alpha_y)\underline{k} + (50.0 + 4.08)\underline{i} + 7.15\underline{j} + 25\underline{k}$$

COEF. OF $\underline{i}$: $-54.08 = -100\alpha_y - 280\alpha_z$ (1)
COEF. OF $\underline{j}$: $a_A - 7.15 = 100\alpha_x - 40\alpha_z$ (2)
COEF. OF $\underline{k}$: $-25 = 280\alpha_x + 40\alpha_y$ (3)

α IS INDETERMINANT: ASSUME $\alpha_y = 0$

EQ. 1: $-54.08 = 0 - 280\alpha_z$ $\quad \alpha_z = 0.19314$

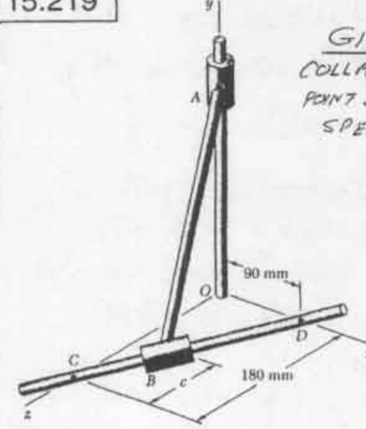
EQ. 3: $-25 = 280\alpha_x + 0$ $\quad \alpha_x = -0.0893$

$$\underline{\alpha} = -(0.0893 \text{ rad/s}^2)\underline{i} + (0.19314 \text{ rad/s}^2)\underline{k}$$

EQ. 2: $a_A - 7.15 = 100(-0.0893) - 40(0.19314)$
$a_A - 7.15 = -8.93 - 7.73$
$a_A = -9.51$

$$\underline{a}_A = -(9.51 \text{ mm/s}^2)\underline{j}$$

15.219



GIVEN: AB = 300 mm
COLLAR B MOVES TOWARD POINT D AT CONSTANT SPEED OF 50 mm/s
$c = 120$ mm

FIND: a_A

$c = 120$ mm: FROM PROB. 15.207 WE RECALL:

$\underline{v}_B = +(22.36 \text{ mm/s})\underline{i} - (44.72 \text{ mm/s})\underline{k}$

$\underline{\omega} = (0.7453 \text{ rad/s})\underline{j} - (0.0777 \text{ rad/s})\underline{k}$

$\underline{r}_{A/B} = -(60 \text{ mm})\underline{i} + (287.75 \text{ mm})\underline{j} - (60 \text{ mm})\underline{k}$

$$\underline{v}_{A/B} = \underline{\omega} \times \underline{r}_{A/B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0.7453 & -0.0777 \\ -60 & 287.75 & -60 \end{vmatrix}$$

$$= (-44.718 + 22.358)\underline{i} + 4.662\underline{j} + 44.718\underline{k}$$

$\underline{v}_{A/B} = -(22.36 \text{ mm/s})\underline{i} + (4.662 \text{ mm/s})\underline{j} + (44.718 \text{ mm/s})\underline{k}$

$\underline{a}_B = 0$

$$\underline{a}_A = \underline{a}_B + \underline{\alpha} \times \underline{r}_{A/B} + \underline{\omega} \times \underline{v}_{A/B}$$

$$a_A\underline{j} = 0 + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \alpha_x & \alpha_y & \alpha_z \\ -60 & 287.75 & -60 \end{vmatrix} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 0.7453 & -0.0777 \\ -22.36 & 4.662 & 44.718 \end{vmatrix}$$

$$a_A\underline{j} = (-60\alpha_y - 287.75\alpha_z)\underline{i} + (-60\alpha_z + 60\alpha_x)\underline{j} + (287.75\alpha_x + 60\alpha_y)\underline{k}$$
$$+ (33.33 + 0.363)\underline{i} + 1.737\underline{j} + 16.66\underline{k}$$

COEF. OF $\underline{i}$: $\quad -33.69 = -60\alpha_y - 287.75\alpha_z \quad (1)$

COEF. OF $\underline{j}$: $\quad a_y - 1.737 = 60\alpha_x - 60\alpha_z \quad (2)$

COEF. OF $\underline{k}$: $\quad -16.66 = 287.75\alpha_x + 60\alpha_y \quad (3)$

α IS INDETERMINANT: ASSUME $\alpha_y = 0$

EQ.1: $-33.69 = 0 - 287.75\alpha_z$: $\quad \alpha_z = 0.1171$

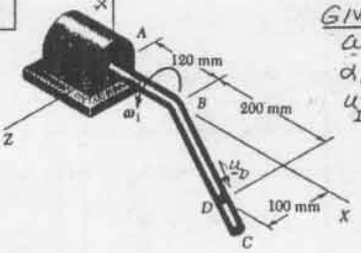
EQ.3: $-16.66 = 287.75\alpha_x + 0$; $\quad \alpha_x = -0.0579$

$\underline{\alpha} = -(0.0579 \text{ rad/s})\underline{i} + (0.1171 \text{ rad/s})\underline{k}$

EQ. 2: $\quad a_y - 1.737 = 60(-0.0579) - 60(0.1171)$
$\quad a_y - 1.737 = -3.474 - 7.026$
$\quad a_y = -8.76$

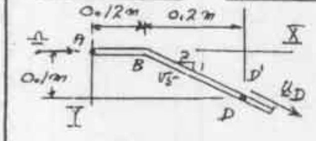
$$\underline{a}_y = -(8.76 \text{ mm/s}^2)\underline{j}$$

15.220



GIVEN:
$\omega_1 = 5$ rad/s
$\alpha_1 = 0$
$u_D = 1.5$ m/s
$\dot{u}_D = 0$

FIND:
(a) $\underline{v}_D$
(b) $\underline{a}_D$



$\underline{v}_{D/\mathcal{F}} = u_D \underline{\lambda}_{DB}$

$v_{D/\mathcal{F}} = 1.5 \dfrac{2\underline{i} + \underline{k}}{\sqrt{5}}$

$\underline{v}_{D/\mathcal{F}} = (1.3416 \text{ m/s})\underline{i} + (0.6708 \text{ m/s})\underline{k}$

$\underline{r}_{D/A} = (0.320 \text{ m})\underline{i} + (0.1 \text{ m})\underline{k}$

$\underline{\Omega} = \underline{\omega}_1 = (5 \text{ rad/s})\underline{i}$

(a) VELOCITY OF D: $\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/A} = 5\underline{i} \times (0.32\underline{i} + 0.1\underline{k})$

$\underline{v}_{D'} = -(0.5 \text{ m/s})\underline{j}$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/\mathcal{F}} = -0.5\underline{j} + 1.3416\underline{i} + 0.6708\underline{k}$

$$\underline{v}_D = (1.342 \text{ m/s})\underline{i} - (0.5 \text{ m/s})\underline{j} + (0.671 \text{ m/s})\underline{k}$$

(b) ACCELERATION OF D: $\quad a_{D/\mathcal{F}} = 0$; $\dot{\Omega} = 0$

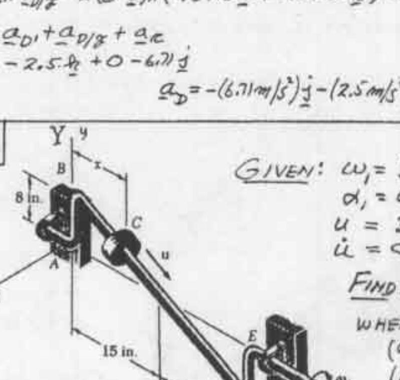
$\underline{a}_{D'} = \dot{\underline{\Omega}} \times \underline{r}_{D/A} + \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{D/A}$
$= 0 + \underline{\Omega} \times \underline{v}_{D'} = 5\underline{i} \times (-0.5\underline{j}) = -(2.5 \text{ m/s}^2)\underline{k}$

$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{D/\mathcal{F}} = 2(5\underline{i}) \times (1.3416\underline{i} + 0.6708\underline{k}) = -(6.71 \text{ m/s}^2)\underline{j}$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$
$= -2.5\underline{k} + 0 - 6.71\underline{j}$

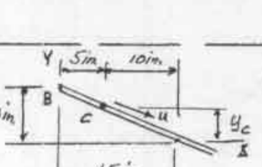
$$\underline{a}_D = -(6.71 \text{ m/s}^2)\underline{j} - (2.5 \text{ m/s}^2)\underline{k}$$

15.221



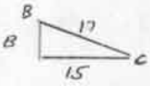
GIVEN: $\omega_1 = 3$ rad/s
$\alpha_1 = 0$
$u = 34$ in./s
$\dot{u} = 0$

FIND: $\underline{v}_C$ AND $\underline{a}_C$
WHEN,
(a) $x = 5$ in.
(b) $x = 15$ in.



(a) $x = 5$ in.

$\dfrac{y_C}{8 \text{ in.}} = \dfrac{10 \text{ in.}}{15 \text{ in.}}$; $\quad y_C = \dfrac{16}{3}$ in



$\underline{v}_{C/\mathcal{F}} = u\,\underline{\lambda}_{BC} = 34 \dfrac{15\underline{i} - 8\underline{j}}{17}$

$\underline{v}_{C/\mathcal{F}} = (30 \text{ in/s})\underline{i} - (16 \text{ in/s})\underline{j}$

$\underline{r}_{C/A} = (5 \text{ in.})\underline{i} + \left(\dfrac{16}{3} \text{ in.}\right)\underline{j}$

$\underline{\Omega} = \omega_1\underline{i} = -(3 \text{ rad/s})\underline{i}$

(CONTINUED)

15.221 continued

$x = 5$ in. VELOCITY:

$$\underline{v}_{C'} = \underline{\Omega} \times \underline{r}_{C/A} = (-3\,\text{rad/s})\underline{i} \times \left[(5\,\text{in.})\underline{i} + (\tfrac{16}{3}\,\text{in.})\underline{j}\right] = -(16\,\text{in./s})\underline{k}$$

$$\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}} = -(16\,\text{in./s})\underline{k} + (30\,\text{in./s})\underline{i} - (16\,\text{in./s})\underline{j}$$

$$\underline{v}_C = (30\,\text{in./s})\underline{i} - (16\,\text{in./s})\underline{j} - (16\,\text{in./s})\underline{k} \blacktriangleleft$$

ACCELERATION: $\underline{a}_{C/\mathcal{F}} = 0;\ \dot{\underline{\Omega}} = 0$

$$\underline{a}_{C'} = \dot{\underline{\Omega}} \times \underline{r}_{C/A} + \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{C/A} = \dot{\underline{\Omega}} \times \underline{r}_{C/A} + \underline{\Omega} \times \underline{v}_{C'}$$

$$\underline{a}_{C'} = 0 + (-3\,\text{rad/s})\underline{i} \times (+16\,\text{in./s})\underline{k} = -(48\,\text{in./s}^2)\underline{j}$$

$$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{C/\mathcal{F}} = 2(-3\,\text{rad/s})\underline{i} \times \left[(30\,\text{in./s})\underline{i} - (16\,\text{in./s})\underline{j}\right]$$

$$\underline{a}_c = (96\,\text{in./s}^2)\underline{k}$$

$$\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_c$$
$$= -(48\,\text{in./s}^2)\underline{j} + 0 + (96\,\text{in./s}^2)\underline{k}$$

$$\underline{a}_C = -(48\,\text{in./s}^2)\underline{j} + (96\,\text{in./s}^2)\underline{k} \blacktriangleleft$$

(b) FOR, $x = 15$ in. (COLLAR C IS IN XZ PLANE)

VELOCITY: FROM PART a: $\underline{v}_{C/\mathcal{F}} = (30\,\text{in./s})\underline{i} - (16\,\text{in./s})\underline{j}$

$\underline{r}_{C/A} = (15\,\text{in.})\underline{i};\ \underline{v}_{C'} = \underline{\Omega} \times \underline{r}_{C/A} = -3\underline{i} \times 15\underline{i} = 0$

$\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}} = 0 + \underline{v}_{C/\mathcal{F}};\ \underline{v}_C = (30\,\text{in./s})\underline{i} - (16\,\text{in./s})\underline{j} \blacktriangleleft$

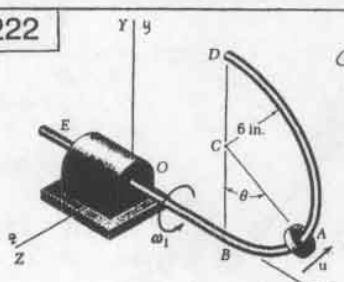
ACCELERATION: $a_{C/\mathcal{F}} = 0;\ \dot{\Omega} = 0$

$\underline{a}_{C'} = \dot{\underline{\Omega}} \times \underline{r}_{C/A} + \underline{\Omega} \times \underline{v}_{C'} = 0 + 0;\quad \underline{a}_{C'} = 0$

a_c IS SAME AS IN PART a: $\underline{a}_c = (96\,\text{in./s}^2)\underline{k}$

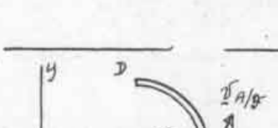
$\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_c = 0 + 0 + \underline{a}_c;\ \underline{a}_C = (96\,\text{in./s}^2)\underline{k} \blacktriangleleft$

15.222



GIVEN:
$\omega_1 = 8$ rad/s
$\alpha_1 = 0$
$u = 30$ in./s
$\dot{u} = 0$
$\theta = 60°$

FIND: (a) $\underline{v}_A$ (b) $\underline{a}_A$



$\underline{r}_{A/O} = x_A\underline{i} + (3\,\text{in.})\underline{j}$

$\underline{v}_{A/\mathcal{F}} = u \measuredangle 60°$
$= (30\,\text{in./s}) \measuredangle 60°$

$\underline{v}_{A/\mathcal{F}} = (15\,\text{in./s})\underline{i} + (25.98\,\text{in./s})\underline{j}$

$\underline{\Omega} = \omega_1\underline{i} = (8\,\text{rad/s})\underline{i}$

$\underline{v}_{A'} = \underline{\Omega} \times \underline{r}_{A/O} = 8\underline{i} \times (x_A\underline{i} + 3\underline{j}) = (24\,\text{in./s})\underline{k}$

(a) VELOCITY: $\underline{v}_A = \underline{v}_{A'} + \underline{v}_{A/\mathcal{F}} = 24\underline{k} + 15\underline{i} + 25.98\underline{j}$

$$\underline{v}_A = (15\,\text{in./s})\underline{i} + (26.0\,\text{in./s})\underline{j} + (24\,\text{in./s})\underline{k} \blacktriangleleft$$

[CONTINUED]

15.222 continued

(b) ACCELERATION: $\dot{\Omega} = 0$

$$\underline{a}_{A/\mathcal{F}} = \frac{u^2}{r} \measuredangle 30° = \frac{(30\,\text{in./s})^2}{6\,\text{in.}} \measuredangle 30° = 150\,\text{in./s}^2 \measuredangle 30°$$

$$\underline{a}_{A/\mathcal{F}} = -(129.9\,\text{in./s}^2)\underline{i} + (75\,\text{in./s}^2)\underline{j}$$

$$\underline{a}_{A'} = \dot{\underline{\Omega}} \times \underline{r}_{A/O} + \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{A/O} = \dot{\underline{\Omega}} \times \underline{r}_{A/O} + \underline{\Omega} \times \underline{v}_{A'}$$

$$\underline{a}_{A'} = 0 + (8\,\text{rad/s})\underline{i} \times (24\,\text{in./s})\underline{k} = -(192\,\text{in./s}^2)\underline{j}$$

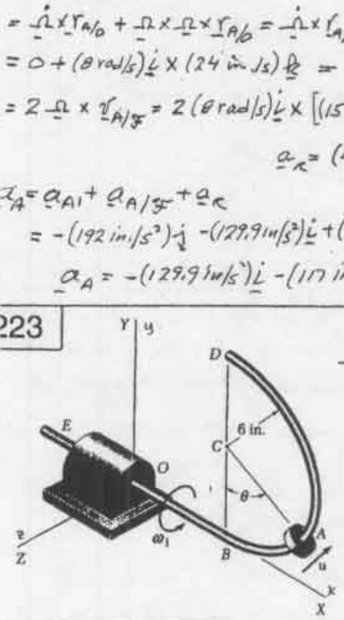
$$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{A/\mathcal{F}} = 2(8\,\text{rad/s})\underline{i} \times \left[(15\,\text{in./s})\underline{i} + (25.98\,\text{in./s})\underline{j}\right]$$

$$\underline{a}_c = (415.7\,\text{in./s}^2)\underline{k}$$

$$\underline{a}_A = \underline{a}_{A'} + \underline{a}_{A/\mathcal{F}} + \underline{a}_c$$
$$= -(192\,\text{in./s}^2)\underline{j} - (129.9\,\text{in./s}^2)\underline{i} + (75\,\text{in./s}^2)\underline{j} + (415.7\,\text{in./s}^2)\underline{k}$$

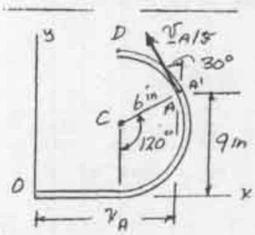
$$\underline{a}_A = -(129.9\,\text{in./s}^2)\underline{i} - (117\,\text{in./s}^2)\underline{j} + (416\,\text{in./s}^2)\underline{k} \blacktriangleleft$$

15.223



GIVEN:
$\omega_1 = 8$ rad/s
$\alpha_1 = 0$
$u = 30$ in./s
$\dot{u} = 0$
$\theta = 120°$

FIND: (a) $\underline{v}_A$ (b) $\underline{a}_A$



$\underline{r}_{A/O} = x_A\underline{i} + (9\,\text{in.})\underline{j}$

$\underline{v}_{A/\mathcal{F}} = u \angle 30°$
$= (30\,\text{in./s}) \angle 30°$

$\underline{v}_{A/\mathcal{F}} = -(15\,\text{in./s})\underline{i} + (25.98\,\text{in./s})\underline{j}$

$\underline{\Omega} = \omega_1\underline{i} = (8\,\text{rad/s})\underline{i}$

$\underline{v}_{A'} = \underline{\Omega} \times \underline{r}_{A/O} = 8\underline{i} \times (x_A\underline{i} + 9\underline{j}) = (72\,\text{in./s})\underline{k}$

(a) VELOCITY: $\underline{v}_A = \underline{v}_{A'} + \underline{v}_{A/\mathcal{F}}$
$= 72\underline{k} - 15\underline{i} + 25.98\underline{j}$

$$\underline{v}_A = -(15\,\text{in./s})\underline{i} + (26.0\,\text{in./s})\underline{j} + (72\,\text{in./s})\underline{k} \blacktriangleleft$$

(b) ACCELERATION: $\dot{\Omega} = 0$

$$\underline{a}_{A/\mathcal{F}} = \frac{u^2}{r} \angle 30° = \frac{(30\,\text{in./s})^2}{6\,\text{in.}} \angle 30° = 150\,\text{in./s}^2 \angle$$

$$\underline{a}_{A/\mathcal{F}} = -(129.9\,\text{in./s}^2)\underline{i} - (75\,\text{in./s}^2)\underline{j}$$

$$\underline{a}_{A'} = \dot{\underline{\Omega}} \times \underline{r}_{A/O} + \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{A/O} = \dot{\underline{\Omega}} \times \underline{r}_{A/O} + \underline{\Omega} \times \underline{v}_{A'}$$

$$\underline{a}_{A'} = 0 + (8\,\text{rad/s})\underline{i} \times (72\,\text{in./s})\underline{k} = -(576\,\text{in./s}^2)\underline{j}$$

$$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{A/\mathcal{F}} = 2(8\,\text{rad/s})\underline{i} \times \left[-(15\,\text{in./s})\underline{i} + (25.98\,\text{in./s})\underline{j}\right]$$

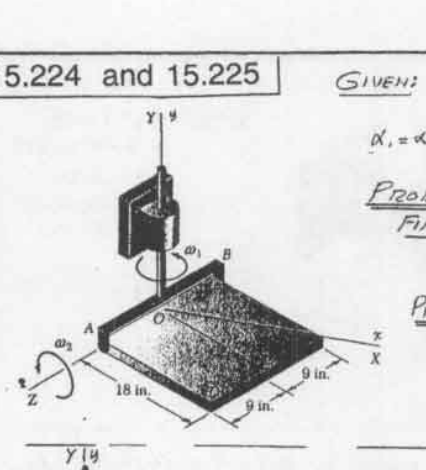
$$\underline{a}_c = (415.7\,\text{in./s}^2)\underline{k}$$

$$\underline{a}_A = \underline{a}_{A'} + \underline{a}_{A/\mathcal{F}} + \underline{a}_c$$
$$= -(576\,\text{in./s}^2)\underline{j} - (129.9\,\text{in./s}^2)\underline{i} - (75\,\text{in./s}^2)\underline{j} + (415.7\,\text{in./s}^2)\underline{k}$$

$$\underline{a}_A = -(129.9\,\text{in./s}^2)\underline{i} - (651\,\text{in./s}^2)\underline{j} + (416\,\text{in./s}^2)\underline{k} \blacktriangleleft$$

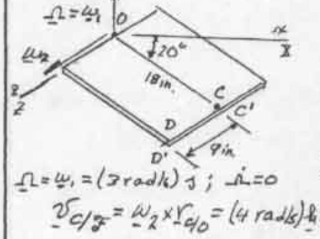
15.224 and 15.225

GIVEN: $\omega_1 = 3$ rad/s, $\omega_2 = 4$ rad/s, $\alpha_1 = \alpha_2 = 0$



PROBLEM 15.224
FIND: (a) $\mathbf{v}_C$
(b) $\mathbf{a}_C$

PROBLEM 15.225
FIND: (a) $\mathbf{v}_D$
(b) $\mathbf{a}_D$



$\mathbf{r}_{C/O} = (18\text{ in.})(\cos 20^\circ\,\mathbf{i} - \sin 20^\circ\,\mathbf{j})$

$\mathbf{r}_{C/O} = (16.914\text{ in.})\mathbf{i} - (6.156\text{ in.})\mathbf{j}$

$\mathbf{r}_{D/O} = \mathbf{r}_{C/O} + (9\text{ in. }\mathbf{k})$

PROBLEM 15.224

$\Omega = \omega_1 = (3\text{ rad/s})\mathbf{j}\,;\ \dot{\Omega} = 0$

$\mathbf{v}_{C/\mathcal{F}} = \omega_2 \times \mathbf{r}_{C/O} = (4\text{ rad/s})\mathbf{k} \times [(16.914\text{ in.})\mathbf{i} - (6.156\text{ in.})\mathbf{j}]$

$\mathbf{v}_{C/\mathcal{F}} = (67.658\text{ in/s})\mathbf{j} + (24.625\text{ in/s})\mathbf{i}$

$\mathbf{v}_{C'} = \Omega \times \mathbf{r}_{C/O} = (3\text{ rad/s})\mathbf{j} \times [(16.914\text{ in})\mathbf{i} - (6.156\text{ in})\mathbf{j}]$

$\mathbf{v}_{C'} = -(50.74\text{ in/s})\mathbf{k}$

$\mathbf{v}_C = \mathbf{v}_{C'} + \mathbf{v}_{C/\mathcal{F}} = -50.74\,\mathbf{k} + 67.658\,\mathbf{j} + 24.625\,\mathbf{i}$

$\mathbf{v}_C = (24.6\text{ in/s})\mathbf{i} + (67.7\text{ in/s})\mathbf{j} - (50.74\text{ in/s})\mathbf{k}$ ◀

$\mathbf{a}_{C/\mathcal{F}} = \omega_2 \times \mathbf{v}_{C/\mathcal{F}} = (4\text{ rad/s})\mathbf{k} \times [(67.658\text{ in/s})\mathbf{j} + (24.625\text{ in/s})\mathbf{i}]$

$\mathbf{a}_{C/\mathcal{F}} = -(270.6\text{ in/s}^2)\mathbf{i} + (98.50\text{ in/s}^2)\mathbf{j}$

$\mathbf{a}_{C'} = \dot{\Omega} \times \mathbf{r}_{C/O} + \Omega \times \mathbf{v}_{C'} = 0 + (3\text{ rad/s})\mathbf{j} \times (-50.74\text{ in/s})\mathbf{k}$

$\mathbf{a}_{C'} = -(152.2\text{ in/s}^2)\mathbf{i}$

$\mathbf{a}_c = 2\Omega \times \mathbf{v}_{C/\mathcal{F}} = 2(3\text{ rad/s})\mathbf{j} \times [(67.658\text{ in/s})\mathbf{j} + (24.625\text{ in/s})\mathbf{i}]$

$\mathbf{a}_c = -(147.75\text{ in/s}^2)\mathbf{k}$

$\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/\mathcal{F}} + \mathbf{a}_c$

$= -(152.2\text{ in/s}^2)\mathbf{i} - (270.6\text{ in/s}^2)\mathbf{i} + (98.5\text{ in/s}^2)\mathbf{j} - (147.75\text{ in/s}^2)\mathbf{k}$

$\mathbf{a}_C = -(423\text{ in/s}^2)\mathbf{i} + (98.5\text{ in/s}^2)\mathbf{j} - (147.8\text{ in/s}^2)\mathbf{k}$ ◀

PROBLEM 15.224 VELOCITY:

$\mathbf{v}_{D/\mathcal{F}} = \omega_2 \times \mathbf{r}_{D/O} = \omega_2\mathbf{k} \times (\mathbf{r}_{C/O} + 9\mathbf{k}) = \omega_2 \times \mathbf{r}_{C/O}$

THUS $\mathbf{v}_{D/\mathcal{F}} = \mathbf{v}_{C/\mathcal{F}} = (67.658\text{ in/s})\mathbf{j} + (24.625\text{ in/s})\mathbf{i}$

$\mathbf{v}_{D'} = \Omega \times \mathbf{r}_{D/O} = \Omega \times [\mathbf{r}_{C/O} + (9\text{ in.})\mathbf{k}]$

$= \Omega \times \mathbf{r}_{C/O} + (3\text{ rad/s})\mathbf{j} \times (9\text{ in.})\mathbf{k} = \mathbf{v}_{C'} + (27\text{ in/s})\mathbf{i}$

SUBSTITUTING FOR $\mathbf{v}_{C'}$ FROM ABOVE

$\mathbf{v}_{D'} = (27\text{ in/s})\mathbf{i} - (50.74\text{ in/s})\mathbf{k}$

$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/\mathcal{F}} = 27\,\mathbf{i} - 50.74\,\mathbf{k} + 67.658\,\mathbf{j} + 24.625\,\mathbf{i}$

$\mathbf{v}_D = (51.6\text{ in/s})\mathbf{i} + (67.7\text{ in/s})\mathbf{j} - (50.7\text{ in/s})\mathbf{k}$ ◀

(CONTINUED)

15.225 continued

ACCELERATION $\dot{\Omega} = 0$

$\mathbf{a}_{D/\mathcal{F}} = \omega_2 \times \mathbf{v}_{D/\mathcal{F}} = (4\text{ rad/s})\mathbf{k} \times [(67.658\text{ in/s})\mathbf{j} + (24.625\text{ in/s})\mathbf{i}]$

$\mathbf{a}_{D/\mathcal{F}} = -(270.63\text{ in/s}^2)\mathbf{i} + (98.5\text{ in/s}^2)\mathbf{j}$

NOTE: $\mathbf{a}_{D/\mathcal{F}} = \mathbf{a}_{C/\mathcal{F}}$ SINCE $\mathbf{v}_{D/\mathcal{F}} = \mathbf{v}_{C/\mathcal{F}}$

$\mathbf{a}_{D'} = \dot{\Omega} \times \mathbf{r}_{D/O} + \Omega \times \mathbf{v}_{D'}$

$= 0 + (3\text{ rad/s})\mathbf{j} \times [(27\text{ in/s})\mathbf{i} - (50.74\text{ in/s})\mathbf{k}]$

$\mathbf{a}_{D'} = -(81\text{ in/s}^2)\mathbf{k} - (152.2\text{ in/s})\mathbf{i}$

$\mathbf{a}_c = 2\Omega \times \mathbf{v}_{D/\mathcal{F}}$ BUT WE KNOW THAT $\mathbf{v}_{D/\mathcal{F}} = \mathbf{v}_{C/\mathcal{F}}$

THUS CORIOLIS ACCELERATION $\mathbf{a}_c$ FOR POINT D IS SAME AS $\mathbf{a}_c$ FOR POINT C.

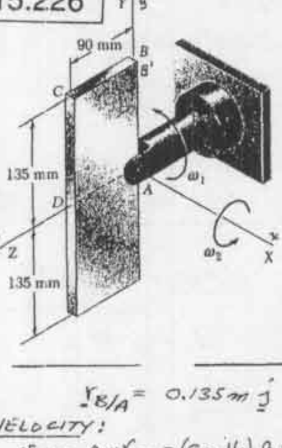
$\mathbf{a}_c = -(147.75\text{ in/s}^2)\mathbf{k}$

$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/\mathcal{F}} + \mathbf{a}_c$

$= -81\,\mathbf{k} - 152.2\,\mathbf{i} - 270.63\,\mathbf{i} + 98.5\,\mathbf{j} - 147.75\,\mathbf{k}$

$\mathbf{a}_D = -(423\text{ in/s}^2)\mathbf{i} + (98.5\text{ in/s}^2)\mathbf{j} - (229\text{ in/s}^2)\mathbf{k}$ ◀

15.226



GIVEN: $\omega_1 = 9$ rad/s, $\omega_2 = 12$ rad/s, $\alpha_1 = \alpha_2 = 0$

FIND: $\mathbf{v}_B$, $\mathbf{a}_B$

$\mathbf{r}_{B/A} = 0.135\text{ m }\mathbf{j}$ $\quad \Omega = \omega_1\mathbf{k} = (9\text{ rad/s})\mathbf{k}$

VELOCITY:

$\mathbf{v}_{B'} = \Omega \times \mathbf{r}_{B/A} = (9\text{ rad/s})\mathbf{k} \times (0.135\text{ m})\mathbf{j} = -(1.215\text{ m/s})\mathbf{i}$

$\mathbf{v}_{B/\mathcal{F}} = \omega_2 \times \mathbf{r}_{B/A} = (12\text{ rad/s})\mathbf{i} \times (0.135\text{ m})\mathbf{j} = (1.62\text{ m/s})\mathbf{k}$

$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/\mathcal{F}}$

$\mathbf{v}_B = -(1.215\text{ m/s})\mathbf{i} + (1.62\text{ m/s})\mathbf{k}$ ◀

ACCELERATION:

$\mathbf{a}_{B'} = \Omega \times \Omega \times \mathbf{r}_{B/A} = \Omega \times \mathbf{v}_{B'}$

$= (9\text{ rad/s})\mathbf{k} \times (-1.215\text{ m/s})\mathbf{i} = -(10.935\text{ m/s}^2)\mathbf{j}$

$\mathbf{a}_{B/\mathcal{F}} = \omega_2 \times \omega_2 \times \mathbf{r}_{B/A} = \omega_2 \times \mathbf{v}_{B/\mathcal{F}}$

$= (12\text{ rad/s})\mathbf{i} \times (1.62\text{ m/s})\mathbf{k} = -(19.44\text{ m/s})\mathbf{j}$

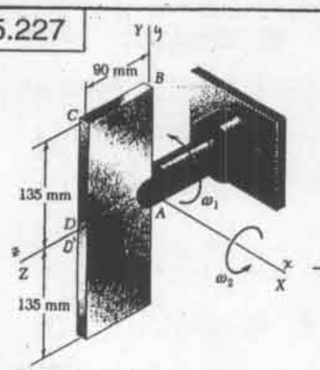
$\mathbf{a}_c = 2\Omega \times \mathbf{v}_{B/\mathcal{F}} = 2(9\text{ rad/s})\mathbf{k} \times (1.62\text{ m/s})\mathbf{k} = 0$

$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/\mathcal{F}} + \mathbf{a}_c$

$= -(10.935\text{ m/s}^2)\mathbf{j} - (19.44\text{ m/s}^2)\mathbf{j}$

$\mathbf{a}_B = -(30.4\text{ m/s}^2)\mathbf{j}$ ◀

15.227



GIVEN: $\omega_1 = 9\ \text{rad/s}$, $\omega_2 = 12\ \text{rad/s}$, $\alpha_1 = \alpha_2 = 0$

FIND: $\underline{v}_D$, $\underline{a}_D$

$\underline{r}_{D/A} = (0.09\ \text{m})\underline{k}$

$\underline{\Omega} = \omega_1 \underline{k} = (9\ \text{rad/s})\underline{k}$

VELOCITY:

$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/A} = (9\ \text{rad/s})\underline{k} \times (0.09\ \text{m})\underline{k} = 0$

$\underline{v}_{D/\mathcal{F}} = \underline{\omega}_2 \times \underline{r}_{D/A} = (12\ \text{rad/s})\underline{i} \times (0.09\ \text{m})\underline{k} = -(1.08\ \text{m/s})\underline{j}$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/\mathcal{F}}$

$\underline{v}_D = -(1.08\ \text{m/s})\underline{j}$ ◀

ACCELERATION

$\underline{a}_{D'} = \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{D/A} = \underline{\Omega} \times \underline{v}_{D'} = \underline{\Omega} \times 0 = 0$

$\underline{a}_{D/\mathcal{F}} = \underline{\omega}_2 \times \underline{\omega}_2 \times \underline{r}_{D/A} = \underline{\omega}_2 \times \underline{v}_{D/\mathcal{F}}$

$\underline{a}_{D/\mathcal{F}} = (12\ \text{rad/s})\underline{i} \times (-1.08\ \text{m/s})\underline{j} = -(12.96\ \text{m/s}^2)\underline{k}$

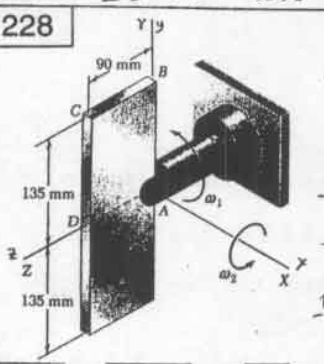
$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{D/\mathcal{F}} = 2(9\ \text{rad/s})\underline{k} \times (-1.08\ \text{m/s})\underline{j}$

$\underline{a}_c = (19.44\ \text{m/s}^2)\underline{i}$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c$
$= 0 - (12.96\ \text{m/s}^2)\underline{k} + (19.44\ \text{m/s}^2)\underline{i}$

$\underline{a}_C = (19.44\ \text{m/s}^2)\underline{i} - (12.96\ \text{m/s}^2)\underline{k}$ ◀

15.228



GIVEN: $\omega_1 = 9\ \text{rad/s}$, $\omega_2 = 12\ \text{rad/s}$, $\alpha_1 = \alpha_2 = 0$

FIND: $\underline{v}_C$, $\underline{a}_C$

$\underline{r}_{C/A} = (0.135\ \text{m})\underline{j} + (0.09\ \text{m})\underline{k}$

$\underline{\Omega} = \omega_1 \underline{k} = (9\ \text{rad/s})\underline{k}$

VELOCITY: $\underline{v}_{C'} = \underline{\Omega} \times \underline{r}_{C/A} = (9\ \text{rad/s})\underline{k} \times [(0.135\ \text{m})\underline{j} + (0.09\ \text{m})\underline{k}]$

$\underline{v}_{C'} = -(1.215\ \text{m/s})\underline{i}$

$\underline{v}_{C/\mathcal{F}} = \underline{\omega}_2 \times \underline{r}_{C/A} = (12\ \text{rad/s})\underline{i} \times [(0.135\ \text{m})\underline{j} + (0.09\ \text{m})\underline{k}]$

$\underline{v}_{C/\mathcal{F}} = (1.62\ \text{m/s})\underline{k} - (1.08\ \text{m/s})\underline{j}$

$\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}}$: $\underline{v}_C = -(1.215\ \text{m/s})\underline{i} - (1.08\ \text{m/s})\underline{j} + (1.62\ \text{m/s})\underline{k}$ ◀

ACCELERATION:

$\underline{a}_{C'} = \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{C/A} = \underline{\Omega} \times \underline{v}_{C'} = (9\ \text{rad/s})\underline{k} \times (-1.215\ \text{m/s})\underline{i} = -(10.95\ \text{m/s})\underline{j}$

$\underline{a}_{C/\mathcal{F}} = \underline{\omega}_2 \times \underline{\omega}_2 \times \underline{r}_{C/A} = \underline{\omega}_2 \times \underline{v}_{C/\mathcal{F}}$

$\underline{a}_{C/\mathcal{F}} = (12\ \text{rad/s})\underline{i} \times [1.62\underline{k} - 1.08\underline{j}] = -(19.44\ \text{m/s}^2)\underline{j} - (12.96\ \text{m/s}^2)\underline{k}$

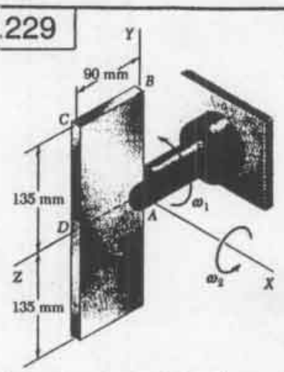
$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{C/\mathcal{F}} = 2(9\ \text{rad/s})\underline{k} \times [(1.62\ \text{m/s})\underline{k} - (1.08\ \text{m/s}^2)\underline{j}]$

$\underline{a}_c = (19.44\ \text{m/s}^2)\underline{i}$

$\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_c$
$= -(10.95\ \text{m/s})\underline{j} - (19.44\ \text{m/s}^2)\underline{j} - (12.96\ \text{m/s}^2)\underline{k} + (19.44\ \text{m/s}^2)\underline{i}$

$\underline{a}_C = (19.44\ \text{m/s}^2)\underline{i} - (30.4\ \text{m/s}^2)\underline{j} - (12.96\ \text{m/s}^2)\underline{k}$ ◀

15.229



GIVEN: $\omega_1 = 9\ \text{rad/s}$, $\alpha_1 = -45\ \text{rad/s}^2$, $\omega_2 = 12\ \text{rad/s}$, $\alpha_2 = -60\ \text{rad/s}^2$

FIND: $\underline{v}_C$, $\underline{a}_C$

$\underline{\Omega} = \omega_1 \underline{k} = (9\ \text{rad/s})\underline{k}$ $\quad$ $\dot{\underline{\Omega}} = \alpha_1 \underline{k} = -(45\ \text{rad/s}^2)\underline{k}$

$\underline{\omega}_2 = \omega_2 \underline{i} = (12\ \text{rad/s})\underline{i}$ $\quad$ $\dot{\underline{\omega}}_2 = \alpha_2 \underline{i} = -(60\ \text{rad/s}^2)\underline{i}$

$\underline{r}_{C/A} = (0.135\ \text{m})\underline{j} + (0.09\ \text{m})\underline{k}$

VELOCITY:

$\underline{v}_{C'} = \underline{\Omega} \times \underline{r}_{C/A} = (9\ \text{rad/s})\underline{k} \times [(0.135\ \text{m})\underline{j} + (0.09\ \text{m})\underline{k}]$

$\underline{v}_{C'} = -(1.215\ \text{m/s})\underline{i}$

$\underline{v}_{C/\mathcal{F}} = \underline{\omega}_2 \times \underline{r}_{C/A} = (12\ \text{rad/s})\underline{i} \times [(0.135\ \text{m})\underline{j} + (0.09\ \text{m})\underline{k}]$

$\underline{v}_{C/\mathcal{F}} = (1.62\ \text{m/s})\underline{k} - (1.08\ \text{m/s})\underline{j}$

$\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}}$

$\underline{v}_C = -(1.215\ \text{m/s})\underline{i} - (1.08\ \text{m/s})\underline{j} + (1.62\ \text{m/s})\underline{k}$ ◀

ACCELERATION

$\underline{a}_{C'} = \dot{\underline{\Omega}} \times \underline{r}_{C/A} + \underline{\Omega} \times \underline{\Omega} \times \underline{r}_{C/A} = \dot{\underline{\Omega}} \times \underline{r}_{C/A} + \underline{\Omega} \times \underline{v}_{C'}$
$= -(45\ \text{rad/s})\underline{k} \times [(0.135\ \text{m})\underline{j} + (0.09\ \text{m})\underline{k}]$
$+ (9\ \text{rad/s})\underline{k} \times (-1.215\ \text{m/s})\underline{i}$

$\underline{a}_{C'} = +(6.075\ \text{m/s}^2)\underline{i} - (10.94\ \text{m/s}^2)\underline{j}$

$\underline{a}_{C/\mathcal{F}} = \dot{\underline{\omega}}_2 \times \underline{r}_{C/A} + \underline{\omega}_2 \times \underline{\omega}_2 \times \underline{r}_{C/A} = \dot{\underline{\omega}}_2 \times \underline{r}_{C/A} + \underline{\omega}_2 \times \underline{v}_{C/\mathcal{F}}$
$= (-60\ \text{rad/s}^2)\underline{i} \times [(0.135\ \text{m})\underline{j} + (0.09\ \text{m})\underline{k}]$
$+ (12\ \text{rad/s})\underline{i} \times [(1.62\ \text{m/s})\underline{k} - (1.08\ \text{m/s})\underline{j}]$
$= -(8.10\ \text{m/s}^2)\underline{k} + (5.4\ \text{m/s}^2)\underline{j} - (19.44\ \text{m/s}^2)\underline{j} - (12.96\ \text{m/s}^2)\underline{k}$

$\underline{a}_{C/\mathcal{F}} = -(14.04\ \text{m/s}^2)\underline{j} - (21.06\ \text{m/s}^2)\underline{k}$

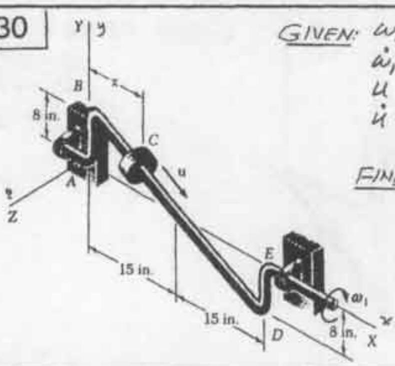
$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{C/\mathcal{F}} = 2(9\ \text{rad/s})\underline{k} \times [(1.62\ \text{m/s})\underline{k} - (1.08\ \text{m/s})\underline{j}]$

$\underline{a}_c = (19.44\ \text{m/s}^2)\underline{i}$

$\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_c$
$= (6.075\ \text{m/s}^2)\underline{i} - (10.94\ \text{m/s}^2)\underline{j}$
$-(14.04\ \text{m/s}^2)\underline{j} - (21.06\ \text{m/s}^2)\underline{k} + (19.44\ \text{m/s}^2)\underline{i}$

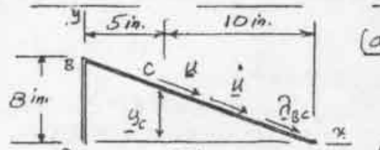
$\underline{a}_C = (25.5\ \text{m/s}^2)\underline{i} - (25.0\ \text{m/s}^2)\underline{j} - (21.1\ \text{m/s}^2)\underline{k}$ ◀

15.230



GIVEN: $\omega_1 = 3$ rad/s
$\dot{\omega}_1 = -12$ rad/s²
$u = 34$ in/s
$\dot{u} = -85$ in/s²

FIND: $\underline{v}_C$ AND $\underline{a}_C$ WHEN,
(a) $x = 5$ in.
(b) $x = 15$ in.



(a) $x = 5$ in.

$\frac{y_c}{8\text{ in.}} = \frac{10\text{ in.}}{15\text{ in.}}$; $y_c = \frac{16}{3}$ in

$\underline{\lambda}_{BC} = \frac{15\underline{i} - 8\underline{j}}{17}$

$\underline{v}_{C/\mathcal{F}} = u\,\underline{\lambda}_{BC} = (34\text{ in./s})\frac{15\underline{i}-8\underline{j}}{17} = (30\text{ in./s})\underline{i} - (16\text{ in./s})\underline{j}$

$\underline{a}_{C/\mathcal{F}} = \dot{u}\,\underline{\lambda}_{BC} = (-85\text{ in/s}^2)\frac{15\underline{i}-8\underline{j}}{17} = -(75\text{ in/s}^2)\underline{i} + (40\text{ in/s}^2)\underline{j}$

$\underline{\Omega} = \omega_1\underline{i} = (3\text{ rad/s})\underline{i}$; $\dot{\underline{\Omega}} = \dot{\omega}\underline{i} = (-12\text{ rad/s}^2)\underline{i}$

$\underline{r}_{C/A} = (5\text{ in.})\underline{i} + (16/3\text{ in.})\underline{j}$

VELOCITY:

$\underline{v}_{C'} = \underline{\Omega}\times\underline{r}_{C/A} = (3\text{ rad/s})\underline{i}\times\left[(5\text{ in.})\underline{i} + (16/3\text{ in.})\underline{j}\right] = -(16\text{ in./s})\underline{k}$

$\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}} = -(16\text{ in./s})\underline{k} + (30\text{ in./s})\underline{i} - (16\text{ in./s})\underline{j}$

$$\underline{v}_C = (30\text{ in./s})\underline{i} - (16\text{ in./s})\underline{j} - (16\text{ in./s})\underline{k}$$

ACCELERATION: $a_{C/\mathcal{F}}$, SEE ABOVE

$\underline{a}_{C'} = \dot{\underline{\Omega}}\times\underline{r}_{C/A} + \underline{\Omega}\times\underline{\Omega}\times\underline{r}_{C/A} = \dot{\underline{\Omega}}\times\underline{r}_{C/A} + \underline{\Omega}\times\underline{v}_{C'}$
$= (-12\text{ rad/s}^2)\underline{i}\times\left[(5\text{ in.})\underline{i} + (16/3\text{ in.})\underline{j}\right] + (3\text{ rad/s})\underline{i}\times(16\text{ in./s})\underline{k}$

$\underline{a}_{C'} = -(64\text{ in./s}^2)\underline{k} - (48\text{ in./s}^2)\underline{j}$

$\underline{a}_c = 2\underline{\Omega}\times\underline{v}_{C/\mathcal{F}} = 2(3\text{ rad/s})\underline{i}\times\left[(30\text{ in./s})\underline{i} - (16\text{ in/s})\underline{j}\right] = (96\text{ in/s}^2)\underline{k}$

$\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_c$
$= -(64\text{ in/s}^2)\underline{k} - (48\text{ in/s}^2)\underline{j} - (75\text{ in/s}^2)\underline{i} + (40\text{ in/s}^2)\underline{j} + (96\text{ in/s}^2)\underline{k}$

$$\underline{a}_C = -(75\text{ in/s}^2)\underline{i} - (8\text{ in/s}^2)\underline{j} + (32\text{ in/s}^2)\underline{k}$$

(b) $x = 15$ in. (COLLAR C IS IN XZ PLANE) $\therefore \underline{v}_{C'} = 0$

VELOCITY: $\underline{v}_{C/\mathcal{F}}$ = SAME AS IN PART a ABOVE

$\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}} = 0 + \underline{v}_{C/\mathcal{F}}$: $\underline{v}_C = (30\text{ in/s})\underline{i} - (16\text{ in/s})\underline{j}$

ACCELERATION:

$\underline{a}_{C/\mathcal{F}}$ = SAME AS IN PART a ABOVE

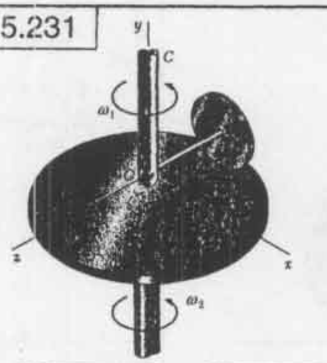
$\underline{a}_{C'} = 0$; SINCE COLLAR LIES ON AXIS OF ROTATION

$\underline{a}_c = 2\underline{\Omega}\times\underline{v}_{C/\mathcal{F}}$ = SAME AS IN PART a ABOVE

$\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_c$
$= 0 - (75\text{ in/s}^2)\underline{i} + (40\text{ in./s}^2)\underline{j} + (96\text{ in/s}^2)\underline{k}$

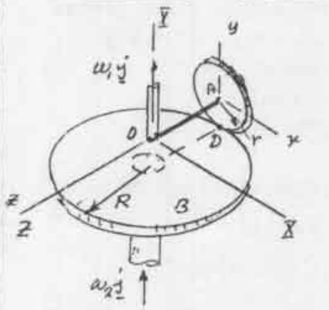
$$\underline{a}_C = -(75\text{ in./s}^2)\underline{i} + (40\text{ in./s}^2)\underline{j} + (96\text{ in./s}^2)\underline{k}$$

15.231



GIVEN: $\underline{\omega}_1 = \omega_1\underline{k}$
$\alpha_1 = 0$
$\underline{\omega}_2 = \omega_2\underline{k}$
$\alpha_2 = 0$

FIND: FOR DISK,
(a) $\underline{\omega}_A$
(b) $\underline{\alpha}_A$



MOVING FRAME Axyz ROTATES WITH ANGULAR VELOCITY $\underline{\Omega} = \omega_1\underline{j}$

$\underline{\omega}_{DISK/\mathcal{F}} = \omega_x\underline{i} + \omega_z\underline{k}$

$\underline{r}_{D/A} = -r\underline{j} - R\underline{k}$

(a) TOTAL ANGULAR VELOCITY OF DISK A:

$$\omega = \omega_1\underline{j} + \omega_{DISK/\mathcal{F}} = \omega_x\underline{i} + \omega_1\underline{j} + \omega_z\underline{k} \qquad (1)$$

DENOTE BY D POINT OF CONTACT OF DISKS

CONSIDER DISK B:

$$\underline{v}_D = \omega_2\underline{j}\times(-R\underline{k}) = -R\omega_2\underline{i} \qquad (2)$$

CONSIDER SYSTEM OC, OA AND DISK A.

$\underline{v}_{D'} = \underline{\Omega}\times\underline{r}_{D/A} = \omega_1\underline{j}\times(-r\underline{j} - R\underline{k}) = -R\omega_1\underline{i}$

$\underline{v}_{D/\mathcal{F}} = \omega_{DISK/\mathcal{F}}\times\underline{r}_{D/A} = (\omega_x\underline{i} + \omega_z\underline{k})\times(-r\underline{j} - R\underline{k})$
$= -r\omega_x\underline{k} + R\omega_x\underline{j} + r\omega_z\underline{i}$

$$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/\mathcal{F}} = -R\omega_1\underline{i} - r\omega_x\underline{k} + R\omega_x\underline{j} + r\omega_z\underline{i} \qquad (3)$$

EQUATE $v_D = v_D$ FROM EQ. 2 AND EQ. 3

$-R\omega_2\underline{i} = -R\omega_1 + r\omega_z + R\omega_x\underline{j} - r\omega_x\underline{k}$

COEF. OF $\underline{j}$: $0 = R\omega_x \rightarrow \omega_x = 0$

COEF. OF $\underline{i}$: $-R\omega_2 = -R\omega_1 + r\omega_z$; $\omega_z = \frac{R}{r}(\omega_1 - \omega_2)$

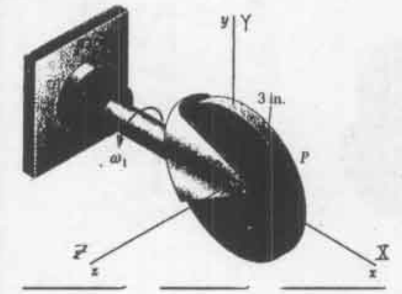
EQ. 3: $\underline{\omega} = \omega_1\underline{j} + \frac{R}{r}(\omega_1 - \omega_2)\underline{k}$

(b) DISK A ROTATES ABOUT y AXIS AT RATE $\underline{\omega}_1$

$\underline{\alpha} = \underline{\omega}_1\times\underline{\omega} = \omega_1\underline{j}\times\left[\omega_1\underline{j} + \frac{R}{r}(\omega_1 - \omega_2)\right]\underline{k}$

$$\underline{\alpha} = \omega_1(\omega_1 - \omega_2)\frac{R}{r}\underline{i}$$

15.232



GIVEN: $\omega_1 = 5\,\text{rad/s}$, $\alpha_1 = 0$; $\omega_2 = 4\,\text{rad/s}$, $\alpha_2 = 0$; $\theta = 30°$

FIND: $\underline{a}_P$

FRAME OXYZ IS FIXED. MOVING FRAME Oxyz ROTATES WITH ANGULAR VELOCITY $\underline{\Omega} = \omega_1 \underline{i} = (5\,\text{rad/s})\underline{i}$

$$\underline{r}_{P/O} = (3\,\text{in.})\cos 30°\,\underline{i} + (3\,\text{in.})\sin 30°\,\underline{j} = (2.598\,\text{in.})\underline{i} + (1.5\,\text{in.})\underline{j}$$

$$\underline{\omega}_{DISK/\mathcal{F}} = \omega_2 \underline{k} = (4\,\text{rad/s})\underline{k}$$

$$\underline{v}_{P/\mathcal{F}} = \underline{\omega}_{DISK/\mathcal{F}} \times \underline{r}_{P/O} = (4\,\text{rad/s})\underline{k} \times [2.598\,\underline{i} + 1.5\,\underline{j}]$$

$$\underline{v}_{P/\mathcal{F}} = (10.392\,\text{in./s})\underline{j} - (6\,\text{in./s})\underline{i}$$

$$\underline{v}_{P'} = \underline{\Omega} \times \underline{r}_{P/O} = (5\,\text{rad/s})\underline{i} \times [2.598\,\underline{i} + 1.5\,\underline{j}]$$

$$\underline{v}_{P'} = (7.5\,\text{in./s})\underline{k}$$

ACCELERATION: $\underline{a}_{P'} = \underline{\Omega} \times \underline{v}_{P'} = (5\,\text{rad/s})\underline{i} \times (7.5\,\text{in/s})\underline{k} = -(37.5\,\text{in./s}^2)\underline{j}$

$$\underline{a}_{P/\mathcal{F}} = \underline{\omega}_{DISK/\mathcal{F}} \times \underline{v}_{P/\mathcal{F}} = (4\,\text{rad/s})\underline{k} \times [(10.392\,\text{in/s})\underline{j} - (6\,\text{in/s})\underline{i}]$$

$$\underline{a}_{P/\mathcal{F}} = -(41.569\,\text{in/s}^2)\underline{i} - (24\,\text{in./s}^2)\underline{j}$$

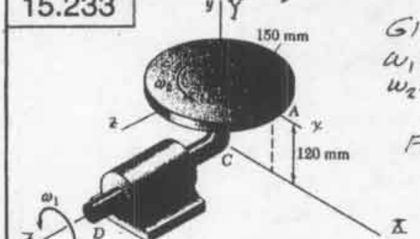
$$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{P/\mathcal{F}} = 2(5\,\text{rad/s})\underline{i} \times [(10.392\,\text{in./s})\underline{j} - (6\,\text{in./s})\underline{i}]$$

$$\underline{a}_C = (103.92\,\text{in./s}^2)\underline{k}$$

$$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/\mathcal{F}} + \underline{a}_C = -37.5\,\underline{j} - 41.569\,\underline{i} - 24\,\underline{j} + 103.92\,\underline{k}$$

$$\underline{a}_P = -(41.6\,\text{in/s}^2)\underline{i} - (61.5\,\text{in/s}^2)\underline{j} + (103.9\,\text{in./s}^2)\underline{k}$$ ◀

15.233



GIVEN: $\omega_1 = 5\,\text{rad/s}$, $\alpha_1 = 0$; $\omega_2 = 4\,\text{rad/s}$, $\alpha_2 = 0$

FIND: α_{DISK}

FRAME CXYZ IS FIXED
MOVING FRAME Bxyz ROTATES WITH ANGULAR VELOCITY $\underline{\Omega} = \omega_1 \underline{k} = (5\,\text{rad/s})\underline{k}$ ABOUT Z AXIS

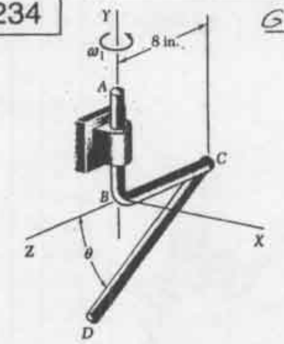
$$\underline{\omega}_{DISK} = \omega_1 \underline{k} + \omega_2 \underline{j}$$

$$\underline{\alpha}_{DISK} = \underline{\Omega} \times \underline{\omega}_{DISK} = \omega_1 \underline{k} \times (\omega_1 \underline{k} + \omega_2 \underline{j}) = -\omega_1 \omega_2 \underline{i}$$

$$\underline{\alpha}_{DISK} = -(5\,\text{rad/s})(4\,\text{rad/s})\,\underline{i}$$

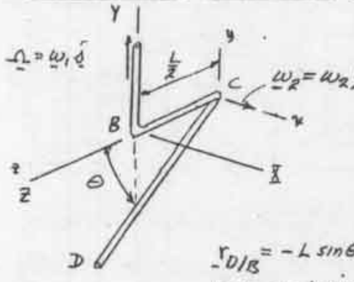
$$\underline{\alpha}_{DISK} = -(20\,\text{rad/s}^2)\,\underline{i}$$ ◀

15.234



GIVEN: $CD = L = 16\,\text{in.}$; $\theta = 30°$; $\omega_1 = 4\,\text{rad/s}$; $\omega_2 = \dfrac{d\theta}{dt} = 3\,\text{rad/s}$

FIND: $\underline{v}_D$ AND $\underline{a}_D$



FRAME BXYZ IS FIXED. MOVING FRAME Cxyz ROTATES ABOUT Y AXIS WITH $\underline{\Omega} = \omega_1 \underline{j}$

$$\underline{r}_{D/B} = -L\sin\theta\,\underline{j} + (L\cos\theta - \tfrac{L}{2})\underline{k}$$

$$\underline{r}_{D/C} = -L\sin\theta\,\underline{j} + L\cos\theta\,\underline{k}$$

VELOCITY: $\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/B} = \omega_1 \underline{j} \times [-L\sin\theta\,\underline{j} + L(\cos\theta - \tfrac{1}{2})\underline{k}]$

$$\underline{v}_{D'} = L\omega_1(\cos\theta - \tfrac{1}{2})\underline{i}$$

$$\underline{v}_{D/\mathcal{F}} = \underline{\omega}_2 \times \underline{r}_{D/C} = \omega_2 \underline{i} \times (-L\sin\theta\,\underline{j} + L\cos\theta\,\underline{k})$$

$$\underline{v}_{D/\mathcal{F}} = -L\omega_2 \sin\theta\,\underline{k} - L\omega_2\cos\theta\,\underline{j}$$

$$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/\mathcal{F}}$$

$$\underline{v}_D = L\omega_1(\cos\theta - \tfrac{1}{2})\underline{i} - L\omega_2\cos\theta\,\underline{j} - L\omega_2\sin\theta\,\underline{k}$$ ◁

ACCELERATION: $\underline{a}_{D'} = \underline{\Omega} \times \underline{v}_{D'} = \omega_1 \underline{j} \times L\omega_1(\cos\theta - \tfrac{1}{2})\underline{i}$

$$\underline{a}_{D'} = -L\omega_1^2(\cos\theta - \tfrac{1}{2})\underline{k}$$

$$\underline{a}_{D/\mathcal{F}} = \underline{\omega}_2 \times \underline{v}_{D/\mathcal{F}} = \omega_2 \underline{i} \times [-L\omega_2\sin\theta\,\underline{k} - L\omega_2\cos\theta\,\underline{j}]$$

$$\underline{a}_{D/\mathcal{F}} = +L\omega_2^2\sin\theta\,\underline{j} - L\omega_2^2\cos\theta\,\underline{k}$$

$$\underline{a}_C = 2\underline{\Omega} \times \underline{v}_{D/\mathcal{F}} = 2\omega_1 \underline{j} \times (-L\omega_2\sin\theta\,\underline{k} - L\omega_2\cos\theta\,\underline{j})$$

$$\underline{a}_C = -2L\omega_1\omega_2\sin\theta\,\underline{i}$$

$$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_C$$
$$= -L\omega_1^2(\cos\theta - \tfrac{1}{2})\underline{k} + L\omega_2^2\sin\theta\,\underline{j} - L\omega_2^2\cos\theta\,\underline{k} - 2L\omega_1\omega_2\sin\theta\,\underline{i}$$

$$\underline{a}_D = -2L\omega_1\omega_2\sin\theta\,\underline{i} + L\omega_2^2\sin\theta\,\underline{j} + (-L\omega_1^2(\cos\theta - \tfrac{1}{2}) - L\omega_2^2\cos\theta)\underline{k}$$ ◁

DATA: $\theta = 30°$, $L = 16\,\text{in.}$, $\omega_1 = 4\,\text{rad/s}$, $\omega_2 = 3\,\text{rad/s}$

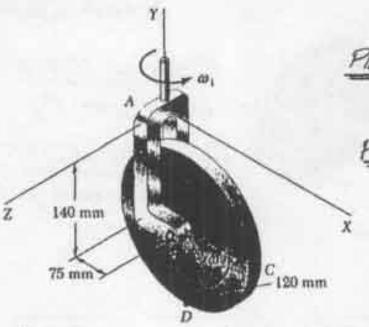
$$\underline{v}_D = 16(4)(\cos 30° - \tfrac{1}{2})\underline{i} - 16(3)\cos 30°\,\underline{j} - 16(3)\sin 30°\,\underline{k}$$

$$\underline{v}_D = (23.4\,\text{in/s})\underline{i} - (41.6\,\text{in/s})\underline{j} - (24\,\text{in/s})\underline{k}$$ ◀

$$\underline{a}_D = -2(16)(3)(4)\sin 30°\,\underline{i} + 16(3)^2\sin 30°\,\underline{j} + (-16(4)^2(\cos 30 - \tfrac{1}{2}) - 16(3)^2\cos 30°)\underline{k}$$

$$\underline{a}_D = -(192\,\text{in/s}^2)\underline{i} + (72\,\text{in/s}^2)\underline{j} - (218\,\text{in/s}^2)\underline{k}$$ ◀

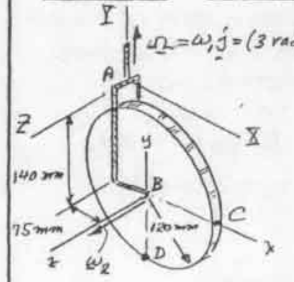
15.235 and 15.236



GIVEN: $\omega_1 = 3$ rad/s
$\omega_2 = 5$ rad/s

PROBLEM 15.235:
FIND: $\mathbf{v}_C$ AND $\mathbf{a}_C$

PROBLEM 15.236:
FIND: $\mathbf{v}_D$ AND $\mathbf{a}_D$



FRAME AXYZ IS FIXED
MOVING FRAME Bxyz
ROTATES ABOUT Y AXIS
WITH $\boldsymbol{\Omega} = (3 \text{ rad/s})\mathbf{j}$
$\boldsymbol{\omega}_2 = (5 \text{ rad/s})\mathbf{k}$

PROBLEM 15.235: FOR POINT C

$\mathbf{r}_{C/A} = (195 \text{ mm})\mathbf{i} - (140 \text{ mm})\mathbf{j}$; $\mathbf{r}_{C/B} = (120 \text{ mm})\mathbf{i}$

VELOCITY:

$\mathbf{v}_{C'} = \boldsymbol{\Omega} \times \mathbf{r}_{C/A} = (3 \text{ rad/s})\mathbf{j} \times (195\mathbf{i} - 140\mathbf{j}) = -(585 \text{ mm/s})\mathbf{k}$

$\mathbf{v}_{C/\mathcal{F}} = \boldsymbol{\omega}_2 \times \mathbf{r}_{C/B} = (5 \text{ rad/s})\mathbf{k} \times (120\mathbf{i}) = +(600 \text{ mm/s})\mathbf{j}$

$\mathbf{v}_C = \mathbf{v}_{C'} + \mathbf{v}_{C/\mathcal{F}}$ $\quad \mathbf{v}_C = (600 \text{ mm/s})\mathbf{j} - (585 \text{ mm/s})\mathbf{k}$ ◀

ACCELERATION

$\mathbf{a}_{C'} = \boldsymbol{\Omega} \times \mathbf{v}_{C'} = (3 \text{ rad/s})\mathbf{j} \times (-585 \text{ mm/s})\mathbf{k} = -(1.755 \text{ m/s}^2)\mathbf{i}$

$\mathbf{a}_{C/\mathcal{F}} = \boldsymbol{\omega}_2 \times \mathbf{v}_{C/\mathcal{F}} = (5 \text{ rad/s})\mathbf{k} \times (600 \text{ mm/s})\mathbf{j} = -(3.00 \text{ m/s}^2)\mathbf{i}$

$\mathbf{a}_c = 2\boldsymbol{\Omega} \times \mathbf{v}_{C/\mathcal{F}} = 2(3 \text{ rad/s})\mathbf{j} \times (600 \text{ mm/s})\mathbf{j} = 0$

$\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/\mathcal{F}} + \mathbf{a}_c = -(1.755 \text{ m/s}^2)\mathbf{i} - (3.00 \text{ m/s}^2)\mathbf{i}$

$\mathbf{a}_C = -(4.76 \text{ m/s}^2)\mathbf{i}$ ◀

PROBLEM 15.236: FOR POINT D

$\mathbf{r}_{D/A} = (75 \text{ mm})\mathbf{i} - (260 \text{ mm})\mathbf{j}$; $\mathbf{r}_{D/B} = -(120 \text{ mm})\mathbf{j}$

VELOCITY

$\mathbf{v}_{D'} = \boldsymbol{\Omega} \times \mathbf{r}_{D/A} = (3 \text{ rad/s})\mathbf{j} \times (75\mathbf{i} - 260\mathbf{j}) = -(225 \text{ mm/s})\mathbf{k}$

$\mathbf{v}_{D/\mathcal{F}} = \boldsymbol{\omega}_2 \times \mathbf{r}_{D/B} = (5 \text{ rad/s})\mathbf{k} \times (-120\mathbf{j}) = (600 \text{ mm/s})\mathbf{i}$

$\mathbf{v}_C = \mathbf{v}_{C'} + \mathbf{v}_{C/\mathcal{F}}$

$\mathbf{v}_C = (600 \text{ mm/s})\mathbf{i} - (225 \text{ mm/s})\mathbf{k}$ ◀

ACCELERATION

$\mathbf{a}_{D'} = \boldsymbol{\Omega} \times \mathbf{v}_{D'} = (3 \text{ rad/s})\mathbf{j} \times (-225 \text{ mm/s})\mathbf{k} = -(0.675 \text{ m/s})\mathbf{i}$

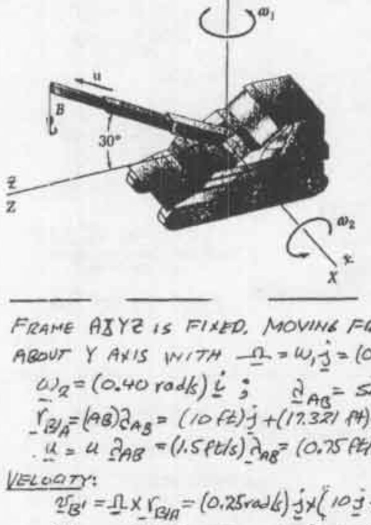
$\mathbf{a}_{D/\mathcal{F}} = \boldsymbol{\omega}_2 \times \mathbf{v}_{D/\mathcal{F}} = (5 \text{ rad/s})\mathbf{k} \times (600 \text{ mm/s})\mathbf{i} = +(3.00 \text{ m/s})\mathbf{j}$

$\mathbf{a}_c = 2\boldsymbol{\Omega} \times \mathbf{v}_{D/\mathcal{F}} = 2(3 \text{ rad/s})\mathbf{j} \times (600 \text{ mm/s})\mathbf{i} = -(3.60 \text{ m/s})\mathbf{k}$

$\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/\mathcal{F}} + \mathbf{a}_c$

$\mathbf{a}_C = -(0.675 \text{ m/s})\mathbf{i} + (3.00 \text{ m/s})\mathbf{j} - (3.60 \text{ m/s})\mathbf{k}$ ◀

15.237



GIVEN:
$\omega_1 = 0.25$ rad/s, $\alpha_1 = 0$
$\omega_2 = 0.40$ rad/s, $\alpha_2 = 0$
$AB = 20$ ft
$u = 1.5$ ft/s
$\dot{u} = 0$

FIND: $\mathbf{v}_B$ AND $\mathbf{a}_B$

FRAME AXYZ IS FIXED. MOVING FRAME Oxyz ROTATES ABOUT Y AXIS WITH $\boldsymbol{\Omega} = \omega_1\mathbf{j} = (0.25 \text{ rad/s})\mathbf{j}$

$\boldsymbol{\omega}_2 = (0.40 \text{ rad/s})\mathbf{i}$; $\quad \boldsymbol{\lambda}_{AB} = \sin 30°\mathbf{j} + \cos 30°\mathbf{k}$

$\mathbf{r}_{B/A} = (AB)\boldsymbol{\lambda}_{AB} = (10 \text{ ft})\mathbf{j} + (17.321 \text{ ft})\mathbf{k}$

$\mathbf{u} = u\,\boldsymbol{\lambda}_{AB} = (1.5 \text{ ft/s})\boldsymbol{\lambda}_{AB} = (0.75 \text{ ft/s})\mathbf{j} + (1.299 \text{ ft/s})\mathbf{k}$

VELOCITY:

$\mathbf{v}_{B'} = \boldsymbol{\Omega} \times \mathbf{r}_{B/A} = (0.25 \text{ rad/s})\mathbf{j} \times (10\mathbf{j} + 17.32\mathbf{k}) = (4.33 \text{ ft/s})\mathbf{i}$

$\mathbf{v}_{B/\mathcal{F}} = \mathbf{u} + \boldsymbol{\omega}_2 \times \mathbf{r}_{B/A} = 0.75\mathbf{j} + 1.299\mathbf{k} + (0.40\mathbf{i}) \times (10\mathbf{j} + 17.32\mathbf{k})$

$= 0.75\mathbf{j} + 1.299\mathbf{k} + 4\mathbf{k} - 6.928\mathbf{j}$

$\mathbf{v}_{B/\mathcal{F}} = -(6.178 \text{ ft/s})\mathbf{j} + (5.299 \text{ ft/s})\mathbf{k}$

$\mathbf{v} = \mathbf{v}_{B'} + \mathbf{v}_{B/\mathcal{F}} = (4.33 \text{ ft/s})\mathbf{i} - (6.178 \text{ ft/s})\mathbf{j} + (5.299 \text{ ft/s})\mathbf{k}$

$\mathbf{v} = (4.33 \text{ ft/s})\mathbf{i} - (6.18 \text{ ft/s})\mathbf{j} + (5.30 \text{ ft/s})\mathbf{k}$ ◀

ACCELERATION:

$\mathbf{a}_{B'} = \boldsymbol{\Omega} \times \mathbf{v}_{B'} = (0.25 \text{ rad/s})\mathbf{j} \times (4.33 \text{ ft/s})\mathbf{i} = -(1.083 \text{ ft/s}^2)\mathbf{k}$

$\mathbf{a}_{B/\mathcal{F}} = \underbrace{\boldsymbol{\omega}_2 \times \boldsymbol{\omega}_2 \times \mathbf{r}_{B/A}} + \underbrace{2\boldsymbol{\omega}_2 \times \mathbf{u}}$

ACCELERATION OF COINCIDING POINT AS AB ROTATES WITH ω_2 ABOUT x AXIS

CORIOLIS ACCELERATION OF B MOVING AT $\mathbf{u}$ AS AB ROTATES ABOUT x AXIS

$\mathbf{a}_{B/\mathcal{F}} = (0.4\mathbf{i}) \times (0.4\mathbf{i} \times (10\mathbf{j} + 17.32\mathbf{k})) + 2(0.4\mathbf{i}) \times (0.75\mathbf{j} + 1.299\mathbf{k})$

$= (0.4\mathbf{i}) \times (4\mathbf{k} - 6.928\mathbf{j}) + 0.6\mathbf{k} - 1.0392\mathbf{j}$

$= -1.6\mathbf{j} - 2.771\mathbf{k} + 0.6\mathbf{k} - 1.0392\mathbf{j}$

$\mathbf{a}_{B/\mathcal{F}} = -(2.639 \text{ ft/s}^2)\mathbf{j} - (2.171 \text{ ft/s}^2)\mathbf{k}$

$\mathbf{a}_c$ = CORIOLIS ACCELERATION OF B MOVING WITH FRAME Axyz IN ROTATION ABOUT Y. WITH ANGULAR VELOCITY $\boldsymbol{\Omega}$

$\mathbf{a}_c = 2\boldsymbol{\Omega} \times \mathbf{v}_{B/\mathcal{F}}$

$= 2(0.25 \text{ rad/s})\mathbf{j} \times [(6.178 \text{ ft/s})\mathbf{j} + (5.299 \text{ ft/s}^2)\mathbf{k}]$

$= (2.650 \text{ ft/s}^2)\mathbf{i}$

$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/\mathcal{F}} + \mathbf{a}_c$

$= -(1.083 \text{ ft/s}^2)\mathbf{k} - (2.639 \text{ ft/s}^2)\mathbf{j} - (2.171 \text{ ft/s}^2)\mathbf{k} + (2.650 \text{ ft/s}^2)\mathbf{i}$

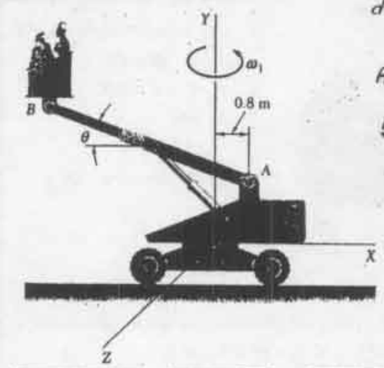
$\mathbf{a}_B = (2.65 \text{ ft/s}^2)\mathbf{i} - (2.64 \text{ ft/s}^2)\mathbf{j} - (3.25 \text{ ft/s}^2)\mathbf{k}$ ◀

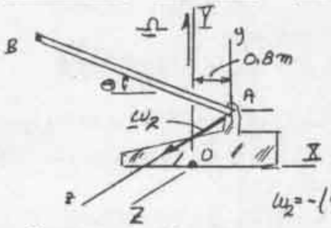
15.238 and 15.239

GIVEN: $\omega_1 = 0.15\,\text{rad/s}$, $\dot{\omega}_1 = 0$
$d\theta/dt = 0.25\,\text{rad/s}$
$d^2\theta/dt^2 = 0$
$AB = 5\,\text{m}$

PROBLEM 15.238
FOR $\theta = 20°$
FIND: $\mathbf{v}_B$ AND $\mathbf{a}_B$

PROBLEM 15.239
FOR $\theta = 40°$
FIND: $\mathbf{v}_B$ AND $\mathbf{a}_B$





FRAME OXYZ IS FIXED. MOVING FRAME Axyz ROTATES ABOUT Y AXIS WITH

$\Omega = \omega_1 \mathbf{j} = (0.15\,\text{rad/s})\,\mathbf{j}$

$\omega_2 = -(d\theta/dt)\,\mathbf{k} = -(0.25\,\text{rad/s})\,\mathbf{k}$

PROBLEM 15.238 $\theta = 20°$

$\mathbf{r}_{B/A} = (5\,\text{m})(-\cos 20°\,\mathbf{i} + \sin 20°\,\mathbf{j}) = -(4.698\,\text{m})\mathbf{i} + (1.710\,\text{m})\mathbf{j}$

$\mathbf{r}_{B/O} = \mathbf{r}_{B/A} + (0.8\,\text{m})\mathbf{i} = -(3.898\,\text{m})\mathbf{i} + (1.710\,\text{m})\mathbf{j}$

VELOCITY:

$\mathbf{v}_{B'} = \Omega \times \mathbf{r}_{B/O} = (0.15\,\text{rad/s})\mathbf{j} \times (-3.898\,\mathbf{i} + 1.710\,\mathbf{j}) = (0.5848\,\text{m/s})\,\mathbf{k}$

$\mathbf{v}_{B/\mathcal{F}} = \omega_2 \times \mathbf{r}_{B/A} = -(0.25\,\text{rad/s})\mathbf{k} \times (-4.698\,\mathbf{i} + 1.710\,\mathbf{j})$

$\mathbf{v}_{B/\mathcal{F}} = +(1.1746\,\text{m/s})\,\mathbf{j} + (0.4275\,\text{m/s})\,\mathbf{i}$

$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/\mathcal{F}}$: $\mathbf{v}_B = +(0.428\,\text{m/s})\mathbf{i} + (1.175\,\text{m/s})\mathbf{j} + (0.585\,\text{m/s})\mathbf{k}$ ◀

ACCELERATION

$\mathbf{a}_{B'} = \Omega \times \mathbf{v}_{B'} = (0.15\,\text{rad/s})\mathbf{j} \times (0.5848\,\text{m/s})\mathbf{k} = (0.0877\,\text{m/s}^2)\,\mathbf{i}$

$\mathbf{a}_{B/\mathcal{F}} = \omega_1 \times \mathbf{v}_{B/\mathcal{F}} = -(0.25\,\text{rad/s})\mathbf{k} \times (+1.1746\,\mathbf{j} + 0.4275\,\mathbf{i})$

$\mathbf{a}_{B/\mathcal{F}} = (0.29365\,\text{m/s}^2)\,\mathbf{i} - (0.10688\,\text{m/s}^2)\,\mathbf{j}$

$\mathbf{a}_c = 2\Omega \times \mathbf{v}_{B/\mathcal{F}} = 2(0.15\,\text{rad/s})\mathbf{j} \times (+1.1746\,\mathbf{j} + 0.4275\,\mathbf{i})$

$\mathbf{a}_c = -(0.1283\,\text{m/s}^2)\,\mathbf{k}$

$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/\mathcal{F}} + \mathbf{a}_c = 0.0877\,\mathbf{i} + 0.29365\,\mathbf{i} - 0.10688\,\mathbf{j} - 0.1283\,\mathbf{k}$

$\mathbf{a}_C = (0.381\,\text{m/s}^2)\mathbf{i} - (0.1069\,\text{m/s}^2)\mathbf{j} - (0.1283\,\text{m/s}^2)\mathbf{k}$ ◀

PROBLEM 15.239 $\theta = 40°$

$\mathbf{r}_{B/A} = -(3.83\,\text{m})\mathbf{i} + (3.214\,\text{m})\mathbf{j}$; $\mathbf{r}_{B/O} = -(3.03\,\text{m})\mathbf{i} + (3.214\,\text{m})\mathbf{j}$

VELOCITY: $\mathbf{v}_{B'} = \Omega \times \mathbf{r}_{B/O} = (0.15\,\text{rad/s})\mathbf{j} \times (-3.03\,\mathbf{i} + 3.214\,\mathbf{j}) = (0.4545\,\text{m/s})\,\mathbf{k}$

$\mathbf{v}_{B/\mathcal{F}} = \omega_2 \times \mathbf{r}_{B/A} = -(0.25\,\text{rad/s})\mathbf{k} \times (-3.83\,\mathbf{i} + 3.214\,\mathbf{j})$

$\mathbf{v}_{B/\mathcal{F}} = +(0.9576\,\text{m/s})\,\mathbf{j} + (0.8035\,\text{m/s})\,\mathbf{i}$

$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/\mathcal{F}}$: $\mathbf{v}_B = +(0.803\,\text{m/s})\mathbf{i} + (0.958\,\text{m/s})\mathbf{j} + (0.455\,\text{m/s})\mathbf{k}$ ◀

ACCELERATION:

$\mathbf{a}_{B'} = \Omega \times \mathbf{v}_{B'} = (0.15\,\text{rad/s})\mathbf{j} \times (0.4545\,\text{m/s})\mathbf{k} = (0.0682\,\text{m/s}^2)\,\mathbf{i}$

$\mathbf{a}_{B/\mathcal{F}} = \omega_1 \times \mathbf{v}_{B/\mathcal{F}} = -(0.25\,\text{rad/s})\mathbf{k} \times (+0.9576\,\mathbf{j} + 0.8035\,\mathbf{i})$

$\mathbf{a}_{B/\mathcal{F}} = (0.2394\,\text{m/s}^2)\,\mathbf{i} - (0.2009\,\text{m/s}^2)\,\mathbf{j}$

$\mathbf{a}_c = 2\Omega \times \mathbf{v}_{B/\mathcal{F}} = 2(0.15\,\text{rad/s})\mathbf{j} \times (+0.9576\,\mathbf{j} + 0.8035\,\mathbf{i})$

$\mathbf{a}_c = -(0.2410\,\text{m/s}^2)\,\mathbf{k}$

$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/\mathcal{F}} + \mathbf{a}_c$
$= (0.0682\,\text{m/s}^2)\mathbf{i} + (0.2394\,\text{m/s}^2)\mathbf{i} - (0.2009\,\text{m/s}^2)\mathbf{j} - (0.2410\,\text{m/s}^2)\mathbf{k}$

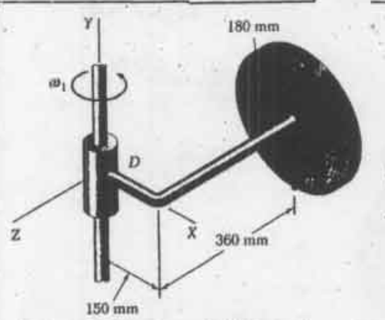
$\mathbf{a}_B = (0.308\,\text{m/s}^2)\mathbf{i} - (0.201\,\text{m/s}^2)\mathbf{j} - (0.241\,\text{m/s}^2)\mathbf{k}$ ◀

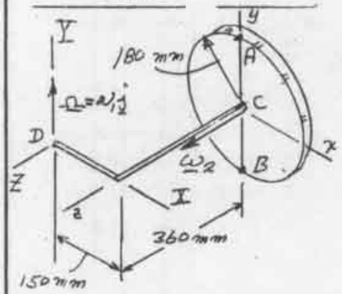
15.240 and 15.241

GIVEN: $\omega_1 = 8\,\text{rad/s}$, $\alpha_1 = 0$
$\omega_2 = 12\,\text{rad/s}$, $\alpha_2 = 0$

PROBLEM 15.240
FIND: $\mathbf{v}_A$ AND $\mathbf{a}_A$

PROBLEM 15.241
FIND: $\mathbf{v}_B$ AND $\mathbf{a}_B$





FRAME DXYZ IS FIXED. MOVING FRAME Cxyz ROTATES ABOUT Y AXIS WITH

$\Omega = \omega_1 \mathbf{j} = (8\,\text{rad/s})\,\mathbf{j}$

$\omega_2 = (12\,\text{rad/s})\,\mathbf{k}$

PROBLEM 15.240: FOR POINT A

$\mathbf{r}_{D/A} = (0.15\,\text{m})\mathbf{i} + (0.18\,\text{m})\mathbf{j} - (0.36\,\text{m})\mathbf{k}$; $\mathbf{r}_{A/C} = (0.18\,\text{m})\,\mathbf{j}$

VELOCITY:

$\mathbf{v}_{A'} = \Omega \times \mathbf{r}_{D/A} = (8\,\text{rad/s})\mathbf{j} \times (0.15\,\mathbf{i} + 0.18\,\mathbf{j} - 0.36\,\mathbf{k})$

$\mathbf{v}_{A'} = -(1.2\,\text{m/s})\mathbf{k} - (2.88\,\text{m/s})\mathbf{i}$

$\mathbf{v}_{A/\mathcal{F}} = \omega_2 \times \mathbf{r}_{A/C} = (12\,\text{rad/s})\mathbf{k} \times (0.18\,\text{m})\mathbf{j} = -(2.16\,\text{m/s})\,\mathbf{i}$

$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/\mathcal{F}} = -(1.2\,\text{m/s})\mathbf{k} - (2.88\,\text{m/s})\mathbf{i} - (2.16\,\text{m/s})\mathbf{i}$

$\mathbf{v}_A = -(5.04\,\text{m/s})\mathbf{i} - (1.2\,\text{m/s})\mathbf{k}$ ◀

ACCELERATION:

$\mathbf{a}_{A'} = \Omega \times \mathbf{v}_{A'} = (8\,\text{rad/s})\mathbf{j} \times (-1.2\,\mathbf{k} - 2.88\,\mathbf{i}) = -(9.6\,\text{m/s}^2)\mathbf{i} + (23.04\,\text{m/s}^2)\mathbf{k}$

$\mathbf{a}_{A/\mathcal{F}} = \omega_2 \times \mathbf{v}_{A/\mathcal{F}} = (12\,\text{rad/s})\mathbf{k} \times (-2.16\,\text{m/s})\mathbf{i} = -(25.92\,\text{m/s}^2)\,\mathbf{j}$

$\mathbf{a}_c = 2\Omega \times \mathbf{v}_{A/\mathcal{F}} = 2(8\,\text{rad/s})\mathbf{j} \times (-2.16\,\text{m/s})\mathbf{i} = (34.56\,\text{m/s}^2)\,\mathbf{k}$

$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/\mathcal{F}} + \mathbf{a}_c = -(9.6\,\text{m/s}^2)\mathbf{i} + (23.04\,\text{m/s}^2)\mathbf{k} - (25.92\,\text{m/s}^2)\mathbf{j} + (34.56\,\text{m/s}^2)\mathbf{k}$

$\mathbf{a}_C = -(9.6\,\text{m/s}^2)\mathbf{i} - (25.9\,\text{m/s}^2)\mathbf{j} + (57.6\,\text{m/s}^2)\mathbf{k}$ ◀

PROBLEM 15.241 FOR POINT B

$\mathbf{r}_{B/A} = (0.15\,\text{m})\mathbf{i} - (0.18\,\text{m})\mathbf{j} - (0.36\,\text{m})\mathbf{k}$; $\mathbf{r}_{B/C} = -(0.18\,\text{m})\,\mathbf{j}$

VELOCITY:

$\mathbf{v}_{B'} = \Omega \times \mathbf{r}_{B/A} = (8\,\text{rad/s})\mathbf{j} \times (0.15\,\mathbf{i} - 0.18\,\mathbf{j} - 0.36\,\mathbf{k})$

$\mathbf{v}_{B'} = -(1.2\,\text{m/s})\mathbf{k} - (2.88\,\text{m/s})\mathbf{i}$

$\mathbf{v}_{B/\mathcal{F}} = \omega_2 \times \mathbf{r}_{B/C} = (12\,\text{rad/s})\mathbf{k} \times (-0.18\,\text{m})\mathbf{j} = (2.16\,\text{m/s})\,\mathbf{i}$

$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/\mathcal{F}} = -(1.2\,\text{m/s})\mathbf{k} - (2.88\,\text{m/s})\mathbf{i} + (2.16\,\text{m/s})\mathbf{i}$

$\mathbf{v}_B = -(0.72\,\text{m/s})\mathbf{i} - (1.2\,\text{m/s})\mathbf{k}$ ◀

ACCELERATION:

$\mathbf{a}_{B'} = \Omega \times \mathbf{v}_{B'} = (8\,\text{rad/s})\mathbf{j} \times (1.2\,\mathbf{k} - 2.88\,\mathbf{i})$

$\mathbf{a}_{B'} = -(9.6\,\text{m/s}^2)\mathbf{i} + (23.04\,\text{m/s}^2)\mathbf{k}$

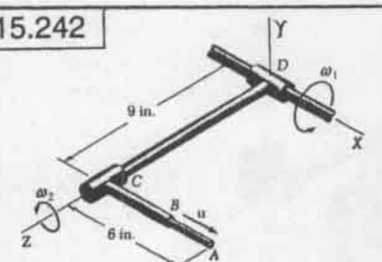
$\mathbf{a}_{B/\mathcal{F}} = \omega_2 \times \mathbf{v}_{B/\mathcal{F}} = (12\,\text{rad/s})\mathbf{k} \times (2.16\,\text{m/s})\mathbf{i} = (25.92\,\text{m/s}^2)\,\mathbf{j}$

$\mathbf{a}_c = 2\Omega \times \mathbf{v}_{B/\mathcal{F}} = 2(8\,\text{rad/s})\mathbf{j} \times (2.16\,\text{m/s})\mathbf{i} = -(34.56\,\text{m/s}^2)\,\mathbf{k}$

$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/\mathcal{F}}$
$= -(9.6\,\text{m/s}^2)\mathbf{i} + (23.04\,\text{m/s}^2)\mathbf{k} + (25.92\,\text{m/s}^2)\mathbf{j} - (34.56\,\text{m/s}^2)\mathbf{k}$

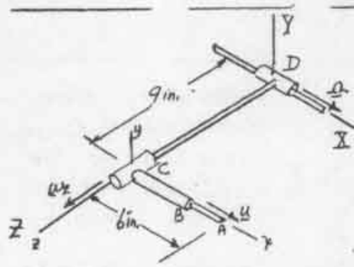
$\mathbf{a}_B = -(9.6\,\text{m/s}^2)\mathbf{i} + (25.9\,\text{m/s}^2)\mathbf{j} - (11.52\,\text{m/s}^2)\mathbf{k}$ ◀

15.242



GIVEN:
$\omega_1 = 1.2\ \text{rad/s},\ \alpha_1 = 0$
$\omega_2 = 1.5\ \text{rad/s},\ \alpha_2 = 0$
$u = 3\ \text{in./s},\ \dot{u} = 0$

FIND: $\mathbf{v}_A$ AND $\mathbf{a}_A$



FRAME DXYZ IS FIXED. MOVING FRAME Cxyz ROTATES ABOUT THE X AXIS WITH $\mathbf{\Omega} = \omega_1 \mathbf{i} = (1.2\ \text{rad/s})\,\mathbf{i}$.

$\boldsymbol{\omega}_2 = \omega_2 \mathbf{k} (1.5\ \text{rad/s})\,\mathbf{k}$
$\mathbf{u} = u\,\mathbf{i} = (3\ \text{in./s})\,\mathbf{i}$

$\mathbf{r}_{A/D} = (6\ \text{in.})\mathbf{i} + (9\ \text{in.})\mathbf{k} \qquad \mathbf{r}_{A/C} = (6\ \text{in.})\mathbf{i}$

VELOCITY:

$$\mathbf{v}_{A'} = \mathbf{\Omega} \times \mathbf{r}_{A/D} = (1.2\ \text{rad/s})\,\mathbf{i} \times [(6\ \text{in.})\mathbf{i} + (9\ \text{in.})\mathbf{k}] = -(10.8\ \text{in./s})\,\mathbf{j}$$

$$\mathbf{v}_{A/\mathcal{F}} = \boldsymbol{\omega}_2 \times \mathbf{r}_{A/C} + \mathbf{u} = (1.5\ \text{rad/s})\mathbf{k} \times (6\ \text{in.})\mathbf{i} + (3\ \text{in./s})\mathbf{i}$$

$$\mathbf{v}_{A/\mathcal{F}} = +(9\ \text{in./s})\,\mathbf{j} + (3\ \text{in./s})\,\mathbf{i}$$

$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/\mathcal{F}} = -(10.8\ \text{in./s})\mathbf{j} + (9\ \text{in./s})\mathbf{j} + (3\ \text{in./s})\mathbf{i}$$

$$\mathbf{v}_A = (3\ \text{in./s})\,\mathbf{i} - (1.8\ \text{in./s})\,\mathbf{j}$$

ACCELERATION

$$\mathbf{a}_{A'} = \mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{r}_{A/D} = \mathbf{\Omega} \times \mathbf{v}_{A'} = (1.2\ \text{rad/s})\,\mathbf{i} \times (-10.8\ \text{in./s})\,\mathbf{j}$$

$$\mathbf{a}_A = -(12.96\ \text{in./s}^2)\,\mathbf{k}$$

$\mathbf{a}_{A/\mathcal{F}}$: NOTE, SINCE POINT A MOVES IN THE ROTATING FRAME Cxyz THERE IS A CORIOLIS ACCELERATION.

$$\mathbf{a}_{A/\mathcal{F}} = \boldsymbol{\omega}_2 \times \boldsymbol{\omega}_2 \times \mathbf{r}_{A/C} + 2\boldsymbol{\omega}_2 \times \mathbf{u}$$
$$= \boldsymbol{\omega}_2 \times (1.5\ \text{rad/s})\mathbf{k} \times (6\ \text{in.})\mathbf{i} + 2(1.5\ \text{rad/s})\mathbf{k} \times (3\ \text{rad/s})\,\mathbf{i}$$
$$= (1.5\ \text{rad/s})\mathbf{k} \times (+9\ \text{in./s})\,\mathbf{j} + (9\ \text{in/s}^2)\,\mathbf{j}$$

$$\mathbf{a}_{A/\mathcal{F}} = -(13.5\ \text{in./s}^2)\,\mathbf{i} + (9\ \text{in/s}^2)\,\mathbf{j}$$

$\mathbf{a}_c$: CORIOLIS ACCELERATION DUE TO A MOVING WITH VELOCITY $\mathbf{v}_{A/\mathcal{F}}$

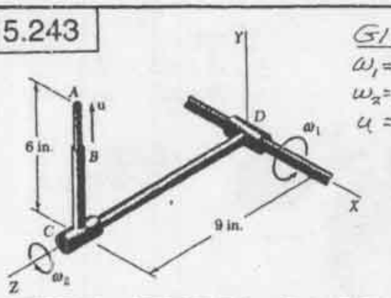
$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{A/\mathcal{F}} = 2(1.2\ \text{rad/s})\,\mathbf{i} \times [(9\ \text{in./s})\mathbf{j} + (3\ \text{in./s})\mathbf{i}]$$

$$\mathbf{a}_c = (21.6\ \text{in./s}^2)\,\mathbf{k}$$

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/\mathcal{F}} + \mathbf{a}_c$$
$$= -(12.96\ \text{in./s}^2)\mathbf{k} - (13.5\ \text{in./s}^2)\mathbf{i} + (9\ \text{in/s}^2)\mathbf{j} + (21.6\ \text{in/s}^2)\mathbf{k}$$

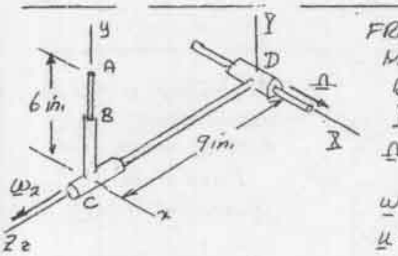
$$\mathbf{a}_A = -(13.5\ \text{in/s}^2)\,\mathbf{i} + (9\ \text{in/s}^2)\,\mathbf{j} + (8.64\ \text{in./s}^2)\,\mathbf{k}$$

15.243



GIVEN:
$\omega_1 = 1.2\ \text{rad/s},\ \alpha_1 = 0$
$\omega_2 = 1.5\ \text{rad/s},\ \alpha_2 = 0$
$u = 3\ \text{in./s},\ \dot{u} = 0$

FIND: $\mathbf{v}_A$ AND $\mathbf{a}_A$



FRAME DXYZ IS FIXED. MOVING FRAME Cxyz ROTATES ABOUT THE X AXIS WITH $\mathbf{\Omega} = \omega_1 \mathbf{i} = (1.2\ \text{rad/s})\,\mathbf{i}$.

$\boldsymbol{\omega}_2 = \omega_2 \mathbf{k} = -(1.5\ \text{rad/s})\,\mathbf{k}$
$\mathbf{u} = u\,\mathbf{j} = (3\ \text{in./s})\,\mathbf{i}$

$\mathbf{r}_{A/D} = (6\ \text{in.})\mathbf{j} + (9\ \text{in.})\mathbf{k} \qquad \mathbf{r}_{A/C} = (6\ \text{in.})\mathbf{j}$

VELOCITY:

$$\mathbf{v}_{A'} = \mathbf{\Omega} \times \mathbf{r}_{A/D} = (1.2\ \text{rad/s})\,\mathbf{i} \times [(6\ \text{in.})\mathbf{j} + (9\ \text{in.})\mathbf{k}]$$

$$\mathbf{v}_{A'} = (7.2\ \text{in./s})\mathbf{k} - (10.8\ \text{in./s})\,\mathbf{j}$$

$$\mathbf{v}_{A/\mathcal{F}} = \boldsymbol{\omega}_2 \times \mathbf{r}_{A/C} = -(1.5\ \text{rad/s})\mathbf{k} \times (6\ \text{in.})\,\mathbf{j} + (3\ \text{in./s})\,\mathbf{j}$$

$$\mathbf{v}_{A/\mathcal{F}} = +(9\ \text{in./s})\,\mathbf{i} + (3\ \text{in./s})\,\mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/\mathcal{F}} = (7.2\ \text{in./s})\mathbf{k} - (10.8\ \text{in./s})\mathbf{j} - (9\ \text{in./s})\mathbf{i} + (3\ \text{in/s})\mathbf{j}$$

$$\mathbf{v}_A = +(9\ \text{in./s})\,\mathbf{i} - (7.8\ \text{in./s})\,\mathbf{j} + (7.2\ \text{in/s})\,\mathbf{k}$$

ACCELERATION:

$$\mathbf{a}_{A'} = \mathbf{\Omega} \times \mathbf{\Omega} \times \mathbf{r}_{A/D} = \mathbf{\Omega} \times \mathbf{v}_{A'} = (1.2\ \text{rad/s})\,\mathbf{i} \times [(7.2\ \text{in./s})\mathbf{k} - (10.8\ \text{in./s})\mathbf{j}]$$

$$\mathbf{a}_A = -(8.64\ \text{in./s}^2)\,\mathbf{j} - (12.96\ \text{in/s}^2)\,\mathbf{k}$$

$\mathbf{a}_{A/\mathcal{F}}$: NOTE SINCE POINT A MOVES IN THE ROTATING FRAME Cxyz THERE IS A CORIOLIS ACCELERATION

$$\mathbf{a}_{A/\mathcal{F}} = \boldsymbol{\omega}_2 \times \boldsymbol{\omega}_2 \times \mathbf{r}_{A/C} + 2\boldsymbol{\omega}_2 \times \mathbf{u}$$
$$= \boldsymbol{\omega}_2 \times (1.5\ \text{rad/s})\mathbf{k} \times (6\ \text{in.})\,\mathbf{j} + 2(1.5\ \text{rad/s})\mathbf{k} \times (3\ \text{rad/s})\,\mathbf{j}$$
$$= (1.5\ \text{rad/s})\mathbf{k} \times (-9\ \text{in./s})\,\mathbf{i} - (9\ \text{in./s}^2)\,\mathbf{i}$$

$$\mathbf{a}_{A/\mathcal{F}} = -(13.5\ \text{in./s}^2)\,\mathbf{j} - (9\ \text{in./s}^2)\,\mathbf{i}$$

$\mathbf{a}_c$: CORIOLIS ACCELERATION DUE TO A MOVING WITH VELOCITY $\mathbf{v}_{A/\mathcal{F}}$

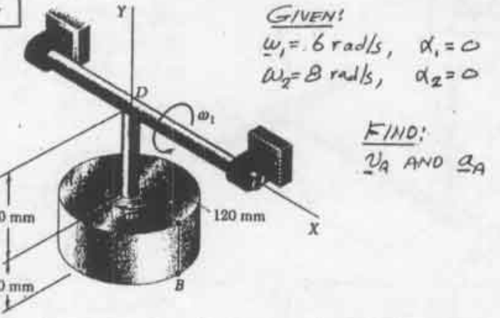
$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{A/\mathcal{F}} = 2(1.2\ \text{rad/s})\,\mathbf{i} \times [+(9\ \text{in/s})\,\mathbf{i} + (3\ \text{in/s})\,\mathbf{j}]$$

$$\mathbf{a}_c = (7.2\ \text{in/s}^2)\,\mathbf{k}$$

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/\mathcal{F}} + \mathbf{a}_c$$
$$= -(8.64\ \text{in./s})\,\mathbf{j} - (12.96\ \text{in./s}^2)\mathbf{k} - (13.5\ \text{in./s}^2)\,\mathbf{j}$$
$$+ (9\ \text{in./s}^2)\,\mathbf{i} + (7.2\ \text{in/s}^2)\,\mathbf{k}$$

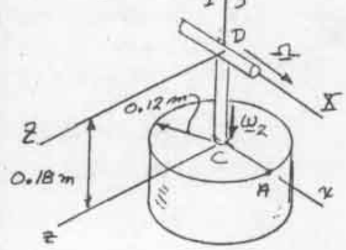
$$\mathbf{a}_A = +(9\ \text{in./s}^2)\,\mathbf{i} - (22.1\ \text{in/s}^2)\,\mathbf{j} - (5.76\ \text{in./s}^2)\,\mathbf{k}$$

15.244



GIVEN:
$\omega_1 = 6$ rad/s, $\alpha_1 = 0$
$\omega_2 = 8$ rad/s, $\alpha_2 = 0$

FIND:
$\mathbf{v}_A$ AND $\mathbf{a}_A$



FRAME DXYZ IS FIXED.
MOVING FRAME, Cxyz, ROTATES ABOUT THE Y AXIS WITH

$$\boldsymbol{\Omega} = \omega_1 \mathbf{i} = (6 \text{ rad/s})\mathbf{i}$$

$$\boldsymbol{\omega}_2 = \omega_2 \mathbf{j} = -(8 \text{ rad/s})\mathbf{j}$$

$$\mathbf{r}_{A/D} = +(0.12 \text{ m})\mathbf{i} - (0.18 \text{ m})\mathbf{j} \qquad \mathbf{r}_{A/C} = (0.12 \text{ m})\mathbf{i}$$

VELOCITY:

$$\mathbf{v}_{A'} = \boldsymbol{\Omega} \times \mathbf{r}_{A/D} = (6 \text{ rad/s})\mathbf{i} \times [+(0.12 \text{ m})\mathbf{i} - (0.18 \text{ m})\mathbf{j}]$$

$$\mathbf{v}_{A'} = -(1.08 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_{A/\mathcal{F}} = \boldsymbol{\omega}_2 \times \mathbf{r}_{A/C} = -(8 \text{ rad/s})\mathbf{j} \times (0.12 \text{ m})\mathbf{i} = (0.96 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/\mathcal{F}} = -(1.08 \text{ m/s})\mathbf{k} + (0.96 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_A = -(0.12 \text{ m/s})\mathbf{k} \quad ◀$$

ACCELERATION:

$$\mathbf{a}_{A'} = \boldsymbol{\Omega} \times \mathbf{v}_{A'} = (6 \text{ rad/s})\mathbf{i} \times (-1.08 \text{ m/s})\mathbf{k}$$

$$\mathbf{a}_{A'} = (6.48 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_{A/\mathcal{F}} = \boldsymbol{\omega}_2 \times \mathbf{v}_{A/\mathcal{F}} = -(8 \text{ rad/s})\mathbf{j} \times (0.96 \text{ m/s})\mathbf{k}$$

$$\mathbf{a}_{A/\mathcal{F}} = -(7.68 \text{ m/s}^2)\mathbf{i}$$

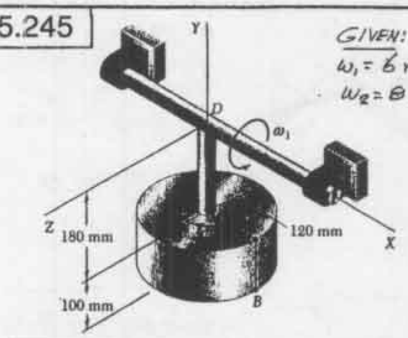
$$\mathbf{a}_c = 2\boldsymbol{\Omega} \times \mathbf{v}_{A/\mathcal{F}} = 2(6 \text{ rad/s})\mathbf{i} \times (0.96 \text{ m/s})\mathbf{k}$$

$$\mathbf{a}_c = -(11.52 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/\mathcal{F}} + \mathbf{a}_c$$
$$= (6.48 \text{ m/s}^2)\mathbf{j} - (7.68 \text{ m/s}^2)\mathbf{i} - (11.52 \text{ m/s}^2)\mathbf{j}$$

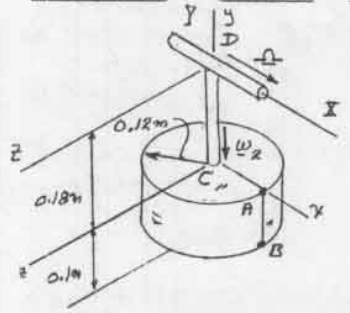
$$\mathbf{a}_A = -(7.68 \text{ m/s}^2)\mathbf{i} - (5.04 \text{ m/s}^2)\mathbf{j} \quad ◀$$

15.245



GIVEN:
$\omega_1 = 6$ rad/s, $\alpha_1 = 0$
$\omega_2 = 8$ rad/s, $\alpha_2 = 0$

FIND:
$\mathbf{v}_B$ AND $\mathbf{a}_B$



FRAME DXYZ IS FIXED.
MOVING FRAME, Cxyz, ROTATES ABOUT THE Y AXIS WITH

$$\boldsymbol{\Omega} = \omega_1 \mathbf{i} = (6 \text{ rad/s})\mathbf{i}$$

$$\boldsymbol{\omega}_2 = \omega_2 \mathbf{j} = -(8 \text{ rad/s})\mathbf{j}$$

$$\mathbf{r}_{B/D} = +(0.12 \text{ m})\mathbf{i} - (0.28 \text{ m})\mathbf{j} \qquad \mathbf{r}_{B/C} = (0.12 \text{ m})\mathbf{i} - (0.1 \text{ m})\mathbf{j}$$

VELOCITY:

$$\mathbf{v}_{B'} = \boldsymbol{\Omega} \times \mathbf{r}_{B/D} = (6 \text{ rad/s})\mathbf{i} \times [(0.12 \text{ m})\mathbf{i} - (0.28 \text{ m})\mathbf{j}]$$

$$\mathbf{v}_{B'} = -(1.68 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_{B/\mathcal{F}} = \boldsymbol{\omega}_2 \times \mathbf{r}_{B/C} = -(8 \text{ rad/s})\mathbf{j} \times [(0.12 \text{ m})\mathbf{i} - (0.1 \text{ m})\mathbf{j}]$$

$$\mathbf{v}_{B/\mathcal{F}} = (0.96 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/\mathcal{F}} = -(1.68 \text{ m/s})\mathbf{k} + (0.96 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_B = -(0.72 \text{ m/s})\mathbf{k} \quad ◀$$

ACCELERATION:

$$\mathbf{a}_{B'} = \boldsymbol{\Omega} \times \mathbf{v}_{B'} = (6 \text{ rad/s})\mathbf{i} \times (-1.68 \text{ m/s})\mathbf{k}$$

$$\mathbf{a}_{B'} = (10.08 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_{B/\mathcal{F}} = \boldsymbol{\omega}_2 \times \mathbf{v}_{B/\mathcal{F}} = -(8 \text{ rad/s})\mathbf{j} \times (0.96 \text{ m/s})\mathbf{k}$$

$$\mathbf{a}_{B/\mathcal{F}} = -(7.68 \text{ m/s}^2)\mathbf{i}$$

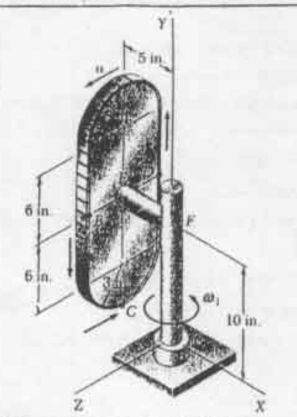
$$\mathbf{a}_c = 2\boldsymbol{\Omega} \times \mathbf{v}_{B/\mathcal{F}} = 2(6 \text{ rad/s})\mathbf{i} \times (0.96 \text{ m/s})\mathbf{k}$$

$$\mathbf{a}_c = -(11.52 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/\mathcal{F}} + \mathbf{a}_c$$
$$= (10.08 \text{ m/s}^2)\mathbf{j} - (7.68 \text{ m/s}^2)\mathbf{i} - (11.52 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_B = -(7.68 \text{ m/s}^2)\mathbf{i} - (1.44 \text{ m/s}^2)\mathbf{j} \quad ◀$$

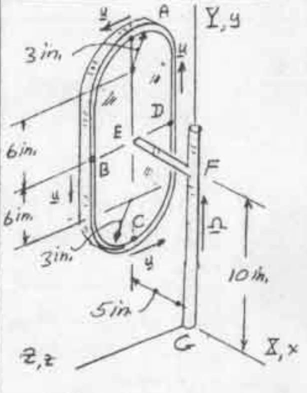
15.246 and 15.247



GIVEN:
$\omega_1 = 1.6$ rad/s, $\alpha_1 = 0$
LINK BELT MOVES AROUND PERIMETER AT CONSTANT SPEED $u = 4.5$ in./s.

PROBLEM 15.246
FIND: (a) $\underline{a}_A$
(b) $\underline{a}_B$

PROBLEM 15.247
FIND: (a) $\underline{a}_C$
(b) $\underline{a}_D$



FRAME GXYZ IS FIXED. MOVING FRAME, Gxyz, ROTATES ABOUT THE Y AXIS WITH
$\underline{\Omega} = \omega_1 \underline{j} = (1.6 \text{ rad/s})\underline{j}$

PROBLEM 15.246: (a) POINT A: $\underline{u} = (4.5 \text{ in./s})\underline{k}$

$\underline{r}_{A/G} = -(5 \text{ in.})\underline{i} + (19 \text{ in.})\underline{j}$

$\underline{v}_{A'} = \underline{\Omega} \times \underline{r}_{A/G} = (1.6 \text{ rad/s})\underline{j} \times [-(5 \text{ in.})\underline{i} + (19 \text{ in.})\underline{j}] = (8 \text{ in/s})\underline{k}$

$\underline{v}_{A/\mathcal{F}} = \underline{u} = (4.5 \text{ in./s})\underline{k}$

$\underline{v}_A = \underline{v}_{A'} + \underline{v}_{A/\mathcal{F}} = (8 \text{ in./s})\underline{k} + (4.5 \text{ in./s})\underline{k}$

$\underline{v}_A = (12.5 \text{ in./s})\underline{k}$

$\underline{a}_{A'} = \underline{\Omega} \times \underline{v}_{A/\mathcal{F}} = (1.6 \text{ rad/s})\underline{j} \times (8 \text{ in./s})\underline{k} = (12.80 \text{ in./s}^2)\underline{i}$

$\underline{a}_{A/\mathcal{F}} = -\dfrac{u^2}{r}\underline{j} = -\dfrac{(4.5 \text{ in./s})^2}{(3 \text{ in.})}\underline{j} = -(6.75 \text{ in./s}^2)\underline{j}$

$\underline{a}_c = 2\underline{\Omega} \times \underline{u} = 2(1.6 \text{ rad/s})\underline{j} \times (4.5 \text{ in./s})\underline{k} = (14.4 \text{ in./s}^2)\underline{i}$

$\underline{a}_A = \underline{a}_{A'} + \underline{a}_{A/\mathcal{F}} + \underline{a}_c$

$\underline{a}_A = (12.80 \text{ in./s}^2)\underline{i} - (6.75 \text{ in./s}^2)\underline{j} + (14.4 \text{ in./s}^2)\underline{i}$

$\underline{a}_A = (27.2 \text{ in./s}^2)\underline{i} - (6.75 \text{ in./s}^2)\underline{j}$ ◀

(CONTINUED)

15.246 and 15.247 continued

PROBLEM 15.246: (b) POINT B: $\underline{u} = -(4.5 \text{ in./s})\underline{j}$

$\underline{r}_{B/G} = -(5 \text{ in.})\underline{i} + (10 \text{ in.})\underline{j} + (3 \text{ in.})\underline{k}$

$\underline{v}_{B'} = \underline{\Omega} \times \underline{r}_{B/G} = (1.6 \text{ in./s})\underline{j} \times [-(5 \text{ in.})\underline{i} + (10 \text{ in.})\underline{j} + (3 \text{ in.})\underline{k}]$

$\underline{v}_{B'} = (8 \text{ in./s})\underline{k} + (4.8 \text{ in./s})\underline{i}$

$\underline{v}_{B/\mathcal{F}} = \underline{u} = -(4.5 \text{ in./s})\underline{j}$

$\underline{v}_B = \underline{v}_{B'} + \underline{v}_{B/\mathcal{F}} = (8 \text{ in./s})\underline{k} + (4.8 \text{ in./s})\underline{i} - (4.5 \text{ in./s})\underline{j}$

$\underline{v}_B = +(4.8 \text{ in./s})\underline{i} - (4.5 \text{ in./s})\underline{j} + (8 \text{ in./s})\underline{k}$

$\underline{a}_{B'} = \underline{\Omega} \times \underline{v}_{B'} = (1.6 \text{ in./s})\underline{j} \times [(8 \text{ in./s})\underline{k} + (4.8 \text{ in./s})\underline{i}]$

$\underline{a}_{B'} = (12.8 \text{ in./s}^2)\underline{i} - (7.68 \text{ in./s}^2)\underline{k}$

$\underline{a}_{B/\mathcal{F}} = 0$

$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{B/\mathcal{F}} = 2(1.6 \text{ in./s})\underline{j} \times (-4.5 \text{ in./s})\underline{j} = 0$

$\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/\mathcal{F}} + \underline{a}_c = (12.8 \text{ in./s}^2)\underline{i} - (7.68 \text{ in./s}^2)\underline{k} + 0 + 0$

$\underline{a}_B = (12.8 \text{ in./s}^2)\underline{i} - (7.68 \text{ in./s}^2)\underline{k}$ ◀

PROBLEM 15.247 (a) POINT C: $\underline{u} = -(4.5 \text{ in./s})\underline{k}$

$\underline{r}_{C/G} = -(5 \text{ in./s})\underline{i} + (1 \text{ in.})\underline{j}$

$\underline{v}_{C'} = \underline{\Omega} \times \underline{r}_{C/G} = (1.6 \text{ in./s})\underline{j} \times [-(5 \text{ in.})\underline{i} + (1 \text{ in})\underline{j}] = (8 \text{ in./s})\underline{k}$

$\underline{v}_{C/\mathcal{F}} = \underline{u} = -(4.5 \text{ in./s})\underline{k}$

$\underline{v}_C = \underline{v}_{C'} + \underline{v}_{C/\mathcal{F}} = (8 \text{ in./s})\underline{k} - (4.5 \text{ in./s})\underline{k}$

$\underline{v}_C = (3.5 \text{ in./s})\underline{k}$

$\underline{a}_{C'} = \underline{\Omega} \times \underline{v}_{C'} = (1.6 \text{ in./s})\underline{j} \times (8 \text{ in./s})\underline{k} = (12.80 \text{ in./s}^2)\underline{i}$

$\underline{a}_{C/\mathcal{F}} = \dfrac{u^2}{r}\underline{j} = \dfrac{(4.5 \text{ in./s})^2}{(3 \text{ in./s})}\underline{j} = (6.75 \text{ in./s}^2)\underline{j}$

$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{C/\mathcal{F}} = 2(1.6 \text{ in./s})\underline{j} \times (-4.5 \text{ in./s})\underline{k} = -(14.40 \text{ in./s}^2)\underline{i}$

$\underline{a}_C = \underline{a}_{C'} + \underline{a}_{C/\mathcal{F}} + \underline{a}_c$

$\underline{a}_C = (12.80 \text{ in./s}^2)\underline{i} + (6.75 \text{ in./s}^2)\underline{j} - (14.40 \text{ in./s}^2)\underline{i}$

$\underline{a}_C = -(1.6 \text{ in./s}^2)\underline{i} + (6.75 \text{ in./s}^2)\underline{j}$ ◀

(b) POINT D: $\underline{u} = (4.5 \text{ in./s})\underline{j}$

$\underline{r}_{D/G} = -(5 \text{ in.})\underline{i} + (10 \text{ in.})\underline{j} - (3 \text{ in.})\underline{k}$

$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/G} = (1.6 \text{ in./s})\underline{j} \times [-(5 \text{ in.})\underline{i} + (10 \text{ in.})\underline{j} - (3 \text{ in.})\underline{k}]$

$\underline{v}_{D'} = (8 \text{ in./s})\underline{k} - (4.8 \text{ in./s})\underline{i}$

$\underline{v}_{D/\mathcal{F}} = \underline{u} = (4.5 \text{ in./s})\underline{j}$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/\mathcal{F}} = (8 \text{ in./s})\underline{k} - (4.8 \text{ in./s})\underline{i} + (4.5 \text{ in./s})\underline{j}$

$\underline{v}_D = -(4.8 \text{ in./s})\underline{i} + (4.5 \text{ in./s})\underline{j} + (8 \text{ in./s})\underline{k}$

$\underline{a}_{D'} = \underline{\Omega} \times \underline{v}_{D'} = (1.6 \text{ rad/s})\underline{j} \times [(8 \text{ in./s})\underline{k} - (4.8 \text{ in./s})\underline{i}]$

$\underline{a}_{D'} = (12.8 \text{ in./s}^2)\underline{i} + (7.68 \text{ in./s}^2)\underline{k}$

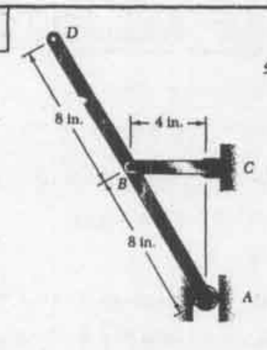
$\underline{a}_{D/\mathcal{F}} = 0$

$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{D/\mathcal{F}} = 2(1.6 \text{ rad/s})\underline{j} \times (4.5 \text{ in./s})\underline{j} = 0$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c = (12.8 \text{ in./s}^2)\underline{i} + (7.68 \text{ in./s}^2)\underline{k} + 0 + 0$

$\underline{a}_D = (12.8 \text{ in./s}^2)\underline{i} + (7.68 \text{ in./s}^2)\underline{k}$ ◀

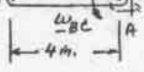
15.248



GIVEN:
$\omega_{AB} = 45\,\text{rpm}$ ↻, α_{AB}

FIND:
(a) $\underline{a}_A$
(b) $\underline{a}_D$

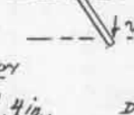
CRANK BC: $\omega_{BC} = (45\,\text{rpm})\frac{2\pi}{60} = 4.7124\,\text{rad/s}$



$a_B = (BC)\omega_{BC}^2 = (4\,\text{in.})(4.7124\,\text{rad/s})^2$

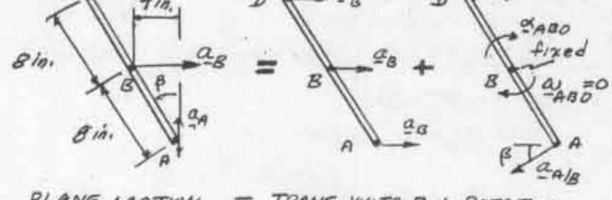
$\underline{a}_B = 88.83\,\text{in./s}^2 \rightarrow$

BAR ABD:
VELOCITY:



INST. CENTER AT ∞
THUS, $\omega_{ABD} = 0$

ACCELERATION

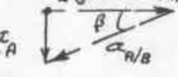


PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

$\beta = \sin^{-1}\frac{4\,\text{in.}}{8\,\text{in.}} = 30°$

$\underline{a}_A = \underline{a}_B + \underline{a}_{A/B}$

$a_A \updownarrow = [88.83\,\text{in./s}^2 \rightarrow] + a_{A/B} \angle \beta$



$\underline{a}_{A/B} = (88.83\,\text{in/s}^2)/\cos 30°$

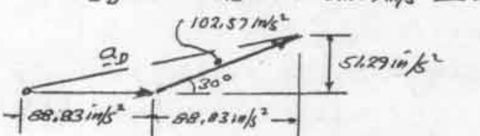
$\underline{a}_{A/B} = 102.57\,\text{in/s}^2 \angle 30°$

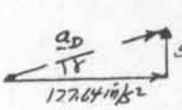
$\underline{a}_A = (102.57)\sin 30° = 51.3\,\text{in/s}^2 \downarrow$ ◀

$a_{A/B} = (AB)\alpha_{ABD} = (8\,\text{in.})\alpha_{ABD}$
$a_{D/B} = (BD)\alpha_{ABD} = (8\,\text{in.})\alpha_{ABD}$ } NOTE $a_{A/D} = a_{A/B}$

POINT D: $\underline{a}_D = \underline{a}_B + \underline{a}_{D/B} = \underline{a}_B + \underline{a}_{D/B} \angle 30°$

$\underline{a}_D = 88.83\,\text{in/s}^2 \rightarrow + 102.57\,\text{in/s}^2 \angle 30°$





$a_D = 184.89\,\text{in/s}^2$
$\gamma = 16.1°$

$\underline{a}_D = 184.9\,\text{in/s}^2 \angle 16.1°$ ◀

15.249

GIVEN: ROTOR IN UNIFORMLY ACCELERATED MOTION
$t = 0$, $\omega_0 = 1800\,\text{rpm}$, $\theta = 0$
$\omega = 0$, $\theta = 1550$ revolutions

FIND: (a) α, (b) t REQUIRED TO COME TO REST.

$\omega_0 = 1800\,\text{rpm}\left(\frac{2\pi}{60}\right) = 188.50\,\text{rad/s}$

$\theta = 1550\,\text{rev}(2\pi) = 9739\,\text{rad}$

(a) ANGULAR ACCELERATION: (USE LAST OF EQS. 15.16)

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

$0 = (188.5\,\text{rad/s})^2 + 2\alpha(9739\,\text{rad} - 0)$

$\alpha = -1.824\,\text{rad/s}^2$ ◀

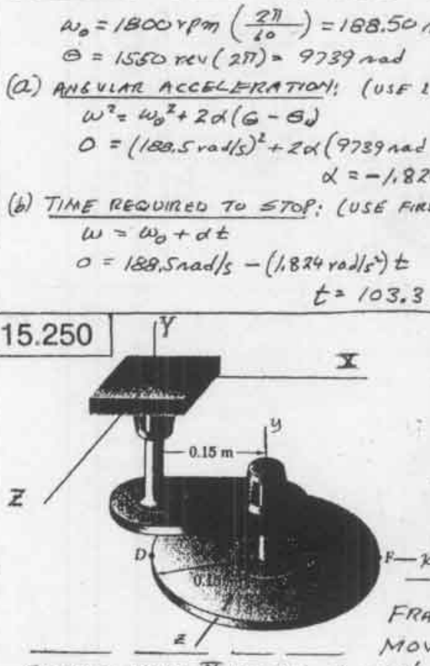
(b) TIME REQUIRED TO STOP: (USE FIRST OF EQS. 15.16)

$\omega = \omega_0 + \alpha t$

$0 = 188.5\,\text{rad/s} - (1.824\,\text{rad/s}^2)t$

$t = 103.3\,\text{s}$ ◀

15.250



GIVEN:
$\omega_1 = \omega_2 = 3\,\text{rad/s}$
$\alpha_1 = \alpha_2 = 0$

FIND:
(a) $\underline{v}_D$ AND $\underline{a}_D$
(b) $\underline{v}_F$ AND $\underline{a}_F$

FRAME AXYZ IS FIXED.
MOVING FRAME, Exyz, ROTATES ABOUT Y AXIS AT $\underline{\Omega} = \omega_1\underline{j} = (3\,\text{rad/s})\underline{j}$.

(a) POINT D: $\underline{\omega}_2 = \omega_2\underline{j} = (3\,\text{rad/s})\underline{j}$

$\underline{r}_{D/A} = 0$; $\underline{r}_{D/E} = -(0.15\,\text{m})\underline{i}$

$\underline{v}_{D'} = \underline{\Omega} \times \underline{r}_{D/A} = 0$

$\underline{v}_{D/\mathcal{F}} = \underline{\omega}_2 \times \underline{r}_{D/E} = (3\,\text{rad/s})\underline{j} \times (-0.15\,\text{m})\underline{i} = (0.45\,\text{m/s})\underline{k}$

$\underline{v}_D = \underline{v}_{D'} + \underline{v}_{D/\mathcal{F}}$; $\underline{v}_D = (0.45\,\text{m/s})\underline{k}$ ◀

$\underline{a}_{D'} = \underline{\Omega} \times \underline{v}_{D'} = 0$

$\underline{a}_{D/\mathcal{F}} = \underline{\omega}_2 \times \underline{v}_{D/\mathcal{F}} = (3\,\text{rad/s})\underline{j} \times (0.45\,\text{m/s})\underline{k} = (1.35\,\text{m/s}^2)\underline{i}$

$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{D/\mathcal{F}} = 2(3\,\text{rad/s})\underline{j} \times (0.45\,\text{m/s})\underline{k} = (2.70\,\text{m/s}^2)\underline{i}$

$\underline{a}_D = \underline{a}_{D'} + \underline{a}_{D/\mathcal{F}} + \underline{a}_c = 0 + (1.35\,\text{m/s}^2)\underline{i} + (2.70\,\text{m/s}^2)\underline{i}$

$\underline{a}_D = (4.05\,\text{m/s}^2)\underline{i}$ ◀

(b) POINT F: $\underline{\omega}_2 = \omega_2\underline{j} = (3\,\text{rad/s})\underline{j}$

$\underline{r}_{F/A} = (0.3\,\text{m})\underline{i}$; $\underline{r}_{F/E} = (0.15\,\text{m})\underline{i}$

$\underline{v}_{F'} = \underline{\Omega} \times \underline{r}_{F/A} = (3\,\text{rad/s})\underline{j} \times (0.3\,\text{m})\underline{i} = -(0.9\,\text{m/s})\underline{k}$

$\underline{v}_{F/\mathcal{F}} = \underline{\omega}_2 \times \underline{r}_{F/E} = (3\,\text{rad/s})\underline{j} \times (0.15\,\text{m})\underline{i} = -(0.45\,\text{m/s})\underline{k}$

$\underline{v}_F = \underline{v}_{F'} + \underline{v}_{F/\mathcal{F}} = -(0.9\,\text{m/s})\underline{k} - (0.45\,\text{m/s})\underline{k}$

$\underline{v}_F = -(1.35\,\text{m/s})\underline{k}$ ◀

$\underline{a}_{F'} = \underline{\Omega} \times \underline{v}_{F'} = (3\,\text{rad/s})\underline{j} \times (-0.9\,\text{m/s})\underline{k} = -(2.7\,\text{m/s}^2)\underline{i}$

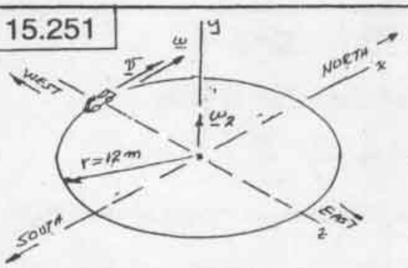
$\underline{a}_{F/\mathcal{F}} = \underline{\omega}_2 \times \underline{v}_{F/\mathcal{F}} = (3\,\text{rad/s})\underline{j} \times (-0.45\,\text{m/s})\underline{k} = -(1.35\,\text{m/s}^2)\underline{i}$

$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{F/\mathcal{F}} = 2(3\,\text{rad/s})\underline{j} \times (-0.45\,\text{m/s})\underline{k} = -(2.7\,\text{m/s}^2)\underline{i}$

$\underline{a}_F = \underline{a}_{F'} + \underline{a}_{F/\mathcal{F}} + \underline{a}_c$
$= -(2.7\,\text{m/s}^2)\underline{i} - (1.35\,\text{m/s}^2)\underline{i} - (2.7\,\text{m/s}^2)\underline{i}$

$\underline{a}_F = -(6.75\,\text{m/s}^2)\underline{i}$ ◀

15.251



GIVEN:
AUTO: $\underline{v} = (12\ km/h)\,\underline{i}$
FAN: $\underline{\omega} = (2500\,rpm)\,\underline{i}$

FIND: α OF FAN

$\underline{v} = (12\ km/h)\,\frac{1000\,m}{km}\cdot\frac{h}{3600\,s}\,\underline{i} = (3.333\,m/s)\,\underline{i}$

$\underline{\omega} = (2500\,rpm)\,\frac{2\pi\,rad}{rev}\cdot\frac{min}{60\,s} = (261.8\,rad/s)\,\underline{i}$

ω_2 = ANGULAR VELOCITY OF AUTO

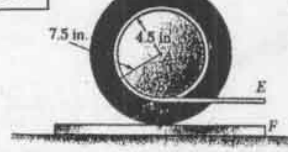
$\omega_2 = -\left(\frac{v}{r}\right)\underline{j} = -\left(\frac{3.333\,m/s}{12\,m}\right)\underline{j} = -(0.2778\,rad/s)\,\underline{j}$

$\underline{\alpha} = \underline{\omega}_2 \times \underline{\omega}_1 = (-0.2778\,rad/s)\,\underline{j} \times (261.8\,rad/s)\,\underline{i}$

$\underline{\alpha} = (72.7\,rad/s^2)\,\underline{k}$

$\underline{\alpha} = 72.7\,rad/s^2$ TOWARD THE EAST ◀

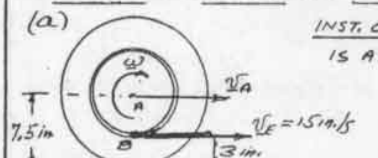
15.252



GIVEN: $v_F = 0$
$\underline{v}_E = 15\ in/s \rightarrow$, $\underline{a}_E = 0$

FIND: (a) $\underline{v}_A$
(b) $\underline{a}_D$

(a)

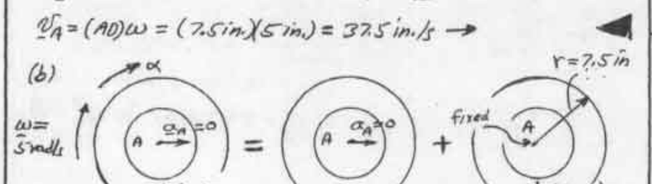


INST. CENTER OF ROTATION IS AT POINT D.

$v_E = (BD)\omega$; $15\,in/s = (3\,in.)\,\omega$; $\underline{\omega} = 5\,rad/s$ ↻

$\underline{v}_A = (AD)\omega = (7.5\,in.)(5\,in.) = 37.5\,in./s \rightarrow$ ◀

(b)



PLANE MOTION = TRANS. WITH A + ROTATION ABOUT A

$a_D = a_A + a_{D/A}$

$(a_D)_t + (a_D)_n\uparrow = a_A \rightarrow + (a_{D/A})_t \leftarrow + (a_{D/A})_n \uparrow$

$0 + (a_D)_n\uparrow = 0 + r\alpha \leftarrow + r\omega^2\uparrow$

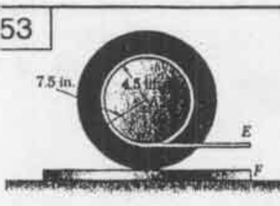
$\xrightarrow{+}$ x COMPONENTS: $r\alpha = 0$, $\alpha = 0$

+↑ y COMPONENTS: $(a_D)_n = r\omega^2 = (7.5\,in.)(5\,in/s)^2$

$(\underline{a}_D)_n = 187.5\,in/s^2\uparrow$

$a_D = (a_D)_n$: $\underline{a}_D = 187.5\,in./s^2\uparrow$ ◀

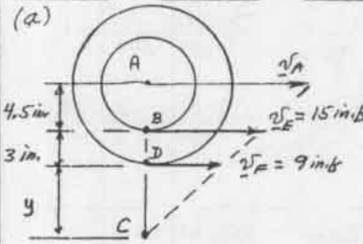
15.253



GIVEN: $\underline{v}_F = 9\,in/s \rightarrow$
$\underline{v}_E = 15\,in/s \rightarrow$
$a_E = a_F = 0$

FIND: (a) $\underline{v}_D$
(b) $\underline{a}_D$

(a)



INST. CENTER OF ROTATION IS AT POINT C.

SIMILAR TRIANGLES

$\frac{y}{9\,in/s} = \frac{y + 3\,in.}{15\,in/s}$

$15y = 9y + 27$

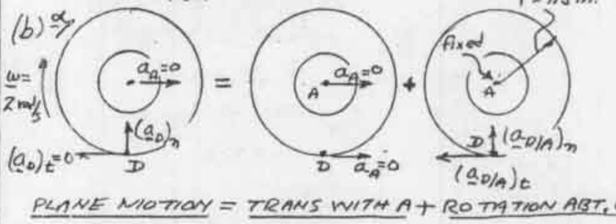
$y = 4.5\,in.$

$\frac{v_A}{AC} = \frac{v_F}{CD}$; $\frac{v_A}{(4.5+3+4.5)\,in.} = \frac{9\,in/s}{4.5\,in}$

$\underline{v}_A = 24\,in./s \rightarrow$ ◀

$\underline{\omega} = \frac{v_F}{y} = \frac{9\,in/s}{4.5\,in} = 2\,rad/s$ ↻

(b)



PLANE MOTION = TRANS WITH A + ROTATION ABT. A

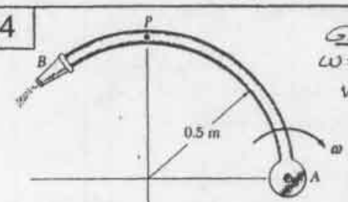
$a_D = a_A + a_{D/A}$: $(a_D)_t + (a_D)_n\uparrow = \underline{a}_A + (a_{D/A})_n\uparrow + (a_{D/A}) \leftarrow$

$0 + (a_D)_n\uparrow = 0 + r\omega^2\uparrow + r\alpha \leftarrow$

+↑ x COMPONENTS: $(a_D)_n = r\omega^2 = (7.5\,in.)(2\,rad/s)^2$

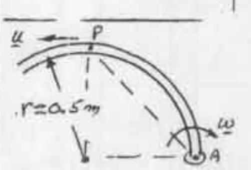
$a_D = (a_D)_n$: $\underline{a}_D = 30\,in./s^2\uparrow$ ◀

15.254



GIVEN:
$\omega = 90\,rpm$, $\alpha = 0$
VELOCITY OF WATER RELATIVE TO PIPE = 8 m/s

FIND: $\underline{a}_P$ OF WATER AT P.



$AP = (0.5\,m)\sqrt{2} = 0.707\,m$

$\underline{u} = 8\,m/s \leftarrow$

$\underline{\omega} = 90\,rpm\,\frac{2\pi}{60} = 9.425\,rad/s$ ↻

$\underline{a}_P = \underline{a}_{P'} + \underline{a}_{P/\mathcal{F}} + \underline{a}_c$

$\underline{a}_{P'} = (AP)\omega^2 \measuredangle 45° = (0.707\,m)(9.425\,rad/s)^2 = 62.8\,m/s^2 \measuredangle 45°$

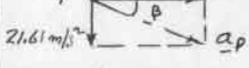
$\underline{a}_{P/\mathcal{F}} = u^2/r\downarrow = (8\,m/s)^2/0.5\,m\downarrow = 128\,m/s^2\downarrow$

$\underline{a}_c = 2\underline{\omega}\times\underline{u} = 2(9.425\,rad/s)(8\,m/s) = 150.8\,m/s^2\uparrow$

$a_P = 62.8\,m/s^2 \measuredangle 45° + 128\,m/s^2\downarrow + 150.8\,m/s^2\uparrow$

+↑ $(a_P)_y = -62.8\sin 45° - 128 + 150.8 = -21.61\,m/s^2$

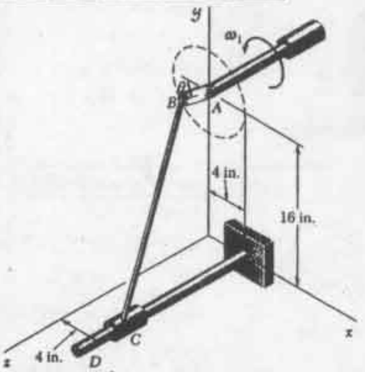
$\xrightarrow{+}$ $(a_P)_x = +62.8\cos 45° = 44.41\,m/s^2$



$\beta = 26.0°$ $a_P = 49.4\,m/s^2$

$\underline{a}_P = 49.4\,m/s^2 \measuredangle 26.0°$ ◀

15.255 and 15.256



GIVEN:
$\omega_1 = 10$ rad/s, $\alpha_1 = 0$
$AB = 4$ in.
$BC = 24$ in.

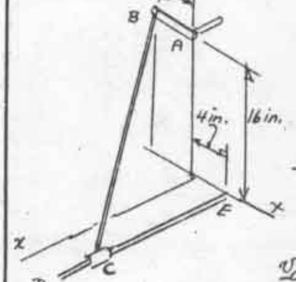
FIND: v_C

PROBLEM 15.255 WHEN $\theta = 0$

PROBLEM 15.256 WHEN $\theta = 90°$

PROBLEM 15.255 $\theta = 0$: $r_{B/A} = -(4\text{ in.})\underline{i}$

$\underline{v}_B = \omega_1 \times r_{B/A} = (10\text{ rad/s})\underline{k} \times (-4\text{ in.})\underline{i} = -(40\text{ in./s})\underline{j}$



$BC = 24$ in
$(BC)_x = 8$ in. $(BC)_y = 16$ in.
$BC^2 = (BC)_x^2 + (BC)_y^2 + (BC)_z^2$
$24^2 = 8^2 + 16^2 + (BC)_z^2$
$(BC)_z = 16$
$\underline{r}_{C/B} = (8\text{ in.})\underline{i} - (16\text{ in.})\underline{j} + (16\text{ in.})\underline{k}$
$\underline{v}_C = v_C\underline{k}$

ROD BC:
$\underline{v}_C = \underline{v}_B + \underline{v}_{C/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{C/B}$

$$v_C\underline{k} = -(40\text{ in/s})\underline{j} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 8 & -16 & 16 \end{vmatrix}$$

$$v_C\underline{k} = -40\underline{j} + (16\omega_y + 16\omega_z)\underline{i} + (8\omega_z - 16\omega_x)\underline{j} + (-16\omega_x - 8\omega_y)\underline{k}$$

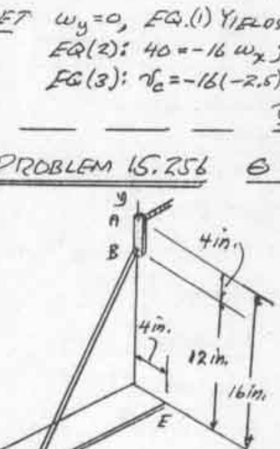
x COMPONENTS:	$0 = \quad 16\omega_y + 16\omega_z$	(1)
y COMPONENTS:	$40 = -16\omega_x \quad + 8\omega_z$	(2)
z COMPONENTS:	$v_C = -16\omega_x - 8\omega_y$	(3)

LET $\omega_y = 0$, EQ.(1) YIELDS: $\omega_z = 0$
EQ(2): $40 = -16\omega_x$; $\omega_x = -2.5$ rad/s
EQ(3): $v_C = -16(-2.5) = 40$

$\underline{v}_C = (40\text{ in./s})\underline{k}$ ◀

PROBLEM 15.256 $\theta = 90°$ $\underline{r}_{B/A} = -(4\text{ in.})\underline{j}$

$\underline{v}_B = \underline{\omega} \times \underline{r}_{B/A} = (10\text{ rad/s})\underline{k} \times (-4\text{ in.})\underline{j}$
$\underline{v}_B = (40\text{ in/s})\underline{i}$



$BC = 24$ in.
$(BC)_x = 4$ in.
$(BC)_y = 12$ in.
$BC^2 = (BC)_x^2 + (BC)_y^2 + (BC)_z^2$
$24^2 = 4^2 + 12^2 + (BC)_z^2$
$(BC)_z = 20.396$ in.
$\underline{r}_{C/B} = (4\text{ in.})\underline{i} - (12\text{ in.})\underline{j} + (20.396\text{ in.})\underline{k}$

(CONTINUED)

15.256 continued

$\underline{v}_C = v_C\underline{k}$

ROD BC: $\underline{v}_C = \underline{v}_B + \underline{v}_{C/B} = \underline{v}_B + \underline{\omega} \times \underline{r}_{C/B}$

$$v_C\underline{k} = (40\text{ in/s})\underline{i} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ 4 & -12 & 20.396 \end{vmatrix}$$

$$v_C\underline{k} = 40\underline{i} + (20.396\omega_y + 12\omega_z)\underline{i} + (4\omega_z - 20.396\omega_x)\underline{j} + (-12\omega_x - 4\omega_y)\underline{k}$$

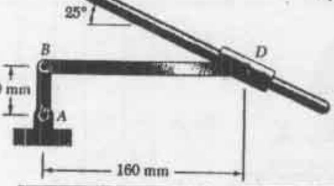
x COMPONENTS:	$-40 = \quad 20.396\omega_y + 12\omega_z$	(1)
y COMPONENTS:	$0 = -20.396\omega_x \quad + 4\omega_z$	(2)
z COMPONENTS:	$v_C = -12\omega_x - 4\omega_y$	(3)

LET: $\omega_z = 0$, EQ(2) YIELDS $\omega_x = 0$
EQ(1): $-40 = 20.396\omega_y$ $\quad \omega_y = -1.9612$ rad/s
EQ(3): $v_C = -4(-1.9612) = 7.8447$ in/s

$\underline{v}_C = (7.84\text{ in./s})\underline{k}$ ◀

15.257 and 15.258

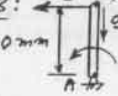
GIVEN:
$\underline{\omega}_{AB} = 1.5$ rad/s ↻, $\alpha_{AB} = 0$



PROBLEM 15.257
FIND: (a) $\underline{\omega}_{BD}$, (b) $\underline{v}_D$

PROBLEM 15.258
FIND: (a) $\underline{\alpha}_{BD}$, (b) $\underline{a}_D$

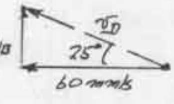
CRANK AB:



$v_B = (AB)\omega_{AB} = (40\text{ mm})(1.5\text{ rad/s})$
$\underline{v}_B = 60$ mm/s ←
$a_B = (AB)\omega_{AB}^2 = (40\text{ mm})(1.5\text{ rad/s})^2$
$\underline{a}_B = 90$ mm/s² ↓

PROBLEM 15.257:
ROD BD:

$\underline{v}_D = \underline{v}_B + \underline{v}_{D/B} = \underline{v}_B + (BD)\omega_{BD}$
$v_D \measuredangle 25° = 60$ mm/s ← $+ (160\text{ mm})\omega_{BD}$ ↕

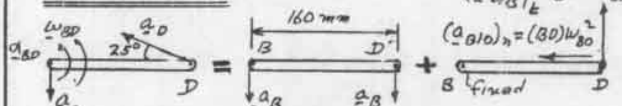


$v_{D/B} = 60 \tan 25° = 27.978$ mm/s ↑
$\omega_{BD} = \dfrac{27.978\text{ mm/s}}{160\text{ mm}} = 0.17487$ rad/s

$\underline{\omega}_{BD} = 0.1749$ rad/s ↻ ◀

$v_D = \dfrac{60\text{ mm/s}}{\cos 25°} = 66.202$ mm/s $\quad \underline{v}_D = 66.2$ mm/s $\measuredangle 25°$ ◀

PROBLEM 15.258



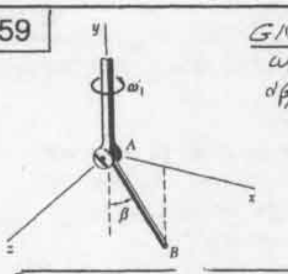
PLANE MOTION = TRANS. WITH B + ROTATION ABOUT B

$\underline{a}_D = \underline{a}_B + \underline{a}_{D/B}$
$a_D \measuredangle 25° = a_B \downarrow + (a_{D/B})_t \updownarrow + (a_{D/B})_n \leftarrow$
$a_D \measuredangle 25° = 90\text{ mm/s}^2 \downarrow + (160\text{ mm})\alpha_{BD} \updownarrow + (160\text{ mm})(0.17487\text{ rad/s})^2 \leftarrow$

$\pm$ COMPONENTS: $+a_D \cos 25° = (160\text{ mm})(0.17487\text{ rad/s})^2$
$a_D \cos 25° = 4.8925$ mm/s²
$a_D = 5.398$ mm/s²: $\quad \underline{a}_D = 5.40$ mm/s² $\measuredangle 25°$ ◀

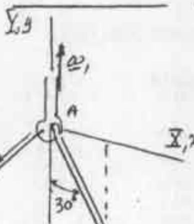
+↑ COMPONENTS: $+a_D \sin 25° = -90\text{ mm/s}^2 + (160\text{ mm})\alpha_{BD}$
$(5.398\text{ mm/s}^2)\sin 25° = -90\text{ mm/s}^2 + (160\text{ mm})\alpha_{BD}$
$\alpha_{BD} = +0.5768$ rad/s²
$\underline{\alpha}_{BD} = 0.577$ rad/s² ↺ ◀

15.259



GIVEN: $AB = 125\ mm$
$\omega_1 = 5\ rad/s,\ \alpha_1 = 0$
$d\beta/dt = 3\ rad/s,\ d^2\beta/dt^2 = 0$
$\beta = 30°$

FIND: $\underline{v}_B$ AND $\underline{a}_B$



FRAME AXYZ IS FIXED.
MOVING FRAME Axyz ROTATES ABOUT THE $\underline{Y}$ AXIS WITH
$\underline{\Omega} = \underline{\omega}_1 = (5\ rad/s)\underline{j}$

$\underline{\omega}_2 = (d\beta/dt)\underline{k} = (3\ rad/s)\underline{k}$

VELOCITY

$$\underline{r}_{B/A} = (0.125\ m)[\sin 30°\,\underline{i} - \cos 30°\,\underline{j}] = (0.0625\ m)\underline{i} - (0.10826\ m)\underline{j}$$

$$\underline{v}_{B'} = \underline{\Omega} \times \underline{r}_{B/A} = (5\ rad/s)\underline{j} \times [(0.0625\ m)\underline{i} - (0.10826\ m)\underline{j}]$$

$$\underline{v}_{B'} = -(0.3125\ m/s)\underline{k}$$

$$\underline{v}_{B/\mathcal{F}} = \underline{\omega}_2 \times \underline{r}_{B/A} = (3\ rad/s)\underline{k} \times [(0.0625\ m)\underline{i} - (0.10826\ m)\underline{j}]$$

$$\underline{v}_{B/\mathcal{F}} = +(0.1875\ m/s)\underline{j} + (0.32476\ m/s)\underline{i}$$

$$\underline{v}_B = \underline{v}_{B'} + \underline{v}_{B/\mathcal{F}} = -0.3125\,\underline{k} + 0.1875\,\underline{j} + 0.32476\,\underline{i}$$

$$\underline{v}_B = (0.325\ m/s)\underline{i} + (0.1875\ m/s)\underline{j} - (0.313\ m/s)\underline{k}$$ ◀

ACCELERATION

$$\underline{a}_{B'} = \underline{\Omega} \times \underline{v}_{B'} = (5\ rad/s)\underline{j} \times (-0.3125\ m/s)\underline{k}$$

$$\underline{a}_{B'} = -(1.5625\ m/s^2)\underline{i}$$

$$\underline{a}_{B/\mathcal{F}} = \underline{\omega}_2 \times \underline{v}_{B/\mathcal{F}} = (3\ rad/s)\underline{k} \times [(0.1875\ m/s)\underline{j} + (0.32476\ m/s)\underline{i}]$$

$$\underline{a}_{B/\mathcal{F}} = -(0.5625\ m/s^2)\underline{i} + (0.9743\ m/s^2)\underline{j}$$

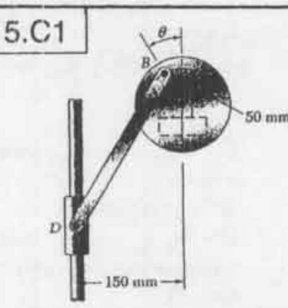
$$\underline{a}_c = 2\underline{\Omega} \times \underline{v}_{B/\mathcal{F}} = 2(5\ rad/s)\underline{j} \times [(0.1875\ m/s)\underline{j} + (0.32476\ m/s)\underline{i}]$$

$$\underline{a}_c = -(3.248\ m/s^2)\underline{k}$$

$$\underline{a}_B = \underline{a}_{B'} + \underline{a}_{B/\mathcal{F}} + \underline{a}_c$$
$$= -1.5625\,\underline{i} - 0.5625\,\underline{i} + 0.9743\,\underline{j} - 3.248\,\underline{k}$$

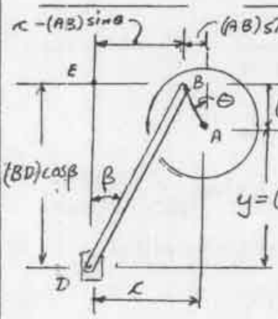
$$\underline{a}_B = -(2.13\ m/s^2)\underline{i} + (0.974\ m/s^2)\underline{j} - (3.25\ m/s^2)\underline{k}$$ ◀

15.C1



GIVEN:
$\omega_{DISK} = \omega = 500\ rpm$
$\dot{\omega} = 0$
$BD = 250\ mm$

FIND: (a) v_D AND ω_{BD} FOR $\theta = 0$ TO $360°$ USING $30°$ INCREMENTS.
(b) TWO VALUES OF θ FOR WHICH $v_D = 0$



ANALYSIS
SIGNS: $+y\downarrow$, $+\omega_{BD}$ ↻

$$y = (BD)\cos\beta - (AB)\cos\theta \qquad (1)$$

FROM ΔBDE

$$\beta = \sin^{-1}\left[\frac{c - (AB)\sin\theta}{BD}\right] \qquad (2)$$

FROM (2): $\sin\beta = \dfrac{c - (AB)\sin\theta}{BD}$

$\dfrac{d}{dt}$: $\cos\beta\,\dfrac{d\beta}{dt} = -\dfrac{AB}{BD}\cos\theta\,\dfrac{d\theta}{dt}$

BUT: $\dfrac{d\theta}{dt} = \omega$ AND $\dfrac{d\beta}{dt} = \omega_{BD}$

$$\omega_{BD} = -\frac{AB}{BD}\frac{\cos\theta}{\cos\beta}\,\omega \qquad (3)$$

FROM (1): $v_D = \dfrac{dy}{dt} = -(BD)\sin\beta\,\dfrac{d\beta}{dt} + (AB)\sin\theta\,\dfrac{d\theta}{dt}$

$$v_D = -(BD)\sin\beta\,\omega_{BD} + (AB)\sin\theta\,\omega \qquad (4)$$

DATA: $\dfrac{d\theta}{dt} = \omega = 500\ rpm = 500\left(\dfrac{2\pi}{60}\right)\ rad/s$

$AB = 50\ mm$; $BD = 250\ mm$; $c = 150\ mm$

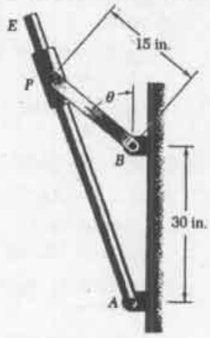
OUTLINE OF PROGRAM: FOR EACH VALUE OF θ,
1. DETERMINE β BY USING EQ 2, THEN
2. USE EQ(3) TO DETERMINE ω_{BD}, FINALLY
3. DETERMINE v_D (+↓) BY USING EQ(4)

theta deg	beta deg	yD mm	vD m/s	omega BD rad/s
0.000	36.870	150.000	1.963	13.090
30.000	30.000	173.205	2.618	10.472
60.000	25.264	201.087	2.885	5.790
90.000	23.578	229.129	2.618	0.000
120.000	25.264	251.087	1.649	-5.790
150.000	30.000	259.808	0.000	-10.472
180.000	36.870	250.000	-1.963	-13.090
210.000	44.427	221.837	-3.531	-12.699
240.000	50.643	183.539	-3.863	-8.257
270.000	53.130	150.000	-2.618	-0.000
300.000	50.643	133.539	-0.671	8.257
330.000	44.427	135.234	0.913	12.699
360.000	36.870	150.000	1.963	13.090

Theta [for vD = 0]			vD
149.900	29.980	259.807	0.006
150.000	30.000	259.808	0.000
150.100	30.020	259.807	-0.006

Theta [for vD = 0]			vD
311.400	48.592	132.288	-0.001
311.410	48.590	132.288	0.000
311.420	48.588	132.288	0.001

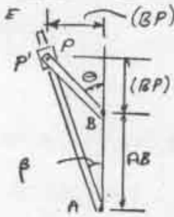
15.C2



GIVEN: $\omega_{BP} = 6$ rad/s ↻, $\alpha_{BP} = 0$

FIND:
(1) ω_{AE} AND α_{AE} FOR $\theta = 0$ TO $180°$ AT $15°$ INCREMENTS.
(2) $(\alpha_{AE})_{minimum}$ AND CORRESPONDING VALUE OF θ.

GEOMETRY:



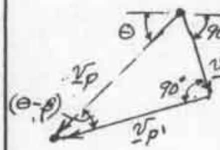
$$\beta = \tan^{-1}\frac{(BP)\sin\theta}{AB + (BP)\cos\theta} \qquad (1)$$

$$AP = [(BP)\sin\theta]/\sin\beta \qquad (2)$$

VELOCITY:

ROD BP: $v_P = (BP)\,\omega_{BP} \measuredangle \theta$ (3)

ROD AE: $v_{P'} = (AP)\,\omega_{AE} \measuredangle \beta$ (4)



$$v_P = v_{P'} + v_{P/\mathcal{F}}$$

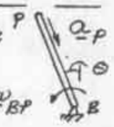
$$[v_P \measuredangle \theta] = [v_{P'} \measuredangle \beta] + [v_{P/\mathcal{F}} \nwarrow \beta]$$

$$v_{P/\mathcal{F}} = v_P \sin(\theta - \beta) \nwarrow \beta$$

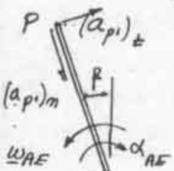
$$v_{P'} = v_P \cos(\theta - \beta) \measuredangle \beta$$

$$\omega_{AE} = v_{P'}/(AP) \; ↻ \qquad (5)$$

ACCELERATION:



$$a_P = (BP)\,\omega_{BP}^2 \nwarrow \theta \qquad (6)$$



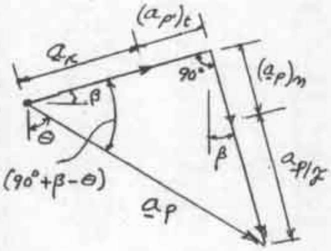
$$(a_{P'})_t = (AP)\,\alpha_{AE} \measuredangle \beta$$

$$(a_{P'})_n = (AP)\,\omega_{AE}^2 \nwarrow \beta$$

$$a_{P/\mathcal{F}} = a_{P/\mathcal{F}} \nwarrow \beta$$

$$a_C = 2\,\omega_{AE}\, v_{P/\mathcal{F}} \measuredangle \beta \qquad (7)$$

$$a_P = a_{P'} + a_{P/\mathcal{F}} + a_C$$



RIGHT TRIANGLE: $a_C + (a_{P'})_t = a_P \cos(90° + \beta - \theta)$

$$a_C + (AP)\,\alpha_{AE} = a_P \cos(90° + \beta - \theta)$$

$$\alpha_{AE} = \frac{1}{AP}\left[a_P \cos(90° + \beta - \theta) - a_C\right] \qquad (8)$$

(CONTINUED)

15.C2 continued

DATA: $\omega_{BP} = 6$ rad/s
$BP = 15$ in.; $AB = 30$ in

OUTLINE OF PROGRAM:

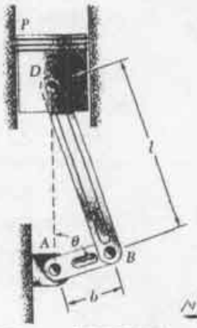
1. USE EQS. (1) AND (2) TO FIND β AND AP.
2. USE EQS. (3) AND (4) TO FIND v_P AND $v_{P'}$
3. DETERMINE ω_{AE} BY USING EQ. (5).
4. USE EQ. (6) TO FIND a_P
5. USE EQ. (7) TO FIND a_C
6. DETERMINE α_{AE} BY USING EQ. (8)

theta deg.	beta deg.	omegaAE rad/s	alpha rad/s^2
0	0.00	2.000	0.000
15	4.99	1.985	0.712
30	9.90	1.937	1.508
45	14.84	1.850	2.492
60	19.11	1.714	3.818
75	23.15	1.509	5.728
90	26.57	1.200	8.640
105	29.02	0.730	13.273
120	30.00	0.000	20.785
135	28.68	-1.144	32.388
150	23.79	-2.860	45.782
165	14.05	-4.920	43.298
180	0.00	-6.000	0.000

theta for maximum alpha

theta deg.	alpha rad/s^2
157.0800	48.58693
157.0900	48.58694
157.1000	48.58694
157.1100	48.58693

15.C3



GIVEN: $\omega_{AB} = 1000$ rpm ↻

$\alpha_{AB} = 0$

$l = 160$ mm

$b = 60$ mm

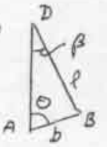
FOR VALUES OF θ FROM 0 to 180° AT 10° INTERVALS:

FIND: (a) ω_{BD} AND α_{BD}

(b) v_D AND a_D

NOTE: MOTION OF D & P ARE EQUAL

GEOMETRY:



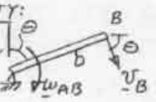
LAW OF SINES

$$\frac{\sin\beta}{b} = \frac{\sin\theta}{l}$$

$$\sin\beta = \frac{b}{l}\sin\theta \qquad (1)$$

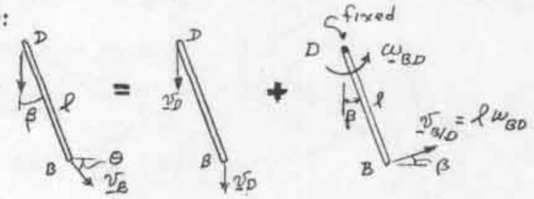
VELOCITY:

ROD AB



$$v_B = b\,\omega_{AB} \measuredangle\theta \qquad (2)$$

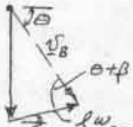
ROD BD:



PLANE MOTION = TRANS. WITH D + ROTATION ABOUT D

$$v_B = v_D + v_{D/B}$$

$$[v_B \measuredangle\theta] = [v_D \downarrow] + [l\omega_{BD} \measuredangle \beta]$$



LAW OF SINES

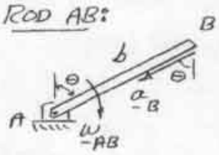
$$\frac{v_D}{\sin(\theta+\beta)} = \frac{v_B}{\sin(90°-\beta)} = \frac{l\omega_{BD}}{\sin(90°-\theta)}$$

$$v_D = v_B\frac{\sin(\theta+\beta)}{\sin(90°-\beta)} = v_B\frac{\sin(\theta+\beta)}{\cos\beta} \qquad (3)$$

$$\omega_{BD} = \frac{v_B}{l}\cdot\frac{\sin(90°-\theta)}{\sin(90-\beta)} = \frac{v_B}{l}\cdot\frac{\cos\theta}{\cos\beta} \qquad (4)$$

ACCELERATION:

ROD AB:

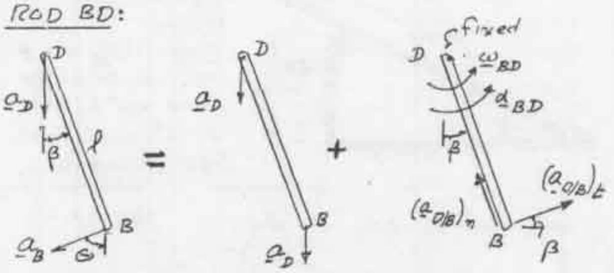


$$a_B = b\,\omega_{AB}^2 \measuredangle\theta \qquad (5)$$

(CONTINUED)

15.C3 continued

ROD BD:



PLANE MOTION = TRANS. WITH D + ROTATION ABOUT D

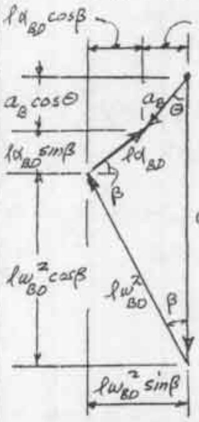
$$(a_{D/B})_t = l\alpha_{BD}$$

$$(a_{D/B})_n = l\omega_{BD}^2$$

$$a_B = a_D + a_{D/B}$$

$$a_B = a_D + (a_{D/B})_t + (a_{D/B})_n$$

$$[a_B \measuredangle\beta] = [a_D\downarrow] + [l\alpha_{BD} \measuredangle\beta] + [l\omega_{BD}^2 \measuredangle\beta]$$



VECTOR DIAGRAM

$\xleftarrow{+}$ COMPONENTS

$$a_B\cos\theta = l\omega_{BD}^2\sin\beta - l\alpha_{BD}\cos\beta$$

$$\alpha_{BD} = \frac{l\omega_{BD}^2\sin\beta - a_B\cos\theta}{l\cos\beta} \qquad (6)$$

$+\downarrow$ COMPONENTS

$$a_B\cos\theta = a_D - l\omega_{BD}^2\cos\beta - l\alpha_{BD}\sin\beta$$

$$a_D = a_B\cos\theta + l\omega_{BD}^2\cos\beta + l\alpha_{BD}\sin\beta \qquad (7)$$

OUTLINE OF PROGRAM: FOR EACH VALUE OF θ:

VELOCITY: USE EQS 1, 2, 3, AND 4, IN SEQUENCE TO OBTAIN β, v_B, v_D, AND ω_{BD}

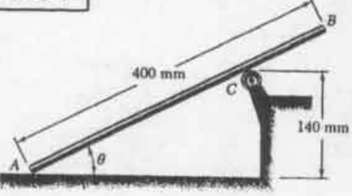
ACCELERATION: USE EQ(5) TO FIND a_B, RECALL VALUES OF β AND ω_{BD}, AND FROM EQS (6) AND (7) FIND α_{BD} AND a_D.

DATA: $\omega_{AB} = 1000$ rpm $= 1000\left(\frac{2\pi}{60}\right)$ rad/s

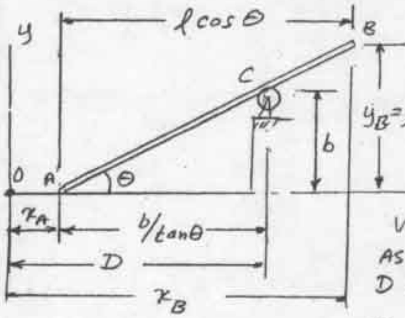
$l = 0.16$ m $\quad b = 0.06$ m

theta deg.	beta deg.	omega rad/s	alpha rad/s^2	vD m/s	aD m/s^2
0	0.00	39.27	0	0.000	905
10	3.73	38.76	-618	1.495	881
20	7.37	37.21	-1238	2.913	813
30	10.81	34.62	-1864	4.180	702
40	13.95	31.00	-2485	5.234	557
50	16.69	26.35	-3081	6.024	388
60	18.95	20.76	-3618	6.520	206
70	20.63	14.35	-4052	6.713	27
80	21.67	7.34	-4337	6.621	-134
90	22.02	0.00	-4436	6.283	-266
100	21.67	-7.34	-4337	5.754	-362
110	20.63	-14.35	-4052	5.095	-423
120	18.95	-20.76	-3618	4.363	-452
130	16.69	-26.35	-3081	3.602	-458
140	13.95	-31.00	-2485	2.843	-451
150	10.81	-34.62	-1864	2.103	-437
160	7.37	-37.21	-1239	1.385	-424
170	3.73	-38.76	-618	0.687	-415
180	0.00	-39.27	-0	0.000	-411

15.C4



GIVEN:
$v_A = 180$ mm/s →
$a_A = 0$
FIND: (1) v_B AND α FOR VALUES OF θ FROM 20° TO 90° AT 5° INCREMENTS.
(2) θ AND α FOR α_{MAX}.



$AB = l$

WE PLACE ORIGIN AS SHOWN AT DISTANCE D TO LEFT OF C

$$x_A = D - \frac{b}{\tan\theta}$$

$$v_A = \dot{x}_A = +\frac{b}{\sin^2\theta}\dot{\theta}\,, \quad \omega = \dot{\theta} = \frac{v_A}{b}\sin^2\theta \qquad (1)$$

$$x_B = D - \frac{b}{\tan\theta} + l\cos\theta$$

$$(v_B)_x = \dot{x}_B = \frac{b}{\sin^2\theta}\dot{\theta} - l\sin\theta\,\dot{\theta} = \left(\frac{b}{\sin^2\theta} - l\sin\theta\right)\frac{v_A}{b}\sin^2\theta$$

$$(v_B)_x = v_A\left(1 - \frac{l}{b}\sin^3\theta\right) \qquad (2)$$

$$y_B = l\sin\theta\,; \quad (v_B)_y = \dot{y}_B = l\cos\theta\,\dot{\theta} = l\cos\theta\,\frac{v_A}{b}\sin^2\theta$$

$$(v_B)_y = v_A\frac{l}{b}\cos\theta\sin\theta \qquad (3)$$

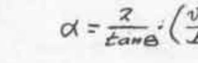
ACCELERATION

$$a_A = \ddot{x}_A = 0: \quad a_A = \frac{b}{\sin^2\theta}\ddot{\theta} - \frac{2\cos\theta}{\sin^3\theta}\dot{\theta}^2$$

$$a_A = 0 = \frac{b}{\sin^2\theta}\left[\alpha - \frac{2}{\tan\theta}\omega^2\right]$$

$$\alpha = \frac{2}{\tan\theta}\left(\frac{v_A}{b}\sin^2\theta\right) \qquad \alpha = 2\left(\frac{v_A}{b}\right)^2\sin^3\theta\cos\theta \quad (4)$$

OUTLINE OF PROGRAM FOR EACH VALUE OF θ:
USE EQS (1) AND (4) TO CALCULATE ω AND α,
USE EQS (2) AND (3) TO CALCULATE $(v_B)_x$ AND $(v_B)_y$

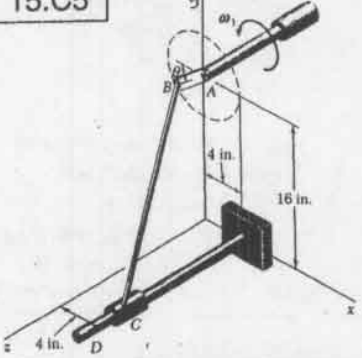


THEN FIND v_B
$v_B^2 = (v_B)_x^2 + (v_B)_y^2$; $\gamma = \tan^{-1}\frac{(v_B)_y}{(v_B)_x}$

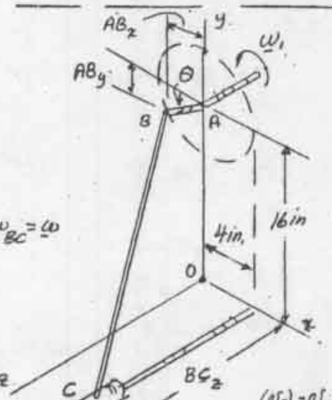
theta deg.	omega rad/s	alpha rad/s^2	vx mm/s	vy mm/s	vel mm/s	gamma deg.
20	0.150	0.124	159.42	56.53	169.15	19.52
25	0.230	0.226	141.18	83.25	163.90	30.53
30	0.321	0.358	115.71	111.35	160.59	43.90
35	0.423	0.511	82.95	138.60	161.52	59.10
40	0.531	0.673	43.41	162.78	168.47	75.07
45	0.643	0.827	-1.83	181.83	181.84	-89.42
50	0.754	0.955	-51.19	193.99	200.63	-75.22
55	0.863	1.042	-102.68	197.94	222.98	-62.58
60	0.964	1.074	-154.04	192.86	246.82	-51.39
65	1.056	1.040	-202.85	178.53	270.22	-41.35
70	1.135	0.938	-246.74	155.32	291.55	-32.19
75	1.200	0.771	-283.49	124.19	309.49	-23.66
80	1.247	0.548	-311.20	86.61	323.03	-15.55
85	1.276	0.285	-328.44	44.48	331.44	-7.71
90	1.286	0.000	-334.29	0.00	334.29	-0.00

theta [deg.]	maximum alpha [rad/s^2]
59.900	1.073682
60.000	1.073695
60.100	1.073682

15.C5

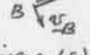


GIVEN: $BC = 24$ in.
$AB = 4$ in.
$\omega_1 = 10$ rad/s
$\alpha_1 = 0$
FIND: (1) v_C FOR VALUES OF θ FROM 0 TO 360° AT 30° INCREMENTS.
(2) TWO VALUES OF θ FOR WHICH $v_C = 0$



$AB = 4$ in.: □
$(AB)_x = (4\text{ in.})\cos\theta$ □
$(AB)_y = (4\text{ in.})\sin\theta$ □
$BC = 24$ in.:
$(BC)_x = 4\text{ in.} + (AB)_x$ □
$(BC)_y = 16\text{ in.} - (AB)_y$ □
$(BC)_z = \left[(24)^2 - (AB)_x^2 - (AB)_y^2\right]$ □

VELOCITY OF B:



$v_B = (AB)\,\omega_1$ □

$(v_B)_x = v_B\sin\theta\,;\ (v_B)_y = v_B\cos\theta$ □

$$\mathbf{r}_{C/B} = (BC)_x\mathbf{i} + (BC)_y\mathbf{j} + (BC)_z\mathbf{k}$$

VELOCITY OF C:

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B} = \mathbf{v}_B + \boldsymbol{\omega}_{BC}\times\mathbf{r}_{C/B}$$

$$v_C\,\mathbf{k} = (v_B)_x\mathbf{i} + (v_B)_y\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ (BC)_x & (BC)_y & (BC)_z \end{vmatrix}$$

x COMPONENTS: $-(v_B)_x = \qquad (BC)_z\omega_y - (BC)_y\omega_z$ (a)
y COMPONENTS: $-(v_B)_y = -(BC)_z\,\omega_x \qquad (BC_x)\,\omega_z$ (b)
z COMPONENTS: $v_C = (BC)_y\,\omega_x - (BC)_x\omega_y$ (c)

DETERMINANT $(\omega_x, \omega_y, \omega_z)$ IS ZERO, CHOOSE $\omega_z = 0$ □
EQ (a) YIELDS: $\omega_y = -(v_B)_x/(BC)_z$ □
EQ (b) YIELDS: $\omega_x = (v_B)_y/(BC)_z$ □
THEN USE EQ (c): $v_C = (BC)_y\,\omega_x - (BC)_x\omega_y$ □

OUTLINE OF PROGRAM:
FOR INITIAL VALUE $\theta = 0$, START AT TOP OF SOLUTION AND IN SEQUENCE SHOWN, PROGRAM EACH EQUATION DESIGNATED BY A "□". EVALUATE AND STORE LEFT-HAND MEMBER OF EACH EQUATION. PRINT VALUES OF θ, COMPONENTS OF BC, AND v_C. INCREASE VALUE OF θ BY 30° AND REPEAT EVALUATION UNTIL $\theta = 360°$.

(CONTINUED)

15.C5 continued

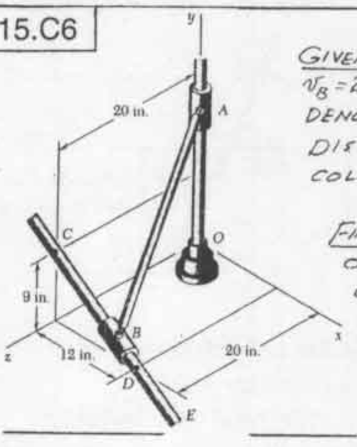
theta deg	Components of rod BC x in.	y in.	z in.	Velocity of C in./s
0.000	8.000	−16.000	16.000	40.000
30.000	7.464	−14.000	18.008	35.221
60.000	6.000	−12.536	19.567	23.436
90.000	4.000	−12.000	20.396	7.645
120.000	2.000	−12.536	20.368	−8.908
150.000	0.536	−14.000	19.486	−24.338
180.000	0.000	−16.000	17.889	−35.777
210.000	0.536	−18.000	15.865	−39.977
240.000	2.000	−19.464	13.898	−32.995
270.000	4.000	−20.000	12.649	−12.649
300.000	6.000	−19.464	12.694	14.293
330.000	7.464	−18.000	14.010	33.851
360.000	8.000	−16.000	16.000	40.000

Determination of values of theta for vC = 0

theta	Components of rod BC x	y	z	Velocity of C
104.034	3.030	−12.119	20.492	0.001
104.035	3.030	−12.119	20.492	0.001
104.036	3.030	−12.119	20.492	0.000
104.037	3.030	−12.119	20.492	−0.000

theta	Components of rod BC x	y	z	Velocity of C
284.020	4.969	−19.881	12.492	−0.015
284.030	4.970	−19.881	12.492	−0.006
284.040	4.970	−19.881	12.492	0.003

15.C6



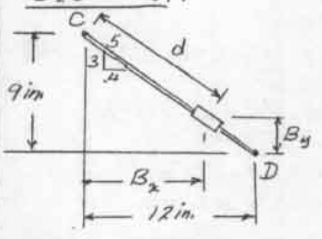
GIVEN: $AB = 25\,in.$

$v_B = 20$ in/s TOWARD POINT E

DENOTE BY d THE DISTANCE BC OF COLLAR FROM POINT C

FIND: v_A FOR VALUES OF d FROM 0 TO 15 in. AT 1-in. INCREMENTS

GEOMETRY:



$CD = 15\,in.$

$$B_x = \frac{4}{5}d = 0.8d$$

ROD AB: $(AB)_x = -B_x = -0.8d$ (1)

$AB = 25\,in.$ $\quad (AB)_z = -20\,in.$ (2)

$$AB^2 = (AB)_x^2 + (AB)_y^2 + (AB)_z^2$$

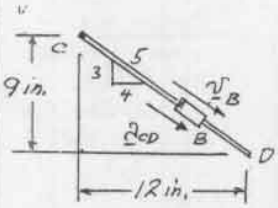
$$(AB)_y = \left[AB^2 - (AB)_x^2 - (AB)_z^2\right]^{1/2}$$

$$(AB)_y = \left[25^2 - (0.8d)^2 - 20^2\right]^{1/2} = \left[225 - 0.64d^2\right]^{1/2} \quad (3)$$

$$\underline{r}_{A/B} = +(AB)_x\underline{i} + (AB)_y\underline{j} + (AB)_z\underline{k}$$

(CONTINUED)

15.C6 continued



VELOCITY OF B:

$v_B = 20$ in/s

$$\underline{\lambda}_{CD} = \frac{4}{5}\underline{i} - \frac{3}{5}\underline{j}$$

$$\underline{v}_B = v_B\,\underline{\lambda}_{CD}$$

$$\underline{v}_B = (20\,in/s)\left(\frac{4}{5}\underline{i} - \frac{3}{5}\underline{j}\right)$$

$$(v_B)_x = 16\,in/s \quad (4)$$

$$(v_B)_y = -12\,in/s \quad (5)$$

VELOCITY OF A:

$$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B} = \underline{v}_B + \underline{\omega}\times\underline{r}_{A/B}$$

$$v_A\underline{j} = (v_B)_x\underline{i} + (v_B)_y\underline{j} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \omega_x & \omega_y & \omega_z \\ (AB)_x & (AB)_y & (AB)_z \end{vmatrix}$$

x COMPONENTS: $\quad -(v_B)_x = \quad +(AB)_z\omega_y - (AB)_y\omega_z$ (a)

y COMPONENTS: $\quad v_A - (v_B)_y = -(AB)_z\omega_x \quad +(AB)_x\omega_z$ (b)

z COMPONENTS: $\quad 0 = (AB)_y\omega_x - (AB)_x\omega_y$ (c)

DETERMINATE OF $(\omega_x, \omega_y, \omega_z)$ IS ZERO.

CHOOSE $\omega_x = 0$

EQ. (c) YIELDS: $\omega_y = 0$

EQ. (a): $-(v_B)_x = 0 + (AB)_y\omega_z$

$$\omega_z = -(v_B)_x/(AB)_y \quad (6)$$

EQ. (b): $v_A - (v_B)_y = 0 + (AB)_x\omega_z$

$$v_A = (v_B)_y + (AB)_x\omega_z \quad (7)$$

OUTLINE OF PROGRAM:

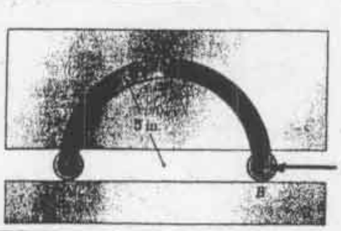
FOR INITIAL VALUE $d = 0$, PROGRAM, IN SEQUENCE EQUATIONS (1) THROUGH (7). EVALUATE LEFT-HAND MEMBER OF EACH EQUATION AND PRINT VALUES OF d, COMPONENTS OF $r_{A/B}$, AND v_C.

INCREASE VALUE OF d BY 1 in. AND REPEAT PROCESS UNTIL $d = CD = 15$ in

d in.	Components of AB x in.	y in.	z in.	Velocity vA in/s
0.000	0.000	15.000	−20.00	−12.000
1.000	−0.800	14.979	−20.00	−12.855
2.000	−1.600	14.914	−20.00	−13.716
3.000	−2.400	14.807	−20.00	−14.593
4.000	−3.200	14.655	−20.00	−15.494
5.000	−4.000	14.457	−20.00	−16.427
6.000	−4.800	14.211	−20.00	−17.404
7.000	−5.600	13.915	−20.00	−18.439
8.000	−6.400	13.566	−20.00	−19.548
9.000	−7.200	13.159	−20.00	−20.754
10.000	−8.000	12.689	−20.00	−22.088
11.000	−8.800	12.147	−20.00	−23.591
12.000	−9.600	11.526	−20.00	−25.327
13.000	−10.400	10.809	−20.00	−27.394
14.000	−11.200	9.978	−20.00	−29.960
15.000	−12.000	9.000	−20.00	−33.333

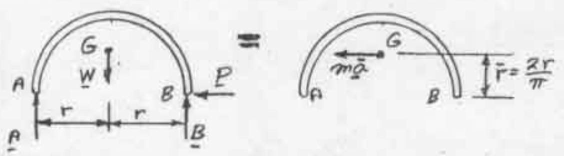
16.1 and 16.2

GIVEN: $W = 3\,lb$



PROBLEM 16.1:
FOR $P = 5\,lb$,
FIND: (a) $\underline{a}$.
(b) REACTIONS.

PROBLEM 16.2:
FOR $A = 0$,
FIND: (a) $\underline{P}$, (b) $\underline{a}$.



$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad P = m\bar{a}$ $\quad$ (1)

$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$: $\quad B(2r) - Wr = m\bar{a}\left(\frac{2r}{\pi}\right)$ $\quad$ (2)

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $\quad A + B - W = 0$ $\quad$ (3)

PROBLEM 16.1: $P = 5\,lb$, $W = 3\,lb$, $m = W/g$

EQ(1): $P = (W/g)\bar{a}$

$\bar{a} = (P/W)g = \frac{5\,lb}{3\,lb}(32.2\,ft/s^2)$

$\bar{a} = 53.67\,ft/s^2$; $\quad \underline{\bar{a}} = 53.7\,ft/s^2 \leftarrow$ ◀

EQ(2): $B(2r) - Wr = \frac{W}{g}\left(\frac{P}{W}g\right)\left(\frac{2r}{\pi}\right)$

$B = \frac{1}{2}W + \frac{P}{\pi} = \frac{1}{2}(3\,lb) + \frac{5\,lb}{\pi}$

$B = 3.092\,lb$ $\qquad \underline{B} = 3.09\,lb \uparrow$ ◀

EQ(3): $A + 3.092\,lb - 3\,lb = 0$

$A = -0.092\,lb$ $\qquad \underline{A} = 0.092\,lb \downarrow$ ◀

PROBLEM 16.2: $A = 0$, $W = 3\,lb$, $m = W/g$

EQ(2): $0 - Wr = \frac{W}{g}\bar{a}\left(\frac{2r}{\pi}\right)$

$\bar{a} = \frac{\pi}{2}g = \frac{\pi}{2}(32.2\,ft/s^2)$

$\bar{a} = 50.58\,ft/s^2$ $\qquad \underline{\bar{a}} = 50.6\,ft/s^2 \leftarrow$ ◀

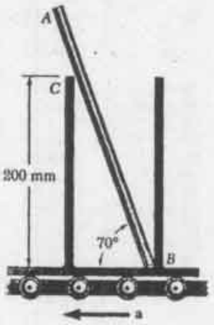
EQ(1): $P = \frac{W}{g}\bar{a}$

$P = \frac{W}{g}\left(\frac{\pi}{2}g\right) = \frac{\pi}{2}W = 4.712\,lb$

$\underline{P} = 4.71\,lb \leftarrow$ ◀

16.3 and 16.4

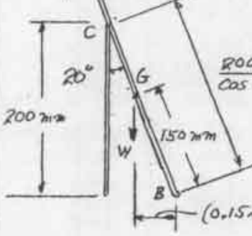
GIVEN:
ROD: $m = 2.5\,kg$, $AB = 300\,mm$



PROBLEM 16.3:
FOR $\underline{a} = 1.5\,m/s^2 \leftarrow$
FIND: (a) $\underline{C}$, (b) $\underline{B}$

PROBLEM 16.4:
FIND: a_{max} FOR ROD TO REMAIN IN POSITION

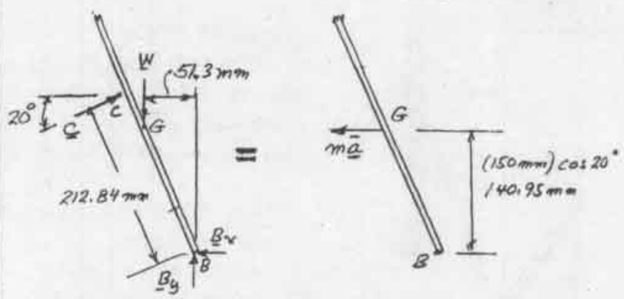
GEOMETRY



$\frac{200\,mm}{\cos 20°} = 212.84\,mm$

$(0.15\,m)\sin 20° = 51.3\,mm$

$W = mg = (2.5\,kg)\,9.81\,m/s^2 = 24.525\,N$



$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$: $\quad C(212.84\,mm) - W(51.3\,mm) = -m\bar{a}(140.95\,mm)$

$C = 0.241\,W - 0.6622\,m\bar{a}$

$C = 0.241(24.525\,N) - 0.6622(2.5\,kg)(a)$

$C = 5.911\,N - 1.656\,a$ $\qquad$ (1)

PROBLEM 16.3: $a = 1.5\,m/s^2$

EQ(1): $C = 5.911 - 1.656(1.5)$; $\quad \underline{C} = 3.43\,N \measuredangle 20°$ ◀

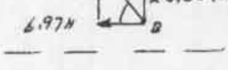
$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $\quad B_y - W + C\sin 20° = 0$

$B_y = 24.525\,N - (3.43\,N)\sin 20° = 23.35\,N \uparrow$

$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad B_x - C\cos 20° = m\bar{a}$

$B_x - (3.43\,N)\cos 20° = (2.5\,kg)(1.5\,m/s^2)$

$B_x = 3.22 + 3.75 = 6.97\,N \leftarrow$



$\underline{B} = 24.4\,N \measuredangle 73.4°$ ◀

PROBLEM 16.4: For a_{max}, $C = 0$

EQ(1) $\quad C = 5.911\,N - 1.656\,a$

$0 = 5.911\,N - 1.656\,a_{max}$

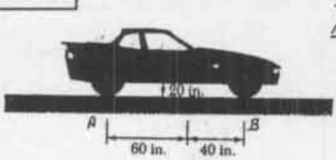
$a_{max} = 3.57\,m/s^2$

$\underline{a}_{max} = 3.57\,m/s^2 \leftarrow$ ◀

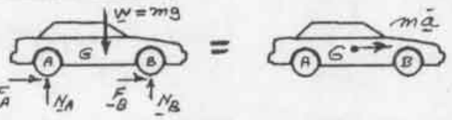
16.5

GIVEN: $\mu_s = 0.80$

FIND: a_{max} ASSUMING
(a) FOUR-WHEEL DRIVE
(b) REAR-WHEEL DRIVE
(c) FRONT-WHEEL DRIVE



(a) FOUR-WHEEL DRIVE:



$+\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0 \qquad N_A + N_B = W = mg$

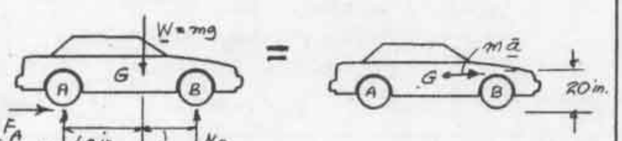
THUS: $F_A + F_B = \mu_k N_A + \mu_k N_B = \mu_k(N_A + N_B) = \mu_k W = 0.80\,mg$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}: \quad F_A + F_B = m\bar{a}$

$0.80\,mg = m\bar{a}$

$\bar{a} = 0.80\,g = 0.80(32.2\ ft/s^2) \qquad \bar{a} = 25.8\ ft/s^2 \rightarrow$ ◀

(b) REAR-WHEEL DRIVE:



$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}: \quad (40\ in.)W - (100\ in.)N_A = -(20\ in.)m\bar{a}$

$N_A = 0.4W + 0.2\,m\bar{a}$

THUS: $F_A = \mu_k N_B = 0.80(0.4W + 0.2\,m\bar{a}) = 0.32\,mg + 0.16\,m\bar{a}$

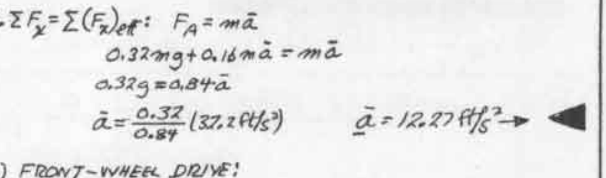
$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}: \quad F_A = m\bar{a}$

$0.32\,mg + 0.16\,m\bar{a} = m\bar{a}$

$0.32\,g = 0.84\,\bar{a}$

$\bar{a} = \frac{0.32}{0.84}(32.2\ ft/s^2) \qquad \bar{a} = 12.27\ ft/s^2 \rightarrow$ ◀

(c) FRONT-WHEEL DRIVE:



$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}: \quad (100\ in.)N_B - (60\ in.)W = -(20\ in.)m\bar{a}$

$N_B = 0.6W - 0.2\,m\bar{a}$

THUS: $F_B = \mu_k N_B = 0.80(0.6W - 0.2\,m\bar{a}) = 0.48\,mg - 0.16\,m\bar{a}$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}: \quad F_B = m\bar{a}$

$0.48\,mg - 0.16\,m\bar{a} = m\bar{a}$

$0.48\,g = 1.16\,\bar{a}$

$\bar{a} = \frac{0.48}{1.16}(32.2\ ft/s^2)$

$\bar{a} = 13.32\ ft/s^2 \rightarrow$ ◀

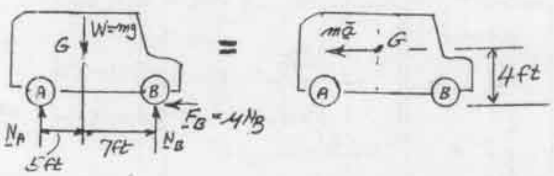
16.6

GIVEN: $v_0 = 30\ ft/s \rightarrow$

FROM SAMPLE PROB 16.1
$\mu_k = 0.699$

FIND: DISTANCE REQUIRED TO STOP IF
(a) REAR-WHEEL BRAKES FAIL TO OPERATE
(b) FRONT-WHEEL BRAKES FAIL TO OPERATE

(a) IF REAR-WHEEL BRAKES FAIL TO OPERATE



$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}: \quad N_B(12\ ft) - W(5\ ft) = m\bar{a}(4\ ft)$

$N_B = \frac{5}{12}W + \frac{1}{3}\frac{W}{g}\bar{a}$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}: \quad F_B = m\bar{a}$

$\mu_k N_B = \frac{W}{g}\bar{a}$

$0.699\left(\frac{5}{12}W + \frac{1}{3}\frac{W}{g}\bar{a}\right) = \frac{W}{g}\bar{a}$

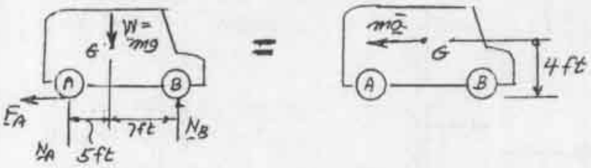
$\bar{a} = \frac{0.699\left(\frac{5}{12}\right)(32.2\ ft/s^2)}{1 - 0.233} \qquad \bar{a} = 12.227\ ft/s^2 \leftarrow$

UNIFORMLY ACCELERATED MOTION

$v^2 = v_0^2 + 2ax \qquad 0 = (30\ ft/s)^2 - 2(12.227\ ft/s^2)x$

$x = 36.8\ ft$ ◀

(b) IF FRONT-WHEEL BRAKES FAIL TO OPERATE



$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}: \quad W(7\ ft) - N_A(12\ ft) = m\bar{a}(4\ ft)$

$N_A = \frac{7}{12}W - \frac{1}{3}\frac{W}{g}\bar{a}$

$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}: \quad F_A = m\bar{a}$

$\mu_k N_A = \frac{W}{g}\bar{a}$

$0.699\left(\frac{7}{12}W - \frac{1}{3}\frac{W}{g}\bar{a}\right) = \frac{W}{g}\bar{a}$

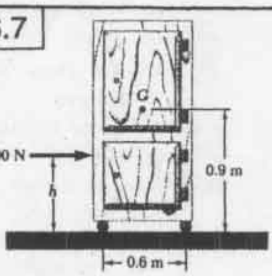
$\bar{a} = \frac{0.699\left(\frac{7}{12}\right)(32.2\ ft/s^2)}{1 + 0.233} \qquad \bar{a} = 10.648\ ft/s^2 \leftarrow$

UNIFORMLY ACCELERATED MOTION

$v^2 = v_0^2 + 2ax \qquad 0 = (30\ ft/s)^2 - 2(10.648\ ft/s^2)x$

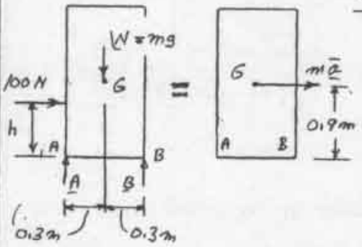
$x = 42.3\ ft$ ◀

16.7



GIVEN: $\mu = 0$
$m = 20$ kg.
FIND: (a) $\bar{a}$
(b) RANGE OF VALUES OF h FOR NO TIPPING

(a) ACCELERATION



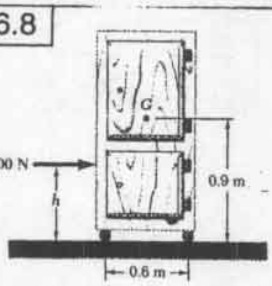
$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$:
$100 N = m\bar{a}$
$100 N = (20 kg)\bar{a}$
$\bar{a} = 5 m/s^2 \rightarrow$ ◀

(b) FOR TIPPING TO IMPEND ↻; $A = 0$
$+↻ \Sigma M_B = \Sigma (M_B)_{eff}$:
$(100N)h - mg(0.3m) = m\bar{a}(0.9m)$
$(100N)h - (20kg)(9.81 m/s^2)(0.3m) = (100N)(0.9m)$
$h = 1.489 m$ ◁

FOR TIPPING TO IMPEND ↺; $B = 0$
$+↺ \Sigma M_A = \Sigma (M_A)_{eff}$:
$(100N)h + mg(0.3m) = m\bar{a}(0.9m)$
$(100N)h + (20kg)(9.81 m/s^2)(0.3m) = (100N)(0.9m)$
$h = 0.311 m$ ◁

CABINET WILL NOT TIP: $0.311 m \leq h \leq 1.489 m$ ◀

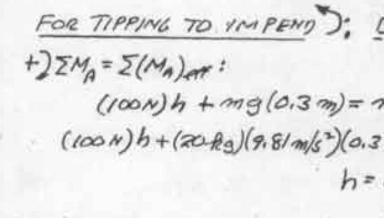
16.8



GIVEN: $\mu = 0.25$
$m = 20$ kg

FIND: (a) $\bar{a}$
(b) RANGE OF VALUES OF h FOR NO TIPPING

(a) ACCELERATION



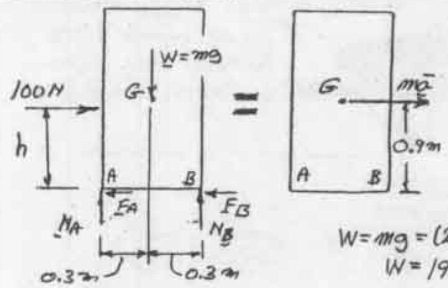
$+\uparrow \Sigma F_y = 0$
$N_A + N_B - W = 0$
$N_A + N_B = mg$
BUT, $F = \mu N$, THUS
$F_A + F_B = \mu(mg)$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $100N - (F_A + F_B) = m\bar{a}$
$100 N - \mu mg = m\bar{a}$
$100 N - 0.25(20 kg)(9.81 m/s^2) = (20 kg)\bar{a}$
$\bar{a} = 2.548 m/s^2$ $\quad \bar{a} = 2.55 m/s^2 \rightarrow$ ◀

(CONTINUED)

16.8 continued

(b) TIPPING OF CABINET



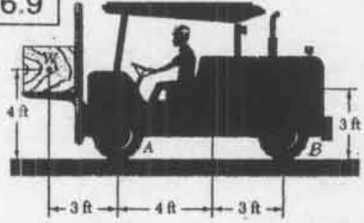
$W = mg = (20 kg)(9.81 m/s^2)$
$W = 196.2 N$

FOR TIPPING TO IMPEND ↻: $N_A = 0$
$+↻ \Sigma M_B = \Sigma (M_B)_{eff}$:
$(100N)h - W(0.3m) = m\bar{a}(0.9m)$
$(100N)h - (196.2N)(0.3m) = (20 kg)(2.548 m/s^2)(0.9m)$
$h = 1.047 m$ ◁

FOR TIPPING TO IMPEND ↺: $N_B = 0$
$+↺ \Sigma M_A = \Sigma (M_A)_{eff}$:
$(100N)h + W(0.3m) = m\bar{a}(0.9m)$
$(100N)h + (196.2N)(0.3m) = (20 kg)(9.81 m/s^2)(0.9m)$
$h = -0.130 m$ (IMPOSSIBLE) ◁

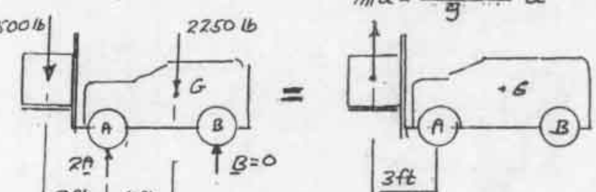
CABINET WILL NOT TIP: $h \leq 1.047 m$ ◀

16.9



GIVEN:
2250-lb TRUCK
2500-lb CRATE
FIND:
(a) $\bar{a}$ OF CRATE FOR $B = 0$
(b) A AT EACH FRONT WHEEL

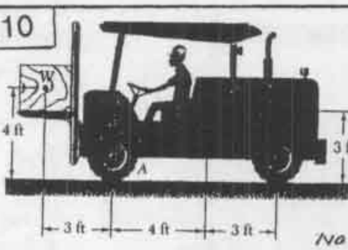
(a) ACCELERATION OF CRATE FOR $B = 0$



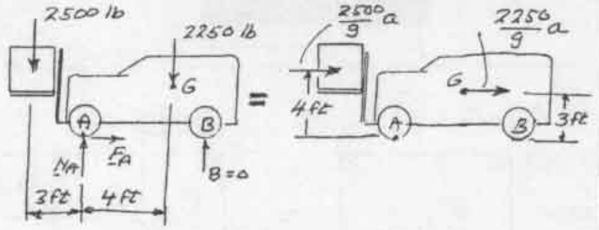
$+↺ \Sigma M_A = \Sigma (M_A)_{eff}$:
$(2500 lb)(3 ft) - (2250 lb)(4 ft) = -m\bar{a}(3 ft)$
$7500 - 9000 = \frac{2500}{g}\bar{a}(3)$
$\bar{a} = \frac{1}{5}g = \frac{1}{5}(32.2 ft/s^2)$
$\bar{a} = 6.44 ft/s^2 \uparrow$ ◀

(b) REACTION AT A:
$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $2A - 2500 lb - 2250 lb = m\bar{a}$
$2A - 4750 lb = \frac{2500 lb}{g}\left(\frac{g}{5}\right)$
$2A = 5250 lb$ FOR ONE WHEEL:
$A = 2625 lb \uparrow$ ◀

16.10



GIVEN: $\mu = 0.30$
2250-lb TRUCK
2500-lb CRATE
$v_0 = 10$ ft/s ←
FIND: SMALLEST DISTANCE FOR TRUCK TO STOP WITH NO TIPPING OR SLIDING.



ASSUME CRATE DOES NOT SLIDE AND THAT TIPPING IMPENDS ABOUT A. ($B = 0$)

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$:

$(2500\,lb)(3\,ft) - (2250\,lb)(4\,ft) = -(2500\frac{a}{g})(4\,ft) - (2250\frac{a}{g})(3\,ft)$

$7500 - 9000 = -(10\,000 + 6750)\frac{a}{g}$

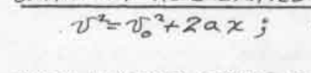
$\frac{a}{g} = 0.09$; $a = 0.09(32.2\ ft/s^2)$; $a = 2.884\ ft/s^2 \rightarrow$

UNIFORMLY ACCELERATED MOTION

$v^2 = v_0^2 + 2ax$; $0 = (10\ ft/s)^2 - 2(2.884\ ft/s^2)x$

$x = 17.34$ ft ◀

CHECK WHETHER CRATE SLIDES



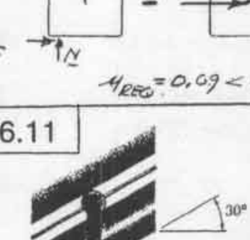
$N = W$

$F = ma = \frac{W}{g}a$

$\mu_{REQ} = \frac{F}{N} = \frac{a}{g} = \frac{2.884\ ft/s^2}{32.2\ ft/s^2}$

$\mu_{REQ} = 0.09 < 0.30$. CRATE DOES NOT SLIDE

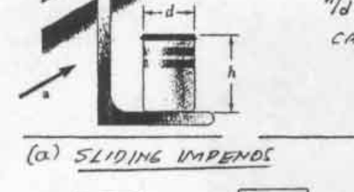
16.11



GIVEN: $\mu_s = 0.25$

FIND: (a) $\underline{a}$ FOR CAN TO SLIDE
(b) SMALLEST RATIO h/d FOR TIPPING BEFORE CAN SLIDES

(a) SLIDING IMPENDS



$\overset{+}{\rightarrow} \Sigma F_x = \Sigma(F_x)_{eff}$:
$F = ma\cos 30°$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$:
$N - mg = ma\sin 30°$
$N = m(g + a\sin 30°)$

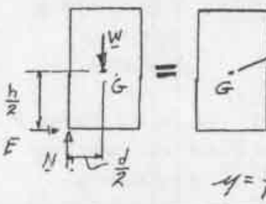
$\mu_s = \frac{F}{N}$; $0.25 = \frac{ma\cos 30°}{m(g + a\sin 30°)}$; $g + a\sin 30° = 4a\cos 30°$

$\frac{a}{g} = \frac{1}{4\cos 30° - \sin 30°}$: $a = 0.337g \angle 30°$ ◀

(CONTINUED)

16.11 continued

(b) TIPPING IMPENDS



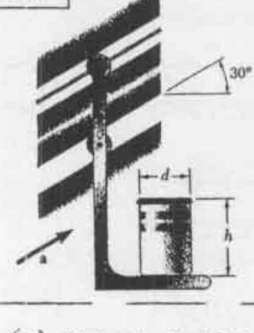
$\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$:

$F(\frac{h}{2}) - N(\frac{d}{2}) = 0$

$\frac{F}{N} = \frac{d}{h}$

$\mu = \frac{F}{N}$; $0.25 = \frac{d}{h}$; $\frac{h}{d} = 4$ ◀

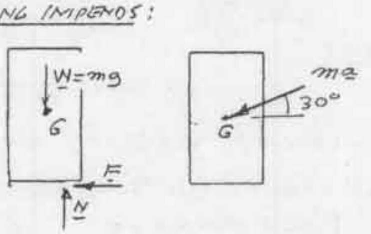
16.12



GIVEN: $\mu = 0.25$

FIND: (a) $\underline{a}$ FOR CAN TO SLIDE
(b) SMALLEST RATIO h/d FOR TIPPING BEFOR CAN SLIDE

(a) SLIDING IMPENDS:



$\overset{+}{\leftarrow} \Sigma F_x = \Sigma(F_x)_{eff}$: $F = ma\cos 30°$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N - mg = -ma\sin 30°$

$N = m(g - a\sin 30°)$

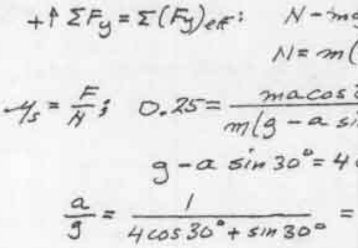
$\mu_s = \frac{F}{N}$; $0.25 = \frac{ma\cos 30°}{m(g - a\sin 30°)}$

$g - a\sin 30° = 4a\cos 30°$

$\frac{a}{g} = \frac{1}{4\cos 30° + \sin 30°} = 0.252$

$a = 0.252g \measuredangle$ ◀

(b) TIPPING IMPENDS:

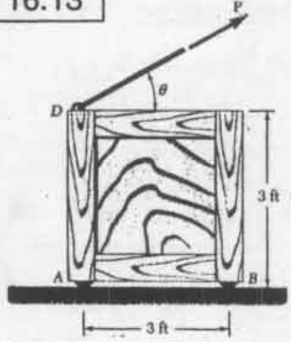


$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$

$F(\frac{h}{2}) = W(\frac{d}{2})$; $\frac{F}{N} = \frac{d}{h}$

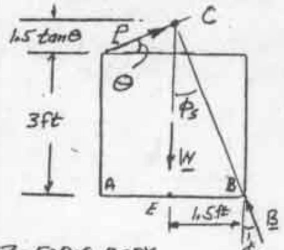
$\mu = \frac{F}{N}$: $0.25 = \frac{d}{h}$; $\frac{h}{d} = 4$ ◀

16.13



GIVEN: 100-lb CRATE
$\mu_s = 0.40$
$\mu_k = 0.30$
FIND:
(a) VALUES OF θ AND P FOR BOTH SLIDING AND TIPPING IMPENDING
(b) ACCELERATION OF CRATE IF P IS SLIGHTLY INCREASED

(a) CRATE IS IN EQUILIBRIUM: FREE-BODY DIAGRAM



3-FORCE BODY

TIPPING IMPENDS: $\underline{A} = 0$

SLIDING IMPENDS:
$\tan\phi_s = \mu_s = 0.40$

IN ΔBCE

$$\tan\phi_s = \frac{1.5}{3 + 1.5\tan\theta}$$

$$0.40 = \frac{1}{2 + \tan\theta}$$

$\tan\theta = \frac{1}{2}$; $\theta = 26.57°$; $\theta = 26.6°$ ◀

$+\circlearrowleft \Sigma M_B = 0$: $(P\cos\theta)(3\text{ ft}) + (P\sin\theta)(3\text{ ft}) - W(1.5\text{ ft}) = 0$

$P(\cos\theta + \sin\theta) = \frac{1}{2}W$

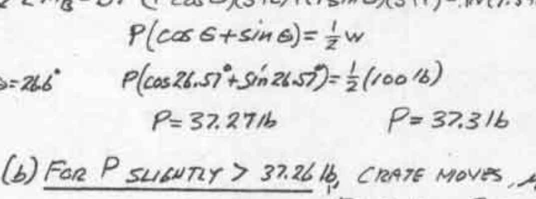
$\theta = 26.6°$ $\quad P(\cos 26.57° + \sin 26.57°) = \frac{1}{2}(100\text{ lb})$

$P = 37.27\text{ lb}$ $\qquad P = 37.3\text{ lb}$ ◀

(b) FOR P SLIGHTLY > 37.26 lb, CRATE MOVES, $\mu = \mu_k$

$F_A = \mu_k N_A$, $F_B = \mu_k N_B$

$\underline{P} = 37.27\text{ lb}$



$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N_A + N_B - 100\text{ lb} - (37.27\text{ lb})\sin 26.57° = 0$

$N_A + N_B = 100 - 16.67 = 83.33\text{ lb}$

$F_A + F_B = \mu_k(N_A + N_B) = 0.30(83.33\text{ lb}) = 25.0\text{ lb}$

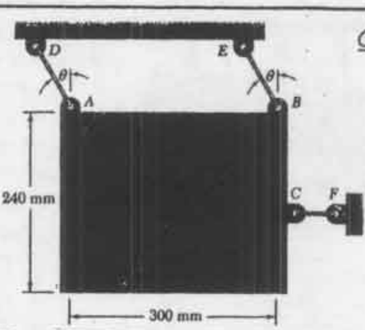
$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$:

$(37.27\text{ lb})\cos 26.57° - (F_A + F_B) = m\bar{a}$

$$33.33 - 25.0 = \frac{100\text{ lb}}{32.2\text{ ft/s}^2}\bar{a}$$

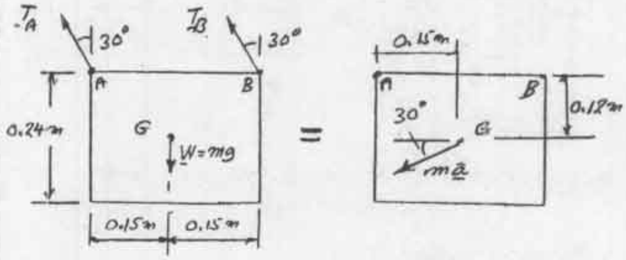
$\bar{a} = 2.68\text{ ft/s}^2 \rightarrow$ ◀

16.14



GIVEN: $\theta = 30°$
$m = 5$ kg

CUT CF:
FIND:
(a) $\bar{a}$
(b) T_{AD} AND T_{BE}



(a) ACCELERATION $\quad +\swarrow 30°\ \Sigma F = \Sigma F_{eff}$:

$mg\sin 30° = m\bar{a}$

$\bar{a} = 0.5g = 4.905\text{ m/s}^2$ $\qquad \bar{a} = 4.91\text{ m/s}^2 \swarrow 30°$ ◀

(b) TENSION IN ROPES

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$:

$(T_B\cos 30°)(0.3\text{ m}) - mg(0.15\text{ m}) = -m\bar{a}(\cos 30°)(0.12\text{ m})$
$\qquad - m\bar{a}(\sin 30°)(0.15\text{ m})$

$0.2598\,T_B - (5\text{ kg})(9.81\text{ m/s}^2)(0.15\text{ m}) = -(5\text{ kg})(4.905\text{ m/s}^2)(0.1039 + 0.075)$

$0.2598\,T_B - 7.3575 = 4.388$

$T_B = +11.43\text{ N}$ $\qquad T_{BE} = 11.43\text{ N}$ ◀

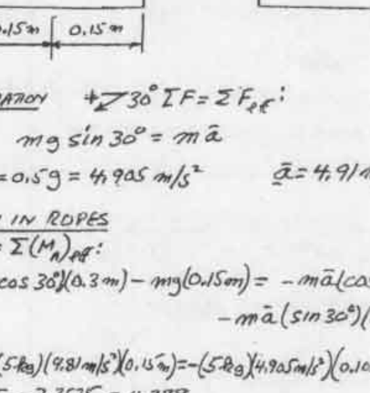
$+\nwarrow 60°\ \Sigma F = \Sigma F_{eff}$: $T_A + 11.43\text{ N} - mg\cos 30° = 0$

$T_A + 11.43\text{ N} - (5\text{ kg})(9.81)\cos 30° = 0$

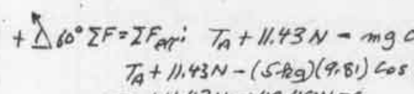
$T_A + 11.43\text{ N} - 42.48\text{ N} = 0$

$T_A = 31.04\text{ N}$ $\qquad T_{AD} = 31.0\text{ N}$ ◀

16.15



FIND: LARGEST θ FOR WHICH ROPE REMAIN TAUT WHEN CF IS CUT.



$\swarrow \Sigma F = \Sigma F_{eff}$
$mg\sin\theta = m\bar{a}$
$\bar{a} = g\sin\theta$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $mg(0.15\text{ m}) = m\bar{a}\cos\theta(0.12\text{ m}) + m\bar{a}\sin\theta(0.15\text{ m})$

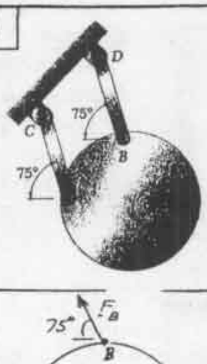
$\cancel{m}\cancel{g}(0.15) = \cancel{m}(\cancel{g}\sin\theta)(0.12\cos\theta + 0.15\sin\theta)$

$1 = 0.8\sin\theta\cos\theta - \sin^2\theta$

$1 - \sin^2\theta = 0.8\sin\theta\cos\theta$; $\cos^2\theta = 0.8\sin\theta\cos\theta$

$1 = 0.8\sin\theta/\cos\theta$; $\tan\theta = 1.25$; $\theta = 51.3°$ ◀

16.16

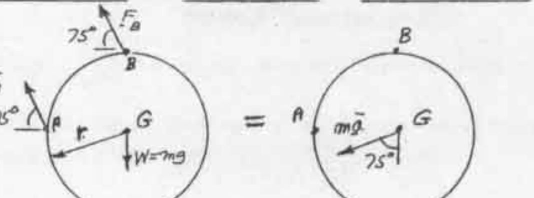


GIVEN: $m = 3$ kg

PLATE IS RELEASED FROM REST.

FIND: (a) $\bar{a}$

(b) TENSION IN EACH LINK



(a) ACCELERATION

$\swarrow \Sigma F = \Sigma F_{eff}$: $mg\cos 75° = m\bar{a}$

$\bar{a} = g\cos 75°$ $\quad \bar{a} = 2.54\ m/s^2 \measuredangle 15°$ ◀

(b) TENSION IN EACH LINK

$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$:

$(F_A\cos 75°)r + (F_A\sin 75°)r = (m\bar{a}\sin 75°)r$

$F_A(\cos 75° + \sin 75°) = (3\ kg)(2.54\ m/s^2)\sin 75°$

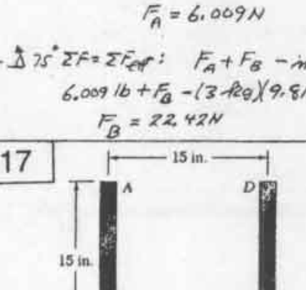
$F_A = 6.009\ N$ $\quad F_{AC} = 6.01\ N$, T ◀

$+\measuredangle 75°\ \Sigma F = \Sigma F_{eff}$: $F_A + F_B - mg\sin 75° = 0$

$6.009\ lb + F_B - (3\ kg)(9.81\ m/s^2)\sin 75° = 0$

$F_B = 22.42\ N$ $\quad F_{BD} = 22.4\ N$ T ◀

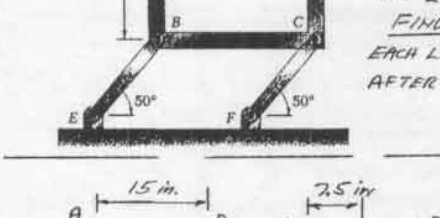
16.17



GIVEN: FOR EACH BAR $W = 8$ lb

NEGLECTING WEIGHT OF LINKS BE AND CF

FIND: FORCE IN EACH LINK IMMEDIATELY AFTER RELEASE



MASS CENTER OF ABCD IS AT G

$\measuredangle 40°\ \Sigma F = \Sigma F_{eff}$: $mg\cos 50° = m\bar{a}$ $\quad \bar{a} = 20.7\ ft/s^2 \measuredangle 40°$

$(24\ lb)\cos 50° = m\bar{a}$ $\quad m\bar{a} = 15.427\ lb$

$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$:

$(F_{CF}\sin 50°)(15\ in.) - (24\ lb)(7.5\ in) =$

$- m\bar{a}\sin 40°(7.5\ in.) - m\bar{a}\cos 40°(5\ in)$

$11.491\ F_{CF} - 180 = -m\bar{a}(4.821 + 3.83)$

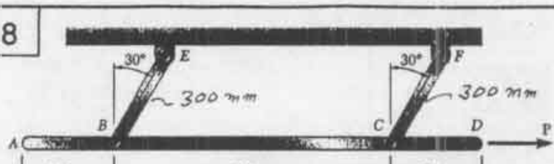
$11.491\ F_{CF} - 180 = -(15.427\ lb)(8.651)$

$F_{CF} = +4.05\ lb$ $\quad F_{CF} = 4.05\ lb$ C ◀

$\measuredangle 50°\ \Sigma F = \Sigma F_{eff}$: $F_{BE} + 4.05\ lb - (24\ lb)\sin 50° = 0$

$F_{BE} = +14.33\ lb$ $\quad F_{BE} = 14.33\ lb$ C ◀

16.18



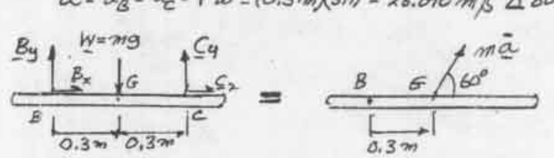
GIVEN: $m_{AD} = 6$ kg, $P = 0$, $\omega_{BE} = \omega_{CF} = 90$ rpm $\circlearrowright$, $\alpha_{BE} = \alpha_{CF} = 0$

FIND: B_y AND C_y

$\omega = 90\ rpm\left(\frac{2\pi}{60}\right) = 3\pi\ rad/s$

BAR AD IN TRANSLATION

$\bar{a} = a_B = a_C = r\omega^2 = (0.3\ m)(3\pi)^2 = 26.648\ m/s^2 \measuredangle 60°$



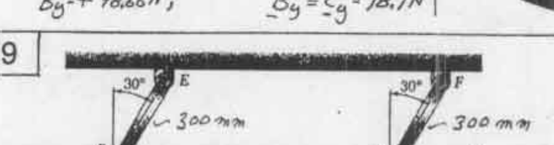
$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$: $C_y(0.3\ m) - B_y(0.3\ m)$; $\quad B_y = C_y$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $B_y + C_y - mg = m\bar{a}\sin 60°$

$2B_y - (6\ kg)(9.81\ m/s^2) = (6\ kg)(26.648\ m/s^2)\sin 60°$

$B_y = +98.66\ N$; $\quad B_y = C_y = 98.7\ N \uparrow$ ◀

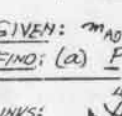
16.19



GIVEN: $m_{AD} = 6$ kg, $\omega_{BE} = \omega_{CF} = 6\ rad/s \circlearrowright$, $\alpha_{BE} = \alpha_{CF} = 12\ rad/s^2 \circlearrowright$

FIND: (a) P, (b) F_{BE} AND F_{CF}.

LINKS:



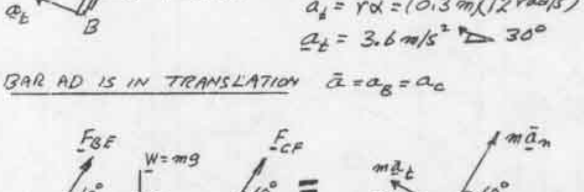
$a_n = r\omega^2 = (0.3\ m)(6\ rad/s)^2$

$a_n = 10.80\ m/s^2 \measuredangle 60°$

$a_t = r\alpha = (0.3\ m)(12\ rad/s^2)$

$a_t = 3.6\ m/s^2 \measuredangle 30°$

BAR AD IS IN TRANSLATION $\quad \bar{a} = a_B = a_C$



$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$: $F_{CF}\cos 60°(0.3\ m) - F_{BE}\cos 60°(0.3\ m)$

$F_{CF} = F_{BE}$

$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$:

$F_{CF}\sin 60°(0.6\ m) - mg(0.3\ m) = +ma_t\sin 30°(0.3\ m) + ma_n\sin 60°(0.3\ m)$

$0.5196\ F_{CF} - (6\ kg)(9.81\ m/s^2)(0.3\ m) = +(6\ kg)(3.6\ m/s^2)\sin 30°(0.3\ m)$

$+(6\ kg)(10.80\ m/s^2)\sin 60°(0.3\ m)$

$0.5196\ F_{CF} - 17.658 = +3.24 + 16.836$

$F_{CF} = +72.62\ N$ $\quad F_{CF} = F_{BE} = 72.6\ N$ (T) ◀

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$: $(F_{BE} + F_{CF})\cos 60° + P = -ma_t\cos 30° + ma_n\cos 60°$

$2(72.62\ N)\cos 60° + P = -(6\ kg)(3.6\ m/s^2)\cos 30° + (6\ kg)(10.8\ m/s^2)\cos 60°$

$72.62 + P = -18.706 + 32.40$

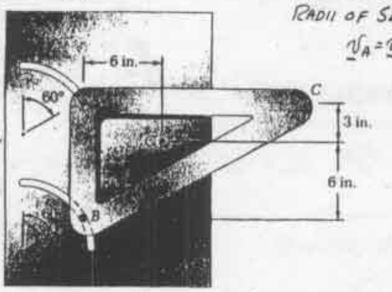
$P = -58.9\ N$ $\quad P = 58.9\ N \leftarrow$ ◀

16.20

GIVEN: $W = 16\ lb$

RADII OF SLOTS: $r = 6\ in.$

$\underline{v}_A = \underline{v}_B = 30\ in/s$ ↘

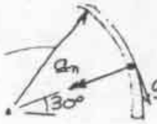


AT INSTANT SHOWN:

FIND: (a) $\bar{\underline{a}}$

(b) REACTIONS AT A AND B

SLOT: $r = 6\ in.$



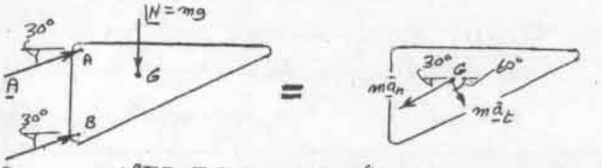
$v = 30\ in/s$

$a_n = \frac{v^2}{r} = \frac{(30\ in/s)^2}{6\ in} = 150\ in/s^2$

$\underline{a}_n = 12.5\ ft/s^2 \measuredangle 30°$

$\underline{a}_t = a_t \measuredangle 60°$

WELDMENT IS IN TRANSLATION $\bar{\underline{a}}_n = 12.5\ ft/s^2$



$\measuredangle 60°\ \Sigma F = \Sigma F_{eff}$: $mg \cos 30° = m a_t$

$\bar{\underline{a}}_t = 27.886\ ft/s^2 \measuredangle 60°$

(a) ACCELERATION

$\beta = \tan^{-1}\frac{\bar{a}_n}{\bar{a}_t} = \tan^{-1}\frac{12.5}{27.886} = 24.14°$

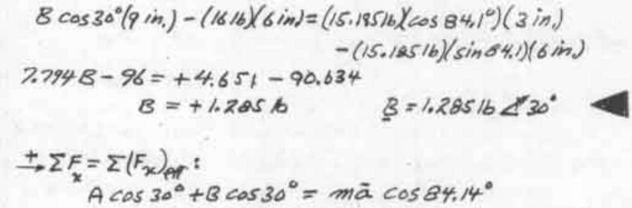
$\bar{a}^2 = a_t^2 + a_n^2 = (27.886)^2 + (12.5)^2$

$\bar{a} = 30.56\ ft/s^2 \measuredangle 84.1°$

$\bar{\underline{a}} = 30.6\ ft/s^2 \measuredangle 84.1°$ ◀

(b) REACTIONS

$m\bar{\underline{a}} = \frac{16\ lb}{32.2\ ft/s^2}(30.56\ ft/s^2) = 15.185\ lb$



$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$:

$B\cos 30°(9\ in.) - (16\ lb)(6\ in) = (15.185\ lb)(\cos 84.1°)(3\ in.) - (15.185\ lb)(\sin 84.1°)(6\ in)$

$7.794B - 96 = +4.651 - 90.634$

$B = +1.285\ lb \qquad \underline{B} = 1.285\ lb \measuredangle 30°$ ◀

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$:

$A\cos 30° + B\cos 30° = m\bar{a}\cos 84.14°$

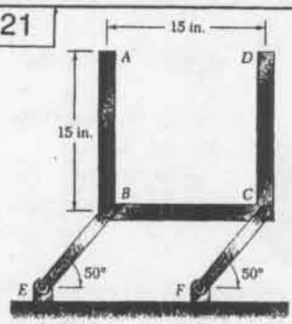
$A\cos 30° + (1.285\ lb)\cos 30° = (15.185\ lb)\cos 84.14°$

$A\cos 30° + 1.113\ lb = 1.550\ lb$

$A = +0.505\ lb$

$\underline{A} = 0.505\ lb \measuredangle 30°$ ◀

* 16.21



GIVEN:

$W_{AB} = 8\ lb$

FROM PROB. 16.17

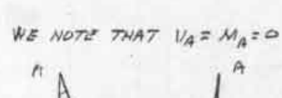
$\bar{\underline{a}} = 20.7\ ft/s^2 \measuredangle 40°$

DRAW V AND M DIAGRAMS FOR AB

DISTRIBUTED WEIGHT PER UNIT LENGTH: $w = \frac{8\ lb}{(15/12\ ft)} = 6.4\ lb/ft$

HORIZONTAL COMP. OF EFFECTIVE FORCES

$\frac{w}{g}\bar{a}_x = \frac{6.4\ lb/ft}{32.2\ ft/s^2}(20.7\ ft/s^2)\cos 40° = 3.15138\ lb/ft$



$\Sigma M_B = \Sigma (M_B)_{eff}$

$M_B = (3.15138\ lb/ft)\frac{(1.25\ ft)^2}{2}$

$M_B = 2.46\ lb\cdot ft$

$\xrightarrow{+} \Sigma F = \Sigma (F)_{eff}$

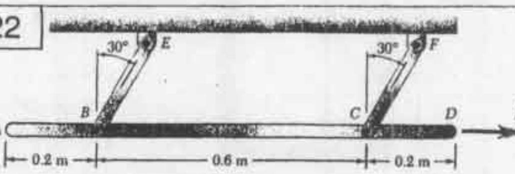
$V_B = (3.15138\ lb/ft)(1.25\ ft)$

$V_B = 3.94\ lb$

WE NOTE THAT $V_A = M_A = 0$ AND SKETCH THE V AND M DIAGRAMS

◀

* 16.22



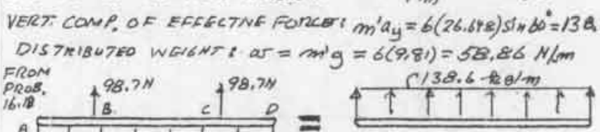
GIVEN: $m_{AD} = 6\ kg$, FROM PROB 16.18 $\underline{a} = 26.648\ m/s^2 \measuredangle 60°$

DRAW V AND M DIAGRAMS FOR BAR AD

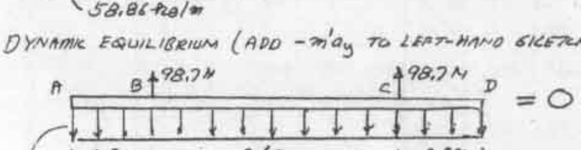
DISTRIBUTED MASS PER UNIT LENGTH: $m' = \frac{6\ kg}{1\ m} = 6\ kg/m$

VERT. COMP. OF EFFECTIVE FORCE: $m'a_y = 6(26.648)\sin 60° = 138.6\ N/m$

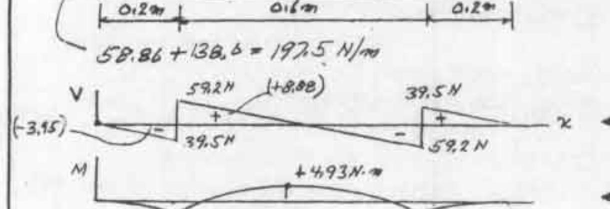
DISTRIBUTED WEIGHT: $w = m'g = 6(9.81) = 58.86\ N/m$



DYNAMIC EQUILIBRIUM (ADD $-m'a_y$ TO LEFT-HAND SKETCH)



$58.86 + 138.6 = 197.5\ N/m$



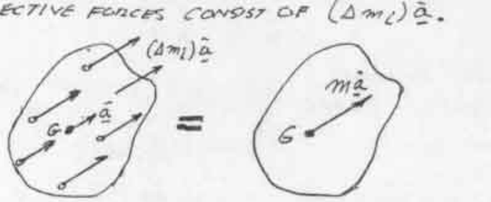
◀

◀

16.23

FOR TRANSLATION SHOW THAT EFFECTIVE FORCES ARE $(\Delta m)\bar{a}$ ATTACHED TO PARTICLES AND ARE REDUCE TO $m\bar{a}$ ATTACHED AT G

SINCE SLAB IS IN TRANSLATION, EACH PARTICLE HAS SAME ACCELERATION AS G, NAMELY $\bar{a}$. THE EFFECTIVE FORCES CONSIST OF $(\Delta m_i)\bar{a}$.



THE SUM OF THESE VECTORS IS: $\Sigma(\Delta m_i)\bar{a} = (\Sigma \Delta m_i)\bar{a}$

OR SINCE $\Sigma \Delta m_i = m$,

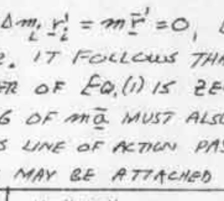
$$\Sigma(\Delta m_i)\bar{a} = m\bar{a}$$

THE SUM OF THE MOMENTS ABOUT G IS:

$$\Sigma \underline{r}_i' \times (\Delta m_i)\bar{a} = (\Sigma \Delta m_i \underline{r}_i') \times \bar{a} \qquad (1)$$

BUT, $\Sigma \Delta m_i \underline{r}_i' = m\bar{\underline{r}}' = 0$, BECAUSE G IS THE MASS CENTER. IT FOLLOWS THAT THE RIGHT-HAND MEMBER OF EQ.(1) IS ZERO. THUS, THE MOMENT ABOUT G OF $m\bar{a}$ MUST ALSO BE ZERO, WHICH MEANS THAT ITS LINE OF ACTION PASSES THROUGH G AND THAT IT MAY BE ATTACHED AT G.

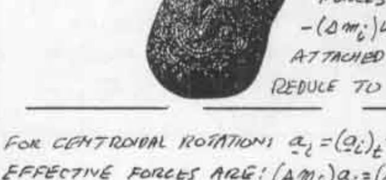
16.24



FOR CENTROIDAL ROTATION, SHOW THAT EFFECTIVE FORCES CONSIST OF VECTORS $-(\Delta m_i)\omega^2 \underline{r}_i'$ AND $(\Delta m_i)(\underline{\alpha} \times \underline{r}_i')$ ATTACHED TO PARTICLES AND REDUCE TO A COUPLE $\bar{I}\underline{\alpha}$.

FOR CENTROIDAL ROTATION: $\underline{a}_i = (\underline{a}_i)_t + (\underline{a}_i)_n = \underline{\alpha} \times \underline{r}_i' - \omega^2 \underline{r}_i'$

EFFECTIVE FORCES ARE: $(\Delta m_i)\underline{a}_i = (\Delta m_i)(\underline{\alpha} \times \underline{r}_i') - (\Delta m_i)\,\omega^2 \underline{r}_i'$



$$\Sigma(\Delta m_i)\underline{a}_i = \Sigma(\Delta m_i)(\underline{\alpha} \times \underline{r}_i') - \Sigma(\Delta m_i)\omega^2 r_i'$$
$$= \alpha \times \Sigma(\Delta m_i)\underline{r}_i' - \omega^2 \Sigma(\Delta m_i) r_i'$$

SINCE G IS THE MASS CENTER, $\Sigma(\Delta m_i) r_i' = 0$

$\therefore$ EFFECTIVE FORCES REDUCE TO A COUPLE.

SUMMING MOMENTS ABOUT G

$$\Sigma(\underline{r}_i' \times \Delta m_i \underline{a}_i) = \Sigma\left[\underline{r}_i' \times (\Delta m_i)(\underline{\alpha} \times \underline{r}_i')\right] - \Sigma \underline{r}_i' \times (\Delta m_i)\omega^2 r_i'$$

BUT, $\underline{r}_i' \times (\Delta m_i)\omega^2 \underline{r}_i' = \omega^2 (\Delta m_i)(\underline{r}_i' \times \underline{r}_i') = 0$

AND, $\underline{r}_i' \times (\Delta m_i)(\underline{\alpha} \times \underline{r}_i') = (\Delta m_i) r_i'^2 \alpha$

THUS, $\Sigma(\underline{r}_i' \times \Delta m_i \underline{a}_i) = \Sigma(\Delta m_i) r_i'^2 \alpha = \left[\Sigma(\Delta m_i) r_i'^2\right]\alpha$

SINCE $\Sigma(\Delta m) r_i'^2 = \bar{I}$,

THE MOMENT OF THE COUPLE IS $\bar{I}\alpha$

16.25

FLYWHEEL: $W = 6000\text{ lb}$ $\bar{k} = 36\text{ in.}$

AT $t = 0$, $\omega_0 = 300\text{ rpm}$, AT $t = 10\text{ min.}$, $\omega = 0$

FIND COUPLE DUE TO KINETIC FRICTION. (UNIF. ACCEL. MOTION)

$$\bar{I} = m\bar{k}^2 = \left(\frac{6000\text{ lb}}{32.2\text{ ft/s}^2}\right)(3\text{ ft}) = 1677.0\text{ lb·ft·s}^2$$

$$\omega_0 = 300\text{ rpm}\left(\frac{2\pi}{60}\right) = 10\pi\text{ rad/s}$$

$$\omega = \omega_0 + \alpha t; \quad 0 = 10\pi\text{ rad/s} + \alpha(600\text{ s})$$
$$\alpha = -0.05236\text{ rad/s}^2$$

$$M = I\alpha = (1677\text{ lb·ft·s}^2)(0.5236\text{ rad/s}^2) = 87.81\text{ lb·ft}$$

$$M = 87.8\text{ lb·ft}$$

16.26

ROTOR: $m = 50\text{ kg}$, $\bar{k} = 180\text{ mm}$

FRICTION COUPLE: $M = 3.5\text{ N·m}$

$\theta = 0$, $\omega_0 = 3600\text{ rpm}$ (UNIF. ACCEL. MOTION)

FIND: REVOLUTIONS AS ROTOR COASTS TO REST

$$\bar{I} = m\bar{k}^2 = (50\text{ kg})(0.180\text{ m})^2 = 1.620\text{ kg·m}^2$$

$M = I\alpha$: $\quad 3.5\text{ N·m} = (1.620\text{ kg·m}^2)\,\alpha$

$\alpha = 2.1605\text{ rad/s}^2$ (DECELERATION)

$$\omega_0 = 3600\text{ rpm}\left(\frac{2\pi}{60}\right) = 120\pi\text{ rad/s}$$

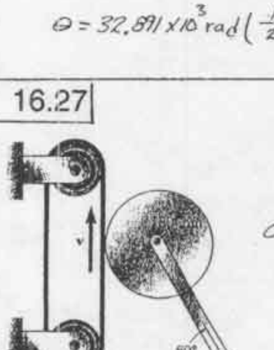
$\omega^2 = \omega_0^2 + 2\alpha\theta$: $\quad 0 = (120\pi\text{ rad/s})^2 + 2(-2.1605\text{ rad/s}^2)\,\theta$

$$\theta = 32.891 \times 10^3\text{ rad}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right); \quad \theta = 5234.8\text{ rev}$$

$$\theta = 5230\text{ rev}$$

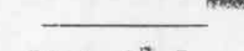
16.27

GIVEN: $\mu_k = 0.40$



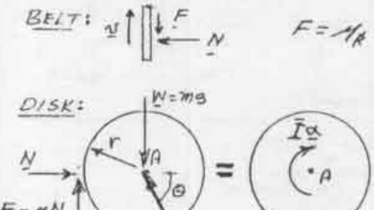
FIND: α FOR DIRECTION OF MOTION OF BELT SHOWN

BELT:



$F = \mu_k N$

DISK:



$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$:

$N - F_{AB}\cos\theta = 0$

$F_{AB}\cos\theta = N \qquad (1)$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$:

$\mu_k N + F_{AB}\sin\theta - mg = 0$

$F_{AB}\sin\theta = mg - \mu_k N \quad (2)$

$\frac{EQ.(2)}{EQ.(1)}$: $\quad \tan\theta = \frac{mg - \mu_k N}{N}$

$N\tan\theta = mg - \mu_k N$; $\quad N = \frac{mg}{\tan\theta + \mu_k}$; $\quad F = \mu_k N = \frac{mg\,\mu_k}{\tan\theta + \mu_k}$

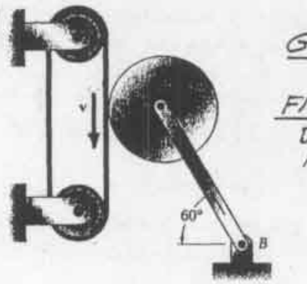
$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $\quad Fr = \bar{I}\alpha$

$$\alpha = \frac{r}{\bar{I}}F = \frac{r}{\frac{1}{2}mr^2} \cdot \frac{mg\,\mu_k}{\tan\theta + \mu_k} = \frac{2g}{r} \cdot \frac{\mu_k}{\tan\theta + \mu_k}$$

DATA: $r = 0.18\text{ m}$, $\theta = 60°$, $\mu_k = 0.40$

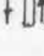
$$\alpha = \frac{2(9.81\text{ m/s}^2)}{0.18\text{ m}} \cdot \frac{0.40}{\tan 60° + 0.40} \qquad \underline{\alpha} = 20.4\frac{\text{rad}}{\text{s}^2}\curvearrowright$$

16.28

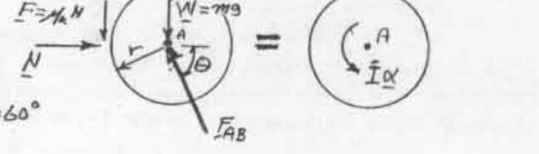


GIVEN: $\mu_k = 0.40$

FIND: α FOR DIRECTION OF MOTION OF BELT SHOWN

BELT:  $F = \mu_k N$

DISK:



$\theta = 60°$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $N - F_{AB}\cos\theta$; $F_{AB}\cos\theta = N$ (1)

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $F_{AB}\sin\theta - mg - \mu_k N = 0$

$F_{AB}\sin\theta = mg + \mu_k N$ (2)

$\frac{EQ.(2)}{EQ.(1)}$: $\tan\theta = \frac{mg + \mu_k N}{N}$

$N\tan\theta = mg + \mu_k N$; $N = \frac{mg}{\tan\theta - \mu_k}$

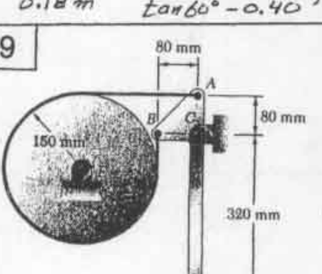
$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$: $Fr = \bar{I}\alpha$

$\alpha = \frac{Fr}{\bar{I}} = \frac{\mu_k N r}{\bar{I}} = \frac{\mu_k r}{\frac{1}{2}mr^2} \cdot \frac{mg}{\tan\theta - \mu_k} = \frac{2g}{r} \cdot \frac{\mu_k}{\tan\theta - \mu_k}$

DATA: $r = 0.18$ m, $\theta = 60°$, $\mu_k = 0.40$

$\alpha = \frac{2(9.81 \text{ m/s}^2)}{0.18 \text{ m}} \cdot \frac{0.40}{\tan 60° - 0.40}$; $\underline{\alpha} = 32.7 \frac{\text{rad}}{\text{s}^2} \circlearrowright$ ◀

16.29



GIVEN: $\bar{I} = 75 \text{ kg}\cdot\text{m}^2$

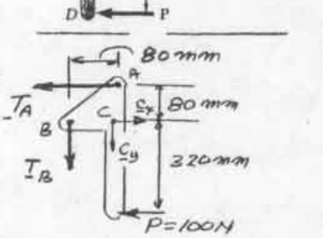
$P = 100$ N

$\mu_k = 0.25$

$\omega_0 = 240$ rpm $\circlearrowright$

FIND: TIME REQUIRED FOR DISK TO COME TO REST

LEVER ABCD



STATIC EQUILIBRIUM:

$+\circlearrowright \Sigma M_C = 0$: $T_A(80 \text{ mm}) + T_B(80 \text{ mm}) - (100\text{N})(320 \text{ mm}) = 0$

$T_A + T_B = 400$ N (1)

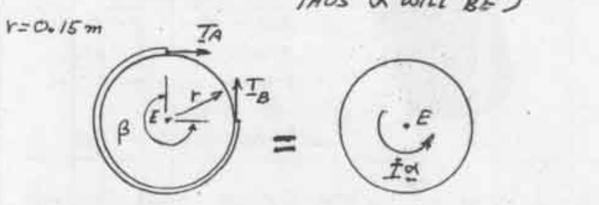
(CONTINUED)

16.29 continued

$\omega_0 = 240$ rpm $\left(\frac{2\pi}{60}\right) = 8\pi$ rad/s $\circlearrowright$

THUS α WILL BE $\circlearrowleft$

$r = 0.15$ m



$+\circlearrowleft \Sigma M_E = \Sigma (M_E)_{eff}$: $T_B r - T_A r = \bar{I}\alpha$

$T_B - T_A = \frac{\bar{I}}{r}\alpha$ (2)

BELT FRICTION:

$\beta = 270° = \frac{3}{2}\pi$ rad $\quad \frac{T_B}{T_A} = e^{\mu_k\beta} = e^{(0.25)\frac{3}{2}\pi} = e^{1.178} = 3.248$

$T_B = 3.248\, T_A$ (3)

EQ(1): $T_A + T_B = 400$N ; $T_A + 3.248\, T_A = 400$ N

$T_A = 94.16$N $\quad T_B = 3.248(94.16\text{N}) = 305.9$N

EQ.(2): $T_B - T_A = \frac{\bar{I}}{r}\alpha$; $305.9\text{N} - 94.16\text{N} = \frac{75 \text{ kg m}^2}{0.15 \text{ m}}\alpha$

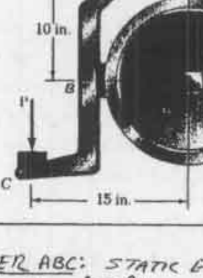
$\alpha = 0.423$ rad/s²

UNIF. ACCEL. MOTION

$\omega_0 = \alpha t$: 8π rad/s $= (0.423 \text{ rad/s}^2)t$; $t = 59.4$s ◀

NOTE: IF α IS REVERSED THEN T_A AND T_B ARE INTERCHANGED. THIS CAUSES NO CHANGE IN EQ.(1) AND EQ(2). THUS FROM EQ(3), α IS NOT CHANGED.

16.30



GIVEN:

$\bar{I} = 14 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

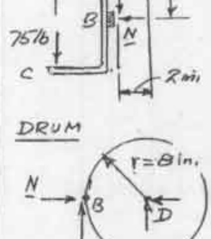
$\mu_k = 0.35$

$P = 75$ lb

$\omega_0 = 360$ rpm $\circlearrowright$

FIND: NUMBER OF REVOLUTIONS OF DRUM BEFORE IT COMES TO REST

LEVER ABC: STATIC EQUILIBRIUM (FRICTION FORCE $\downarrow$)



$F = \mu_k N = 0.35N$

$+\circlearrowright \Sigma M_A = 0$:

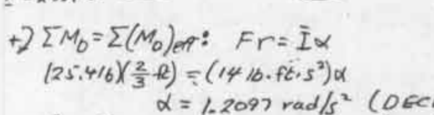
$N(10 \text{ in.}) - F(2 \text{ in.}) - (75\text{lb})(9 \text{ in.}) = 0$

$10N - 2(0.35N) - 675 = 0$

$N = 72.58$ lb

$F = \mu_k N = 0.35(72.58 \text{ lb}) = 25.40$ lb

DRUM



$r = 8$ in $= \frac{2}{3}$ ft

$\omega_0 = 360$ rpm$\left(\frac{2\pi}{60}\right)$

$\omega_0 = 12\pi$ rad/s

$+\circlearrowleft \Sigma M_D = \Sigma (M_D)_{eff}$: $Fr = \bar{I}\alpha$

$(25.4\text{lb})(\frac{2}{3}\text{ft}) = (14 \text{ lb}\cdot\text{ft}\cdot\text{s}^2)\alpha$

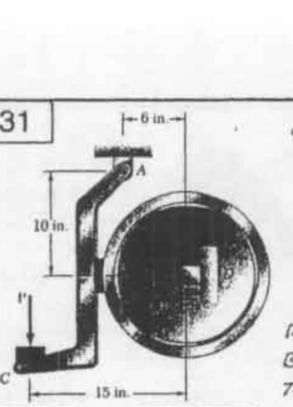
$\alpha = 1.2097$ rad/s² (DECELERATION)

$\omega^2 = \omega_0^2 + 2\alpha\theta$; $0 = (12\pi \text{ rad/s})^2 + 2(-1.2097 \text{ rad/s}^2)\theta$

$\theta = 587.4$ rad

$\theta = 587.4 \text{ rad}\left(\frac{1}{2\pi}\right) = 93.49$ rev $\quad \theta = 93.5$ rev ◀

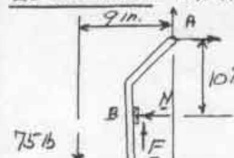
16.31



GIVEN:
$\bar{I} = 14\ lb\cdot ft\cdot s^2$
$\mu_k = 0.35$
$P = 75\ lb$
$\omega_0 = 360\ rpm$ ↻

FIND: NUMBER OF REVOLUTIONS OF DRUM BEFORE IT COMES TO REST.

LEVER ABC: STATIC EQUILIBRIUM (FRICTION FORCE ↑)



$F = \mu_k N = 0.35N$

$+\circlearrowright \Sigma M_A = 0$

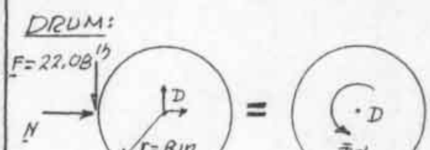
$N(10in.) + F(2in) - (75lb)(9in.) = 0$

$10N + 2(0.35N) - 675 = 0$

$N = 63.08\ lb$

$F = \mu_k N = 0.35(63.08\ lb) = 22.08\ lb$

DRUM:



$r = 8\ in. = \frac{2}{3}\ ft$

$\omega_0 = 360\ rpm\left(\frac{2\pi}{60}\right)$

$\omega_0 = 12\pi\ rad/s$

$+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{eff} = 0 \qquad Fr = \bar{I}\alpha$

$(22.08\ lb)(\frac{2}{3}ft) = (14\ lb\cdot ft\cdot s^2)\alpha$

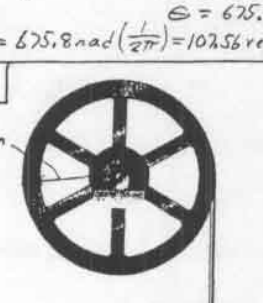
$\alpha = 1.5015\ rad/s^2$ (DECELERATION)

$\omega^2 = \omega_0^2 + 2\alpha\theta: \quad 0 = (12\pi\ rad/s)^2 + 2(-1.5015\ rad/s^2)\theta$

$\theta = 675.8\ rad$

$\theta = 675.8\ rad\left(\frac{1}{2\pi}\right) = 107.56\ rev; \quad \theta = 107.6\ rev$ ◀

16.32



GIVEN:
FLYWHEEL $m_F = 120\ kg$
$k = 375\ mm$
$\mu = 0$
$v_0 = 0$ AT $S = 0$

FIND:
(a) $\underline{a}$ OF BLOCK.
(b) $\underline{v}_A$ AFTER IT HAS MOVED 1.5 m.

$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{eff}:$

$(m_A g)r = \bar{I}\alpha + (m_A a)r$

$m_A g r = m_F k^2\left(\frac{a}{r}\right) + m_A a r$

$a = \dfrac{m_A g}{m_A + m_F\left(\frac{k}{r}\right)^2}$

$a = \dfrac{(15\ kg)(9.81\ m/s^2)}{15\ kg + (120\ kg)\left(\frac{375\ mm}{500\ mm}\right)^2} = 1.7836\ m/s$

(CONTINUED)

16.32 continued

(a) $\underline{a} = 1.7836\ m/s^2$ ↓ ◀

$\alpha = \frac{a}{r} = \frac{1.7836\ m/s}{0.5\ m} = 3.567\ rad/s^2$

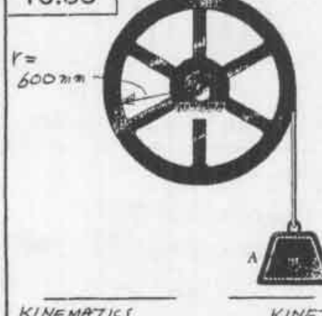
$\underline{\alpha} = 3.57\ rad/s^2$ ↻

(b) $v_A^2 = v_0^2 + 2aS$

FOR $S = 1.5\ m$: $v_A^2 = 0 + 2(1.7836\ m/s^2)(1.5\ m)$

$v_A = 2.313\ m/s \qquad \underline{v}_A = 2.31\ m/s$ ↓ ◀

16.33



GIVEN: SYSTEM RELEASED FROM REST:
1. IF $m_A = 12\ kg$, BLOCK FALLS 3 m IN 4.6 s
2. IF $m_A = 24\ kg$, BLOCK FALLS 3 m IN 3.1 s
ASSUME CONSTANT M_f DUE TO AXLE FRICTION.

FIND: $\bar{I}$

$+\circlearrowright \Sigma M_B = \Sigma(M_B)_{eff}:$

$(m_A g)r - M_f = \bar{I}\alpha + (m_A a)r$

$m_A g r - M_f = \bar{I}\frac{a}{r} + m_A a r \qquad (1)$

CASE 1: $y = 3\ m,\ t = 4.6\ s$

$y = \frac{1}{2}at^2; \quad 3\ m = \frac{1}{2}a(4.6s)^2; \quad a = 0.2836\ m/s^2$

$m_A = 12\ kg$

SUBSTITUTE INTO EQ(1)

$(12\ kg)(9.81\ m/s^2)(0.6\ m) - M_f = \bar{I}\left(\frac{0.2836\ m/s^2}{0.6\ m}\right) + (12\ kg)(0.2836\frac{m}{s^2})(0.6\ m)$

$70.632 - M_f = I(0.4727) + 2.0419 \qquad (2)$

CASE 2: $y = 3\ m,\ t = 3.1\ s$

$y = \frac{1}{2}at^2; \quad 3\ m = \frac{1}{2}a(3.1s)^2; \quad a = 0.6243\ m/s^2$

$m_A = 24\ kg$

SUBSTITUTE INTO EQ(1):

$(24\ kg)(9.81\ m/s^2)(0.6\ m) - M_f = \bar{I}\left(\frac{0.6243\ m/s^2}{0.6\ m}\right) + (24\ kg)(0.6243\frac{m}{s^2})(0.6\ m)$

$141.264 - M_f = \bar{I}(1.0406) + 8.9899 \qquad (3)$

SUBTRACT EQ(1) FROM EQ(2), TO ELIMINATE M_f

$70.632 = \bar{I}(1.0406 - 0.4727) + 6.948$

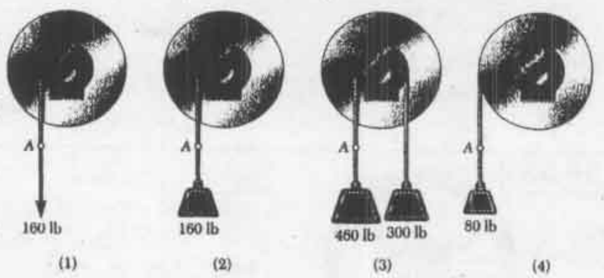
$63.684 = \bar{I}(0.5679)$

$\bar{I} = 112.14\ kg\cdot m^2$

$\bar{I} = 112.1\ kg\cdot m^2$ ◀

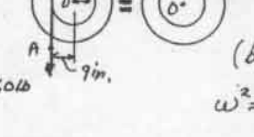
16.34

GIVEN: FOR EACH PULLEY, $\bar{I} = 15\ lb\cdot ft\cdot s^2$
INNER RADIUS = 9 in.; OUTER RADIUS = 18 in.



FIND: FOR EACH PULLEY: (a) $\underline{\alpha}$, (b) $\underline{\omega}$ WHEN $y_A = 10 ft$

CASE 1:



(a) $+\circlearrowright \Sigma M_o = \Sigma (M_o)_{eff}$

$(160\ lb)(\frac{9}{12} ft) = (15\ lb\cdot ft\cdot s^2)\alpha$

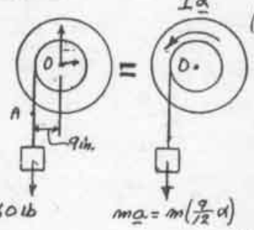
$\underline{\alpha} = 8\ rad/s^2 \circlearrowright$ ◀

(b) $\theta = \frac{10 ft}{(9/12\ ft)} = 13.333\ rad$

$\omega^2 = 2\alpha\theta = 2(8\ rad/s^2)(13.33\ rad)$

$\underline{\omega} = 14.61\ rad/s \circlearrowright$ ◀

CASE 2:



(a) $+\circlearrowright \Sigma M_o = \Sigma (M_o)_{eff}$

$(160)(\frac{9}{12}) = 15\alpha + ma(\frac{9}{12})$

$120 = 15\alpha + \frac{160}{32.2}(\frac{9}{12}\alpha)(\frac{9}{12})$

$120 = (15 + 2.795)\alpha$

$\alpha = 6.7435\ rad/s^2$

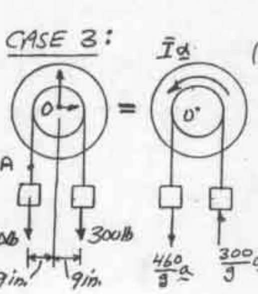
$\underline{\alpha} = 6.74\ rad/s^2 \circlearrowright$ ◀

(b) $\theta = \frac{10 ft}{(9/12 ft)} = 13.333\ rad$

$\omega^2 = 2\alpha\theta = 2(6.7435\ rad/s^2)(13.333\ rad)$

$\underline{\omega} = 13.41\ rad/s \circlearrowright$ ◀

CASE 3:



(a) $+\circlearrowright \Sigma M_o = \Sigma (M_o)_{eff}$

$(460)(\frac{9}{12}) - (300)(\frac{9}{12}) = 15\alpha + \frac{460}{32.2}a(\frac{9}{12}) + \frac{300}{32.2}a(\frac{9}{12})$

$120 = 15\alpha + \frac{460}{32.2}(\frac{9}{12})^2\alpha + \frac{300}{32.2}(\frac{9}{12})^2\alpha$

$\alpha = 4.2437\ rad/s^2$

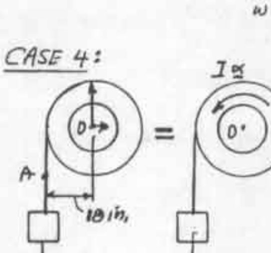
$\underline{\alpha} = 4.24\ rad/s^2 \circlearrowright$ ◀

(b) $\theta = \frac{10 ft}{9/12} = 13.333\ rad$

$\omega^2 = 2\alpha\theta = 2(4.2437)(13.333)$

$\underline{\omega} = 10.64\ rad/s \circlearrowright$ ◀

CASE 4:



(a) $+\circlearrowright \Sigma M_o = \Sigma (M_o)_{eff}$:

$(80)(\frac{18}{12}) = 15\alpha + \frac{80}{32.2}a(\frac{18}{12})$

$120 = 15\alpha + \frac{80}{32.2}(\frac{18}{12})^2\alpha$

$120 = (15 + 5.5901)\alpha$

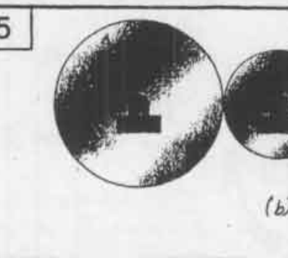
$\alpha = 5.828\ rad/s^2$

$\underline{\alpha} = 5.83\ rad/s^2 \circlearrowright$ ◀

(b) $\theta = \frac{10 ft}{(18/12\ ft)} = 6.6667\ rad$

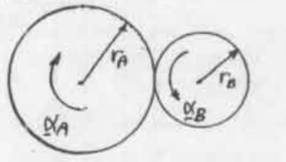
$\omega^2 = 2\alpha\theta = 2(5.828\ rad/s^2)(6.6667\ rad)$; $\underline{\omega} = 8.82\ \frac{rad}{s} \circlearrowright$ ◀

16.35



GIVEN:
$W_A = 10\ lb$, $r_A = 4.5\ in.$
$W_B = 4\ lb$, $r_B = 3\ in.$
$M = 5\ lb\cdot in.$ ⟲
FIND:
(a) $\underline{\alpha}_A$ AND $\underline{\alpha}_B$
(b) FRICTION FORCE EXERTED ON B.

KINEMATICS:



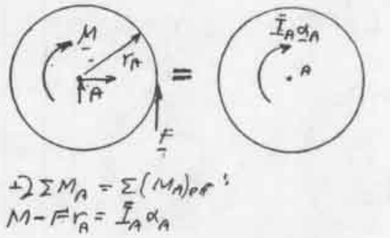
SINCE THE TANGENTIAL ACCELERATIONS OF THE OUTSIDE OF THE DISKS ARE EQUAL,

$r_A\alpha_A = r_B\alpha_B$

$\alpha_B = \frac{r_A}{r_B}\alpha_A \qquad (1)$

KINETICS:

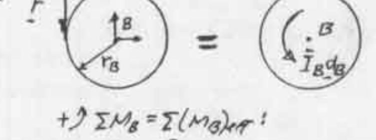
DISK A:



$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$:

$M - Fr_A = \bar{I}_A\alpha_A \qquad (2)$

DISK B:



$\bar{I}_B = \frac{1}{2}m_B r_B^2$

$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$:

$Fr_B = \bar{I}_B\alpha_B$

$Fr_B = (\frac{1}{2}m_B r_B^2)\alpha_B \qquad F = \frac{1}{2}m_B r_B\alpha_B \qquad (3)$

SUBSTITUTE FOR F FROM EQ(3) INTO EQ(2):

$M - (\frac{1}{2}m_B r_B\alpha_B)r_A = \bar{I}_A\alpha_A$

SUBSTITUTE FOR F FROM EQ(3), AND FOR α_B FROM EQ(1).

$M - \frac{1}{2}m_B r_B r_A(\frac{r_A}{r_B}\alpha_A) = \frac{1}{2}m_A r_A^2\alpha_A$

$M = \frac{1}{2}(m_A + m_B)r_A^2\alpha_A$

$\alpha_A = \frac{2M}{(m_A + m_B)r_A^2} = \frac{2Mg}{(W_A + W_B)r_A^2}$

DATA: $W_A = 10\ lb$, $W_B = 4\ lb$

(a) $r_A = 4.5\ in. = 0.375\ ft$; $M = 5\ lb\cdot in. = \frac{5}{12}\ lb\cdot ft$

$\alpha_A = \frac{2(5/12\ lb\cdot ft)(32.2\ ft/s^2)}{(10\ lb + 4\ lb)(0.375\ ft)^2}$ $\qquad \underline{\alpha}_A = 13.63\ rad/s^2 \circlearrowleft$ ◀

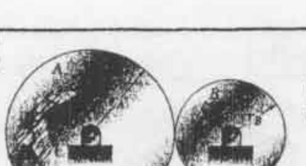
EQ(1): $\alpha_B = \frac{r_A}{r_B}\alpha_A = \frac{4.5\ in.}{3\ in.}(13.63\ rad/s^2)$

$\alpha_B = 20.44\ rad/s^2$ $\qquad \underline{\alpha}_B = 20.4\ rad/s^2 \circlearrowright$ ◀

(b) EQ.3: $F = \frac{1}{2}m_B r_B\alpha_B = \frac{1}{2}\frac{W_B}{g}r_B\alpha_B$

$F = \frac{1}{2}\frac{4\ lb}{32.2\ ft/s^2}(\frac{3}{12}\ ft)(20.44\ rad/s^2)$; $\underline{F} = 0.317\ lb \downarrow$ ◀

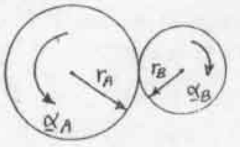
16.36



GIVEN:
$W_A = 10\,lb$, $r_A = 4.5\,in.$
$W_B = 4\,lb$, $r_B = 3\,in.$
$M = 5\,lb\cdot in.$ ↻

FIND: (a) α_A AND α_B
(b) FRICTION FORCE EXERTED ON B.

KINEMATICS:



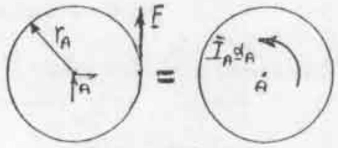
SINCE THE TANGENTIAL ACCELERATION OF THE OUTSIDE OF THE DISKS ARE EQUAL.

$$r_A \alpha_A = r_B \alpha_B$$

$$\alpha_A = \frac{r_B}{r_A}\alpha_B \qquad (1)$$

KINETICS:

DISK A:



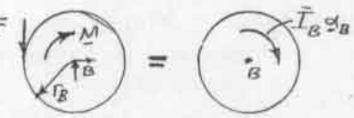
$\bar{I}_A = \frac{1}{2} m_A r_A^2$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$$

$$F r_A = \bar{I}_A \alpha_A$$

$$F r_A = \tfrac{1}{2} m_A r_A^2 \alpha_A \qquad F = \tfrac{1}{2} m_A r_A \alpha_A \qquad (2)$$

DISK B:



$$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$$

$$M - F r_B = \bar{I}_B \alpha_B \qquad (3)$$

SUBSTITUTE FOR F FROM EQ (2) INTO EQ (3)

$$M - \left(\tfrac{1}{2} m_A r_A \alpha_A\right) r_B = \bar{I}_B \alpha_B$$

SUBSTITUTE FOR α_A FROM EQ (1), AND $\bar{I}_B = \frac{1}{2} m_B r_B^2$

$$M - \tfrac{1}{2} m_A r_A r_B \left(\frac{r_B}{r_A}\alpha_B\right) = \tfrac{1}{2} m_B r_B^2 \alpha_B$$

$$M = \tfrac{1}{2}(m_A + m_B) r_B^2 \alpha_B$$

$$\alpha_B = \frac{2M}{(m_A + m_B) r_B^2} = \frac{2Mg}{(W_A + W_B) r_B^2}$$

DATA: $W_A = 10\,lb$, $W_B = 4\,lb$, $r_A = 4.5\,in.$
$r_B = 3\,in. = 0.25\,ft$; $M = 5\,lb\cdot in. = \frac{5}{12}\,lb\cdot ft$

(a)
$$\alpha_B = \frac{2(5/12\,lb\cdot ft)(32.2\,ft/s^2)}{(10\,lb + 4\,lb)(0.25\,ft)^2} = 30.667\,\frac{rad}{s^2}$$

$$\alpha_B = 30.7\,\frac{rad}{s^2} \circlearrowright \quad ◀$$

EQ(1): $\alpha_A = \frac{r_B}{r_A}\alpha_B = \left(\frac{3\,in.}{4.5\,in.}\right)(30.667\,\frac{rad}{s^2}) = 20.44\,rad/s^2$

$$\alpha_A = 20.4\,rad/s^2 \circlearrowleft \quad ◀$$

(b) EQ(2) $F = \frac{1}{2} m_A r_A \alpha_A = \frac{1}{2}\frac{W_A}{g} r_A \alpha_A$

$$F = \tfrac{1}{2}\left(\frac{10\,lb}{32.2\,ft/s^2}\right)\left(\frac{4.5}{12}\,ft\right)(20.44\,rad/s^2) = 1.190\,lb$$

FRICTION FORCE ON DISK B: $F = 1.190\,lb \downarrow$ ◀

16.37

GIVEN:
DISK A: $W = 20\,lb$, $r_A = 8\,in.$
DISK B: $W = 12\,lb$, $r_B = 6\,in.$

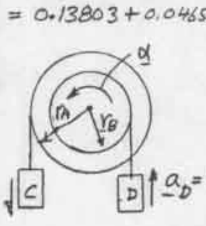
FIND: (a) a_C
(b) a_D

TOTAL $\bar{I}$: $\bar{I} = \frac{1}{2} m_A r_A^2 + \frac{1}{2} m_B r_B^2$

$$= \tfrac{1}{2}\frac{20\,lb}{32.2\,ft/s^2}\left(\tfrac{8}{12}\,ft\right)^2 + \tfrac{1}{2}\frac{12\,lb}{32.2\,ft/s^2}\left(\tfrac{6}{12}\,ft\right)^2$$

$$= 0.13803 + 0.04658 = 0.18461\,lb\cdot ft\cdot s^2$$

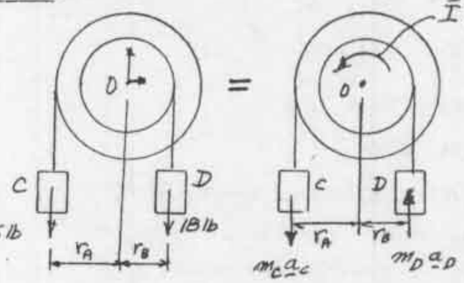
KINEMATICS:



$a_C = r_A\alpha = \left(\frac{8}{12}\,ft\right)\alpha$
$a_C = \frac{2}{3}\alpha$

$a_D = r_B\alpha = \left(\frac{6}{12}\,ft\right)\alpha = \frac{1}{2}\alpha$

KINETICS



$$+\circlearrowleft \Sigma M_O = \Sigma (M_O)_{eff}$$

$$(15\,lb) r_A - (18\,lb) r_B = \bar{I}\alpha + m_C a_C r_A + m_D a_D r_B$$

$$(15\,lb)\left(\tfrac{8}{12}\,ft\right) - (18\,lb)\left(\tfrac{6}{12}\,ft\right) = 0.18461\,\alpha + \frac{15\,lb}{32.2\,ft/s^2}\left(\tfrac{2}{3}\alpha\right)\left(\tfrac{8}{12}\,ft\right) + \frac{18\,lb}{32.2\,ft/s^2}\left(\tfrac{1}{2}\alpha\right)\left(\tfrac{6}{12}\,ft\right)$$

$$10 - 9 = (0.18461 + 0.20704 + 0.13975)\alpha$$

$$1 = 0.5314\,\alpha$$

$$\alpha = 1.8818\,rad/s^2$$

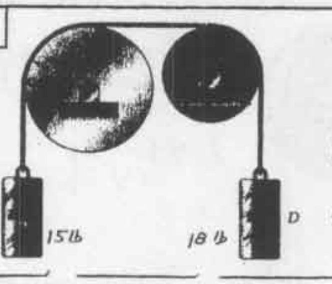
(a) $a_C = \frac{2}{3}\alpha = \frac{2}{3}(1.8818\,rad/s^2)$

$$a_C = 1.255\,ft/s^2 \downarrow \quad ◀$$

(b) $a_D = \frac{1}{2}\alpha = \frac{1}{2}(1.8818\,rad/s^2)$

$$a_D = 0.941\,ft/s^2 \uparrow \quad ◀$$

16.38



GIVEN:
DISK A:
$W_A = 20\,lb,\ r_A = 8\,in$
DISK B:
$W_B = 12\,lb,\ r_B = 6\,in.$

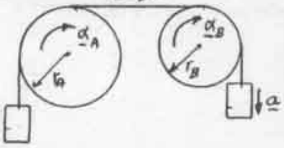
FIND: (a) $\underline{a}_C$
(b) $\underline{a}_D$

MOMENTS OF INERTIA

$$\bar{I}_A = \tfrac{1}{2} m_A r_A^2 = \tfrac{1}{2}\,\frac{20\,lb}{32.2\,ft/s^2}\left(\tfrac{8}{12}\,ft\right)^2 = 0.13803\ lb\cdot ft\cdot s^2$$

$$\bar{I}_B = \tfrac{1}{2} m_B r_B^2 = \tfrac{1}{2}\,\frac{12\,lb}{32.2\,ft/s^2}\left(\tfrac{6}{12}\,ft\right)^2 = 0.04658\ lb\cdot ft\cdot s^2$$

KINEMATICS

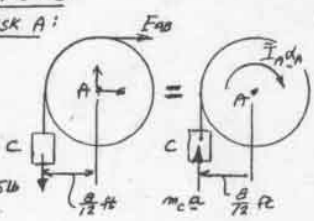


$$\alpha_A = \frac{a}{r_A} = \frac{a}{(8/12\,ft)} = 1.5a$$

$$\alpha_B = \frac{a}{r_B} = \frac{a}{(6/12\,ft)} = 2a$$

KINETICS:

DISK A:



F_{AB} = TENSION IN CORD BETWEEN DISKS

$$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$$

$$F_{AB}\left(\tfrac{8}{12}\,ft\right) - 15\,lb\left(\tfrac{8}{12}\,ft\right) = \bar{I}_A\alpha_A + m_C a\left(\tfrac{8}{12}\,ft\right)$$

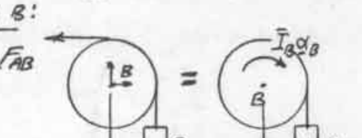
$$\tfrac{2}{3}F_{AB} - 10 = (0.13803)(1.5a) + \frac{15\,lb}{32.2\,ft/s^2}\,a\left(\tfrac{2}{3}\right)$$

$$\tfrac{2}{3}F_{AB} - 10 = 0.20705a + 0.31056a$$

$$\tfrac{2}{3}F_{AB} - 10 = 0.51760a$$

$$F_{AB} = 15 + 0.77641a \qquad (1)$$

DISK B:



$$\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$$

$$(18\,lb)\left(\tfrac{6}{12}\,ft\right) - F_{AB}\left(\tfrac{6}{12}\,ft\right) = I_B\alpha_B + m_D a\left(\tfrac{6}{12}\,ft\right)$$

$$9 - 0.5F_{AB} = (0.04658)(2a) + \frac{18\,lb}{32.2\,ft/s^2}\,a\left(\tfrac{1}{2}\right)$$

$$9 - 0.5F_{AB} = 0.0931a + 0.2795a$$

SUBSTITUTE FOR F_{AB} FROM EQ(1)

$$9 - 0.5(15 + 0.77641a) = 0.37266a$$

$$9 - 7.5 - 0.3882a = 0.37266a$$

$$1.5 = 0.76086a$$

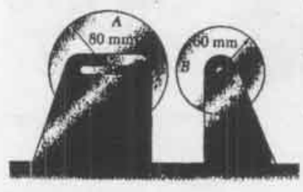
$$a = 1.971\ ft/s^2$$

BOTH a_C AND a_D HAVE THE SAME MAGNITUDE

$$\underline{a}_C = 1.971\ ft/s^2 \uparrow$$

$$\underline{a}_D = 1.971\ ft/s^2 \downarrow$$

16.39 and 16.40



GIVEN: $m_A = 6\,kg;\ m_B = 3\,kg$
$N = 20\,N,\ \mu_k = 0.15$

PROBLEM 16.39:
$(\omega_A)_0 = 360\,rpm \circlearrowright;\ (\omega_B)_0 = 0$

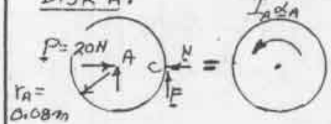
PROBLEM 16.40:
$(\omega_A)_0 = 0;\ (\omega_B)_0 = 360\,rpm \circlearrowright$

FOR EACH PROBLEM:

FIND: (a) $\underline{\alpha}_A$ AND $\underline{\alpha}_B$. (b) FINAL VELOCITIES $\underline{\omega}_A$ AND $\underline{\omega}_B$

WHILE SLIPPING OCCURS, A FRICTION FORCE F↑ IS APPLIED TO DISK A, AND F↓ TO DISK B.

DISK A:



$$I_A = \tfrac{1}{2} m_A r_A^2 = \tfrac{1}{2}(6\,kg)(0.08\,m)^2 = 0.0192\ kg\cdot m^2$$

$\Sigma F:\ N = P = 20\,N$

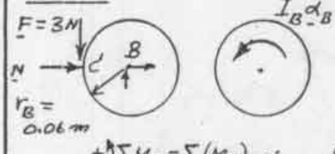
$F = \mu N = 0.15(20) = 3\,N$

$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}:\ F r_A = \bar{I}_A\alpha_A$$

$$(3N)(0.08\,m) = (0.0192\,kg\cdot m^2)\alpha_A$$

$$\alpha_A = 12.5\ rad/s^2 \qquad \underline{\alpha}_A = 12.5\ rad/s^2 \circlearrowleft$$

DISK B:



$$\bar{I}_B = \tfrac{1}{2} m_B r_B^2 = \tfrac{1}{2}(3\,kg)(0.06\,m)^2 = 0.0054\ kg\cdot m^2$$

$$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}:\quad F r_B = \bar{I}_B\alpha_B$$

$$(3N)(0.06\,m) = (0.0054\,kg\cdot m^2)\alpha_B$$

$$\alpha_B = 33.33\ rad/s^2$$

$$\underline{\alpha}_B = 33.3\ rad/s^2 \circlearrowleft$$

PROBLEM 16.39: $(\omega_A)_0 = 360\,rpm\left(\frac{2\pi}{60}\right) = 12\pi\,rad/s \circlearrowright;\ (\omega_B)_0 = 0$

DISKS WILL STOP SLIDING, WHEN $v_C = v_{C'}$, THAT IS WHEN

$$\omega_A r_A = \omega_B r_B$$

$$[(\omega_A)_0 - \alpha_A t]\, r_A = \alpha_B t\, r_B$$

$$(12\pi - 12.5t)(0.08) = (33.33t)(0.06)$$

$$3.0159 - t = 2t; \qquad t = 1.00531\,s$$

$$+\circlearrowright\ \omega_A = (\omega_A)_0 - \alpha_A t = 12\pi - 12.5(1.00531) = 25.132\ rad/s$$

$$\omega_A = 25.132\ rad/s\left(\tfrac{60}{2\pi}\right) = 240\,rpm \qquad \underline{\omega}_A = 240\,rpm \circlearrowright$$

$$+\circlearrowleft\ \omega_B = \alpha_B t = (33.33)(1.00531) = 33.507\ rad/s$$

$$\omega_B = 33.507\ rad/s\left(\tfrac{60}{2\pi}\right) = 320\,rpm \qquad \underline{\omega}_B = 320\,rpm \circlearrowleft$$

PROBLEM 16.40: $(\omega_A)_0 = 0;\ (\omega_B)_0 = 360\,rpm\left(\frac{2\pi}{60}\right) = 12\pi\,\frac{rad}{s}$

SLIDING STOPS WHEN $v_C = v_{C'}$ THAT IS WHEN

$$\omega_A r_A = \omega_B r_B$$

$$(\alpha_A t) r_A = [(\omega_B)_0 - \alpha_B t]\, r_B$$

$$(12.5t)(0.08) = (12\pi - 33.33t)(0.06)$$

$$t = 2.26195 - 2t; \qquad t = 0.75398\,s$$

$$+\circlearrowleft\ \omega_A = \alpha_A t = (12.5)(0.75398) = 9.4248\ rad/s$$

$$\omega_A = 9.4248\ rad/s\left(\tfrac{60}{2\pi}\right) = 90\,rpm$$

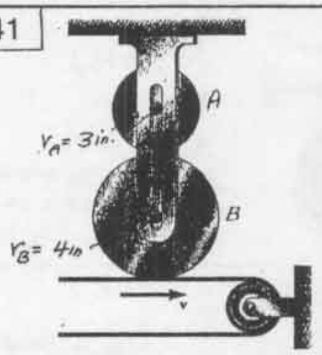
$$\underline{\omega}_A = 90\,rpm \circlearrowleft$$

$$+\circlearrowright\ \omega_B = (\omega_B)_0 - \alpha_B t = 12\pi - (33.33)(0.75398) = 12.569\ rad/s$$

$$\omega_B = 12.569\ rad/s\left(\tfrac{60}{2\pi}\right) = 120\,rpm$$

$$\underline{\omega}_B = 120\,rpm \circlearrowright$$

16.41

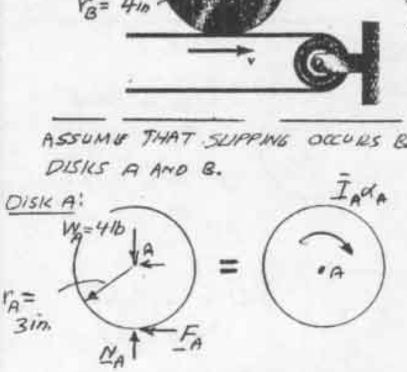


GIVEN:
$W_A = 4\ lb$
$W_B = 9\ lb$
$\mu_k = 0.20$ AT ALL SURFACES

FIND: INITIAL ANGULAR ACCELERATION OF EACH DISK

ASSUME THAT SLIPPING OCCURS BETWEEN DISKS A AND B.

DISK A:



$\bar{I}_A = \frac{1}{2} m_A r_A^2 = \frac{1}{2} \frac{4\ lb}{32.2} \left(\frac{3}{12} ft\right)^2$

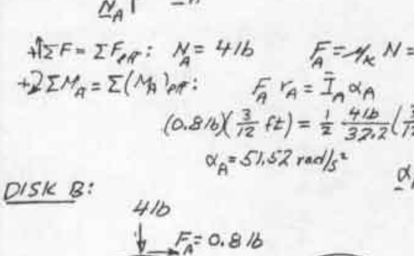
$+\uparrow \Sigma F = \Sigma F_{eff}$: $N_A = 4\ lb$ $\quad F_A = \mu_k N = 0.2(4\ lb) = 0.8\ lb$

$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$: $\quad F_A r_A = \bar{I}_A \alpha_A$

$(0.8\ lb)\left(\frac{3}{12} ft\right) = \frac{1}{2} \frac{4\ lb}{32.2} \left(\frac{3}{12} ft\right)^2 \alpha_A$

$\alpha_A = 51.52\ rad/s^2$ $\quad \underline{\alpha}_A = 51.5\ \frac{rad}{s^2} \circlearrowright$ ◀

DISK B:



$\bar{I}_B = \frac{1}{2} m_B r_B^2 = \frac{1}{2} \frac{9\ lb}{32.2} \left(\frac{4}{12} ft\right)^2$

$+\uparrow \Sigma F = \Sigma F_{eff}$: $\quad N_B = 4 + 9 = 13\ lb$

$F_B = \mu_k N_B = 0.20(13\ lb) = 2.6\ lb$

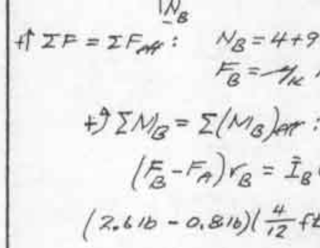
$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$:

$(F_B - F_A) r_B = \bar{I}_B \alpha_B$

$(2.6\ lb - 0.8\ lb)\left(\frac{4}{12} ft\right) = \frac{1}{2} \frac{9\ lb}{32.2} \left(\frac{4}{12} ft\right)^2 \alpha_B$

$\alpha_B = 38.64\ rad/s^2$ $\quad \underline{\alpha}_B = 38.6\ rad/s^2 \circlearrowleft$ ◀

KINEMATICS:



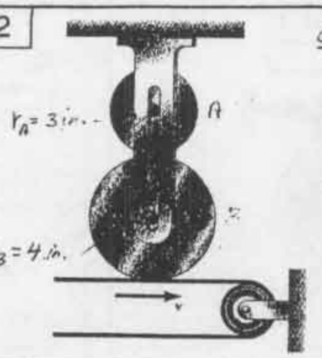
WE CALCULATE THE TANGENTIAL COMPONENTS OF POINTS OF CONTACT

$(a_C)_t = r_A \alpha_A = \left(\frac{3}{12} ft\right)(51.52\ rad/s^2) = 12.88\ ft/s^2$

$(a_{C'})_t = r_B \alpha_B = \left(\frac{4}{12} ft\right)(38.64\ rad/s^2) = 12.88\ ft/s^2$

WE FIND THAT SLIPPING DOES NOT OCCUR BETWEEN DISKS. BUT SINCE $(a_C)_t = (a_{C'})_t$ SLIPPING IMPENDS AND THAT $F_A = \mu_k N_A = 0.8\ lb$ AND ABOVE RESULTS ARE VALID.

16.42

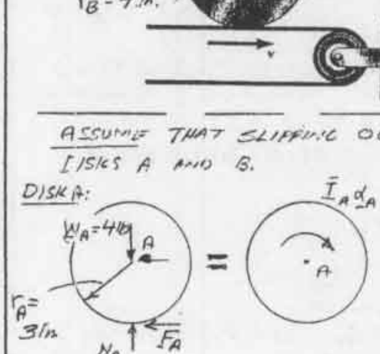


GIVEN:
$W_A = 4\ lb$
$W_B = 9\ lb$
$\mu_k = 0.10$ BETWEEN THE DISKS
$\mu_k = 0.20$ BETWEEN BELT AND DISK B

FIND: INITIAL ANGULAR ACCELERATION OF EACH DISK.

ASSUME THAT SLIPPING OCCURS BETWEEN DISKS A AND B.

DISK A:



$\bar{I}_A = \frac{1}{2} m_A r_A^2 \alpha_A = \frac{1}{2} \frac{4\ lb}{32.2} \left(\frac{3}{12} ft\right)^2$

$+\uparrow \Sigma F = \Sigma (F)_{eff}$: $\quad N_A = 4\ lb$ $\quad F_A = \mu_k N_A = 0.1(4\ lb) = 0.4\ lb$

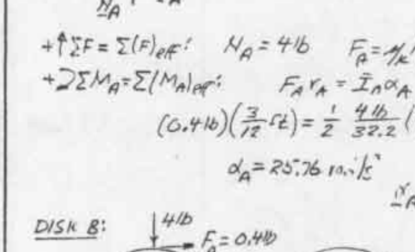
$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$: $\quad F_A r_A = \bar{I}_A \alpha_A$

$(0.4\ lb)\left(\frac{3}{12} ft\right) = \frac{1}{2} \frac{4\ lb}{32.2} \left(\frac{3}{12} ft\right)^2 \alpha_A$

$\alpha_A = 25.76\ rad/s^2$

$\underline{\alpha}_A = 25.8\ rad/s^2 \circlearrowright$ ◀

DISK B:



$\bar{I}_B = \frac{1}{2} m_B r_B^2 = \frac{1}{2} \frac{9\ lb}{32.2} \left(\frac{4}{12} ft\right)^2$

$\Sigma F = \Sigma F_{eff}$: $\quad N_B = 4 + 9 = 13\ lb$

$F_B = \mu_k N_B = 0.20(13\ lb) = 2.6\ lb$

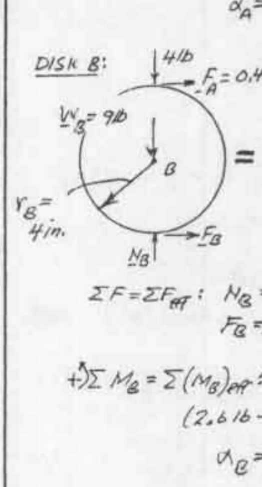
$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$: $\quad (F_B - F_A) r_B = \bar{I}_B \alpha_B$

$(2.6\ lb - 0.4\ lb)\left(\frac{4}{12} in.\right) = \frac{1}{2} \frac{9\ lb}{32.2} \left(\frac{4}{12} in.\right)^2 \alpha_B$

$\alpha_B = 47.23\ rad/s^2$

$\underline{\alpha}_B = 47.2\ rad/s^2 \circlearrowleft$ ◀

KINEMATICS:



WE CALCULATE THE TANGENTIAL COMPONENTS OF POINTS OF CONTACT

$(a_C)_t = r_A \alpha_A = \left(\frac{3}{12} ft\right)(25.76\ rad/s^2) = 6.44\ ft/s^2 \leftarrow$

$(a_{C'})_t = r_B \alpha_B = \left(\frac{4}{12} ft\right)(47.23\ rad/s^2) = 15.74\ ft/s^2 \leftarrow$

SINCE $(\overleftarrow{a}_{C'})_t > (\overleftarrow{a}_C)_t$, WE CONFIRM THAT ASSUMPTION OF SLIPPING BETWEEN DISKS IS TRUE

16.43 and 16.44

GIVEN:
$P = 2.5\ lb$, $\mu_k = 0.25$
$W_A = 6\ lb$, $r_A = 3\ in.$
$W_B = 15\ lb$, $r_B = 5\ in.$

FIND:
(a) α_A AND α_B
(b) FINAL ω_A AND ω_B

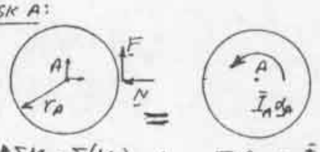
PROBLEM 16.43: $(\omega_A)_0 = 375\ rpm$ ↻, $(\omega_B)_0 = 0$

PROBLEM 16.44: $(\omega_B)_0 = 0$, $(\omega_B)_0 = 375\ rpm$ ↺

WHILE SLIPPING OCCURS:

$$F = \mu_k N = \mu_k P = 0.25(2.5\ lb) = 0.625\ lb$$

DISK A:



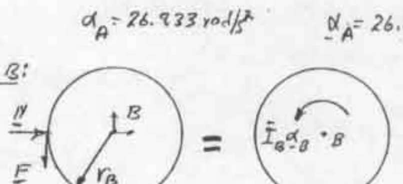
$$+↻\Sigma M_A = \Sigma (M_A)_{eff}: \quad F r_A = \bar{I}_A \alpha = \frac{1}{2} m_A r_A^2 \alpha_A$$

$$F = \frac{1}{2} m_A r_A \alpha$$

$$0.625\ lb = \frac{1}{2}\ \frac{6\ lb}{32.2\ ft/s^2}\left(\frac{3}{12}\ ft\right)\alpha_A$$

$$\alpha_A = 26.833\ rad/s^2 \qquad \alpha_A = 26.8\ rad/s^2 ↻ \blacktriangleleft$$

DISK B:



$$+↻\Sigma M_B = \Sigma (M_B)_{eff}: \quad F r_B = \bar{I}_B \alpha_B = \frac{1}{2} m_B r_B^2 \alpha_B$$

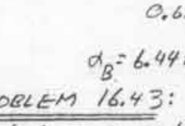
$$F = \frac{1}{2} m_B r_B \alpha_B$$

$$0.625\ lb = \frac{1}{2}\ \frac{15\ lb}{32.2\ ft/s^2}\left(\frac{5}{12}\ ft\right)\alpha_B$$

$$\alpha_B = 6.44\ rad/s^2 \qquad \alpha_B = 6.44\ rad/s^2 ↻ \blacktriangleleft$$

PROBLEM 16.43:

$$(\omega_A)_0 = 375\ rpm\left(\frac{2\pi}{60}\right) = 39.27\ rad/s ↺\ ; \ (\omega_B)_0 = 0$$



WHEN DISKS STOP SLIDING

$$v_P = v_P: \quad \omega_A r_A = \omega_B r_B \qquad (1)$$

$$[(\omega_A)_0 - \alpha_A t]\, r_A = (\alpha_B t)\, r_B$$

$$(39.27 - 26.833\,t)(3\ in.) = (6.44\,t)(5\ in.)$$

$$t = 1.0454\ s$$

$$\omega_A = (\omega_A)_0 - \alpha_A t = 39.27 - (26.833)(1.0454)$$

$$\omega_A = 11.22\ rad/s\left(\frac{60}{2\pi}\right) \qquad \omega_A = 107.1\ rpm ↺ \blacktriangleleft$$

$$EQ.(1)\ (107.2\ rpm)(3\ in.) = \omega_B(5\ in.) \qquad \omega_B = 64.3\ rpm ↻ \blacktriangleleft$$

PROBLEM 16.44: $(\omega_A)_0 = 0$; $(\omega_B)_0 = 375\ rpm\left(\frac{2\pi}{60}\right) = 39.27\ rad/s$ ↺

$$EQ(1): \ \omega_A r_A = \omega_B r_B; \quad \alpha_A t\, r_A = [(\omega_B)_0 - \alpha_B t]\, r_B$$

$$(26.833\,t)(3\ in.) = [39.27 - 6.44\,t](5\ in.)$$

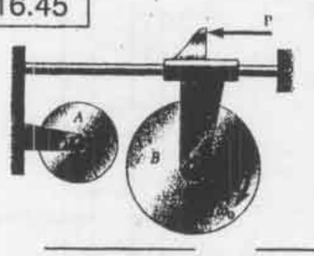
$$t = 1.742\ s$$

$$\omega_A = \alpha_A t = (26.833)(1.742) = 46.74\ rad/s\left(\frac{60}{2\pi}\right)$$

$$\omega_A = 446\ rpm ↻ \blacktriangleleft$$

$$EQ(1): \ (446\ rpm)(3\ in) = \omega_B(5\ in) \qquad \omega_B = 268\ rpm ↺ \blacktriangleleft$$

16.45

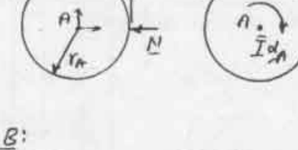


GIVEN:
$(\omega_B)_0 = \omega_0$ ↺
$(\omega_A)_0 = 0$

SHOW THAT:
(a) FINAL ω_A AND ω_B ARE INDEPENDENT OF μ_k
(b) FINAL $\omega_B = f(\omega_0, m_A/m_B)$

WHILE SLIPPING OCCURS: $F = \mu_k N = \mu_k P$

DISK A



$$↻\Sigma M_A = \Sigma (M_A)_{eff}:$$

$$F r_A = \bar{I}_A \alpha_A$$

$$\mu_k P r_A = \frac{1}{2} m_A r_A^2 \alpha_A$$

$$\alpha_A = \frac{2P\mu_k}{m_A r_A} \qquad (1)$$

DISK B:

$$+\Sigma M_B = \Sigma (M_B)_{eff}$$

$$F r_B = \bar{I}_B \alpha_B$$

$$\mu_k P r_B = \frac{1}{2} m_B r_B^2 \alpha_B$$

$$\alpha_B = \frac{2P\mu_k}{m_B r_B} \qquad (2)$$

AT ANY TIME t

$$+↻\ \omega_A = 0 + \alpha_A t = \frac{2P\mu_k}{m_A r_A}\, t \qquad (3)$$

$$+↺\ \omega_B = \omega_0 - \alpha_B t = \omega_0 - \frac{2P\mu_k}{m_B r_B}\, t \qquad (4)$$

SLIPPING ENDS WHEN $\omega_A r_A = \omega_B r_B$

$$\alpha_A t\, r_A = (\omega_0 - \alpha_B t)\, r_B$$

$$(\alpha_A r_A + \alpha_B r_B)\, t = \omega_0 r_B$$

SUBSTITUTE FROM EQS (1)+(2): $\left[\frac{2P\mu_k}{m_A r_A} r_A + \frac{2P\mu_k}{m_B r_B} r_B\right] t = \omega_0 r_B$

$$2P\mu_k\left(\frac{1}{m_A} + \frac{1}{m_B}\right) t = \omega_0 r_B\ ; \quad t = \frac{\omega_0 r_B}{2P\mu_k}\cdot\frac{1}{\frac{1}{m_A} + \frac{1}{m_B}}$$

$$t = \frac{\omega_0 r_B}{2P\mu_k}\cdot\frac{m_A m_B}{m_A + m_B}$$

$$EQ(3): \ \omega_A = \alpha_A t = \frac{2P\mu_k}{m_A r_A}\cdot\left[\frac{\omega_0 r_B}{2P\mu_k}\cdot\frac{m_A m_B}{m_A + m_B}\right]$$

$$\omega_A = \frac{r_B}{r_A}\cdot\frac{m_B}{m_A + m_B}\,\omega_0$$

(ω_A IS INDEPENDENT OF μ_k, QED)

$$EQ(4) \ \omega_B = \omega_0 - \alpha_B t = \omega_0 - \frac{2P\mu_k}{m_B r_B}\cdot\left[\frac{\omega_0 r_B}{2P\mu_k}\cdot\frac{m_A m_B}{m_A + m_B}\right]$$

$$\omega_B = \omega_0\left\{1 - \frac{m_A}{m_A + m_B}\right\}$$

$$\omega_B = \omega_0\,\frac{m_A + m_B - m_A}{m_A + m_B} = \omega_0\,\frac{m_B}{m_A + m_B}$$

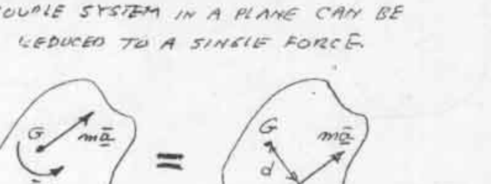
$$\omega_B = \frac{\omega_0}{\frac{m_A}{m_B} + 1}$$

ω_B DEPENDS ONLY UPON ω_0 AND $\frac{m_A}{m_B}$ (QED)

16.46

SHOW THAT SYSTEM OF EFFECTIVE FORCES FOR A SLAB REDUCES TO $m\bar{a}$ AND EXPRESS DISTANCE FROM ITS LINE OF ACTION TO G IN TERMS OF $\bar{k}$, $\bar{a}$, AND α.

WE KNOW THAT THE SYSTEM OF EFFECTIVE FORCES CAN BE REDUCED TO THE VECTOR $m\bar{a}$ AT G AND THE COUPLE $\bar{I}\alpha$. WE FURTHER KNOW FROM CHAPTER 3 OF STATICS THAT A FORCE-COUPLE SYSTEM IN A PLANE CAN BE FURTHER REDUCED TO A SINGLE FORCE.

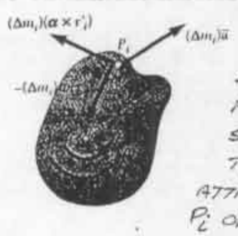


THE PERPENDICULAR DISTANCE d FROM G TO THE LINE OF ACTION OF THE SINGLE VECTOR $m\bar{a}$ IS EXPRESSED BY WRITING

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}: \quad \bar{I}\alpha = (m\bar{a})d$

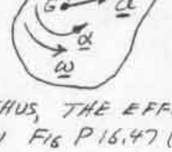
$$d = \frac{\bar{I}\alpha}{m\bar{a}} = \frac{m\bar{k}^2\alpha}{m\bar{a}} \qquad d = \frac{\bar{k}^2\alpha}{\bar{a}}$$

16.47



SHOW THAT THE SYSTEM OF EFFECTIVE FORCES OF A RIGID SLAB CONSISTS OF THE VECTORS SHOWN ATTACHED TO THE PARTICLES P_i OF THE SLAB. FURTHER SHOW THAT THE EFFECTIVE FORCES REDUCE TO $m\bar{a}$ ATTACHED AT G AND A COUPLE $\bar{I}\alpha$.

KINEMATICS THE ACCELERATION OF P_i IS



$$a_i = \bar{a} + a_{P_i/G}$$

$$a_i = \bar{a} + \alpha \times r_i' + \omega \times (\omega \times r_i')$$

$$= \bar{a} + \alpha \times r_i' - \omega^2 r_i'$$

NOTE THAT $\alpha \times r_i'$ IS $\perp$ TO r_i'

THUS, THE EFFECTIVE FORCES ARE AS SHOWN IN FIG P16.47 (also shown above). WE WRITE

$$(\Delta m_i)a_i = (\Delta m_i)\bar{a} + (\Delta m_i)(\alpha \times r_i') - (\Delta m_i)\,\omega^2 r_i'$$

THE SUM OF THE EFFECTIVE FORCES IS

$$\Sigma(\Delta m_i)a_i = \Sigma(\Delta m_i)\bar{a} + \Sigma(\Delta m_i)(\alpha \times r_i') - \Sigma(\Delta m_i)\,\omega^2 r_i'$$

$$\Sigma(\Delta m_i)a_i = \bar{a}\,\Sigma(\Delta m_i) + \alpha \times \Sigma(\Delta m_i)r_i' - \omega^2\,\Sigma(\Delta m_i)r_i'$$

(CONTINUED)

16.47 continued

WE NOTE THAT

$\Sigma(\Delta m_i) = m$. AND SINCE G IS THE MASS CENTER

$$\Sigma(\Delta m_i)r_i' = m\bar{r}_i' = 0$$

THUS, $\Sigma(\Delta m_i)a_i = m\bar{a}$ (1)

THE SUM OF THE MOMENTS ABOUT G OF THE EFFECTIVE FORCES IS:

$$\Sigma(r_i' \times \Delta m_i a_i) = \Sigma r_i' \times \Delta m_i \bar{a} + \Sigma r_i' \times (\Delta m_i)(\alpha \times r_i') - \Sigma r_i' \times (\Delta m_i)\,\omega^2 r_i'$$

$$\Sigma(r_i' \times \Delta m_i a_i) = (\Sigma r_i' \Delta m_i)\bar{a} + \Sigma\left[r_i' \times (\alpha \times r_i')\Delta m_i\right] - \omega^2\Sigma(r_i' \times r_i')\Delta m_i$$

SINCE G IS THE MASS CENTER, $\Sigma r_i' \Delta m_i = 0$

ALSO, FOR EACH PARTICLE, $r_i' \times r_i' = 0$

THUS

$$\Sigma(r_i' \times \Delta m_i a_i) = \Sigma\left[r_i' \times (\alpha_i \times r_i')\Delta m_i\right]$$

SINCE $\alpha \perp r_i'$, WE HAVE $r_i' \times (\alpha \times r_i') = r_i^2\alpha$ AND

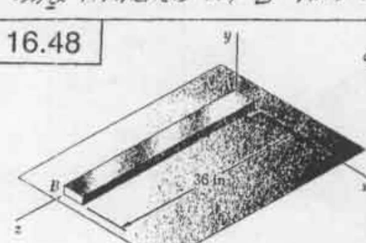
$$\Sigma(r_i' \times \Delta m_i a_i) = \Sigma r_i'^2(\Delta m_i)\alpha = (\Sigma r_i'^2 \Delta m_i)\alpha$$

SINCE $\Sigma r_i'^2 \Delta m_i = \bar{I}$

$$\Sigma(r_i' \times \Delta m_i a_i) = \bar{I}\alpha \qquad (2)$$

FROM EQS. (1) AND (2) WE CONCLUDE THAT SYSTEM OF EFFECTIVE FORCES REDUCE TO $m\bar{a}$ ATTACHED AT G AND A COUPLE $\bar{I}\alpha$.

16.48



GIVEN: 1.75-lb ROD AB

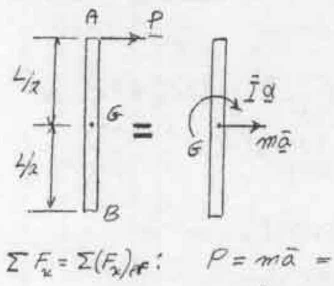
$P = 0.25$ lb

$L = 36$ in

FIND: ACCELERATION

(a) OF A.

(b) OF B.



$$m = \frac{W}{g} \qquad \bar{I} = \frac{1}{12}\frac{W}{g}L^2$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}: \quad P = m\bar{a} = \frac{W}{g}\bar{a}$

$$\bar{a} = \frac{P}{W}g = \frac{0.25\text{ lb}}{1.75\text{ lb}}g = \frac{1}{7}g \rightarrow$$

$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{eff}: \quad P\frac{L}{2} = \bar{I}\alpha = \frac{1}{12}\frac{W}{g}L^2\alpha$

$$\alpha = 6\frac{P}{W}\frac{g}{L} = 6\,\frac{0.25\text{ lb}}{1.75\text{ lb}}\,\frac{g}{L} = \frac{6}{7}\frac{g}{L} \circlearrowright$$

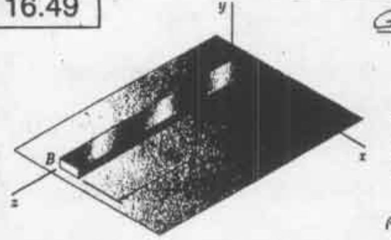
(a) $\xrightarrow{+} a_A = \bar{a} + \frac{L}{2}\alpha = \frac{1}{7}g + \frac{L}{2}\cdot\frac{6}{7}\frac{g}{L} = \frac{4}{7}g = \frac{4}{7}(32.2\text{ ft/s}^2)$

$$a_A = 18.40\text{ ft/s}^2 \rightarrow$$

(b) $\xrightarrow{+} a_B = \bar{a} - \frac{L}{2}\alpha = \frac{1}{7}g - \frac{L}{2}\cdot\frac{6}{7}\frac{g}{L} = -\frac{2}{7}g = -\frac{2}{7}(32.2\text{ ft/s}^2)$

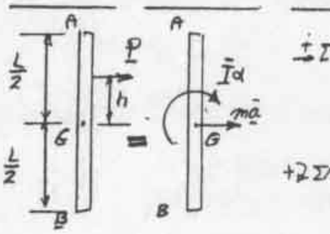
$$a_B = 9.2\text{ ft/s}^2 \leftarrow$$

16.49



GIVEN: 1.75-lb ROD AB
$P = 0.25\ lb$
$L = 3\ ft$

FIND: (a) WHERE $\underline{P}$ SHOULD BE APPLIED FOR $\underline{a}_B = 0$.
(b) CORRESPONDING ACCEL. OF POINT A.



$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$

$P = m\bar{a} = \frac{W}{g}\bar{a}$

$\underline{\bar{a}} = \frac{P}{W} g \rightarrow$

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$:

$Ph = \bar{I}\alpha = \frac{1}{12}\frac{W}{g}L^2\alpha$

$\underline{\alpha} = \frac{12Ph}{WL^2} g \circlearrowleft$

(a) $\xrightarrow{+} \underline{a}_B = \bar{a} - \frac{L}{2}\alpha$

$0 = \frac{P}{W}g - \frac{L}{2}\cdot\frac{12Ph}{WL^2}g$; $h =$

$h = \frac{L}{6} = \frac{36\ in}{6} = 6\ in.$

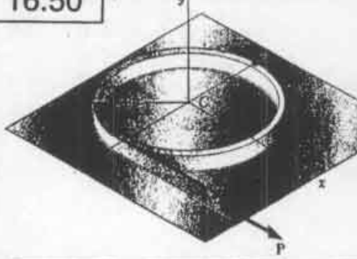
THUS, P IS LOCATED 12 in. FROM END A. ◀

FOR $h = L/6$: $\alpha = \frac{12P(L/6)}{WL^2}g = 2\frac{P}{W}\cdot\frac{g}{L} \circlearrowleft$

(b) $\xrightarrow{+} a_A = \bar{a} + \frac{L}{2}\alpha = \frac{P}{W}g + \frac{L}{2}\cdot 2\frac{P}{W}\frac{g}{L} = 2\frac{P}{W}g$

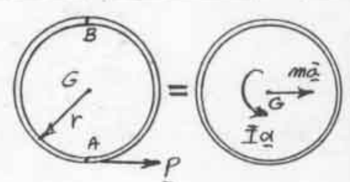
$a_A = 2\,\frac{0.25\ lb}{1.75\ lb}(32.2\ ft/s^2)$; $\underline{a}_A = 9.2\ ft/s^2 \rightarrow$ ◀

16.50



GIVEN: $P = 3N$
$m = 2.4\ kg$

FIND: (a) $\underline{a}_A$
(b) $\underline{a}_B$



HOOP: $\bar{I} = mr^2$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$:

$P = m\bar{a}$

$\underline{\bar{a}} = \frac{P}{m} \rightarrow$

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $Pr = \bar{I}\alpha = mr^2\alpha$

$\underline{\alpha} = \frac{P}{mr} \circlearrowleft$

(a) $\xrightarrow{+} a_A = \bar{a} + r\alpha = \frac{P}{m} + r\left(\frac{P}{mr}\right) = 2\frac{P}{m}$

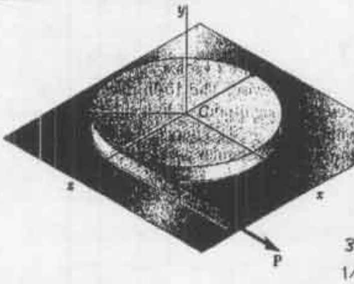
$a_A = 2\,\frac{3N}{2.4\ kg} = 2.5\ m/s^2$

$\underline{a}_A = 2.5\ m/s^2 \rightarrow$ ◀

(b) $\xrightarrow{+} a_B = \bar{a} - r\alpha = \frac{P}{m} - r\left(\frac{P}{mr}\right) = 0$

$\underline{a}_B = 0$ ◀

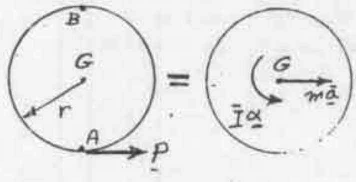
16.51 and 16.52



GIVEN: $P = 3N$
$m = 2.4\ kg$

PROBLEM 16.51
FIND: (a) $\underline{a}_A$
(b) $\underline{a}_B$

PROBLEM 16.52
SHOW THAT FOR 360° ROTATION DISK WILL MOVE DISTANCE πr.



DISK: $\bar{I} = \frac{1}{2}mr^2$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$:

$P = m\bar{a}$

$\underline{\bar{a}} = \frac{P}{m} \rightarrow$

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$

$Pr = \bar{I}\alpha$

$Pr = \frac{1}{2}mr^2\alpha$

$\alpha = \frac{2P}{mr} \circlearrowleft$

PROBLEM 16.51

(a) $\xrightarrow{+} a_A = \bar{a} + r\alpha = \frac{P}{m} + r\cdot\frac{2P}{mr} = 3\frac{P}{m}$

$a_A = 3\,\frac{3N}{2.4\ kg} = 3.75\ m/s^2$

$\underline{a}_A = 3.75\ m/s^2 \rightarrow$ ◀

(b) $\xrightarrow{+} a_B = \bar{a} - r\alpha = \frac{P}{m} - r\cdot\frac{2P}{mr} = -\frac{P}{m}$

$a_B = -\frac{3N}{2.4\ kg} = -1.25\ m/s^2$

$\underline{a}_B = 1.25\ m/s^2 \leftarrow$ ◀

PROBLEM 16.52

LET t_1 = TIME REQUIRED FOR 360° ROTATION

$\theta = \frac{1}{2}\alpha t_1^2$; $2\pi\ rad = \frac{1}{2}\left(\frac{2P}{mr}\right)t_1^2$

$t_1^2 = \frac{2\pi mr}{P}$

LET x_1 = DISTANCE G MOVES DURING 360° ROTATION

$x_1 = \frac{1}{2}\bar{a}t_1^2 = \frac{1}{2}\,\frac{P}{m}\left(\frac{2\pi mr}{P}\right)$

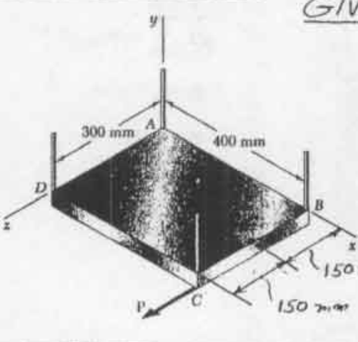
$x_1 = \pi r$ Q.E.D. ◀

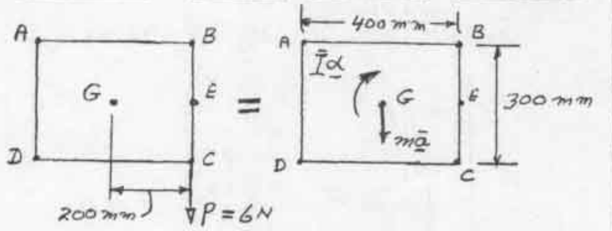
16.53 and 16.54

GIVEN: $m = 5\ \text{kg}$, $P = 6\ \text{N}$

PROBLEM 16.53
FIND: (a) $\underline{a}_E$
(b) $\underline{a}_B$

PROBLEM 16.54
FIND: (a) POINT OF ZERO ACCELERATION.
(b) $\underline{\alpha}$





$$\bar{I} = \frac{1}{12} m(b^2+h^2) = \frac{1}{12}(5\ \text{kg})\left[(0.4\ \text{m})^2+(0.3\ \text{m})^2\right] = 0.10417\ \text{kg}\cdot\text{m}^2$$

$+\downarrow \Sigma F = \Sigma F_{eff}$: $P = m\bar{a}$

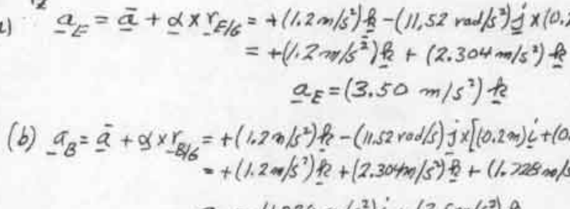
$6\ \text{N} = (5\ \text{kg})\bar{a}$ $\quad \bar{a} = +(1.2\ \text{m/s}^2)\underline{k}$

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $P(0.2\ \text{m}) = \bar{I}\alpha$

$(6\ \text{N})(0.2\ \text{m}) = (0.10417\ \text{kg}\cdot\text{m}^2)\alpha$

$\underline{\alpha} = -(11.52\ \text{rad/s}^2)\underline{j}$

PROBLEM 16.53:



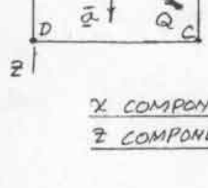
(a) $\underline{a}_E = \underline{\bar{a}} + \underline{\alpha} \times \underline{r}_{E/G} = +(1.2\ \text{m/s}^2)\underline{k} - (11.52\ \text{rad/s}^2)\underline{j} \times (0.2\ \text{m})\underline{i}$
$= +(1.2\ \text{m/s}^2)\underline{k} + (2.304\ \text{m/s}^2)\underline{k}$

$\underline{a}_E = (3.50\ \text{m/s}^2)\underline{k}$

(b) $\underline{a}_B = \underline{\bar{a}} + \underline{\alpha} \times \underline{r}_{B/G} = +(1.2\ \text{m/s}^2)\underline{k} - (11.52\ \text{rad/s})\underline{j} \times [(0.2\ \text{m})\underline{i} + (0.15\ \text{m})\underline{k}]$
$= +(1.2\ \text{m/s}^2)\underline{k} + (2.304\ \text{m/s}^2)\underline{k} + (1.728\ \text{m/s}^2)\underline{i}$

$\underline{a}_B = (1.728\ \text{m/s}^2)\underline{i} + (3.5\ \text{m/s}^2)\underline{k}$

PROBLEM 16.54: FOR POINT Q WE SEEK $\underline{a}_Q = 0$



$r_{Q/G} = x\underline{i} + z\underline{k}$

$\underline{a}_Q = \underline{\bar{a}} + \alpha \times r_{Q/G}$

$0 = (1.2\ \text{m/s}^2)\underline{k} - (11.52\ \text{rad/s})\underline{j} \times (x\underline{i} + z\underline{k})$

$0 = 1.2\underline{k} + 11.52x\underline{k} - 11.52z\underline{i}$

x COMPONENTS: $0 = 11.52z$; $z = 0$

z COMPONENTS: $0 = 1.2 + 11.52x$; $x = -0.1042\ \text{m}$

POINT OF ZERO ACCELERATION IS 104.2 mm TO LEFT OF G

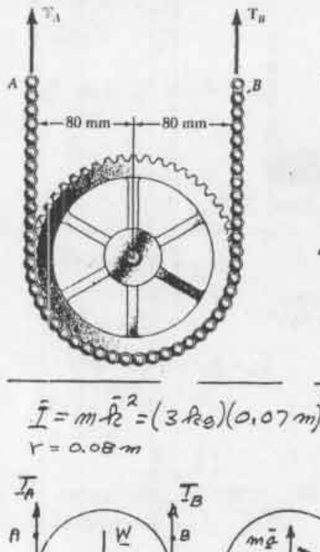
16.55 and 16.56

GIVEN: $m = 3\ \text{kg}$, $\bar{k} = 70\ \text{mm}$

FIND: $\underline{a}_A$ AND a_B

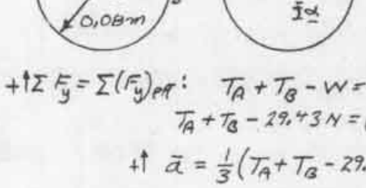
PROBLEM 16.55
FOR $T_A = 14\ \text{N}$, $T_B = 18\ \text{N}$

PROBLEM 16.56
FOR $T_A = 14\ \text{N}$, $T_B = 12\ \text{N}$



$\bar{I} = m\bar{k}^2 = (3\ \text{kg})(0.07\ \text{m})^2 = 14.7 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$

$r = 0.08\ \text{m}$



$W = mg = (3\ \text{kg})(9.81\ \text{m/s}^2)$

$W = 29.43\ \text{N}$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $T_A + T_B - W = m\bar{a}$

$T_A + T_B - 29.43\ \text{N} = (3\ \text{kg})\bar{a}$

$+\uparrow\ \bar{a} = \frac{1}{3}(T_A + T_B - 29.43) \qquad (1)$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$: $T_B(0.08\ \text{m}) - T_A(0.08\ \text{m}) = \bar{I}\alpha$

$(T_B - T_A)(0.08\ \text{m}) = (14.7 \times 10^{-3}\ \text{kg}\cdot\text{m}^2)\alpha$

$+\circlearrowright\ \alpha = 5.442(T_B - T_A) \qquad (2)$

PROBLEM 16.55 $\quad T_A = 14\ \text{N}$, $T_B = 18\ \text{N}$

EQ(1): $+\uparrow\ \bar{a} = \frac{1}{3}(14 + 18 - 29.43) = 0.8567\ \text{m/s}^2$

EQ.(2): $+\circlearrowright\ \alpha = 5.442(18 - 14) = 21.769\ \text{rad/s}^2$

$+\uparrow\ \underline{a}_A = (a_{A'})_t = \bar{a} + r\alpha = 0.8567 - (0.08)(21.769) = -0.885\ \text{m/s}^2$

$\underline{a}_A = 0.885\ \text{m/s}^2 \downarrow$

$+\uparrow\ \underline{a}_B = (a_{B'})_t = \bar{a} + r\alpha = 0.8567 + (0.08)(21.769) = +2.60\ \text{m/s}^2$

$\underline{a}_B = 2.60\ \text{m/s}^2 \uparrow$

PROBLEM 16.56 $\quad T_A = 14\ \text{N}$, $T_B = 12\ \text{N}$

EQ.(1): $+\uparrow\ \bar{a} = \frac{1}{3}(14 + 12 - 29.43) = -1.1433\ \text{m/s}^2$

$\underline{\bar{a}} = 1.1433\ \text{m/s}^2 \downarrow$

EQ.(2): $+\circlearrowright\ \alpha = 5.442(12 - 14) = -10.884\ \text{rad/s}^2$

$\underline{\alpha} = 10.884\ \text{rad/s}^2 \circlearrowleft$

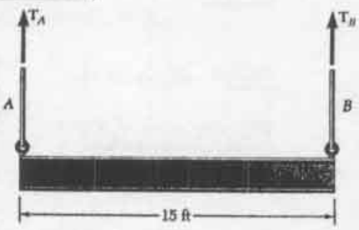
$+\uparrow\ \underline{a}_A = (a_{A'})_t = \bar{a} + r\alpha = -1.1433 + (0.08)(10.884) = -0.273\ \text{m/s}^2$

$\underline{a}_A = 0.273\ \text{m/s}^2 \downarrow$

$+\uparrow\ \underline{a}_B = (a_{B'})_t = \bar{a} + r\alpha = -1.1433 - (0.08)(10.884) = -2.01\ \text{m/s}^2$

$\underline{a}_B = 2.01\ \text{m/s}^2 \downarrow$

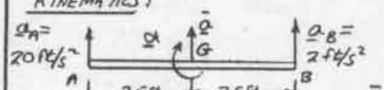
16.57



GIVEN: $W = 500\ lb$
$\underline{a}_A = 20\ ft/s^2 \uparrow$
$\underline{a}_B = 2\ ft/s^2 \uparrow$

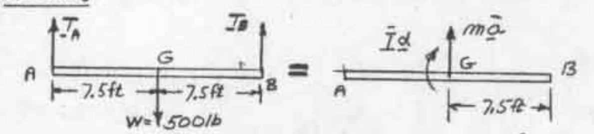
FIND: T_A AND T_B

KINEMATICS:



$a_B = a_A + (15\ ft)\alpha$
$2\uparrow = 20\uparrow + 15\alpha$
$\underline{\alpha} = 1.2\ rad/s^2 \circlearrowright$
$\bar{a} = \frac{1}{2}(a_A + a_B) = \frac{1}{2}(2+20)$
$\underline{\bar{a}} = 11\ ft/s^2 \uparrow$

KINETICS:



$\bar{I} = \frac{1}{12} mL^2 = \frac{1}{12} \frac{500\ lb}{32.2\ ft/s^2}(15\ ft)^2 = 291.15\ lb \cdot ft \cdot s^2$

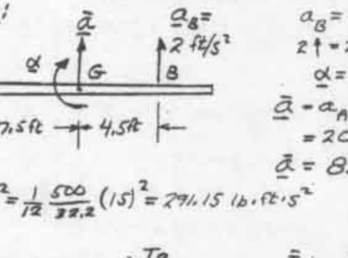
$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$: $T_A(15\ ft) - W(7.5\ ft) = m\bar{a}(7.5\ ft) + \bar{I}\alpha$

$T_A(15\ ft) - (500\ lb)(7.5\ ft) = \frac{500\ lb}{32.2\ ft/s^2}(11\ ft/s^2)(7.5\ ft) + (291.15\ lb \cdot ft \cdot s^2)(1.2\ \frac{rad}{s^2})$

$15T_A - 3750 = 1281 + 349.3$
$T_A = 358.7\ lb$ $\qquad T_A = 359\ lb$ ◀

$+\uparrow \Sigma F = \Sigma F_{eff}$: $T_A + T_B - W = m\bar{a}$
$358.7\ lb + T_B - 500\ lb = \frac{500\ lb}{32.2\ ft/s^2}(11\ ft/s^2)$
$T_B = 312.2\ lb$ $\qquad T_B = 312\ lb$ ◀

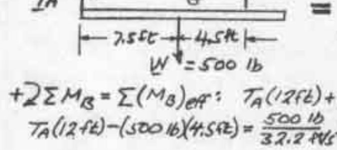
16.58



GIVEN:
$W = 500\ lb$
$\underline{a}_A = 20\ ft/s^2 \uparrow$
$\underline{a}_B = 2\ ft/s^2 \uparrow$

FIND:
T_A AND T_B

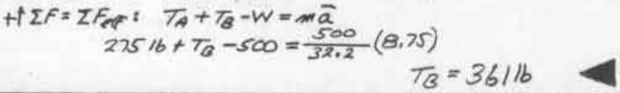
KINEMATICS:



$a_B = a_A + 12\alpha$
$2\uparrow = 20\uparrow + 12\alpha$
$\underline{\alpha} = 1.5\ rad/s^2 \circlearrowright$
$\bar{a} = a_A + 7.5\alpha$
$= 20\uparrow + (7.5)(1.5)\downarrow$
$\underline{\bar{a}} = 8.75\ ft/s^2 \uparrow$

$\bar{I} = \frac{1}{12} mL^2 = \frac{1}{12} \frac{500}{32.2}(15)^2 = 291.15\ lb \cdot ft \cdot s^2$

KINETICS:



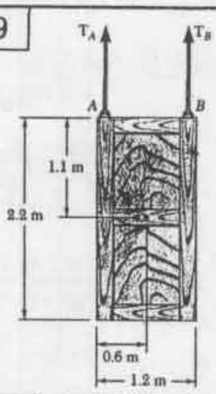
$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$: $T_A(12\ ft) + W(4.5\ ft) = m\bar{a}(4.5\ ft) + \bar{I}\alpha$

$T_A(12\ ft) - (500\ lb)(4.5\ ft) = \frac{500\ lb}{32.2\ ft/s^2}(8.75\ ft/s^2)(4.5\ ft) + (291)(1.5\ rad/s^2)$

$12T_A - 2250 = 611.4 + 437$; $\qquad T_A = 275\ lb$ ◀

$+\uparrow \Sigma F = \Sigma F_{eff}$: $T_A + T_B - W = m\bar{a}$
$275\ lb + T_B - 500 = \frac{500}{32.2}(8.75)$
$T_B = 361\ lb$ ◀

16.59

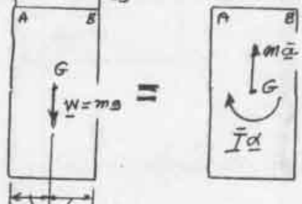


GIVEN:
$(\underline{a}_A)_y = 9\ m/s^2 \uparrow$
$(\underline{a}_B)_y = 3\ m/s^2 \uparrow$
$m = 180\ kg$

FIND:
T_A AND T_B

KINETICS $\quad m = 180\ kg$

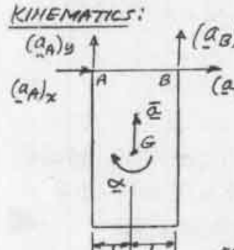
$\bar{I} = \frac{1}{12} m(b^2 + c^2) = \frac{1}{12}(180\ kg)(2.2^2 + 1.2^2) = 94.2\ kg \cdot m^2$



$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $T_A + T_B - mg = m\bar{a}$
$T_A + T_B - (180)(9.81) = (180)\bar{a}$
$T_A + T_B = 1765.8 + 180\bar{a}$ $\qquad (1)$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$: $T_A(0.6\ m) - T_B(0.6\ m) = \bar{I}\alpha$
$0.6(T_A - T_B) = 94.2\alpha$
$T_A - T_B = 157\alpha$ $\qquad (2)$

KINEMATICS:



$(a = \bar{a} + r\alpha)$

$(a_A)_y = \bar{a} + 0.6\alpha$
$9\ m/s^2 = \bar{a} + 0.6\alpha$ $\qquad (3)$
$(a_B)_y = \bar{a} - 0.6\alpha$
$3\ m/s^2 = \bar{a} - 0.6\alpha$ $\qquad (4)$

EQ(3) + EQ(4): $12 = 2\bar{a}$
$\bar{a} = +6$; $\qquad \underline{\bar{a}} = 6\ m/s^2 \uparrow$
EQ(3): $9 = 6 + 0.6\alpha$
$\alpha = +5$; $\qquad \underline{\alpha} = 5\ rad/s^2 \circlearrowright$

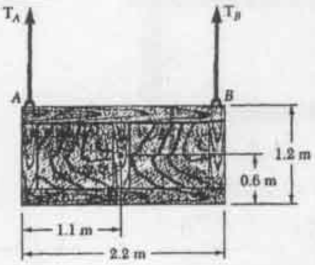
EQ(1) $\qquad T_A + T_B = 1765.8 + 180(6)$
$T_A + T_B = 2845.8$ $\qquad (5)$

EQ(2) $\qquad T_A - T_B = (157)(5\ rad/s^2)$
$T_A - T_B = +785$ $\qquad (6)$

EQ(5) + EQ(6): $\qquad 2T_A = 3630.8$
$T_A = 1815.4\ N$
$T_A = 1815\ N$ ◀

EQ(1) $\qquad T_A + 1815.4 = 2845.8$
$T_A = 1030.4\ N$
$T_B = 1030\ N$ ◀

16.60

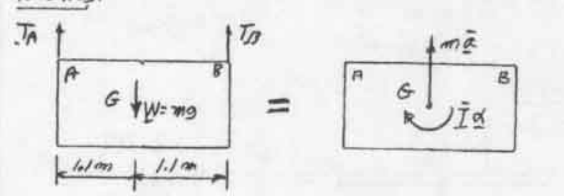


GIVEN:
$(a_A)_y = 9\, m/s^2 \uparrow$
$(a_B)_y = 3\, m/s^2 \uparrow$
$m = 180\, kg$

FIND
T_A AND T_B

$$\bar{I} = \frac{1}{12} m(a^2+b^2) = \frac{1}{12}(180\, kg)(2.2^2+1.2^2) = 94.2\ kg \cdot m^2$$

KINETICS:



$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $\quad T_A + T_B - mg = m\bar{a}$

$$T_A + T_B + (180\, kg)(9.81\, m/s^2) = (180\, kg)\bar{a}$$

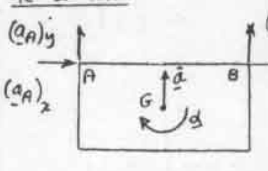
$$T_A + T_B = 1765.8 + 180\bar{a} \qquad (1)$$

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$: $\quad T_A(1.1\, m) - T_B(1.1\, m) = \bar{I}\alpha$

$$1.1(T_A - T_B) = (94.2\, kg \cdot m^2)\alpha$$

$$T_A - T_B = 85.636\, \alpha \qquad (2)$$

KINEMATICS



$(a = \bar{a} + r\alpha)$

$$(a_A)_y = \bar{a} + 1.1\alpha$$

$$9\, m/s^2 = \bar{a} + 1.1\alpha \qquad (3)$$

$$(a_B)_y = \bar{a} - 1.1\alpha$$

$$3\, m/s^2 = \bar{a} - 1.1\alpha \qquad (4)$$

EQ(3) + EQ(4): $\quad 12 = 2\bar{a}$

$$\bar{a} = +6\, m/s^2 \qquad \bar{a} = 6\, m/s^2 \uparrow$$

EQ(3) − EQ(4): $\quad 6 = 2.2\alpha$

$$\alpha = 2.727\, rad/s^2 \qquad \alpha = 2.73\, rad/s^2 \circlearrowleft$$

EQ(1): $\quad T_A + T_B = 1765.8 + 180(6)$

$$T_A + T_B = 2845.8 \qquad (5)$$

EQ.(2): $\quad T_A - T_B = 85.636(2.727)$

$$T_A - T_B = 233.5 \qquad (6)$$

EQ(5) + EQ(6): $\quad 2T_A = 3079.3$

$$T_A = 1539.7\, N$$

$$T_A = 1540\, N \blacktriangleleft$$

EQ(1) − EQ(2): $\quad 2T_B = 2612.3$

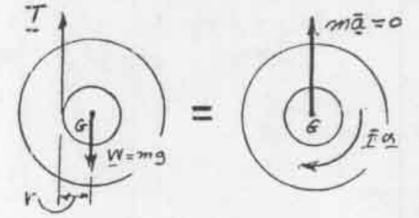
$$T_B = 1306.2\, N$$

$$T_B = 1306\, N \blacktriangleleft$$

16.61

GIVEN:
INNER AXLE: r
CENTROIDAL RADIUS OF GYRATION: $\bar{k}$
MASS: m

FIND: T FOR $\bar{a} = 0$



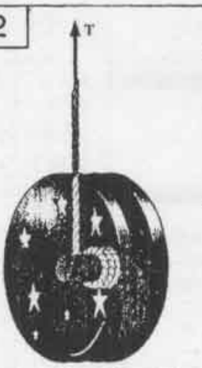
$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $\quad T - mg = 0\ ; \ T = mg$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$: $\quad Tr = \bar{I}\alpha$

$$mgr = m\bar{k}^2\alpha$$

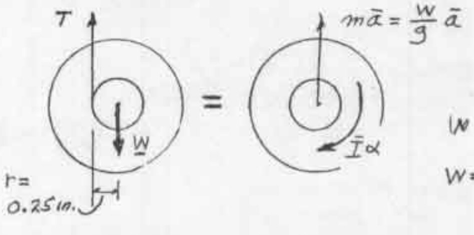
$$\alpha = \frac{rg}{\bar{k}^2} \qquad \alpha = \frac{rg}{\bar{k}^2} \circlearrowright \blacktriangleleft$$

16.62



GIVEN: $W = 3\, oz.$
$\bar{k} = 1.25\, in.$
RADIUS OF AXLE $= 0.25\, in.$
$\bar{a} = 3\, ft/s^2 \uparrow$

FIND: (a) T
(b) α



$$W = 3\, oz\left(\frac{lb}{16\, oz}\right)$$

$$W = \frac{3}{16}\, lb$$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $\quad T - W = \frac{W}{g}\bar{a}$

$$T - \frac{3}{16}\, lb = \left(\frac{3}{16}\, lb\right)\frac{3\, ft/s^2}{32.2\, ft/s^2}$$

$$T = 0.205\, lb \qquad T = 0.205\, lb \blacktriangleleft$$

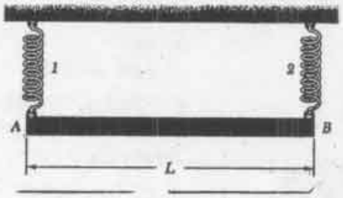
$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$: $\quad Tr = \bar{I}\alpha$

$$(0.205\, lb)\left(\frac{0.25}{12}\, ft\right) = m\bar{k}^2\alpha$$

$$4.271 \times 10^{-3}\, lb \cdot ft = \frac{3/16\, lb}{32.2\, ft/s^2}\left(\frac{1.25}{12}\, ft\right)^2 \alpha$$

$$\alpha = 67.6\, rad/s^2 \qquad \alpha = 67.6\, rad/s^2 \circlearrowright \blacktriangleleft$$

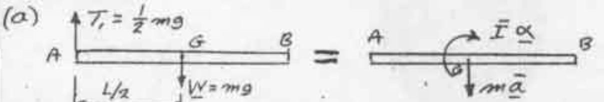
16.63



JUST AFTER SPRING 2 BREAKS, FIND: (a) $\underline{\alpha}$ (b) $\underline{a}_A$, (c) $\underline{a}_C$

STATICS: $T_1 = T_2 = \frac{1}{2}W = \frac{1}{2}mg$

(a)



$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{eff}$: $\quad T(L/2) = \bar{I}\alpha$

$\frac{1}{2}mg(L/2) = \frac{1}{12}mL^2\alpha$

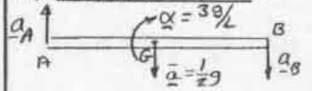
$\alpha = 3g/L \qquad \underline{\alpha} = \frac{3g}{L} \circlearrowright$ ◀

$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$: $\quad W - T_1 = m\bar{a}$

$mg - \frac{1}{2}mg = m\bar{a}$

$\bar{a} = \frac{1}{2}g \qquad \underline{\bar{a}} = \frac{1}{2}g\downarrow$

(b) ACCELERATION OF A:



$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G}$

$+\downarrow \underline{a}_A = \frac{1}{2}g - \frac{L}{2}\alpha$

$= \frac{1}{2}g - \frac{L}{2}\left(\frac{3g}{L}\right)$

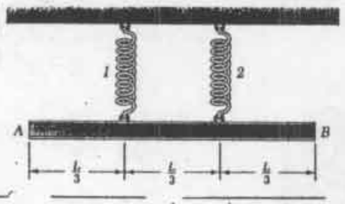
$a_A = -g; \quad \underline{a}_A = g\uparrow$ ◀

(c) ACCELERATION OF B:

$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G}$

$+\downarrow a_B = \bar{a} + \frac{L}{2}\alpha = \frac{1}{2}g + \frac{L}{2}\left(\frac{3g}{L}\right) = +2g; \quad \underline{a}_B = 2g\downarrow$ ◀

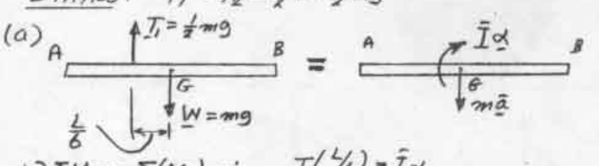
16.64



JUST AFTER SPRING 2 BREAKS, FIND: (a) $\underline{\alpha}$ (b) $\underline{a}_A$, (c) $\underline{a}_B$

STATICS: $T_1 = T_2 = \frac{1}{2}W = \frac{1}{2}mg$

(a)



$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{eff}$: $\quad T(L/6) = \bar{I}\alpha$

$\frac{1}{2}mg(L/6) = \frac{1}{12}mL^2\alpha$

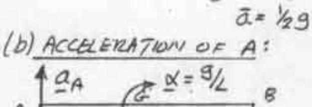
$\alpha = g/L \qquad \underline{\alpha} = g/L \circlearrowright$ ◀

$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$: $\quad W - T_1 = m\bar{a}$

$mg - \frac{1}{2}mg = m\bar{a}$

$\bar{a} = \frac{1}{2}g \qquad \underline{\bar{a}} = \frac{1}{2}g\downarrow$

(b) ACCELERATION OF A:



$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G}$

$+\downarrow \underline{a}_A = \frac{1}{2}g - \frac{L}{2}\alpha$

$= \frac{1}{2}g - \frac{L}{2}\left(\frac{g}{L}\right) = 0$

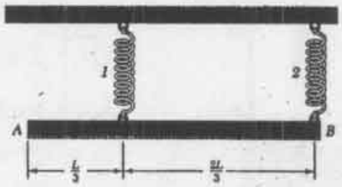
$\underline{a}_A = 0$ ◀

(c) ACCELERATION OF B:

$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G}$

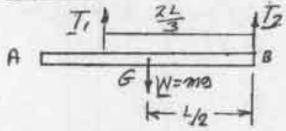
$+\downarrow \underline{a}_B = \bar{a} + \frac{L}{2}\alpha = \frac{1}{2}g + \frac{L}{2}\left(\frac{g}{L}\right) = +g \qquad \underline{a}_B = g\downarrow$ ◀

16.65



JUST AFTER SPRING 2 BREAKS, FIND: (a) $\underline{\alpha}$, (b) $\underline{a}_A$, (c) $\underline{a}_B$

STATICS:

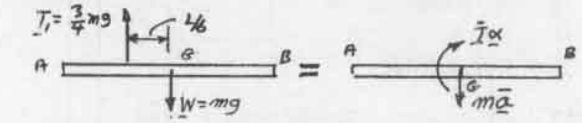


$\circlearrowright \Sigma M_B = 0$

$T_1\left(\frac{2L}{3}\right) - mg\left(\frac{L}{2}\right) = 0$

$T_1 = \frac{3}{4}mg$

(a) KINETICS



$+\circlearrowright \Sigma M_G = \Sigma(M_G)_{eff}$: $\quad T(L/6) = \bar{I}\alpha$

$\frac{3}{4}mg(L/6) = \frac{1}{12}mL^2\alpha$

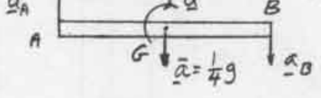
$\alpha = \frac{3}{2}\frac{g}{L} \qquad \underline{\alpha} = \frac{3}{2}\frac{g}{L}\circlearrowright$ ◀

$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$: $\quad W - T_1 = m\bar{a}$

$mg - \frac{3}{4}mg = m\bar{a}$

$\bar{a} = \frac{1}{4}g \qquad \underline{\bar{a}} = \frac{1}{4}g\downarrow$

(b) ACCELERATION OF A:



$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G}$

$+\downarrow a_A = \frac{1}{4}g - \frac{L}{2}\alpha$

$= \frac{1}{4}g - \frac{L}{2}\left(\frac{3}{2}\frac{g}{L}\right)$

$a_A = -\frac{1}{2}g; \quad \underline{a}_A = \frac{1}{2}g\uparrow$ ◀

(c) ACCELERATION OF B:

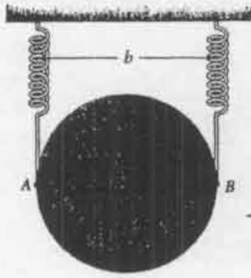
$\underline{a}_B = \underline{a}_G + \underline{a}_{B/G}$

$+\downarrow a_B = \frac{1}{4}g + \frac{L}{2}(\alpha)$

$= \frac{1}{4}g + \frac{L}{2}\left(\frac{3g}{2L}\right)$

$a_A = +g \qquad \underline{a}_A = g\downarrow$ ◀

16.66



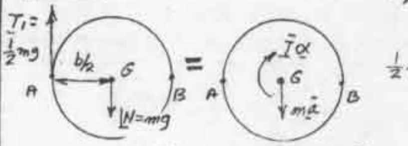
JUST AFTER SPRING 2 BREAKS,

FIND: (a) $\underline{a}_A$ (b) $\underline{a}_B$

$\bar{I} = \frac{1}{2} m\left(\frac{b}{2}\right)^2 = \frac{1}{8} m b^2$

STATICS: $T_1 = T_2 = \frac{1}{2} W = \frac{1}{2} mg$

KINETICS:



$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$

$T_1 \frac{b}{2} = \bar{I}\alpha$

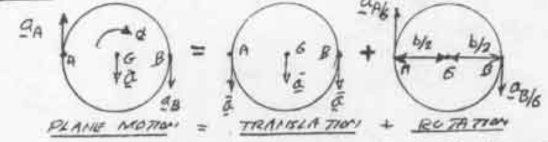
$\frac{1}{2} mg\left(\frac{b}{2}\right) = \frac{1}{8} m b^2 \alpha$

$\alpha = 2\, g/b \circlearrowright$

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $W - T_1 = m\bar{a}$

$mg - \frac{1}{2} mg = m\bar{a}$ $\quad \bar{a} = \frac{1}{2} g \downarrow$

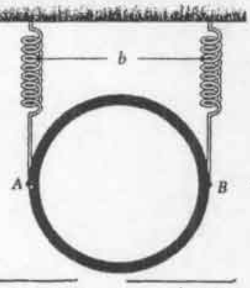
KINEMATICS



(a) $\underline{a}_A = \underline{a}_G + \underline{a}_{A/G} = \bar{a}\downarrow + \frac{b}{2}\alpha\uparrow = \frac{1}{2} g\downarrow + \frac{b}{2}\left(2\frac{g}{b}\right)\uparrow$; $\underline{a}_A = \frac{1}{2} g \uparrow$ ◀

(b) $\underline{a}_B = \underline{a}_G + \underline{a}_{B/G} = \bar{a}\downarrow + \frac{b}{2}\alpha\downarrow = \frac{1}{2} g\downarrow + \frac{b}{2}\left(\frac{2g}{b}\right)\downarrow$; $\underline{a}_B = \frac{3}{2} g \downarrow$ ◀

16.67



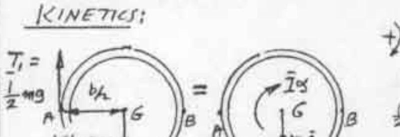
JUST AFTER SPRING 2 BREAKS,

FIND: (a) $\underline{a}_A$ (b) $\underline{a}_B$

$\bar{I} = m\left(\frac{b}{2}\right)^2 = \frac{1}{4} m b^2$

STATICS: $T_1 = T_2 = \frac{1}{2} W = \frac{1}{2} mg$

KINETICS:



$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$:

$T_1\left(\frac{b}{2}\right) = \bar{I}\alpha$

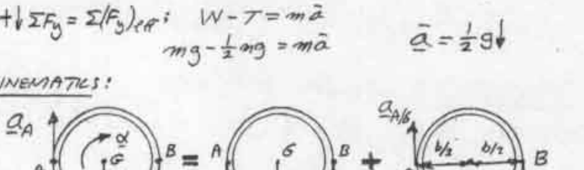
$\frac{1}{2} mg\left(\frac{b}{2}\right) = \frac{1}{4} m b^2 \alpha$

$\alpha = g/b \circlearrowright$

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $W - T = m\bar{a}$

$mg - \frac{1}{2} mg = m\bar{a}$ $\quad \bar{a} = \frac{1}{2} g \downarrow$

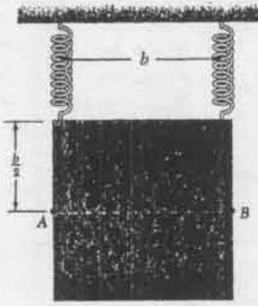
KINEMATICS:



(a) $\underline{a}_A = \underline{a}_G + \underline{a}_{A/G} = \bar{a}\downarrow + \frac{b}{2}\alpha\uparrow = \frac{1}{2} g\downarrow + \frac{b}{2}\left(\frac{g}{b}\right)\uparrow$; $\underline{a}_A = 0$ ◀

(b) $\underline{a}_B = \underline{a}_G + \underline{a}_{B/G} = \bar{a}\downarrow + \frac{b}{2}\alpha\downarrow = \frac{1}{2} g\downarrow + \frac{b}{2}\left(g/b\right)\downarrow$; $\underline{a}_B = g \downarrow$ ◀

16.68



JUST AFTER SPRING 2 BREAKS

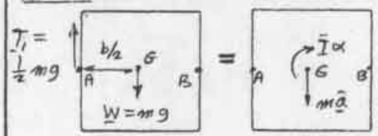
FIND: (a) $\underline{a}_A$ (b) $\underline{a}_B$

$\bar{I} = \frac{1}{12} m\left(b^2 + b^2\right)$

$\bar{I} = \frac{1}{6} m b^2$

STATICS: $T_1 = T_2 = \frac{1}{2} W = \frac{1}{2} mg$

KINETICS:



$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$

$T\, b/2 = \bar{I}\alpha$

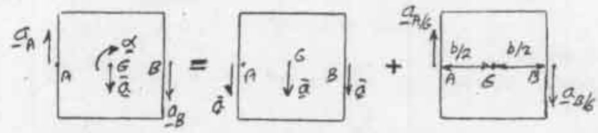
$\frac{1}{2} mg\left(\frac{b}{2}\right) = \frac{1}{6} m b^2 \alpha$

$\alpha = \frac{3g}{2b} \circlearrowright$

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $W - T_1 = m\bar{a}$

$mg - \frac{1}{2} mg = m\bar{a}$ $\quad \bar{a} = \frac{1}{2} g \downarrow$

KINEMATICS:



PLANE MOTION = TRANSLATION + ROTATION

(a) $\underline{a}_A = \underline{a}_G + \underline{a}_{A/G} = \bar{a}\downarrow + \frac{b}{2}\alpha\uparrow$

$\underline{a}_A = \frac{g}{2}\downarrow + \frac{b}{2}\left(\frac{3g}{2b}\right)\uparrow = \frac{g}{4}\uparrow$

$\underline{a}_A = \frac{1}{4} g \uparrow$ ◀

(b) $\underline{a}_B = \underline{a}_G + \underline{a}_{B/G} = \bar{a}\downarrow + \frac{b}{2}\alpha\downarrow$

$\underline{a}_B = \frac{1}{2} g\downarrow + \frac{b}{2}\left(\frac{3g}{2b}\right)\downarrow = \frac{5}{4} g \downarrow$

$\underline{a}_B = \frac{5}{4} g \downarrow$ ◀

16.69 and 16.70

GIVEN: $\bar{v}_0 = 15\ \text{ft/s} \rightarrow$

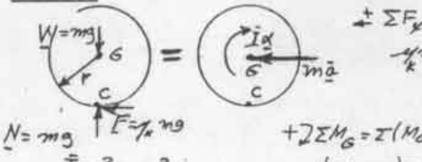
$r = 4\ \text{in.}$, $\mu_k = 0.10$.

PROBLEM 16.69: $\omega_0 = 9\ \text{rad/s}$

PROBLEM 16.70: $\omega_0 = 18\ \text{rad/s}$

FIND: (a) t_1 WHEN ROLLING STARTS,
(b) v AT t_1,
(c) DISTANCE TRAVELED AT t_1

KINETICS:



$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$

$-\mu_k mg = -m\bar{a}$

$\bar{a} = \mu_k g \leftarrow$

$N = mg$

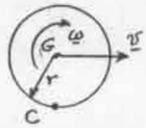
$\bar{I} = \frac{2}{5} m r^2$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$: $Fr = \bar{I}\alpha$

$(\mu_k mg) r = \frac{2}{5} m r^2$

$\alpha = \frac{5}{2} \frac{\mu_k g}{r} \circlearrowright$

KINEMATICS:



WHEN SPHERE ROLLS, INSTANT CENTER OF ROTATION IS AT C AND WHEN $t = t_1$, $v = r\omega$ (1)

$\bar{v} = \bar{v}_0 - \bar{a}t = \bar{v}_0 - \mu_k g t$ (2)

$\omega = -\omega_0 + \alpha t = -\omega_0 + \frac{5}{2} \frac{\mu_k g}{r} t$

WHEN $t = t_1$:

EQ(1): $v = r\omega$: $\bar{v}_0 - \mu_k g t_1 = \left(-\omega_0 + \frac{5}{2}\frac{\mu_k g}{r} t_1\right) r$

$\bar{v}_0 - \mu_k g t_1 = -\omega_0 r + \frac{5}{2} \mu_k g t_1$

$t_1 = \frac{2}{7} \frac{(\bar{v}_0 + r\omega_0)}{\mu_k g}$ (3)

PROBLEM 16.69: $\bar{v}_0 = 15\ \text{ft/s}$, $\omega_0 = 9\ \text{rad/s}$, $r = 4\ \text{in.} = \frac{1}{3}\ \text{ft}$

(a) $t_1 = \frac{2}{7} \frac{(15 + \frac{1}{3}(9))}{0.1(32.2)} = 1.5972\ \text{s}$ $\quad t_1 = 1.597\ \text{s}$ ◀

(b) EQ.(2): $\bar{v}_1 = v_0 - \mu_k g t_1 = 15 - 0.1(32.2)(1.5972)$

$\bar{v}_1 = 15 - 5.1429 = 9.857\ \text{ft/s}$

$\bar{v}_1 = 9.86\ \text{ft/s} \rightarrow$ ◀

(c)

$\bar{a} = \mu_k g = 0.1(32.2\ \text{ft/s}^2) = 3.22\ \text{ft/s}^2 \leftarrow$

$\xrightarrow{+} s_1 = \bar{v}_0 t_1 - \frac{1}{2}\bar{a} t_1^2$

$= (15\ \text{ft/s})(1.597\ \text{s}) - \frac{1}{2}(3.22\ \text{ft/s}^2)(1.597\ \text{s})^2$

$= 23.96 - 4.11 = 19.85\ \text{ft}$

$s_1 = 19.85\ \text{ft} \rightarrow$ ◀

PROBLEM 16.70: $\bar{v}_0 = 15\ \text{ft/s}$, $\omega_0 = 18\ \text{rad/s}$, $r = \frac{1}{3}\ \text{ft}$

(a) EQ(3): $t_1 = \frac{2}{7} \frac{(15 + \frac{1}{3}(18))}{0.1(32.2)} = 1.8634\ \text{s}$

$t_1 = 1.863\ \text{s}$ ◀

(b) EQ(2): $v_1 = v_0 - \mu_k g t = 15 - 0.1(32.2)(1.8634)$

$v_1 = 15 - 6.000 = 9\ \text{ft/s}$

$\bar{v}_1 = 9\ \text{ft/s} \rightarrow$ ◀

(c)

$\bar{a} = \mu_k g = 0.1(32.2\ \text{ft/s}^2) = 3.22\ \text{ft/s}^2 \leftarrow$

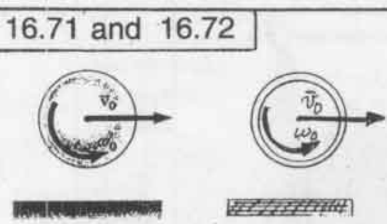
$\xrightarrow{+} s_1 = \bar{v}_0 t_1 - \frac{1}{2}\bar{a} t_1^2$

$= (15\ \text{ft/s})(1.8634\ \text{s}) - \frac{1}{2}(3.22\ \text{ft/s}^2)(1.8634\ \text{s})^2$

$= 27.95 - 5.59 = 22.36\ \text{ft}$

$\bar{v}_1 = 22.4\ \text{ft} \rightarrow$ ◀

16.71 and 16.72



SPHERE
PROBLEM 16.71

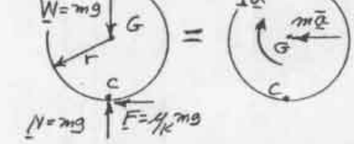
HOOP
PROBLEM 16.72

GIVEN:
m = MASS
r = RADIUS
μ_k = COEFF. OF KINETIC FRICTION

FIND: FOR EACH PROBLEM
(a) ω_0 SO FINAL VELOCITY IS ZERO
(b) TIME t_1 WHEN VELOCITY BECOMES ZERO
(c) DISTANCE s_1 MOVED BEFORE v BECOMES ZERO

KINETICS:

$\bar{I} = m\bar{k}^2$



$N = mg$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad F = m\bar{a}$

$\mu_k mg = m\bar{a}$ $\quad \bar{a} = \mu_k g \leftarrow$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$: $\quad Fr = \bar{I}\alpha$

$(\mu_k mg) r = m\bar{k}^2 \alpha$

$\alpha = \frac{\mu_k g r}{\bar{k}^2} \circlearrowright$

KINEMATICS: $\xrightarrow{+} v = v_0 - \bar{a}t$

$v = v_0 - \mu_k g t$

FOR $v = 0$ WHEN $t = t_1$

$0 = v_0 - \mu_k g t_1$; $\quad t_1 = \frac{v_0}{\mu_k g}$ (1)

$+\circlearrowright \omega = \omega_0 - \alpha t$

$\omega = \omega_0 - \frac{\mu_k g r}{\bar{k}^2} t$

FOR $\omega = 0$ WHEN $t = t_1$

$0 = \omega_0 - \frac{\mu_k g r}{\bar{k}^2} t_1$; $\quad t_1 = \frac{\bar{k}^2}{\mu_k g r} \omega_0$ (2)

SET EQ(1) = EQ(2)

$\frac{v_0}{\mu_k g} = \frac{\bar{k}^2}{\mu_k g r} \omega_0$; $\quad \omega_0 = \frac{r}{\bar{k}^2} v_0$ (3)

DISTANCE TRAVELED: $s_1 = v_0 t_1 - \frac{1}{2}\bar{a} t_1^2$

$s_1 = v_0\left(\frac{v_0}{\mu_k g}\right) - \frac{1}{2}(\mu_k g)\left(\frac{v_0}{\mu_k g}\right)^2$; $\quad s_1 = \frac{v_0^2}{2\mu_k g}$ (4)

PROBLEM 16.71 SPHERE $\bar{k}^2 = \frac{2}{5} r^2$

(a) EQ(3): $\omega_0 = \frac{r}{\frac{2}{5}r^2} v_0 = \frac{5}{2}\frac{v_0}{r}$ $\quad \omega_0 = \frac{5}{2}\frac{v_0}{r} \circlearrowleft$ ◀

(b) EQ(1) $\quad t_1 = v_0/\mu_k g$ ◀

(c) EQ(4) $\quad s_1 = v_0^2/2\mu_k g$ ◀

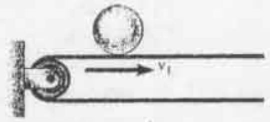
PROBLEM 16.72 HOOP $\bar{k} = r$

(a) EQ(3): $\omega_0 = \frac{r}{r^2} v_0 = \frac{v_0}{r}$ $\quad \omega_0 = v_0/r \circlearrowleft$ ◀

(b) AND (c) SAME AS ABOVE: $\quad t_1 = v_0/\mu_k g$ ◀

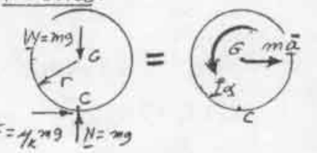
$s_1 = v_0^2/2\mu_k g$ ◀

16.73



GIVEN: SPHERE PLACED ON BELT WITH NO VELOCITY.
r = RADIUS,
μ_k = COEF. KINETIC FRICTION
FIND: (a) t_1 WHEN SPHERE ROLLS
(b) $\bar{v}$ AND ω WHEN $t = t_1$

KINETICS:



$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $F = m\bar{a}$

$\mu_k mg = m\bar{a}$

$\bar{a} = \mu_k g \rightarrow$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$: $Fr = \bar{I}\alpha$

$(\mu_k mg) r = \frac{2}{5} m r^2 \alpha$

$\alpha = \frac{5}{2}\frac{\mu_k g}{r} \circlearrowright$

KINEMATICS: $\xrightarrow{+} \bar{v} = \bar{a}t = \mu_k g t$ (1)

$+\circlearrowright \omega = \alpha t = \frac{5}{2}\frac{\mu_k g}{r} t$ (2)

C = POINT OF CONTACT WITH BELT

$\xrightarrow{+} v_C = \bar{v} + \omega r = \mu_k g t + \left(\frac{5}{2}\frac{\mu_k g}{r} t\right) r$

$v_C = \frac{7}{2}\mu_k g t$

(a) WHEN SPHERE STARTS ROLLING ($t = t_1$), WE HAVE

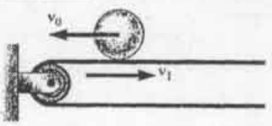
$v_C = v_1$; $v_1 = \frac{7}{2}\mu_k g t_1$ $\quad t_1 = \frac{2}{7}\frac{v_1}{\mu_k g}$ ◀

(b) VELOCITIES WHEN $t = t_1$

EQ(1): $\bar{v} = \mu_k g\left(\frac{2}{7}\frac{v_1}{\mu_k g}\right)$ $\quad \bar{v} = \frac{2}{7} v_1 \rightarrow$ ◀

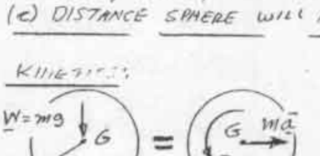
EQ(2): $\omega = \left(\frac{5}{2}\frac{\mu_k g}{r}\right)\left(\frac{2}{7}\frac{v_1}{\mu_k g}\right)$ $\quad \omega = \frac{5}{7}\frac{v_1}{r} \circlearrowright$ ◀

16.74



GIVEN: SPHERE WITH $v_0 \leftarrow$ AND $\omega_0 = 0$ PLACED ON BELT. r = RADIUS
μ_k = COEF. KINETIC FRICTION
FIND: (a) v_0 SO THAT SPHERE WILL HAVE NO LINEAR VELOCITY AFTER IT STARTS ROLLING ON BELT, (b) t_1 WHEN SPHERE STARTS ROLLING, (c) DISTANCE SPHERE WILL HAVE MOVED WHEN $t = t_1$

KINETICS:



$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $F = m\bar{a}$

$\mu_k mg = m\bar{a}$

$\bar{a} = \mu_k g \rightarrow$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$: $Fr = \bar{I}\alpha$

$(\mu_k mg) r = \frac{2}{5} m r^2 \alpha$

$\alpha = \frac{5}{2}\frac{\mu_k g}{r} \circlearrowright$

KINEMATICS:

$\xrightarrow{+} \bar{v} = v_0 - \bar{a}t = v_0 - \mu_k g t$ (1)

$+\circlearrowright \omega = \alpha t = \frac{5}{2}\frac{\mu_k g}{r} t$ (2)

(CONTINUED)

16.74 continued

C = POINT OF CONTACT WITH BELT

$\xrightarrow{+} v_C = -\bar{v} + r\omega$

$v_C = -\bar{v} + r\frac{5}{2}\frac{\mu_k g}{r} t$

$v_C = -\bar{v} + \frac{5\mu_k g}{2} t$ (3)

BUT, WHEN $t = t_1$, $\bar{v} = 0$ AND $v_C = v_1$

EQ(3): $v_1 = \frac{5\mu_k g}{2} t_1$; $\quad t_1 = \frac{2 v_1}{5\mu_k g}$ ◀

EQ(1): $\bar{v} = v_0 - \mu_k g t$

WHEN $t = t_1$, $\bar{v} = 0$,

$0 = v_0 - \mu_k g\left(\frac{2v_1}{5\mu_k g}\right)$; $\quad v_0 = \frac{2}{5} v_1$ ◀

DISTANCE WHEN $t = t_1$:

$\xleftarrow{+} s = v_0 t_1 - \frac{1}{2} a t_1^2$

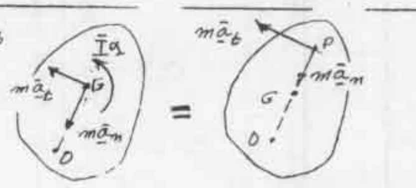
$s = \left(\frac{2}{5}v_1\right)\left(\frac{2v_1}{5\mu_k g}\right) - \frac{1}{2}(\mu_k g)\left(\frac{2v_1}{5\mu_k g}\right)^2$

$s = \frac{v_1^2}{\mu_k g}\left(\frac{4}{25} - \frac{2}{25}\right)$; $\quad s = \frac{2}{25}\frac{v_1^2}{\mu_k g} \leftarrow$ ◀

16.75

SHOW THAT $\bar{I}\alpha$ (Fig 16.15) CAN BE ELIMINATED BY ATTACHING $m\bar{a}_t$ AND $m\bar{a}_n$ AT POINT P ON OG WHERE $GP = \bar{k}^2/\bar{r}$

FIG 16.15b



$OG = \bar{r}$ $\quad \bar{a}_t = \bar{r}\alpha$

WE FIRST OBSERVE THAT THE SUM OF THE VECTORS IS THE SAME IN BOTH FIGURES

TO HAVE THE SAME SUM OF MOMENTS ABOUT G, WE MUST HAVE

$+\circlearrowright \Sigma M_G = \Sigma M_G$: $\quad \bar{I}\alpha = (m\bar{a}_t)(GP)$

$m\bar{k}^2\alpha = m\bar{r}\alpha\,(GP)$

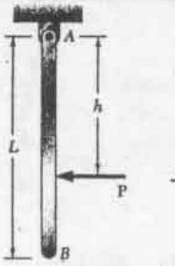
$GP = \frac{\bar{k}^2}{\bar{r}}$ (Q.E.D.) ◀

NOTE: THE CENTER OF ROTATION AND THE CENTER OF PERCUSSION ARE INTERCHANGEABLE. INDEED, SINCE $OG = \bar{r}$, WE MAY WRITE

$GP = \frac{\bar{k}^2}{GO}$ OR $GO = \frac{\bar{k}^2}{GP}$

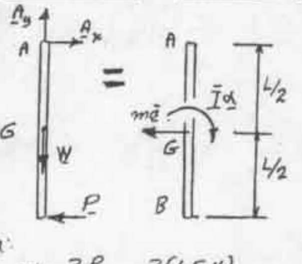
THUS, IF POINT P IS SELECTED AS CENTER OF ROTATION, THEN POINT O IS THE CENTER OF PERCUSSION.

16.76



GIVEN: $L = 36$ in.
$W = 4$ lb
$P = 1.5$ lb
$h = L = 36$ in.

FIND: (a) $\underline{\alpha}$
(b) $\underline{A}_x$ AND $\underline{A}_y$



$\bar{a} = \frac{L}{2}\alpha \qquad \bar{I} = \frac{1}{12} mL^2$

$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$

$PL = (m\bar{a})\frac{L}{2} + \bar{I}\alpha$

$= (m\frac{L}{2}\alpha)\frac{L}{2} + \frac{1}{12} mL^2\alpha$

$PL = \frac{1}{3} mL^2\alpha$

(a)

$\alpha = \frac{3P}{mL} = \frac{3(1.5\text{ lb})}{(4\text{lb}/32.2\text{ft/s}^2)(3\text{ft})} = 12.08 \text{ rad/s}^2$

$\underline{\alpha} = 12.08 \text{ rad/s}^2 \circlearrowright$ ◀

(b) $+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $A_y - W = 0$

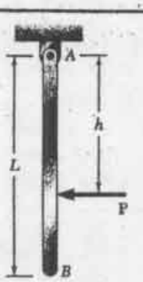
$A_y = W = 4$ lb $\qquad \underline{A}_y = 4\text{lb}\uparrow$ ◀

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $A_x - P = -m\bar{a}$

$A_x = P - m(\frac{L}{2}\alpha) = P - m\frac{L}{2}(\frac{3P}{mL}) = -\frac{P}{2}$

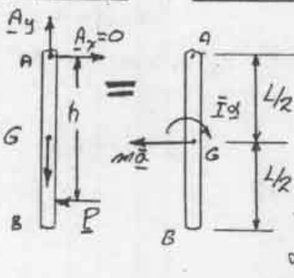
$A_x = -\frac{P}{2} = -\frac{1.5\text{lb}}{2} = -0.75$ lb $\qquad \underline{A}_x = 0.75\text{lb} \leftarrow$ ◀

16.78



GIVEN: $L = 36$ in.
$W = 4$ lb
$P = 1.5$ lb

FIND: (a) h FOR $\underline{A}_x = 0$.
(b) CORRESPONDING ANGULAR ACCEL. $\underline{\alpha}$.



$\bar{a} = \frac{L}{2}\alpha \qquad \bar{I} = \frac{1}{12} mL^2$

$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$:

$P = m\bar{a}$

$P = m(\frac{L}{2}\alpha)$

$\alpha = \frac{2P}{mL}$

$\alpha = \frac{2(1.5\text{ lb})}{(\frac{4\text{lb}}{32.2\text{ft/s}^2})(3\text{ft})}$

$\underline{\alpha} = 8.05 \text{ rad/s}^2 \circlearrowright$ ◀

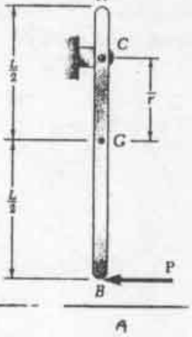
$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$:

$P(h - \frac{L}{2}) = \bar{I}\alpha$: $\cancel{P(h - L}$

$P(h - \frac{L}{2}) = \frac{1}{12} mL^2 (\frac{2P}{mL}) = \frac{PL}{6}$

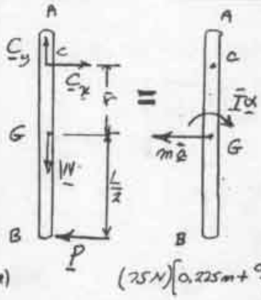
$(h - \frac{L}{2}) = \frac{L}{6}$; $h = \frac{L}{2} + \frac{L}{6} = \frac{2}{3}L \qquad h = 24$ in. ◀

16.77



GIVEN: $L = 900$ mm
$m = 4$ kg
$P = 75$ N
$\bar{r} = 225$ mm

FIND: (a) $\underline{\alpha}$
(b) $\underline{C}_x$ AND $\underline{C}_y$



$\bar{a} = \bar{r}\alpha \qquad \bar{I} = \frac{1}{12} mL^2$

$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}$:

$P(\bar{r} + \frac{L}{2}) = (m\bar{a})\bar{r} + \bar{I}\alpha$

$= (m\bar{r}\alpha)\bar{r} + \frac{1}{12} mL^2\alpha$

$P(\bar{r} + \frac{L}{2}) = m(\bar{r}^2 + \frac{1}{12}L^2)\alpha$

SUBSTITUTE DATA:

(a) $(75N)\left[0.225\text{m} + \frac{0.9\text{m}}{2}\right] = (4\text{kg})\left[(0.225\text{m})^2 + \frac{1}{12}(0.9\text{m})^2\right]\alpha$

$50.625 = 0.4725\alpha \qquad \alpha = 107.14 \text{ rad/s}^2$

$\underline{\alpha} = 107.1 \text{ rad/s}^2 \circlearrowright$ ◀

(b) $+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $A_y - W = 0$

$A_y = W = mg = (4\text{kg})(9.81\text{m/s}^2)$; $\underline{A}_y = 39.2\text{N}\uparrow$ ◀

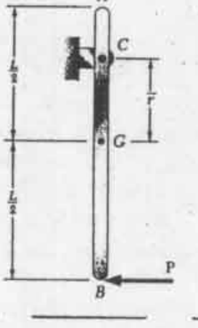
$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $A_x - P = -m\bar{a}$

$A_x = P - m\bar{a} = P - m(\bar{r}\alpha)$

$= 75\text{N} - (4\text{kg})(0.225\text{m})(107.14\text{rad/s}^2)$

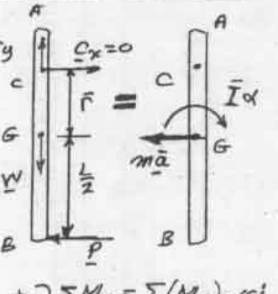
$A_x = 75\text{N} - 96.4\text{N} = -21.4\text{N}$; $\underline{A}_x = 21.4\text{N} \leftarrow$ ◀

16.79



GIVEN: $L = 900$ mm
$m = 4$ kg
$P = 75$ N

FIND: (a) $\bar{r}$ FOR $\underline{C}_x = 0$
(b) CORRESPONDING ANGULAR ACCEL. $\underline{\alpha}$.



$\bar{a} = \bar{r}\alpha \qquad \bar{I} = \frac{1}{12} mL^2$

$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$:

$P = m\bar{a}$

$P = m(\bar{r}\alpha)$

$\alpha = \frac{P}{m\bar{r}} \qquad (1)$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$: $\quad P\frac{L}{2} = \bar{I}\alpha$

$P\frac{L}{2} = \frac{1}{12} mL^2\alpha$

$P\frac{L}{2} = \frac{1}{12} mL^2 (\frac{P}{m\bar{r}})$

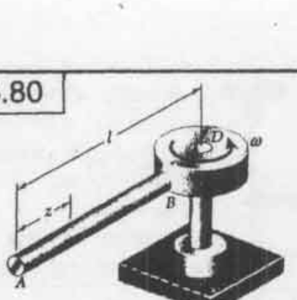
$\frac{L}{2} = \frac{L^2}{12\bar{r}}$; $\bar{r} = \frac{1}{6}L \qquad \bar{r} = \frac{900\text{mm}}{6}$

$\bar{r} = 150$ mm ◀

EQ(1): $\alpha = \frac{P}{m\bar{r}} = \frac{P}{m(L/6)} = \frac{6P}{mL}$

$\alpha = \frac{6(75\text{N})}{(4\text{kg})(0.9\text{m})} = 125 \text{ rad/s}^2$; $\underline{\alpha} = 125 \frac{\text{rad}}{\text{s}^2} \circlearrowright$ ◀

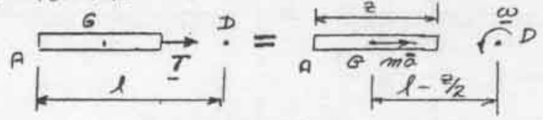
16.80



GIVEN: $w = 0.25$ lb/ft
$l = 1.2$ ft
$\omega = 150$ rpm
$z = 0.9$ ft

FIND: TENSION IN ROD
(a) IN TERMS OF w, l, z, AND ω.
(b) FOR GIVEN DATA

IN HORIZONTAL PLANE: $\bar{a} = r\omega^2 = (l - \frac{z}{2})\omega^2$



$\overset{+}{\rightarrow} \Sigma F = \Sigma F_{eff}$: $T = m\bar{a}$ $\quad m = \frac{w}{g} z$

$= \left(\frac{w}{g} z\right)\left(l - \frac{z}{2}\right)\omega^2$

$$T = \frac{w}{g}\left(lz - \frac{z^2}{2}\right)\omega^2$$

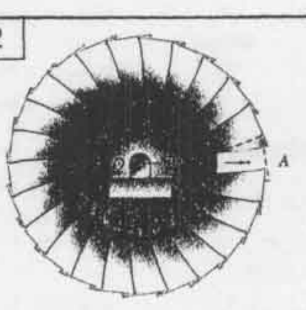
SUBSTITUTE DATA:

$\omega = 150 \text{ rpm}\left(\frac{2\pi}{60}\right) = 5\pi$ rad/s, $z = 0.9$ ft

$$T = \frac{0.25 \text{ lb/ft}}{32.2 \text{ ft/s}^2}\left[(1.2\text{ft})(0.9\text{ft}) - \frac{(0.9\text{ft})^2}{2}\right](5\pi \text{ rad/s})^2$$

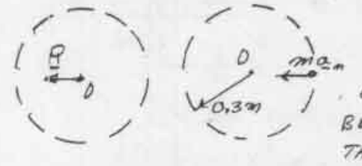
$$T = 1.293 \text{ lb}$$

16.82



GIVEN: A 45-g VANE IS THROW OFF FROM BALANCED TURBIN DISK.
$\omega = 9600$ rpm

FIND: REACTION AT O



$\omega = 9600 \text{ rpm}\left(\frac{2\pi}{60}\right)$

$\omega = 320\pi$ rad/s

CONSIDER VANE BEFORE IT IS THROWN OFF

$\overset{+}{\rightarrow} \Sigma F = \Sigma F_{eff}$: $R = m a_n = m \bar{r}\omega^2$

$= (45 \times 10^{-3} \text{ kg})(0.3 \text{ m})(320\pi)^2$

$R = 13.64$ kN

BEFORE VANE WAS THROWN OFF DISK WAS BALANCED ($R = 0$). REMOVING VANE AT A ALSO REMOVES ITS REACTION, SO DISK IS UNBALANCED AND REACTION IS $\quad \underline{R} = 13.64 \text{ kN} \rightarrow$

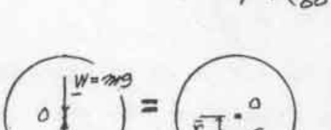
16.81

GIVEN: FLYWHEEL, CENTER OF ROTATION AT O, AND MASS CENTER AT G
$\omega = 1200$ rpm. MAXIMUM FORCE EXERTED ON SHAFT IS 55 kN↑ AND 85 kN↓.

FIND: (a) MASS OF FLYWHEEL
(b) DISTANCE $\bar{r}$

$\omega = 1200 \text{ rpm}\left(\frac{2\pi}{60}\right) = 40\pi$ rad/s

$a_n = \bar{r}\omega^2$



$+\uparrow \Sigma F = \Sigma F_{eff}$

$85 \text{ kN} - mg = ma_n$

$85 - mg = m\bar{r}\omega^2 \qquad (1)$

$+\downarrow \Sigma F = \Sigma F_{eff}$

$55 \text{ kN} + mg = ma_n$

$55 + mg = m\bar{r}\omega^2 \qquad (2)$

EQ(2) − EQ(1): $\quad 30 \text{ kN} - 2mg = 0$

$30 \times 10^3 \text{ N} = 2m(9.81 \text{ m/s}^2)$

$m = 1529$ kg

$$m = 1529 \text{ kg}$$

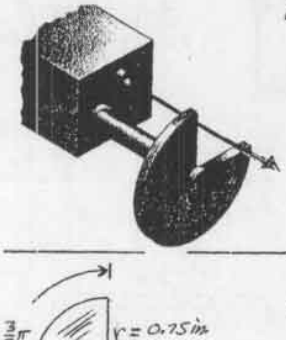
EQ(1) + EQ(2): $\quad 140 \text{ kN} = 2m\bar{r}\omega^2$

$140 \times 10^3 \text{ N} = 2(1529 \text{ kg})\bar{r}(40\pi)^2$

$\bar{r} = 2.90 \times 10^{-3}$ m

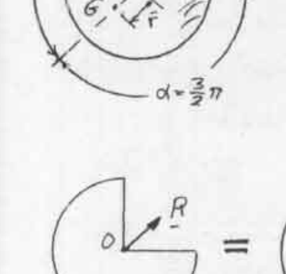
$$\bar{r} = 2.90 \text{ mm}$$

16.83



GIVEN: 0.125-lb SHUTTER OF RADIUS 0.75 in.
$\omega = 24$ cycles per second

FIND: MAGNITUDE OF FORCE EXERTED ON SHAFT BY SHUTTER

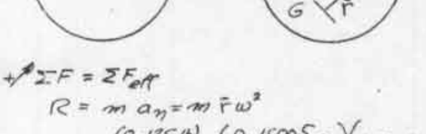


SEE INSIDE FRONT COVER FOR CENTROID OF A CIRCULAR SECTOR

$\bar{r} = \frac{2r\sin\alpha}{3\alpha}$

$\bar{r} = \frac{2(0.75 \text{ in.})\sin\left(\frac{3}{4}\pi\right)}{3\left(\frac{3}{4}\pi\right)}$

$\bar{r} = 0.15005$ in.



$a_n = \bar{r}\omega^2$

$\omega = 24 \frac{\text{rev}}{\text{s}}$

$= 24(2\pi)\frac{\text{rad}}{\text{s}}$

$\omega = 150.8$ rad/s

$+\nearrow \Sigma F = \Sigma F_{eff}$

$R = m a_n = m\bar{r}\omega^2$

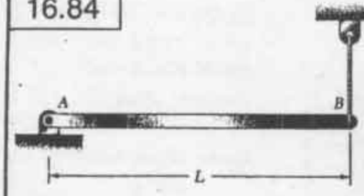
$= \frac{(0.125 \text{ lb})}{32.2 \text{ ft/s}^2}\left(\frac{0.15005}{12} \text{ ft}\right)(150.8 \text{ rad/s})^2$

$\underline{R} = 1.1038 \text{ lb} \nearrow$

FORCE ON SHAFT IS $\underline{R} = 1.104 \text{ lb} \swarrow$

MAGNITUDE: $R = 1.104$ lb

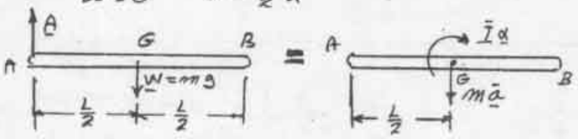
16.84



GIVEN: CABLE SUDDENLY BREAKS.

FIND: (a) $\underline{a}_B$ (b) REACTION AT A

$\omega = 0 \qquad \bar{a} = \frac{L}{2}\alpha$



$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}: \quad W\frac{L}{2} = \bar{I}\alpha + m\bar{a}\frac{L}{2}$

$mg\frac{L}{2} = \frac{1}{12}mL^2\alpha + m\left(\frac{L}{2}\alpha\right)\frac{L}{2}$

$mg\frac{L}{2} = \frac{1}{3}mL^2\alpha \qquad \underline{\alpha} = \frac{3}{2}\frac{g}{L} \circlearrowright$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad A - mg = -m\bar{a} = -m\frac{L}{2}\alpha$

$A - mg = -m\left(\frac{L}{2}\right)\left(\frac{3}{2}\frac{g}{L}\right)$

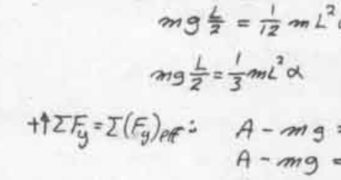
$A - mg = -\frac{3}{4}mg$

$A = \frac{1}{4}mg; \qquad \underline{A} = \frac{1}{4}mg\uparrow$ ◀

$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A} = 0 + L\alpha\downarrow$

$\underline{a}_B = L\left(\frac{3}{2}\frac{g}{L}\right) = \frac{3}{2}g\downarrow; \qquad \underline{a}_B = \frac{3}{2}g\downarrow$ ◀

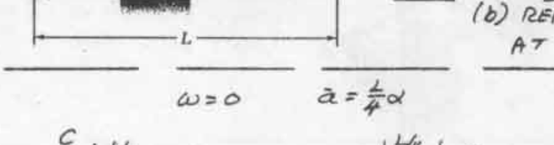
16.85



GIVEN: CABLE SUDDENLY BREAKS.

FIND: (a) a_B (b) REACTION AT C.

$\omega = 0 \qquad \bar{a} = \frac{L}{4}\alpha$



$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}: \quad W\frac{L}{4} = \bar{I}\alpha + m\bar{a}\frac{L}{4}$

$mg\frac{L}{4} = \frac{1}{12}mL^2\alpha + m\left(\frac{L}{4}\alpha\right)\frac{L}{4}$

$mg\frac{L}{4} = \frac{7}{48}mL^2\alpha$

$\underline{\alpha} = \frac{12g}{7L} \circlearrowright$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad C - mg = -m\bar{a} = -m\frac{L}{4}\alpha$

$C - mg = -m\left(\frac{L}{4}\right)\left(\frac{12g}{7L}\right)$

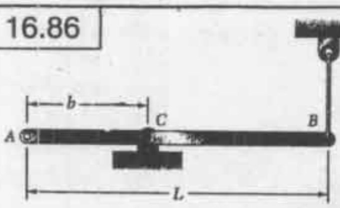
$C - mg = -\frac{3}{7}mg$

$C = \frac{4}{7}mg; \qquad \underline{C} = \frac{4}{7}mg\uparrow$ ◀

$a_B = a_C + a_{C/B} = 0 + \frac{3L}{4}\alpha$

$a_B = \frac{3L}{4}\left(\frac{12g}{7L}\right) = \frac{9}{7}g; \qquad \underline{a}_B = \frac{9}{7}g\downarrow$ ◀

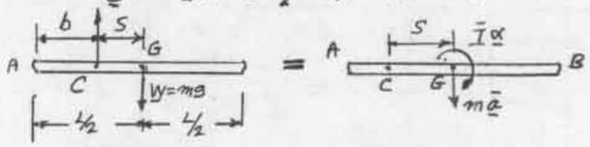
16.86



GIVEN: CABLE SUDDENLY BREAKS

FIND: (a) DISTANCE b FOR WHICH $\underline{a}_A$ IS MAXIMUM.
(b) CORRESPONDING $\underline{a}_A$ AND REACTION AT C.

LET $s = \frac{L}{2} - b, \qquad \bar{a} = s\alpha$



$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}: \quad Ws = \bar{I}\alpha + mas$

$mgs = \frac{1}{12}mL^2 + m(s\alpha)s$

$mgs = m\left(\frac{1}{12}L^2 + s^2\right)\alpha$

$$\alpha = \frac{sg}{\frac{L^2}{12} + s^2} \qquad (1)$$

FOR ROTATION ABOUT C: $a_A = \frac{L}{2} - s$

$$a_A = \frac{\left(\frac{L}{2} - s\right)sg}{\frac{L^2}{12} + s^2} = \frac{\frac{L}{2}s - s^2}{\frac{L^2}{12} + s^2}g$$

$$a_A = \frac{Ls - 2s^2}{L^2 + 12s^2}(6g)$$

DIFFERENTIATE WITH RESPECT TO s.

$$\frac{da_A}{ds} = \frac{(L^2 + 12s^2)(L - 4s) - (Ls - 2s^2)(24s)}{(L^2 + 12s^2)^2}(6g)$$

SET NUMERATOR EQUAL TO ZERO

$L^3 - 4sL^2 + 12s^2L - \cancel{48s^3} - 24s^2L + \cancel{48s^3} = 0$

$L^3 - 4sL^2 - 12s^2L = 0$

$L(L^2 - 4sL - 12s^2) = 0$

$L(L - 6s)(L + 2s) = 0$

$s = -\frac{L}{2} \quad \text{AND} \quad s = \frac{L}{6}$

(a)

FOR $s = -\frac{L}{2}$, $b = L$ AND SUPPORT WAS AT B, IMPOSSIBLE

FOR $\underline{s = \frac{L}{6}}$, $\underline{b = \frac{L}{3}}$ THIS RESULTS IN MAX $\underline{a}_A$

$b = \frac{L}{3}$ ◀

(b) EQ. 1 WITH $s = L/6$

$$\alpha = \frac{\frac{L}{6}g}{\frac{L^2}{12} + \left(\frac{L}{6}\right)^2} = \frac{\frac{1}{6}}{\frac{1}{9}}\frac{g}{L} \qquad \alpha = \frac{3}{2}\frac{g}{L}$$

$a_A = s\alpha = \frac{L}{6}\left(\frac{3}{2}\frac{g}{L}\right) = \frac{1}{4}g \qquad \text{MAX: } a_A = \frac{1}{4}g\uparrow$ ◀

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad C - mg = -m\bar{a}$

$C - mg = -ms\alpha$

$C - mg = -m\left(\frac{L}{6}\right)\left(\frac{3}{2}\frac{g}{L}\right)$

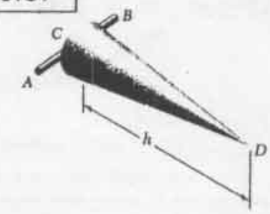
$C - mg = -\frac{1}{4}mg$

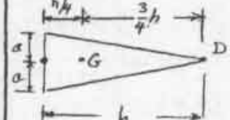
$\underline{C} = \frac{3}{4}mg\uparrow$ ◀

16.87

GIVEN: m = MASS OF CONE
IMMEDIATELY AFTER CONE IS RELEASED

FIND: (a) a_D
(b) REACTION AT C





FROM INSIDE COVER

$I_D = \frac{3}{5}m\left(\frac{1}{4}a^2 + h^2\right)$

FOR SLENDER CONE, NEGLECT 'a'

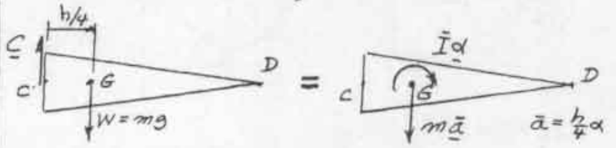
$I_D = \frac{3}{5}mh^2$

PARALLEL AXIS THEOREM

$I_D = \bar{I} + m\left(\frac{3}{4}h\right)^2 \; ; \; \frac{3}{5}mh^2 = \bar{I} + \frac{9}{16}mh^2$

$\bar{I} = \left(\frac{3}{5} - \frac{9}{16}\right)mh^2 \qquad \bar{I} = \frac{3}{80}mh^2$

SAME RESULT CAN BE OBTAINED FROM SAMPLE PROB. 9.11, PAGE 503.



$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{eff}: \quad W\frac{h}{4} = \bar{I}\alpha + m\bar{a}\frac{h}{4}$

$mg\frac{h}{4} = \frac{3}{80}mh^2\alpha + m\left(\frac{h}{4}\alpha\right)\frac{h}{4}$

$mg\frac{h}{4} = \left(\frac{3}{80} + \frac{1}{16}\right)mh^2\alpha$

$mg\frac{h}{4} = \frac{1}{10}mh^2\alpha$

$\alpha = \frac{5}{2}\frac{g}{h}$

(a) $a_D = h\alpha = h\left(\frac{5}{2}\frac{g}{h}\right) \qquad a_D = 2.50g \downarrow$ ◀

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}: \quad C - mg = -m\bar{a}$

$C - mg = -m\frac{h}{4}\alpha$

$C - mg = -m\frac{h}{4}\left(\frac{5}{2}\frac{g}{h}\right)$

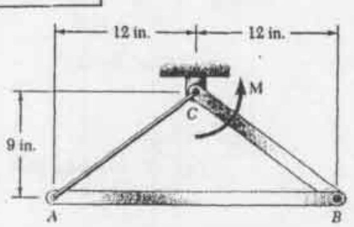
$C - mg = -\frac{5}{8}mg$

$\underline{C} = \frac{3}{8}mg \uparrow$ ◀

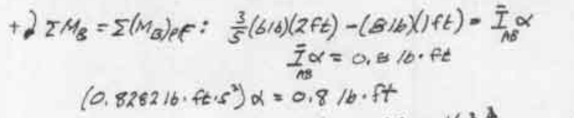
16.88

GIVEN: $W_{AB} = 8\text{ lb}$
$W_{BC} = 5\text{ lb}$
$\omega = 0$
$T_{AC} = 6\text{ lb}$

FIND:
(a) α
(b) $\underline{M}$



ROD AB: $\bar{I}_{AB} = \frac{1}{12}\frac{W}{g}L_{AB}^2 = \frac{1}{12}\frac{8\text{ lb}}{32.2}(2\text{ ft})^2 = 0.8282\text{ lb}\cdot\text{ft}\cdot\text{s}^2$



$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}: \quad \frac{3}{5}(6\text{ lb})(2\text{ ft}) - (8\text{ lb})(1\text{ ft}) = \bar{I}_{AB}\alpha$

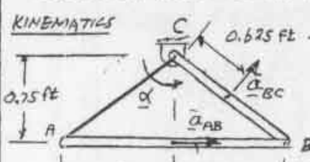
$\bar{I}_{AB}\alpha = 0.8\text{ lb}\cdot\text{ft} \qquad (1)$

$(0.8282\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\alpha = 0.8\text{ lb}\cdot\text{ft}$

$\alpha = 9.66\text{ rad/s}^2 \circlearrowleft$ ◀

ENTIRE ASSEMBLY: SINCE AC IS TAUT, ASSEMBLY ROTATES ABOUT C AS A RIGID BODY.

KINEMATICS



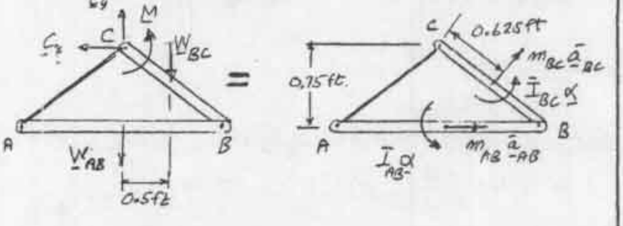
$CB = \sqrt{(0.75)^2 + 1^2} = 1.25\text{ ft}$

$CG_{BC} = \frac{1}{2}CB = 0.625\text{ ft}$

$\bar{a}_{BC} = (0.625\text{ ft})\alpha \nearrow$

$\bar{a}_{AB} = (0.75\text{ ft})\alpha \rightarrow$

KINETICS $\quad \bar{I}_{BC} = \frac{1}{12}\frac{W_{BC}}{g}(CB)^2 = \frac{1}{12}\frac{5\text{ lb}}{32.2}(1.25\text{ ft})^2 = 0.0202\text{ lb}\cdot\text{ft}\cdot\text{s}^2$



$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}:$

$M - W_{BC}(0.5\text{ ft}) = m_{BC}\bar{a}_{BC}(0.625\text{ ft}) + \bar{I}_{BC}\alpha + m_{AB}\bar{a}_{AB}(1\text{ ft}) + \bar{I}_{AB}\alpha$

$M - (5\text{ lb})(0.5\text{ ft}) = \frac{5\text{ lb}}{32.2}(0.625\text{ ft})^2\alpha + (0.0202\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\alpha + \frac{8\text{ lb}}{32.2}(0.75\text{ ft})^2\alpha + \bar{I}_{AB}\alpha$

SUBSTITUTE $\alpha = 9.66\text{ rad/s}^2$ AND FROM EQ(1), $\bar{I}_{AB}\alpha = 0.8\text{ lb}\cdot\text{ft}$

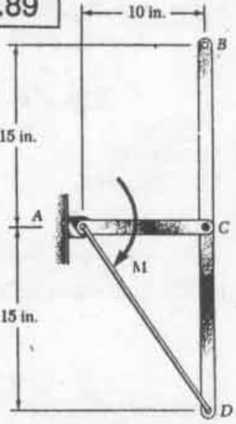
$M - 2.5 = (0.06066)(9.66) + (0.0202)(9.66) + (0.1398)(9.66) + 0.8$

$M - 2.5 = 0.5860 + 0.1950 + 1.35 + 0.8$

$M - 2.5 = 2.93$

$M = +5.43\text{ lb}\cdot\text{ft} \qquad \underline{M} = 5.43\text{ lb}\cdot\text{ft} \circlearrowleft$ ◀

16.89



GIVEN: $W_{AC} = 8\ lb$

$W_{BD} = 20\ lb$

$\underline{M} = 6\ lb \cdot ft \circlearrowright$

FIND: (a) $\underline{\alpha}$

(b) T IN CORD AD

ENTIRE ASSEMBLY:

KINEMATICS

$\omega = 0$

$\bar{a}_{AC} = \left(\frac{5}{12}\ ft\right)\alpha$

$\underline{\alpha}$

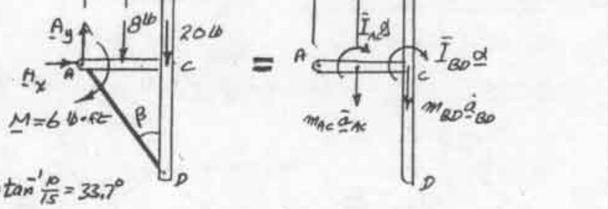
$\bar{a}_{AC}$

$\bar{a}_{BD} = \left(\frac{10}{12}\ ft\right)\alpha$

KINETICS

$$\bar{I}_{AC} = \frac{1}{12}\frac{W_{AC}}{g}(AC)^2 = \frac{1}{12}\cdot\frac{8\ lb}{32.2}\left(\frac{10}{12}\ ft\right)^2 = 0.014378\ lb\cdot ft\cdot s^2$$

$$\bar{I}_{BD} = \frac{1}{12}\frac{W_{BD}}{g}(BD)^2 = \frac{1}{12}\ \frac{20\ lb}{32.2}\left(\frac{30}{12}\ ft\right)^2 = 0.3235\ lb\cdot ft\cdot s^2$$



$\beta = \tan^{-1}\frac{10}{15} = 33.7°$

$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$:

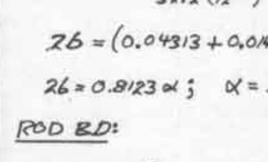
$$(8\ lb)\left(\tfrac{5}{12}\ ft\right) + (20\ lb)\left(\tfrac{10}{12}\ ft\right) + 6\ lb\cdot ft = m_{AC}\bar{a}_{AC}\left(\tfrac{5}{12}\ ft\right) + \bar{I}_{AC}\alpha + m_{BD}\bar{a}_{BD}\left(\tfrac{10}{12}\ ft\right) + \bar{I}_{BD}\alpha$$

$$26\ ft\cdot lb = \frac{8\ lb}{32.2}\left(\frac{5}{12}\ ft\right)^2\alpha + \bar{I}_{AC}\alpha + \frac{20\ lb}{32.2}\left(\frac{10}{12}\ ft\right)^2\alpha + \bar{I}_{BD}\alpha$$

$$26 = (0.04313 + 0.014378 + 0.4313 + 0.3235)\ \alpha$$

$26 = 0.8123\ \alpha$; $\alpha = 32.01\ rad/s^2$ $\qquad \underline{\alpha} = 32.0\ rad/s^2 \circlearrowright$ ◀

ROD BD:



$\beta = 33.7°$

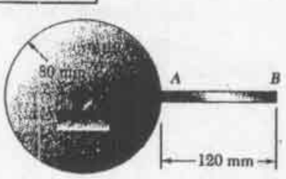
$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}$:

$$(T_{AC}\sin\beta)\left(\tfrac{15}{12}\ ft\right) = \bar{I}_{BD}\alpha$$

$$(T_{AC}\sin 33.7°)\left(\tfrac{15}{12}\ ft\right) = (0.3235\ lb\cdot ft\cdot s^2)(32.01\ rad/s^2)$$

$T_{AC} = 14.93\ lb$ $\qquad T_{AC} = 14.93\ lb$ ◀

16.90



GIVEN: $m_{AB} = 1.5\ kg$

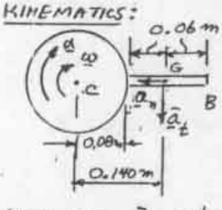
$m_{DISK} = 5\ kg$

$\underline{\omega} = 10\ rad/s \circlearrowright$

FIND: (a) $\underline{\alpha}$

(b) COMPONENTS OF REACTION AT C

KINEMATICS:



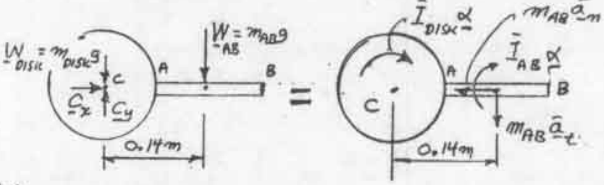
$$\bar{a}_n = (CG)\omega^2 = (0.14\ m)(10\ rad/s)^2$$

$$\bar{a}_n = 14\ m/s^2 \leftarrow$$

$$\bar{a}_t = (CG)\alpha = (0.14\ m)\alpha \downarrow$$

KINETICS: $\bar{I}_{DISK} = \frac{1}{2} m_{DISK}(CG)^2 = \frac{1}{2}(5\ kg)(0.08\ m)^2 = 16\times10^{-3}\ kg\cdot m^2$

$\bar{I}_{AB} = \frac{1}{12} m_{AB}(AB)^2 = \frac{1}{12}(1.5\ kg)(0.12\ m)^2 = 1.8\times10^{-3}\ kg\cdot m^2$



(a)

$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}$:

$$W_{AB}(0.14\ m) = \bar{I}_{DISK}\alpha + m_{AB}\bar{a}_t(0.14\ m) + \bar{I}_{AB}\alpha$$

$$(1.5\ kg)(9.81\ m/s^2)(0.14\ m) = \bar{I}_{DISK}\alpha + (1.5\ kg)(0.14\ m)^2\alpha + \bar{I}_{AB}\alpha$$

$$2.060\ N\cdot m = (16\times10^{-3} + 29.4\times10^{-3} + 1.8\times10^{-3})\ \alpha$$

$$2.060\ N\cdot m = (47.2\times10^{-6}\ kg\cdot m^2)\ \alpha$$

$\alpha = 43.64\ rad/s^2$ $\qquad \underline{\alpha} = 43.6\ rad/s^2 \circlearrowright$ ◀

(b) $\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$

$$C_x = -m_{AB}\bar{a}_n = -(1.5\ kg)(14\ m/s^2)$$

$C_x = -21.0\ N$ $\qquad \underline{C}_x = 21.0\ N \leftarrow$ ◀

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $\qquad a_t = (0.14\ m)(\alpha)$

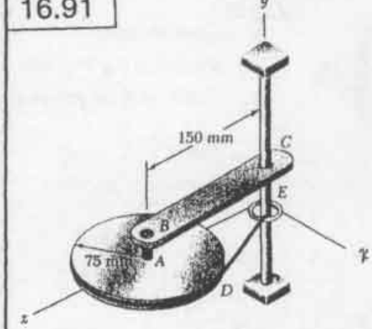
$$C_y - m_{DISK}g - m_{AB}g = -m_{AB}\bar{a}_t$$

$$C_y - (5\ kg)9.81 - (1.5\ kg)9.81 = -(1.5\ kg)(0.14\ m)(43.64\ rad/s^2)$$

$$C_y - 49.05\ N - 14.715\ N = -9.164\ N$$

$C_y = +54.6\ N$ $\qquad \underline{C}_y = 54.6\ N \uparrow$ ◀

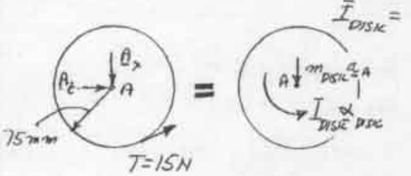
16.91



GIVEN:
$m_{DISK} = 5\ kg$
$m_{BC} = 3\ kg$
CORD: $T = 15\ N$

IMMEDIATELY AFTER RELEASE FROM REST,
FIND: (a) α_{DISK}
(b) a_A

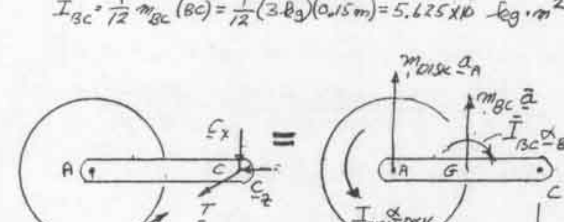
DISK: $\bar{I}_{DISK} = \frac{1}{2} m_{DISK} r^2 = \frac{1}{2}(5\ kg)(0.075\ m)^2$
$\bar{I}_{DISK} = 14.06 \times 10^{-3}\ kg \cdot m^2$



$T = 15\ N$

$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$: $(15\ N)(0.075\ m) = \bar{I}_{DISK} \alpha_{DISK}$
$1.125\ N \cdot m = (14.06 \times 10^{-3}\ kg \cdot m^2)\alpha_{DISK}$
$\alpha_{DISK} = 80.0\ rad/s^2$ $\quad \alpha_{DISK} = 80.0\ rad/s^2 \circlearrowleft$

ENTIRE ASSEMBLY
$\bar{I}_{BC} = \frac{1}{12} m_{BC} (BC)^2 = \frac{1}{12}(3\ kg)(0.15\ m)^2 = 5.625 \times 10^{-3}\ kg \cdot m^2$



ASSUME α_{BC} IS $\circlearrowright$
$a_A = (0.15\ m)\alpha_{BC} \uparrow$
$\bar{a} = (0.075\ m)\alpha_{BC}$

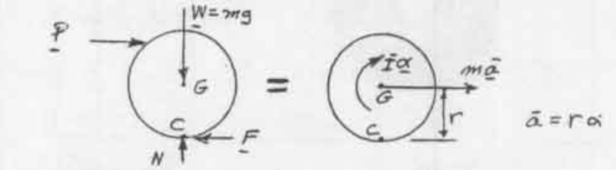
$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$:
$0 = I_{DISK}\alpha_{DISK} - m_{DISK} a_A (0.15\ m) - m_{BC}\bar{a}(0.075\ m) - \bar{I}_{BC}\alpha_{BC}$
$0 = (14.06 \times 10^{-3}\ kg \cdot m^2)(80\ rad/s^2) - (5\ kg)(0.15\ m)^2 \alpha_{BC}$
$\quad - (3\ kg)(0.075\ m)^2 \alpha_{BC} - (5.625 \times 10^{-3}\ kg\, m^2)\alpha_{BC}$
$0 = 1.125 - 0.1125\alpha_{BC} - 16.875 \times 10^{-3}\alpha_{BC} - 5.625 \times 10^{-3}\alpha_{BC}$
$0 = 1.125 - 0.135\alpha_{BC}$
$\alpha_{BC} = +8.333\ rad/s^2$ $\quad \alpha_{BC} = 8.33\ rad/s^2 \circlearrowright$

$a_A = (AC)\alpha_{BC} = (0.15\ m)(8.333\ rad/s^2)$
$a_A = +1.25\ m/s^2$ $\quad a_A = 1.25\ m/s^2 \uparrow$

NOTE: ANSWERS CAN ALSO BE WRITTEN
$\alpha_{DISK} = (80\ rad/s^2)\mathbf{j}$ $\quad a_A = -(1.25\ m/s^2)\mathbf{i}$

16.92

DERIVE $\Sigma M_C = I_C \alpha$ FOR THE ROLLING DISK OF FIG. 16.17.

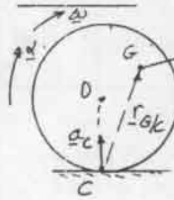


$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$: $\quad \Sigma M_C = (m\bar{a})r + \bar{I}\alpha$
$= (m r\alpha)r + \bar{I}\alpha$
$\Sigma M_C = (mr^2 + \bar{I})\alpha$

BUT, WE KNOW THAT $I_C = mr^2 + \bar{I}$
THUS: $\Sigma M_C = I_C \alpha$ $\quad$ (Q.E.D.)

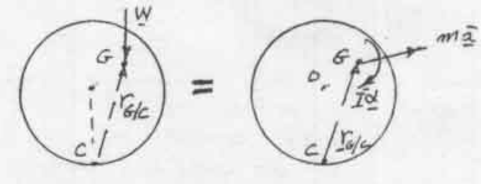
16.93

FOR AN UNBALANCED DISK SHOW THAT $\Sigma M = I_C \alpha$ IS VALID ONLY WHEN THE MASS CENTER G, THE GEOMETRIC CENTER O, AND THE INSTANTANEOUS CENTER C HAPPEN TO LIE IN A STRAIGHT LINE.



KINEMATICS:
$\bar{a} = a_C + a_{G/C}$
$= a_C + \alpha \times r_{G/C} + \omega \times (\omega \times r_{G/C})$
OR, SINCE $\omega \perp r_{G/C}$
$\bar{a} = a_C + \alpha \times r_{G/C} - \omega^2 r_{G/C}$ $\quad (1)$

KINETICS



$\Sigma M_C = \Sigma (M_C)_{eff}$: $\quad \Sigma M_C = \bar{I}\alpha + r_{G/C} \times m\bar{a}$
RECALL EQ(1): $\Sigma M_C = \bar{I}\alpha + r_{G/C} \times m(a_C + \alpha \times r_{G/C} - \omega^2 r_{G/C})$

$\Sigma M_C = \bar{I}\alpha + r_{G/C} \times m a_C + m\, r_{G/C} \times (\alpha \times r_{G/C}) - m\omega^2 r_{G/C} \times r_{G/C}$

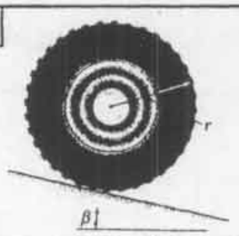
BUT: $r_{G/C} \times r_{G/C} = 0$ AND $\alpha \perp r_{G/C}$
$r_{G/C} \times m(\alpha \times r_{G/C}) = m\, r_{G/C}^2 \alpha$

THUS: $\Sigma M_C = (\bar{I} + m r_{G/C}^2)\alpha + r_{G/C} \times m a_C$
SINCE $I_C = \bar{I} + m r_{G/C}^2$
$\Sigma M_C = I_C \alpha + r_{G/C} \times m a_C$ $\quad (2)$

EQ(2) REDUCES TO $\Sigma M_C = I_C \alpha$ WHEN $r_{G/C} \times m a_C = 0$
THAT IS, WHEN $r_{G/C}$ AND a_C ARE COLLINEAR.

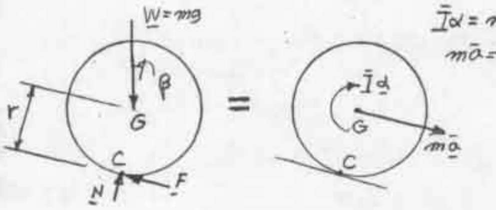
REFERRING TO THE FIRST DIAGRAM, WE NOTE THAT THIS WILL OCCUR ONLY WHEN POINTS G, O AND C LIE IN A STRAIGHT LINE.
(Q.E.D.)

16.94



GIVEN: ROLLING WHEEL

FIND: $\bar{a}$ IN TERMS OF r, $\bar{k}$, β, AND g.



$\bar{I}\alpha = m\bar{k}^2\alpha$

$m\bar{a} = mr\alpha$

$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}: \quad (W \sin\beta) r = (m\bar{a}) r + \bar{I}\alpha$

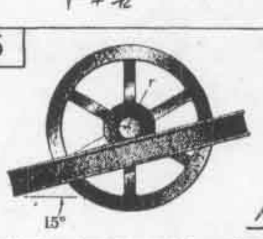
$(mg \sin\beta) r = (mr\alpha) r + m\bar{k}^2\alpha$

$rg \sin\beta = (r^2 + \bar{k}^2)\alpha$

$\alpha = \dfrac{rg \sin\beta}{r^2 + \bar{k}^2}$

$\bar{a} = r\alpha = r\,\dfrac{rg\sin\beta}{r^2 + \bar{k}^2} \qquad \bar{a} = \dfrac{r^2}{r^2+\bar{k}^2}\, g \sin\beta$ ◀

16.95



GIVEN: STARTING FROM REST, FLYWHEEL MOVES 16 ft IN 40 s

$r = 1.5\,in.$

FIND: $\bar{I}$

KINEMATICS: $\quad s = v_0 t + \frac{1}{2}\bar{a}t^2$

$16\,ft = 0 + \frac{1}{2}\bar{a}(40\,s)^2$

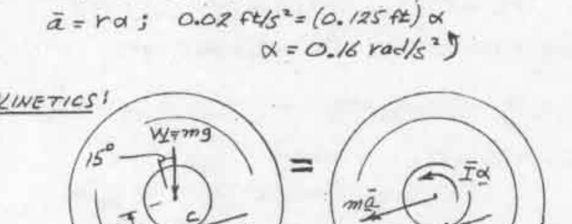
$\bar{a} = 0.02\,ft/s^2$

SINCE $r = 1.5\,in = 0.125\,ft$

$\bar{a} = r\alpha; \quad 0.02\,ft/s^2 = (0.125\,ft)\alpha$

$\alpha = 0.16\,rad/s^2$

KINETICS:



$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}:$

$(mg \sin 15°) r = \bar{I}\alpha + (m\bar{a}) r$

$(\cancel{m}g \sin 15°) r = \cancel{m}\bar{k}^2\alpha + (\cancel{m}r\alpha) r$

$g r \sin 15° = (\bar{k}^2 + r^2)\alpha$

DATA: $r = 0.125\,ft, \quad \alpha = 0.16\,rad/s^2$

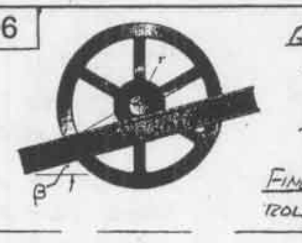
$(32.2\,ft/s^2)(0.125\,ft)\sin 15° = (\bar{k}^2 + r^2)(0.16\,rad/s^2)$

$\bar{k}^2 + r^2 = 6.511\,ft^2$

$\bar{k}^2 + (0.125\,ft)^2 = 6.511\,ft^2$

$\bar{k}^2 = 6.4953 \qquad \bar{k} = 2.55\,ft$ ◀

16.96

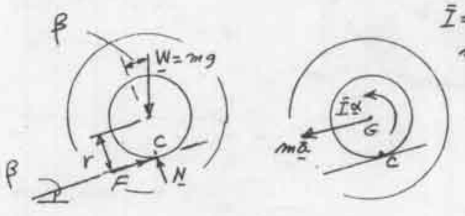


GIVEN:

$\bar{k}$ = CENTROIDAL RADIUS OF GYRATION

μ_s = COEF. STATIC FRICTION

FIND: LARGEST β FOR ROLLING WITHOUT SLIPPING



$\bar{I} = m\bar{k}^2$

$m = r\alpha$

$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}: \quad (mg \sin\beta) r = \bar{I}\alpha + (m\bar{a}) r$

$mg \sin\beta\, r = m\bar{k}^2\alpha + mr^2\alpha$

$\alpha = \dfrac{gr}{r^2 + \bar{k}^2}\sin\beta \qquad (1)$

$+\nearrow \Sigma F = \Sigma F_{eff}: \quad F - mg\sin\beta = -m\bar{a}$

$F - mg\sin\beta = -mr\alpha$

$F = mg\sin\beta - mr\alpha$

$+\nwarrow \Sigma F = \Sigma F_{eff}: \quad N - mg\cos\beta = 0$

$N = mg\cos\beta$

IF SLIPPING IMPENDS $F = \mu_s N$ OR $\mu_s = \dfrac{F}{N}$

$\mu_s = \dfrac{F}{N} = \dfrac{mg\sin\beta - mr\alpha}{mg\cos\beta} = \dfrac{\sin\beta - \frac{r}{g}\alpha}{\cos\beta}$

SUBSTITUTING FOR α FROM EQ(1)

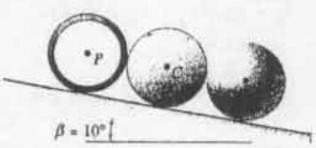
$\mu_s = \dfrac{\sin\beta - \frac{r}{g}\cdot\frac{gr}{r^2+\bar{k}^2}\sin\beta}{\cos\beta}$

$\mu_s = \tan\beta\left[1 - \dfrac{r^2}{r^2+\bar{k}^2}\right] = \tan\beta\left[\dfrac{\bar{k}^2}{r^2+\bar{k}^2}\right]$

$\tan\beta = \mu_s\,\dfrac{r^2+\bar{k}^2}{\bar{k}^2}$

$\tan\beta = \mu_s\left[1 + \left(\dfrac{r}{\bar{k}}\right)^2\right]$ ◀

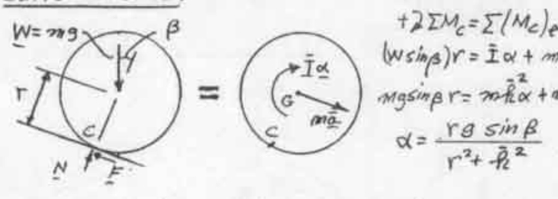
16.97



$\beta = 10°$

GIVEN: PIPE, CYLINDER AND SPHERE ARE RELEASED FROM REST. AFTER 4 S, FIND DISTANCE BETWEEN
(a) PIPE AND CYLINDER
(b) CYLINDER AND SPHERE.

GENERAL CASE: $\bar{I} = m\bar{k}^2 \quad \bar{a} = r\alpha$



$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}$

$(W\sin\beta)r = \bar{I}\alpha + m\bar{a}r$

$mg\sin\beta\, r = m\bar{k}^2\alpha + mr^2\alpha$

$\alpha = \dfrac{rg\sin\beta}{r^2 + \bar{k}^2}$

$\bar{a} = r\alpha = r\dfrac{rg\sin\beta}{r^2+\bar{k}^2} \qquad \bar{a} = \dfrac{r^2}{r^2+\bar{k}^2} g\sin\beta \qquad (1)$

FOR PIPE: $\bar{k} = r \qquad \bar{a}_p = \dfrac{r^2}{r^2+r^2} g\sin\beta = \dfrac{1}{2} g\sin\beta$

FOR CYLINDER: $\bar{k}^2 = \frac{1}{2} \qquad \bar{a}_c = \dfrac{r^2}{r^2+r^2/2} g\sin\beta = \dfrac{2}{3} g\sin\beta$

FOR SPHERE: $\bar{k}^2 = \frac{2}{5} \qquad \bar{a}_s = \dfrac{r^2}{r^2+\frac{2}{5}r^2} g\sin\beta = \dfrac{5}{7} g\sin\beta$

(a) BETWEEN PIPE AND CYLINDER

$a_{c/p} = a_c - a_p = \left(\frac{2}{3} - \frac{1}{2}\right) g\sin\beta = \frac{1}{6} g\sin\beta$

$x_{c/p} = \frac{1}{2} a_{c/p} t^2 = \frac{1}{2}\left(\frac{1}{6} g\sin\beta\right)t^2$

SI UNITS: $x_{c/p} = \frac{1}{2}\left(\frac{1}{6}\, 9.81\, m/s^2\right)\sin 10°(4\,s)^2 = 2.27\,m$ ◀

US UNITS: $x_{c/p} = \frac{1}{2}\left(\frac{1}{6}\, 32.2\, ft/s^2\right)\sin 10°(4\,s)^2 = 7.46\,ft$ ◀

(b) BETWEEN SPHERE AND CYLINDER

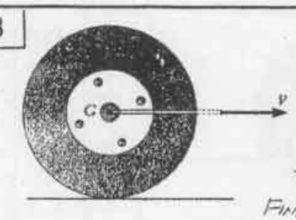
$a_{s/c} = a_s - a_c = \left(\frac{5}{7} - \frac{2}{3}\right) g\sin\beta = \frac{1}{21} g\sin\beta$

$x_{s/c} = \frac{1}{2} a_{s/c} t^2 = \frac{1}{2}\left(\frac{1}{21} g\sin\beta\right)t^2$

SI UNITS: $x_{s/c} = \frac{1}{2}\left(\frac{1}{21}\, 9.81\, m/s^2\right)\sin 10°(4\,s)^2 = 0.649\,m$ ◀

US UNITS: $x_{s/c} = \frac{1}{2}\left(\frac{1}{21}\, 32.2\, ft/s^2\right)\sin 10°(4\,s)^2 = 2.13\,ft$ ◀

16.98



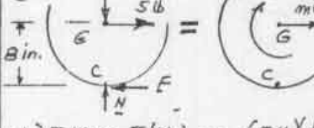
GIVEN: $W = 10\,lb$
$r_o = 8\,in.\quad r_i = 4\,in.$
$\bar{k} = 6\,in.$
$P = 5\,lb$
$\mu_s = 0.25,\ \mu_k = 0.20$

FIND: (a) DOES DISK SLIDE, (b) α AND $\bar{a}$.

ASSUME DISK ROLLS: $\bar{a} = r\alpha = \left(\frac{8}{12}\,ft\right)\alpha$

$\bar{I} = m\bar{k}^2 = \frac{10\,lb}{32.2}\left(\frac{6}{12}\,ft\right)^2$

$\bar{I} = 0.07764\ lb\cdot ft\cdot s^2$



$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}: \quad (5\,lb)\left(\frac{8}{12}\,ft\right) = (m\bar{a})r + \bar{I}\alpha$

$3.333\,lb\cdot ft = \frac{10\,lb}{32.2}\left(\frac{8}{12}\,ft\right)^2\alpha + 0.07764\alpha$

$3.333 = 0.21566\,\alpha$

$\alpha = 15.456\ rad/s^2 \qquad \alpha = 15.46\ rad/s^2$ ↻ ◀

$\bar{a} = r\alpha = \left(\frac{8}{12}\,ft\right)(15.456\ rad/s^2) \qquad \bar{a} = 10.30\ ft/s^2 \rightarrow$ ◀

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}: \quad -F + 5\,lb = m\bar{a}$

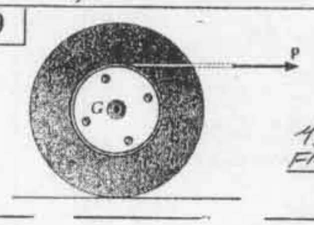
$-F + 5\,lb = \frac{10\,lb}{32.2}(10.30\,ft/s^2); \quad F = 1.801\,lb$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad N - 10\,lb = 0 \qquad N = 10\,lb$

$F_m = \mu_s N = 0.25(10\,lb) = 2.5\,lb$

SINCE $F < F_m$, DISK ROLLS WITH NO SLIDING ◀

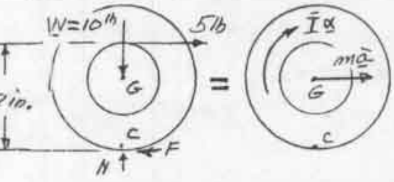
16.99



GIVEN: $W = 10\,lb$
$r_o = 8\,in.,\ r_i = 4\,in.$
$\bar{k} = 6\,in.,\ P = 5\,lb$
$\mu_s = 0.25,\ \mu_k = 0.20$

FIND: (a) DOES DISK SLIDE. (b) α AND $\bar{a}$

ASSUME DISK ROLLS: $\bar{a} = r\alpha = \left(\frac{8}{12}\,ft\right)\alpha$



$\bar{I} = m\bar{k}^2 = \frac{10\,lb}{32.2}\left(\frac{6}{12}\,ft\right)^2$

$\bar{I} = 0.07764\ lb\cdot ft\cdot s^2$

$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}: \quad (5\,lb)(1\,ft) = (m\bar{a})r + \bar{I}\alpha$

$5\,lb\cdot ft = \frac{10\,lb}{32.2}\left(\frac{8}{12}\,ft\right)^2\alpha + 0.07764\alpha$

$5 = 0.21566\,\alpha$

$\alpha = 23.184\ rad/s^2 \qquad \alpha = 23.2\ rad/s^2$ ↻

$\bar{a} = r\alpha = \left(\frac{8}{12}\,ft\right)\left(23.184\ \frac{rad}{s^2}\right) \qquad \bar{a} = 15.46\ ft/s^2 \rightarrow$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}: \quad -F + 5\,lb = m\bar{a}$

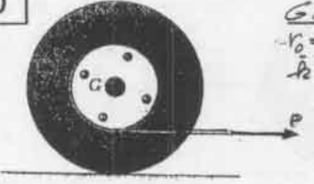
$-F + 5\,lb = \frac{10\,lb}{32.2}(15.46\,ft/s^2); \quad F = 0.20\,lb$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad N - 10\,lb = 0 \qquad N = 10\,lb$

$F_m = \mu_s N = 0.25(10\,lb) = 2.5\,lb$

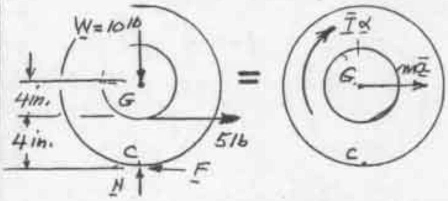
SINCE $F < F_m$, DISK ROLLS WITH NO SLIDING ◀

16.100



GIVEN: $W = 10\text{ lb}$
$r_o = 8\text{ in.}$, $r_i = 4\text{ in.}$
$\bar{k} = 6\text{ in.}$, $P = 5\text{ lb}$
$\mu_s = 0.25$, $\mu_k = 0.20$
FIND:
(a) DOES DISK SLIDE
(b) $\underline{\alpha}$ AND $\underline{\bar{a}}$.

ASSUME DISK ROLLS: $\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)\alpha$



$\bar{I} = m\bar{k}^2$
$= \frac{10\text{ lb}}{32.2}\left(\frac{6}{12}\text{ ft}\right)^2$
$\bar{I} = 0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}$: $(5\text{ lb})\left(\frac{4}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$

$1.6667\text{ lb}\cdot\text{ft} = \frac{10\text{ lb}}{32.2}\left(\frac{8}{12}\text{ ft}\right)^2\alpha + 0.07764\alpha$

$1.6667 = 0.21566\alpha$
$\alpha = 7.728\text{ rad/s}^2$ $\qquad \underline{\alpha} = 7.73\text{ rad/s}^2 \circlearrowright$
$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)7.728\text{ rad/s}^2$ $\qquad \underline{\bar{a}} = 5.153\text{ ft/s}^2 \rightarrow$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $-F + 5\text{ lb} = m\bar{a}$
$-F + 5\text{ lb} = \frac{10\text{ lb}}{32.2}(5.153\text{ ft/s}^2)$
$F = 3.40\text{ lb}$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $N - 10\text{ lb} = 0$ $\qquad N = 10\text{ lb}$

$F_m = \mu_s N = 0.25(10\text{ lb}) = 2.5\text{ lb}$

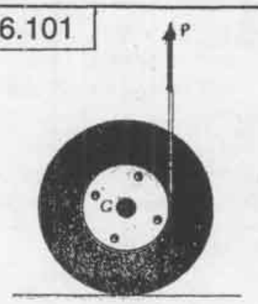
SINCE $F > F_m$, DISK SLIDES ◀

KNOWING THAT DISK SLIDES
$F = \mu_k N = 0.20(10\text{ lb}) = 2\text{ lb}$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$:
$F\left(\frac{8}{12}\text{ ft}\right) - (5\text{ lb})\left(\frac{4}{12}\text{ ft}\right) = \bar{I}\alpha$
$(2\text{ lb})\left(\frac{8}{12}\text{ ft}\right) - 1.6667\text{ lb}\cdot\text{ft} = (0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\alpha$
$-0.3333 = 0.07764\alpha$
$\alpha = -4.29\text{ rad/s}^2$ $\qquad \underline{\alpha} = 4.29\text{ rad/s}^2 \circlearrowleft$ ◀

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$:
$-F + 5\text{ lb} = m\bar{a}$
$-2\text{ lb} + 5\text{ lb} = \frac{10\text{ lb}}{32.2}\bar{a}$
$\bar{a} = 9.66\text{ ft/s}^2$ $\qquad \underline{\bar{a}} = 9.66\text{ ft/s}^2 \rightarrow$ ◀

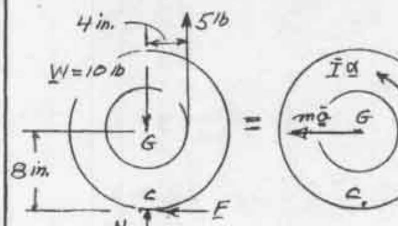
16.101



GIVEN: $W = 10\text{ lb}$
$r_o = 8\text{ in.}$, $r_i = 4\text{ in.}$
$\bar{k} = 6\text{ in.}$
$P = 5\text{ lb}$
$\mu_s = 0.25$, $\mu_k = 0.20$

FIND:
(a) DOES DISK SLIDE
(b) $\underline{\alpha}$ AND $\underline{\bar{a}}$

ASSUME DISK ROLLS $\quad \bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)\alpha$



$\bar{I} = m\bar{k}^2$
$= \frac{10\text{ lb}}{32.2}\left(\frac{6}{12}\text{ ft}\right)^2$
$\bar{I} = 0.07764\text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$: $(5\text{ lb})\left(\frac{4}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$

$1.6667\text{ lb}\cdot\text{ft} = \frac{10\text{ lb}}{32.2}\left(\frac{8}{12}\text{ ft}\right)^2\alpha + 0.07764\alpha$

$1.6667 = 0.21566\alpha$
$\alpha = 7.728\text{ rad/s}^2$ $\qquad \underline{\alpha} = 7.73\text{ rad/s}^2 \circlearrowleft$

$\bar{a} = r\alpha = \left(\frac{8}{12}\text{ ft}\right)(7.728\text{ rad/s}^2)$ $\qquad \underline{\bar{a}} = 5.153\text{ ft/s}^2 \leftarrow$

$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $F = m\bar{a}$
$F = \frac{10\text{ lb}}{32.2}(5.153\text{ ft/s}^2)$; $F = 1.60\text{ lb}$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $N - 10\text{ lb} + 5\text{ lb} = 0$ $\qquad N = 5\text{ lb}$

$F_m = \mu_s N = 0.25(5\text{ lb}) = 1.25\text{ lb}$

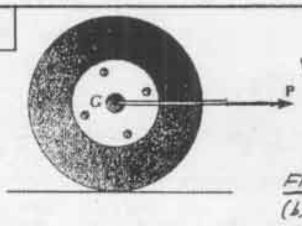
SINCE $F > F_m$, DISK SLIDES ◀

KNOWING THAT DISK SLIDES $\quad F = \mu_k N = 0.2(5)$
$F = 1.00\text{ lb}$

$+\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff}$:
$(5\text{ lb})\left(\frac{4}{12}\text{ ft}\right) - F\left(\frac{8}{12}\text{ ft}\right) = \bar{I}\alpha$
$(5\text{ lb})\left(\frac{4}{12}\text{ ft}\right) - (1.00\text{ lb})\left(\frac{8}{12}\text{ ft}\right) = 0.07764\alpha$
$1.000 = 0.07764\alpha$
$\alpha = 12.88\text{ rad/s}^2$ $\qquad \underline{\alpha} = 12.88\text{ rad/s}^2 \circlearrowleft$ ◀

$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $F = m\bar{a}$
$1.00\text{ lb} = \frac{10\text{ lb}}{32.2}\bar{a}$
$\bar{a} = 3.22\text{ ft/s}^2$
$\underline{\bar{a}} = 3.22\text{ ft/s}^2 \leftarrow$ ◀

16.102



GIVEN: $m = 6\text{ kg}$

$r_0 = 120\text{ mm},\ r_i = 60\text{ mm}$

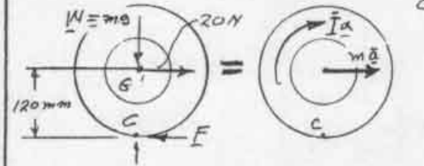
$\bar{k} = 90\text{ mm}$

$P = 20\text{ N}$

DISK ROLLS;

FIND: (a) α AND $\bar{a}$

(b) MINIMUM μ_s



$\bar{a} = r\alpha = (0.12\text{ m})\,\alpha$

$\bar{I} = m\bar{k}^2 = (6\text{ kg})(0.09\text{ m})^2$

$\bar{I} = 40.5\times10^{-3}\text{ kg}\cdot\text{m}^2$

$+\circlearrowright\Sigma M_C = \Sigma(M_C)_{eff}:\quad (20\text{ N})(0.12\text{ m}) = (m\bar{a})r + \bar{I}\alpha$

$2.4\text{ N}\cdot\text{m} = (6\text{ kg})(0.12\text{ m})^2\alpha + 40.5\times10^{-3}\text{ kg}\cdot\text{m}^2$

$2.4 = 135.0\times10^{-3}\alpha$

(a) $\alpha = 17.778\text{ rad/s}^2;\qquad \underline{\alpha = 17.78\text{ rad/s}^2 \circlearrowright}$ ◀

$\bar{a} = r\alpha = (0.12\text{ m})(17.778\text{ rad/s}^2) = 2.133\text{ m/s}^2;\qquad \underline{\bar{a} = 2.13\text{ m/s}^2 \rightarrow}$ ◀

(b) $+\uparrow\Sigma F_y = \Sigma(F_y)_{eff}:\quad N - mg = 0$

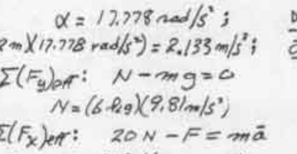
$N = (6\text{ kg})(9.81\text{ m/s}^2)\qquad \underline{N} = 58.86\text{ N}\uparrow$

$\xrightarrow{+}\Sigma F_x = \Sigma(F_x)_{eff}:\quad 20\text{ N} - F = m\bar{a}$

$20\text{ N} - F = (6\text{ kg})(2.133\text{ m/s}^2)\qquad F = 7.20\text{ N}\leftarrow$

$(\mu_s)_{min} = \dfrac{F}{N} = \dfrac{7.20\text{ N}}{58.86\text{ N}}\qquad (\mu_s)_{min} = 0.122$ ◀

16.103



GIVEN: $m = 6\text{ kg}$

$r_0 = 120\text{ mm},\ r_i = 60\text{ mm}$

$\bar{k} = 90\text{ mm}$

$P = 20\text{ N}$

DISK ROLLS

FIND: (a) α AND $\bar{a}$

(b) MINIMUM μ_s

$\bar{a} = r\alpha = (0.12\text{ m})\alpha$

$\bar{I} = m\bar{k}^2 = (6\text{ kg})(0.09\text{ m})^2$

$\bar{I} = 40.5\times10^{-3}\text{ kg}\cdot\text{m}^2$

$+\circlearrowright\Sigma M_C = \Sigma(M_C)_{eff}:\quad (20\text{ N})(0.18\text{ m}) = (m\bar{a})r + \bar{I}\alpha$

$3.6\text{ N}\cdot\text{m} = (6\text{ kg})(0.12\text{ m})^2\alpha + 40.5\text{ kg}\cdot\text{m}^2$

$3.6 = 135\times10^{-3}\alpha$

(a) $\alpha = 26.667\text{ rad/s}^2\qquad \underline{\alpha = 26.7\text{ rad/s}^2 \circlearrowright}$ ◀

$\bar{a} = r\alpha = (0.12\text{ m})(26.667\text{ rad/s}^2) = 3.2\text{ m/s}^2\qquad \underline{\bar{a} = 3.2\text{ m/s}^2 \rightarrow}$ ◀

(b) $+\uparrow\Sigma F_y = \Sigma(F_y)_{eff}:\quad N - mg = 0$

$N = (6\text{ kg})(9.81\text{ m/s}^2)\qquad \underline{N} = 58.86\text{ N}\uparrow$

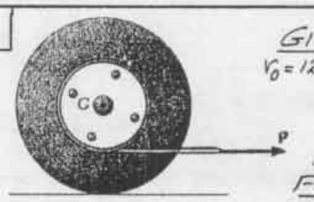
$\xrightarrow{+}\Sigma F_x = \Sigma(F_x)_{eff}:\quad 20\text{ N} - F = m\bar{a}$

$20\text{ N} - F = (6\text{ kg})(3.2\text{ m/s}^2)$

$F = 0.8\text{ N}\leftarrow$

$(\mu_s)_{min} = \dfrac{F}{N} = \dfrac{0.8\text{ N}}{58.86\text{ N}}\qquad (\mu_s)_{min} = 0.0136$ ◀

16.104



GIVEN: $m = 6\text{ kg}$

$r_0 = 120\text{ mm},\ r_i = 60\text{ mm}$

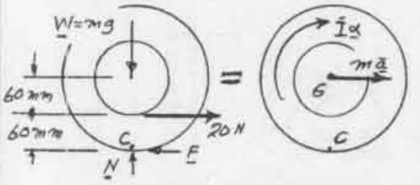
$\bar{k} = 90\text{ mm}$

$P = 20\text{ N}$

DISK ROLLS:

FIND: (a) α AND $\bar{a}$

(b) MINIMUM μ_s



$\bar{a} = r\alpha = (0.12\text{ m})\alpha$

$\bar{I} = m\bar{k}^2 = (6\text{ kg})(0.09\text{ m})^2$

$\bar{I} = 40.5\text{ kg}\cdot\text{m}^2$

$+\circlearrowright\Sigma M_C = \Sigma(M_C)_{eff}:\quad (20\text{ N})(0.06\text{ m}) = (m\bar{a})r + \bar{I}\alpha$

$1.2\text{ N}\cdot\text{m} = (6\text{ kg})(0.12\text{ m})^2\alpha + 40.5\times10^{-3}\text{ kg}\cdot\text{m}^2$

$1.2 = 135\times10^{-3}\alpha$

(a) $\alpha = 8.889\text{ rad/s}^2\qquad \underline{\alpha = 8.89\text{ rad/s}^2 \circlearrowright}$ ◀

$\bar{a} = r\alpha = (0.12\text{ m})(8.889\text{ rad/s}^2 = 1.0667\text{ m/s}^2$

$\underline{\bar{a} = 1.067\text{ m/s}^2 \rightarrow}$ ◀

(b) $+\uparrow\Sigma F_y = \Sigma(F_y)_{eff}:\quad N - mg = 0$

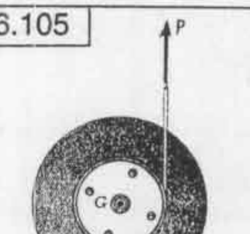
$N = (6\text{ kg})(9.81\text{ m/s}^2)\qquad \underline{N} = 58.86\text{ N}\uparrow$

$\xrightarrow{+}\Sigma F_x = \Sigma(F_x)_{eff}:\quad 20\text{ N} - F = m\bar{a}$

$(20\text{ N}) - F = (6\text{ kg})(1.0667\text{ m/s}^2);\quad \underline{F} = 13.6\text{ N}\leftarrow$

$(\mu_s) = \dfrac{F}{N} = \dfrac{13.6\text{ N}}{58.86\text{ N}}\qquad (\mu_s)_{min} = 0.231$ ◀

16.105



GIVEN: $m = 6\text{ kg}$

$r_0 = 120\text{ mm},\ r_i = 60\text{ mm}$

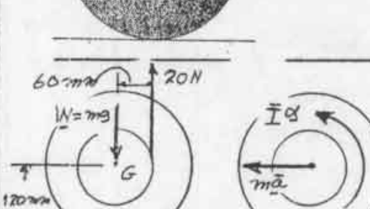
$\bar{k} = 90\text{ mm}$

$P = 20\text{ N}$

DISK ROLLS:

FIND: (a) α AND $\bar{a}$

(b) MINIMUM μ_s



$\bar{a} = r\alpha = (0.12\text{ m})\alpha$

$\bar{I} = m\bar{k}^2 = (6\text{ kg})(0.09\text{ m})^2$

$\bar{I} = 40.5\text{ kg}\cdot\text{m}^2$

$+\circlearrowleft\Sigma M_C = \Sigma(M_C)_{eff}:\quad (20\text{ N})(0.06\text{ m}) = (m\bar{a})r + \bar{I}\alpha$

$1.2\text{ N}\cdot\text{m} = (6\text{ kg})(0.12\text{ m})^2\alpha + 40.5\times10^{-3}\text{ kg}\cdot\text{m}^2$

$1.2 = 135\times10^{-3}\alpha$

(a) $\alpha = 8.889\text{ rad/s}^2\qquad \underline{\alpha = 8.89\text{ rad/s}^2 \circlearrowleft}$ ◀

$\bar{a} = r\alpha = (0.12\text{ m})(8.889\text{ rad/s}^2) = 1.0667\text{ m/s}^2$

$\underline{\bar{a} = 1.067\text{ m/s}^2 \leftarrow}$ ◀

(b) $+\uparrow\Sigma F_y = \Sigma(F_y)_{eff}:\quad N + 20\text{ N} - mg = 0$

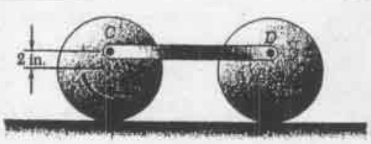
$N + 20\text{ N} - (6\text{ kg})(9.81\text{ m/s}^2)\qquad \underline{N} = 38.86\text{ N}\uparrow$

$\xleftarrow{+}\Sigma F_x = \Sigma(F_x)_{eff}:\quad F = m\bar{a}$

$F = (6\text{ kg})(1.0667\text{ m/s}^2)\qquad F = 6.4\text{ N}\leftarrow$

$(\mu_s)_{min} = \dfrac{F}{N} = \dfrac{6.4\text{ N}}{38.86\text{ N}}\qquad (\mu_s)_{min} = 0.165$ ◀

16.106

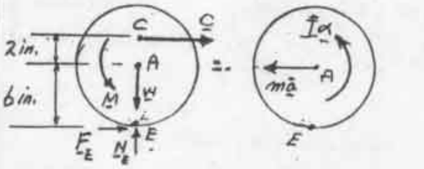


GIVEN: 4-lb DISKS
3-lb ROD
$M = 1.5\text{ lb}\cdot\text{ft} \circlearrowleft$
DISKS ROLL
FIND: (a) $\bar{a}$ OF DISKS
(b) HORIZ COMP. OF D ACTING ON DISK B

$\alpha_A = \alpha_B = \alpha \qquad \bar{a}_A = \bar{a}_B = \bar{a} = r\alpha$

DISK A:

$W = 4\text{ lb}$



$\bar{I} = \frac{1}{2} m r^2$

$+\circlearrowleft \Sigma M_E = \Sigma (M_E)_{eff}: \quad M - C\left(\frac{8}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha$

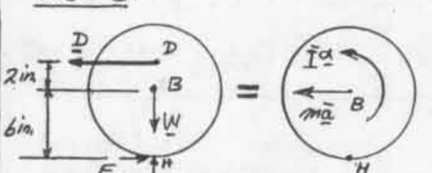
$1.5\text{ lb}\cdot\text{ft} - \frac{8}{12} C = (mr\alpha)r + \frac{1}{2} m r^2 \alpha$

$= \frac{3}{2} m r^2 \alpha$

$1.5\text{ lb}\cdot\text{ft} - \frac{8}{12} C = \frac{3}{2} \frac{4\text{ lb}}{32.2} \left(\frac{6}{12}\text{ ft}\right)^2 \alpha$

$1.5 - \frac{2}{3} C = 0.046584\alpha \qquad (1)$

DISK B:



$\bar{I} = \frac{1}{2} m r^2$

$+\circlearrowleft \Sigma M_H = \Sigma (M_H)_{eff}: \quad D\left(\frac{8}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha = mr^2\alpha + \frac{1}{2} m r^2 \alpha$

$D\left(\frac{8}{12}\right) = \frac{3}{2} \frac{4\text{ lb}}{32.2} \left(\frac{6}{12}\text{ ft}\right)^2 \alpha$

$\frac{2}{3} D = 0.046584\alpha \qquad (2)$

ROD CD:

$\xrightarrow{+} \bar{a}_R = a_C = a_A + a_{C/A}$

$\bar{a}_R = \frac{6}{12}\alpha + \frac{2}{12}\alpha = \frac{8}{12}\alpha$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}: \quad C - D = m_R \bar{a}_R$

$C - D = \frac{3\text{ lb}}{32.2}\left(\frac{8}{12}\alpha\right)$

MULTIPLY BY $\frac{2}{3}$: $\quad \frac{2}{3} C - \frac{2}{3} D = 0.041408\alpha \qquad (3)$

ADD (1), (2), (3): $\quad 1.5 - \cancel{\frac{2}{3}C} + \cancel{\frac{2}{3}D} + \cancel{\frac{2}{3}C} - \cancel{\frac{2}{3}D} = 0.134576\alpha$

$\alpha = 11.146\text{ rad/s}^2 \circlearrowleft$

(a) $\bar{a}_A = \bar{a}_B = r\alpha = \left(\frac{6}{12}\text{ ft}\right)(11.146\text{ rad/s}^2) = 5.573\text{ ft/s}^2$

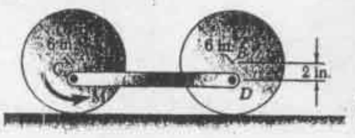
$\bar{a}_A = \bar{a}_B = 5.57\text{ ft/s}^2 \leftarrow$ ◀

(b) SUBSTITUTE FOR α IN (2)

$\frac{2}{3} D = 0.046584(11.146)$

$D = 0.779\text{ lb} \leftarrow$ ◀

16.107

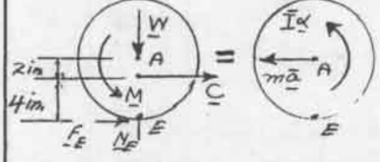


GIVEN: 4-lb DISKS
3-lb ROD
$M = 1.5\text{ lb}\cdot\text{ft}$
DISKS ROLL
FIND: (a) $\bar{a}$ OF DISKS
(b) HORIZ COMP. OF D ACTING ON DISK B

$\alpha_A = \alpha_B = \alpha \qquad \bar{a}_A = \bar{a}_B = \bar{a} = r\alpha$

DISK A:

$W = 4\text{ lb}$



$\bar{I} = \frac{1}{2} m r^2$

$+\circlearrowleft \Sigma M_E = \Sigma (M_E)_{eff}: \quad M - C\left(\frac{4}{12}\text{ in.}\right) = (m\bar{a})r + \bar{I}\alpha$

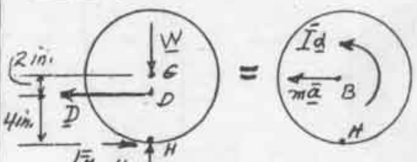
$1.5\text{ lb}\cdot\text{ft} - \frac{4}{12} C = (mr\alpha)r + \frac{1}{2} m r^2 \alpha$

$= \frac{3}{2} m r^2 \alpha$

$1.5\text{ lb}\cdot\text{ft} - \frac{4}{12} C = \frac{3}{2} \frac{4\text{ lb}}{32.2} \left(\frac{6}{12}\text{ ft}\right)^2 \alpha$

$1.5 - \frac{1}{3} C = 0.046584\alpha \qquad (1)$

DISK B:



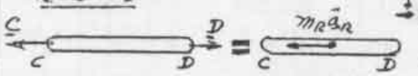
$\bar{I} = \frac{1}{2} m r^2$

$+\circlearrowleft \Sigma M_H = \Sigma (M_H)_{eff}: \quad D\left(\frac{4}{12}\text{ ft}\right) = (m\bar{a})r + \bar{I}\alpha = mr^2\alpha + \frac{1}{2} m r^2 \alpha$

$D\left(\frac{4}{12}\right) = \frac{3}{2} \frac{4\text{ lb}}{32.2} \left(\frac{6}{12}\text{ ft}\right)\alpha$

$\frac{1}{3} D = 0.046584\alpha \qquad (2)$

ROD CD



$\xrightarrow{+} \bar{a}_R = a_C = a_A + a_{C/A}$

$\bar{a}_R = \frac{6}{12}\alpha - \frac{2}{12}\alpha = \frac{4}{12}\alpha$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}: \quad C - D = m_R \bar{a}_R$

$C - D = \frac{3\text{ lb}}{32.2}\left(\frac{4}{12}\alpha\right)$

MULTIPLY BY $\frac{1}{3}$: $\quad \frac{1}{3} C - \frac{1}{3} D = 0.010352\alpha \qquad (3)$

ADD (1), (2), (3): $\quad 1.5 - \cancel{\frac{C}{3}} + \cancel{\frac{D}{3}} + \cancel{\frac{C}{3}} - \cancel{\frac{D}{3}} = 0.10352\alpha$

$\alpha = 14.490\text{ rad/s}^2 \circlearrowleft$

(a) $\bar{a}_A = \bar{a}_B = r\alpha = \left(\frac{6}{12}\text{ ft}\right)(0.10352\text{ rad/s}) = 7.245\text{ ft/s}^2$

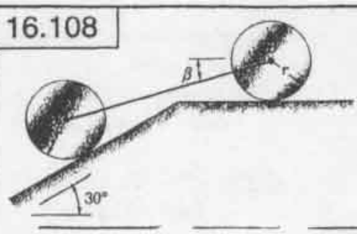
$\bar{a}_A = \bar{a}_B = 7.24\text{ ft/s}^2 \leftarrow$ ◀

(b) SUBSTITUTE FOR α IN (2)

$\frac{D}{3} = 0.046584(14.490); \quad D = 2.025\text{ lb}$

$D = 2.02\text{ lb} \leftarrow$ ◀

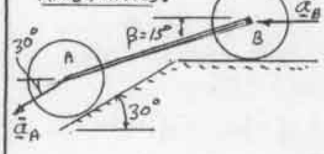
16.108



GIVEN: DISKS OF MASS m AND ROLL ON SURFACES. RELEASE FROM REST WHEN $\beta = 15°$.

FIND: (a) $\bar{a}_A$, (b) $\bar{a}_B$

KINEMATICS:

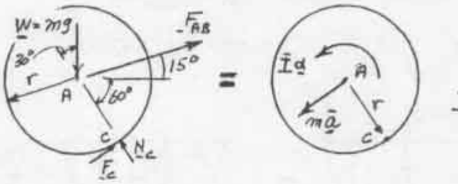


$\bar{a}_A \measuredangle 30° = \bar{a}_B \leftarrow + a_{A/B} \measuredangle 15°$

ISOSCELES TRIANGLE $\therefore \bar{a}_A = \bar{a}_B$

DENOTE BY $\bar{a} = \bar{a}_A = \bar{a}_B$

KINETICS: DISK A:



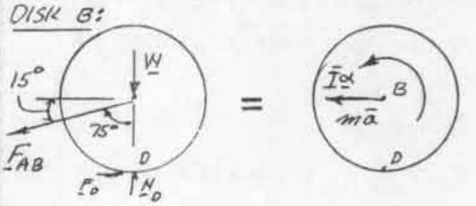
$\bar{a} = r\alpha$

$\bar{I} = \frac{1}{2}mr^2$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:

$$(mg \sin 30°)r - (F_{AB} \sin 75°)r = (m\bar{a})r + \bar{I}\alpha$$
$$= (mr\alpha)r + \tfrac{1}{2}mr^2\alpha$$
$$mgr \sin 30° - F_{AB}\, r \sin 75° = \tfrac{3}{2}mr^2 \qquad (1)$$

DISK B:



$\bar{I} = \frac{1}{2}mr^2$

$+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{eff}$: $(F_{AB} \sin 75°)r = (m\bar{a})r + \bar{I}\alpha$

$$= (mr\alpha)r + \tfrac{1}{2}mr^2\alpha$$
$$F_{AB}\, r \sin 75° = \tfrac{3}{2}mr^2\alpha \qquad (2)$$

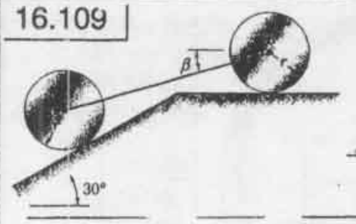
EQ(1) + EQ(2): $mgr \sin 30° = 3mr^2\alpha$

$$\alpha = \frac{g}{3r} \sin 30° = \frac{1}{6}\frac{g}{r}$$
$$\bar{a} = r\alpha = r\left(\frac{1}{6}\frac{g}{r}\right) = \frac{1}{6}g$$

RECALL $\bar{a}_A = \bar{a}_B = a$

$$\bar{a}_A = \tfrac{1}{6}g \measuredangle 30° \quad ◀$$
$$\bar{a}_B = \tfrac{1}{6}g \leftarrow \quad ◀$$

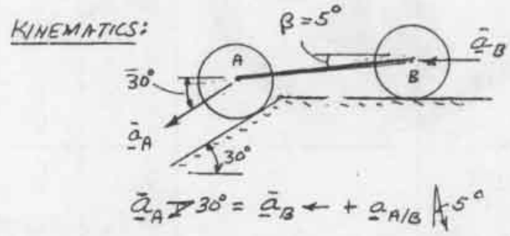
16.109



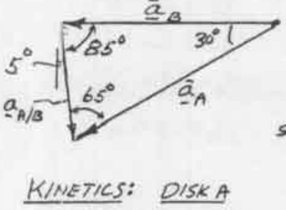
GIVEN: DISKS OF MASS m AND ROLL ON SURFACES. RELEASE FROM REST WHEN $\beta = 5°$.

FIND: (a) $\bar{a}_A$, (b) $\bar{a}_B$.

KINEMATICS:



$\bar{a}_A \measuredangle 30° = \bar{a}_B \leftarrow + a_{A/B} \measuredangle 5°$



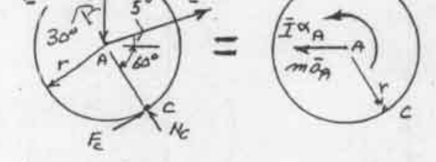
LAW OF SINES

$$\frac{\bar{a}_B}{\sin 65°} = \frac{\bar{a}_A}{\sin 85°}$$
$$\bar{a}_B = 0.90852\, a_A$$

SINCE $\bar{a}_B = r\alpha_B$ AND $\bar{a}_A = r\alpha_A$

$$\alpha_B = 0.90852\,\alpha_A \qquad (1)$$

KINETICS: DISK A



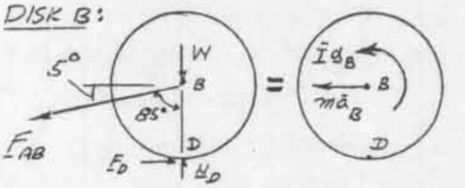
$\bar{a}_A = r\alpha_A$

$\bar{I} = \frac{1}{2}mr^2$

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:

$$(mg \sin 30°)r - (F_{AB} \sin 65°)r = (ma)r + \bar{I}\alpha_A$$
$$= (mr\alpha_A)r + \tfrac{1}{2}mr^2\alpha_A$$
$$mg \sin 30° - F_{AB} \sin 65° = \tfrac{3}{2}mr\alpha_A \qquad (2)$$

DISK B:



$\bar{I} = \frac{1}{2}mr^2$

$\bar{a}_B = r\alpha_B$

$+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{eff}$: $(F_{AB} \sin 85°)r = (m\bar{a}_B)r + \bar{I}\alpha_B$

$$= (mr\alpha_B)r + \tfrac{1}{2}mr^2\alpha_B$$
$$F_{AB} = \frac{3}{2}\frac{mr}{\sin 85°}\alpha_B \qquad (3)$$

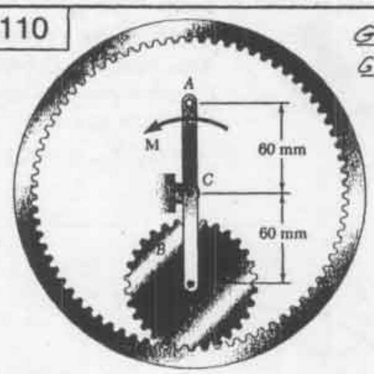
SUBSTITUTE FOR $\bar{a}_B$ FROM EQ(1) AND F_{AB} FROM EQ(3) INTO EQ(2)

$$mg \sin 30° - \tfrac{3}{2}mr \frac{\sin 65°}{\sin 85°}(0.90852\alpha_A) = \tfrac{3}{2}mr\alpha_A$$
$$0.5\frac{g}{r} = \tfrac{3}{2}(0.82654 + 1)\alpha_A$$
$$\alpha_A = 0.1825\frac{g}{r} \;:\; \bar{a}_A = r\alpha_A = 0.1825g$$
$$\bar{a}_A = 0.1825g \measuredangle 30° \quad ◀$$

EQ(1) $\bar{a}_B = 0.90852\,\bar{a}_A = (0.90852)(0.1826g) = 0.1659g$

$$\bar{a}_B = 0.1659g \leftarrow \quad ◀$$

16.110



GIVEN: $M = 1.25\ \text{N·m}$

GEAR, $m = 1.8\ \text{kg}$

$\bar{k} = 32\ \text{mm}$

BAR AD: $m_{AD} = 2.5\ \text{kg}$

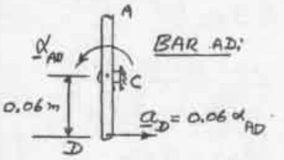
FIND:

(a) $\underline{\alpha}_{AD}$

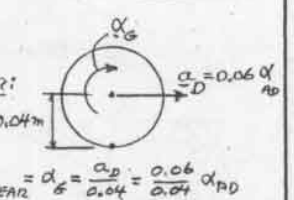
(b) $\underline{a}_D$

KINEMATICS:

BAR AD:



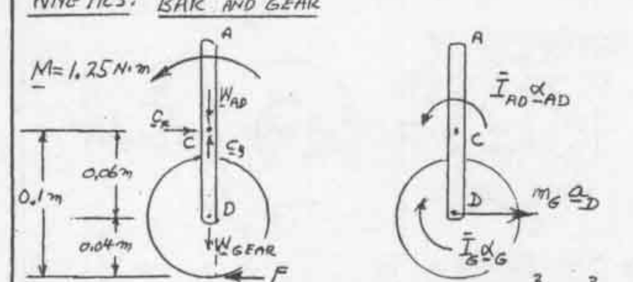
GEAR:



$$\alpha_{GEAR} = \alpha_G = \frac{a_D}{0.04} = \frac{0.06}{0.04}\alpha_{AD}$$

$$\underline{\alpha}_G = 1.5\,\alpha_{AD} \circlearrowright$$

KINETICS: BAR AND GEAR



$$\bar{I}_{AD} = \frac{1}{12}(2.5\ \text{kg})(0.12\ \text{m})^2 = 3\times10^{-3}\ \text{kg·m}^2$$

$$\bar{I}_G = m\bar{k}^2 = (1.8\ \text{kg})(0.032\ \text{m})^2 = 1.843\times10^{-3}\ \text{kg·m}^2$$

$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$

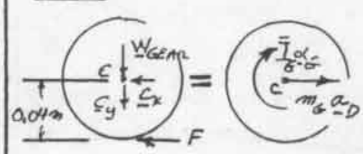
$$M - F(0.1\ \text{m}) = \bar{I}_{AD}\alpha_{AD} + (m_G a_D)(0.06\ \text{m}) - \bar{I}_G\alpha_G$$

$$M - 0.1F = (3\times10^{-3}\ \text{kg·m}^2)\alpha_{AD} + [(1.8\ \text{kg})(0.06\,\alpha_{AD})](0.06\ \text{m}) - [(1.843\times10^{-3}\ \text{kg·m}^2)(1.5\alpha_{AD})]$$

$$M - 0.1F = 3\times10^{-3}\alpha_{AD} + 6.48\times10^{-3}\alpha_{AD} - 2.765\times10^{-3}\alpha_{AD}]$$

$$1.25\ \text{N·m} - 0.1F = 6.715\times10^{-3}\alpha_{AD} \qquad (1)$$

GEAR



$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}$

$$F(0.04\ \text{m}) = \bar{I}_G\alpha_G$$

$$0.04F = (1.843\times10^{-3})\alpha_G$$

$$0.04F = (1.843\times10^{-3})\,1.5\alpha_{AD}$$

$$F = 69.12\times10^{-3}\alpha_{AD}$$

SUBSTITUTE FOR F IN EQ(1)

$$1.25 - (0.1)(69.12\times10^{-3})\alpha_{AD} = 6.715\times10^{-3}\alpha_{AD}$$

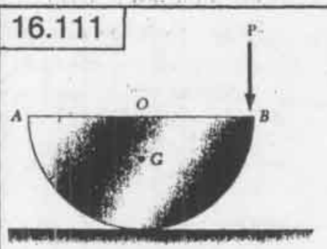
$$1.25 = 13.627\times10^{-3}\alpha_{AD}$$

$$\alpha_{AD} = 91.73\ \text{rad/s}^2 \qquad \underline{\alpha}_{AD} = 91.7\ \text{rad/s}^2 \circlearrowleft \blacktriangleleft$$

$$a_D = (CD)\alpha_{AD} = (0.06\ \text{m})(91.73\ \text{rad/s}^2) = 5.50\ \text{m/s}^2$$

$$\underline{a}_D = 5.50\ \text{m/s}^2 \rightarrow \blacktriangleleft$$

16.111



GIVEN: HALF CYLINDER

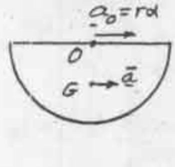
MASS $= m$

ROLLING WITH NO SLIPPING

FIND: (a) $\underline{\alpha}$

(b) $(\mu_s)_{min}$

KINEMATICS: ASSUME $\alpha \circlearrowright$, $\underline{a}_O = r\alpha \rightarrow$



FROM INSIDE COVER OF TEXT

$$OG = 4r/3\pi$$

$$\underline{\bar{a}} = \underline{a}_O + \underline{a}_{G/O} = [a_O \rightarrow] + [(OG)\alpha \leftarrow]$$

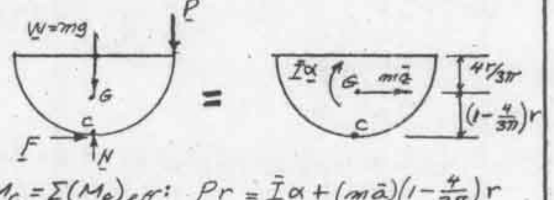
$$\underline{\bar{a}} = [r\alpha \rightarrow] + \left[\frac{4r}{3\pi}\alpha \leftarrow\right] = r\left(1 - \frac{4}{3\pi}\right)\alpha \rightarrow$$

KINETICS: m = MASS OF HALF CYLINDER, $\therefore I_O = \frac{1}{2}mr^2$

$$I_O = \bar{I} + m(OG)^2;\quad \frac{1}{2}mr^2 = \bar{I} + \left(\frac{4r}{3\pi}\right)^2 m$$

$$\bar{I} = mr^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)$$

(a)



$+\circlearrowright \Sigma M_C = \Sigma (M_C)_{eff}$: $\quad Pr = \bar{I}\alpha + (m\bar{a})\left(1 - \frac{4}{3\pi}\right)r$

$$Pr = mr^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)\alpha + mr\left(1 - \frac{4}{3\pi}\right)\alpha\left(1 - \frac{4}{3\pi}\right)r$$

$$Pr = mr^2\left(\frac{1}{2} - \cancel{\frac{16}{9\pi^2}}\right)\alpha + mr^2\left(1 - \frac{8}{3\pi} + \cancel{\frac{16}{9\pi^2}}\right)\alpha$$

$$Pr = mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\alpha$$

$$Pr = mr^2(0.6511)\alpha$$

$$\alpha = 1.5357\frac{P}{mr} \qquad \underline{\alpha} = 1.536\frac{P}{mr} \circlearrowright \blacktriangleleft$$

(b) $\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad F = m\bar{a}$

$$F = mr\left(1 - \frac{4}{3\pi}\right)\alpha = mr(0.57559)\alpha$$

$$F = mr(0.57559)\left(1.5357\frac{P}{mr}\right) = 0.8839\,P$$

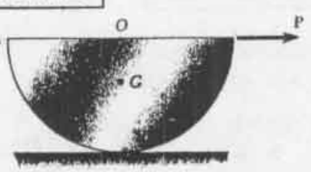
$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $\quad N - P - mg = 0$

$$N = mg + P$$

$$(\mu_s)_{min} = \frac{F}{N} = \frac{0.8839P}{mg + P}$$

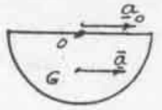
$$(\mu_s)_{min} = 0.884\frac{P}{mg + P} \blacktriangleleft$$

16.112



GIVEN: HALF CYLINDER
MASS = m
ROLLING WITH NO SLIPPING
FIND: (a) $\underline{\alpha}$
(b) $(\mu_s)_{min}$

KINEMATICS: ASSUME $\alpha \circlearrowright$, $\underline{a}_o = r\alpha \rightarrow$



FROM INSIDE COVER OF TEXT

$OG = 4r/3\pi$

$\bar{\underline{a}} = \underline{a}_o + \underline{a}_{G/O} = [a_o \rightarrow] + [(OG)\alpha \leftarrow]$

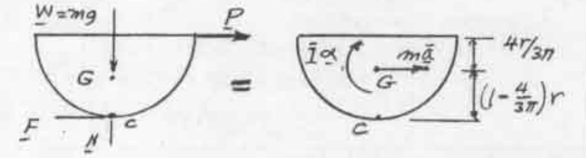
$\bar{\underline{a}} = [r\alpha \rightarrow] + \left[\frac{4r}{3\pi}\alpha \leftarrow\right] = r\left(1 - \frac{4}{3\pi}\right)\alpha \rightarrow$

KINETICS: m = MASS OF HALF CYLINDER $\therefore \bar{I}_o = \frac{1}{2}mr^2$

$\bar{I}_o = \bar{I} + m(OG)^2; \quad \frac{1}{2}mr^2 = \bar{I} + \left(\frac{4r}{3\pi}\right)^2 m$

$\bar{I} = mr^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)$

(a)



$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{eff}: \; Pr = \bar{I}\alpha + (m\bar{a})\left(1 - \frac{4}{3\pi}\right)r$

$Pr = mr^2\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)\alpha + mr\left(1 - \frac{4}{3\pi}\right)\alpha\left(1 - \frac{4}{3\pi}\right)r$

$Pr = mr^2\left(\frac{1}{2} - \cancel{\frac{16}{9\pi^2}}\right)\alpha + mr^2\left(1 - \frac{8}{3\pi} + \cancel{\frac{16}{9\pi^2}}\right)\alpha$

$Pr = mr^2\left(\frac{3}{2} - \frac{8}{3\pi}\right)\alpha$

$Pr = mr^2(0.65117)\alpha$

$\alpha = 1.5357\frac{P}{mr} \qquad \underline{\alpha} = 1.536\frac{P}{mr} \circlearrowright$ ◀

(b) $\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$

$P - F = m\bar{a} = mr\left(1 - \frac{4}{3\pi}\right)\alpha$

$P - F = mr\left(1 - \frac{4}{3\pi}\right)\left(1.5357\frac{P}{mr}\right) = 0.8839P$

$F = P - 0.8839P = 0.1161P$

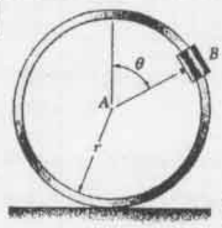
$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}: \quad N - W = 0$

$N = W$

$(\mu_s)_{min} = \frac{F}{N} = \frac{0.1161P}{W}$

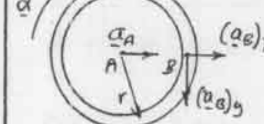
$(\mu_s)_{min} = 0.116\frac{P}{W}$ ◀

16.113



GIVEN:
m_B = MASS OF CLAMP B
m_h = MASS OF HOOP
$m_h = 3m_B$
$\theta = 90°$
SYSTEM IS RELEASED AND ROLLS WITHOUT SLIDING
FIND: (a) $\underline{\alpha}$
(b) $(\underline{a}_B)_x$ AND $(\underline{a}_B)_y$

KINEMATICS:



$\underline{a}_A = r\alpha \rightarrow, \quad a_{B/A} = r\alpha \downarrow$

$\underline{a}_B = \underline{a}_A \rightarrow + a_{B/A} \downarrow$

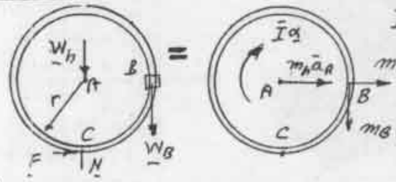
$\underline{a}_B = r\alpha \rightarrow + r\alpha \downarrow$

$(\underline{a}_B)_x = r\alpha \rightarrow \quad (a_B)_y = r\alpha \downarrow$

KINETICS:

$m_h = 3m_B$

$\bar{I} = m_h r^2 = 3m_B r^2$



(a)

$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{eff}:$

$W_B r = \bar{I}\alpha + m_h\bar{a}_A r + m_B(a_B)_x r + m_B(a_B)_y r$

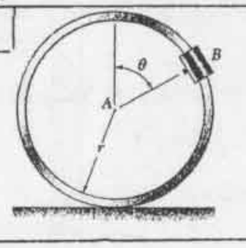
$m_B g r = 3m_B r^2\alpha + (3m_B)r^2\alpha + m_B r^2\alpha + m_B r^2\alpha$

$gr = 8r^2\alpha$

$\underline{\alpha} = \frac{1}{8}\frac{g}{r} \circlearrowright$ ◀

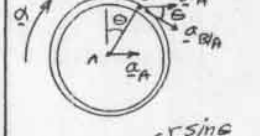
(b) $(\underline{a}_B)_x = r\alpha = \frac{1}{8}g \rightarrow, \; (a_B)_y = r\alpha = \frac{1}{8}g \downarrow$ ◀

16.114



GIVEN: m_h = MASS OF HOOP
m_B = MASS OF CLAMP
SYSTEM IS RELEASED AND ROLLS WITHOUT SLIDING.
FIND: α IN TERMS OF m_B, m_h, r, AND θ.

KINEMATICS:



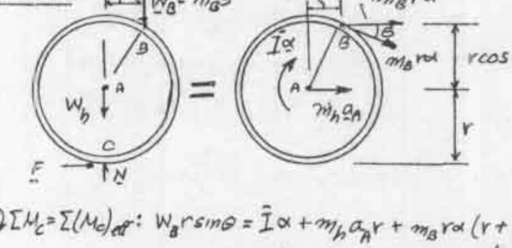
$\underline{a}_A = r\alpha \rightarrow \qquad a_{B/A} = r\alpha \measuredangle \theta$

$\underline{a}_B = a_A \rightarrow + a_{B/A} \measuredangle \theta$

$\underline{a}_B = r\alpha \rightarrow + r\alpha \measuredangle \theta$

$\bar{I} = m_h r^2$

KINETICS:



$+\circlearrowright \Sigma M_C = \Sigma(M_C)_{eff}: \; W_B r\sin\theta = \bar{I}\alpha + m_h a_A r + m_B r\alpha(r + r\cos\theta)$
$+ m_B r\alpha\sin\theta(r\sin\theta) + m_B r\alpha\cos\theta(r + r\cos\theta)$

(CONTINUED)

16.114 continued

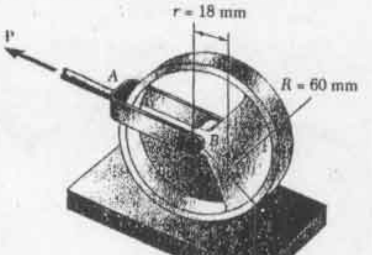
$$m_B g r\sin\theta = m_h r^2\alpha + m_h(r\alpha)r + m_B r\alpha(1+\cos\theta)(r+r\cos\theta) + m_B r\alpha\sin\theta(r\sin\theta)$$

$$m_B g r\sin\theta = 2m_h r^2\alpha + m_B r^2\alpha\left[(1+\cos\theta)^2+\sin^2\theta\right]$$
$$= 2m_h r^2\alpha + m_B r^2\alpha\left[1+2\cos\theta+\cos^2\theta+\sin^2\theta\right]$$

$$m_B g r\sin\theta = r^2\alpha\left[2m_h + m_B(2+2\cos\theta)\right]$$

$$\alpha = \frac{g}{2r}\,\frac{m_B\sin\theta}{m_h+m_B(1+\cos\theta)}$$ ◀

16.115 and 16.116

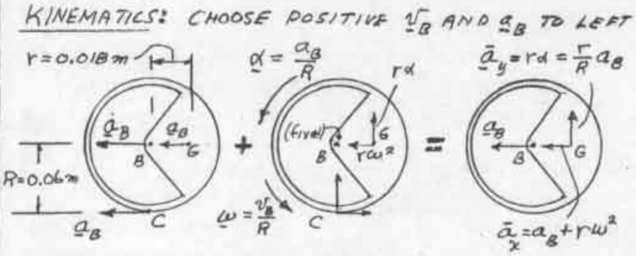


GIVEN: $m = 1.5$ kg, $\bar{k} = 44$ mm

PROBLEM 16.115: FIND: P WHEN $v_B = 0.35$ m/s ←, $a_B = 1.2$ m/s² ←

PROBLEM 16.116: FIND: P WHEN $v_B = 0.35$ m/s →, $a_B = 1.2$ m/s² →

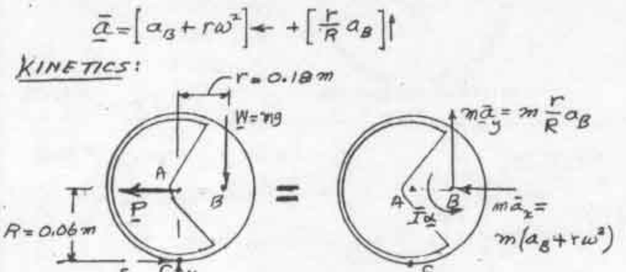
KINEMATICS: CHOOSE POSITIVE v_B AND a_B TO LEFT



TRANS. WITH B + ROTATION ABT B = ROLLING MOTION

$$\bar{a} = \left[a_B + r\omega^2\right]\leftarrow + \left[\frac{r}{R}a_B\right]\uparrow$$

KINETICS:



$+\circlearrowleft\ \Sigma M_C = \Sigma(M_C)_{eff}$:

$$PR - Wr = (m\bar{a}_y)r + (m\bar{a}_x)R + \bar{I}\alpha$$

$$PR - mgr = m\left(\frac{r}{R}a_B\right)r + m(a_B + r\omega^2)R + m\bar{k}^2\frac{a_B}{R}$$

$$= ma_B\left(\frac{r^2}{R} + R + \frac{\bar{k}^2}{R}\right) + mr\left(\frac{v_B}{r}\right)^2 R$$

$$P = mg\left(\frac{r}{R}\right) + ma_B\left(1 + \frac{r^2+\bar{k}^2}{R^2}\right) + m\frac{r}{R^2}v_B^2 \qquad (1)$$

(CONTINUED)

16.115 and 16.116 continued

SUBSTITUTE: $m = 1.5$ kg, $r = 0.018$ m, $R = 0.06$ m, $\bar{k} = 0.044$ m AND $g = 9.81$ m/s² IN EQ(1)

$$P = 1.5(9.81)\frac{0.018}{0.06} + 1.5(a_B)\left(1 + \frac{0.018^2+0.044^2}{0.06^2}\right) + 1.5\frac{0.018}{0.06^2}v_B^2$$

$$P = 4.4145 + 2.4417\,a_B + 7.5\,v_B^2 \qquad (2)$$

PROBLEM 16.115: $v_B = 0.35$ m/s ← ; $v_B = +0.35$ m/s
$a_B = 1.2$ m/s² ← ; $a_B = +1.2$ m/s²

SUBSTITUTE IN EQ(2):

$$P = 4.4145 + 2.4417(+1.2) + 7.5(+0.35)^2$$
$$= 4.4145 + 2.9300 + 0.9188 = +8.263\text{ N}$$

$$P = 8.26\text{ N} \leftarrow$$ ◀

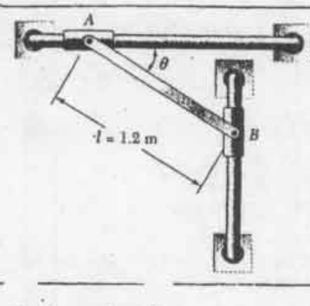
PROBLEM 16.116: RECALL WE ASSUMED POSITIVE TO LEFT
$v_B = 0.35$ m/s → ; $v_B = -0.35$ m/s
$a_B = 1.2$ m/s → ; $a_B = -1.2$ m/s²

SUBSTITUTE INTO EQ(2):

$$P = 4.4145 + 2.4417(-1.2) + 7.5(-0.35)^2$$
$$= 4.4145 - 2.9300 + 0.9188 = +2.403\text{ N}$$

$$P = 2.40\text{ N} \leftarrow$$ ◀

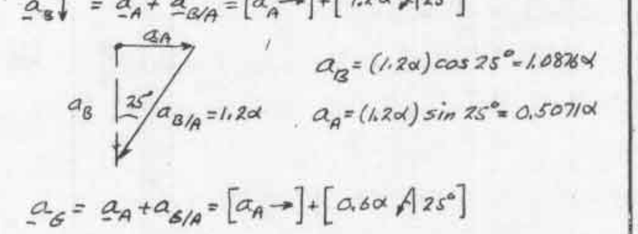
16.117



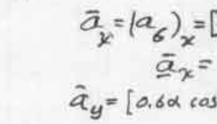
GIVEN: $m = 10$ kg, $\theta = 25°$, RELEASE FROM REST

FIND: (a) A (b) B

KINEMATICS: ASSUME $\alpha \circlearrowright$ $\qquad \omega = 0$



$$a_B\downarrow = a_A + a_{B/A} = \left[a_A \rightarrow\right] + \left[1.2\alpha \measuredangle 25°\right]$$



$a_B = (1.2\alpha)\cos 25° = 1.0876\alpha$

$a_A = (1.2\alpha)\sin 25° = 0.5071\alpha$

$$a_G = a_A + a_{G/A} = \left[a_A \rightarrow\right] + \left[0.6\alpha \measuredangle 25°\right]$$

$$a_G = \left[0.5071\alpha \rightarrow\right] + \left[0.6\alpha \measuredangle 25°\right]$$

$$\bar{a}_x = (a_G)_x = \left[0.5071\alpha \rightarrow\right] + \left[0.2536\alpha \leftarrow\right]$$

$$\bar{a}_x = 0.2535\alpha \rightarrow$$

$$\bar{a}_y = \left[0.6\alpha\cos 25° \leftarrow\right] = 0.5438\alpha\downarrow$$

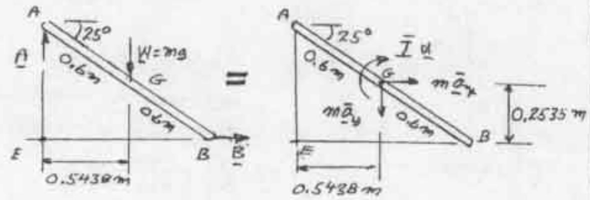
(CONTINUED)

16.117 continued

WE HAVE FOUND FOR $\alpha\circlearrowright$.

$\bar{a}_x = 0.2535\alpha \rightarrow;\ \bar{a}_y = 0.5438\alpha\downarrow$

KINETICS: $\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}m(1.2\,m)^2$



$+\circlearrowright \Sigma M_E = \Sigma(M_E)_{eff}$:

$mg(0.5438\,m) = \bar{I}\alpha + m\bar{a}_x(0.2535\,m) + m\bar{a}_y(0.5438\,m)$

$mg(0.5438) = \frac{1}{12}m(1.2)^2 + m(0.2535)^2\alpha + m(0.5438)^2\alpha$

$g(0.5438) = 0.48\alpha \qquad \underline{\alpha} = 1.133g = 11.11\ \text{rad/s}^2 \circlearrowright$ ◀

(a) $+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $A - mg = -m\bar{a}_y = -m(0.5438\alpha)$

$A - 10(9.81) = -(10)(0.5438)(11.115)$

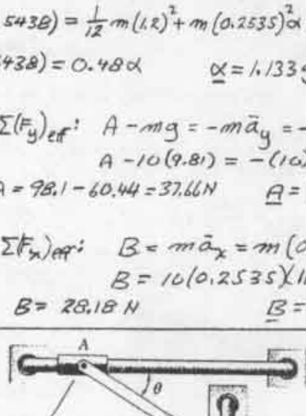
$A = 98.1 - 60.44 = 37.66\,N \qquad \underline{A} = 37.7\ N\uparrow$ ◀

(b) $\overset{+}{\rightarrow} \Sigma F_x = \Sigma(F_x)_{eff}$: $B = m\bar{a}_x = m(0.2535\alpha)$

$B = 10(0.2535)(11.115)$

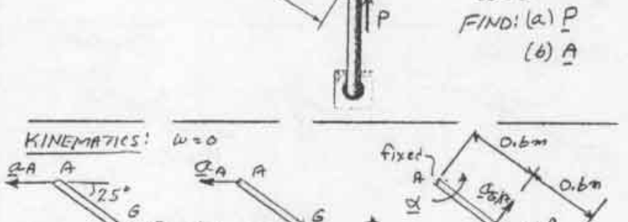
$B = 28.18\ N \qquad \underline{B} = 28.2\ N \rightarrow$ ◀

16.118

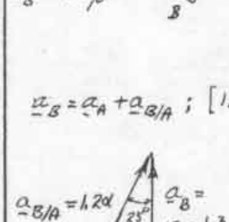


GIVEN:
$m = 10\ kg$
$\theta = 25°$
$\underline{a}_B = 12\ m/s^2\uparrow$
$\omega = 0$
FIND: (a) $\underline{P}$
(b) $\underline{A}$

KINEMATICS: $\omega = 0$



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$; $[12\ m/s^2\uparrow] = [a_A \leftarrow] + [1.2\alpha \angle 25°]$



$a_B = a_{B/A}\cos 25°$

$12\ m/s^2 = (1.2\alpha)\cos 25°$

$\alpha = 11.034\ \text{rad/s}^2 \circlearrowright$

$\underline{a}_A = 12\tan 25° = 5.596\ m/s^2 \leftarrow$

$\underline{a}_G = \underline{a}_A + \underline{a}_{G/A}$: $\quad \underline{a}_G = [5.596 \leftarrow] + [0.6\alpha \angle 25°]$

$\underline{a}_G = [5.596 \leftarrow] + [0.6(11.034) \angle 25°]$

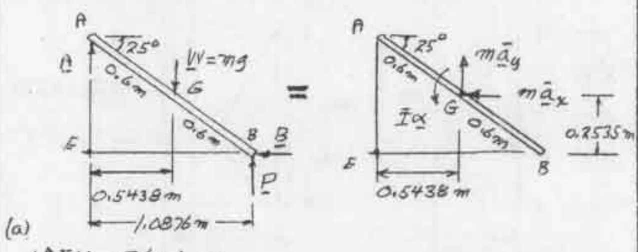
$\bar{a}_x = (a_G)_x = 2.798\ m/s^2 \leftarrow$

$\bar{a}_y = (a_G)_y = 6.00\ m/s^2\uparrow$

(CONTINUED)

16.118 continued

KINETICS: $\bar{I} = \frac{1}{12}mL^2 = \frac{1}{12}m(1.2)^2$



(a)

$+\circlearrowright \Sigma M_E = \Sigma(M_E)_{eff}$:

$P(1.0876) - W(0.5438) = \bar{I}\alpha + m\bar{a}_x(0.2535) + m\bar{a}_y(0.5438)$

$W = mg = 10(9.81) = 98.1\,N$

$\bar{I}\alpha = \frac{1}{12}mL^2\alpha = \frac{1}{12}(10)(1.2)^2(11.034) = 13.24\ N\cdot m$

$m\bar{a}_x = (10)(2.798) = 27.98\ N$

$m\bar{a}_y = (10)(6) = 60\ N$

$P(1.0876) - (98.1)(0.5438) = 13.24 + (27.98)(0.2535) + (60)(0.5438)$

$P(1.0876) - 53.347 = 13.24 + 7.096 + 32.628$

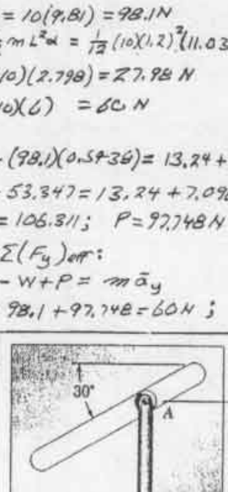
$P(1.0876) = 106.311; \quad P = 97.748\,N \qquad \underline{P} = 97.7\,N\uparrow$ ◀

(b) $+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$:

$A - W + P = m\bar{a}_y$

$A - 98.1 + 97.748 = 60\,N; \qquad A = 60.4\,N\uparrow$ ◀

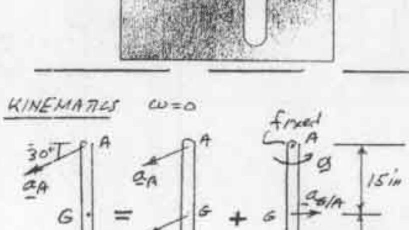
16.119



GIVEN:
$W = 8\ lb$
ROD RELEASED FROM REST

FIND: (a) $\underline{\alpha}$
(b) $\underline{B}$

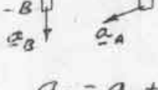
KINEMATICS $\omega = 0$



$\underline{a}_{G/A} = \left(\frac{15}{12}ft\right)\alpha \rightarrow = 1.25\alpha \rightarrow$

$\underline{a}_{B/A} = \left(\frac{30}{12}ft\right)\alpha \rightarrow = 2.5\alpha \rightarrow$

$a_B = a_A + a_{B/A}$: $[a_B\downarrow] = [a_A \measuredangle 30°] + [2.5\alpha \rightarrow]$



$a_A = \frac{2.5\alpha}{\cos 30°} = 2.887\alpha \measuredangle 30°$

$a_B = (2.5\alpha)\tan 30° = 1.443\alpha\downarrow$

$\bar{a} = a_G = a_A + a_{G/A}$; $\bar{a} = [2.887\alpha \measuredangle 30°] + [1.25\alpha \rightarrow]$

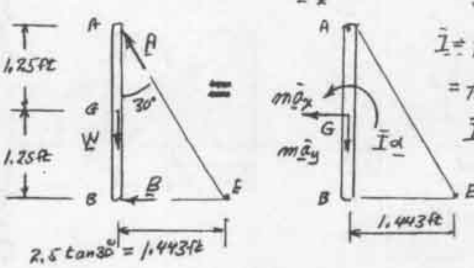
$\bar{a}_x = [2.5\alpha \leftarrow] + [1.25\alpha \rightarrow] = 1.25\alpha \leftarrow$

$\bar{a}_y = [1.443\alpha\downarrow] \qquad = 1.443\alpha\downarrow$

(CONTINUED)

16.119 continued

WE HAVE:
$\underline{\bar{a}}_x = 1.25\alpha \leftarrow$; $\underline{\bar{a}}_y = 1.443\alpha \downarrow$



$\bar{I} = \frac{1}{12}\cdot\frac{W}{g}L^2$
$= \frac{1}{12}\cdot\frac{8\,lb}{32.2}(2.5\,ft)$
$\bar{I} = 0.1294\ lb\cdot ft\cdot s^2$

$+\circlearrowright \Sigma M_E = \Sigma(M_E)_{eff}$: $W(1.443\,ft) = \bar{I}\alpha + m\bar{a}_x(1.25\,ft) + m\bar{a}_y(1.443\,ft)$

$8(1.443) = 0.1294\,\alpha + \frac{8}{32.2}(1.25)^2\alpha + \frac{8}{32.2}(1.443)^2\alpha$

$11.544 = 1.035\,\alpha$

$\alpha = 11.154\ rad/s^2$ $\qquad \underline{\alpha} = 11.15\ rad/s^2 \circlearrowright$ ◀

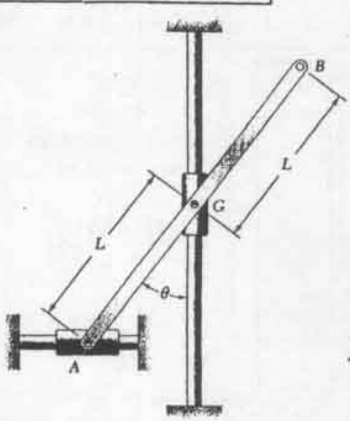
$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $B(2.5\,ft) = -\bar{I}\alpha + m\bar{a}_x(1.25\,ft)$

$2.5B = -(0.1294)(11.154) + \frac{8}{32.2}(1.25)(11.154)(1.25)$

$2.5B = -1.443 + 4.330$

$B = 1.155\ lb$ $\qquad \underline{B} = 1.155\ lb \leftarrow$ ◀

16.120 and 16.121



GIVEN:
W = WEIGHT OF AB

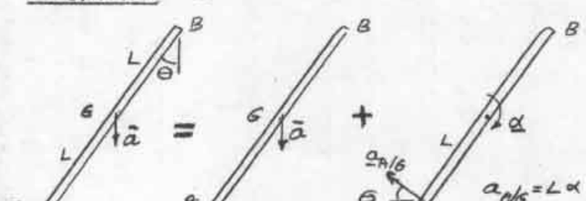
ROD IS RELEASED FROM REST.

FIND: (a) $\underline{\alpha}$
(b) $\underline{A}$

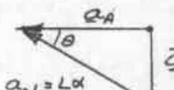
PROBLEM 16.120
SOLVE IN TERMS OF W, L, AND θ

PROBLEM 16.121
SOLVE FOR W = 14 lb
L = 15 in. AND θ = 30°

KINEMATICS: $\omega = 0$



$\underline{a}_A = \underline{a}_G + \underline{a}_{A/G}$: $[a_A \leftarrow] = [\bar{a}\downarrow] + [L\alpha \measuredangle \theta]$

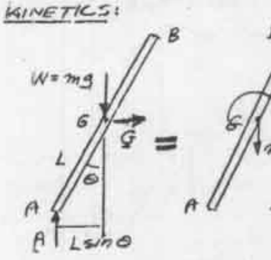


$\bar{a} = L\alpha\ \sin\theta \downarrow$

(CONTINUED)

16.120 and 16.121 continued

KINETICS:



$\bar{I} = \frac{1}{12}m(2L)^2 = \frac{1}{3}mL^2$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$

$G = 0$

$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$

$mg(L\sin\theta) = \bar{I}\alpha + m\bar{a}(L\sin\theta)$

$mgL\sin\theta = \frac{1}{3}mL^2\alpha + m(L\alpha\sin\theta)(L\sin\theta)$

$\underline{\alpha} = \frac{g}{L}\left[\frac{\sin\theta}{\frac{1}{3}+\sin^2\theta}\right] \circlearrowright$ ◀

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $A - mg = -m\bar{a} = -mL\alpha\sin\theta$

$A = mg - mL\frac{g}{L}\left[\frac{\sin\theta}{\frac{1}{3}+\sin^2\theta}\right]\sin\theta$

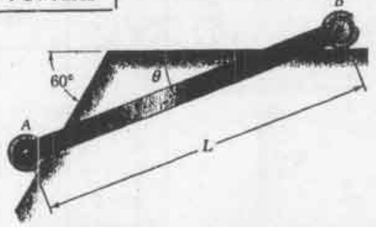
$A = mg\,\frac{\frac{1}{3}+\cancel{\sin^2\theta}-\cancel{\sin^2\theta}}{\frac{1}{3}+\sin^2\theta}$ $\qquad A = \frac{mg}{1+3\sin^2\theta}$ ◀

PROBLEM 16.121: $W = mg = 14\ lb$, $L = 15\ in. = 1.25\ ft$, $\theta = 30°$

$\alpha = \frac{32.2}{1.25}\left[\frac{\sin 30°}{\frac{1}{3}+\sin^2 30°}\right]$ $\qquad \underline{\alpha} = 22.1\ rad/s^2 \circlearrowright$ ◀

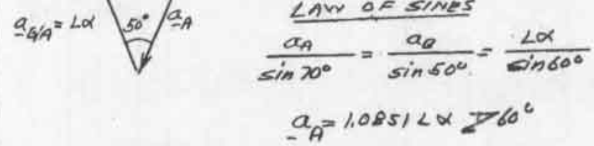
$A = \frac{14\ lb}{1+3\sin^2 30°} = \frac{14}{1+3/4}$; $\qquad \underline{A} = 8\ lb\uparrow$ ◀

16.122



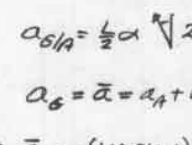
GIVEN:
$m = 5\ kg$
$L = 750\ mm$
$\theta = 20°$
RELEASE FROM REST
FIND:
(a) α
(b) $\bar{A}$

KINEMATICS



$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$

$[a_B \leftarrow] = [a_A \measuredangle 60°] + [L\alpha \measuredangle 20°]$



LAW OF SINES

$\frac{a_A}{\sin 70°} = \frac{a_B}{\sin 50°} = \frac{L\alpha}{\sin 60°}$

$\underline{a}_A = 1.0851\,L\alpha \measuredangle 60°$

$\underline{a}_B = 0.88455\,L\alpha \leftarrow$

$a_{G/A} = \frac{L}{2}\alpha \measuredangle 20°$

$a_G = \bar{a} = a_A + a_{G/A} = [1.0851L\alpha \measuredangle 60°] + [\frac{L}{2}\alpha \measuredangle 20°]$

$\xleftarrow{+} \bar{a}_x = (1.0851L\alpha)\cos 60° + (0.5L\alpha)\sin 20°$
$= 0.54254L\alpha + 0.17101L\alpha$; $\quad \underline{\bar{a}}_x = 0.71355L\alpha \leftarrow$

$+\downarrow \bar{a}_y = (1.0851L\alpha)\sin 60° - (0.5L\alpha)\cos 20°$
$= 0.93972L\alpha - 0.46985L\alpha$; $\quad \underline{\bar{a}}_y = 0.46985L\alpha \downarrow$

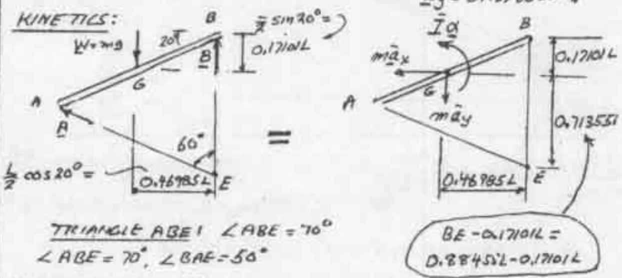
(CONTINUED)

16.122 continued

WE HAVE: $\bar{a}_x = 0.71355L\alpha \leftarrow$

$\bar{a}_y = 0.46985L\alpha \downarrow$

KINETICS:



TRIANGLE ABE: $\angle ABE = 70°$

$\angle ABE = 70°$, $\angle BAE = 50°$

LAW OF SINES AB = L

$$\frac{BE}{\sin 50°} = \frac{L}{\sin 60°}; \quad BE = 0.88455L$$

$+\circlearrowright \Sigma M_E = \Sigma(M_E)_{eff}$

$$mg(0.46985L) = \bar{I}\alpha + m\bar{a}_x(0.71355L) + m\bar{a}_y(0.46985L)$$

$$0.46985\,mgL = \tfrac{1}{12}mL^2\alpha + m(0.71355L\alpha)(0.71355L) + m(0.46985L\alpha)(0.46985L)$$

$$0.46985\,mgL = mL^2(0.81325)\alpha$$

$$\alpha = 0.57775\frac{g}{L} = 0.57775\frac{9.81\ m/s^2}{0.75\ m} = 7.557\ rad/s^2$$

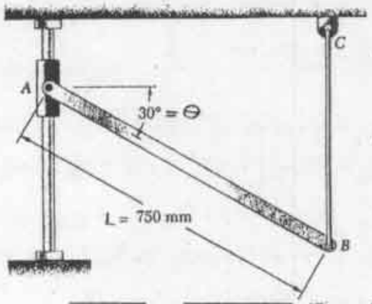
$$\alpha = 7.56\ rad/s^2$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$: $A\sin 60° = ma_x = m(0.71355L\alpha)$

$$A\sin 60° = (5\ kg)(0.71355)(0.75\ m)(7.557\ rad/s^2)$$

$$A = 23.3\ N \measuredangle 30°$$

16.123



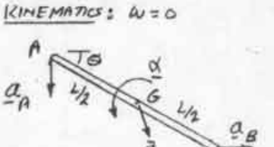
GIVEN: $m = 8\ kg$

RELEASE FROM REST.

FIND: (a) α (b) A

KINEMATICS: $\omega = 0$

$a_B = a_A + a_{B/A}$; $[a_B \rightarrow] = [a_A \downarrow] + [L\alpha \measuredangle \theta]$



$+\downarrow\ 0 = a_A - L\alpha\cos\theta$

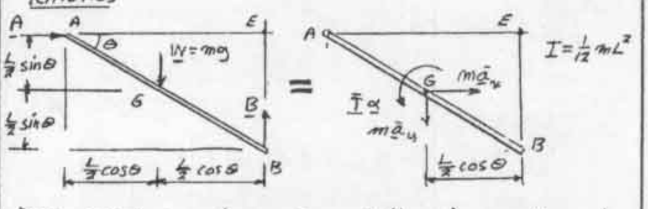
$a_A = L\alpha\cos\theta \downarrow$

$\bar{a} = a_A + a_{G/A} = a_A + \frac{L}{2}\alpha$

$\bar{a} = [L\alpha\cos\theta \downarrow] + [\frac{L}{2}\alpha \measuredangle \theta]$

$\bar{a}_x = \frac{L}{2}\alpha\sin\theta \rightarrow$; $\bar{a}_y = \frac{L}{2}\alpha\cos\theta \uparrow$

KINETICS



$+\circlearrowright \Sigma M_E = \Sigma(M_E)_{eff}$: $mg\frac{L}{2}\cos\theta = I\alpha + m\bar{a}_x(\frac{L}{2}\sin\theta) + m\bar{a}_y(\frac{L}{2}\cos\theta)$

$$mg\frac{L}{2}\cos\theta = \tfrac{1}{12}mL^2\alpha + m(\tfrac{L}{2}\sin\theta)^2\alpha + m(\tfrac{L}{2}\cos\theta)^2\alpha$$

$$mg\frac{L}{2}\cos\theta = \tfrac{1}{3}mL^2\alpha \qquad \alpha = \frac{3}{2}\frac{g}{L}\cos\theta \circlearrowright$$

(CONTINUED)

16.123 continued

$$\alpha = \frac{3}{2}\frac{g}{L}\cos\theta \circlearrowright$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$: $A = m\bar{a}_x = m\frac{L}{2}\alpha\sin\theta$

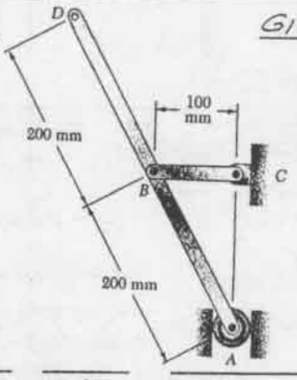
$$A = m\frac{L}{2}\left(\frac{3}{2}\frac{g}{L}\cos\theta\right)\sin\theta; \quad A = \frac{3}{4}mg\sin\theta\cos\theta \rightarrow$$

DATA: $m = 8\ kg$, $\theta = 30°$, $L = 0.75\ m$

$$\alpha = \frac{3}{2}\frac{9.81\ m/s^2}{0.75\ m}\cos 30°; \quad \alpha = 16.99\ rad/s^2 \circlearrowright$$

$$A = \tfrac{3}{4}(8\ kg)(9.81\ m/s^2)\sin 30\cos 30°; \quad A = 25.5\ N \rightarrow$$

16.124



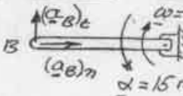
GIVEN: $m_{ABD} = 4\ kg$

$\omega_{BC} = 6\ rad/s \circlearrowleft$

$\alpha_{BC} = 15\ rad/s^2 \circlearrowright$

FIND: REACTION AT A.

CRANK BC: $BC = 0.1\ m$

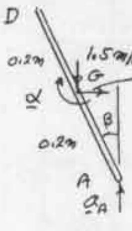


$(a_B)_t = (BC)\alpha = (0.1\ m)(15\ rad/s^2) = 1.5\ m/s^2 \downarrow$

$(a_B)_n = (BC)\omega^2 = (0.1\ m)(6\ rad/s)^2 = 3.6\ m/s^2 \rightarrow$

ROD ABD:

$$\beta = \sin^{-1}\frac{BC}{AB} = \sin^{-1}\frac{0.1\ m}{0.2\ m} = 30°$$



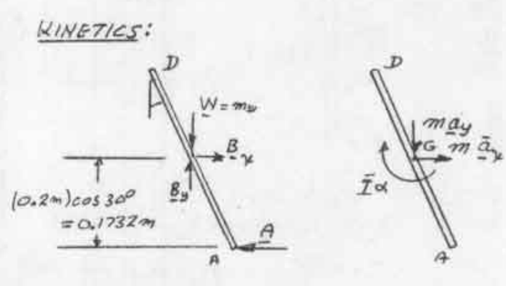
$a_A = a_G + a_{A/G}$

$[a_A \uparrow] = [1.5 \downarrow + 3.6 \rightarrow] + [0.2\alpha \measuredangle \beta]$

$\xrightarrow{+}\ 0 = 3.6 - (0.2\alpha)\cos\beta$

$$\alpha = \frac{3.6}{0.2\cos\beta} = \frac{18}{\cos 30°} = 20.78\ \frac{rad}{s^2} \circlearrowleft$$

KINETICS:



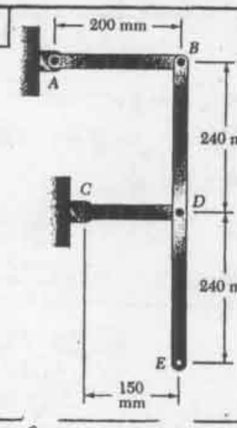
$+\circlearrowleft \Sigma M_G = \Sigma(M_G)_{eff}$:

$$A(0.1732\ m) = \bar{I}\alpha = \tfrac{1}{12}mL^2\alpha$$

$$= \tfrac{1}{12}(4\ kg)(0.4\ m)^2(20.78\ rad/s^2)$$

$$A = 6.399\ N \qquad A = 6.40\ N \leftarrow$$

16.125



GIVEN: $m_{BDE} = 5\ kg$

$\omega_{AB} = 6\ rad/s$ ↻

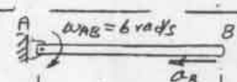
$\alpha_{AB} = 0$

FIND: HORIZONTAL COMPONENT OF REACTION AT

(a) B

(b) D

ROD AB:



$v_B = r\omega = (0.2\ m)(6\ rad/s)$

$v_B = 1.2\ m/s \downarrow$

$a_B = (0.2\ m)(6\ rad/s)^2$; $\quad a_B = 7.2\ m/s^2 \leftarrow$

ROD CD:

$v_D = v_B = 1.2\ m/s \downarrow$

$v_D = (CD)\omega_{CD}$

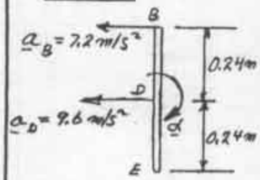
$1.2\ m/s = (0.15\ m)\ \omega_{CD}$

$\omega_{CD} = 8\ rad/s$

$a_D = (0.15\ m)(8\ rad/s)^2$

$a_D = 9.6\ m/s^2 \leftarrow$

ROD BDE:



$a_D = a_B + a_{D/B}$

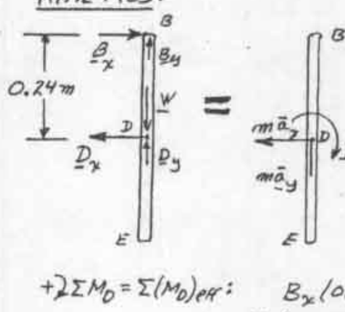
$[a_D \leftarrow] = [a_B \leftarrow] + [(BD)\alpha \rightarrow]$

$[7.2 \leftarrow] = [9.6 \leftarrow] + [(0.24)\alpha \rightarrow]$

$\xleftarrow{+}\ 7.2 = 9.6 - 0.24\alpha$

$\alpha = 10\ rad/s^2$ ↻

KINETICS:



$m\bar{a}_x = m a_D$

$\bar{I} = \frac{1}{12} m (BE)^2$

$= \frac{1}{12}(5\ kg)(0.48\ m)^2$

$\bar{I} = 96 \times 10^{-3}\ kg \cdot m^2$

$+↻\ \Sigma M_D = \Sigma (M_D)_{eff}$: $\quad B_x(0.24\ m) = \bar{I}\alpha$

$B_x(0.24\ m) = (96 \times 10^{-3}\ kg \cdot m^2)(10\ rad/s^2)$

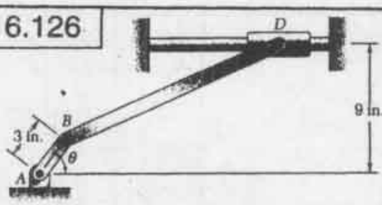
$B_x = +4\ N \qquad B_x = 4\ N \rightarrow$ ◀

$\xleftarrow{+}\ \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad D_x - B_x = m\bar{a}_x$

$D_x - 4\ N = (5\ kg)(9.6\ m/s^2)$

$D_x = +52\ N \qquad D_x = 52\ N \leftarrow$ ◀

16.126



GIVEN:

$BD = 15\ in.,\ W = 8\ lb$

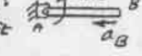
$\omega_{AB} = 300\ rpm$ ↻

$\alpha_{AB} = 0$

$\theta = 0$

FIND: D

CRANK AB:



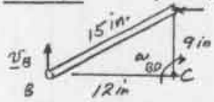
$AB = 3\ in. = 0.25\ ft$

$\omega_{AB} = 300\ rpm\left(\frac{2\pi}{60}\right) = 10\pi\ \frac{rad}{s}$

$v_B = (AB)\omega_{AB} = (0.25)(10\pi) = 7.854\ ft/s \uparrow$

$a_B = (AB)\omega_{AB}^2 = (0.25)(10\pi)^2 = 246.74\ ft/s^2 \leftarrow$

ROD BD:



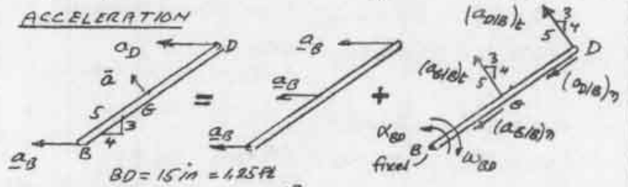
VELOCITY: INSTANT CTR AT C

$v_B = (BC)\ \omega_{BD}$

$7.854\ ft/s = (1\ ft)\ \omega_{BD}$

$\omega_{BD} = 7.854\ rad/s$ ↻

ACCELERATION



$BD = 15\ in = 1.25\ ft$

$(a_{D/B})_t = (BD)\alpha_{BD} = 1.25\alpha_{BD}$ ↖$^3_4$

$(a_{D/B})_n = (BD)\omega_{BD}^2 = (1.25)(7.854)^2 = 77.11\ ft/s^2$ ↙$^5_4{}^3$

$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$

$[a_D \leftarrow] = [246.74\ ft/s^2 \leftarrow] + [1.25\alpha$ ↖$^3_4] + [77.11\ ft/s^2$ ↙$^5_4{}^3]$

$+\uparrow\ 0 = (1.25\alpha)\frac{4}{5} - (77.11)\frac{3}{5} \qquad \alpha_{BD} = 46.266\ rad/s^2$ ↺

$(a_{G/B})_t = (BG)\alpha_{BD} = \left(\frac{1.25}{2}\ ft\right)(46.266\ rad/s^2) = 28.92\ ft/s^2$ ↖$^3_4$

$(a_{G/B})_n = (BG)\omega_{BD}^2 = \left(\frac{1.25}{2}\ ft\right)(7.854\ rad/s)^2 = 38.55\ ft/s^2$ ↙$^5_4{}^3$

$\bar{a} = a_B + a_{G/B} = a_B + (a_{G/B})_t + (a_{G/B})_n$

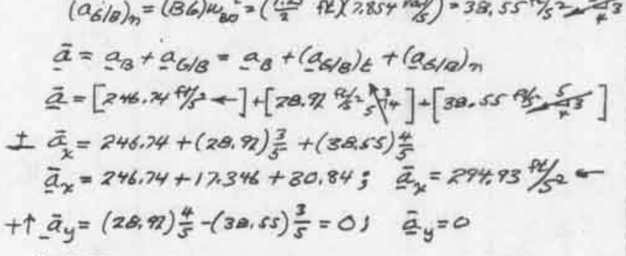
$\bar{a} = [246.74\ ft/s^2 \leftarrow] + [28.92\ ft/s^2$ ↖$^3_4] + [38.55\ ft/s^2$ ↙$^5_4{}^3]$

$\xleftarrow{+}\ \bar{a}_x = 246.74 + (28.92)\frac{3}{5} + (38.55)\frac{4}{5}$

$\bar{a}_x = 246.74 + 17.346 + 30.84$; $\quad \bar{a}_x = 294.93\ ft/s^2 \leftarrow$

$+\uparrow\ \bar{a}_y = (28.92)\frac{4}{5} - (38.55)\frac{3}{5} = 0$; $\quad \bar{a}_y = 0$

KINETICS



$\bar{I} = \frac{1}{12}\frac{W}{g}L^2 = \frac{1}{12}\ \frac{8\ lb}{32.2}\left(\frac{15}{12}\ ft\right)^2$

$\bar{I} = 0.03235\ lb \cdot ft \cdot s^2$

$+↺\ \Sigma M_B = \Sigma (M_B)_{eff}$:

$D\left(\frac{12}{12}\ ft\right) - W\left(\frac{6}{12}\ ft\right) = \bar{I}\alpha_{BD} + m\bar{a}_x\left(\frac{4.5}{12}\ ft\right)$

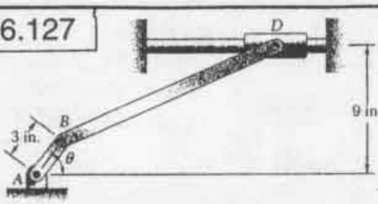
$D - (8\ lb)\frac{6}{12} = (0.03235\ lb \cdot ft \cdot s^2)(46.266\ rad/s^2)$

$+ \frac{8\ lb}{32.2}(294.93\ ft/s^2)\left(\frac{4.5}{12}\ ft\right)$

$D - 4 = 1.4967 + 27.478$

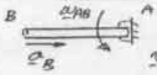
$D = 32.97\ lb \qquad D = 33.0\ lb \uparrow$ ◀

16.127



GIVEN: $BD = 15\ in.$, $W = 8\ lb$, $\omega_{AB} = 300\ rpm$ ↻, $\alpha_{AB} = 0$, $\theta = 180°$

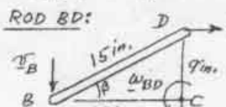
FIND: $\underline{D}$

CRANK AB: $AB = 3\ in. = 0.25\ ft$

$\omega = 300\ rpm\left(\frac{2\pi}{60}\right) = 10\pi\ \frac{rad}{s}$

$v_B = (AB)\omega_{AB} = (0.25)(10\pi) = 7.854\ ft/s$ ↓

$a_B = (AB)\omega_{AB}^2 = (0.25)(10\pi)^2 = 246.74\ ft/s^2$ →

ROD BD:



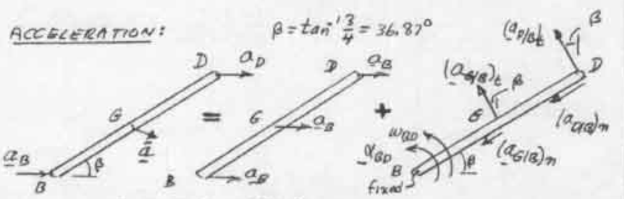
VELOCITY: INSTANT CTR. AT C.

$v_B = (BC)\omega_{BD}$

$7.854\ ft/s = (1\ ft)\omega_{BD}$

$\omega_{BD} = 7.854\ rad/s$ ↻

ACCELERATION: $\beta = \tan^{-1}\frac{3}{4} = 36.87°$



$BD = 15\ in. = 1.25\ ft$

$(a_{D/B})_t = (BD)\alpha_{BD} = (1.25\ ft)\alpha$ ↖β

$(a_{D/B})_n = (BD)\omega_{BD}^2 = (1.25)(7.854)^2 = 77.11\ ft/s^2$ ↙β

$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$

$[a_D \rightarrow] = [246.74\ ft/s^2 \rightarrow] + [1.25\alpha_{BD}$ ↖β$] + [77.11\ ft/s^2$ ↙β$]$

$+\uparrow\ 0 = 1.25\alpha_{BD}\cos\beta = 77.11\sin\beta$

$\alpha_{BD} = \frac{77.11}{1.25}\cdot\frac{\sin\beta}{\cos\beta} = 61.688\tan\beta = 61.688\left(\frac{3}{4}\right)$

$\alpha_{BD} = 46.266\ rad/s^2$ ↻

$(a_{G/B})_t = (BG)\alpha_{BD} = \left(\frac{1.25}{2}\ ft\right)\left(46.266\ \frac{rad}{s^2}\right) = 28.92\ ft/s^2$ ↖β

$(a_{G/B})_n = (BG)\omega_{BD}^2 = \left(\frac{1.25}{2}\ ft\right)\left(7.854\ \frac{rad}{s}\right)^2 = 38.55\ ft/s^2$ ↙β

$\bar{a} = a_B + a_{G/B} = a_B + (a_{G/B})_t + (a_{G/B})_n$

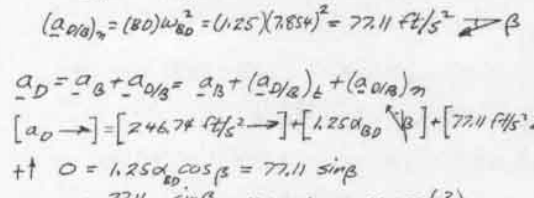
$\bar{a} = [246.74\ ft/s^2 \rightarrow] + [28.92\ ft/s^2$ ↖β$] + [38.55\ ft/s^2$ ↙β$]$

$\xrightarrow{+}\ \bar{a}_x = 246.74 - (28.92)\sin\beta - (38.55)\cos\beta$

$\bar{a}_x = 246.74 - 17.346 - 30.84$; $\bar{a}_x = 198.55\ ft/s^2$ →

$+\uparrow\ \bar{a}_y = (28.92)\cos\beta - (38.55)\sin\beta = 0$; $\bar{a}_y = 0$

KINETICS



$\bar{I} = \frac{1}{12}\frac{W}{g}(BD)^2 = \frac{1}{12}\frac{8\ lb}{32.2}\left(\frac{15}{12}\ ft\right)^2$

$\bar{I} = 0.03235\ lb\cdot ft\cdot s^2$

$+\circlearrowright\ \Sigma M_B = \Sigma(M_B)_{eff}$: $D(1\ ft) + W(0.5\ ft) = -\bar{I}\alpha_{BD} + m\bar{a}_x\left(\frac{4.5}{12}\ ft\right)$

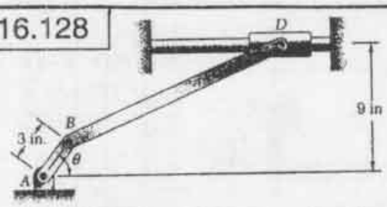
$D + (8\ lb)(0.5\ ft) = -(0.03235\ lb\cdot ft\cdot s^2)(46.266\ \frac{rad}{s^2}) + \frac{8\ lb}{32.2}(198.55\ ft/s^2)\left(\frac{4.5}{12}\ ft\right)$

$D + 4 = -1.4967 + 18.498$

$D = +13.00\ lb$

$\underline{D} = 13.00\ lb$ ↓ ◀

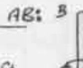
16.128



GIVEN: $BD = 15\ in.$, $W = 8\ lb$, $\omega_{AB} = 300\ rpm$ ↻, $\alpha_{AB} = 0$, $\theta = 90°$

FIND: $\underline{D}$

CRANK AB: $AB = 3\ in. = 0.25\ ft$

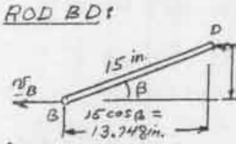


$\omega = 300\ rpm\left(\frac{2\pi}{60}\right) = 10\pi\ rad/s$

$v_B = (AB)\omega_{AB} = (0.25)(10\pi) = 7.854\ ft/s$ ←

$a_B = (AB)\omega_{AB}^2 = (0.25)(10\pi)^2 = 246.74\ ft/s^2$ ↓

ROD BD:

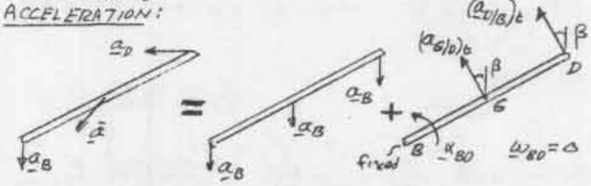


INSTANT CENTER AT ∞

$\therefore \omega_{BD} = 0$

$\beta = \sin^{-1}\frac{6\ in.}{15\ in.} = 23.58°$

ACCELERATION:



$BD = 15\ in. = 1.25\ ft$: $(a_{D/B})_t = (BD)\alpha_{BD} = (1.25\ ft)$ ↖β

$a_D = a_B + a_{D/B} = a_B + (a_{D/B})_t + (a_{D/B})_n$

$[a_D \leftarrow] = [246.74\ ft/s^2 \downarrow] + [1.25\alpha_{BD}$ ↖β$] + 0$

$+\uparrow\ 0 = -246.74 + (1.25\alpha_{BD})\cos\beta$

$\alpha_{BD} = \frac{246.74}{1.25\cos 23.57°}$ $\qquad \alpha_{BD} = 215.36\ rad/s^2$ ↻

$(a_{G/B})_t = (BG)\alpha_{BD} = \left(\frac{1.25}{2}\ ft\right)(215.36) = 134.6\ ft/s^2$

$\bar{a} = a_B + a_{G/B} = a_B + (a_{G/B})_t + (a_{G/B})_n$

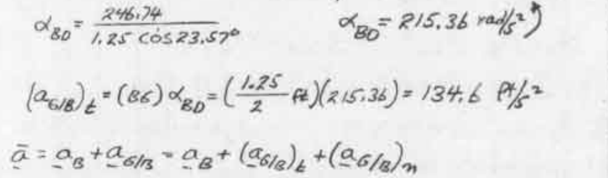
$\bar{a} = [246.74\ ft/s^2 \downarrow] + [134.6\ ft/s^2$ ↖β$] + 0$

$\xleftarrow{+}\ \bar{a}_x = [134.6]\sin 23.57° = 53.82\ ft/s^2$; $\bar{a}_x = 53.82\ ft/s^2$ ←

$+\downarrow\ \bar{a}_y = 246.74 - [134.6]\cos 23.57° = 246.74 - 123.37$

$\bar{a}_y = 123.37\ ft/s^2$ $\qquad \bar{a}_y = 123.37\ ft/s^2$ ↓

KINETICS



$\bar{I} = \frac{1}{12}\frac{W}{g}(BD)^2 = \frac{1}{12}\frac{8\ lb}{32.2}\left(\frac{15}{12}\ ft\right)^2 = 0.03235\ lb\cdot ft\cdot s^2$

$+\circlearrowright\ \Sigma M_B = \Sigma(M_B)_{eff}$:

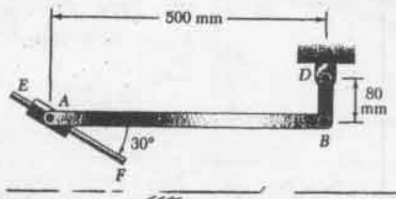
$D\left(\frac{13.748}{12}\ ft\right) - W\left(\frac{6.874}{12}\ ft\right) = \bar{I}\alpha + m\bar{a}_x\left(\frac{3}{12}\ ft\right) - m\bar{a}_y\left(\frac{6.874}{12}\ ft\right)$

$1.1456D - (8\ lb)(0.5728) = (0.03235)(215.36) + \left(\frac{8}{32.2}\right)(53.82)(0.25) - \left(\frac{8}{32.2}\right)(123.37)(0.5728)$

$1.1456D - 4.583 = 6.967 + 3.343 - 17.557$

$D = -2.325\ lb$ $\qquad \underline{D} = 2.32\ lb$ ↓ ◀

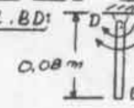
16.129



GIVEN:
ROD AB = 3 kg
$\underline{\omega}_{BD} = 15$ rad/s ↻
$\underline{\alpha}_{BD} = 60$ rad/s² ↻

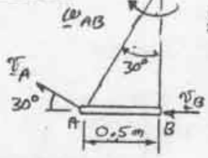
FIND: $\underline{A}$

CRANK BD:



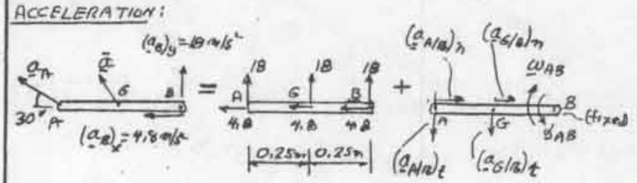
$\underline{\omega}_{BD} = 15$ rad/s, $\underline{v}_B = (0.08\,m)(15\,rad/s) = 1.2\,m/s$ ←
$\underline{\alpha}_{BD} = 60$ rad/s²
$(\underline{a}_B)_x = (0.08\,m)(60\,rad/s^2) = 4.8\,m/s^2$ ←
$(\underline{a}_B)_y = (0.08\,m)(15\,rad/s)^2 = 18\,m/s^2$ ↑

ROD AB:



VELOCITY: INSTANT. CTR. AT C.
$CB = (0.5\,m)/\tan 30° = 0.86603\,m$
$\underline{\omega}_{AB} = \frac{v_B}{CB} = \frac{1.2\,m/s}{0.86603\,m} = 1.3856\,\frac{rad}{s}$ ↻

ACCELERATION:

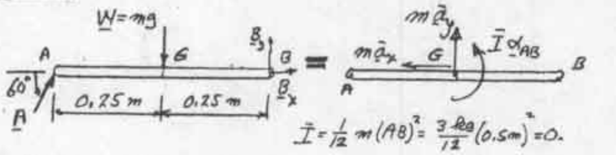


$(\underline{a}_{A/B})_t = (AB)\alpha_{AB} = 0.5\,\alpha_{AB}$ ↓
$(\underline{a}_{A/B})_n = (AB)\omega_{AB}^2 = (0.5)(1.3856)^2 = 0.96\,m/s^2$ →
$(\underline{a}_{G/B})_t = (GB)\alpha_{AB} = 0.25\,\alpha_{AB}$ ↓
$(\underline{a}_{A/B})_n = (GB)\omega_{AB}^2 = (0.25)(1.3856)^2 = 0.48\,m/s^2$ →

$\underline{a}_A = \underline{a}_B + \underline{a}_{B/A} = \underline{a}_B + (\underline{a}_{B/A})_t + (\underline{a}_{B/A})_n$
$[a_A \measuredangle 30°] = [4.8 \leftarrow] + [18 \uparrow] + [0.5\alpha_{AB} \downarrow] + [0.96 \rightarrow]$
$\pm\; a_A \cos 30° = 4.8 - 0.96$; $\underline{a}_A = 4.434\,m/s^2 \measuredangle 30°$
$+\uparrow\; (4.434)\sin 30° = 18 - 0.5\,\alpha_{AB}$; $\underline{\alpha}_{AB} = 31.566\,rad/s^2$ ↺

$\bar{a} = a_B + a_{G/B} = a_B + (a_{G/B})_t + (a_{G/B})_n$
$\bar{\underline{a}} = [4.8 \leftarrow] + [18 \uparrow] + [0.25(31.566) \downarrow] + [0.48 \rightarrow]$
$\pm\; \bar{a}_x = 4.8 - 0.48 = 4.32$; $\bar{\underline{a}}_x = 4.32\,m/s^2$ ←
$+\uparrow\; \bar{a}_y = 18 - 7.892 = 10.108$; $\bar{\underline{a}}_y = 10.108\,m/s^2$ ↑

KINETICS: $\bar{I} = \frac{1}{12}m(AB)^2 = \frac{3\,kg}{12}(0.5\,m)^2 = 0.0625\,kg\cdot m^2$



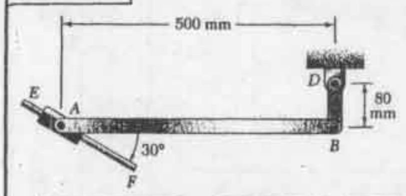
$\bar{I} = \frac{1}{12}m(AB)^2 = \frac{3\,kg}{12}(0.5\,m)^2 = 0.$

$+↻\Sigma M_B = \Sigma(M_B)_{eff}$:
$(A \sin 60°)(0.5\,m) - mg(0.25\,m) = -\bar{I}\alpha_{AB} + m\bar{a}_y(0.25\,m)$
$0.433\,A - (3\,kg)(9.81\,m/s^2)(0.25\,m) = -(0.0625\,kg\,m^2)(31.566\,\frac{rad}{s^2})$
$+ (3\,kg)(10.108\,m/s^2)(0.25\,m)$

$0.433\,A - 7.358 = -1.973 + 7.581$
$A = 29.94\,N$

$\underline{A} = 29.9\,N \measuredangle 60°$ ◀

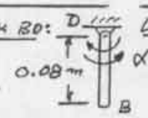
16.130



GIVEN:
ROD AB = 3 kg
$\underline{\omega}_{BD} = 15$ rad/s ↻
$\underline{\alpha}_{BD} = 60$ rad/s² ↺

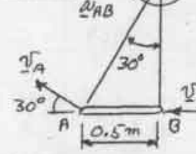
FIND: $\underline{A}$

CRANK BD:



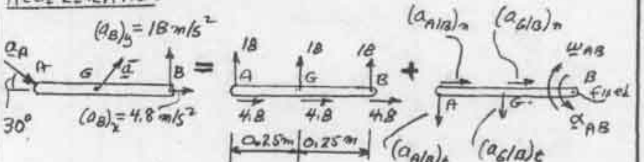
$\underline{\omega}_{BD} = 15$ rad/s; $\underline{v}_B = (0.08\,m)(15\,rad/s) = 1.2\,m/s$
$\alpha_{BD} = 60$ rad/s²
$(\underline{a}_B)_x = (0.08\,m)(60\,rad/s^2) = 4.8\,m/s^2$ →
$(\underline{a}_B)_y = (0.08\,m)(15\,rad/s)^2 = 18\,m/s^2$ ↑

ROD AB:



VELOCITY: INSTANT. CTR. AT C
$CB = (0.5\,m)/\tan 30° = 0.86603\,m$
$\omega_{AB} = \frac{v_B}{CB} = \frac{1.2\,m/s}{0.86603\,m} = 1.3856\,\frac{rad}{s}$ ↻

ACCELERATION:

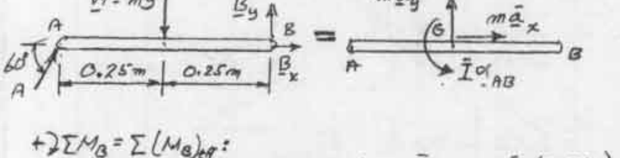


$(a_{A/B})_t = (AB)\,\alpha_{AB} = 0.5\,\alpha_{AB}$ ↓
$(a_{A/B})_n = (AB)\,\omega_{AB}^2 = (0.5)(1.3856)^2 = 0.96\,m/s^2$ →
$(a_{G/B})_t = (GB)\,\alpha_{AB} = 0.25\,\alpha_{AB}$ ↓
$(a_{G/B})_n = (GB)\,\omega_{AB}^2 = (0.25)(1.3856)^2 = 0.48\,m/s^2$ →

$\underline{a}_A = \underline{a}_B + \underline{a}_{B/A} = \underline{a}_B + (\underline{a}_{B/A})_t + (\underline{a}_{B/A})_n$
$[a_A \measuredangle 30°] = [4.8 \rightarrow] + [18 \uparrow] + [0.5\alpha_{AB} \downarrow] + [0.96 \rightarrow]$
$\pm\; a_A \cos 30° = 4.8 + 0.96$; $\underline{a}_A = 6.651\,m/s^2 \measuredangle 30°$
$+\downarrow\; (6.651)\sin 30° = -18 + 0.5\,\alpha_{AB}$; $\underline{\alpha}_{AB} = 42.65\,rad/s^2$ ↺

$\bar{a} = \underline{a}_B + \underline{a}_{G/B} = \underline{a}_B + (\underline{a}_{G/B})_t + (\underline{a}_{G/B})_n$
$\bar{\underline{a}} = [4.8 \rightarrow] + [18 \uparrow] + [0.25(42.65) \downarrow] + [0.48 \rightarrow]$
$\pm\; \bar{a}_x = 4.8 + 0.48 = 5.28$; $\bar{\underline{a}}_x = 5.28\,m/s^2$ →
$+\uparrow\; \bar{a}_y = 18 - 10.663 = 7.337$; $\bar{\underline{a}}_y = 7.337\,m/s^2$ ↑

KINETICS: $\bar{I} = \frac{1}{12}m(AB)^2 = \frac{3\,kg}{12}(0.5\,m)^2 = 0.0625\,kg\cdot m^2$

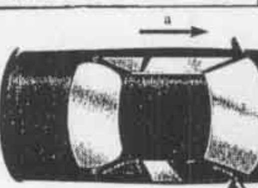


$+↻\Sigma M_B = \Sigma(M_B)_{eff}$:
$(A \sin 60°)(0.5\,m) - mg(0.25\,m) = -\bar{I}\alpha_{AB} + m\bar{a}_y(0.25\,m)$
$0.433\,A - (3\,kg)(9.81\,m/s^2)(0.25\,m) = -(0.0625\,kg\cdot m^2)(42.65\,\frac{rad}{s^2})$
$+ (3\,kg)(7.337\,m/s^2)(0.25\,m)$

$0.433\,A - 7.358 = -2.666 + 5.503$
$A = 23.55\,N$

$\underline{A} = 23.5\,N \measuredangle 60°$ ◀

16.131 and 16.132

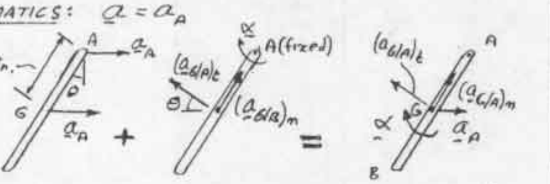


GIVEN: 80-lb DOOR WITH MASS CENTER $\bar{r} = 22$ in. FROM A AND $\bar{k} = 12.5$ in. INITIALLY $\theta = 0$.

PROBLEM 16.131
$\underline{a} = 6$ ft/s² →
FIND: ANGULAR VELOCITY ω WHEN $\theta = 90°$

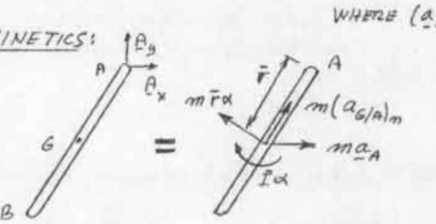
PROBLEM 16.132
FIND: $\underline{a}$ SO THAT $\omega = 2$ rad/s WHEN $\theta = 90°$

KINEMATICS: $\underline{a} = \underline{a}_A$



WHERE $(\underline{a}_{G/A})_t = \bar{r}\alpha \measuredangle \theta$

KINETICS:



$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$: $\quad 0 = \bar{I}\alpha + (m\bar{r}\alpha)\bar{r} - m a_A(\bar{r}\cos\theta)$

$$m\bar{k}^2\alpha + m\bar{r}^2\alpha = m a_A \bar{r}\cos\theta$$

$$\alpha = \frac{a_A \bar{r}}{\bar{k}^2 + \bar{r}^2}\cos\theta$$

SETTING $\alpha = \omega\frac{d\omega}{d\theta}$, AND USING $\bar{r} = \frac{22}{12}$ ft, $\bar{k} = \frac{12.5}{12}$ ft

$$\omega\frac{d\omega}{d\theta} = \frac{\left(\frac{22}{12}\text{ ft}\right)a_A}{\left[\left(\frac{12.5}{12}\text{ ft}\right)^2 + \left(\frac{22}{12}\text{ ft}\right)^2\right]}\cos\theta = 0.41234 a_A\cos\theta$$

$$\omega\,d\omega = 0.41234\, a_A\cos\theta\, d\theta$$

$$\int_0^{\omega_f}\omega\,d\omega = \int_0^{\pi/2}(0.4124 a_A)\cos\theta\,d\theta$$

$$\left|\frac{1}{2}\omega^2\right|_0^{\omega_f} = 0.41234\, a_A\left[\sin\theta\right]_0^{\pi/2}$$

$$\omega_f^2 = 0.82468\, a_A \qquad (1)$$

PROBLEM 16.131

GIVEN DATA: $\underline{a}_A = 6$ ft/s² →

$\omega_f^2 = 0.82468(6) = 4.948$

$\omega_f = 2.22$ rad/s ◀

PROBLEM 16.132

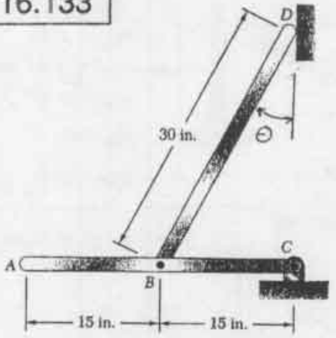
GIVEN DATA: $\omega_f = 2$ rad/s

EQ(1): $\quad \omega_f^2 = 0.82468\, a_A$

$(2)^2 = 0.82468\, a_A$

$a_A = 4.85$ ft/s² $\qquad \underline{a}_A = 4.85$ ft/s² → ◀

16.133



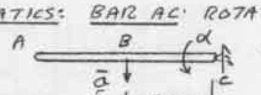
GIVEN: $W_{AC} = W_{BD} = 8$ lb IMMEDIATELY AFTER SYSTEM IS RELEASED FROM REST.

FIND: $\underline{D}$

NOTE: $\theta = \sin^{-1}\frac{15\text{ in.}}{30\text{ in.}}$

$\theta = 30°$

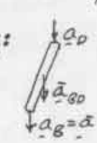
KINEMATICS: BAR AC: ROTATION ABOUT C



$\bar{a} = (BC)\alpha = \left(\frac{15}{12}\text{ ft}\right)\alpha$

$\bar{a} = 1.25\alpha \downarrow$

BAR BC:

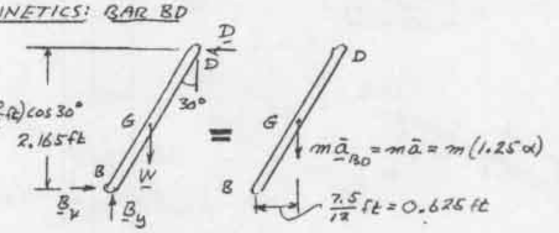


$\underline{a}_{D/B} = L\alpha$ MUST BE ZERO SINCE a_D↓

$\therefore \alpha_{BD} = 0$ AND

$\bar{a}_{BD} = \bar{a}$

KINETICS: BAR BD



$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $\quad B_y - W = -m\bar{a}$

$B_y - 8\text{ lb} = -\frac{8\text{ lb}}{32.2}(1.25\alpha)$

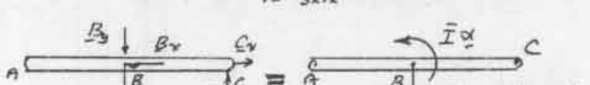
$$B_y = 8 - 0.3105\alpha \qquad (1)$$

$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$: $\quad D(2.165\text{ ft}) - W(0.625\text{ ft}) = -m\bar{a}(0.625\text{ ft})$

$D(2.165\text{ ft}) - (8\text{ lb})(0.625\text{ ft}) = -\frac{8\text{ lb}}{32.2}(1.25\alpha)(0.625\text{ ft})$

$$D = 2.309 - 0.08965\alpha \qquad (2)$$

BAR AC: $\bar{I} = \frac{1}{12}m(AC)^2 = \frac{1}{12}\frac{8\text{ lb}}{32.2}(2.5\text{ ft})^2 = 0.1294$ lb·ft·s²



$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$: $\quad W(1.25\text{ ft}) + B_y(1.25\text{ ft}) = \bar{I}\alpha + m(1.25\alpha)(1.25)$

SUBSTITUTE FROM EQ(1) FOR B_y

$8(1.25) + (8 - 0.3105\alpha)(1.25) = (0.1294)\alpha + \frac{8}{32.2}(1.25)^2\alpha$

$10 + 10 - 0.3881\alpha = 0.1294\alpha + 0.3882\alpha$

$20 = 0.9057\alpha$

$\alpha = 22.08$ rad/s²

EQ(2): $\quad D = 2.309 - 0.08965\alpha$

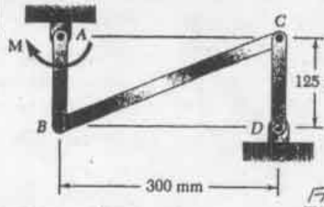
$= 2.309 - 0.08965(22.08)$

$= 2.309 - 1.966$

$D = 0.330$ lb

$\underline{D} = 0.330$ lb ← ◀

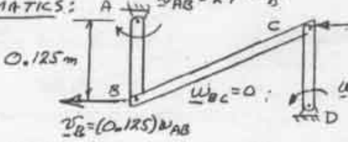
16.134



GIVEN: $m_{BC} = 3\ kg$, $m_{AB} = m_{CD} = 1.5\ kg$, $\omega_{AB} = 24\ \frac{rad}{s}$ ↻, $\alpha_{AB} = 0$

FIND: (a) M, (b) B_x, B_y

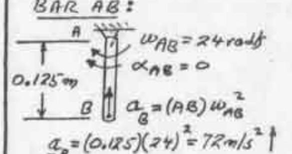
KINEMATICS:



$\omega_{AB} = 24\ rad/s$; $v_C = v_B = (0.125\ m)\omega_{AB}$; $v_B = (0.125)\omega_{AB}$

$\omega_{BC} = 0$; $\omega_{CD} = \frac{v_C}{0.125\ m} = \omega_{AB}$

BAR AB:

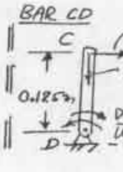


$\omega_{AB} = 24\ rad/s$, $\alpha_{AB} = 0$

$a_B = (AB)\omega_{AB}^2$

$a_B = (0.125)(24)^2 = 72\ m/s^2$ ↑

BAR CD

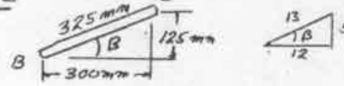


$(a_C)_x = 0.125\,\alpha_{CD}$

$(a_C)_y = (0.125)(24)^2 = 72\ m/s^2$

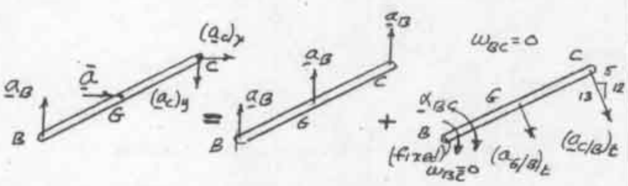
α_{CD}, $\omega_{CD} = 24\ rad/s$

BAR BC



$\beta = \tan^{-1}\frac{5}{12}$

$\beta = 22.62°$



$(a_{G/B})_t = (BG)\alpha_{BC} = \left(\frac{0.325}{2}\ m\right)\alpha_{BC} = 0.1625\,\alpha_{BC}$

$(a_{C/B})_t = (BC)\alpha_{BC} = (0.325\ m)\alpha_{BC} = 0.325\,\alpha_{BC}$

$a_C = a_B + (a_{C/B})_t$; $(a_C)_x + (a_C)_y = a_B + (a_{C/B})_t$

$[0.125\,\alpha_{CD} \rightarrow] + [72\ m/s^2 \downarrow] = [72\ m/s^2 \uparrow] + [0.325\,\alpha_{BC}\ \text{(5-12-13)}]$

$+\downarrow\ 72 = -72 + (0.325\,\alpha_{BC})\frac{12}{13}$; $\alpha_{BC} = 480\ rad/s^2$ ↻

$\xrightarrow{+}\ 0.125\,\alpha_{CD} = (0.325)(480)\left(\frac{5}{13}\right)$; $\alpha_{CD} = 480\ rad/s^2$ ↻

$\bar{a} = a_B + a_{G/B} = a_B + (a_{G/B})_t$

$= [72\ m/s^2 \uparrow] + [0.1625\,\alpha_{BC}\ \text{(5-12-13)}]$

$= [72\ m/s^2 \uparrow] + [0.1625(480)\ \text{(5-12-13)}]$

$\bar{a} = [72\ m/s^2 \uparrow] + [78\ m/s^2\ \text{(5-12-13)}]$

$\xrightarrow{+}\ \bar{a}_x = 78\left(\frac{5}{13}\right) = 30\ m/s^2 \longrightarrow$

$+\uparrow\ \bar{a}_y = 72 - 78\left(\frac{12}{13}\right) = 72 - 72 = 0$

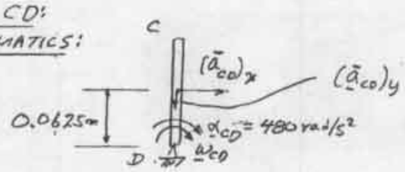
TOTAL ACCELERATION OF G IS $30\ m/s^2 \longrightarrow$

(CONTINUED)

16.134 continued

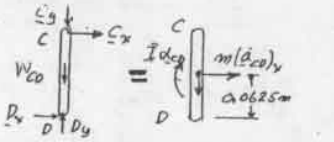
BAR CD:

KINEMATICS:



$(\bar{a}_{CD})_x = (0.0625\ m)(480\ rad/s^2) = 30\ m/s^2 \longrightarrow$

KINETICS



$\bar{I}_{CD} = \frac{1}{12}m(CD)^2$

$= \frac{1}{12}(1.5\ kg)(0.125\ m)^2$

$\bar{I}_{CD} = 1.953 \times 10^{-3}\ kg \cdot m^2$

$+\circlearrowleft \Sigma M_D = \Sigma (M_D)_{eff}$:

$C_x(0.125\ m) = \bar{I}\alpha_{CD} + m(\bar{a}_{CD})_x(0.0625\ m)$

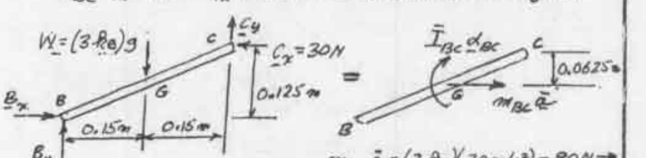
$0.125\,C_x = (1.953 \times 10^{-3}\ kg \cdot m^2)(480\ rad/s^2) + (1.5\ kg)(30\ m/s^2)(0.0625\ m)$

$0.125\,C_x = 3.75$

$C_x = 30\ N$

BAR BC:

$\bar{I}_{BC} = \frac{1}{12}m(BC)^2 = \frac{1}{12}(3\ kg)(0.325\ m)^2 = 26.406 \times 10^{-3}\ kg \cdot m^2$



$m_{BC}\bar{a} = (3\ kg)(30\ m/s^2) = 90\ N \rightarrow$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $B_x - C_x = m_{BC}\bar{a}$

$B_x - 30\ N = 90\ N$

$B_x = +120\ N$ $\qquad$ $B_x = 120\ N \rightarrow$ ◀

$+\circlearrowleft \Sigma M_C = \Sigma (M_C)_{eff}$:

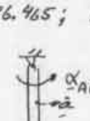
$B_y(0.3\ m) - B_x(0.125\ m) - W(0.15\ m) = \bar{I}_{BC}\alpha_{BC} - (m_{BC}\bar{a})(0.0625\ m)$

$0.3\,B_y - (120)(0.125) - (3)(9.81)(0.15) = (26.406 \times 10^{-3}\ kg \cdot m^2)(480\ rad/s^2) - (90\ N)(0.0625)$

$0.3\,B_y - 15 - 4.4145 = 12.675 - 5.625$

$0.3\,B_y = 26.465$; $B_y = +88.22\ N$; $\qquad$ $B_y = 88.2\ N$ ↑ ◀

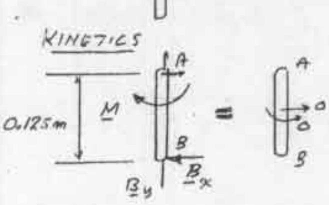
BAR AB:



$\alpha_{AB} = 0$ (GIVEN)

$\bar{a} = 0$

KINETICS



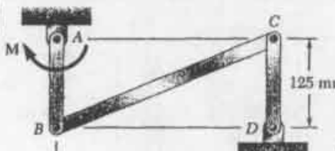
$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$

$M + B_x(0.125\ m) = 0$

$M + (120\ N)(0.125\ m) = 0$

$M = -15\ N \cdot m$ $\qquad$ $M = 15\ N \cdot m$ ↻ ◀

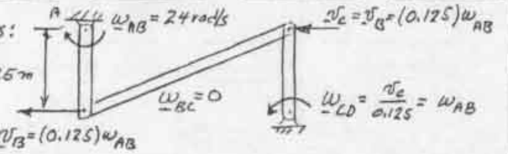
16.135



GIVEN:
$m_{BC} = 3\ kg$
$m_{AB} = m_{CD} = 1.5\ kg$
$\omega_{AB} = 24\ rad/s$ ↻
$\alpha_{AB} = 160\ rad/s^2$ ↻

FIND: (a) $\underline{M}$, (b) $\underline{B}_x$, B_y

KINEMATICS:

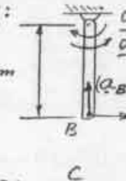


$v_C = v_B = (0.125)\omega_{AB}$

$\omega_{BC} = 0$ $\omega_{CD} = \frac{v_C}{0.125} = \omega_{AB}$

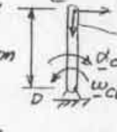
$v_B = (0.125)\omega_{AB}$

BAR AB:



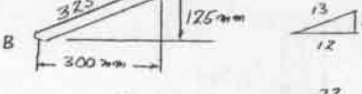
$(a_B)_x = (AB)\alpha_{AB} = (0.125)(160)$
$(a_B)_x = 20\ m/s^2 \rightarrow$
$(a_B)_y = (AB)\omega_{AB}^2 = (0.125)(24)^2$
$(a_B)_y = 72\ m/s^2 \uparrow$

BAR CD:

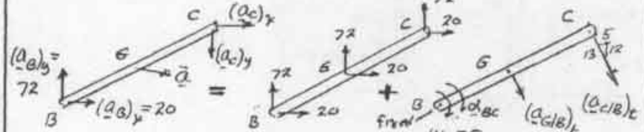


$(a_C)_x = 0.125\alpha_{CD}$
$(a_C)_y = (CD)(\omega_{CD})^2 = (0.125)(24)^2$
$(a_C)_y = 72\ m/s^2 \downarrow$

BAR BC:



$\beta = \tan^{-1}\frac{5}{12}$
$\beta = 22.62°$



$(a_{C/B})_t = (BC)\alpha_{BC} = (0.325\ m)\alpha_{BC} = 0.325\alpha_{BC}$

$(a_{G/B}) = (BG)\alpha_{BC} = \left(\frac{0.325\ m}{2}\right)\alpha_{BC} = 0.1625\alpha_{BC}$

$a_C = a_B + a_{C/B}$: $(a_C)_x + (a_C)_y = (a_B)_x + (a_B)_y + (a_{C/B})_t$

$[0.125\alpha_{CD} \rightarrow] + [72\ m/s^2 \downarrow] = [20\ m/s^2 \rightarrow] + [72\ m/s^2 \uparrow] + [0.325\alpha_{BC}]$

$+\downarrow\ 72 = -72 + (0.325\alpha_{BC})\frac{12}{13}$; $\underline{\alpha}_{BC} = 480\ rad/s^2$ ↻

$\xrightarrow{+} 0.125\alpha_{CD} = 20 + (0.325)(480)(\frac{5}{13})$; $\underline{\alpha}_{CD} = 640\ rad/s^2$ ↻

$\bar{a} = a_B + a_{G/B} = (a_B)_x + (a_B)_y + (a_{G/B})_t$

$\bar{a} = [20\ m/s^2 \rightarrow] + [72\ m/s^2 \uparrow] + [0.1625\alpha_{BC}]$
$= [20\ m/s^2 \rightarrow] + [72\ m/s^2 \uparrow] + [0.1625(480)]$

$\bar{a} = [20\ m/s^2 \rightarrow] + [72\ m/s^2 \uparrow] + [78\ m/s^2]$

$\xrightarrow{+} \bar{a}_x = 20 + 78(\frac{5}{13}) = 20 + 30 = 50\ m/s^2$

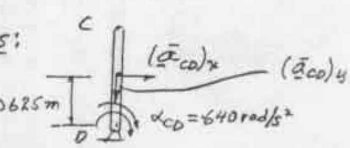
$+\uparrow \bar{a}_y = 72 - 78(\frac{12}{13}) = 0$

TOTAL ACCELERATION OF G IS $50\ m/s^2 \rightarrow$

(CONTINUED)

16.135 continued

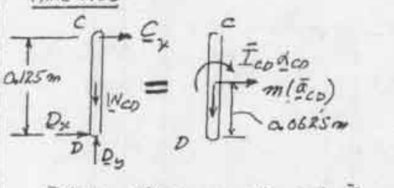
BAR CD:
KINEMATICS:



$\alpha_{CD} = 640\ rad/s^2$

$(\bar{a}_{CD})_x = (0.0625\ m)(640\ rad/s^2) = 40\ m/s^2 \rightarrow$

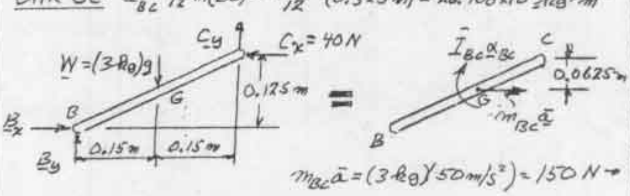
KINETICS



$\bar{I}_{CD} = \frac{1}{12}m(CD)^2$
$= \frac{1.5\ kg}{12}(0.125\ m)^2$
$\bar{I}_{CD} = 1.953 \times 10^{-3}\ kg \cdot m^2$

$+\circlearrowleft \Sigma M_D = \Sigma(M_D)_{eff}$: $C_x(0.125\ m) = \bar{I}_{CD}\alpha_{CD} + m(\bar{a}_{CD})_x(0.0625\ m)$
$0.125C_x = (1.953 \times 10^{-3}\ kg \cdot m^2)(640\ rad/s^2)$
$+ (1.5\ kg)(40\ m/s^2)(0.0625\ m)$
$0.125C_x = 5.00$ $C_x = 40\ N$

BAR BC: $\bar{I}_{BC} = \frac{1}{12}m(BC)^2 = \frac{3\ kg}{12}(0.325\ m)^2 = 26.406 \times 10^{-3}\ kg \cdot m^2$



$m_{BC}\bar{a} = (3\ kg)(50\ m/s^2) = 150\ N \rightarrow$

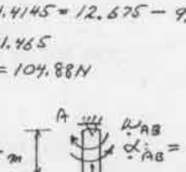
$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$: $B_x - C_x = m_{BC}\bar{a}$
$B_x - 40\ N = 150\ N$
$B_x = 190\ N$ $\underline{B}_x = 190\ N \rightarrow$ ◀

$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:
$B_y(0.3\ m) - B_x(0.125\ m) - W(0.15\ m) = \bar{I}_{BC}\alpha_{BC} - (m_{BC}\bar{a})(0.0625\ m)$

$0.3B_y - (190)(0.125) - (3\ kg)(9.81)(0.15\ m) = (26.406 \times 10^{-3}\ kg \cdot m^2)(480\ rad/s^2)$
$- (150\ N)(0.0625\ m)$

$0.3B_y - 23.75 - 4.4145 = 12.675 - 9.375$
$0.3B_y = 31.465$
$B_y = 104.88\ N$ $\underline{B}_y = 104.9\ N \uparrow$ ◀

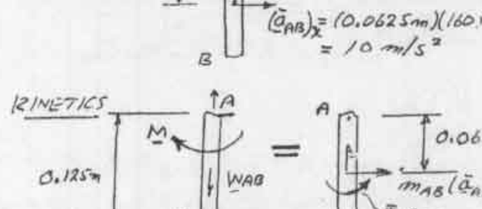
BAR AB:



$\alpha_{AB} = 160\ rad/s^2$ (GIVEN)

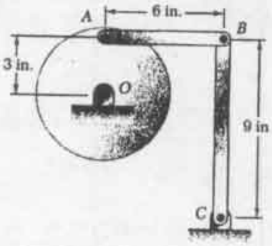
$(\bar{a}_{AB})_x = (0.0625\ m)(160\ rad/s^2)$
$= 10\ m/s^2$

KINETICS



$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $M - B_x(0.125\ m) = -\bar{I}_{AB}\alpha_{AB} - m_{AB}(a_{AB})_x(0.0625\ m)$
$M + (190)(0.125) = -\frac{1}{12}(1.5)(0.125)^2(160) - (1.5)(10)(0.0625)$
$M + 23.75 = -0.3125 - 0.9375$
$M = -25.0\ N \cdot m$ $\underline{M} = 25.0\ N \cdot m$ ↻ ◀

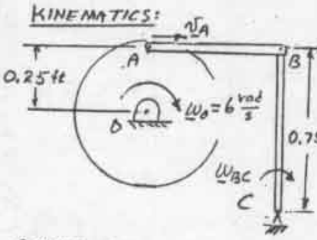
16.136



GIVEN:
$W_{AB} = 4\,lb$
$W_{BC} = 6\,lb$
DISK: $\omega_0 = 6\,rad/s$ ↻
$\alpha_0 = 0$

FIND: FORCES ON AB AT A AND B

KINEMATICS:

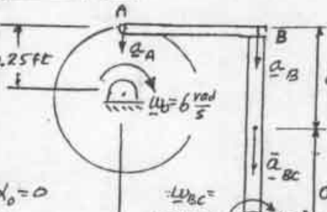


VELOCITY
$\omega_{AB} = 0$
$v_B = v_A = (0.25\,ft)(6\,rad/s)$
$= 1.5\,ft/s^2$

$\omega_{BC} = \frac{v_B}{0.75\,ft} = \frac{1.5\,m/s^2}{0.75\,ft}$

$\omega_{BC} = 2\,rad/s$ ↻

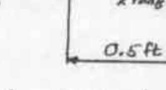
ACCELERATION



$a_A = (0.25\,ft)(6\,rad/s)^2$
$a_A = 9\,ft/s^2$ ↓

$a_B = (0.75)(2\,rad/s)^2$
$= 3\,ft/s^2$ ↓

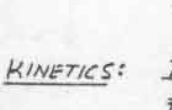
$\bar{a}_{BC} = (0.375\,ft)(2\,rad/s)^2$
$\bar{a}_{BC} = 1.5\,ft/s^2$



$\bar{a}_{AB} = \frac{1}{2}(a_A + a_B) = \frac{1}{2}(9+3)$
$\bar{a}_{AB} = 6\,ft/s^2$ ↓
$a_A = a_B + (0.5\,ft)\alpha_{AB}$
$9\,ft/s^2 = 3\,ft/s^2 + (0.5\,ft)\alpha_{AB}$
$\alpha_{AB} = 12\,rad/s^2$ ↺

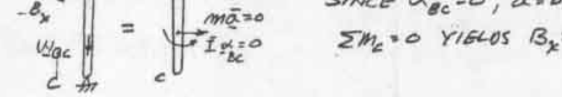
KINETICS: $\bar{I}_{AB} = \frac{1}{12} m_{AB}(AB)^2 = \frac{1}{12}\frac{4\,lb}{32.2}(0.5\,ft)^2$
$\bar{I}_{AB} = 2.588 \times 10^{-3}\,lb \cdot ft \cdot s^2$

ROD BC:



SINCE $\alpha_{BC} = 0$, $\bar{a} = 0$
$\Sigma M_C = 0$ YIELDS $B_x = 0$

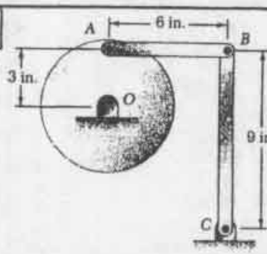
ROD AB:



$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$: $A_x = 0$
$+↺ \Sigma M_A = \Sigma(M_A)_{eff}$:
$B_y(0.5\,ft) - W_{AB}(0.25\,ft) = \bar{I}_{AB}\alpha_{AB} - m_{AB}\bar{a}_{AB}(0.25\,ft)$
$0.5B_y - (4\,lb)(0.25\,ft) = (2.588 \times 10^{-3}\,lb \cdot ft \cdot s^2)(12\,rad/s)$
$- \frac{4\,lb}{32.2}(6\,ft/s^2)(0.25\,ft)$

$0.5B_y - 1 = 0.03106 - 0.1863$
$0.5B_y = 0.8447$ $B_y = 1.689\,lb$; $B = 1.689\,lb$ ↑ ◀
$+↑ \Sigma F_y = \Sigma(F_y)_{eff}$: $A_y - W_{AB} + B_y = -m_{AB}\bar{a}_{AB}$
$A_y - 4\,lb + 1.689\,lb = -\frac{4\,lb}{32.2}(6\,ft/s^2)$; $A_y = 1.565\,lb$; $A = 1.565\,lb$ ↑ ◀

16.137

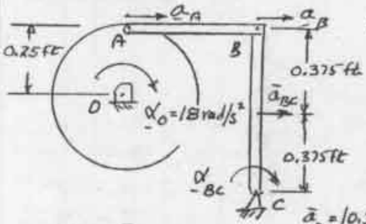


GIVEN:
$W_{AB} = 4\,lb$
$W_{BC} = 6\,lb$
Disk: $\omega_0 = 0$
$\alpha_0 = 18\,rad/s^2$ ↻

FIND: FORCES ON AB AT A AND B

KINEMATICS: VELOCITY OF ALL ELEMENTS = 0

ACCELERATION:

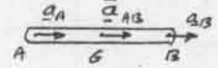


$a_B = a_A = (0.25\,ft)(18\,rad/s^2)$
$= 4.5\,ft/s^2$ →

$\alpha_{BC} = \frac{a_B}{0.75\,ft} = \frac{4.5\,ft/s^2}{0.75\,ft}$

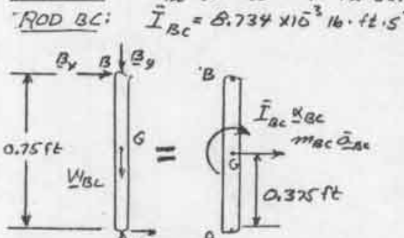
$\alpha_{BC} = 6\,rad/s^2$ ↻

$\bar{a}_{BC} = (0.375\,ft)(6\,rad/s^2) = 2.25\,ft/s^2$ →



$\bar{a}_{AB} = a_A = a_B = 4.5\,ft/s^2$ →

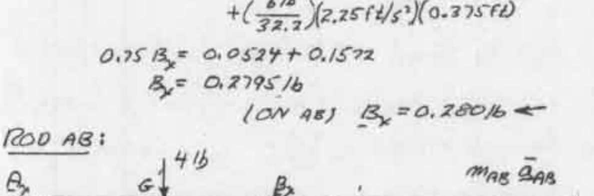
KINETICS: $\bar{I}_{BC} = \frac{1}{12} m_{BC}(BC)^2 = \frac{1}{12}\frac{6\,lb}{32.2}(0.75\,ft)^2$
ROD BC: $\bar{I}_{BC} = 8.734 \times 10^{-3}\,lb \cdot ft \cdot s^2$



$+↻ \Sigma M_A = \Sigma(M_A)_{eff}$:
$B_x(0.75\,ft) = \bar{I}_{BC}\alpha_{BC} + m_{BC}\bar{a}_{BC}(0.375\,ft)$
$0.75B_x = (8.734 \times 10^{-3}\,lb \cdot ft \cdot s^2)(6\,rad/s^2)$
$+ \left(\frac{6\,lb}{32.2}\right)(2.25\,ft/s^2)(0.375\,ft)$

$0.75B_x = 0.0524 + 0.1572$
$B_x = 0.2795\,lb$
(ON AB) $B_x = 0.280\,lb$ ←

ROD AB:

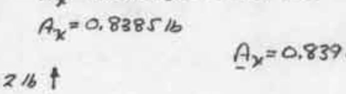


$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$: $A_x - B_x = m_{AB}\bar{a}_{AB}$
$A_x - 0.2795\,lb = \left(\frac{4\,lb}{32.2}\right)(4.5\,ft/s^2)$

$A_x - 0.2795\,lb = 0.5590\,lb$
$A_x = 0.8385\,lb$

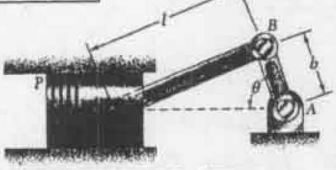
$A_x = 0.839\,lb$ →

ΣM_A: $B_y = 2\,lb$ ↑
ΣM_B: $A_y = 2\,lb$ ↑



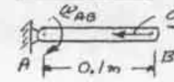
◀

16.138



GIVEN: $\omega_{AB} = 600$ rpm ↻
$l = 250$ mm, $b = 100$ mm
$m_{BD} = 1.2$ kg, $m_P = 1.8$ kg
$\theta = 180°$
FIND: FORCES ON BD AT B AND D

KINEMATICS: CRANK AB:



$\omega_{AB} = 600 \text{ rpm}\left(\frac{2\pi}{60}\right) = 62.832 \text{ rad/s}$ ↻

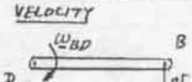
$a_B = (AB)\omega_{AB}^2 = (0.1 \text{ m})(62.832 \text{ rad/s})^2$

$a_B = 394.78 \text{ m/s}^2 \leftarrow$

ALSO: $v_B = (AB)\omega_{AB} = (0.1 \text{ m})(62.832 \text{ rad/s}) = 6.2832 \text{ m/s} \downarrow$

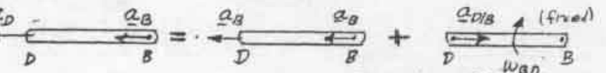
CONNECTING ROD BD:

VELOCITY — INSTANT CENTER AT D.



$\omega_{BD} = \frac{v_B}{BD} = \frac{6.2832 \text{ m/s}}{0.25 \text{ m}} = 25.133 \text{ rad/s}$ ↻

ACCELERATION:



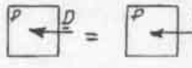
$a_D = a_B + a_{D/B} = [a_B \leftarrow] + [(BD)\omega_{BD}^2 \rightarrow]$

$a_D = [394.78 \text{ m/s}^2 \leftarrow] + [(0.25 \text{ m})(25.133 \text{ rad/s})^2 \rightarrow]$

$a_D = [394.78 \text{ m/s}^2 \leftarrow] + [157.92 \text{ m/s}^2 \rightarrow] = 236.86 \text{ m/s}^2 \leftarrow$

$\bar{a}_{BD} = \frac{1}{2}(a_B + a_D) = \frac{1}{2}(394.78 \leftarrow + 236.86 \leftarrow) = 315.82 \text{ m/s}^2 \leftarrow$

KINETICS OF PISTON



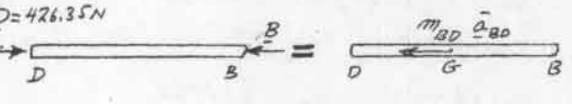
$m_P a_D = (1.8 \text{ kg})(236.86 \text{ m/s}^2)$

$D = 426.35 \text{ N} \leftarrow$

FORCE EXERTED ON CONNECTING ROD AT D IS:

$D = 426.35 \rightarrow$

KINETICS OF CONNECTING ROD: (NEGLECT WEIGHT)



$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$:

$B - D = m_{BD}\bar{a}_{BD}$

$B - 426.35 \text{ N} = (1.2 \text{ kg})(315.82 \text{ m/s}^2)$

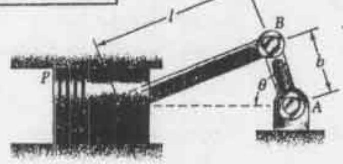
$B = 426.35 \text{ N} + 378.98 \text{ N} = 805.33 \text{ N}$

FORCES EXERTED ON CONNECTING ROD

$B = 805 \text{ N} \leftarrow$ ◀

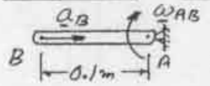
$D = 426 \text{ N} \rightarrow$ ◀

16.139



GIVEN: $\omega_{AB} = 600$ rpm ↺
$l = 250$ mm, $b = 150$ mm
$m_{BD} = 1.2$ kg, $m_P = 1.8$ kg
$\theta = 0$
FIND: FORCES ON BD AT B AND D

KINEMATICS: CRANK AB:



$\omega_{AB} = 600 \text{ rpm}\left(\frac{2\pi}{60}\right) = 62.832 \text{ rad/s}$ ↺

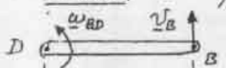
$a_B = (AB)\omega_{AB}^2 = (0.1 \text{ m})(62.832 \text{ rad/s})^2$

$a_B = 394.78 \text{ m/s}^2 \rightarrow$

ALSO: $v_B = (AB)\omega_{AB} = (0.1 \text{ m})(62.832 \text{ rad/s}) = 6.2832 \text{ m/s} \uparrow$

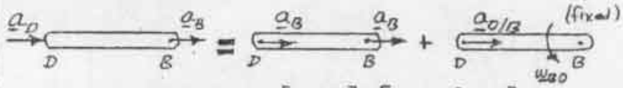
CONNECTING ROD BD:

VELOCITY — INSTANT CENTER AT D:



$\omega_{BD} = \frac{v_B}{BD} = \frac{6.2832 \text{ m/s}}{0.25 \text{ m}} = 25.133 \text{ rad/s}$ ↺

ACCELERATION:



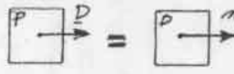
$a_D = a_B + a_{D/B} = [a_B \rightarrow] + [(BD)\omega_{BD}^2 \rightarrow]$

$a_D = [394.78 \text{ m/s}^2 \rightarrow] + [(0.25 \text{ m})(25.133 \text{ rad/s})^2 \rightarrow]$

$a_D = [394.78 \text{ m/s}^2 \rightarrow] + [157.92 \text{ m/s}^2 \rightarrow] = 552.70 \text{ m/s}^2 \rightarrow$

$\bar{a}_{BD} = \frac{1}{2}(a_B + a_D) = \frac{1}{2}(394.78 \rightarrow + 552.70 \rightarrow) = 473.74 \text{ m/s}^2 \rightarrow$

KINETICS OF PISTON



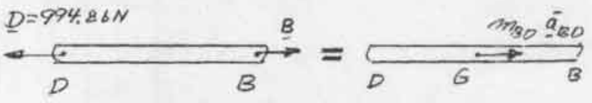
$m_P a_D = (1.8 \text{ kg})(552.70 \text{ m/s}^2)$

$D = 994.86 \text{ N} \rightarrow$

FORCE EXERTED ON CONNECTING ROD AT D IS:

$D = 994.86 \text{ N} \leftarrow$

KINETICS OF CONNECTING ROD (NEGLECT WEIGHT)



$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$:

$B - D = m_{BD}\bar{a}_{BD}$

$B - 994.86 \text{ N} = (1.2 \text{ kg})(473.74 \text{ m/s}^2)$

$B = 994.86 \text{ N} + 568.44 \text{ N} = 1563.3 \text{ N}$

FORCES ACTING ON CONNECTING ROD

$B = 1563 \text{ N} \rightarrow$ ◀

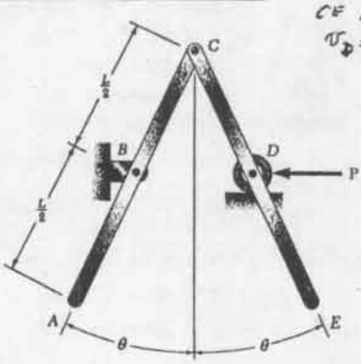
$D = 995 \text{ N} \leftarrow$ ◀

16.140 and 16.141

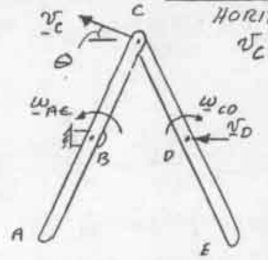
GIVEN: RODS AC AND CE EACH WEIGH W.
v_D = CONSTANT TO LEFT

PROBLEM 16.140
FIND: P IN TERMS OF L, W, v_D AND θ

PROBLEM 16.141
FIND: P IF
$L = 3$ ft, $W = 24$ lb, $v_D = 6$ ft/s, $\theta = 55°$.



KINEMATICS: VELOCITY:



HORIZ. COMPONENT OF v_C EQUALS $v_D/2$ ←

$v_C \cos\theta = \frac{1}{2} v_D$

$v_C = \frac{v_D}{2\cos\theta}$ ⦣ θ

ROD AC:

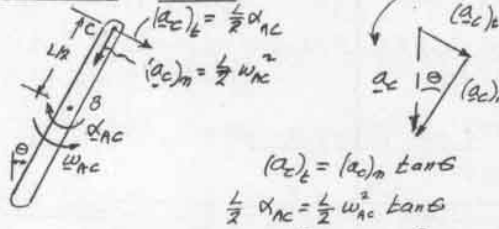
$v_C = \frac{L}{2}\omega_{AC}$

$\frac{v_D}{2\cos\theta} = \frac{L}{2}\omega_{AC}$

$\omega_{AC} = \frac{v_D}{L\cos\theta}$

BY SYMMETRY: $|\omega_{CE}| = |\omega_{AC}|$
ALSO SINCE $a_D = 0$, HORIZONTAL COMPONENT OF a_C IS ZERO. THUS a_C IS VERTICAL

ACCELERATION: ROD AC



$(a_C)_t = (a_C)_n \tan\theta$

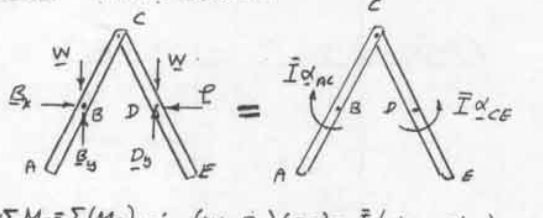
$\frac{L}{2}\alpha_{AC} = \frac{L}{2}\omega_{AC}^2 \tan\theta$

$\alpha_{AC} = \omega_{AC}^2 \tan\theta = \left(\frac{v_D}{L\cos\theta}\right)^2 \tan\theta = \frac{v_D^2}{L^2}\frac{\tan\theta}{\cos^2\theta}$ ↻

BY SYMMETRY: $|\alpha_{AC}| = |\alpha_{CD}|$

$\alpha_{CE} = \frac{v_D^2}{L^2}\frac{\tan\theta}{\cos^2\theta}$ ↺

KINETICS: ENTIRE SYSTEM

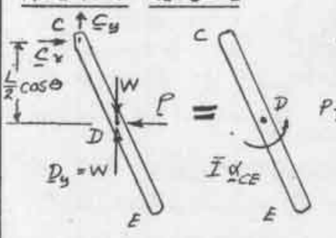


$\Sigma M_B = \Sigma(M_B)_{eff}$: $(W - D_y)(BD) = \bar{I}(\alpha_{CE} - \alpha_{AC})$
SINCE $\alpha_{CE} = \alpha_{AC}$, WE FIND $D_y = W$↑

(CONTINUED)

16.140 and 16.141 continued

KINETICS: ROD CE



$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$:

$P\frac{L}{2}\cos\theta = -\bar{I}\alpha_{CE}$

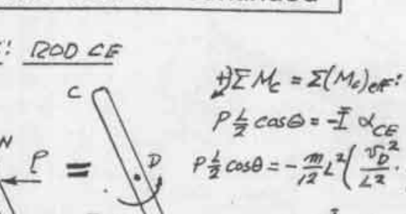
$P\frac{L}{2}\cos\theta = -\frac{m}{12}L^2\left(\frac{v_D^2}{L^2}\cdot\frac{\tan\theta}{\cos^2\theta}\right)$

$P = \frac{m v_D^2}{6L}\cdot\frac{\tan\theta}{\cos^3\theta}$ → ◀

DATA: $L = 3$ ft, $W = 24$ lb, $v_D = 6$ ft/s, $\theta = 55°$

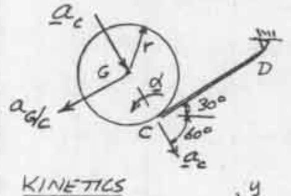
$P = \frac{24\text{ lb}}{32.2\text{ ft/s}^2}\cdot\frac{(6\text{ ft/s})^2}{6(3\text{ ft})}\cdot\frac{\tan 55°}{\cos^3 55°}$; $P = 11.28$ lb → ◀

*16.142



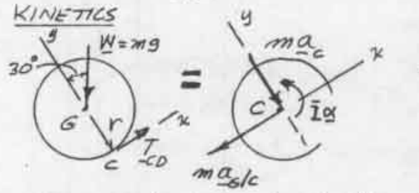
GIVEN:
$m = 4$ kg
$r = 150$ mm
BELT AB THEN BREAKS
FIND: (a) $\bar{a}$, (b) T_{CD}

KINEMATICS: $\omega = 0$



$\bar{a} = a_G = a_C + a_{G/C}$
WHERE $a_{G/C} = r\alpha$

KINETICS



$+\nearrow \Sigma F_y = \Sigma(F_y)_{eff}$:
$-W\cos 30° = -ma_C$
$a_C = 0.866g$ ⦨ $60°$

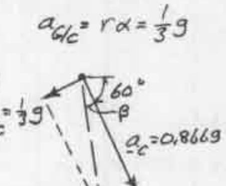
$+\circlearrowleft \Sigma M_C = \Sigma(M_C)_{eff}$: $(W\sin 30°)r = \bar{I}\alpha + (ma_{G/C})r$

$mgr\sin 30° = (\frac{1}{2}mr^2)\alpha + (mr\alpha)r$

$\frac{1}{2}g = \frac{3}{2}r\alpha$ $\qquad \alpha = \frac{1}{3}\frac{g}{r}$

$a_{G/C} = r\alpha = \frac{1}{3}g$ $\qquad a_{G/C} = \frac{1}{3}g$ ⦩ $30°$

(a)



$\beta = \tan^{-1}\frac{1/3\,g}{0.866g} = 21.052°$

$\bar{a} = \frac{0.866g}{\cos\beta} = \frac{0.866g}{\cos 21.052}$

$\bar{a} = 0.9279g = 0.9279(9.81)$

$\bar{a} = 9.10$ m/s² ⦨ $81.1°$ ◀

(b) $\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$:

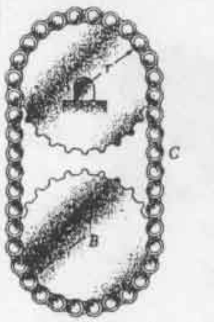
$T - W\sin 30° = -ma_{G/C}$

$T = 0.5mg - m(g/3) = \frac{1}{6}mg$

$T = \frac{1}{6}(4\text{ kg})(9.81\text{ m/s}^2) = 6.54$ N

$T = 6.54$ N ◀

*16.143

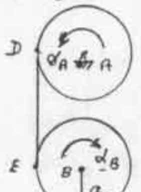


GIVEN:
DISK OF MASS m AND RADIUS r
PIN AT C IS REMOVED

FIND:
(a) α_A AND α_B
(b) TENSION IN CHAIN
(c) a_B

KINEMATICS: ASSUME $\alpha_A \circlearrowleft$ AND $\alpha_B \circlearrowright$

$\omega_A = \omega_B = 0$



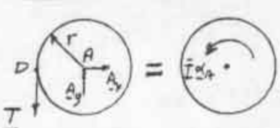
$a_D = r\alpha_A \downarrow$

$a_E = a_D = r\alpha_A \downarrow$

$\bar{a}_B = a_E + a_{B/E}$
$= (r\alpha_A + r\alpha_B)$

$a_B = r(\alpha_A + \alpha_B)\downarrow$

KINETICS: DISK A:



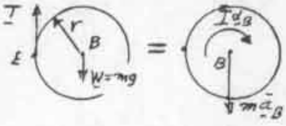
$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$:

$Tr = \bar{I}\alpha_A$

$Tr = \frac{1}{2}mr^2\alpha_A$

$\alpha_A = \frac{2T}{mr} \circlearrowleft \qquad (1)$

DISK B:



$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$:

$Tr = \bar{I}\alpha_B$

$Tr = \frac{1}{2}mr^2\alpha_B$

$\alpha_B = \frac{2T}{mr} \circlearrowright \qquad (2)$

FROM (1) AND (2) WE NOTE THAT $\alpha_A = \alpha_B$

$+\circlearrowright \Sigma M_E = \Sigma (M_E)_{eff}$: $\quad Wr = \bar{I}\alpha_B + (m\bar{a}_B)r$

$Wr = \frac{1}{2}mr^2\alpha_B + mr(\alpha_A + \alpha_B)r$

$\alpha_A = \alpha_B$: $\quad Wr = \frac{5}{2}mr^2\alpha_A \qquad \alpha_A = \frac{2}{5}\frac{g}{r} \circlearrowleft$ ◀

$\alpha_B = \frac{2}{5}\frac{g}{r} \circlearrowright$ ◀

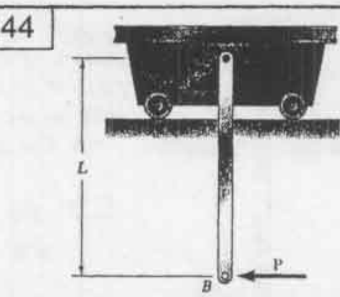
SUBSTITUTE FOR α_A INTO (1):

$\frac{2}{5}\frac{g}{r} = \frac{2T}{mr} \qquad T = \frac{1}{5}mg$ ◀

$a_B = r(\alpha_A + \alpha_B) = r(2\alpha_A) = 2r\left(\frac{2}{5}\frac{g}{r}\right)$

$a_B = \frac{4}{5}g \downarrow$ ◀

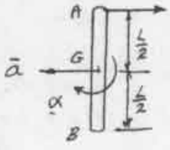
*16.144



GIVEN:
CART OF MASS m
ROD OF MASS m
CART AT REST WHEN P IS APPLIED

FIND: a_A
a_B

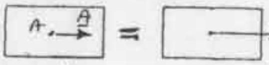
KINEMATICS:



$a_{CART} = a_A$

$a_A = \frac{L}{2}\alpha - \bar{a}$

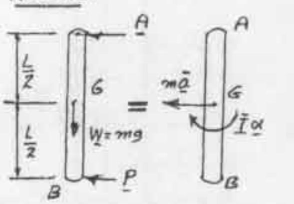
KINETICS: CART



$m a_A = m\left(\frac{L}{2}\alpha - \bar{a}\right)$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad A = m\left(\frac{L}{2}\alpha - \bar{a}\right) \qquad (1)$

ROD AB:



$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$:

$PL = m\bar{a}\frac{L}{2} + \bar{I}\alpha$

$PL = \frac{1}{2}mL\bar{a} + \frac{1}{12}mL^2\alpha \qquad (2)$

$\xleftarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad P + A = m\bar{a}$

FROM (1) $\quad P + m\left(\frac{L}{2}\alpha - \bar{a}\right) = m\bar{a}$

$P = 2m\bar{a} - \frac{1}{2}mL\bar{a} \qquad (3)$

MULTIPLY (3) BY $\frac{L}{6}$: $\quad \frac{1}{6}PL = \frac{1}{3}mL\bar{a} - \frac{1}{12}mL^2\alpha \qquad (4)$

ADD (2) AND (4): $\quad \frac{7}{6}PL = \frac{5}{6}mL\bar{a}$

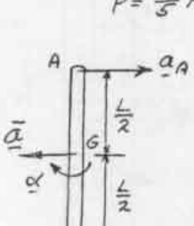
$\bar{a} = \frac{7}{5}\frac{P}{m} \leftarrow$

SUBSTITUTE (5) INTO (3):

$P = 2m\left(\frac{7}{5}\frac{P}{m}\right) - \frac{1}{2}mL\alpha$

$P = \frac{14}{5}P - \frac{1}{2}mL\alpha$

$\alpha = \frac{18}{5}\frac{P}{mL} \circlearrowright$



$\xrightarrow{+} a_A = \frac{L}{2}\alpha - \bar{a}$

$= \frac{L}{2}\left(\frac{18}{5}\frac{P}{mL}\right) - \frac{7}{5}\frac{P}{m}$

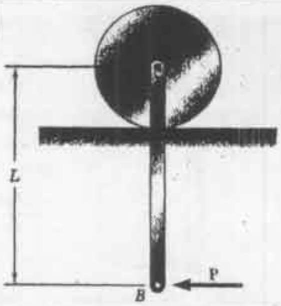
$a_A = \frac{2}{5}\frac{P}{m} \rightarrow$ ◀

$\xleftarrow{+} a_B = \frac{L}{2}\alpha + \bar{a}$

$= \frac{L}{2}\left(\frac{18}{5}\cdot\frac{P}{mL}\right) + \frac{7}{5}\frac{P}{m}$

$a_B = \frac{16}{5}\frac{P}{m} \leftarrow$ ◀

*16.145



GIVEN:
BAR AB OF MASS m
CYLINDER OF MASS m
SYSTEM AT REST WHEN P IS APPLIED

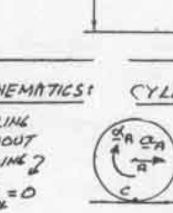
FIND: $\underline{a}_A$
$\underline{a}_B$

KINEMATICS: CYLINDER: $\omega = 0$

ROLLING WITHOUT SLIDING: $(a_C)_x = 0$

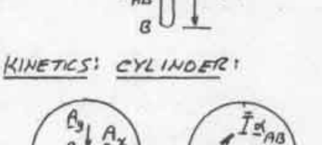
$\pm\ \underline{a}_A = (\underline{a}_C)_x + \underline{a}_{A/C}$
$= 0 + r\alpha_A$
$\underline{a}_A = r\alpha_A \rightarrow$
$\alpha_A = \frac{a_A}{r}$ ↻

ROD AB:



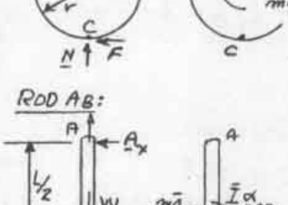
$\pm\ a_A = \frac{L}{2}\alpha_{AB} - \bar{a}_{AB}$

KINETICS: CYLINDER:



$+↺\ \Sigma M_C = \Sigma(M_C)_{eff}$:
$A_x r = m a_A r + \bar{I}\alpha_A$
$A_x r = m a_A r + \frac{1}{2} m r^2 \left(\frac{a_A}{r}\right)$
$A_x = \frac{3}{2} m a_A$
$A_x = \frac{3}{2} m \left(\frac{L}{2}\alpha_{AB} - \bar{a}_{AB}\right)$ (1)

ROD AB:



$+↺\ \Sigma M_A = \Sigma(M_A)_{eff}$:
$PL = m\bar{a}_{AB}\frac{L}{2} + \bar{I}\alpha_{AB}$
$PL = m\bar{a}_{AB}\frac{L}{2} + \frac{m}{12}L^2\alpha_{AB}$ (2)

$\pm\ \Sigma F_x = \Sigma(F_x)_{eff}$: $P + A_x = m\bar{a}_{AB}$

SUBSTITUTE FROM (1): $P + \frac{3}{2} m \left(\frac{L}{2}\alpha_{AB} - \bar{a}_{AB}\right) = m\bar{a}_{AB}$

$P = \frac{5}{2} m\,\bar{a}_{AB} - \frac{3}{4} mL\alpha_{AB}$ (3)

MULTIPLY BY $\frac{L}{9}$: $\frac{1}{9}PL = \frac{5L}{18} m\bar{a}_{AB} - \frac{1}{12} mL^2\alpha_{AB}$ (4)

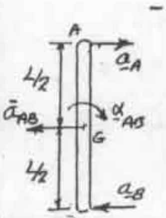
(4)+(2): $\frac{10}{9}PL = \left(\frac{1}{2} + \frac{5}{18}\right) mL\,\bar{a}_{AB} = \frac{7}{9} mL\bar{a}_{AB}$

$\bar{a}_{AB} = \frac{10}{7}\frac{P}{m} \leftarrow$ (5)

(5) → (3) $P = \frac{5}{2} m \left(\frac{10}{7}\frac{P}{m}\right) - \frac{3}{4} mL\alpha_{AB}$

$P = \frac{25}{7}P - \frac{3}{4} mL\alpha_{AB}$

$-\frac{18}{7}P = \frac{3}{4} mL\alpha_{AB}$ $\quad \underline{\alpha}_{AB} = \frac{24}{7}\frac{P}{mL}$ ↻ ◀



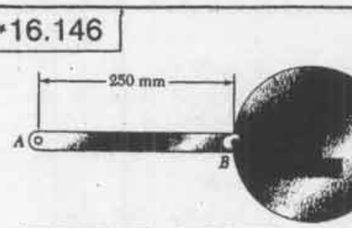
$\pm\ a_A = \frac{L}{2}\alpha_{AB} - \bar{a}_{AB} = \frac{L}{2}\left(\frac{24}{7}\frac{P}{mL}\right) - \frac{10}{7}\frac{P}{m}$

$a_A = \left(\frac{12}{7} - \frac{10}{7}\right)\frac{P}{m}$; $\quad \underline{a}_A = \frac{2}{7}\frac{P}{m} \rightarrow$ ◀

$\pm\ a_B = \frac{L}{2}\alpha_{AB} + \bar{a}_{AB} = \frac{L}{2}\left(\frac{24}{7}\frac{P}{mL}\right) + \frac{10}{7}\frac{P}{m}$

$a_B = \left(\frac{12}{7} + \frac{10}{7}\right)\frac{P}{m}$; $\quad \underline{a}_B = \frac{22}{7}\frac{P}{m} \leftarrow$ ◀

*16.146



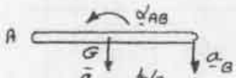
GIVEN: $m_{AB} = 5$ kg
$m_C = 8$ kg

RELEASE FROM REST

FIND: (a) $\underline{a}_A$
(b) $\underline{a}_B$

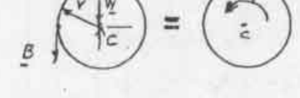
KINEMATICS: $\omega = 0$

$\underline{a}_B = r\alpha_C \downarrow$



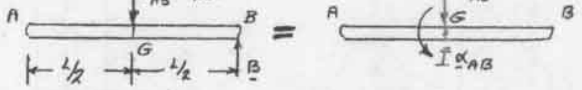
$+\downarrow\ \bar{a} = a_B + a_{G/B}$
$\bar{a} = r\alpha_C + \frac{L}{2}\alpha_{AB} \downarrow$

KINETICS: DISK



$+↺\ \Sigma M_C = \Sigma(M_C)_{eff}$:
$B r = \bar{I}\alpha_C$
$B r = \frac{1}{2} m_C r^2 \alpha_C$
$B = \frac{1}{2} m_C r\alpha_C$

ROD AB:



$+↺\ \Sigma M_G = \Sigma(M_G)_{eff}$: $\quad B\frac{L}{2} = \bar{I}\alpha_{AB}$

$\left(\frac{1}{2} m_C r\alpha_C\right)\frac{L}{2} = \frac{1}{12} m_{AB} L^2 \alpha_{AB}$

$\alpha_C = \frac{1}{3}\frac{m_{AB}}{m_C}\cdot\frac{L}{r}\alpha_{AB}$ (1)

$+\downarrow\ \Sigma F_y = \Sigma(F_y)_{eff}$:
$m_{AB} g - B = m_{AB}\bar{a}$
$m_{AB} g - \frac{1}{2} m_C r\alpha_C = m_{AB}\left(r\alpha_C + \frac{L}{2}\alpha_{AB}\right)$

$g = \frac{L}{2}\alpha_{AB} + \left(\frac{1}{2}\frac{m_C}{m_{AB}} + 1\right) r\alpha_C$

$\frac{g}{L} = \frac{1}{2}\alpha_{AB} + \left(\frac{1}{2}\frac{m_C}{m_{AB}} + 1\right)\frac{r}{L}\cdot\left(\frac{1}{3}\frac{m_{AB}}{m_C}\cdot\frac{L}{r}\right)\alpha_{AB}$

$\frac{g}{L} = \left(\frac{1}{2} + \frac{1}{6} + \frac{1}{3}\frac{m_{AB}}{m_C}\right)\alpha_{AB} = \frac{1}{3}\left(2 + \frac{m_{AB}}{m_C}\right)\alpha_{AB}$

$\alpha_{AB} = \frac{3g}{L}\frac{1}{\left(2 + \frac{m_{AB}}{m_C}\right)}$ (2)

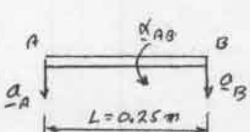
$m_{AB} = 5$ kg, $m_C = 8$ kg, $r = 0.1$ m, $L = 0.25$ m

EQ(1): $\underline{\alpha}_{AB} = \frac{3(9.81\ m/s^2)}{0.25\ m}\cdot\frac{1}{2 + \frac{5\ kg}{8\ kg}} = 44.846\ rad/s^2$ ↻

EQ(2): $\alpha_C = \frac{1}{3}\frac{5\ kg}{8\ kg}\cdot\frac{0.25\ m}{0.1\ m}\cdot(44.846\ rad/s^2) = 23.357\ \frac{rad}{s^2}$ ↻

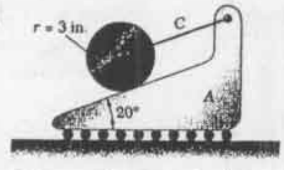
$a_B = r\alpha_C = (0.1\ m)\left(23.357\ \frac{rad}{s^2}\right) = 2.336\ m/s^2$

$\underline{a}_B = 2.34\ m/s^2 \downarrow$ ◀



$+\downarrow\ a_A = a_B + a_{B/A}$
$a_A = a_B + L\alpha_{AB}$
$a_A = 2.336\ m/s^2 + (0.25\ m)\left(44.846\ \frac{rad}{s^2}\right)$
$a_A = 2.336 + 11.212$
$\underline{a}_A = 13.55\ m/s^2 \downarrow$ ◀

*16.147

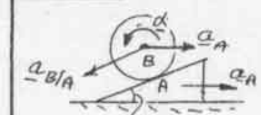


GIVEN: $W_B = 6\text{ lb}$, $W_A = 4\text{ lb}$

AFTER CORD IS CUT CYLINDER ROLLS.

FIND: (a) $\underline{a}_A$ (b) $\underline{\alpha}$

KINEMATICS: WE RESOLVE a_B INTO $\underline{a}_A$ AND A COMPONENT PARALLEL TO THE INCLINE

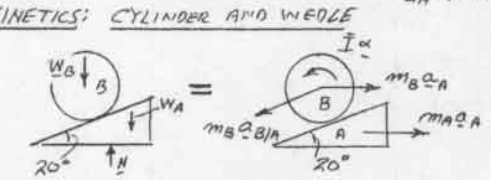


$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$

WHERE $a_{B/A} = r\alpha$, SINCE THE CYLINDER ROLLS ON WEDGE A.

$$a_{B/A} = (0.25\text{ ft})\alpha$$

KINETICS: CYLINDER AND WEDGE



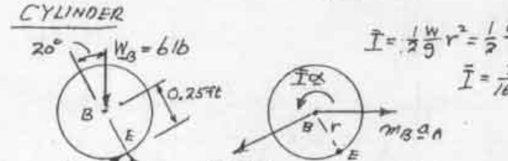
$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$: $\quad 0 = m_A a_A + m_B a_A - m_B a_{B/A}\cos 20°$

$$0 = \frac{(4+6)\text{ lb}}{g} a_A - \frac{6\text{ lb}}{g}\left(\frac{3}{12}\text{ ft}\right)\alpha \cos 20°$$

$$a_A = (0.15\cos 20°)\alpha \qquad (1)$$

CYLINDER

$$\bar{I} = \frac{1}{2}\frac{W}{g}r^2 = \frac{1}{2}\frac{6\text{ lb}}{g}(0.25\text{ ft})^2 \qquad \bar{I} = \frac{3}{16g}$$



$+\circlearrowright \Sigma M_E = \Sigma(M_E)_{eff}$:

$$(6\text{ lb})\sin 20°(0.25\text{ ft}) = \bar{I}\alpha + m_B a_{B/A}(0.25\text{ ft}) - m_B a_A \cos 20°(0.25\text{ ft})$$

$$1.5\sin 20° = \frac{3}{16(32.2)}\alpha + \frac{6\text{ lb}}{32.2}(0.25\alpha)(0.25) - \frac{6\text{ lb}}{32.2}a_A\cos 20°(0.25)$$

$$0.51303 = 0.00582\alpha + 0.01165\alpha - 0.04378\,a_A$$

SUBSTITUTE FROM (1):

$$0.51303 = 0.01747\alpha - 0.04378(0.15\cos 20°)\alpha$$

$$0.51303 = (0.01747 - 0.00617)\alpha$$

$$\alpha = 45.41\text{ rad/s}^2 \qquad \underline{\alpha} = 45.4\text{ rad/s}^2 \circlearrowright$$ ◀

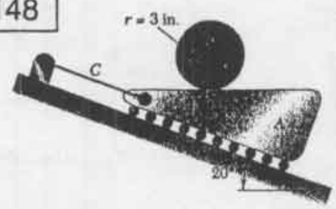
EQ(1): $\quad a_A = (0.15\cos 20°)\alpha$

$$= (0.15\cos 20°)(45.41)$$

$$a_A = 6.401\text{ ft/s}^2$$

$$\underline{a}_A = 6.40\text{ ft/s}^2 \longrightarrow$$ ◀

*16.148



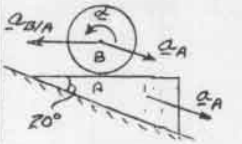
GIVEN: $W_B = 6\text{ lb}$, $W_A = 4\text{ lb}$

AFTER CORD IS CUT CYLINDER ROLLS

FIND: (a) $\underline{a}_A$ (b) $\underline{\alpha}$

KINEMATICS: WE RESOLVE $\underline{a}_B$ INTO $\underline{a}_A$ AND A HORIZONTAL COMPONENT $\underline{a}_{B/A}$

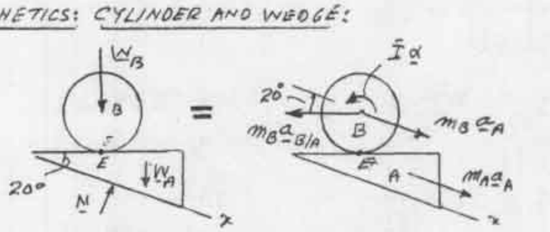
$$\underline{a}_B = \underline{a}_A + \underline{a}_{B/A}$$



WHERE $a_{B/A} = r\alpha$, SINCE THE CYLINDER B ROLLS ON WEDGE A.

$$a_{B/A} = (0.25\text{ ft})\alpha$$

KINETICS: CYLINDER AND WEDGE:



$\overset{+}{\searrow} \Sigma F_x = \Sigma(F_x)_{eff}$:

$$(W_A + W_B)\sin 20° = (m_A + m_B)a_A - m_B a_{B/A}\cos 20°$$

$$(10\text{ lb})\sin 20° = \left(\frac{10}{g}\right)a_A - \left(\frac{6}{g}\right)(0.25\alpha)\cos 20°$$

$$a_A = g\sin 20° + \frac{6}{10}(0.25)\cos 20°\,\alpha$$

$$a_A = g\sin 20° + 0.15\cos 20°\,\alpha \qquad (1)$$

CYLINDER: $+\circlearrowleft \Sigma M_E = \Sigma(M_E)_{eff}$

$$0 = \bar{I}\alpha + (m_B a_{B/A})(0.25\text{ ft}) - (m_B a_A\cos 20°)(0.25\text{ ft})$$

$$0 = \frac{1}{2}\frac{6\text{ lb}}{g}(0.25\text{ ft})^2\alpha + \frac{6\text{ lb}}{g}(0.25\alpha)(0.25) - \frac{6\text{ lb}}{g}a_A\cos 20°(0.25)$$

$$0 = \frac{1}{g}\left[0.1875\alpha + 0.375\alpha - 1.4095\,a_A\right]$$

$$0 = 0.5625\alpha - 1.4095\,a_A; \quad \alpha = 2.506\,a_A \qquad (2)$$

SUBSTITUTE FOR α FROM (2) INTO (1):

$$a_A = g\sin 20° + 0.15\cos 20°(2.506\,a_A)$$

$$a_A = 11.013 + 0.3532\,a_A$$

$$(1 - 0.3532)a_A = 11.013$$

$$a_A = 17.027\text{ ft/s}^2$$

$$\underline{a}_A = 17.03\text{ ft/s}^2 \measuredangle 20°$$ ◀

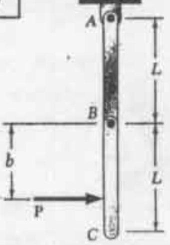
EQ(2) $\quad \alpha = 2.506\,a_A$

$$= 2.506(17.027)$$

$$\alpha = 42.7\text{ rad/s}^2$$

$$\underline{\alpha} = 42.7\text{ rad/s}^2 \circlearrowleft$$ ◀

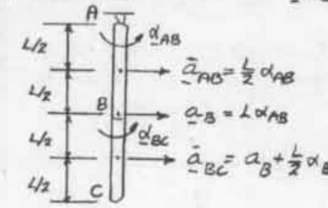
*16.149



GIVEN: $P = 20\ N$
$m_{AB} = m_{BC} = m = 3\ kg$
$L = 500\ mm$
$b = L = 500\ mm$

FIND: α_{AB} AND α_{BC}

KINEMATICS: ASSUME $\alpha_{AB}\circlearrowright$, $\alpha_{BC}\circlearrowright$, AND $\omega_{AB} = \omega_{BC} = 0$

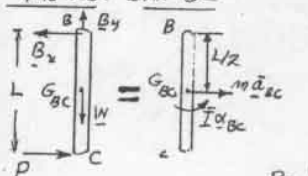


$\bar{a}_{AB} = \frac{L}{2}\alpha_{AB}$

$a_B = L\alpha_{AB}$

$\bar{a}_{BC} = a_B + \frac{L}{2}\alpha_{BC} = L\alpha_{AB} + \frac{1}{2}L\alpha_{BC}$

KINETICS: BAR BC



$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$:

$PL = \bar{I}\alpha_{BC} + (m\bar{a}_{BC})\frac{L}{2}$

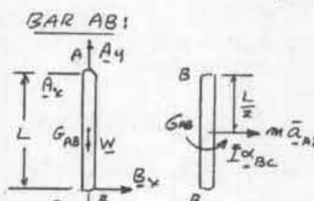
$= \frac{m}{12}L^2\alpha_{BC} + m(L\alpha_{AB} + \frac{L}{2}\alpha_{BC})\frac{L}{2}$

$$P = \frac{1}{2}mL\alpha_{AB} + \frac{1}{3}mL\alpha_{BC} \qquad (1)$$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad P - B_x = m\bar{a}_{BC}$

$$P - B_x = m(L\alpha_{AB} + \frac{1}{2}L\alpha_{BC}) \qquad (2)$$

BAR AB:



$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$:

$B_x L = \bar{I}\alpha_{AB} + (m\bar{a}_{AB})\frac{L}{2}$

$= \frac{m}{12}L^2\alpha_{AB} + m(\frac{L}{2}\alpha_{AB})\frac{L}{2}$

$$B_x = \frac{1}{3}mL\alpha_{AB} \qquad (3)$$

ADD (2) AND (3): $\quad P = \frac{4}{3}mL\alpha_{AB} + \frac{1}{2}mL\alpha_{BC} \qquad (4)$

SUBTRACT (1) FROM (4)

$$0 = \frac{5}{6}mL\alpha_{AB} + \frac{1}{6}mL\alpha_{BC}$$

$$\alpha_{BC} = -5\alpha_{AB} \qquad (5)$$

SUBSTITUTE FOR α_{BC} IN (1):

$$P = \frac{1}{2}mL\alpha_{AB} + \frac{1}{3}mL(-5\alpha_{AB}) = -\frac{7}{6}mL\alpha_{AB}$$

$$\alpha_{AB} = -\frac{6}{7}\frac{P}{mL} \qquad (6)$$

EQ(5) $\quad \alpha_{BC} = -5(-\frac{6}{7}\frac{P}{mL}) \qquad \alpha_{BC} = \frac{30}{7}\frac{P}{mL} \qquad (7)$

DATA: $L = 0.5\ m$, $m = 3\ kg$, $P = 20\ N$

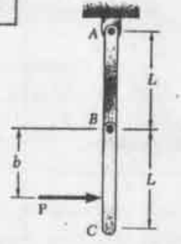
EQ(6): $\alpha_{AB} = -\frac{6}{7}\frac{20\ N}{(3\ kg)(0.5\ m)} = -11.43\ rad/s^2$

$\alpha_{AB} = 11.43\ rad/s^2 \circlearrowleft$ ◀

EQ(7): $\alpha_{BC} = \frac{30}{7}\frac{20\ N}{(3\ kg)(0.5\ m)} = 57.14\ rad/s^2$

$\alpha_{BC} = 57.1\ rad/s^2 \circlearrowright$ ◀

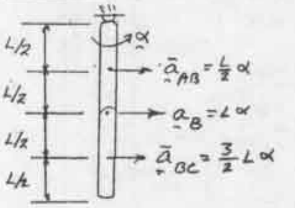
*16.150



GIVEN: $P = 20\ N$
$m_{AB} = m_{BC} = m = 3\ kg$
$L = 500\ mm$

FIND: (a) DISTANCE b FOR WHICH BARS MOVE AS A SINGLE RIGID BODY
(b) α OF BARS

KINEMATICS: WE CHOOSE $\alpha = \alpha_{AB} = \alpha_{BC}\circlearrowright$

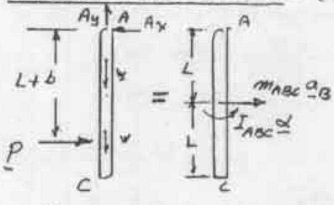


$\bar{a}_{AB} = \frac{L}{2}\alpha$

$a_B = L\alpha$

$\bar{a}_{BC} = \frac{3}{2}L\alpha$

KINETICS: BARS AB AND BC (ACTING AS RIGID BODY)



$m_{ABC} = 2m$

$\bar{I} = \frac{1}{12}(2m)(2L)^2$

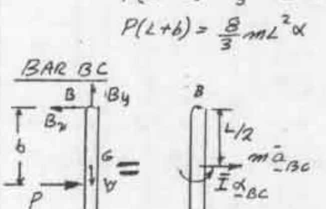
$\bar{I} = \frac{2}{3}mL^2$

$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$: $\quad P(L+b) = \bar{I}_{ABC}\alpha + m_{ABC}a_B L$

$P(L+b) = \frac{2}{3}mL^2\alpha + (2m)(L\alpha)L$

$$P(L+b) = \frac{8}{3}mL^2\alpha \qquad (1)$$

BAR BC



$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$:

$Pb = \bar{I}_{BC}\alpha + (m\bar{a}_{BC})\frac{L}{2}$

$= \frac{m}{12}L^2\alpha + m(\frac{3}{2}L\alpha)\frac{L}{2}$

$Pb = \frac{5}{6}mL^2\alpha$

$$\alpha = \frac{6}{5}\frac{Pb}{mL^2} \qquad (2)$$

SUBSTITUTE FOR α INTO (1)

$$P(L+b) = \frac{8}{3}mL^2(\frac{6}{5}\frac{Pb}{mL^2})$$

$$PL + Pb = \frac{16}{5}Pb \quad ; \quad L = (\frac{16}{5} - 1)b = \frac{11}{5}b$$

$$b = \frac{5}{11}L$$

EQ(2) $\quad \alpha = \frac{6}{5}\frac{P}{mL^2}(\frac{5}{11}L) \qquad \alpha = \frac{6}{11}\frac{P}{mL}$

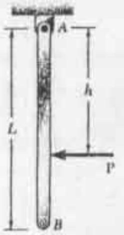
DATA: $L = 0.5\ m$, $m = 3\ kg$, $P = 20\ N$

(a) $\quad b = \frac{5}{11}L = \frac{5}{11}(500\ mm); \qquad b = 227\ mm$ ◀

(b) $\quad \alpha = \frac{6}{11}\frac{P}{mL} = \frac{6}{11}\frac{20\ N}{(3\ kg)(0.5\ m)} = 7.273\ rad/s^2$

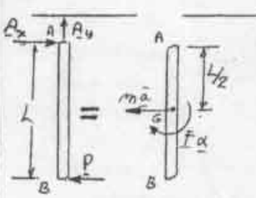
$\alpha = 7.27\ rad/s^2 \circlearrowright$ ◀

*16.151



GIVEN: $L = 36 in.$
$W = 4 in.$
$P = 1.5 lb$
$b = L = 36 in.$

FIND: M_{max} AND SHOW THAT M_{max} IS INDEPENDENT OF W



ROD AB: $\bar{a} = \frac{L}{2}\alpha$

$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{eff}$

$PL = (m\bar{a})\frac{L}{2} + \bar{I}\alpha$

$= (m\frac{L}{2}\alpha)\frac{L}{2} + \frac{1}{12}mL^2\alpha$

$\alpha = \frac{3P}{mL} \circlearrowright$ (1)

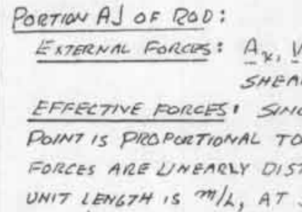
$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$: $A_x - P = -m\bar{a}$

$A_x = P - m\frac{L}{2}\alpha = P - m\frac{L}{2}\left(\frac{3P}{mL}\right) = -\frac{P}{2}$; $\quad A_x = \frac{1}{2}P \leftarrow$

PORTION AJ OF ROD:

EXTERNAL FORCES: A_x, W_{AJ}, AXIAL FORCE F_J, SHEAR V_J, AND BENDING MOMENT M_J

EFFECTIVE FORCES: SINCE ACCELERATION AT ANY POINT IS PROPORTIONAL TO DISTANCE FROM A, EFFECTIVE FORCES ARE LINEARLY DISTRIBUTED. SINCE MASS PER UNIT LENGTH IS m/L, AT POINT J WE FIND



$\left(\frac{m}{L}\right)a_J = \frac{m}{L}(x\alpha)$

USING (1): $\frac{m}{L}a_J = \frac{m}{L}\left(\frac{3P}{mL}\right)$

$\frac{m}{L}a_J = \frac{3Px}{L^2}$

$+\circlearrowright \Sigma M_J = \Sigma(M_J)_{eff}$: $M_J - A_x x = -\frac{1}{2}\left(\frac{3Px}{L^2}\right)x\left(\frac{2x}{3}\right)$

$M_J = \frac{1}{2}Px - \frac{1}{2}\frac{P}{L^2}x^3$ (2)

FOR M_{max}: $\frac{dM_J}{dx} = \frac{P}{2} - \frac{3}{2}\frac{P}{L^2}x^2 = 0$

$x = \frac{L}{\sqrt{3}}$ (3)

SUBSTITUTING INTO (2)

$(M_J)_{max} = \frac{1}{2}\frac{PL}{\sqrt{3}} - \frac{1}{2}\frac{P}{L^2}\left(\frac{L}{\sqrt{3}}\right)^3 = \frac{1}{2}\frac{PL}{\sqrt{3}}\left(\frac{2}{3}\right)$

$(M_J)_{max} = \frac{PL}{3\sqrt{3}}$ (4)

NOTE: EQS. (3) AND (4) ARE INDEPENDENT OF W

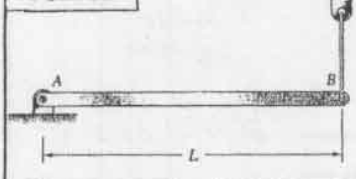
DATA: $L = 36 in.$, $P = 1.5 lb$

EQ(3): $x = \frac{L}{\sqrt{3}} = \frac{36 in.}{\sqrt{3}} = 20.78 in.$

EQ(4): $(M_J)_{max} = \frac{(1.5 lb)(36 in.)}{3\sqrt{3}} = 10.392 lb\cdot in$

$M_{max} = 10.39 lb\cdot in.$ 20.8 in. BELOW A ◀

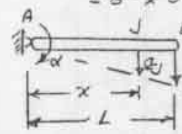
*16.152



GIVEN:
m = MASS OF AB
CORD BREAKS
DRAW:
V AND M DIAGRAMS

FROM ANSWERS TO PROB. 16.84:

$a_B = \frac{3}{2}g \qquad A = \frac{1}{4}mg \uparrow$



WE NOW FIND

$\alpha = \frac{a_B}{L} = \frac{3g}{2L} \circlearrowright$

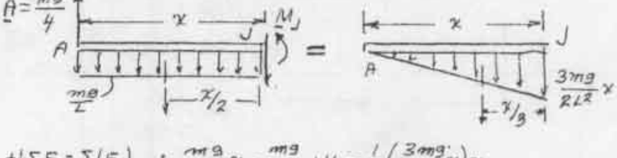
$a_J = x\alpha = \frac{3g}{2L}x \downarrow$

PORTION AJ OF ROD:

EXTERNAL FORCES: REACTION A, DISTRIBUTED LOAD PER UNIT LENGTH mg/L, SHEAR V_J, BENDING MOMENT M_J.

EFFECTIVE FORCES: SINCE $a \sim x$, THE EFFECTIVE FORCES ARE LINEARLY DISTRIBUTED. THE EFFECTIVE FORCE PER UNIT LENGTH AT J IS:

$$\frac{m}{L}a_J = \frac{m}{L}\cdot\frac{3g}{2L}x = \frac{3mg}{2L^2}x$$



$+\downarrow \Sigma F_y = \Sigma(F_y)_{eff}$: $\frac{mg}{L}x - \frac{mg}{4} + V_J = \frac{1}{2}\left(\frac{3mg}{2L^2}x\right)x$

$V_J = \frac{mg}{4} - \frac{mg}{L}x + \frac{3}{4}\frac{mg}{L^2}x^2$

$+\circlearrowright \Sigma M_J = \Sigma(M_J)_{eff}$: $\left(\frac{mg}{L}x\right)\frac{x}{2} - \frac{mg}{4}x + M_J = \frac{1}{2}\left(\frac{3mg}{2L^2}x\right)x\left(\frac{x}{3}\right)$

$M_J = \frac{mg}{4}x - \frac{1}{2}\frac{mg}{L}x^2 + \frac{1}{4}\frac{mg}{L^2}x^3$

FIND V_{min}: $\frac{dV_J}{dx} = -\frac{mg}{L} + \frac{3}{2}\frac{mg}{L^2}x = 0$; $x = \frac{2}{3}L$

$V_{min} = \frac{mg}{4} - \frac{mg}{L}\left(\frac{2}{3}L\right) + \frac{3}{4}\frac{mg}{L^2}\left(\frac{2}{3}L\right)^2$; $V_{min} = -\frac{mg}{12}$

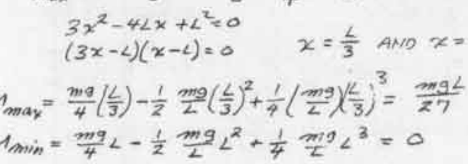
FIND M_{max} WHERE $V_J = 0$: $V_J = \frac{mg}{4} - \frac{mg}{L}x + \frac{3}{4}\frac{mg}{L}x^2 = 0$

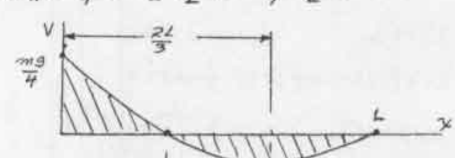
$3x^2 - 4Lx + L^2 = 0$

$(3x - L)(x - L) = 0 \qquad x = \frac{L}{3}$ AND $x = L$

$M_{max} = \frac{mg}{4}\left(\frac{L}{3}\right) - \frac{1}{2}\frac{mg}{L}\left(\frac{L}{3}\right)^2 + \frac{1}{4}\left(\frac{mg}{L}\right)\left(\frac{L}{3}\right)^3 = \frac{mgL}{27}$

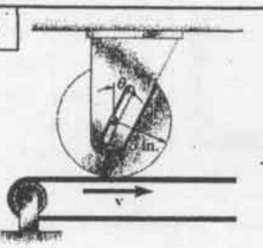
$M_{min} = \frac{mg}{4}L - \frac{1}{2}\frac{mg}{L}L^2 + \frac{1}{4}\frac{mg}{L}L^3 = 0$





$M_{max} = \frac{mgL}{27}$ AT $\frac{L}{3}$ FROM A ◀

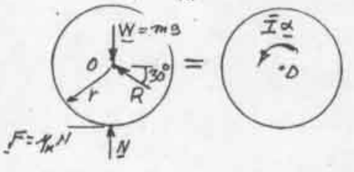
16.153



GIVEN:
$\theta = 30°$
$\mu_k = 0.20$

FIND: α WHILE SLIPPING OCCURS

$r = \frac{5}{12}$ ft



$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$
$\mu_k N - R\cos\theta = 0$
$R\cos\theta = \mu_k N \quad (1)$
$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$
$R\sin\theta + N - mg = 0$
$R\sin\theta = mg - N \quad (2)$

DIVIDE (2) BY (1): $\tan\theta = \frac{mg - N}{\mu_k N}$; $0.5774 = \frac{mg-N}{0.2N}$

$0.1155N = mg - N$; $N = \frac{mg}{1.1155} = 0.8965\,mg$

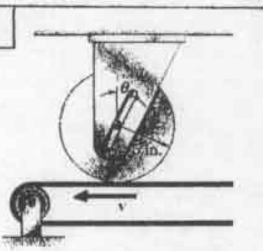
$+\circlearrowright \Sigma M_O = \Sigma (M_O)_{eff}$: $\mu_k N r = \bar{I}\alpha$

$(0.2)(0.8965mg)r = \frac{1}{2}mr^2\alpha$

$\alpha = 0.35858\frac{g}{r} = 0.35858\frac{32.2 \text{ ft/s}^2}{(5/12 \text{ ft})} = 27.71$ rad/s²

$\alpha = 27.7$ rad/s² ↺ ◀

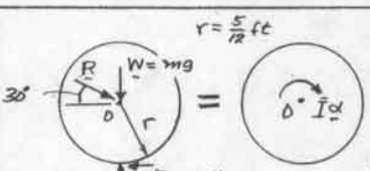
16.154



GIVEN:
$\theta = 30°$
$\mu_k = 0.20$

FIND: α WHILE SLIPPING OCCURS

$r = \frac{5}{12}$ ft



$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$:
$R\cos\theta - \mu_k N = 0$
$R\cos\theta = \mu_k N \quad (1)$
$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$:
$R\sin\theta + mg - N = 0$
$R\sin\theta = N - mg \quad (2)$

DIVIDE (2) BY (1): $\tan\theta = \frac{N - mg}{\mu_k N}$; $0.5774 = \frac{N-mg}{0.2N}$

$0.1155N = N - mg$: $N = \frac{mg}{0.8845} = 1.1306\,mg$

$+\circlearrowright \Sigma M_O = \Sigma (M_O)_{eff}$: $\mu_k N r = \bar{I}\alpha$

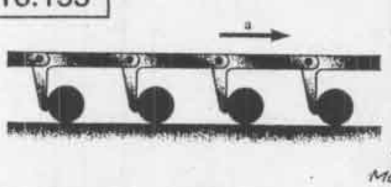
$(0.2)(1.1306mg)r = \frac{1}{2}mr^2\alpha$

$\alpha = 0.4522\frac{g}{r} = 0.4522\frac{32.2 \text{ ft/s}^2}{(5/12 \text{ ft})}$

$\alpha = 34.948$ rad/s²

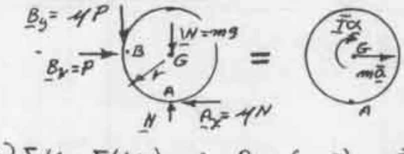
$\alpha = 34.9$ rad/s² ↻ ◀

16.155



GIVEN: CYLINDERS
FIND: (a) MAXIMUM a FOR ROLLING WITH NO SLIDING
(b) MINIMUM a FOR CYLINDER TO MOVE → WITH NO ROTATING

(a) CYLINDER ROLLS WITHOUT SLIDING: $a = r\alpha$ OR $\alpha = \frac{a}{r}$



P IS HORIZ. COMPONENT OF FORCE ARM EXERTS ON CYLINDER

$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$: $Pr - (\mu_k P)r = \bar{I}\alpha + (m\bar{a})r$

$P(1-\mu)r = \frac{1}{2}mr^2\left(\frac{\bar{a}}{r}\right) + (m\bar{a})r$

$P = \frac{3}{2}\frac{m\bar{a}}{(1-\mu)} \quad (1)$

$+\uparrow \Sigma F_y = 0$: $N - \mu P - mg = 0 \quad (2)$
$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $P - \mu N = m\bar{a} \quad (3)$

SOLVE (2) FOR N AND SUBSTITUTE FOR N INTO (3).

$P - \mu^2 P - \mu mg = m\bar{a}$

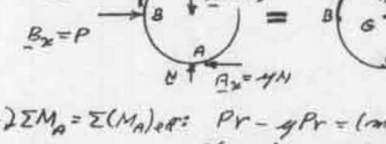
SUBSTITUTE P FROM (1): $(1-\mu^2)\frac{3}{2}\frac{m\bar{a}}{(1-\mu)} - \mu mg = m\bar{a}$

$3(1+\mu)\bar{a} - 2\mu g = 2\bar{a}$

$\bar{a}(1+3\mu) - 2\mu g = 0 \qquad \bar{a} = \frac{2\mu}{1+3\mu}g$ ◀

(b) CYLINDER TRANSLATES: $\alpha = 0$

SLIDING OCCURS AT A: $A_x = \mu N$
ASSUME SLIDING IMPENDS AT B: $B_y = \mu P$



$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$: $Pr - \mu Pr = (m\bar{a})r$

$P(1-\mu)r = m\bar{a}r$

$P = \frac{m\bar{a}}{1-\mu} \quad (4)$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $P - \mu N = m\bar{a} \quad (5)$
$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $N - \mu P - mg = 0 \quad (6)$

SOLVE (5) FOR N AND SUBSTITUTE FOR N INTO (6).

$P - P\mu^2 - \mu mg = m\bar{a}$

SUBSTITUTE FOR P FROM (4):

$\frac{m\bar{a}}{1-\mu}(1-\mu^2) - \mu mg = m\bar{a}$

$\bar{a}(1+\mu) - \mu g = \bar{a}$

$\bar{a}\mu - \mu g = 0$

$\bar{a} = g$ ◀

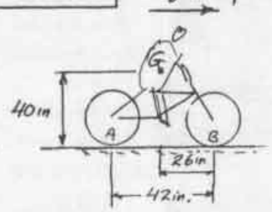
SUMMARY: $a < \frac{2\mu}{1+3\mu}g$: ROLLING

$\frac{2\mu}{1+3\mu}g < a < g$: ROTATING AND SLIDING

$a > g$: TRANSLATION

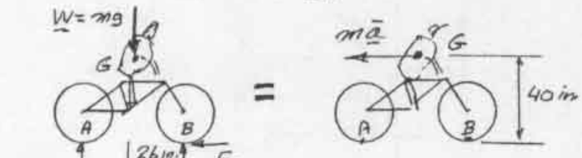
16.156

$v_0 = 20\text{ mph}$



FIND: SHORTEST STOPPING DISTANCE IF CYCLIST IS NOT TO BE THROWN OVER FRONT WHEEL

WHEN CYCLIST IS ABOUT TO BE THROWN OVER THE FRONT WHEEL, $\underline{N}_A = 0$



$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}: \quad mg(26\text{ in.}) = m\bar{a}(40\text{ in.})$

$$a = \frac{26}{40}g = \frac{26}{40}(32.2\text{ ft/s}^2) = 20.93\text{ ft/s}^2$$

UNIFORMLY ACCELERATED MOTION:

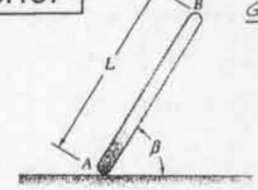
$v_0 = 20\text{ mph} = 29.333\text{ ft/s}^2$

$v^2 - v_0^2 = 2as: \quad 0 - (29.333\text{ ft/s}^2) = 2(-20.93\text{ ft/s}^2)s$

$s = 20.555\text{ ft}$

$$s = 20.6\text{ ft}$$

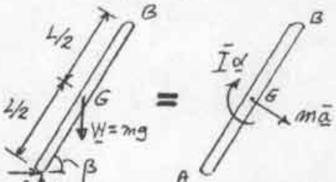
16.157



GIVEN: $\beta = 70°$. UNIFORM ROD RELEASED FROM REST. FRICTION IS SUFFICIENT TO PREVENT SLIDING AT A.

FIND: (a) $\underline{\alpha}$ (b) $\underline{N}_A$, (c) $\underline{F}_A$

WE NOTE ROD ROTATES ABOUT A. $\omega = 0$



$\bar{I} = \frac{1}{12}mL^2$

$\bar{a} = \frac{L}{2}\alpha$

$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}: \quad mg\left(\frac{L}{2}\cos\beta\right) = \bar{I}\alpha + (m\bar{a})\frac{L}{2}$

$$\frac{1}{2}mgL\cos\beta = \frac{1}{12}mL^2\alpha + \left(m\frac{L}{2}\alpha\right)\frac{L}{2}$$

$$= \frac{1}{3}mL^2\alpha$$

$$\alpha = \frac{3}{2}\frac{g\cos\beta}{L} \qquad (1)$$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}: \quad F_A = m\bar{a}\sin\beta$

$$F_A = m\frac{L}{2}\alpha\sin\beta = m\frac{L}{2}\left(\frac{3}{2}\frac{g\cos\beta}{L}\right)\sin\beta$$

$$F_A = \frac{3}{4}mg\sin\beta\cos\beta \qquad (2)$$

(CONTINUED)

16.157 continued

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad N_A - mg = -m\bar{a}\cos\beta = -m\left(\frac{L}{2}\alpha\right)\cos\beta$

$$N_A - mg = -m\frac{L}{2}\left(\frac{3}{2}\frac{g\cos\beta}{L}\right)\cos\beta$$

$$N_A = mg\left(1 - \frac{3}{4}\cos^2\beta\right) \qquad (3)$$

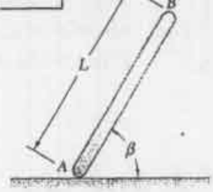
FOR $\beta = 70°$:

(a) EQ.(1): $\alpha = \frac{3}{2}\frac{g\cos 70°}{L}$; $\quad \underline{\alpha} = 0.513\frac{g}{L}\circlearrowright$

(b) EQ.(3): $N_A = mg\left(1 - \frac{3}{4}\cos^2 70°\right)$; $\quad \underline{N}_A = 0.912\,mg\uparrow$

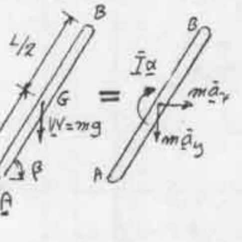
(c) EQ.(2) $F_A = \frac{3}{4}mg\sin 70°\cos 70°$; $\quad \underline{F}_A = 0.241\,mg \rightarrow$

16.158



GIVEN: $\beta = 70°$. UNIFORM ROD RELEASED FROM REST. FRICTION AT SURFACE EQUALS ZERO.

FIND: (a) $\underline{\alpha}$, (b) $\underline{\bar{a}}$, (c) REACTION AT $\underline{A}$.



$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}:$

$0 = m\bar{a}_x \quad \bar{a}_x = 0$

$\underline{a}_A = \underline{\bar{a}}_y + \frac{L}{2}\alpha \measuredangle\beta$

$+\downarrow 0 = a_y - \frac{L}{2}\alpha\cos\beta$

$\bar{a}_y = \frac{L}{2}\alpha\cos\beta$

$\xrightarrow{+} a_A = \frac{L}{2}\sin\beta$

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}:$

$$mg - A = m\bar{a}_y = m\left(\frac{L}{2}\alpha\cos\beta\right) \qquad (1)$$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}:$

$$A\left(\frac{L}{2}\cos\beta\right) = \bar{I}\alpha = \frac{1}{12}mL^2\alpha$$

$$A = \frac{mL}{6}\frac{\alpha}{\cos\beta} \qquad (2)$$

SUBSTITUTE (2) INTO (1):

$$mg - \frac{mL}{6}\frac{\alpha}{\cos\beta} = m\frac{L}{2}\alpha\cos\beta$$

$$g = \left(\frac{L}{2}\cos\beta + \frac{L}{6\cos\beta}\right)\alpha$$

$$g = \frac{L}{6}\left(3\cos\beta + \frac{1}{\cos\beta}\right)\alpha$$

$$g = \frac{L}{6}\left(\frac{3\cos^2\beta + 1}{\cos\beta}\right)\alpha$$

$$\underline{\alpha} = \frac{6g}{L}\left(\frac{\cos\beta}{1 + 3\cos^2\beta}\right)\circlearrowright$$

$$\underline{\bar{a}} = \frac{L}{2}\alpha\cos\beta = \frac{L}{2}\left(\frac{6g}{L}\cdot\frac{\cos\beta}{1 + 3\cos^2\beta}\right)\cos\beta = 3g\left(\frac{\cos^2\beta}{1 + 3\cos^2\beta}\right)\downarrow$$

$$\underline{A} = \frac{mL}{6}\cdot\frac{\alpha}{\cos\beta} = \frac{mL}{6}\cdot\left(\frac{6g}{L}\cdot\frac{\cos\beta}{1 + 3\cos^2\beta}\right)\frac{1}{\cos\beta} = mg\frac{1}{1 + 3\cos^2\beta}\uparrow$$

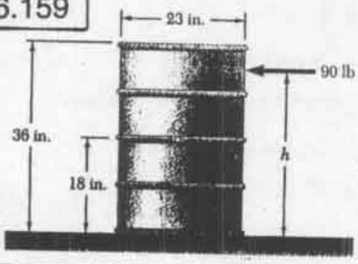
FOR $\beta = 70°$:

(a) $\alpha = \frac{6g}{L}\frac{\cos 70°}{1 + 3\cos^2 70°}$; $\quad \underline{\alpha} = 1.519\frac{g}{L}\circlearrowright$

(b) $\bar{a} = 3g\frac{\cos^2 70}{1 + 3\cos^2 70°}$; $\quad \underline{\bar{a}} = 0.260\,g\downarrow$

(c) $A = mg\frac{1}{1 + 3\cos^2 70°}$; $\quad \underline{A} = 0.740\,mg\uparrow$

16.159

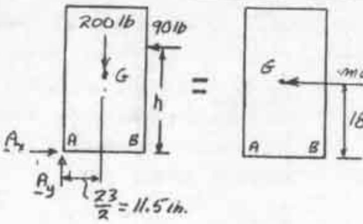


GIVEN: $W = 200$ lb

$\mu_s = 0.40$, $\mu_k = 0.35$.

FIND: (a) $\bar{a}$

(b) RANGE OF h FOR WHICH BARREL WILL NOT TIP.



WEIGHT = 200 lb

FOR TIPPING ABOUT A IMPENDING REACTION IS AT A

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $A_y - 200\text{ lb} = 0$; $A_y = 200\text{ lb}\uparrow$

FOR SLIDING: $A_x = \mu_k A_y = 0.35(200) = 70\text{ lb} \rightarrow$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $90\text{ lb} - A_x = m\bar{a}$

$$90\text{ lb} - 70\text{ lb} = \frac{200\text{ lb}}{g}\bar{a}$$

$$\bar{a} = \frac{20\text{ lb}}{200\text{ lb}} g = 0.1g = 0.1(32.2) \qquad \bar{a} = 3.22\text{ ft/s}^2 \leftarrow$$

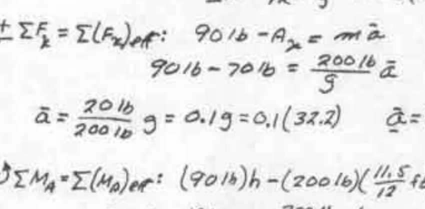
$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{eff}$: $(90\text{ lb})h - (200\text{ lb})\left(\frac{11.5}{12}\text{ ft}\right) = m\bar{a}\left(\frac{18}{12}\text{ ft}\right)$

$$90h - 191.67 = \frac{200\text{ lb}}{32.2}(3.22\text{ ft/s}^2)(1.5\text{ ft})$$

$$90h - 191.67 = 30; \quad 90h = 221.67$$

$$h = 2.463\text{ ft} \qquad h = 29.6\text{ in.}$$

FOR TIPPING IMPENDING ABOUT B, REACTION IS AT B



$B_x = \mu_k B_y = 0.35(200) = 70\text{ lb}$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: SAME AS ABOVE: $\bar{a} = 3.22\text{ ft/s}^2 \leftarrow$

$+\circlearrowleft \Sigma M_B = \Sigma (M_B)_{eff}$: $(90h) + (200\text{ lb})\left(\frac{11.5}{12}\text{ ft}\right) = m\bar{a}\left(\frac{18}{12}\text{ ft}\right)$

$$90h + 191.67 = \frac{200\text{ lb}}{32.2}(3.22\text{ ft/s}^2)(1.5\text{ ft})$$

$$90h + 191.67 = 30$$

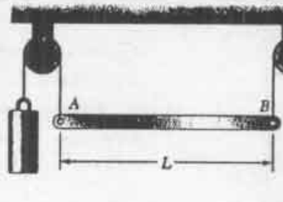
$$90h = -161.67$$

$$h < 0 \quad \text{IMPOSSIBLE}$$

THUS RANGE FOR NO TIPPING IS

$$h < 29.6\text{ in}$$

16.160



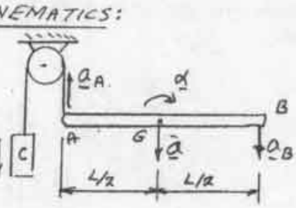
GIVEN: WEIGHTS BAR AB: W

COUNTERWEIGHTS $= \frac{1}{2}W$.

IMMEDIATELY AFTER WIRE AT B IS CUT.

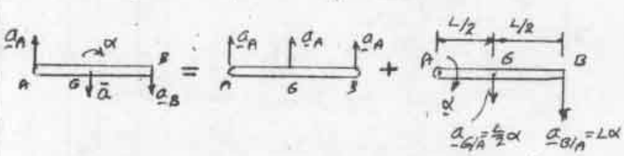
FIND: (a) a_A, (b) a_B.

KINEMATICS:



$\omega = 0$

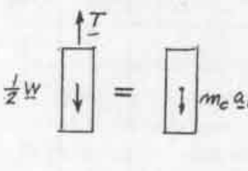
$a_C = a_A$



$$[\bar{a}\downarrow] = [a_A\uparrow] + [\tfrac{L}{2}\alpha\downarrow] \qquad [a_B\downarrow] = [a_A\uparrow] + [L\alpha\downarrow]$$

$$\bar{a} = (\tfrac{1}{2}L\alpha - a_A)\downarrow \qquad a_B = (L\alpha - a_A)\downarrow$$

KINETICS: COUNTERWEIGHT $\quad m$ = MASS OF BAR AB



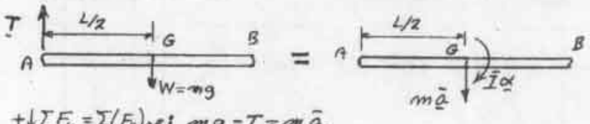
$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$:

$$\tfrac{1}{2}W - T = m_C a_C = \tfrac{1}{2} m a_A$$

$$\tfrac{1}{2}mg - T = \tfrac{1}{2} m a_A$$

$$T = \tfrac{1}{2} m (g - a_A) \qquad (1)$$

KINETICS BAR AB



$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $mg - T = m\bar{a}$

$$mg - \tfrac{1}{2}m(g - a_A) = m(\tfrac{1}{2}L\alpha - a_A)$$

$$2g - g + a_A = L\alpha - 2a_A \qquad g + 3a_A = L\alpha \qquad (2)$$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$: $T\frac{L}{2} = \bar{I}\alpha$

$$\tfrac{1}{2}m(g - a_A)\tfrac{L}{2} = \tfrac{1}{12} mL^2\alpha$$

$$3g - 3a_A = L\alpha \qquad (3)$$

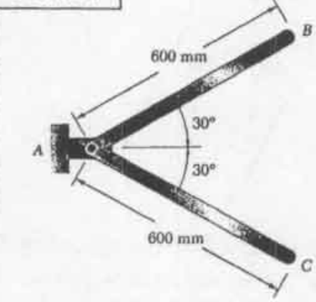
ADD EQS. (2) AND (3): $4g = 2L\alpha \qquad \alpha = \frac{2g}{L}$

SUBSTITUTE INTO (2): $g + 3a_A = L\left(\frac{2g}{L}\right)$; $a_A = \frac{1}{3}g\uparrow$

$\bar{a} = (\frac{1}{2}L\alpha - a_A) = \frac{1}{2}L\left(\frac{2g}{L}\right) - \frac{1}{3}g$; $\quad \bar{a} = \frac{2}{3}g\downarrow$

$a_B = (L\alpha - a_A) = L\left(\frac{2g}{L} - \frac{1}{3}g\right)$; $\quad a_B = \frac{5}{3}g\downarrow$

16.161

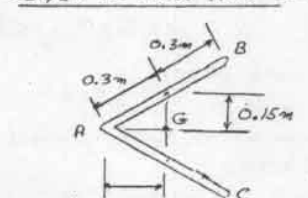


GIVEN: $m_{AB} = m_{AC} = m = 3\ kg$

SYSTEM RELEASED FROM REST

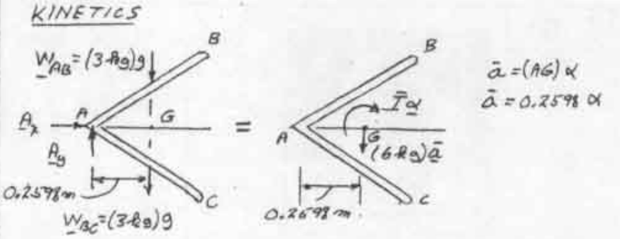
FIND: (a) $\underline{a}_B$ (b) $\underline{A}$

CENTER OF MASS AND $\bar{I}$:



$$\bar{I} = 2\left[\bar{I}_{AB} + m_{AB}(0.15\,m)^2\right]$$
$$\bar{I} = 2\left[\tfrac{1}{12}(3\,kg)(0.6\,m)^2 + (3\,kg)(0.15\,m)^2\right]$$
$$\bar{I} = 2\left[0.09\ kg\cdot m^2 + 0.0675\ kg\cdot m^2\right]$$
$$\bar{I} = 0.315\ kg\cdot m^2$$

KINETICS



$\bar{a} = (AG)\alpha$
$\bar{a} = 0.2598\,\alpha$

$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$:

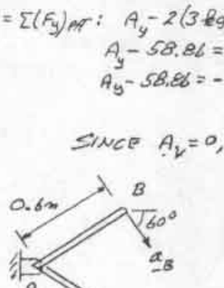
$$2(3\,kg)(9.81\,m/s^2)(0.2598\,m) = \bar{I}\alpha + (m\bar{a})(0.2598\,m)$$
$$15.292 = (0.315\,kg\cdot m^2)\alpha + (6\,kg)(0.2598\,m)^2\alpha$$
$$15.292 = 0.720\ \alpha$$
$$\underline{\alpha} = 21.24\ rad/s^2 \circlearrowright$$
$$\bar{a} = 0.2598(21.24) = 5.518\ m/s^2$$

$\xrightarrow{+} \Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $A_y - 2(3\,kg)(9.81\,m/s^2) = -(6\,kg)\bar{a}$

$$A_y - 58.86 = -(6\,kg)(5.518\,m/s^2)$$
$$A_y - 58.86 = -33.11 \qquad A_y = 25.75\ N$$

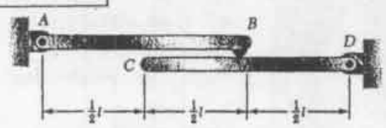
SINCE $A_x = 0$, $\quad \underline{A} = 25.8\ N \uparrow$ ◀



$$a_B = (0.6\,m)\alpha = (0.6\,m)(21.24\ rad/s^2)$$

$$\underline{a}_B = 12.74\ m/s^2 \ \measuredangle 60°$$ ◀

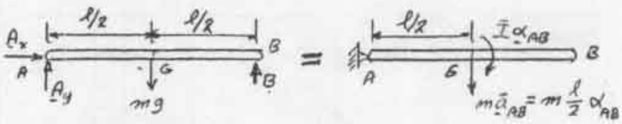
16.162



GIVEN: RODS OF MASS m RELEASED FROM REST

FIND: (a) $\underline{a}_C$, (b) $\underline{B}$

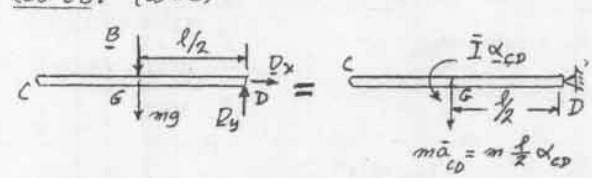
ROD AB: $(\omega = 0)$



$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}$:

$$mg\left(\tfrac{l}{2}\right) - Bl = \bar{I}\alpha + m\bar{a}_{AB}\left(\tfrac{l}{2}\right)$$
$$\tfrac{1}{2}mgl - Bl = \tfrac{1}{12}ml^2\alpha_{AB} + m\left(\tfrac{l}{2}\alpha_{AB}\right)\tfrac{l}{2}$$
$$\tfrac{1}{2}mgl - Bl = \tfrac{1}{3}ml^2\alpha_{AB} \qquad (1)$$

ROD CD: $(\omega = 0)$



$+\circlearrowleft \Sigma M_D = \Sigma (M_D)_{eff}$:

$$mg\left(\tfrac{l}{2}\right) + B\left(\tfrac{l}{2}\right) = \bar{I}\alpha_{CD} + m\,a_{CD}\tfrac{l}{2}$$
$$\tfrac{1}{2}mgl + \tfrac{1}{2}Bl = \tfrac{1}{12}ml^2\alpha_{CD} + m\left(\tfrac{l}{2}\alpha_{CD}\right)\tfrac{l}{2}$$

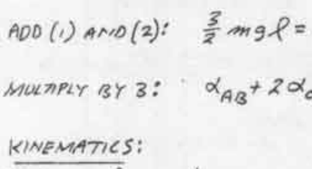
MULTIPLY BY 2:

$$mgl + Bl = \tfrac{2}{3}ml^2\alpha_{CD} \qquad (2)$$

ADD (1) AND (2): $\quad \tfrac{3}{2}mgl = ml^2\left(\tfrac{1}{3}\alpha_{AB} + \tfrac{2}{3}\alpha_{CD}\right) \qquad (3)$

MULTIPLY BY 3: $\quad \alpha_{AB} + 2\alpha_{CD} = \tfrac{9}{2}\tfrac{g}{l} \qquad (4)$

KINEMATICS:



WE MUST HAVE

$$(a_B \downarrow) = (a_{B'} \downarrow)$$
$$l\alpha_{AB} = \tfrac{l}{2}\alpha_{CD}$$
$$\alpha_{AB} = \tfrac{1}{2}\alpha_{CD} \qquad (5)$$

SUBSTITUTE FOR α_{AB} FROM (5) INTO (4)

$$\tfrac{1}{2}\alpha_{CD} + 2\alpha_{CD} = \tfrac{9}{2}\tfrac{g}{l}$$
$$\tfrac{5}{2}\alpha_{CD} = \tfrac{9}{2}\tfrac{g}{l}; \qquad \alpha_{CD} = 1.8\tfrac{g}{l} \qquad (6)$$

(a) ACCELERATION OF C:

$$a_C = l\alpha_{CD} = l\left(1.8\tfrac{g}{l}\right); \quad \underline{a}_C = 1.8g\downarrow$$ ◀

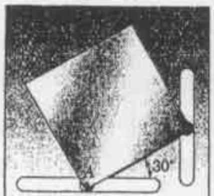
(b) FORCE ON KNOB B:

SUBSTITUTE FOR α_{CD} FROM (6) INTO (2)

$$mgl + Bl = \tfrac{2}{3}ml^2\left(1.8\tfrac{g}{l}\right)$$
$$B = 1.2\,mg - mg$$

(ON ROD AB): $\underline{B} = 0.2\,mg\uparrow$ ◀

16.163

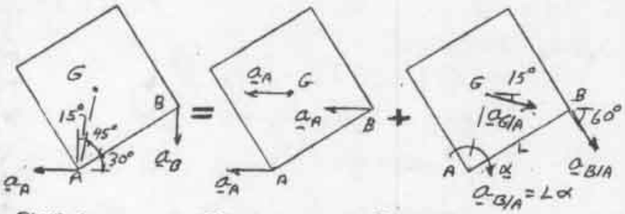


GIVEN: SQUARE PLATE OF SIDE $L = 150$ mm AND $m = 2.5$ kg IS RELEASED FROM REST

FIND: (a) $\underline{\alpha}$ (b) $\underline{A}$

KINEMATICS:

$AG = \frac{L}{2}\sqrt{2} = \frac{L}{\sqrt{2}} \qquad a_{G/A} = (AG)\alpha = \frac{L\alpha}{\sqrt{2}}$

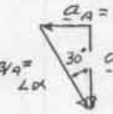


$\underline{a}_{B/A} = L\alpha$

PLANE MOTION = TRANSLATION + ROTATION

$\underline{a}_B \downarrow = a_A \leftarrow + a_{B/A} \measuredangle 60^\circ$

$\underline{a}_B \downarrow = \underline{a}_A \leftarrow + L\alpha \measuredangle 60^\circ$



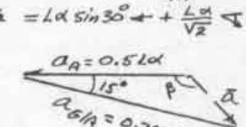
$a_A = L\alpha \sin 30^\circ$

$a_{B/A} = L\alpha$

$a_B = L\alpha \cos 30^\circ$

$\bar{a} = a_A \leftarrow + a_{G/A} \measuredangle 15^\circ$

$\bar{a} = L\alpha \sin 30^\circ \leftarrow + \frac{L\alpha}{\sqrt{2}} \measuredangle 15^\circ = 0.5L\alpha \leftarrow + 0.707L\alpha \measuredangle 15^\circ$



LAW OF COSINES

$\bar{a}^2 = a_A^2 + a_{G/A}^2 - 2a_A a_{G/A} \cos 15^\circ$

$\bar{a}^2 = (0.5L\alpha)^2 + (0.707L\alpha)^2 - 2(0.5L\alpha)(0.707L\alpha)\cos 15^\circ$

$\bar{a}^2 = L^2\alpha^2(0.25 + 0.5 - 0.68301)$

$\bar{a}^2 = L^2\alpha^2(0.06699)\,; \qquad \bar{a} = 0.25882L\alpha$

LAW OF SINES

$\frac{\bar{a}}{\sin 15^\circ} = \frac{a_{G/A}}{\sin\beta}\,; \quad \sin\beta = \frac{a_{G/A}}{\bar{a}}\sin 15^\circ = \frac{0.707L\alpha}{0.25882L\alpha}\sin 15^\circ$

$\sin\beta = 0.707\,; \quad \beta = 135^\circ$

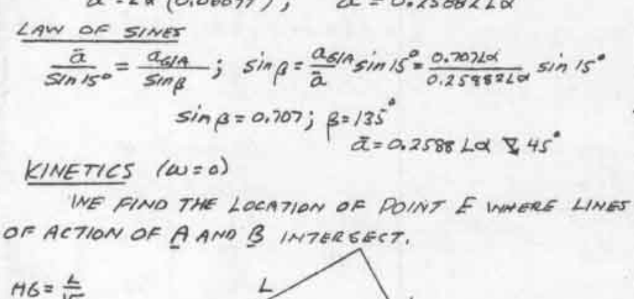
$\bar{a} = 0.2588\,L\alpha \measuredangle 45^\circ$

KINETICS ($\omega = 0$)

WE FIND THE LOCATION OF POINT E WHERE LINES OF ACTION OF $\underline{A}$ AND $\underline{B}$ INTERSECT.

$AG = \frac{L}{\sqrt{2}}$

$\angle EAG = 15^\circ$



$(AG)\cos 15^\circ = \frac{L}{\sqrt{2}}\cos 15^\circ = 0.683L$

$(EG)_x = (AG)\sin 15^\circ = \frac{L}{\sqrt{2}}\sin 15^\circ = 0.183L$

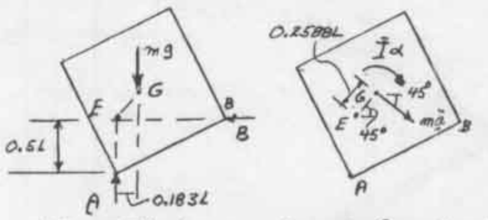
$(EG)_y = 0.6829L - 0.5L = 0.183L$

$EG = (0.183L)\sqrt{2} = 0.2588L$

(CONTINUED)

16.163 continued

$\bar{I} = \frac{1}{6}mL^2$



$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}: \quad mg(0.183L) = \bar{I}\alpha + (m\bar{a})(0.2588L)$

$0.183mgL = \frac{1}{6}mL^2\alpha + m(0.2588L\alpha)(0.2588L)$

$0.183gL = L^2\alpha\left(\frac{1}{6} + 0.06698\right)$

$0.183\frac{g}{L} = 0.2336\alpha\,; \quad \alpha = 0.7834\frac{g}{L}$

$\alpha = 0.7834\,\frac{9.81\text{ m/s}^2}{0.15\text{ m}} \qquad \underline{\alpha} = 51.2\text{ rad/s}^2 \circlearrowright$ ◀

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad A - mg = -m\bar{a}\sin 45^\circ$

$= -m(0.2588\,L\alpha)\sin 45^\circ$

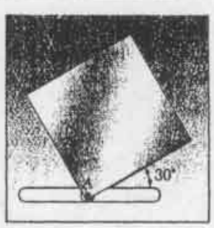
$= -m(0.2588L)(0.7834\frac{g}{L})\sin 45^\circ$

$A - mg = 0.1434mg$

$A = 0.8566mg = 0.8566(2.5\text{ kg})(9.81\text{ m/s}^2) = 21.01\text{N}$

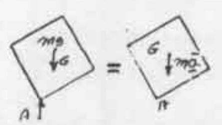
$\underline{A} = 21.0\text{N}\uparrow$ ◀

16.164



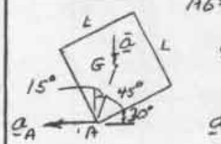
GIVEN: SQUARE PLATE OF SIDE $L = 150$ mm AND $m = 2.5$ kg IS RELEASED FROM REST.

FIND: (a) $\underline{\alpha}$ (b) $\underline{A}$



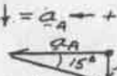
SINCE BOTH $\underline{A}$ AND mg ARE VERTICAL, $\bar{a}_x = 0$ AND $\bar{a}$ IS $\downarrow$

KINEMATICS



$AG = \frac{L}{\sqrt{2}} \; \measuredangle 15^\circ \qquad a_{G/A} = (AG)\alpha \measuredangle 15^\circ$

$\underline{\bar{a}}\downarrow = \underline{a}_A \leftarrow + \underline{a}_{G/A} \measuredangle 15^\circ$



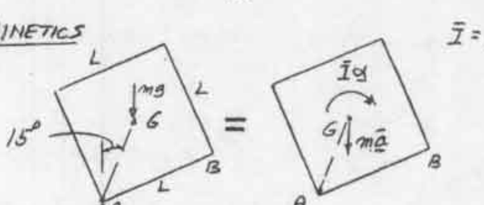
$\underline{a}_{G/A} = \frac{L}{\sqrt{2}}\alpha$

$\underline{\bar{a}} = \frac{L\alpha}{\sqrt{2}}\sin 15^\circ$

$\underline{\bar{a}} = 0.183L\alpha \measuredangle 15^\circ$

KINETICS

$\bar{I} = \frac{1}{6}mL^2$



$+\circlearrowright \Sigma M_A = \Sigma (M_A)_{eff}: \quad mg(AG)\sin 15^\circ = \bar{I}\alpha + m\bar{a}(AG)\sin 15^\circ$

$mg\left(\frac{L}{\sqrt{2}}\right)\sin 15^\circ = \frac{1}{6}mL^2\alpha + m(0.183L\alpha)\left(\frac{L}{\sqrt{2}}\right)\sin 15^\circ$

$0.183\frac{g}{L} = \left(\frac{1}{6} + 0.033494\right)$

$0.183\frac{g}{L} = 0.2002\alpha\,; \quad \alpha = 0.9143\frac{g}{L} = 0.9143\,\frac{9.81\text{ m/s}^2}{0.15\text{ m}}$

$\underline{\alpha} = 59.8\text{ rad/s}^2 \circlearrowright$ ◀

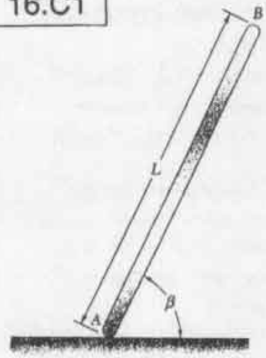
$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}: \quad A - mg = -m\bar{a}$

$A - mg = -m(0.183L\alpha) = -m(0.183L)(0.9143\frac{g}{L})$

$A - mg = -0.1673mg\,; \quad A = 0.8326\,mg$

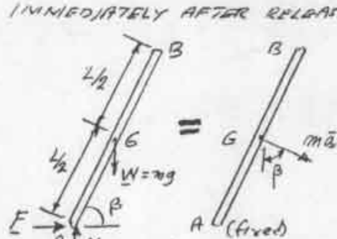
$A = 0.8326(2.5\text{ kg})(9.81\text{ m/s}^2)\,; \qquad \underline{A} = 20.4\text{ N}\uparrow$ ◀

16.C1



GIVEN: $W = 5\,lb$
ROD AB RELEASED FROM REST.
(a) FOR NO SLIPPING AT A, FIND N_A AND F_A IMMEDIATELY AFTER RELEASE FOR $\beta = 0$ TO $85°$ USING $5°$ INCREMENTS.
(b) FOR $\mu_s = 0.50$, FIND RANGE OF VALUES OF β FOR WHICH ROD WILL SLIP IMMEDIATELY AFTER RELEASE.

WE NOTE THAT ROD ROTATES ABOUT A AND THAT IMMEDIATELY AFTER RELEASE $\omega = 0$.



$\bar{I} = \frac{1}{12} mL^2$

$\bar{a} = \frac{L}{2}\alpha$

$+\circlearrowleft \Sigma(M_A) = \Sigma(M_A)_{eff}$: $mg\left(\frac{L}{2}\cos\beta\right) = \bar{I}\alpha + m\bar{a}\left(\frac{L}{2}\right)$

$$\frac{1}{2} mgL\cos\beta = \frac{1}{12} mL^2\alpha + m\left(\frac{L}{2}\alpha\right)\frac{L}{2}$$
$$= \frac{1}{3} mL^2\alpha$$
$$\alpha = \frac{3}{2}\cdot\frac{g}{L}\cos\beta \qquad (1)$$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$: $F = m\bar{a}\sin\beta$

$$F = m\frac{L}{2}\alpha\sin\beta = m\frac{L}{2}\left(\frac{3}{2}\frac{g}{L}\cos\beta\right)\sin\beta$$
$$F = \frac{3}{4} mg\sin\beta\cos\beta \qquad (2)$$

$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $N - mg = m\bar{a}\cos\beta$

$$= m\frac{L}{2}\alpha\cos\beta = m\frac{L}{2}\left(\frac{3}{2}\frac{g}{L}\cos\beta\right)\cos\beta$$
$$N = mg\left(1 - \frac{3}{4}\cos^2\beta\right) \qquad (3)$$

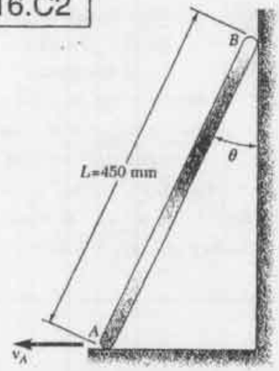
OUTLINE OF PROGRAM:

(a) FOR $\beta = 0$ TO $85°$ AT $5°$ INCREMENTS, DETERMINE F (from EQ(2)) AND N (from EQ.(3)). ALSO DETERMINE REQUIRED VALUE OF $\mu = F/N$

(b) USE SMALLER INCREMENTS TO FIND TWO VALUES OF β CORRESPONDING TO $\mu_s = 0.50$.

beta	F	N	mu	?slip?
0.000	0.000	1.250	0.000	no slip
5.000	0.326	1.278	0.255	no slip
10.000	0.641	1.363	0.470	no slip
15.000	0.938	1.501	0.624	slip
20.000	1.205	1.689	0.714	slip
25.000	1.436	1.920	0.748	slip
30.000	1.624	2.188	0.742	slip
35.000	1.762	2.484	0.709	slip
40.000	1.847	2.799	0.660	slip
45.000	1.875	3.125	0.600	slip
50.000	1.847	3.451	0.535	slip
55.000	1.762	3.766	0.468	no slip
60.000	1.624	4.063	0.400	no slip
65.000	1.436	4.330	0.332	no slip
70.000	1.205	4.561	0.264	no slip
75.000	0.938	4.749	0.197	no slip
80.000	0.641	4.887	0.131	no slip
85.000	0.326	4.972	0.065	no slip
------Seek start of range------				
10.810	0.691	1.382	0.500	no slip
10.820	0.691	1.382	0.500	slip
------Seek end of range------				
52.620	1.809	3.618	0.500	slip
52.630	1.809	3.618	0.500	no slip
52.640	1.809	3.619	0.500	no slip

16.C2

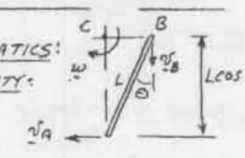


GIVEN: $m = 5\,kg$
$v_A = 1.5\,m/s \leftarrow$
$a_A = 0$,

FIND:
NORMAL REACTIONS AT A AND B FOR $\theta = 0$ TO $50°$ USING $5°$ INCREMENTS.
VALUE OF θ AT WHICH END B LOSES CONTACT WITH WALL.

KINEMATICS:

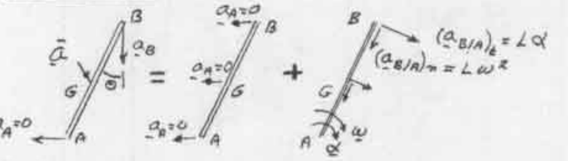
VELOCITY:



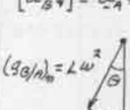
INSTANT CTR. AT C.

$$\omega = \frac{v_A}{L\cos\theta} \qquad (1)$$

ACCELERATION



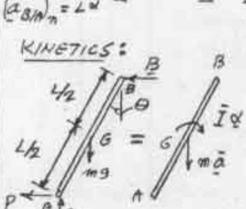
$$[a_B\downarrow] = a_A + [(a_{B/A})_t \measuredangle \theta] + [(a_{B/A})_n \measuredangle \theta]$$



$$a_B = \frac{L\omega^2}{\cos\theta}\downarrow$$

$$\bar{a} = \frac{1}{2}(a_A + a_B) = \frac{1}{2}a_B\,;\ \bar{a} = \frac{L\omega^2}{2\cos\theta} \qquad (2)$$

KINETICS:



$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$:
$A - mg = m\bar{a}$
$$A = m(g - \bar{a}) \qquad (3)$$

$\bar{I} = \frac{1}{12} mL^2$

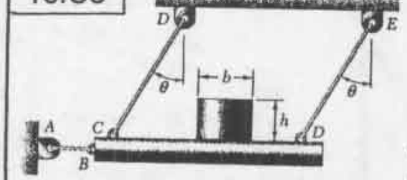
$+\circlearrowleft \Sigma M_A = \Sigma(M_A)_{eff}$: $B(L\cos\theta) - mg\left(\frac{L}{2}\sin\theta\right) = -\bar{I}\alpha - m\bar{a}\left(\frac{L}{2}\sin\theta\right)$

$$B = \frac{m(g-\bar{a})\frac{L}{2}\sin\theta + \frac{1}{12}mL^2\alpha}{L\cos\theta} \qquad (4)$$

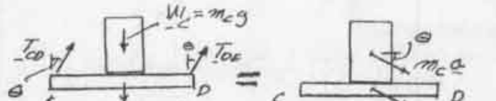
OUTLINE OF PROGRAM: DATA $m = 5\,kg$, $L = 0.45\,m$.
FOR EACH VALUE OF θ EVALUATE ω AND $\bar{a}$, THEN USE ω AND $\bar{a}$ TO EVALUATE A AND B.
USING SMALLER INCREMENTS FIND VALUE OF θ FOR WHICH $B = 0$.

theta deg.	omega rad/s	alpha rad/s²	$\bar{a}$ m/s²	A N	B N
0.000	3.333	0.000	2.500	36.550	0.000
5.000	3.346	0.980	2.529	36.406	1.408
10.000	3.385	2.020	2.617	35.963	2.786
15.000	3.451	3.191	2.774	35.180	4.094
20.000	3.547	4.580	3.013	33.986	5.271
25.000	3.678	6.308	3.358	32.259	6.216
30.000	3.849	8.553	3.849	29.805	6.752
35.000	4.069	11.595	4.548	26.309	6.557
40.000	4.351	15.888	5.561	21.243	5.024
45.000	4.714	22.222	7.071	13.695	0.955
50.000	5.186	32.049	9.413	1.984	-8.166
----	Find theta for B = 0		------------		
45.747	4.777	23.420	7.357	12.265	0.002
45.748	4.777	23.422	7.357	12.264	0.001
45.749	4.777	23.423	7.358	12.262	-0.001

16.C3



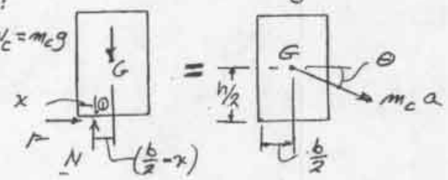
GIVEN: $b=8$ in, $h=6$ in.
30-lb CYLINDER
10-lb PLATFORM
AFTER AB IS CUT,
FIND: μ_{min} FOR WHICH CYLINDER DOES NOT SLIP FOR $\theta=0$ TO $30°$ USING $5°$ INCREMENTS. THEN FOR $\mu_s=0.60$, FIND θ FOR WHICH SLIPPING IMPENDS. IN ALL CASES, CHECK WHETHER CYLINDER TIPS.



$+\searrow \Sigma F = \Sigma F_{eff}$: $(m_c+m_p)g\sin\theta = (m_c+m_p)a$

$a = g\sin\theta$

CYLINDER:



RESULTANT OF FORCES EXERTED BY PLATFORM ON TO CYLINDER ACTS AT DISTANCE x FROM CORNER.

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $F = m_c a\cos\theta$

$+\uparrow \Sigma F_y = \Sigma (F_y)_{eff}$: $N - m_c g = -m_c a\sin\theta$

$N = m_c(g - a\sin\theta)$

$$(\mu_s) = \frac{F}{N} = \frac{\cancel{m_c}\, a\cos\theta}{\cancel{m_c}(g-a\sin\theta)} = \frac{(g\sin\theta)\cos\theta}{g-(g\sin\theta)\sin\theta}$$

$$\mu_s = \frac{\sin\theta\cos\theta}{1-\sin^2\theta}$$

$+\circlearrowleft \Sigma M_{\textcircled{\scriptsize 0}} = \Sigma (M_0)_{eff}$: $\cancel{m_c}g\left(\frac{b}{2}-x\right) = \cancel{m_c}a\cos\theta\left(\frac{h}{2}\right) + \cancel{m_c}a\sin\theta\left(\frac{b}{2}-x\right)$

$\cancel{g}\left(\frac{b}{2}-x\right) = (\cancel{g}\sin\theta)\cos\theta\,\frac{h}{2} + (\cancel{g}\sin\theta)\sin\theta\left(\frac{b}{2}-x\right)$

$\left(\frac{b}{2}-x\right)(1-\sin^2\theta) = \frac{1}{2}\sin\theta\cos\theta$; $\left(\frac{b}{2}-x\right)\cos^2\theta = \frac{1}{2}\sin\theta\cos\theta$

$\frac{b}{2}-x = \frac{h}{2}\frac{\sin\theta}{\cos\theta}$; $\quad x = \frac{1}{2}(b - h\tan\theta)$

CYLINDER TIPS IF $x \le 0$; $\tan\theta \ge \frac{b}{h} = \frac{8\text{ in}}{6\text{ in}}$; $\theta \ge 53.1°$

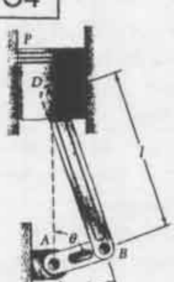
OUTLINE OF PROGRAM For $b = 8$ in AND $h = 6$ in.,
EVALUATE μ_s AND x FOR EACH VALUE OF θ. PRINT μ_s AS MINIMUM VALUE OF μ_s FOR NO SLIDING.

theta	x	mu req.	?slip?	?tip?
0.000	4.000	0.000	no slip	no tip
5.000	3.738	0.087	no slip	no tip
10.000	3.471	0.176	no slip	no tip
15.000	3.196	0.268	no slip	no tip
20.000	2.908	0.364	no slip	no tip
25.000	2.601	0.466	no slip	no tip
30.000	2.268	0.577	no slip	no tip
35.000	1.899	0.700	slips	no tip

-- Find theta for mu = 0.60 --------------------------------

30.960	2.200	0.5999
30.980	2.199	0.6004

16.C4

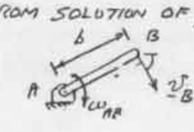


GIVEN: ENGINE SYSTEM OF PROB 15.C3.
$\omega_{AB} = 1000$ rpm $\circlearrowright$, $\alpha_{AB}=0$
$l = 160$ mm, $b = 60$ mm
$m_p = 2.5$ kg, $m_{BD} = 3$ kg

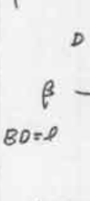
FIND: COMPONENTS OF DYNAMIC REACTIONS ON BD AT POINTS B AND D FOR $\theta = 0$ TO $180°$ USING $10°$ INCREMENTS.

VELOCITY: FROM SOLUTION OF PROB 16.C3

$$\beta = \sin^{-1}\frac{b\sin\theta}{l} \quad (1)$$

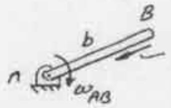


$v_B = b\,\omega_{AB}\ \searrow\theta \quad (2)$

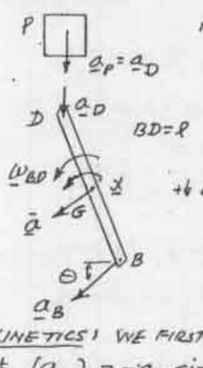


$$\omega_{BD} = \frac{v_B}{l}\cdot\frac{\cos\theta}{\cos\beta} \quad (3)$$

ACCELERATION:



$a_B = b\,\omega_{AB}^2 \measuredangle\theta \quad (4)$



BD = l

$$\alpha = \frac{l\omega_{BD}^2\sin\beta - a_B\cos\theta}{l\cos\beta} \quad (5)$$

$$+\downarrow a_D = a_B\cos\theta + l\omega_{BD}^2\cos\beta + l\alpha_{BD}\sin\beta \quad (6)$$

KINETICS: WE FIRST FIND $\bar{a}_x$ AND $\bar{a}_y$:

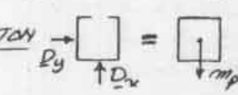
$\xrightarrow{+} (a_B)_x = -a_B\sin\theta$ AND $+\downarrow (a_B)_y = a_B\cos\theta$

SINCE G IS AT THE MIDDLE OF BD

$$\xrightarrow{+} \bar{a}_x = \tfrac{1}{2}(a_B)_x \quad (7)$$

$$+\downarrow \bar{a}_y = \tfrac{1}{2}\left[(a_B)_y + a_D\right] \quad (8)$$

PISTON



$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$

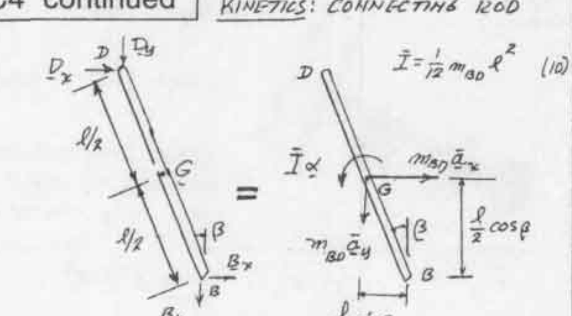
$$D_y = m_p a_D \quad (9)$$

NOTE: SINCE WE SEEK THE DYNAMIC REACTIONS, WE OMIT THE WEIGHT OF THE PISTON AND CONNECTING ROD

(CONTINUED)

16.C4 continued

KINETICS: CONNECTING ROD



$$\bar{I} = \frac{1}{12} m_{BD} \ell^2 \qquad (10)$$

$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$:

$$D_x \ell \cos\beta - D_y \ell \sin\beta = -\bar{I}\alpha + m_{BD} a_x \left(\frac{\ell}{2}\cos\beta\right) - m_{BD}\left(\frac{\ell}{2}\sin\beta\right)$$

DIVIDE BY ℓ SOLVE FOR D_x

$$D_x = D_y \frac{\sin\beta}{\cos\beta} - \frac{\bar{I}\alpha}{\ell\cos\beta} + \frac{m_{BD}}{2}\bar{a}_x - \frac{m_{BD}}{2}\bar{a}_y \frac{\sin\beta}{\cos\beta}$$

$$D_x = D_y \tan\beta - \frac{\bar{I}\alpha}{\ell\cos\beta} + \frac{m_{BD}}{2}\bar{a}_x - \frac{m_{BD}}{2}\bar{a}_y \tan\beta \qquad (11)$$

$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$:

$$B_x + D_x = m_{BD}\bar{a}_x$$

$$B_x = m_{BD}\bar{a}_x - D_x \qquad (12)$$

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$:

$$B_y + D_y = m_{BD}\bar{a}_y$$

$$B_y = m_{BD}\bar{a}_y - D_y \qquad (13)$$

OUTLINE OF PROGRAM:

ENTER DATA: $\omega_{AB} = 1000\,\text{rpm}\left(\frac{2\pi}{60}\right) = \frac{100}{3}\pi$ rad/s

$m_p = 2.5$ kg, $m_{BD} = 3$ kg

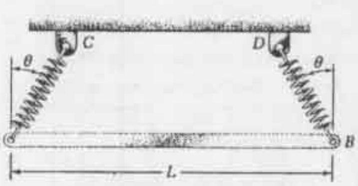
$\ell = 0.1$ m, $b = 0.06$ m

PROGRAM, IN SEQUENCE, EQS. (1) THROUGH (13).
EVALUATE AND PRINT B_x, B_y, D_x, AND D_y FOR VALUES OF θ FROM 0 TO 180° AT 5° INCREMENTS:

Positive directions of force components are:
DOWN and TO THE RIGHT

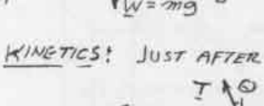
theta deg	Bx N	By N	Dx N	Dy N
0	0.00	4605.82	0.00	-2261.78
10	108.19	4497.37	-279.57	-2203.38
20	182.74	4177.66	-520.30	-2031.39
30	194.59	3663.87	-688.07	-1755.71
40	124.19	2985.61	-758.59	-1393.47
50	-33.57	2185.52	-722.48	-969.45
60	-265.48	1318.43	-589.25	-515.59
70	-539.76	447.34	-387.68	-68.61
80	-811.62	-364.52	-160.35	334.94
90	-1034.81	-1064.65	47.85	665.41
100	-1174.86	-1621.34	202.89	906.22
110	-1217.65	-2028.11	290.21	1056.59
120	-1169.20	-2300.43	314.47	1129.34
130	-1048.31	-2466.79	292.25	1145.24
140	-877.31	-2558.80	242.90	1126.72
150	-675.57	-2604.17	182.09	1093.40
160	-456.95	-2623.56	119.39	1060.08
170	-230.09	-2630.38	58.71	1036.51
180	-0.00	-2631.89	0.00	1028.08

16.C5



GIVEN: UNIFORM BAR OF MASS m SUPPORTED BY SPRINGS OF CONSTANT k. IMMEDIATELY AFTER SPRING AC BREAKS
FIND: a_A AND a_B FOR VALUES OF θ FROM 0 TO 90°, USING 10° INCREMENTS

STATICS: INITIAL SPRING TENSIONS

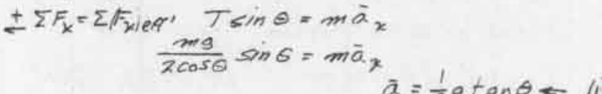


$$+\uparrow \Sigma F_y = 0$$

$$2T\cos\theta - mg = 0$$

$$T = \frac{mg}{2\cos\theta}$$

KINETICS: JUST AFTER AC BREAKS



$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{eff}$: $\quad T\sin\theta = m\bar{a}_x$

$$\frac{mg}{2\cos\theta}\sin\theta = m\bar{a}_x$$

$$\bar{a}_x = \frac{1}{2} g \tan\theta \leftarrow \qquad (1)$$

$+\downarrow \Sigma F_y = \Sigma (F_y)_{eff}$: $\quad mg - T\cos\theta = m\bar{a}_y$

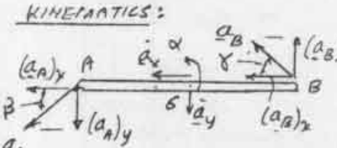
$$mg - \frac{mg}{2\cos\theta}\cos\theta = m\bar{a}_y$$

$$\bar{a}_y = \frac{1}{2} g \downarrow \qquad (2)$$

$+\circlearrowright \Sigma M_G = \Sigma (M_G)_{eff}$: $\quad (T\cos\theta)\frac{L}{2} = \bar{I}\alpha$

$$\frac{mg}{2\cos\theta}\cos\theta \frac{L}{2} = \frac{1}{12} m L^2 \alpha \qquad \alpha = \frac{3g}{L} \circlearrowright \qquad (3)$$

KINEMATICS:



$$(a_A)_x = \bar{a}_x = \frac{1}{2} g \tan\theta \qquad (4)$$

$$(a_B)_x = \bar{a}_x = \frac{1}{2} g \tan\theta \qquad (5)$$

$$+\downarrow (a_A)_y = \bar{a}_y + \frac{L}{2}\alpha = \frac{1}{2}g + \frac{L}{2}\left(\frac{3g}{L}\right) = 2g \downarrow \qquad (6)$$

$$+\uparrow (a_B)_y = -\bar{a}_y + \frac{L}{2}\alpha = -\frac{1}{2} + \frac{L}{2}\left(\frac{3g}{L}\right) = g \uparrow \qquad (7)$$

END A: $\quad \beta = \tan^{-1}\frac{(a_A)_y}{(a_A)_x}; \quad a_A = \frac{(a_A)_x}{\cos\beta} \qquad (8, 9)$

END B: $\quad \gamma = \tan^{-1}\frac{(a_B)_y}{(a_B)_x}; \quad a_B = \frac{(a_B)_x}{\cos\gamma} \qquad (10, 11)$

OUTLINE OF PROGRAM:

PROGRAM, IN SEQUENCE, EQS. (1) THROUGH (11).
EVALUATE AND PRINT a_A, β, a_B, γ FOR VALUES OF θ FROM 0 TO 90° USING 10° INCREMENTS.

theta	[$\frac{aA}{g}$	beta]	[$\frac{aB}{g}$	gamma]
0.000	2.000	90.000	1.000	90.000
10.000	2.002	87.476	1.004	84.962
20.000	2.008	84.801	1.016	79.686
30.000	2.021	81.787	1.041	73.898
40.000	2.044	78.153	1.084	67.240
50.000	2.087	73.409	1.164	59.210
60.000	2.179	66.587	1.323	49.107
70.000	2.426	55.516	1.699	36.052
80.000	3.470	35.196	3.007	19.425
90.000	infinite	0.000	infinite	0.000

17.1 GIVEN: 6000-lb FLYWHEEL, $\bar{k} = 36$ in., $\omega_0 = 300$ rpm.

FIND: MAGNITUDE OF COUPLE DUE TO FRICTION KNOWING FLYWHEEL ROTATES 1500 REVOLUTIONS WHILE COASTING TO REST.

$\omega_0 = 300 \text{ rpm}\left(\frac{2\pi}{60}\right) = 10\pi$ rad

$\bar{I} = m\bar{k}^2 = \frac{6000 \text{ lb}}{32.2 \text{ ft/s}^2}(3 \text{ ft})^2 = 1677 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$T_1 = \frac{1}{2}\bar{I}\omega_0^2 = \frac{1}{2}(1677)(10\pi)^2 = 827{,}600 \text{ ft}\cdot\text{lb}, \quad T_2 = 0$

$U_{1\to2} = -M\theta = -M(1500 \text{ rev})\left(2\pi \frac{\text{rad}}{\text{rev}}\right) = -9424.7M$

$T_1 + U_{1\to2} = T_2$: $827{,}600 - 9424.7M = 0$

$M = 87.81 \text{ lb}\cdot\text{ft}$ $\qquad M = 87.8 \text{ lb}\cdot\text{ft}$

17.2 GIVEN: 50-kg ROTOR, $\bar{k} = 180$ mm, $\omega_0 = 3600$ rpm, $M_f = 3.5$ N·m

FIND: NUMBER OF REVOLUTIONS AS ROTOR COASTS TO REST

$\omega_0 = 3600 \text{ rpm}\left(\frac{2\pi}{60}\right) = 120\pi$ rad/s

$\bar{I} = m\bar{k}^2 = (50 \text{ kg})(0.180 \text{ m})^2 = 1.620 \text{ kg}\cdot\text{m}^2$

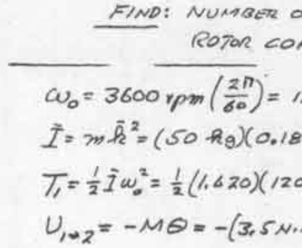
$T_1 = \frac{1}{2}\bar{I}\omega_0^2 = \frac{1}{2}(1.620)(120\pi)^2 = 115.12 \text{ kJ}, \quad T_2 = 0$

$U_{1\to2} = -M\theta = -(3.5 \text{ N}\cdot\text{m})\theta$

$T_1 + U_{1\to2} = T_2$: $115.12 \text{ kJ} - (3.5 \text{ N}\cdot\text{m})\theta = 0$

$\theta = 32.891 \times 10^3$ rad $\qquad \theta = 5230$ rev

17.3 GIVEN: 8-lb DISK OF 9-in. DIAMETER, ROD AB WEIGHS 3 lb/ft, $M = 4$ lb·ft



FIND: LENGTH L IF ω IS 300 rpm AFTER 2 REVOLUTIONS

$r = 4.5 \text{ in.} = \frac{3}{8}$ ft: $\qquad \omega = 300 \text{ rpm}\left(\frac{2\pi}{60}\right) = 10\pi$ rad/s

$W_{DISK} = 8 \text{ lb}, \quad W_{ROD} = (3 \text{ lb/ft})L$

$\bar{I} = \frac{1}{2}m_{DISK}r^2 + \frac{1}{12}m_{ROD}L^2$

$= \frac{1}{2}\frac{8}{g}\left(\frac{3}{8}\right)^2 + \frac{1}{12}\frac{3L}{g}L^2 = \frac{1}{g}\left(\frac{9}{16} + \frac{L^3}{4}\right)$

$T_1 = 0, \quad T_2 = \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2g}\left(\frac{9}{16} + \frac{L^2}{4}\right)(10\pi)^2$

$U_{1\to2} = M\theta = (4 \text{ lb}\cdot\text{ft})(2 \text{ rev})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = 16\pi$

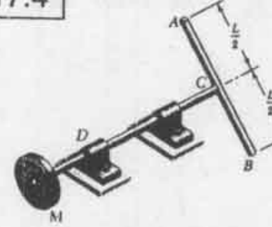
$T_1 + U_{1\to2} = T_2$: $0 + 16\pi = \frac{1}{2g}\left(\frac{9}{16} + \frac{L^3}{4}\right)(10\pi)^2$

$\frac{16\pi(2g)}{(10\pi)^2} = \frac{9}{16} + \frac{L^3}{4}$

$3.2799 = \frac{9}{16} + \frac{L^3}{4}; \quad \frac{L^3}{4} = 2.717$

$L^3 = 10.869 \text{ ft}^3 \qquad L = 2.22$ ft

17.4 GIVEN: $\omega_0 = 0$, $\bar{I}_{DISK} = I_0$, w = WEIGHT / UNIT LENGTH OF ROD



FIND: LENGTH L FOR MAXIMUM v_A AFTER COUPLE M IS APPLIED FOR ONE REVOLUTION

$T_1 = 0 \qquad T_2 = \frac{1}{2}\left(I_0 + \frac{1}{12}\frac{wL}{g}L^2\right)\omega_2^2$

$U_{1\to2} = M\theta = M(2\pi \text{ rad})$

$T_1 + U_{1\to2} = T_2$: $0 + 2\pi M = \frac{1}{2}\left(I_0 + \frac{wL^3}{12g}\right)\omega_2^2$

$$\omega_2^2 = \frac{4\pi M}{I_0 + \frac{wL^3}{12g}}$$

$v_A = \frac{L}{2}\omega_2$: $\quad v_A^2 = \frac{L^2}{4}\omega_2^2 = \frac{\pi M L^2}{I_0 + \frac{wL^3}{12g}}$

DIFFERENTIATING WITH RESPECT TO L,

$$2v_A\left(\frac{dv_A}{dL}\right) = \left[2L\left(I_0 + \frac{wL^3}{12g}\right) - L^2\left(\frac{3wL^2}{12g}\right)\right]\left[\frac{\pi M}{\left(I_0 + \frac{wL^3}{12g}\right)^2}\right]$$

$\frac{dv_A}{dL} = 0$: $\quad 2L\left(I_0 + \frac{wL^3}{12g}\right) - L^2\left(\frac{3wL^2}{12g}\right) = 0$

$2I_0L - \frac{wL^4}{12g}$: $\qquad L^3 = \frac{24g\,I_0}{w}$

17.5 GIVEN: 300-kg PUNCHING MACHINE FLYWHEEL, $\bar{k} = 600$ mm. $\omega_1 = 300$ rpm. EACH PUNCH REQUIRES 2500 J.

FIND: (a) ω_2 IMMEDIATELY AFTER A PUNCH (b) IF $M = 25$ N·m, FIND REVOLUTIONS BEFORE ω IS AGAIN 300 rpm.

$\bar{I} = m\bar{k}^2 = (300 \text{ kg})(0.6 \text{ m})^2 = 108 \text{ kg}\cdot\text{m}^2$

(a) $\omega_1 = 300 \text{ rpm}\left(\frac{2\pi}{60}\right) = 10\pi$ rad/s

$T_1 = \frac{1}{2}m\omega_1^2 = \frac{1}{2}(108 \text{ kg}\cdot\text{m}^2)(10\pi \text{ rad/s})^2$

$T_1 = 53.296$ kJ

$U_{1\to2} = -2500 \text{ J} = 2.5$ kJ

$T_2 = \frac{1}{2}m\omega_2^2 = \frac{1}{2}(108 \text{ kg}\cdot\text{m}^2)\omega_2^2$

$T_1 + U_{1\to2} = T_2$: $53.296 \text{ kJ} - 2.5 \text{ kJ} = \frac{1}{2}(108 \text{ kg}\cdot\text{m}^2)\omega_2^2$

$\omega_2 = 30.67 \text{ rad/s}\left(\frac{60}{2\pi}\right) = 292.9$ rpm

$\omega_2 = 293$ rpm

(b) $U_{2\to1} = M\theta$

$2500 \text{ J} = (25 \text{ N}\cdot\text{m})\,\theta$

$\theta = 100 \text{ rad}\left(\frac{\text{rev}}{2\pi \text{ rad}}\right) = 15.9155$ rev

$\theta = 15.92$ rev

17.6 GIVEN: $\omega_1 = 360$ rpm OF PUNCHING MACHINE FLYWHEEL. EACH PUNCH REQUIRES 1500 ft·lb. AFTER EACH PUNCH $\omega_2 = 0.95\,\omega_1$.
FIND: (a) $\bar{I}$ OF FLYWHEEL
(b) REVOLUTIONS REQUIRED FOR ANGULAR VELOCITY TO AGAIN BE 360 rpm IF CONSTANT 18 lb·ft COUPLE IS APPLIED

(a)
$\omega_1 = 360\text{ rpm}\left(\frac{2\pi}{60}\right) = 12\pi$ rad/s
$\omega_2 = 0.95\,\omega_1 = 0.95(12\pi \text{ rad/s}) = 11.4\pi$ rad/s
$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}\bar{I}(12\pi)^2$
$T_2 = \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\bar{I}(11.4\pi)^2$ $\qquad U_{1\to2} = -1500$ ft·lb

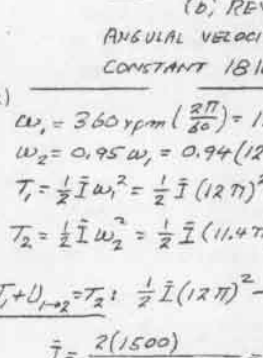
$T_1 + U_{1\to2} = T_2$: $\frac{1}{2}\bar{I}(12\pi)^2 - 1500 = \frac{1}{2}\bar{I}(11.4\pi)^2$

$$\bar{I} = \frac{2(1500)}{\pi^2(12^2 - 11.4^2)} = \frac{3000}{138.57} = 21.649 \text{ lb·ft·s}^2$$

$\bar{I} = 21.6$ lb·ft·s² ◀

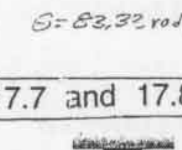
(b) $U_{2\to1} = M\theta$: $1500\text{ lb·ft} = (18\text{ ft·lb})\theta$
$\theta = 83.33\text{ rad}\left(\frac{\text{rev}}{2\pi\text{ rad}}\right) = 13.263$ rev
$\theta = 13.26$ rev ◀

17.7 and 17.8



GIVEN: DISH PLACED ON BELT WHEN $\omega_0 = 0$. COEFFICIENT OF KINETIC FRICTION $= \mu_s$.
FIND: REVOLUTIONS BEFORE ω = CONSTANT.
PROBLEM 17.7: IN TERMS OF v, r, AND μ_s
PROBLEM 17.8: FOR $r = 6$ in, $v = 40$ ft/s, AND $\mu_s = 0.20$.

ONLY FORCE DOING WORK IS F. SINCE ITS MOMENT ABOUT A IS $M = rF$, WE HAVE



$U_{1-2} = M\theta = rF\theta = r(\mu_s mg)\theta$

ANGULAR VELOCITY BECOMES CONSTANT WHEN $\omega_2 = \frac{v}{r}$

$T_1 = 0$

$T_2 = \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{mv^2}{4}$

$T_1 + U_{1-2} = T_2$: $0 + r\mu_s mg\theta = \frac{mv^2}{4}$

$\theta = \frac{v^2}{4r\mu_s g}$ rad $\qquad \theta = \frac{v^2}{8\pi r\mu_s g}$ rev ◀

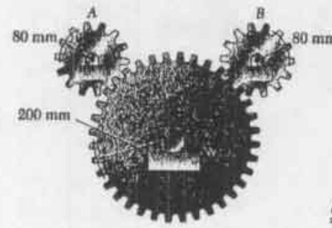
PROBLEM 17.8: $r = 0.5$ ft, $\mu_s = 0.20$, $v = 40$ ft/s

$$\theta = \frac{(40\text{ ft/s})^2}{8\pi(0.5\text{ ft})(0.20)(32.2\text{ ft/s}^2)}$$

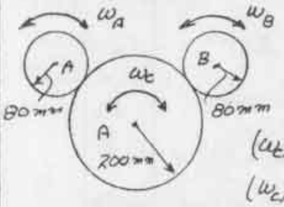
$\theta = 19.77$ rev ◀

NOTE: RESULT IS INDEPENDENT OF W.

17.9 and 17.10



GIVEN: $m_C = 12$ kg, $\bar{k}_C = 150$ mm
$m_A = m_B = 2.4$ kg
$\bar{k}_A = \bar{k}_B = 60$ mm
$M = 10$ N·m
FIND: (a) REVOLUTIONS OF C AS ω_C INCREASES FROM 100 rpm TO 450 rpm
(b) TANGENTIAL FORCE ON A
PROBLEM 17.9
M IS APPLIED TO GEAR C
PROBLEM 17.10
M IS APPLIED TO GEAR B



KINEMATICS:

$\omega_A = \omega_B = \frac{200\text{ mm}}{80\text{ mm}}\omega_C = 2.5\,\omega_C$

$(\omega_C)_1 = 100\text{ rpm}\left(\frac{2\pi}{60}\right) = 10.472$ rad/s
$(\omega_C)_2 = 450\text{ rpm}\left(\frac{2\pi}{60}\right) = 47.124$ rad/s

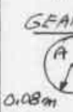
WORK AND ENERGY:
$\bar{I}_A = \bar{I}_B = m\bar{k}^2 = (2.4\text{ kg})(0.06\text{ m})^2 = 8.64\times10^{-3}$ kg·m²
$\bar{I}_C = m\bar{k}^2 = (12\text{ kg})(0.150\text{ m})^2 = 0.270$ kg·m²

POSITION 1: $(\omega_C)_1 = 10.472$ rad/s;
$(\omega_A)_1 = (\omega_B)_1 = 2.5(\omega_C)_1 = 26.18$ rad/s
$T_1 = 2\left[\frac{1}{2}\bar{I}_A(\omega_A)_1^2\right] + \frac{1}{2}\bar{I}_C(\omega_C)_1^2$
$= 2\left[\frac{1}{2}(8.64\times10^{-3})(26.18)^2\right] + \frac{1}{2}(0.270)(10.472)^2 = 20.726$ J

POSITION 2: $(\omega_C)_2 = 47.124$ rad/s
$(\omega_A)_2 = (\omega_B)_2 = 2.5(\omega_C)_2 = 117.81$ rad/s
$T_2 = 2\left[\frac{1}{2}(8.64\times10^{-3})(117.81)^2\right] + \frac{1}{2}(0.270)(47.124)^2 = 419.71$ J

PROBLEM 17.9: $M = 10$ N·m APPLIED TO GEAR C
$U_{1\to2} = M\theta_C = 10\,\theta_C$:
$T_1 + U_{1\to2} = T_2$: $20.726\text{ J} + 10\theta_C = 419.71$ J
$\theta_C = 39.90$ rad $\qquad \theta_C = 6.35$ rev ◀

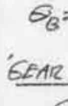
GEAR A: $\theta_A = 2.5\,\theta_C = 2.5(39.90) = 99.75$ rad



$T_1 + U_{1-2} = T_2$: $\frac{1}{2}m_A(\omega_A)_1^2 + F(0.08)\theta_A = \frac{1}{2}m_A(\omega_A)_2^2$
$\frac{1}{2}(8.64\times10^{-3})(26.18)^2 + F(0.08)(99.75) = \frac{1}{2}(8.64\times10^{-3})(117.81)^2$
$2.961 + 7.98F = 59.96$
$F = 7.14$ N $\qquad F = 7.14$ N ↑ ◀

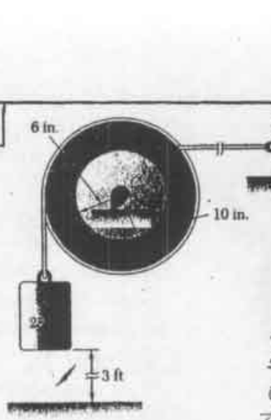
PROBLEM 17.10: $M = 10$ N·m APPLIED TO GEAR B
NOTE: ANGULAR SPEEDS ARE SAME AS IN PROB 17.9, THUS T_1 AND T_2 ARE ALSO THE SAME
$T_1 = 20.726$ J $\qquad T_2 = 419.71$ J
WE HAVE $U_{1-2} = M\theta_B = 10\,\theta_B$
$T_1 + U_{1-2} = T_2$: $20.726\text{ J} + 10\,\theta_B = 419.71$ J
$\theta_B = 39.90$ rad
$\theta_B = 2.5\theta_C$; $39.90\text{ rad} = 2.5\theta_C$; $\theta_C = 15.96$ rad
$\theta_C = 2.54$ rev ◀

GEAR A: $\theta_A = \theta_B = 39.90$ rad



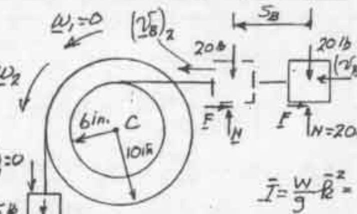
$T_1 + U_{1-2} = T_2$: $\frac{1}{2}m_A(\omega_A)_1^2 + F(0.08)\theta_A = \frac{1}{2}m_A(\omega_A)_2^2$
$\frac{1}{2}(8.64\times10^{-3})(26.18)^2 + F(0.08)(39.90) + \frac{1}{2}(8.64\times10^{-3})(117.81)^2$
$2.961 + 3.192F = 59.96$
$F = 17.86$ N $\qquad F = 17.86$ N ↙ ◀

17.11



GIVEN: 30-lb PULLEY
$\bar{k} = 6.5$ in.
$\mu_k = 0.25$
SYSTEM IS RELEASED FROM REST

FIND: (a) v_A AS IT STRIKES THE GROUND
(b) TOTAL DISTANCE THAT BLOCK B MOVES



$s_B = \frac{6 \text{ in.}}{10 \text{ in.}} h = \frac{6}{10}(3 \text{ ft})$

$s_B = 1.8$ ft

$F = \mu N = 0.25(20 \text{ lb})$

$F = 5$ lb

$\bar{I} = \frac{W}{g}\bar{k}^2 = \frac{30 \text{ lb}}{32.2}\left(\frac{6.5}{12} \text{ ft}\right)^2$

$\bar{I} = 0.27336$ lb·ft·s^2

$v_A = \omega\left(\frac{10}{12} \text{ ft}\right) = \frac{5}{6}\omega$

$v_B = \omega\left(\frac{6}{12} \text{ ft}\right) = \frac{1}{2}\omega$

(a)

$T_1 = 0$: $\quad T_2 = \frac{1}{2}m_A(v_A)_2^2 + \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m_B(v_B)_2^2$

$= \frac{1}{2}\frac{25 \text{ lb}}{32.2}\left(\frac{5}{6}\omega_2\right)^2 + \frac{1}{2}(0.27336)\omega_2^2 + \frac{1}{2}\frac{20 \text{ lb}}{32.2}\left(\frac{1}{2}\omega_2\right)^2$

$= 0.26958\,\omega_2^2 + 0.13668\,\omega_2^2 + 0.07764\,\omega_2^2$

$T_2 = 0.48390\,\omega_2^2$

$U_{1\to2} = W_A h - F(s_B) = (25 \text{ lb})(3 \text{ ft}) - (5 \text{ lb})(1.8 \text{ ft})$

$U_{1\to2} = 66$ ft·lb

$T_1 + U_{1\to2} = T_2$: $\quad 0 + 66 \text{ ft·lb} = 0.48390\,\omega_2^2$

$\omega_2^2 = 136.39 \qquad \omega_2 = 11.679$ rad/s

$(v_A)_2 = \frac{5}{6}\omega_2 = \frac{5}{6}(11.679) \qquad (v_A)_2 = 9.73$ ft/s $\downarrow$ ◀

(b) BLOCK B COASTS TO REST

TOTAL ENERGY OF BLOCKS AND PULLEY JUST BEFORE IMPACT = 66 ft·lb

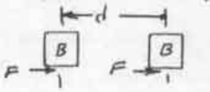
KINETIC ENERGY OF BLOCK A JUST BEFORE IMPACT

$T_A = \frac{1}{2}\frac{W_A}{g}(v_A)_2^2 = \frac{1}{2}\frac{25 \text{ lb}}{32.2}(9.73 \text{ ft/s})^2 = 36.75$ ft·lb

AFTER BLOCK A STRIKES THE GROUND, WE FIND THAT THE KINETIC ENERGY OF THE PULLEY C AND BLOCK B IS

$T_{C+B} = 66 \text{ ft·lb} - 36.75 \text{ ft·lb} = 29.25$ ft·lb

FOR SYSTEM TO STOP, 29.25 ft·lb OF ENERGY MUST BE DISSIPATED BY THE FRICTION FORCE, F = 5 lb.

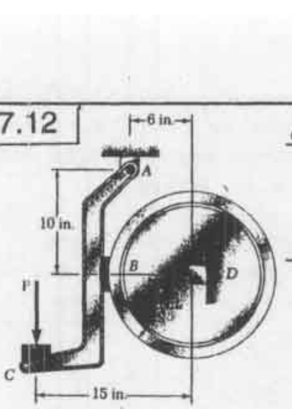


$29.25 \text{ ft·lb} = (5 \text{ lb})d$

$d = 5.85$ ft

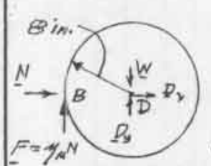
TO FIND TOTAL DISTANCE MOVED BY B, WE ADD $s_B = 1.8$ ft. TOTAL DISTANCE = 1.8 + 5.85 = 7.65 ft ◀

17.12



GIVEN: $\bar{I} = 14$ lb·ft·s^2
$\omega_1 = 360$ rpm
$\mu_k = 0.35$

FIND: P SO THAT FLYWHEEL STOPS IN 100 REVOLUTIONS.



$\omega_1 = 360 \text{ rpm}\left(\frac{2\pi}{60}\right) = 12\pi$ rad/s

$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}(14 \text{ lb·ft·s}^2)(12\pi \text{ rad/s})^2$

$T_1 = 9948.6$ ft·lb $\qquad T_2 = 0$

$\theta = 100 \text{ rev}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = 628.32$ rad

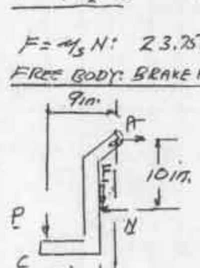
$U_{1\to2} = -M\theta = -Fr\theta = -F\left(\frac{2}{3} \text{ ft}\right)(628.32 \text{ rad})$

$T_1 + U_{1\to2} = T_2$: $\quad 9948.6 - F\left(\frac{2}{3}\right)(628.32) = 0$

$F = 23.75$ lb

$F = \mu_k N$: $\quad 23.75 \text{ lb} = (0.35)N$; $\quad N = 67.86$ lb

FREE BODY: BRAKE AC;



$+\circlearrowright \Sigma M_A = 0$

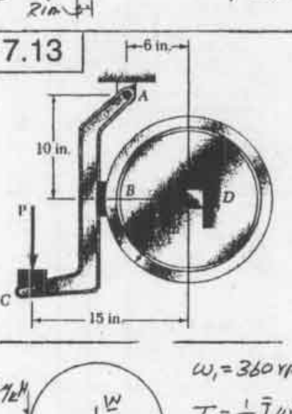
$P(9 \text{ in.}) + F(2 \text{ in.}) - N(10 \text{ in.}) = 0$

$9P + (23.75)(2) - (67.86)(10) = 0$

$9P - 631.1 = 0$

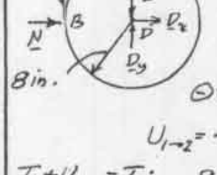
$P = 70.12$ lb $\qquad P = 70.1$ lb $\downarrow$ ◀

17.13



GIVEN: $\bar{I} = 14$ lb·ft·s^2
$\omega_1 = 360$ rpm
$\mu_k = 0.35$

FIND: P SO THAT FLYWHEEL STOPS IN 100 REVOLUTIONS



$\omega_1 = 360 \text{ rpm}\left(\frac{2\pi}{60}\right) = 12\pi$ rad/s

$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}(14 \text{ lb·ft·s}^2)(12\pi \text{ rad/s})^2$

$T_1 = 9948.6$ ft·lb $\qquad T_2 = 0$

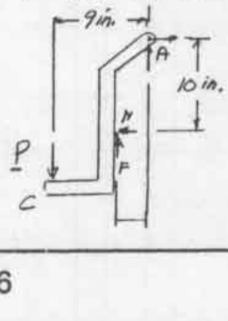
$\theta = 100 \text{ rev}\left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = 628.32$ rad

$U_{1\to2} = -M\theta = -Fr\theta = -F\left(\frac{2}{3} \text{ ft}\right)(628.32 \text{ rad})$

$T_1 + U_{1\to2} = T_2$: $\quad 9948.6 - F\left(\frac{2}{3}\right)(628.32) = 0$

$F = 23.75$ lb

$F = \mu_k N$: $\quad 23.75 \text{ lb} = 0.35N$; $\quad N = 67.86$ lb



FREE BODY: BRAKE AC

$+\circlearrowright \Sigma M_A = 0$

$P(9 \text{ in.}) - F(2 \text{ in.}) - N(10 \text{ in.}) = 0$

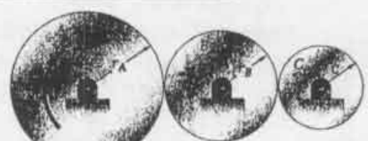
$9P - (23.75)(2) - (67.86)(10) = 0$

$9P - 726.1 = 0$

$P = 80.68$ lb $\qquad P = 80.7$ lb $\downarrow$ ◀

17.14 and 17.15

GIVEN: FRICTION DISKS A, B, AND C ARE MADE OF SAME MATERIAL AND HAVE SAME THICKNESS

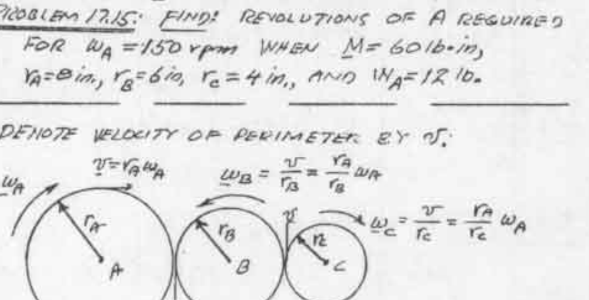


PROBLEM 17.14: FIND: EXPRESSION FOR ω_A AFTER THE COUPLE $\underline{M}$ IS APPLIED FOR ONE REVOLUTION

PROBLEM 17.15: FIND: REVOLUTIONS OF A REQUIRED FOR $\omega_A = 150$ rpm WHEN $\underline{M} = 60$ lb·in, $r_A = 8$ in, $r_B = 6$ in, $r_C = 4$ in., AND $W_A = 12$ lb.

DENOTE VELOCITY OF PERIMETER BY v:

$v = r_A \omega_A$ $\quad \omega_B = \frac{v}{r_B} = \frac{r_A}{r_B}\omega_A$ $\quad \omega_C = \frac{v}{r_C} = \frac{r_A}{r_C}\omega_A$



DENOTE MASS DENSITY OF MATERIAL BY ρ AND THICKNESS OF DISKS BY t.

THEN MASS OF A DISK IS $m = (\text{VOLUME})\rho = (\pi r^2 t)\rho$

AND $\bar{I} = \frac{1}{2} m r^2 = \frac{\pi \rho t}{2} r^4$

KINETIC ENERGY: $T = \sum \frac{1}{2} \bar{I} \omega^2$

$$T = \frac{1}{2}\left(\frac{\pi \rho t}{2}\right)\left[r_A^4 \omega_A^2 + r_B^4 \omega_B^2 + r_C^4 \omega_C^2\right]$$

$$\frac{1}{2}\left(\frac{\pi \rho t}{2}\right)\left[r_A^4 \omega_A^2 + r_B^4\left(\frac{r_A}{r_B}\right)^2 \omega_A^2 + r_C^4\left(\frac{r_A}{r_C}\right)^2 \omega_A^2\right]$$

$$T = \frac{1}{2}\left(\frac{\pi \rho t \omega_A^2}{2}\right) r_A^2 \left[r_A^2 + r_B^2 + r_C^2\right] \qquad (1)$$

WORK: $U_{1-2} = M\theta$ $\qquad \omega_1 = 0;\ \omega_2 = \omega_A$

$T_1 + U_{1-2} = T_2$: $\quad 0 + M\theta = \frac{\pi \rho t}{4}\omega_A^2 r_A^2\left[r_A^2 + r_B^2 + r_C^2\right]$

PROBLEM 17.14 FOR $\theta = 2\pi$: $M(2\pi) = \frac{\pi \rho t}{4}\omega_A^2 r_A^4\left[1 + \left(\frac{r_B}{r_A}\right)^2 + \left(\frac{r_C}{r_A}\right)^2\right]$

$$\omega_A^2 = \frac{8 M_0}{\rho t\, r_A^4\left[1 + \left(\frac{r_B}{r_A}\right)^2 + \left(\frac{r_C}{r_A}\right)^2\right]}$$

PROBLEM 17.15: RECALL THAT $m_A = \pi r_A^2 t \rho$ AND WRITE EQ.(1) AS:

$$T = \frac{1}{4}(\pi r_A^2 t \rho)(r_A^2 + r_B^2 + r_C^2)$$

$$T = \frac{1}{4}\left(\frac{W_A}{g}\right) r_A^2\left[1 + \left(\frac{r_B}{r_A}\right)^2 + \left(\frac{r_C}{r_A}\right)^2\right]\omega_A^2$$

DATA: $\omega_A = 150 \text{ rpm}\left(\frac{2\pi}{60}\right) = 5\pi$ rad/s

$W_A = 12$ lb, $r_A = 8$ in., $r_B = 6$ in., $r_C = 4$ in.,

$M = 60$ lb·in. $= 5$ ft·lb

$U_{1-2} = M\theta = (5 \text{ ft·lb})\theta$

$T_1 + U_{1-2} = T_2$:

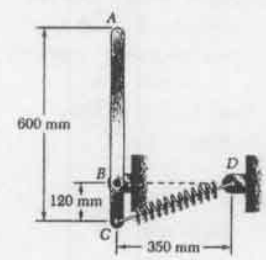
$$0 + 5\theta = \frac{1}{4}\left(\frac{12 \text{ lb}}{32.2}\right)\left(\frac{8}{12}\text{ft}\right)^2\left[1 + \left(\frac{6 \text{in}}{8 \text{in}}\right)^2 + \left(\frac{4 \text{in}}{8 \text{in.}}\right)^2\right](5\pi)^2$$

$$5\theta = 0.041408\left[1 + \frac{4}{9} + \frac{1}{4}\right](5\pi)^2$$

$5\theta = 18.518$; $\quad \theta = 3.704 \text{ rad}\left(\frac{\text{rev}}{2\pi \text{ rad}}\right) = 0.5894$ rev

$\theta = 0.589$ rev

17.16



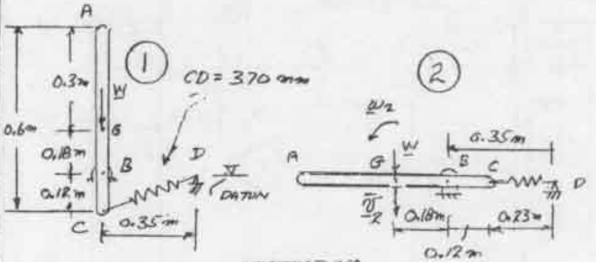
GIVEN: 4-kg ROD AC

SPRING: $k = 400$ N/m

UNSTRETCHED LENGTH $= 150$ mm.

ROD IS RELEASED FROM REST.

FIND: ω AFTER ROD HAS ROTATED 90°



POSITION 1:

SPRING: $x_1 = CD - (150 \text{ mm}) = 370 - 150 = 220 \text{ mm} = 0.22$ m (UNSTRETCHED LENGTH)

$V_e = \frac{1}{2} k x_1^2 = \frac{1}{2}(400 \text{ N/m})(0.22 \text{ m})^2 = 9.68$ J

GRAVITY $V_g = Wh = mgh = (4 \text{ kg})(9.81 \text{ m/s}^2)(0.18 \text{ m}) = 7.063$ J

$V_1 = V_e + V_g = 9.68 \text{ J} + 7.063 \text{ J} = 16.743$ J

KINETIC ENERGY: $T_1 = 0$

POSITION 2:

SPRING: $x_2 = 230 \text{ mm} - 150 \text{ mm} = 80 \text{ mm} = 0.08$ m

$V_e = \frac{1}{2} k x_2^2 = \frac{1}{2}(400 \text{ N/m})(0.08 \text{ m})^2 = 1.28$ J

GRAVITY: $V_g = Wh = 0$

$V_2 = V_e + V_g = 1.28$ J

KINETIC ENERGY: $\bar{v}_2 = r\omega_2 = (0.18 \text{ m})\omega_2$

$\bar{I} = \frac{1}{12} m L^2 = \frac{1}{12}(4 \text{ kg})(0.6 \text{ m})^2 = 0.12 \text{ kg·m}^2$

$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$

$= \frac{1}{2}(4 \text{ kg})(0.18\,\omega_2)^2 + \frac{1}{2}(0.12)\omega_2^2$

$T_2 = 0.1248\,\omega_2^2$

CONSERVATION OF ENERGY:

$$T_1 + V_1 = T_2 + V_2$$

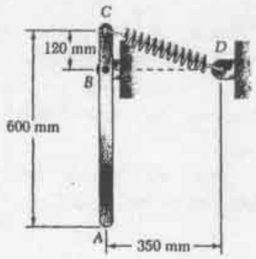
$$0 + 16.743 \text{ J} = 0.1248\,\omega_2^2 + 1.28 \text{ J}$$

$$\omega_2^2 = 123.9$$

$$\omega_2 = 11.131 \text{ rad/s}$$

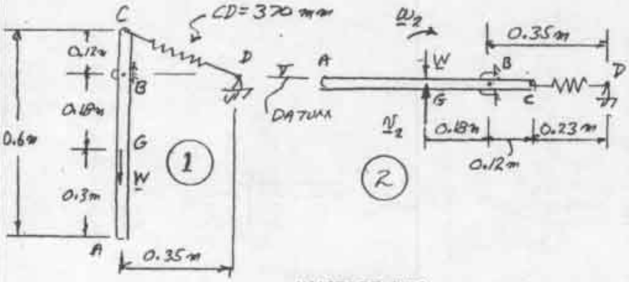
$\omega_2 = 11.13$ rad/s ↻

17.17



GIVEN: 4-kg ROD AC
SPRING: $k = 400\ N/m$
UNSTRETCHED LENGTH $= 150\ mm$
ROD IS RELEASED FROM REST.

FIND: ω AFTER ROD HAS ROTATED 90°



POSITION 1:

UNSTRETCHED LENGTH

SPRING: $x_1 = CD - (150\ mm) = 370 - 150 = 220\ mm = 0.22\ m$

$$V_e = \frac{1}{2} k x_1^2 = \frac{1}{2}(400\ N/m)(0.22\ m)^2 = 9.68\ J$$

GRAVITY: $V_g = Wh = mgh = (4\ kg)(9.81\ m/s^2)(-0.18\ m) = -7.063\ J$

$$V_1 = V_e + V_g = 9.68\ J - 7.063\ J = 2.617\ J$$

KINETIC ENERGY $T_1 = 0$

POSITION 2:

SPRING: $x_2 = 230\ mm - 150\ mm = 80\ mm = 0.08\ m$

$$V_e = \frac{1}{2} k x_2^2 = \frac{1}{2}(400\ N/m)(0.08\ m)^2 = 1.28\ J$$

GRAVITY: $V_g = Wh = 0$

$$V_2 = V_e + V_g = 1.28\ J$$

KINETIC ENERGY: $\bar{v}_2 = r\omega_2 = (0.18\ m)\omega_2$

$$\bar{I} = \frac{1}{12} m L^2 = \frac{1}{12}(4\ kg)(0.6\ m)^2 = 0.12\ kg \cdot m^2$$

$$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}(4\ kg)(0.18\,\omega_2)^2 + \frac{1}{2}(0.12)\,\omega_2^2$$
$$T_2 = 0.1248\,\omega_2^2$$

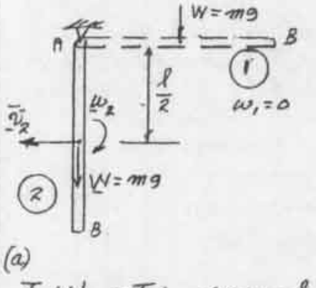
CONSERVATION OF ENERGY

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 2.617\ J = 0.1248\,\omega_2^2 + 1.28\ J$$
$$\omega_2^2 = 10.713$$
$$\omega_2 = 3.273\ rad/s$$

$\omega_2 = 3.27\ rad/s$ ↻ ◀

17.18

GIVEN: ROD OF WEIGHT W IS RELEASED FROM REST.
FIND: (a) ω AND A AS ROD PASSES THROUGH THE VERTICAL
(b) SOLVE PART a FOR $W = 1.8\ lb$, $l = 3\ ft$.



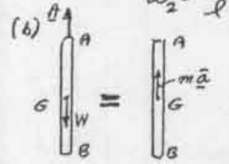
$$\bar{v}_2 = \frac{l}{2}\omega_2$$
$$T_1 = 0$$
$$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2} m\left(\frac{l}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12} m l^2\right)\omega_2^2$$
$$T_2 = \frac{1}{6} m l^2 \omega_2^2$$
$$U_{1\to 2} = mg\frac{l}{2}$$

(a)

$T_1 + U_{1-2} = T_2$: $\quad 0 + mg\frac{l}{2} = \frac{1}{6} m l^2 \omega_2^2$

$$\omega_2^2 = \frac{3g}{l} \qquad \omega_2 = \sqrt{\frac{3g}{l}}\ ↻ \ ◀$$

(b)



$$\bar{a} = \frac{l}{2}\omega_2^2 = \frac{l}{2}\cdot\frac{3g}{l} = \frac{3}{2}g$$

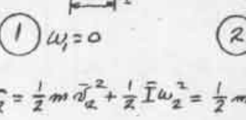
$+\uparrow \Sigma F = \Sigma(F)_{eff}$: $A - W = m\bar{a}$

$$A - mg = m\frac{3}{2}g$$
$$A = \frac{5}{2}mg \qquad A = \frac{5}{2}W \uparrow \ ◀$$

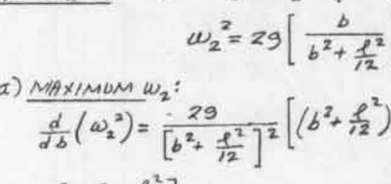
(b) $W = 1.8\ lb$, $l = 3\ ft$

$$\omega_2^2 = \frac{3g}{l} = \frac{3g}{3} = 32.2 \qquad \omega_2 = 5.67\ rad/s\ ↻ \ ◀$$
$$A = \frac{5}{2}W = \frac{5}{2}(1.8\ lb) \qquad A = 4.5\ lb \uparrow \ ◀$$

17.19



GIVEN: ROD AB RELEASED FROM REST. AS ROD PASSES THROUGH VERTICAL,
FIND: (a) DISTANCE b FOR WHICH ω IS MAXIMUM.
(b) CORRESPONDING ω AND C



$$\bar{v}_2 = b\omega_2 \qquad U_{1\to 2} = Wb = mgb$$

$$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2} m (b\omega_2)^2 + \frac{1}{2}\left(\frac{1}{12} m l^2\right)\omega_2^2 = \frac{1}{2} m\left[b^2 + \frac{l^2}{12}\right]\omega_2^2$$

$T_1 + U_{1-2} = T_2$: $\quad 0 + mgb = \frac{1}{2} m\left[b^2 + \frac{l^2}{12}\right]\omega_2^2$

$$\omega_2^2 = 2g\left[\frac{b}{b^2 + \frac{l^2}{12}}\right] \qquad (1)$$

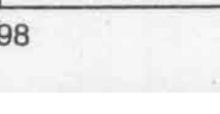
(a) MAXIMUM ω_2:

$$\frac{d}{db}\left(\omega_2^2\right) = \frac{2g}{\left[b^2 + \frac{l^2}{12}\right]^2}\left[\left(b^2 + \frac{l^2}{12}\right) - b(2b)\right] = 0$$

$$\left[-b^2 + \frac{l^2}{12}\right] = 0 \qquad b = \frac{l}{\sqrt{12}} \ ◀$$

(b) EQ. 1:

$$\omega_2^2 = 2g\left[\frac{l/\sqrt{12}}{\frac{l^2}{12} + \frac{l^2}{12}}\right] = \sqrt{12}\,\frac{g}{l}\ ; \quad (\omega_2)_{max} = 1.861\sqrt{\frac{g}{l}} \ ◀$$

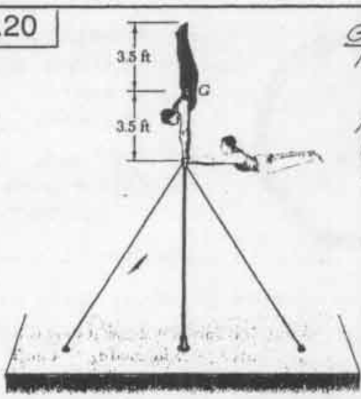


$$\bar{a} = b\,\omega_2^2 = \frac{l}{\sqrt{12}}\cdot\sqrt{12}\,\frac{g}{l} = g \uparrow$$

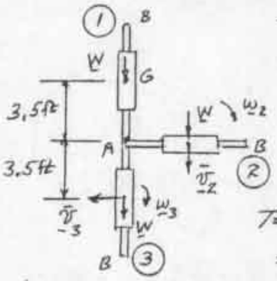
$+\uparrow \Sigma F = \Sigma F_{eff}$: $\quad C - W = mg$

$$C = 2mg \qquad C = 2W \uparrow \ ◀$$

17.20



GIVEN:
160-lb GYMNAST WITH $\bar{k} = 1.5$ ft
HE IS ROTATING VERY SLOWLY ($\omega_1 \approx 0$) IN POSITION SHOWN.
FIND: ω AND FORCE EXERTED ON HIS HANDS AFTER HE HAS ROTATED THROUGH (a) 90°, (b) 180°



$\bar{I} = \frac{W}{g}\bar{k}^2 = \frac{160 \text{ lb}}{32.2}(1.5 \text{ ft})^2$

$\bar{I} = 11.18 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$\bar{v}_2 = (3.5 \text{ ft})\omega_2$

$\bar{v}_3 = (3.5 \text{ ft})\omega_3$

$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

$= \frac{1}{2}\frac{160}{32.2}(3.5\omega)^2 + \frac{1}{2}(11.18)\omega^2$

$T = (30.435 + 5.59)\omega^2 = 36.025\,\omega^2$

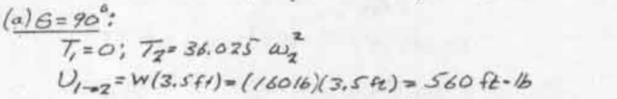
(a) $\theta = 90°$:

$T_1 = 0;\ T_2 = 36.025\,\omega_2^2$

$U_{1\to2} = W(3.5 \text{ ft}) = (160 \text{ lb})(3.5 \text{ ft}) = 560 \text{ ft-lb}$

$T_1 + U_{1\to2} = T_2$: $\quad 0 + 560 = 36.025\,\omega_2^2$

$\omega_2^2 = 15.545 \qquad \omega_2 = 3.94 \text{ rad/s}$ ◄



$+\circlearrowright \Sigma M_A = \Sigma(M_A)_{eff}$: $\quad (160)(3.5) = \frac{W}{g}(3.5\alpha)(3.5) + \bar{I}\alpha$

$560 = \frac{160}{32.2}\,3.5^2\alpha + 11.18\alpha$

$560 = 72.05\alpha \qquad \alpha = 7.772 \text{ rad/s}^2$

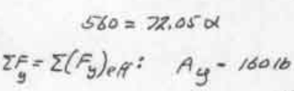
$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $\quad A_y - 160 \text{ lb} = -m(3.5\alpha)$

$A_y - 160 = -\frac{160}{32.2}(3.5)(7.772)$

$A_y - 160 = -135.17 \qquad A_y = 24.837 \text{ lb}\uparrow$

$\xrightarrow{+} \Sigma F_x = \Sigma(F_x)_{eff}$: $A_x = m\bar{a}_n = m(3.5 \text{ ft})\omega_2^2$

$A_x = \frac{160}{32.2}(3.5)(15.545); \quad A_x = 270.3 \text{ lb} \leftarrow$



$\beta = \tan^{-1}\frac{24.837}{270.3} = 5.247°$

$A = \frac{A_x}{\cos\beta} = \frac{270.3}{\cos 5.247°} = 271.48 \text{ lb}$

$A = 271 \text{ lb} \measuredangle 5.2°$ ◄

(CONTINUED)

17.20 continued

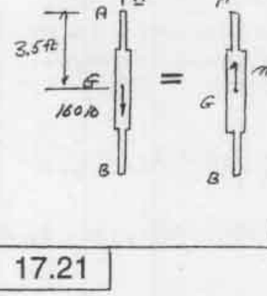
(b) $\theta = 180°$:

$T_1 = 0 \quad T_3 = 36.025\,\omega_3^2$

$U_{1\to3} = W(2\times3.5 \text{ ft}) = (160 \text{ lb})(7 \text{ ft}) = 1120 \text{ ft-lb}$

$T_1 + U_{1\to3} = T_3$: $\quad 0 + 1120 = 36.025\,\omega_3^2$

$\omega_3^2 = 31.09 \qquad \omega_2 = 5.58 \text{ rad/s}$ ◄



$m\bar{a} = m(3.5 \text{ ft})\omega_3^2$

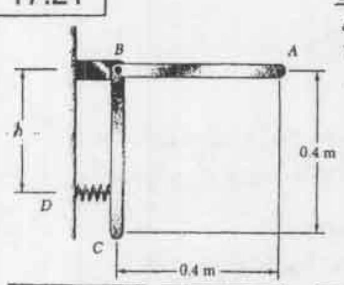
$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$

$A - 160 \text{ lb} = \frac{160}{32.2}(3.5)(31.09)$

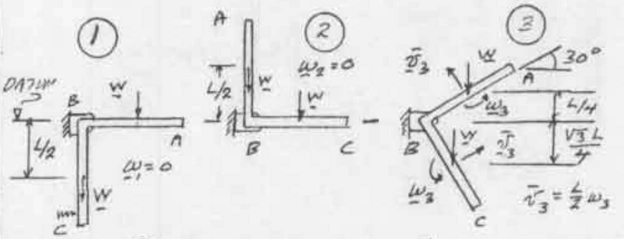
$A - 160 = 540.7$

$A = 700.7 \text{ lb} \qquad A = 701 \text{ lb}\uparrow$ ◄

17.21



GIVEN: TWO RODS EACH OF MASS m ARE WELDED TOGETHER AND PRESSED AGAINST SPRING AT D. AFTER RELEASE RODS ROTATE THROUGH MAX. ANGLE OF 90°
FIND: ANGULAR VELOCITY WHEN AB FORMS 30° WITH HORIZONTAL.



POSITION ①: $T_1 = 0$, $(V_e)_1$, $(V_g)_1 = -W\frac{L}{2}$

POSITION ②: $T_2 = 0$, $(V_e)_2 = 0$, $(V_g)_2 = +W\frac{L}{2}$

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + (V_e)_1 - W\frac{L}{2} = 0 + W\frac{L}{2}$

$(V_e)_1 = WL$

POSITION ③: $(V_e)_3 = 0$; $(V_g)_3 = W\left(\frac{L}{4}\right) - W\left(\frac{\sqrt{3}L}{4}\right) = -0.183\,WL$

$T_3 = 2\left\{\frac{1}{2}m\bar{v}_3^2 + \frac{1}{2}\bar{I}\omega_3^2\right\}$

$= 2\left\{\frac{1}{2}\frac{W}{g}\left(\frac{L}{2}\omega_3\right)^2 + \frac{1}{2}\left(\frac{1}{12}\frac{W}{g}L^2\right)\omega_3^2\right\} = \frac{1}{3}\frac{W}{g}L^2\omega_3^2$

$T_1 + V_1 = T_3 + V_3$:

$0 + (V_e)_1 + (V_g)_1 = T_3 + (V_g)_3$

$0 + WL - \frac{1}{2}WL = \frac{1}{3}\frac{W}{g}L^2\omega_3^2 - 0.183\,WL$

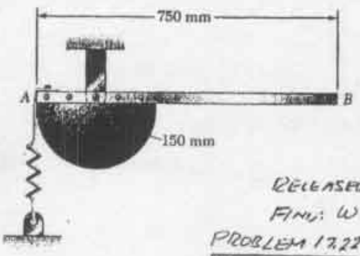
$WL\left(1 - \frac{1}{2} + 0.183\right) = \frac{1}{3}\frac{W}{g}L^2\omega_3^2$

$\omega_3^2 = 3(0.683)\frac{g}{L} = 2.049\frac{g}{L} \qquad \omega_2 = 1.431\sqrt{\frac{g}{L}}$

For $L = 0.4$ m: $\omega_3 = 1.431\sqrt{\frac{9.81 \text{ m/s}^2}{0.4 \text{ m}}} = 7.086 \text{ rad/s}$

$\omega_3 = 7.09 \text{ rad/s}$ ◄

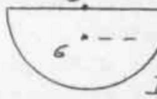
17.22 and 17.23



GIVEN: $m_{AB} = 6$ kg
1.8-kg SEMICIRCULAR DISK.
SPRING OF $k = 160$ N/m UNSTRETCHED WHEN AB IS HORIZONTAL
IF SYSTEM IS RELEASED FROM REST,
FIND: ω AFTER 90° ROTATION
PROBLEM 17.22: WITH SPRING ATTACHED
PROBLEM 17.23: SPRING REMOVED

MOMENT OF INERTIA ABOUT C.

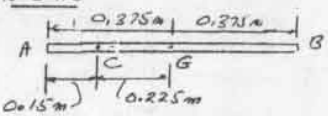
DISK:



$\bar{y} = \frac{4r}{3\pi} = \frac{4(0.15)}{3\pi} = 0.06366$ m

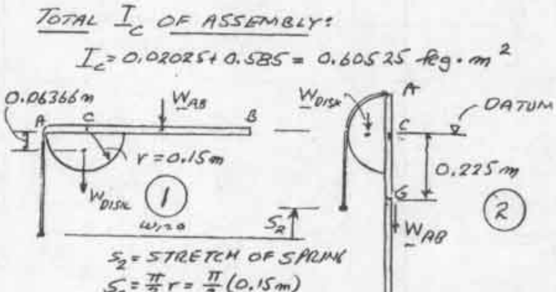
$I_C = \frac{1}{2} m r^2 = \frac{1}{2}(1.8\text{ kg})(0.15\text{ m})^2 = 0.02025$ kg·m²

ROD AB:



$I_C = \bar{I} + m\bar{x}^2 = \frac{1}{12}(6\text{ kg})(0.75\text{ m})^2 + (6\text{ kg})(0.225\text{ m})^2$
$= 0.28125 + 0.30375 = 0.585$ kg·m²

TOTAL I_C OF ASSEMBLY:
$I_C = 0.02025 + 0.585 = 0.60525$ kg·m²



S_2 = STRETCH OF SPRING
$S_2 = \frac{\pi}{2} r = \frac{\pi}{2}(0.15\text{ m})$
$S_2 = 0.2356$ m

POSITION 1: $T_1 = 0$, $V_1 = W_{DISK}(-0.06366\text{ m})$
$V_1 = (1.8\text{ kg})(9.81)(-0.06336) = -1.1188$ J

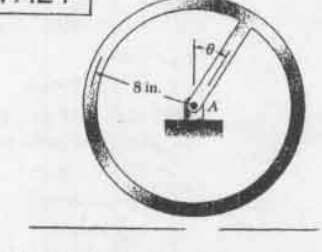
POSITION 2: $(V_e)_2 = \frac{1}{2} k S_2^2 = \frac{1}{2}(160\text{ N/m})(0.2356\text{ m})^2 = 4.44$ J
$(V_g)_2 = W_{AB}(-0.225\text{ m}) = (6\text{ kg})(9.81)(-0.225) = -13.24$ J

FOR NON CENTROIDAL ROTATION WE USE EQ.(17.10)
$T_2 = \frac{1}{2} I_C \omega_2^2 = \frac{1}{2}(0.60525)\omega_2^2 = 0.3026\,\omega_2^2$

PROBLEM 17.22: $T_1 + V_1 = T_2 + V_2$
$0 - 1.1188\text{ J} = 0.3026\,\omega_2^2 + 4.44\text{ J} - 13.24\text{ J}$
$7.681 = 0.3026\,\omega_2^2$
$\omega_2^2 = 25.38$ $\quad \omega_2 = 5.04$ rad/s

PROBLEM 17.23: SPRING IS REMOVED. THUS $(V_e)_2 = 4.44$ J IS REMOVED FROM POTENTIAL ENERGY IN POSITION 2. WE NOW WRITE
$T_1 + V_1 = T_2 + V_2$
$0 - 1.1188\text{ J} = 0.3026\,\omega_2^2 - 13.24\text{ J}$
$12.121 = 0.3026\,\omega_2^2$
$\omega_2^2 = 40.05$ $\quad \omega_2 = 6.33$ rad/s ◀

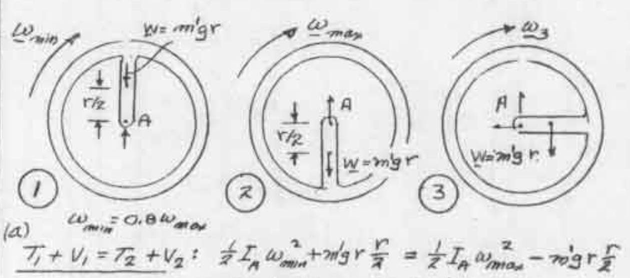
17.24



GIVEN: ASSEMBLY MADE OF $w = 0.25$ lb/ft ROD.
KNOWING THAT $\omega_{min} = 0.80\,\omega_{max}$
FIND: (a) ω_{max}
(b) ω WHEN $\theta = 90°$.

DENOTE MASS PER UNIT LENGTH BY m' AND RADIUS BY r
$I_A = \bar{I}_{ROD} + I_{RING} = \frac{1}{3}(m' r)r^2 + (2\pi r m')r^2 = 6.6165\,m' r^3$

FOR NON CENTROIDAL ROTATION: THE KINETIC ENERGY OF THE ASSEMBLY IS $\frac{1}{2}\bar{I}_A \omega^2$



$\omega_{min} = 0.8\,\omega_{max}$

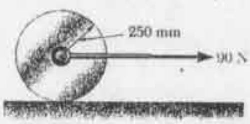
(a) $T_1 + V_1 = T_2 + V_2$: $\frac{1}{2} I_A \omega_{min}^2 + m'gr\frac{r}{2} = \frac{1}{2} I_A \omega_{max}^2 - m'gr\frac{r}{2}$
$\frac{1}{2} I_A(\omega_{max}^2 - \omega_{min}^2) = m'g r^2$
$\frac{1}{2} 6.6165\,m' r^3(1 - 0.8^2)\,\omega_{max}^2 = m' g r^2$

$\omega_{max}^2 = 0.83965\frac{g}{r} = 0.83965\frac{32.2\text{ ft/s}^2}{(8/12\text{ ft})} = 40.555$
$\omega_{max} = 6.37$ rad/s ◀

(b) $T_2 + V_2 = T_3 + V_3$: $\frac{1}{2} I_A \omega_{max}^2 - m'gr\left(\frac{r}{2}\right) = \frac{1}{2} I_A \omega_3^2$
$\frac{1}{2}(6.6165\,m' r^3)\left(0.83965\frac{g}{r}\right) - \frac{m'gr^2}{2} = \frac{1}{2}(6.6165\,m' r^3)\omega_3^2$
$2.7736\,m'g r^2 - 0.5\,m'g r^2 = 3.3162\,m' r^3 \omega_3^2$
$\omega_3^2 = \frac{2.27736}{3.3162}\frac{g}{r} = 0.6865\frac{g}{r}$
$\omega_3^2 = 0.6885\frac{32.2\text{ ft/s}^2}{(8/12\text{ ft})} = 33.26$
$\omega_3 = 5.77$ rad/s ◀

NOTE: RESULTS ARE INDEPENDENT OF WEIGHT PER UNIT LENGTH OF THE ROD USED TO MAKE THE ASSEMBLY.

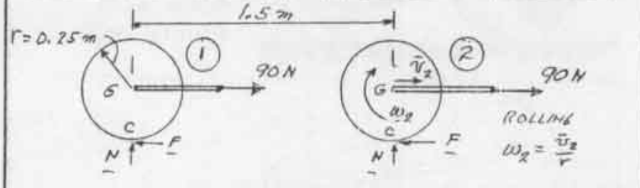
17.25



GIVEN: 20-kg ROLLER ROLLS WITHOUT SLIPPING

FIND: (a) $\bar{v}$ AFTER 1.5 m motion.

(b) FRICTION FORCE REQUIRED TO PREVENT SLIPPING.



INSTANT CENTER AT C: THUS F DOES NO WORK

$T_1 = 0 \qquad U_{1\to 2} = (90\,N)(1.5\,m) = 135\,J$

$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2} m\bar{v}^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\bar{v}_2}{r}\right)^2$

(a) $T_2 = \frac{3}{4} m \bar{v}_2^2 = \frac{3}{4}(20\,kg)(\bar{v}_2^2) = 15\,\bar{v}_2^2$

$T_1 + U_{1\to 2} = T_2$: $\quad 0 + 135\,J = 15\,\bar{v}_2^2$

$\bar{v}_2^2 = 9 \qquad \bar{v}_2 = 3\,m/s \rightarrow$

(b) CONSIDER MOTION ABOUT MASS CENTER.

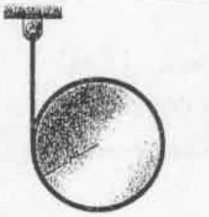
$T_1 = 0 \qquad T_2 = \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\bar{v}_2}{r}\right)^2 = \frac{1}{4} m \bar{v}_2^2$

$U_{1\to 2} = F(1.5\,m)$

$T_1 + U_{1-2} = T_2$: $\quad 0 + 1.5F = \frac{1}{4} m \bar{v}_2^2$

$1.5F = \frac{1}{2}(20\,kg)(3\,m/s)^2$: $\qquad \underline{F} = 30\,lb \leftarrow$

17.26 and 17.27

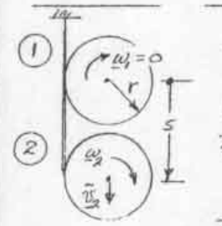


GIVEN: OBJECT SHOWN IS RELEASED FROM REST

FIND: $\bar{v}$ AFTER DOWNWARD MOVEMENT s

PROBLEM 17.26 FOR A CYLINDER

PROBLEM 17.27 FOR A THIN-WALLED PIPE



$\bar{k}$ = RADIUS OF GYRATION

$\bar{v} = r\omega \qquad \omega = \frac{\bar{v}}{r}$

$T_1 = 0$

$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2$

$= \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2}(m\bar{k}^2)\left(\frac{\bar{v}}{r}\right)^2$

$T_2 = \frac{1}{2} m\left(1 + \frac{\bar{k}^2}{r^2}\right)\bar{v}_2^2 \qquad U_{1-2} = mgs$

$T_1 + U_{1-2} + T_2$: $\quad 0 + mgs = \frac{1}{2} m\left(1 + \frac{\bar{k}^2}{r^2}\right)\bar{v}_2^2$

$$\bar{v}_2^2 = \frac{2gs}{1 + \frac{\bar{k}^2}{r^2}} \qquad (1)$$

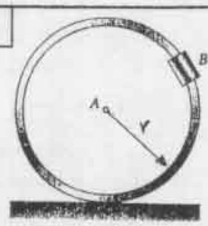
PROBLEM 17.26: CYLINDER $\quad \bar{k}^2 = \frac{1}{2}r^2$

$\bar{v}_2^2 = \frac{2gs}{1 + \frac{1}{2}} = \frac{4gs}{3} \qquad \bar{v}_2 = \sqrt{\frac{4gs}{3}} \downarrow$

PROBLEM 17.27: THIN-WALLED PIPE $\quad \bar{k}^2 = r^2$

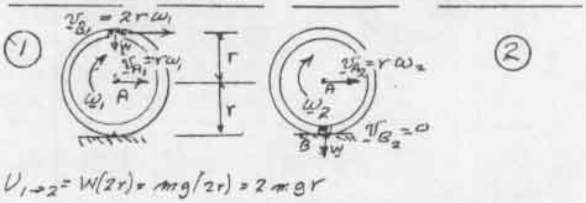
$\bar{v}_2^2 = \frac{2gs}{1+1} = gs \qquad \bar{v}_2 = \sqrt{gs} \downarrow$

17.28



GIVEN: HOOP OF MASS m ROLLS TO RIGHT. WITH COLLAR B OF MASS m AT TOP $\omega = \omega_1$ ↻ AND AT BOTTOM $\omega = 3\omega_1$ ↻.

FIND: ω_1 IN TERMS OF g AND r.



$U_{1\to 2} = W(2r) = mg(2r) = 2mgr$

$T_1 = \frac{1}{2} m v_{A_1}^2 + \frac{1}{2}\bar{I}\omega_1^2 + \frac{1}{2} m v_{B_1}^2$

$= \frac{1}{2} m (r\omega_1)^2 + \frac{1}{2}(mr^2)\omega_1^2 + \frac{1}{2} m (2r\omega_1)^2 = 3mr^2\omega_1^2$

$T_2 = \frac{1}{2} m v_{A_2}^2 + \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2} m v_{B_2}^2$

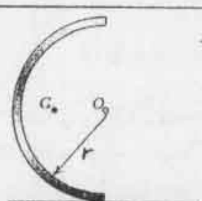
$= \frac{1}{2} m (r\omega_2)^2 + \frac{1}{2} mr^2\omega_2^2 + 0 = mr^2\omega_2^2$

$T_1 + U_{1\to 2} = T_2$: $\quad 3mr^2\omega_1^2 + 2mgr = mr^2\omega_2^2$

GIVEN: $\omega_2 = 3\omega_1 \quad 3mr^2\omega_1^2 + 2mgr = mr^2(3\omega_1)^2$

$2mgr = 6mr^2\omega_1^2$; $\quad \omega_1^2 = \frac{g}{3r}$; $\quad \omega_1 = \sqrt{g/3r}$

17.29

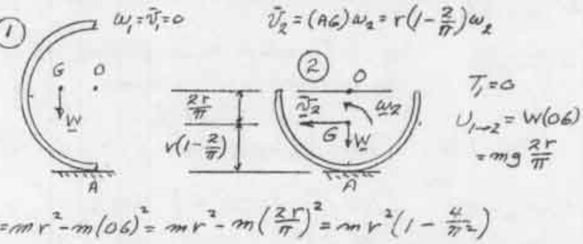


GIVEN: HALF SECTION OF PIPE OF MASS m, RELEASED FROM REST. AFTER ROLLING THROUGH 90°

FIND: (a) $\underline{\omega}$

(b) REACTION

$\omega_1 = \bar{v}_1 = 0 \qquad \bar{v}_2 = (AG)\omega_2 = r\left(1 - \frac{2}{\pi}\right)\omega_2$



$T_1 = 0$

$U_{1\to 2} = W(OG) = mg\frac{2r}{\pi}$

$\bar{I} = mr^2 - m(OG)^2 = mr^2 - m\left(\frac{2r}{\pi}\right)^2 = mr^2\left(1 - \frac{4}{\pi^2}\right)$

$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2} m\left(1 - \frac{2}{\pi}\right)^2 r^2\omega_2^2 + \frac{1}{2} mr^2\left(1 - \frac{4}{\pi^2}\right)\omega_2^2$

(a) $= \frac{1}{2} mr^2\left[\left(1 - \frac{4}{\pi} + \frac{4}{\pi^2}\right) + \left(1 - \frac{4}{\pi^2}\right)\right] = \frac{1}{2} mr^2\left(2 - \frac{4}{\pi}\right)$

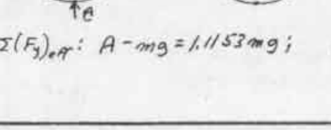
$T_1 + U_{1-2} = T_2$: $\quad 0 + mg\frac{2r}{\pi} = \frac{1}{2} mr^2\left(2 - \frac{4}{\pi}\right)\omega_2^2$

$\omega_2^2 = \frac{2}{\pi\left(1 - \frac{2}{\pi}\right)}\frac{g}{r} = 1.7519\frac{g}{r} \qquad \underline{\omega}_2 = 1.324\sqrt{g/r}$ ↻

(b) KINEMATICS: SINCE $\underline{O}$ MOVES HORIZONTALLY, $(a_O)_y = 0$

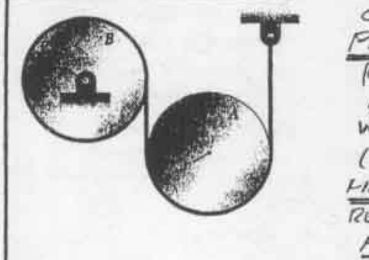
$\underline{a}_n = (OG)\omega_2^2 = \frac{2r}{\pi}\left(1.7519\frac{g}{r}\right) = 1.1153g \uparrow$

KINETICS:



$+\uparrow \Sigma F_y = \Sigma(F_y)_{eff}$: $\quad A - mg = 1.1153mg$; $\qquad \underline{A} = 2.12mg \uparrow$

17.30 and 17.31



GIVEN: 14-lb CYLINDERS OF 5-in. RADIUS.

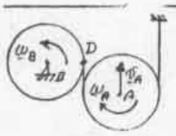
PROBLEM 17.30: $(\omega_B)_0 = 30$ rad/s ↻

FIND: (a) DISTANCE A WILL RISE BEFORE $\omega_B = 5$ rad/s

(b) TENSION IN CORD A-B

PROBLEM 17.31: SYSTEM IS RELEASED FROM REST

FIND: (a) $\bar{v}_A$ AFTER 3 ft OF MOTION. (b) T IN CORD A-B



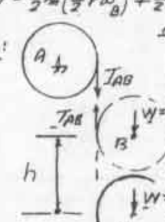
$v_D = r\omega_B \quad \omega_A = \frac{v_D}{2r} = \frac{r\omega_B}{2r} = \frac{1}{2}\omega_B$

$\bar{v}_A = r\omega_A = \frac{1}{2}r\omega_B \quad (1)$

KINETIC ENERGY: $T = \frac{1}{2}m\bar{v}_A^2 + \frac{1}{2}\bar{I}\omega_A^2 + \frac{1}{2}\bar{I}\omega_B^2$

$T = \frac{1}{2}m\left(\frac{1}{2}r\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{1}{2}\omega_B\right)^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega_B^2 = \frac{7}{16}mr^2\omega_B^2$

WORK:



SINCE CORD IS INEXTENSIBLE, WORK IS DONE ONLY BY THE WEIGHT OF CYLINDER B

$U_{1-2} = -Wh = -mgh$

$r = \frac{5}{12}$ ft

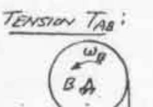
PROBLEM 17.30: $(\omega_B)_1 = 30$ rad/s ↻; $(\omega_B)_2 = 5$ rad/s ↻

$T_1 + U_{1-2} = T_2$: $\frac{7}{16}mr^2(\omega_B)_1^2 - mgh = \frac{7}{16}mr^2(\omega_B)_2^2$

$h = \frac{7}{16}\frac{r^2}{g}\left[(\omega_B)_1^2 - (\omega_B)_2^2\right] \quad (2)$

$h = \frac{7}{16}\left(\frac{5}{12}\text{ ft}\right)^2\frac{(30)^2 - (5)^2}{32.2\text{ ft/s}^2} = 2.064$ ft $\quad h = 2.06$ ft ↑ ◀

TENSION T_{AB}: WE NOTE THAT POINT D MOVE TWICE THE DISTANCE THAT A MOVES



$U_{1-2} = -T_{AB}(2h)$

FOR ONLY CYLINDER B, $T = \frac{1}{2}\bar{I}\omega_B^2$

$T_1 + U_{1-2} = T_2$: $\frac{1}{2}\bar{I}(\omega_B)_1^2 - 2hT_{AB} = \frac{1}{2}\bar{I}(\omega_B)_2^2$

$T_{AB} = \frac{1}{4}I\left[(\omega_B)_1^2 - (\omega_B)_2^2\right]\frac{1}{h} = \frac{1}{4}\left(\frac{1}{2}mr^2\right)\frac{(\omega_B)_1^2 - (\omega_B)_2^2}{\frac{7}{16}\frac{r^2}{g}\left[(\omega_B)_1^2 - (\omega_B)_2^2\right]}$

$T_{AB} = \frac{1}{8}\frac{16}{7}mg = \frac{2}{7}W = \frac{2}{7}(14\text{ lb}) \quad T_{AB} = 4$ lb ◀

NOTE: T_{AB} IS INDEPENDENT OF $(\omega_B)_1$ AND $(\omega_B)_2$

PROBLEM 17.31 $(\omega_B)_1 = 0$, $h = 3$ ft, $r = \frac{5}{12}$ ft

SINCE h AND $\bar{v}_A$ ARE NOW DOWNWARD, $U_{1-2} = +Wh = +mgh$ AND EQ. 2 IS:

$h = -\frac{7}{16}\frac{r^2}{g}\left[(\omega_B)_1^2 - (\omega_B)_2^2\right]$

$3\text{ ft} = -\frac{7}{16}\left(\frac{5}{12}\text{ ft}\right)^2\frac{1}{32.2\text{ ft/s}^2}\left[0 - (\omega_B)_2^2\right]$

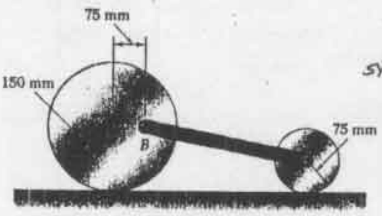
$(\omega_B)_2^2 = 1271.8 \quad (\omega_B)_2 = 35.66$ rad/s

EQ(1): $\bar{v}_A = \frac{1}{2}r(\omega_B)_2 = \frac{1}{2}\left(\frac{5}{12}\text{ ft}\right)(35.66\text{ rad/s}) = 7.430$ ft/s

$\bar{v}_A = 7.43$ ft/s ↓ ◀

TENSION T_{AB}: SINCE T_{AB} IS INDEPENDENT OF VELOCITY, WE AGAIN HAVE $\quad T_{AB} = 4$ lb ◀

17.32



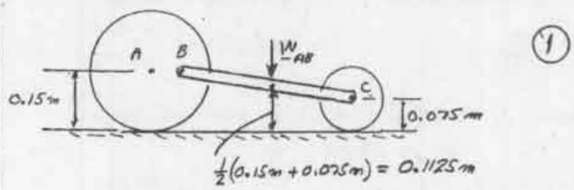
GIVEN: $m_{BC} = 5$ kg

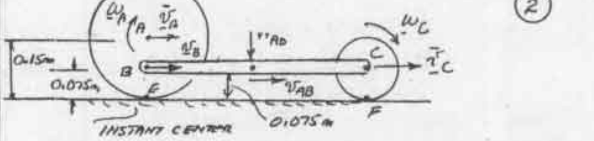
$m_A = 6$ kg

$m_C = 1.5$ kg

SYSTEM IS RELEASED FROM REST

FIND: v_{AB} AFTER DISK A HAS ROTATED 90°.





$v_B = v_{AB} \quad \omega_A = \frac{v_B}{BE} = \frac{v_{AB}}{0.075\text{ m}} \quad \bar{v}_A = 2v_B = 2v_{AB}$

$\bar{v}_C = v_{AB} \quad \omega_C = \frac{v_C}{CF} = \frac{v_{AB}}{0.075\text{ m}}$

$U_{1-2} = W_{AB}(0.1125\text{ m} - 0.075\text{ m}) = (5\text{ kg})(9.8)(0.0375\text{ m})$

$U_{1-2} = 1.8394$ J

$T_1 = 0$

$T_2 = \frac{1}{2}m_A\bar{v}_A^2 + \frac{1}{2}\bar{I}_A\omega_A^2 + \frac{1}{2}m_{AB}v_{AB}^2 + \frac{1}{2}m_B\bar{v}_B^2 + \frac{1}{2}\bar{I}_B\omega_B^2$

$= \frac{1}{2}\left[(6\text{ kg})(2v_{AB})^2 + \frac{1}{2}(6\text{ kg})(0.15\text{ m})^2\left(\frac{v_{AB}}{0.075}\right)^2 + (5\text{ kg})v_{AB}^2\right.$
$\left. + (1.5\text{ kg})(v_{AB})^2 + \frac{1}{2}(1.5\text{ kg})(0.075\text{ m})^2\left(\frac{v_{AB}}{0.075}\right)^2\right]$

$= \frac{1}{2}\left[24 + 12 + 5 + 1.5 + 0.75\right]v_{AB}^2$

$T_2 = 21.625\,v_{AB}^2$

WORK ENERGY

$T_1 + U_{1-2} = T_2$

$0 + 1.8394\text{ J} = 21.625\,v_{AB}^2$

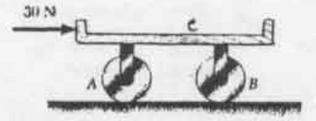
$v_{AB}^2 = 0.08506 \quad v_{AB} = 0.2916$ m/s

$v_{AB} = 292$ mm/s → ◀

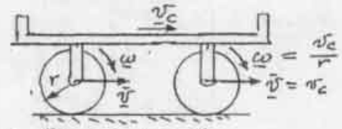
17.33

GIVEN: 9-kg CRADLE WITH 6-kg WHEELS OF $r = 80$ mm INITIALLY AT REST

FIND: v_C OF CRADLE AFTER 250 mm MOVEMENT



KINEMATICS:



$T_1 = 0$: $T_2 = \frac{1}{2} m_C v_C^2 + 2\left[\frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2\right]$

$= \frac{1}{2}(9\text{ kg}) v_C^2 + 2\left[\frac{1}{2}(6\text{ kg}) v_C^2 + \frac{1}{2}\left(\frac{1}{2}(6\text{ kg}) r^2 \left(\frac{v_C}{r}\right)^2\right)\right] = 13.5 v_C^2$

$U_{1\to2} = (30\text{ N})(0.25\text{ m}) = 7.5\text{ J}$

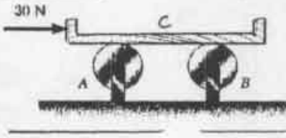
$T_1 + U_{1-2} = T_2$: $0 + 7.5\text{ J} = 13.5 v_C^2$

$v_C^2 = 0.5556$ $\quad v_C = 0.745\text{ m/s} \rightarrow$ ◄

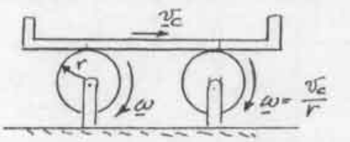
17.34

GIVEN: 9-kg CRADLE WITH 6-kg WHEELS OF $r = 80$ mm INITIALLY AT REST.

FIND: v_C OF CRADLE AFTER 250 mm OF MOVEMENT



KINEMATICS



$T_1 = 0$; $T_2 = \frac{1}{2} m_C v_C^2 + 2\left[\frac{1}{2} \bar{I} \omega^2\right]$

$= \frac{1}{2}(9\text{ kg}) v_C^2 + 2\left[\frac{1}{2}\left(\frac{1}{2}(6\text{ kg}) r^2 \left(\frac{v_C}{r}\right)^2\right)\right] = 7.5 v_C^2$

$U_{1\to2} = (30\text{ N})(0.25\text{ m}) = 7.5\text{ J}$

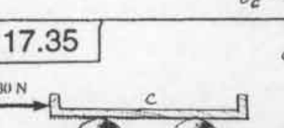
$T_1 + U_{1-2} = T_2$: $0 + 7.5\text{ J} = 7.5 v_C^2$

$v_C^2 = 1.000$ $\quad v_C = 1.000\text{ m/s} \rightarrow$ ◄

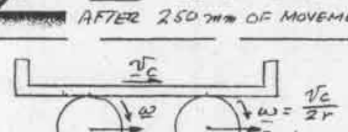
17.35

GIVEN: 9-kg CRADLE WITH 6-kg WHEELS OF $r = 80$ mm INITIALLY AT REST

FIND: v_C OF CRADLE AFTER 250 mm OF MOVEMENT



KINEMATICS:



$T_1 = 0$: $T_2 = \frac{1}{2} m_C v_C^2 + 2\left[\frac{1}{2} m \bar{v}^2 + \frac{1}{2} \bar{I} \omega^2\right]$

$= \frac{1}{2}(9\text{ kg}) v_C^2 + 2\left[\frac{1}{2}(6\text{ kg})\left(\frac{1}{2} v_C\right)^2 + \frac{1}{2}\left(\frac{1}{2}(6\text{ kg}) r^2\right)\left(\frac{v_C}{2r}\right)^2\right]$

$T_2 = 6.75 v_C^2$

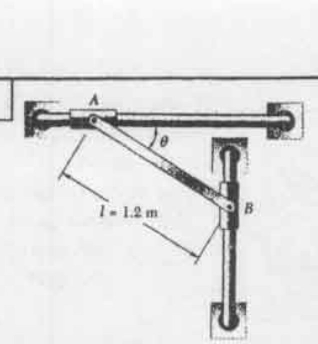
$U_{1-2} = (30\text{ N})(0.25\text{ m}) = 7.5\text{ J}$

$T_1 + U_{1-2} = T_2$: $0 + 7.5\text{ J} = 6.75 v_C^2$

$v_C^2 = 1.1111$

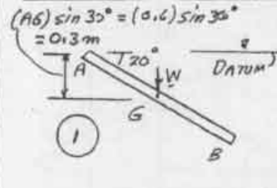
$v_C = 1.054\text{ m/s} \rightarrow$ ◄

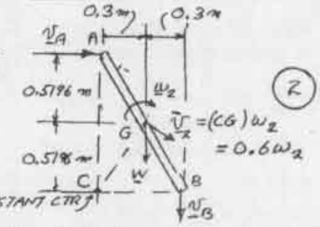
17.36



GIVEN: $m = 10$ kg ROD RELEASED FROM REST WHEN $\theta = 30°$

FIND: v_A AND v_B WHEN $\theta = 60°$





POSITION 1: $T_1 = 0$

$V_1 = -W(0.3\text{ m}) = -(10\text{ kg})(9.81)(0.3) = -29.43\text{ J}$

POSITION 2: $V_2 = -W(0.5196\text{ m}) = -(10\text{ kg})(9.81)(0.5196) = -50.974\text{ J}$

$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2$

$= \frac{1}{2}(10\text{ kg})(0.6\omega_2)^2 + \frac{1}{2}\left(\frac{1}{12}(10\text{ kg})(1.2\text{ m})^2 \omega_2^2\right) = 2.4\omega_2^2$

$T_1 + V_1 = T_2 + V_2$: $0 - 29.43\text{ J} = 2.4\omega_2^2 - 50.974$

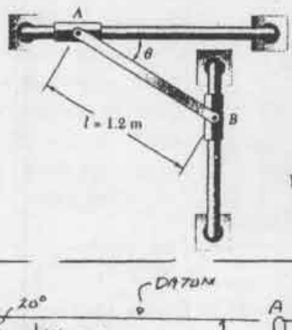
$\omega_2^2 = 8.9768$ $\quad \omega_2 = 2.996\text{ rad/s}$ ↻

VELOCITY OF COLLARS WHEN $\theta = 60°$

$v_A = (AC)\omega_2 = (2 \times 0.5196\text{ m})(2.996\text{ rad/s})$ $\quad v_A = 3.11\text{ m/s} \rightarrow$ ◄

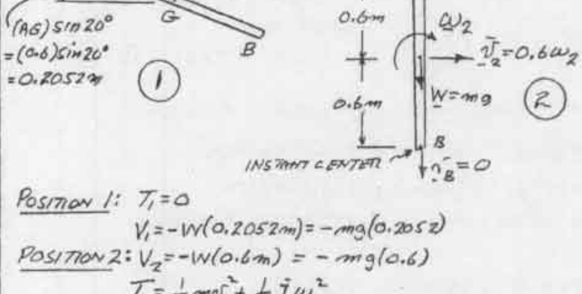
$v_B = (BC)\omega_2 = (2 \times 0.3\text{ m})(2.996\text{ rad/s})$ $\quad v_B = 1.798\text{ m/s} \downarrow$ ◄

17.37



GIVEN: $m = 10$ kg ROD RELEASED FROM REST WHEN $\theta = 20°$

FIND: v_A AND v_B WHEN $\theta = 90°$



POSITION 1: $T_1 = 0$

$V_1 = -W(0.2052\text{ m}) = -mg(0.2052)$

POSITION 2: $V_2 = -W(0.6\text{ m}) = -mg(0.6)$

$T_2 = \frac{1}{2} m \bar{v}_2^2 + \frac{1}{2} \bar{I} \omega_2^2$

$= \frac{1}{2} m (0.6\omega_2)^2 + \frac{1}{2}\left(\frac{1}{12} m (1.2)^2\right)\omega_2^2$

$T_2 = 0.24 m \omega_2^2$

$T_1 + V_1 = T_2 + V_2$: $0 - 0.2052 mg = 0.24 m \omega_2^2 - 0.6 mg$

$\omega_2^2 = 1.645 g = 1.645(9.81) = 16.137$

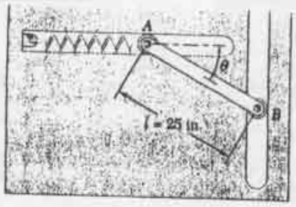
$\omega_2 = 4.017\text{ rad/s}$

VELOCITY OF COLLARS WHEN $\theta = 90°$

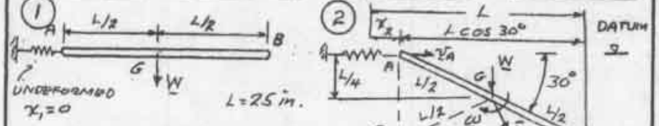
$v_A = (AB)\omega_2 = (1.2\text{ m})(4.017\text{ rad/s})$ $\quad v_A = 4.82\text{ m/s} \rightarrow$ ◄

$v_B = 0$ $\quad v_B = 0$ ◄

17.38



GIVEN: $W_{AB} = 9\text{ lb}$, $k = 3\text{ lb/in.}$
SPRING TENSION IS ZERO WHEN $\theta = 0$.
ROD IS RELEASED FROM REST WHEN $\theta = 0$
FIND: ω AND v_B WHEN $\theta = 30°$



$x_2 = L - L\cos 30° = (25\text{ in})(1-\cos 30°)$
$x_2 = 3.349\text{ in.}$

$\bar{v}_2 = (CG)\omega = \frac{L}{2}\omega_2$

POSITION 1: $T_1 = 0$, $V_1 = 0$
POSITION 2: $V_2 = -W\frac{L}{4} + \frac{1}{2}kx_2^2$

$= -(9\text{ lb})\frac{25\text{ in}}{4} + \frac{1}{2}(3\text{ lb/in})(3.349\text{ in})^2 = -39.42\text{ in·lb} = -3.285\text{ ft·lb}$

$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\cdot\frac{1}{12}mL^2\omega_2^2$

$= \frac{1}{6}mL^2\omega_2^2 = \frac{1}{6}\left(\frac{9\text{ lb}}{32.2}\right)\left(\frac{25}{12}\text{ ft}\right)^2\omega_2^2 = 0.2022\,\omega_2^2$

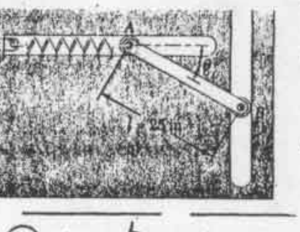
$T_1 + V_1 = T_2 + V_2$: $\quad 0 + 0 = 0.2022\,\omega_2^2 - 3.285\text{ ft·lb}$

$\omega_2^2 = 16.25 \quad \omega_2 = 4.03 \qquad \omega_2 = 4.03\text{ rad/s}$ ◀

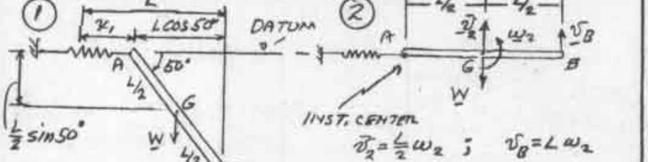
VELOCITY OF B: $v_B = (BC)\omega_2 = (L\cos 30°)\omega_2 = \left(\frac{25}{12}\text{ ft}\right)\cos 30°(4.03\text{ rad/s})$

$v_B = 7.27\text{ ft/s}\downarrow$ ◀

17.39



GIVEN: $W_{AB} = 9\text{ lb}$, $k = 3\text{ lb/in.}$
SPRING TENSION IS ZERO WHEN $\theta = 0$.
ROD IS RELEASED FROM REST WHEN $\theta = 50°$.
FIND: ω AND v_B WHEN $\theta = 0$



$x_1 = L - L\cos 50° = (25\text{ in.})(1-\cos 50°) = 8.9303\text{ in.}$

POSITION 1: $V_1 = -W\frac{L}{2}\sin 50° + \frac{1}{2}kx_1^2$

$V_1 = -(9\text{ lb})\left(\frac{25\text{ in}}{2}\right)\sin 50° + \frac{1}{2}(3\text{ lb/in})(8.9303\text{ in})^2$

$= -86.18 + 119.63 = 33.45\text{ in·lb} = 2.787\text{ ft·lb}$

$T_1 = 0$

POSITION 2: $V_2 = (V_g)_2 + (V_e)_2 = 0$

$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$

$= \frac{1}{6}mL^2\omega_2^2 = \frac{1}{6}\left(\frac{9\text{ lb}}{32.2}\right)\left(\frac{25\text{ in}}{12}\right)^2\omega_2^2 = 0.2022\,\omega_2^2$

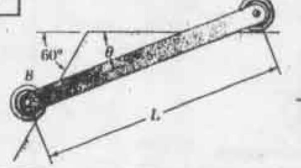
$T_1 + V_1 = T_2 + V_2 \qquad 0 + 2.787\text{ ft·lb} = 0.2022\,\omega_2^2$

$\omega_2^2 = 13.7849 \quad \omega_2 = 3.713\text{ rad/s} \qquad \omega_2 = 3.71\text{ rad/s}$ ◀

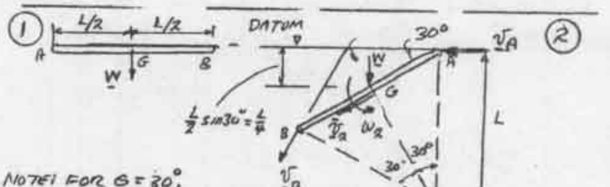
VELOCITY OF B: $v_B = L\omega_2 = \left(\frac{25\text{ in.}}{12}\right)(3.713\text{ rad/s}) = 7.735\text{ ft/s}$

$v_B = 7.74\text{ ft/s}\uparrow$ ◀

17.40



GIVEN: ROD IS RELEASED FROM REST WHEN $\theta = 0$
FIND: v_A AND v_B WHEN $\theta = 30°$



NOTE: FOR $\theta = 30°$,
$\triangle ABC$ IS EQUILATERAL.
IN $\triangle AGC$: $CG = L\cos 30°$; $\bar{v}_2 = L\omega_2\cos 30°$
POSITION 1: $V_1 = 0 \quad T_1 = 0$
POSITION 2: $V_2 = -W\frac{L}{4} = -\frac{1}{4}mgL$

$T_2 = \frac{1}{2}m\bar{v}_2^2 + \frac{1}{2}\bar{I}\omega_2^2 = \frac{1}{2}m(L\omega_2\cos 30)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$

$T_2 = \left(\frac{1}{2}\cos^2 30 + \frac{1}{24}\right)mL^2\omega_2^2 = \left(\frac{3}{8} + \frac{1}{24}\right)mL^2\omega_2^2 = \frac{5}{12}mL^2\omega_2^2$

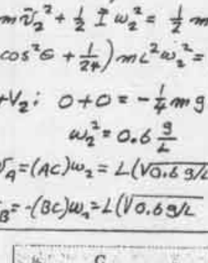
$T_1 + V_1 = T_2 + V_2$: $\quad 0 + 0 = -\frac{1}{4}mgL + \frac{5}{12}mL^2\omega_2^2$

$\omega_2^2 = 0.6\frac{g}{L} \qquad \omega_2 = \sqrt{0.6\,g/L}$

$v_A = (AC)\omega_2 = L(\sqrt{0.6\,g/L}) \qquad v_A = \sqrt{0.6gL}\leftarrow$ ◀

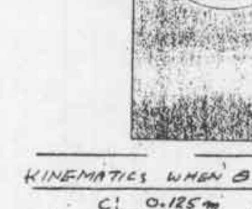
$v_B = (BC)\omega_2 = L(\sqrt{0.6g/L} \qquad v_B = \sqrt{0.6gL}\ \measuredangle 60°$ ◀

17.41



GIVEN: 250-mm ROD AB IS RELEASED WHEN $\theta = 0$
FIND: v_B WHEN $\theta = 90°$

KINEMATICS WHEN $\theta = 90°$



IN $\triangle ABD$: $\beta = \cos^{-1}\frac{0.125\text{ m}}{0.25\text{ m}} = 60°$

$AG = GB = DG = 0.125\text{ m}$

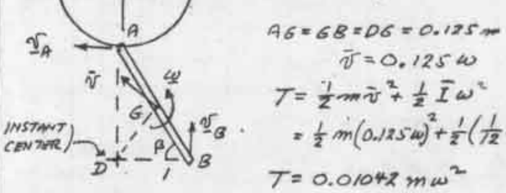
$\bar{v} = 0.125\,\omega \qquad (1)$

$T = \frac{1}{2}m\bar{v}^2 + \frac{1}{2}\bar{I}\omega^2$

$= \frac{1}{2}m(0.125\omega)^2 + \frac{1}{2}\left(\frac{1}{12}m(0.25)^2\right)\omega^2$

$T = 0.01042\,m\omega^2$

CONSERVATION OF ENERGY



POSITION 1: $T_1 = V_1 = 0$
POSITION 2: $V_2 = -mg(0.10825)$

$T_2 = 0.01042\,m\omega_2^2$

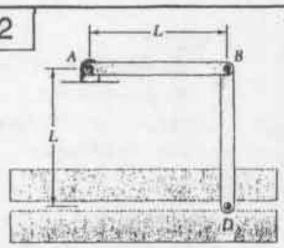
$T_1 + V_1 = T_2 + V_2$: $\quad 0 + 0 = -mg(0.10825) + 0.01042\,m\omega_2^2$

$\omega_2^2 = 10.389g = 10.389(9.81) = 101.916$

VELOCITY OF B WHEN $\theta = 90°$ $\qquad \omega_2 = 10.095\text{ rad/s}$

EQ(1): $v_B = 0.125\,\omega_2 = 0.125(10.095) \qquad v_B = 1.262\text{ m/s}\uparrow$ ◀

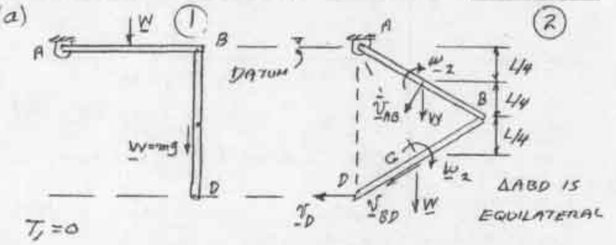
17.42



GIVEN: IDENTICAL RODS RELEASED FROM POSITION SHOWN WITH D MOVED SLIGHTLY TO THE LEFT.

FIND: v_D WHEN
(a) D IS BELOW A,
(b) AB IS VERTICAL.

(a)



$T_1 = 0$

IN POSITION 2 POINT A IS THE INSTANTANEOUS CENTER OF BOTH AB AND BD. ANGULAR VELOCITY OF EACH ROD IS ω.

$AG_{BD} = 0.866L, \quad \bar{v}_{BD} = (AG_{BD})\omega_2, \quad \bar{v}_{AB} = \frac{L}{2}\omega_2$

$T_2 = \frac{1}{2}m\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\bar{v}_{BD}^2 + \frac{1}{2}\bar{I}\omega_2^2$

$= \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2 + \frac{1}{2}m(0.866L\omega_2)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2$

$T_2 = \frac{7}{12}mL^2\omega_2^2$

$V_1 = -\frac{L}{2}mg \qquad V_2 = -mg\frac{L}{4} - mg\frac{3L}{4} = -mgL$

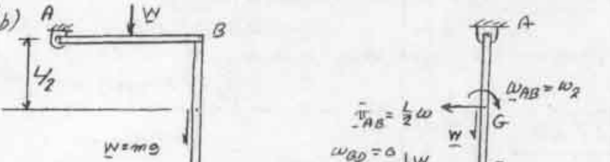
CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2: \quad 0 - \frac{1}{2}mgL = \frac{7}{12}mL^2\omega_2^2 - mgL$

$\omega_2^2 = \frac{6}{7}\frac{g}{L} \qquad \omega_2 = 0.9258\sqrt{g/L}$

$v_D = (AD)\omega_2 = L(0.9258\sqrt{g/L}) \qquad v_D = 0.926\sqrt{gL} \leftarrow$

(b)



IN POSITION 2: $v_D = v_B = \bar{v}_{BD} = L\omega_2 \leftarrow$, AND $\omega_2 = 0$

POSITION 1: $T_1 = 0 \qquad V_1 = -mg\frac{L}{2}$

POSITION 2: $V_2 = -mg\frac{L}{2} - mgL = -\frac{3}{2}mgL$

$T_2 = \frac{1}{2}m\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}_{BD}^2$

$= \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 + \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega_2^2 + \frac{1}{2}m(L\omega_2)^2$

$T_2 = \frac{2}{3}mL^2\omega_2^2$

CONSERVATION OF ENERGY

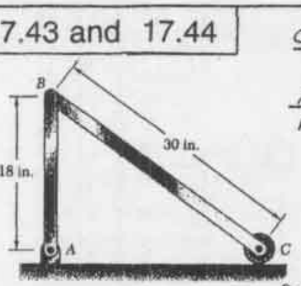
$T_1 + V_1 = T_2 + V_2: \quad 0 - mg\frac{L}{2} = \frac{2}{3}mL^2\omega_2^2 - \frac{3}{2}mgL$

$mgL = \frac{2}{3}mL^2\omega_2^2$

$\omega_2^2 = \frac{3}{2}\frac{g}{L} \qquad \omega_2 = 1.225\sqrt{g/L}$

$v_D = L\omega_2 = L(1.225\sqrt{g/L}) \qquad v_D = 1.225\sqrt{gL} \leftarrow$

17.43 and 17.44



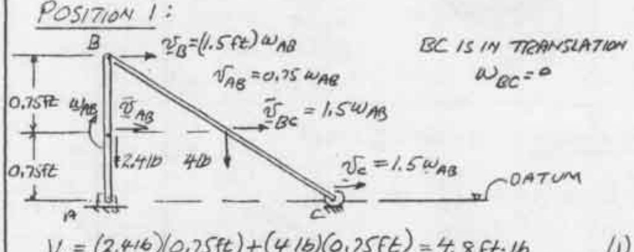
GIVEN: $W_{AB} = 2.4$ lb
$W_{BC} = 4$ lb

FIND: v_B AFTER AB HAS ROTATED THROUGH 90°

PROBLEM 17.43
IF WHEEL IS MOVED SLIGHTLY TO RIGHT AND RELEASED.

PROBLEM 17.44: IF IN POSITION SHOWN $v_C = 6$ ft/s $\rightarrow$

POSITION 1:



BC IS IN TRANSLATION
$\omega_{BC} = 0$

$V_1 = (2.4\text{ lb})(0.75\text{ ft}) + (4\text{ lb})(0.75\text{ ft}) = 4.8\text{ ft·lb} \qquad (1)$

IF $v_C = 0$, THEN $\omega_{AB} = 0$ AND T_1

IF $v_C = 6$ ft/s, $6\text{ ft/s} = (1.5\text{ ft})\omega_{AB} \qquad \omega_{AB} = 4$ rad/s

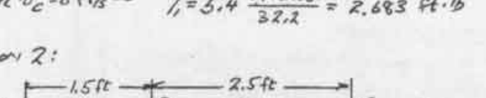
$T_1 = \frac{1}{2}m_{AB}\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}m_{BC}\bar{v}_{BC}^2$

$T_1 = \frac{1}{2}\left(\frac{2.4\text{ lb}}{g}\right)(0.75\omega_{AB})^2 + \frac{1}{2}\left(\frac{1}{12}\frac{2.4\text{ lb}}{g}(1.5\text{ ft})^2\right)\omega_{AB}^2 + \frac{1}{2}\left(\frac{4\text{ lb}}{g}\right)(1.5\omega_{AB})^2$

$T_1 = 0.675\frac{\omega_{AB}^2}{g} + 0.225\frac{\omega_{AB}^2}{g} + 4.5\frac{\omega_{AB}^2}{g} = 5.4\frac{\omega_{AB}^2}{g}$

For $v_C = 6$ ft/s $\rightarrow \quad T_1 = 5.4\frac{(4\text{ rad/s})^2}{32.2} = 2.683$ ft·lb $\qquad (2)$

POSITION 2:



$V_2 = 0$

$\bar{v}_{AB} = 0.75\omega_{AB}\downarrow \qquad v_B = 1.5\omega_{AB}\downarrow \qquad (3)$

$\bar{v}_{BC} = \frac{1}{2}v_B = 0.75\omega_{AB}\downarrow \qquad \omega_{BC} = \frac{v_B}{2.5} = \frac{1.5\omega_{AB}}{2.5} = 0.6\omega_{AB}$

$T_2 = \frac{1}{2}m_{AB}\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}_{AB}\omega_{AB}^2 + \frac{1}{2}m_{BC}\bar{v}_{BC}^2 + \frac{1}{2}\bar{I}_{BC}\omega_{BC}^2$

$= \frac{1}{2}\left(\frac{2.4\text{ lb}}{g}\right)(0.75\omega_{AB})^2 + \frac{1}{2}\left(\frac{1}{12}\frac{2.4\text{ lb}}{g}(1.5)^2\right)(\omega_{AB})^2$

$+ \frac{1}{2}\left(\frac{4\text{ lb}}{g}\right)(0.75\omega_{AB})^2 + \frac{1}{2}\left(\frac{1}{12}\frac{4\text{ lb}}{g}(2.5)^2\right)(0.6\omega_{AB})^2$

$T_2 = (0.675 + 0.225 + 1.125 + 0.375)\frac{\omega_{AB}^2}{g} = \frac{2.4}{32.2}\omega_{AB}^2 = 0.07453\,\omega_{AB}^2$

PROBLEM 17.43 $\quad v_C = 0 \quad T_1 = 0,$

EQ(1) $\quad V_1 = 4.8$ ft·lb, $\quad V_2 = 0 \qquad T_2 = 0.07453\,\omega_{AB}^2$

$T_1 + V_1 = T_2 + V_2 \qquad 0 + 4.8\text{ ft·lb} = 0.07453\,\omega_{AB}^2 + 0$

$\omega_{AB}^2 = 64.4 \qquad \omega_{AB} = 8.025$

EQ(3): $\quad v_B = 1.5\omega_{AB} = 1.5(8.025) \qquad v_B = 12.04$ ft/s $\downarrow$

PROBLEM 17.44 $\quad v_C = 6$ ft/s $\rightarrow$

EQ(2): $\quad T_1 = 2.683$ ft·lb $\qquad T_2 = 0.07453\,\omega_{AB}^2$

EQ(1): $\quad V_1 = 4.8$ ft·lb $\qquad V_2 = 0$

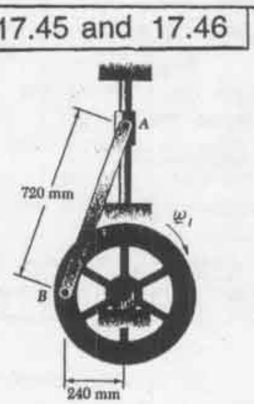
$T_1 + V_1 = T_2 + V_2 \qquad 2.683\text{ ft·lb} + 4.8\text{ ft·lb} = 0.07453\,\omega_{AB}^2 + 0$

$7.483 = 0.07453\,\omega_{AB}^2$

$\omega_{AB}^2 = 100.4 \qquad \omega_{AB} = 10.02$ rad/s

EQ(3): $\quad v_B = 1.5\omega_{AB} = 1.5(10.02) \qquad v_B = 15.03$ ft/s $\downarrow$

17.45 and 17.46

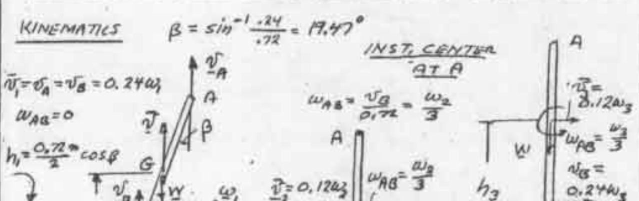


GIVEN: $m_{AB} = 4$ kg
$m_{WHEEL} = 16$ kg
$\bar{k} = 180$ mm

PROBLEM 17.45:
IF $\omega_1 = 60$ rpm ↻,
FIND: ω_2 WHEN B IS DIRECTLY BELOW C

PROBLEM 17.46:
FIND: ω_1 SO ANGULAR VELOCITY IS THE SAME IN POSITION SHOWN AND WHEN B IS DIRECTLY ABOVE C.

KINEMATICS $\beta = \sin^{-1}\frac{.24}{.72} = 19.47°$



POTENTIAL ENERGY:

$V_1 = Wh_1 = mg\,h_1 = (4\,kg)(9.81)(0.3394\,m) = 13.318$ J ◁
$V_2 = W h_2 = mg\,h_2 = (4\,kg)(9.81)(0.12\,m) = 4.709$ J ◁
$V_3 = W h_3 = mg\,h_3 = (4\,kg)(9.81)(0.6\,m) = 23.544$ J ◁

KINETIC ENERGY

$\bar{I}_C = I_{FLYWHEEL} = (16\,kg)(0.18\,m)^2 = 0.5184\ kg\cdot m^2$
$\bar{I}_{AB} = \frac{1}{12} m_{AB} L^2 = \frac{1}{12}(4\,kg)(0.72\,m)^2 = 0.1728\ kg\cdot m^2$
$T_1 = \frac{1}{2}\bar{I}_C\omega_1^2 + \frac{1}{2}m_{AB}\bar{v}_1^2 = \frac{1}{2}(0.5184)\omega_1^2 + \frac{1}{2}(4)(0.24\omega_1)^2$
$= 0.2592\,\omega_1^2 + 0.1152\,\omega_1^2$ $\quad T_1 = 0.3744\,\omega_1^2$ ◁
$T_2 = \frac{1}{2}\bar{I}_C\omega_2^2 + \frac{1}{2}m_{AB}\bar{v}_2^2 + \frac{1}{2}\bar{I}_{AB}\,\omega_{AB}^2$
$= \frac{1}{2}(0.5184)\omega_2^2 + \frac{1}{2}(4)(0.12\omega_2)^2 + \frac{1}{2}(0.1728)\left(\frac{\omega_2}{3}\right)^2$
$= 0.2592\,\omega_2^2 + 0.0288\,\omega_2^2 + 0.0096\,\omega_2^2$ $\quad T_2 = 0.2976\,\omega_2$ ◁
T_3 = SAME COEFFICIENT AS T_2: $\quad T_3 = 0.2976\,\omega_3$ ◁

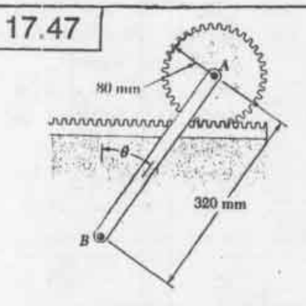
PROBLEM 17.45: POSITION 1 TO POSITION 2

$\omega_1 = 60\text{ rpm}\left(\frac{2\pi}{60}\right) = 2\pi$ rad/s
$T_1 + V_1 = T_2 + V_2$: $\quad 0.3744\omega_1^2 + 13.318\,J = 0.2976\,\omega_2^2 + 4.709\,J$
$0.3744(2\pi)^2 + 13.318 = 0.2976\,\omega_2^2 + 4.709$
$0.2976\,\omega_2^2 = 23.39 \qquad \omega_2^2 = 78.60$ rad/s
$\omega_2 = 8.865\text{ rad/s}\left(\frac{60}{2\pi}\right) \qquad \omega_2 = 84.7$ rpm ↻ ◀

PROBLEM 17.46: POSITION 1 TO POSITION 3 WITH $\omega_1 = \omega_3$

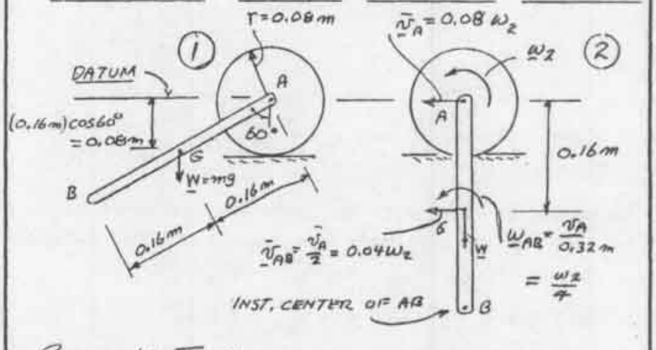
$T_1 + V_1 = T_3 + V_3$: $\quad 0.3744\,\omega_1^2 + 13.318\,J = 0.2976\,\omega_3^2 + 23.544\,J$
$0.3744\,\omega_1^2 + 13.318 = 0.2976\,\omega_3^2 + 23.544$
$0.0768\,\omega_1^2 = 10.226 \qquad \omega_1^2 = 133.2$
$\omega_1 = 11.54\text{ rad/s}\left(\frac{60}{2\pi}\right) \qquad \omega_1 = 110.2$ rpm ↻ ◀

17.47



GIVEN:
5-kg GEAR, $\bar{k} = 60$ mm
4-kg ROD AB
SYSTEM IS RELEASED FROM REST WHEN $\theta = 60°$

FIND:
$\bar{v}_A$ WHEN $\theta = 0$

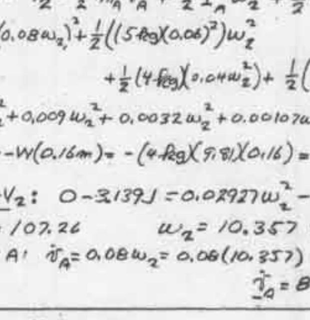


POSITION 1: $T_1 = 0$
$V_1 = -W(0.08\,m) = -(4\,kg)(9.81)(0.08) = -3.139$ J

POSITION 2: $T_2 = \frac{1}{2}m_A\bar{v}_A^2 + \frac{1}{2}\bar{I}_A\,\omega_2^2 + \frac{1}{2}m_{AB}\bar{v}_{AB}^2 + \frac{1}{2}\bar{I}_{AB}\,\omega_{AB}^2$
$T_2 = \frac{1}{2}(5\,kg)(0.08\omega_2)^2 + \frac{1}{2}\left((5\,kg)(0.06)^2\right)\omega_2^2$
$+ \frac{1}{2}(4\,kg)(0.04\omega_2)^2 + \frac{1}{2}\left(\frac{1}{12}(4\,kg)(0.32\,m)^2\right)\left(\frac{\omega_2}{4}\right)^2$
$T_2 = 0.016\omega_2^2 + 0.009\,\omega_2^2 + 0.0032\,\omega_2^2 + 0.00107\omega_2^2 = 0.02927\,\omega_2^2$
$V_2 = -W(0.16\,m) = -(4\,kg)(9.81)(0.16) = -6.278$ J

$T_1 + V_1 = T_2 + V_2$: $\quad 0 - 3.139\,J = 0.02927\,\omega_2^2 - 6.278\,J$
$\omega_2^2 = 107.26 \qquad \omega_2 = 10.357$ rad/s
VELOCITY OF A: $\bar{v}_A = 0.08\,\omega_2 = 0.08(10.357) = 0.829$ m/s
$\bar{v}_A = 829$ mm/s ← ◀

17.48



GIVEN:
$\omega_A = 22.5$ Hz
MOTOR DEVELOPS 3 kW
FIND:
(a) M_A
(b) M_B

$\omega_A = 22.5\,Hz\left(\frac{2\pi\text{ rad}}{\text{cycle}}\right) = 45\pi$ rad/s
$r_A\,\omega_A = r_B\,\omega_B$: $\quad (0.03\,m)(45\pi\text{ rad/s}) = (0.180\,m)\,\omega_B$
$\omega_B = 7.5\pi$ rad/s

(a) PULLEY A: POWER = $M_A\,\omega_A$
$3000\,W = M_A(45\pi\text{ rad/s})$
$M_A = 21.2$ N·m ◀

(b) PULLEY B: POWER = $M_B\,\omega_B$
$3000\,W = M_B(7.5\pi\text{ rad/s})$
$M_B = 127.3$ N·m ◀

17.49

GIVEN: MAXIMUM COUPLE THAT CAN BE APPLIED TO A SHAFT IS 15.5 kip·in.
FIND: MAXIMUM HORSEPOWER THAT CAN BE TRANSMITTED AT (a) 180 rpm, (b) 480 rpm.

$M = 15.5\ \text{kip}\cdot\text{in.} = 1.2917\ \text{kip}\cdot\text{ft} = 1291.7\ \text{lb}\cdot\text{ft}$

(a) $\omega = 180\ \text{rpm}\left(\frac{2\pi}{60}\right) = 6\pi\ \text{rad/s}$

$\text{Power} = M\omega = (1291.7\ \text{lb}\cdot\text{ft})(6\pi\ \text{rad/s}) = 24{,}348\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$

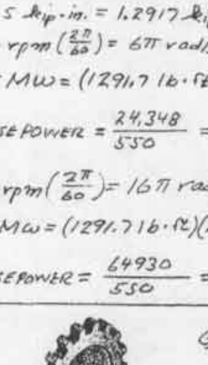
$\text{HORSEPOWER} = \frac{24{,}348}{550} = 44.3\ \text{hp}$ ◀

(b) $\omega = 480\ \text{rpm}\left(\frac{2\pi}{60}\right) = 16\pi\ \text{rad/s}$

$\text{POWER} = M\omega = (1291.7\ \text{lb}\cdot\text{ft})(16\pi\ \text{rad/s}) = 64930\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}$

$\text{HORSEPOWER} = \frac{64930}{550} = 118.1\ \text{hp}$ ◀

17.50



GIVEN: MOTOR ATTACHED TO SHAFT AB DEVELOPES 4.5 hp WHEN $\omega_{AB} = 720$ rpm

FIND: MAGNITUDE OF COUPLE EXERTED ON (a) SHAFT AB (b) SHAFT CD

(a) SHAFT AB: $\omega_{AB} = 720\ \text{rpm}\left(\frac{2\pi}{60}\right) = 75.398\ \text{rad/s}$

$\text{POWER} = 4.5\ \text{hp}\left(\frac{550\ \text{ft}\cdot\text{lb/s}}{\text{hp}}\right) = 2475\ \text{ft}\cdot\text{lb/s}$

$\text{POWER} = M_{AB}\,\omega_{AB};\quad 2475\ \text{ft}\cdot\text{lb/s} = M_{AB}(75.398\ \text{rad/s})$

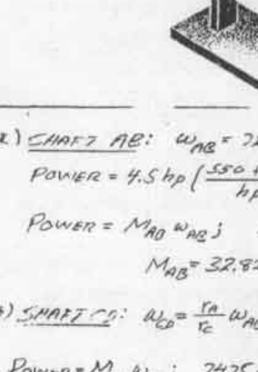
$M_{AB} = 32.826\ \text{lb}\cdot\text{ft} \qquad M_{AB} = 32.8\ \text{lb}\cdot\text{ft}$ ◀

(b) SHAFT CD: $\omega_{CD} = \frac{r_A}{r_C}\,\omega_{AB} = \frac{4\ \text{in.}}{2.5\ \text{in.}}(75.398) = 120.64\ \text{rad/s}$

$\text{POWER} = M_{CD}\,\omega_{CD};\quad 2475\ \text{ft}\cdot\text{lb/s} = M_{CD}(120.64\ \text{rad/s})$

$M_{CD} = 20.5\ \text{lb}\cdot\text{ft}$ ◀

17.51



GIVEN: 2.4 kW TO BE TRANSMITTED FROM A TO D
ALLOWABLE COUPLES ARE
$M_{AB} = 25\ \text{N}\cdot\text{m}$
$M_{CD} = 80\ \text{N}\cdot\text{m}$
FIND: REQUIRED MINIMUM SPEED OF SHAFT AB

SHAFT AB: $\text{POWER} = M_{AB}\,\omega_{AB}$

$2400\ \text{W} = (25\ \text{N}\cdot\text{m})\,\omega_{AB} \qquad \omega_{AB} = 96\ \text{rad/s}$ ◁

SHAFT CD: $\text{POWER} = M_{CD}\,\omega_{CD}$

$2400\ \text{W} = (80\ \text{N}\cdot\text{m})\,\omega_{CD} \qquad \omega_{CD} = 30\ \text{rad/s}$

FOR $\omega_{CD} = 30\ \text{rad/s}$, $\omega_{AB} = \frac{r_C}{r_B}\,\omega_{CD} = \frac{120\ \text{mm}}{30\ \text{mm}}(30\ \text{rad/s})$

$\omega_{AB} = 120\ \text{rad/s}$ ◁

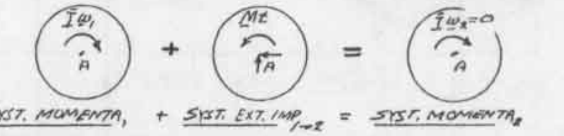
WE CHOOSE THE LARGER ω_{AB}: $\omega_{AB} = (120\ \text{rad/s})\left(\frac{60}{2\pi}\right)$

$\omega_{AB} = 1146\ \text{rpm}$ ◀

17.52

GIVEN: 30-kg ROTOR WITH $\bar{k} = 200$ mm COAST TO REST IN 5.3 min FROM INITIAL ANGULAR VELOCITY OF 3600 rpm.
FIND: MAGNITUDE OF COUPLE DUE TO FRICTION

$\bar{I} = m\bar{k}^2 = (30\ \text{kg})(0.2\ \text{m})^2 = 1.2\ \text{kg}\cdot\text{m}^2,\quad \omega_1 = 3600\ \text{rpm}\left(\frac{2\pi}{60}\right) = 377\ \text{rad/s}$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $\bar{I}\omega_1 - Mt = 0$

$(1.2\ \text{kg}\cdot\text{m}^2)(377\ \text{rad/s}) - M\left(5.3\ \text{min}\times\frac{60\ \text{s}}{\text{min}}\right) = 0$

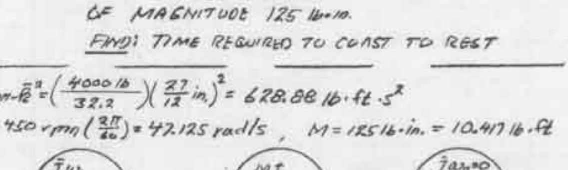
$M = 1.423\ \text{N}\cdot\text{m}$ ◀

17.53

GIVEN: 4000-lb FLYWHEEL WITH $\bar{k} = 27$ in. COASTS TO REST FROM ANGULAR VELOCITY OF 450 rpm. FRICTION COUPLE IS OF MAGNITUDE 125 lb·in.
FIND: TIME REQUIRED TO COAST TO REST

$\bar{I} = m\bar{k}^2 = \left(\frac{4000\ \text{lb}}{32.2}\right)\left(\frac{27}{12}\ \text{in.}\right)^2 = 628.88\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

$\omega_1 = 450\ \text{rpm}\left(\frac{2\pi}{60}\right) = 47.125\ \text{rad/s},\quad M = 125\ \text{lb}\cdot\text{in.} = 10.417\ \text{lb}\cdot\text{ft}$



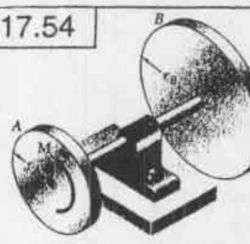
SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $\bar{I}\omega_1 - Mt = 0$

$t = \frac{\bar{I}\omega_1}{M} = \frac{(628.88\ \text{lb}\cdot\text{ft}\cdot\text{s}^2)(47.125\ \text{rad/s})}{10.417\ \text{lb}\cdot\text{ft}} = 2845\ \text{s}$

$t = 2845\ \text{s}\left(\frac{\text{min}}{60\ \text{s}}\right) \qquad t = 47.4\ \text{min.}$ ◀

17.54

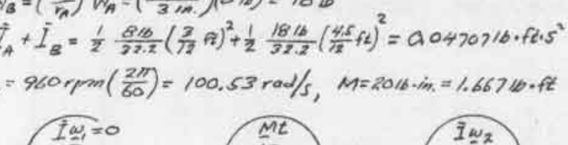


GIVEN: $W_A = 8$ lb, $r_A = 3$ in., $r_B = 4.5$ in.
DISKS OF SAME MATERIAL AND THICKNESS.
$M = 20$ lb·in., $\omega_1 = 0$
FIND: TIME UNTIL $\omega_2 = 960$ rpm

$W_B = \left(\frac{r_B}{r_A}\right)^2 W_A = \left(\frac{4.5\ \text{in.}}{3\ \text{in.}}\right)^2(8\ \text{lb}) = 18\ \text{lb}$

$\bar{I} = \bar{I}_A + \bar{I}_B = \frac{1}{2}\,\frac{8\ \text{lb}}{32.2}\left(\frac{3}{12}\ \text{ft}\right)^2 + \frac{1}{2}\,\frac{18\ \text{lb}}{32.2}\left(\frac{4.5}{12}\ \text{ft}\right)^2 = 0.04707\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

$\omega_2 = 960\ \text{rpm}\left(\frac{2\pi}{60}\right) = 100.53\ \text{rad/s},\quad M = 20\ \text{lb}\cdot\text{in.} = 1.667\ \text{lb}\cdot\text{ft}$



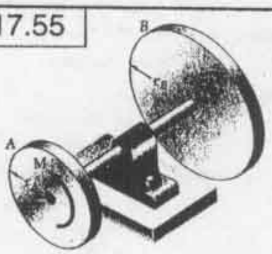
SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C: $0 + Mt = \bar{I}\omega_2$

$t = \frac{\bar{I}\,\omega_2}{M} = \frac{(0.04707\ \text{lb}\cdot\text{ft}\cdot\text{s}^2)(100.53\ \text{rad/s})}{1.667\ \text{lb}\cdot\text{ft}}$

$t = 2.839\ \text{s} \qquad t = 2.84\ \text{s}$ ◀

17.55

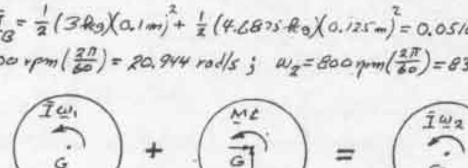


GIVEN: $m_A = 3\text{ kg}$, $r_A = 100\text{ mm}$, $r_B = 125\text{ mm}$. DISKS OF SAME MATERIAL AND THICKNESS.
$\omega_1 = 200\text{ rpm}$, $\omega_2 = 800\text{ rpm}$
$t_{1\to 2} = 3\text{ s}$.
FIND: MAGNITUDE OF COUPLE M

$$m_B = \left(\frac{r_B}{r_A}\right)^2 m_A = \left(\frac{125\text{ mm}}{100\text{ mm}}\right)^2 3\text{ kg} = 4.6875\text{ kg}$$

$$\bar{I} = \bar{I}_A + \bar{I}_B = \tfrac{1}{2}(3\text{ kg})(0.1\text{ m})^2 + \tfrac{1}{2}(4.6875\text{ kg})(0.125\text{ m})^2 = 0.05162\text{ kg}\cdot\text{m}^2$$

$$\omega_1 = 200\text{ rpm}\left(\tfrac{2\pi}{60}\right) = 20.944\text{ rad/s};\quad \omega_2 = 800\text{ rpm}\left(\tfrac{2\pi}{60}\right) = 83.776\text{ rad/s}$$



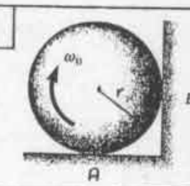
SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to 2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT G: $\bar{I}\omega_1 + Mt = \bar{I}\omega_2$

$$M = \frac{\bar{I}}{t}(\omega_2 - \omega_1) = \frac{0.05162\text{ kg}\cdot\text{m}^2}{3\text{ s}}(83.776\text{ rad/s} - 20.944\text{ rad/s})$$

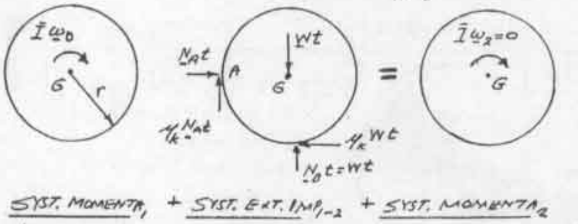
$$M = 1.081\text{ N}\cdot\text{m}$$ ◀

17.56



GIVEN: SPHERE OF WEIGHT W
μ_k = COEF. OF KINETIC FRICTION
FIND: EXPRESSION FOR TIME REQUIRED FOR SPHERE TO COME TO REST.

$$\bar{I} = \frac{2}{5}mr^2 = \frac{2}{5}\frac{W}{g}r^2$$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ + SYST. MOMENTA$_2$

+y↑ COMPONENTS: $0 + N_Bt + \mu_k N_At - Wt = 0$ (1)

$\underrightarrow{+x}$ COMPONENTS: $0 + N_At - \mu_k N_Bt = 0$ (2)

FROM EQ(2): $N_A = \mu_k N_B$ (3)

SUBSTITUTE INTO EQ(1): $N_Bt + \mu_k(\mu_k N_B)t - Wt$

$$N_B = \frac{1}{1+\mu_k^2}W$$

EQ(2): $$N_A = \frac{\mu_k}{1+\mu_k^2}W$$

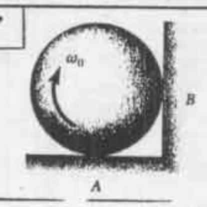
+↻ MOMENTS ABOUT G: $\bar{I}\omega_0 - (\mu_k N_At)r - (\mu_k N_Bt)r = 0$

$$\frac{2}{5}\frac{W}{g}r^2\omega_0 - \frac{\mu_k^2}{1+\mu_k^2}rWt - \frac{\mu_k}{1+\mu_k^2}rWt = 0$$

$$\frac{2}{5}\frac{r}{g}\omega_0 - \frac{\mu_k + \mu_k^2}{1+\mu_k^2}t = 0$$

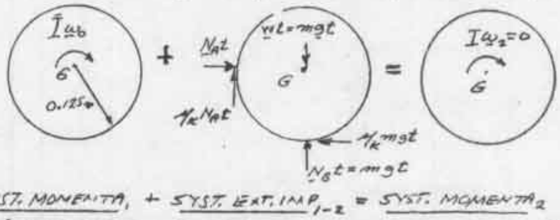
$$t = \frac{1+\mu_k^2}{\mu_k + \mu_k^2}\cdot\frac{2}{5}\frac{r}{g}\omega_0$$ ◀

17.57



GIVEN: $m = 3\text{ kg}$, $r = 125\text{ mm}$
$\omega_0 = 90\text{ rad/s}$
$\mu_k = 0.10$
FIND: TIME REQUIRED FOR SPHERE TO COME TO REST

$$\bar{I} = \frac{2}{5}mr^2 = \frac{2}{5}(3\text{ kg})(0.125\text{ m})^2 = 18.75\times 10^{-3}\text{ kg}\cdot\text{m}^2$$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+y↑ COMPONENTS: $0 + N_Bt - \mu_k N_At - mgt = 0$ (1)

$\underrightarrow{+x}$ COMPONENTS: $0 + N_At - \mu_k N_Bt = 0$ (2)

EQ(2): $N_A = \mu_k N_B$

EQ(1): $N_Bt - \mu_k(\mu_k N_B)t - mgt = 0$

$$N_B = \frac{mg}{1+\mu_k^2} = \frac{(3\text{ kg})\,9.81\text{ m/s}^2}{1+(0.10)^2} = 29.139\text{ N}$$

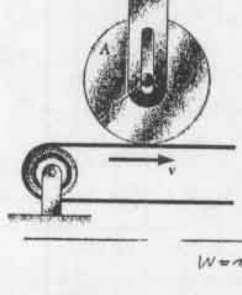
$$N_A = \mu_k N_B = 0.1(29.139\text{ N}) = 2.9139\text{ N}$$

+↻ MOMENTS ABOUT G: $\bar{I}\omega_0 - (\mu_k N_At)r - (\mu_k N_Bt)r = 0$

$$t = \frac{\bar{I}\omega_0}{\mu_k r(N_A + N_B)} = \frac{(18.75\times 10^{-3}\text{ kg}\cdot\text{m}^2)(90\text{ rad/s})}{(0.10)(0.125\text{ m})(29.139\text{ N} + 2.9139\text{ N})}$$

$t = 4.212\text{ s}$ $\qquad t = 4.21\text{ s}$ ◀

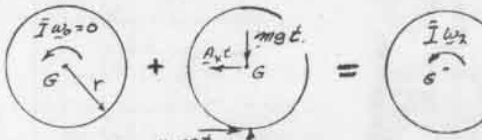
17.58 and 17.59



GIVEN: DISK AT REST PLACED IN CONTACT WITH BELT.
COEF. OF KINETIC FRICTION = μ_k.

FIND: TIME REQUIRED FOR DISK TO REACH CONSTANT ω.
PROBLEM 17.58: IN TERMS OF v, g, AND μ_k
PROBLEM 17.59: FOR $r = 3\text{ in}$, $W = 6\text{ lb}$, $v = 50\text{ ft/s}$, $\mu_k = 0.20$.

$W = mg$ $\qquad \bar{I} = \frac{1}{2}mr^2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT G: $0 + (\mu_k mgt)r = \bar{I}\omega_2$ (1)

FINAL ANGULAR VELOCITY: $v = r\omega_2$; $\omega_2 = v/r$

EQ(1): $(\mu_k mgt)r = \frac{1}{2}mr^2\left(\frac{v}{r}\right)$

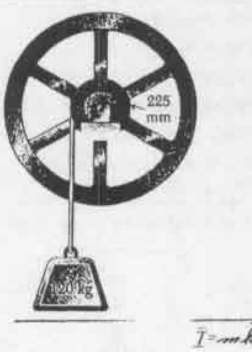
PROBLEM 17.58: $$t = \frac{v}{2g\mu_k}$$ ◀

NOTE: RESULT IS INDEPENDENT OF W AND r.

PROBLEM 17.59: DATA: $v = 50\text{ ft/s}$, $\mu_k = 0.20$

$$t = \frac{v}{2g\mu_k} = \frac{50\text{ ft/s}}{2(32.2\text{ ft/s}^2)(0.20)};\qquad t = 3.88\text{ s}$$ ◀

17.60 and 17.61

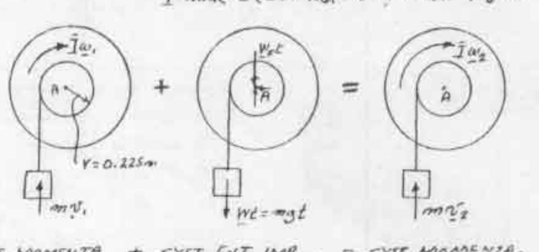


GIVEN: 350-kg FLYWHEEL OF $\bar{k} = 600$ mm.
$\omega_1 = 100$ rpm ↻ WHEN POWER IS CUT OFF AND SYSTEM COASTS.

PROBLEM 17.60: FIND TIME REQUIRED FOR SYSTEM TO COME TO REST

PROBLEM 17.61: FIND TIME WHEN $\omega_2 = 40$ rpm ↻

$\bar{I} = m\bar{k}^2 = (350\text{ kg})(0.6\text{ m})^2 = 126\text{ kg}\cdot\text{m}^2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $mv_1 r + \bar{I}\omega_1 - mgt\,r = mv_2 r + \bar{I}\omega_2$

SUBSTITUTE: $v_1 = r\omega_1$ AND $v_2 = r\omega_2$

$$(mr^2+\bar{I})\omega_1 - mgt\,r = (mr^2+\bar{I})\omega_2$$

$$t = \frac{mr^2+\bar{I}}{mgr}(\omega_1-\omega_2) = \frac{(120\text{ kg})(0.225\text{ m})^2 + 126\text{ kg}\cdot\text{m}^2}{(120\text{ kg})(9.81)(0.225\text{ m})}(\omega_1-\omega_2)$$

$$t = \frac{6.075+126}{264.87}(\omega_1-\omega_2) \qquad t = 0.49864(\omega_1-\omega_2) \qquad (1)$$

PROBLEM 17.60:

$\omega_1 = 100\text{ rpm}\left(\frac{2\pi}{60}\right) = 10.472$ rad/s $\qquad \omega_2 = 0$

EQ(1): $t = 0.49864(10.472\text{ rad/s})$ $\qquad t = 5.22$ s ◀

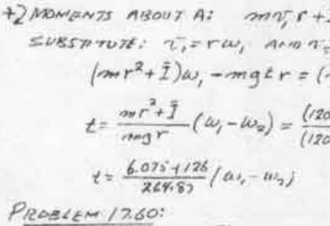
PROBLEM 17.61: $\omega_1 = 100\text{ rpm}\left(\frac{2\pi}{60}\right) = 10.472$ rad/s

$\omega_2 = 40\text{ rpm}\left(\frac{2\pi}{60}\right) = 4.189$ rad/s

EQ(1): $t = 0.49864(10.472 - 4.189)$ $\qquad t = 3.13$ s ◀

17.62

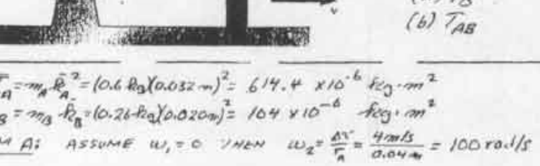
GIVEN: $m_A = 600$ g, $\bar{k}_A = 32$ mm
$m_B = 260$ g, $\bar{k}_B = 20$ mm
SPEED OF TAPE INCREASES 4 m/s IN 0.3 s
FIND:
(a) T_B
(b) T_{AB}



$\bar{I}_A = m_A \bar{k}_A^2 = (0.6\text{ kg})(0.032\text{ m})^2 = 614.4\times10^{-6}\text{ kg}\cdot\text{m}^2$

$\bar{I}_B = m_B \bar{k}_B^2 = (0.26\text{ kg})(0.020\text{ m})^2 = 104\times10^{-6}\text{ kg}\cdot\text{m}^2$

DRUM A: ASSUME $\omega_1 = 0$ THEN $\omega_2 = \frac{\Delta v}{r_A} = \frac{4\text{ m/s}}{0.04\text{ m}} = 100$ rad/s



+↻ MOMENTS ABOUT A: $0 - 3.5t(0.04\text{ m}) + T_{AB}t(0.04\text{ m}) = \bar{I}_A\omega_2$

$t = 0.3$ s: $\quad -3.5(0.3)(0.04) + T_{AB}(0.3)(0.04) = (614.4\times10^{-6})(100\text{ rad/s})$

$-0.042 + 0.012\,T_{AB} = 0.06144$

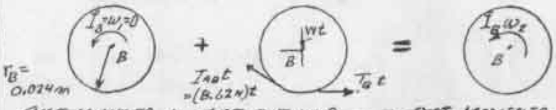
$T_{AB} = 8.62$ N ◀

(CONTINUED)

17.62 continued

DRUM B: WE RECALL: $T_{AB} = 8.62$ N

$\omega_1 = 0 \qquad \omega_2 = \frac{\Delta v}{r_B} = \frac{4\text{ m/s}}{0.024\text{ m}} = 166.67$ rad/s



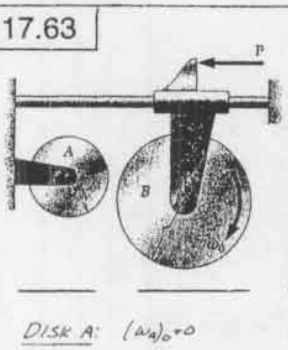
SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT B: $0 + T_B t r - T_{AB} t r = \bar{I}_B\omega_2$

$t = 0.3$ s: $\quad T_B(0.3\text{ s})(0.024\text{ m}) - (8.62\text{ N})(0.3\text{ s})(0.024\text{ m}) = (104\times10^{-6}\text{ kg}\cdot\text{m}^2)(166.67\text{ rad/s})$

$T_B = 11.03$ N ◀

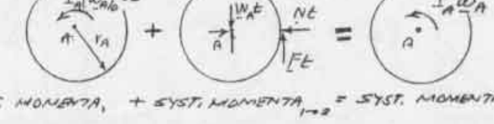
17.63



GIVEN: DISK A IS AT REST WHEN DISKS A AND B ARE BROUGHT INTO CONTACT

SHOW THAT FINAL ω_B DEPENDS ON ONLY ω_0 AND $\frac{m_A}{m_B}$

DISK A: $(\omega_A)_0 = 0$

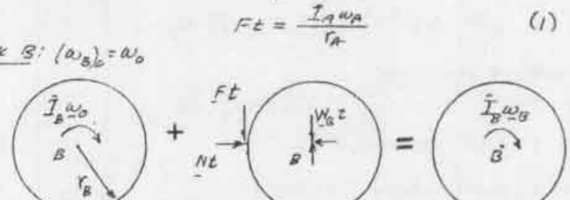


SYST. MOMENTA$_1$ + SYST. MOMENTA$_{1\to2}$ = SYST. MOMENTA$_2$

+↺ MOMENTS ABOUT A: $0 + (Ft)r_A = \bar{I}_A\omega_A$

$$Ft = \frac{\bar{I}_A\omega_A}{r_A} \qquad (1)$$

DISK B: $(\omega_B)_0 = \omega_0$



+↻ MOMENTS ABOUT B: $\bar{I}_B\omega_0 - (Ft)r_B = \bar{I}_B\omega_B$

SUBSTITUTE FOR Ft FROM EQ(1)

$$\bar{I}_B\omega_0 - \bar{I}_A\omega_A\frac{r_B}{r_A} = \bar{I}_B\omega_B \qquad (2)$$

FOR FINAL ANGULAR VELOCITIES: $r_A\omega_A = r_B\omega_B$; $\quad \omega_A = \frac{r_B}{r_A}\omega_B$

EQ(2) $\quad \bar{I}_B\omega_0 - \bar{I}_A\omega_B\left(\frac{r_B}{r_A}\right)^2 = \bar{I}_B\omega_B$

$$\omega_B = \frac{\omega_0}{1+\frac{\bar{I}_A}{\bar{I}_B}\left(\frac{r_B}{r_A}\right)^2} \qquad (3)$$

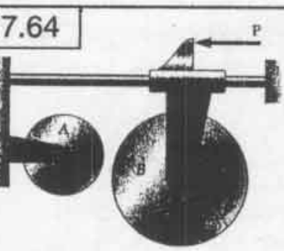
BUT FOR UNIFORM DISKS: $\frac{\bar{I}_A}{\bar{I}_B} = \frac{\frac{1}{2}m_A r_A^2}{\frac{1}{2}m_B r_B^2} = \frac{m_A}{m_B}\left(\frac{r_A}{r_B}\right)^2$

SUBSTITUTE INTO EQ(3):

$$\omega_B = \frac{\omega_0}{1+\frac{m_A}{m_B}}$$ ◀

THUS, ω_B DEPENDS ON ONLY ω_0 AND $\frac{m_A}{m_B}$

17.64



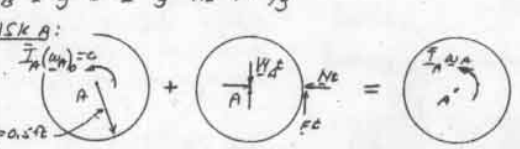
GIVEN: $W_A = 7.5\,lb$, $r_A = 6\,in.$
$W_B = 10\,lb$, $r_B = 8\,in.$
$\omega_0 = 900\,rpm$

FIND: (a) FINAL ω_A AND ω_B
(b) IMPULSE OF FRICTION FORCE EXERTED ON DISK A.

$\bar{I}_A = \frac{1}{2}\frac{W_A}{g}r_A^2 = \frac{1}{2}\frac{7.5\,lb}{g}\left(\frac{6}{12}ft\right)^2 = \frac{7.5}{8g}$

$\bar{I}_B = \frac{1}{2}\frac{W_B}{g}r_B^2 = \frac{1}{2}\frac{10\,lb}{g}\left(\frac{8}{12}ft\right)^2 = \frac{20}{9g}$

DISK A:

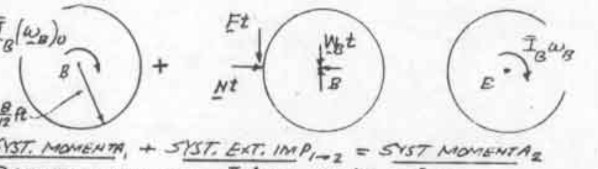


SYST MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $0 + (Ft)r_A = \bar{I}\omega_A$

$(Ft)(0.5\,ft) = \frac{7.5}{8g}\omega_A \qquad Ft = \frac{7.5}{4g}\omega_A \qquad (1)$

DISK B: $(\omega_B)_0 = \omega_0 = 900\,rpm\left(\frac{2\pi}{60}\right) = 30\pi\,rad/s$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST MOMENTA$_2$

+↻ MOMENTS ABOUT B: $\bar{I}_B(\omega_B)_0 - (Ft)r_B = \bar{I}_B\omega_B$

$\frac{20}{9g}\cdot 30\pi - (Ft)\left(\frac{8}{12}ft\right) = \frac{20}{9g}\omega_B$

SUBSTITUTE FROM EQ(1):

$\frac{20}{9g}\cdot 30\pi - \left(\frac{7.5}{4g}\omega_A\right)\left(\frac{8}{12}\right) = \frac{20}{9g}\omega_B$

$30\pi - 0.5625\,\omega_A = \omega_B \qquad (2)$

FINAL VELOCITIES OCCUR WHEN:

$r_A\omega_A = r_B\omega_B; \quad \omega_B = \frac{r_A}{r_B}\omega_A = \frac{6\,in.}{8\,in.}\omega_A = 0.75\,\omega_A \qquad (3)$

SUBSTITUTE FOR ω_B FROM (2) INTO (3)

$30\pi - 0.5625\,\omega_A = 0.75\,\omega_A$

$30\pi = 1.3125\,\omega_A \qquad \omega_A = 71.807\,rad/s$

$\omega_A = 71.807\,rad/s\left(\frac{60}{2\pi}\right) \qquad \omega_A = 685.7\,rpm$

EQ(3): $\omega_B = 0.75\,\omega_A = 0.75(685.7\,rpm) \qquad \omega_B = 514.3\,rpm$

IMPULSE OF Ft EXERTED ON DISK A:

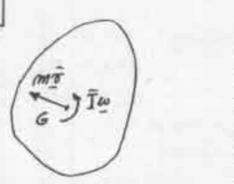
EQ(1) $\quad Ft = \frac{7.5}{4g}\omega_A = \frac{7.5}{4(32.2)}(71.807\,rad/s)$

$Ft = 4.18\,lb\cdot s \uparrow$ ◀

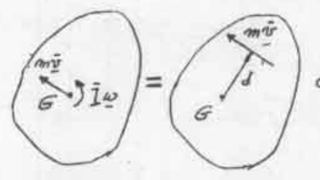
$\omega_A = 686\,rpm$ ↺ ◀

$\omega_B = 514\,rpm$ ↻ ◀

17.65



SHOW THAT SYSTEM OF MOMENTA IS EQUIVALENT TO A SINGLE VECTOR AND EXPRESS THE DISTANCE FROM G TO THE LINE OF ACTION OF THE VECTOR IN TERMS OF $\bar{k}$, $\bar{v}$, AND ω.



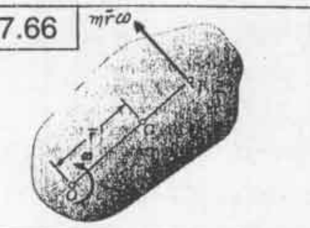
+↺ MOMENTS ABOUT G

$\bar{I}\omega = (m\bar{v})d$

$d = \frac{\bar{I}\omega}{m\bar{v}} = \frac{m\bar{k}^2}{m\bar{v}}$

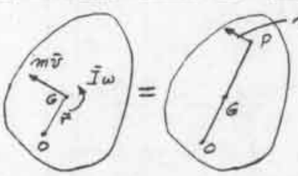
$d = \frac{\bar{k}^2}{\bar{v}}$ ◀

17.66



SHOW THAT SYSTEM OF MOMENTA IS EQUIVALENT TO $m\bar{r}\omega$ LOCATED AT P WHERE

$GP = \bar{k}^2/\bar{r}$



+↺ MOMENTS ABOUT G

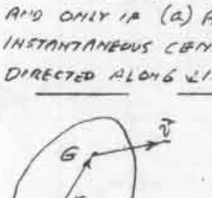
$\bar{I}\omega = (m\bar{r}\omega)GP$

$GP = \frac{\bar{I}\omega}{m\bar{r}\omega} = \frac{m\bar{k}^2}{m\bar{r}}$

$GP = \frac{\bar{k}^2}{\bar{r}}$ ◀

17.67

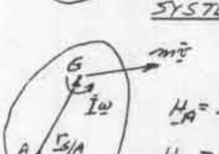
FOR A RIGID SLAB IN PLANE MOTION, SHOW THAT $\underline{H}_A$ IS EQUAL TO $I_A\omega$, IF AND ONLY IF (a) A IS THE MASS CENTER, (b) A IS THE INSTANTANEOUS CENTER OF ROTATION, (c) $\underline{v}_A$ IS DIRECTED ALONG LINE AG.



FOR GENERAL PLANE MOTION

$\underline{\bar{v}} = \underline{v}_A + \underline{v}_{G/A} = \underline{v}_A + \underline{\omega}\times\underline{r}_{G/A}$

SYSTEM OF MOMENTA



MOMENTS ABOUT A

$\underline{H}_A = \bar{I}\underline{\omega} + \underline{r}_{G/A}\times m\underline{\bar{v}}$

$\underline{H}_A = \bar{I}\underline{\omega} + m\,\underline{r}_{G/A}\times(\underline{v}_A + \underline{\omega}\times\underline{r}_{G/A})$

$\underline{H}_A = \bar{I}\underline{\omega} + m\,\underline{r}_{G/A}\times\underline{v}_A + m\,\underline{r}_{G/A}\times(\underline{\omega}\times\underline{r}_{G/A})$

SINCE $\underline{\omega} \perp \underline{r}_{G/A}$ THE TRIPLE VECTOR PRODUCT CAN BE WRITTEN: $\underline{r}_{G/A}\times(\underline{\omega}\times\underline{r}_{G/A}) = r_{G/A}^2\underline{\omega}$

THUS $\underline{H}_A = \bar{I}\underline{\omega} + m\,\underline{r}_{G/A}\times\underline{v}_A + m\,r_{G/A}^2\,\underline{\omega}$

BY PARALLEL-AXIS THEOREM: $I_A = \bar{I} + m\,r_{G/A}^2$

WE NOW HAVE $\underline{H}_A = I_A\,\omega + m\,\underline{r}_{G/A}\times\underline{v}_A$

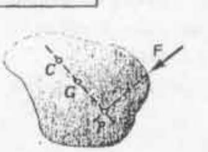
$\therefore\ \underline{H}_A = I_A\,\omega$, ONLY WHEN $\underline{r}_{G/A}\times\underline{v}_A = 0$

(a) $r_{G/A} = 0$: A COINCIDES WITH G

(b) $v_A = 0$: A IS INSTANT. CENTER

(c) $\underline{r}_{G/A}$ AND $\underline{v}_A$ ARE COLLINEAR: $\underline{v}_A$ IS DIRECTED ALONG AG.

17.68



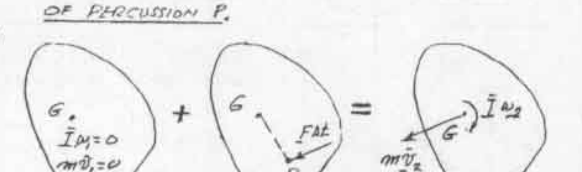
GIVEN: IMPULSIVE FORCE $\underline{F}$ IS APPLIED TO SLAB.

SHOW THAT: (a) INST. CENTER IS AT C AND $GC = \bar{k}^2/GP$.

(b) IF $\underline{F}$ WERE APPLIED AT C THEN P IS THE INST. CENTER

Δt = TIME OF APPLICATION OF F AT THE CENTER OF PERCUSSION P.

(a)

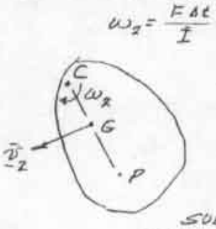


SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+ COMPONENTS: $F\Delta t = m\bar{v}_2$; $\bar{v}_2 = \frac{F\Delta t}{m}$ (1)

+ MOMENTS ABOUT G: $(F\Delta t)(GP) = \bar{I}\omega_2$

$$\omega_2 = \frac{F\Delta t}{\bar{I}}(GP) = \frac{F\Delta t}{m\bar{k}^2}(GP) \quad (2)$$



KINEMATICS: THE INSTANTANEOUS CENTER MUST BE LOCATED ON A LINE ⊥ TO $\bar{v}_2$, THAT IS, ON GP.

ALSO, $\bar{v}_2 = (GC)\omega_2$ $\quad GC = \frac{\bar{v}_2}{\omega_2}$

SUBSTITUTE FROM (1) AND (2)

$$GC = \frac{\frac{F\Delta t}{m}}{\frac{F\Delta t}{m\bar{k}^2}(GP)} \qquad GC = \frac{\bar{k}^2}{GP}$$

(b)

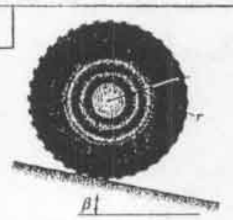
WE NOW ASSUME THAT F IS APPLIED SO THAT THE NEW CENTER OF PERCUSSION P' IS LOCATED AT C.

FROM PART a, WE NOTE THAT NEW INST. CENTER WILL BE LOCATED AT C' WHERE

$$GC' = \frac{\bar{k}^2}{GP'} = \frac{\bar{k}^2}{GC} = \frac{\bar{k}^2}{\bar{k}^2/GP} = GP$$

THUS, NEW INSTANTANEOUS CENTER IS LOCATED AT P

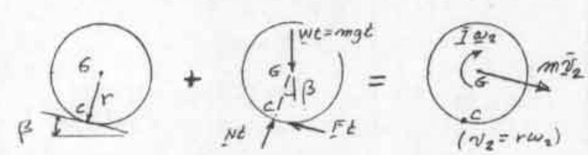
17.69



GIVEN: $\bar{k}$ = RADIUS OF GYRATION $v_1 = 0$

FIND: (a) $\bar{v}_2$ AT TIME t

(b) μ_s REQUIRED TO PREVENT SLIPPING



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+ MOMENTS ABOUT C: $(Wt\sin\beta)r = \bar{I}\omega_2 + m\bar{v}_2 r$

$mgt\sin\beta = m\bar{k}^2\omega_2 + mr^2\omega_2$

$$\omega_2 = \frac{rgt\sin\beta}{r^2+\bar{k}^2} \quad (1)$$

(CONTINUED)

17.69 continued

(a) $\bar{v}_2 = r\omega_2$: $\quad \bar{v}_2 = \frac{r^2}{r^2+\bar{k}^2}\,gt\sin\beta$ ∡ β

(b) + COMPONENTS: $Nt - mgt\cos\beta$

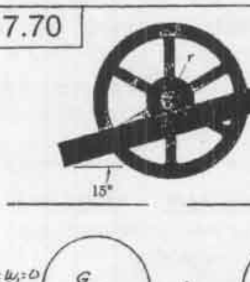
$N = mg\cos\beta$

+ MOMENTS ABOUT G: $(Ft)r = \bar{I}\omega_2$

$$F = \frac{\bar{I}}{rt}\omega_2 = \frac{m\bar{k}^2}{rt}\cdot\frac{r^2gt}{r^2+\bar{k}^2}\sin\beta = \frac{\bar{k}^2}{r^2+\bar{k}^2}mg\sin\beta$$

$$\mu_s = \frac{F}{N} = \frac{\bar{k}^2}{r^2+\bar{k}^2}\cdot\frac{mg\sin\beta}{mg\cos\beta}; \quad \mu_s = \frac{\bar{k}^2}{r^2+\bar{k}^2}\tan\beta$$

17.70

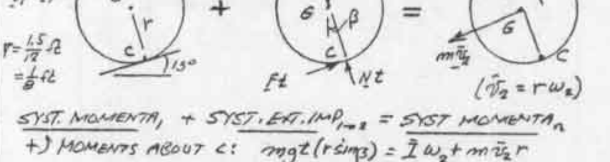


GIVEN: $r = 1.5$ in.

WHEEL STARTS FROM REST AND ROLLS WITHOUT SLIDING.

$\bar{v}_2 = 6$ in./s AT $t = 30$ s.

FIND: $\bar{k}$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST MOMENTA$_2$

+ MOMENTS ABOUT C: $mgt(r\sin\beta) = \bar{I}\omega_2 + m\bar{v}_2 r$

$\cancel{m}gtr\sin\beta = \cancel{m}\bar{k}^2\omega_2 + \cancel{m}r^2\omega_2$

$gtr\sin\beta = (\bar{k}^2 + r^2)\omega_2 \quad (1)$

DATA: $r = \frac{1}{8}$ ft, $\bar{v}_2 = 6$ in./s $= 0.5$ ft/s, $t = 30$ s

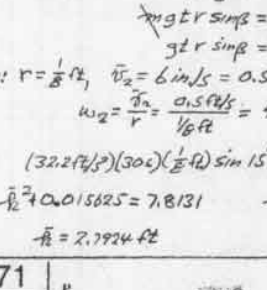
$\omega_2 = \frac{\bar{v}_2}{r} = \frac{0.5\text{ ft/s}}{1/8\text{ ft}} = 4$ rad/s

EQ(1) $(32.2\text{ ft/s}^2)(30\text{s})(\frac{1}{8}\text{ft})\sin 15° = \left[\bar{k}^2 + (0.125\text{ ft})^2\right](4\text{ rad/s})$

$\bar{k}^2 + 0.015625 = 7.8131 \qquad \bar{k}^2 = 7.7975$

$\bar{k} = 2.7924$ ft $\qquad \bar{k} = 2.79$ ft

17.71



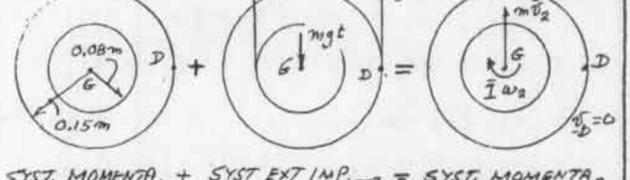
GIVEN: $m = 3$ kg, $\bar{k} = 100$ mm

PULLEY IS AT REST WHEN $P = 24$ N IS APPLIED TO B

FIND: (a) $\bar{v}$ AFTER 1.5 S

(b) TENSION IN CORD C

$(\bar{v}_2 = 0.15\omega_2)$



SYST. MOMENTA$_1$ + SYST EXT IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+ MOMENTS ABOUT D: $Pt(0.08+0.15) - mgt(0.15) = \bar{I}\omega_2 + m\bar{v}_2(0.15)$

$(24\text{N})(1.5\text{s})(0.23) - (3\text{kg})(9.81)(1.5\text{s})(0.15) = (3\text{kg})(0.1\text{m})^2\omega_2 + (3\text{kg})(0.15)^2\omega_2$

$1.6583 = (0.03 + 0.0675)\omega_2$; $\omega_2 = 17.008$ rad/s

$\bar{v}_2 = (0.15)\omega_2 = (0.15)(17.008) = 2.551$ m/s $\quad \bar{v}_2 = 2.55$ m/s ↑

(CONTINUED)

17.71 continued

WE HAVE FOUND $\bar{v}_2 = 2.551\,m/s\uparrow$

$+\uparrow$ COMPONENTS: $Pt + T_c t - mgt = m\bar{v}_2$

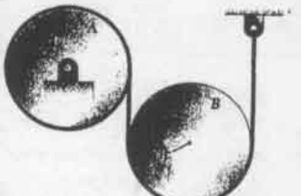
$(24N)(1.5s) + T_c(1.5s) - (3kg)(9.81)(1.5s) = (3kg)(2.551\,m/s)$

$36 + 1.5T_c - 44.145 = 7.653$

$1.5T_c = 15.798$

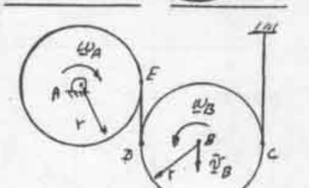
$T_c = 10.53\,N$ ◀

17.72



GIVEN: TWO 14-lb CYLINDERS OF RADIUS $r = 5$ in. SYSTEM IS RELEASED FROM REST WHEN $t = 0$.

FIND: (a) $\bar{v}_B$ AT $t = 3s$.
(b) TENSION IN BELT CONNECTING CYLINDERS



KINEMATICS CYLINDER B

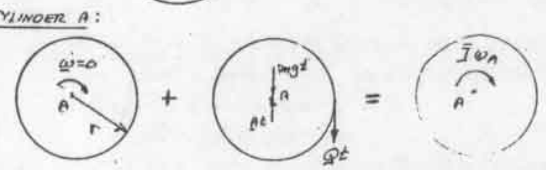
INSTANT. CENTER OF B IS AT C.

$\bar{v}_B = r\omega_B$

$v_D = v_E = 2r\omega_B$

CYLINDER A $\quad \omega_A = \frac{v_E}{r} = 2\omega_B$

CYLINDER A:



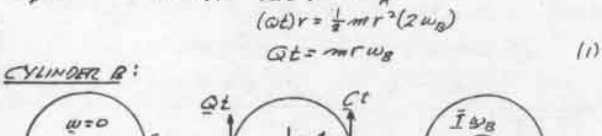
SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

$+\circlearrowright$ MOMENTS ABOUT A: $(Qt)r = \bar{I}\omega_A$

$(Qt)r = \frac{1}{2}mr^2(2\omega_B)$

$Qt = mr\omega_B \qquad (1)$

CYLINDER B:



$+\circlearrowright$ MOMENTS ABOUT C: $(mgt)r - (Qt)2r = \bar{I}\omega_B + m\bar{v}_B r$

$mgtr - (Qt)(2r) = \frac{1}{2}mr^2\omega_B + m(r\omega_B)r$

$mgt - 2(Qt) = \frac{3}{2}mr\omega_B \qquad (2)$

SUBSTITUTE FOR (Qt) FROM (1) $\quad mgt - 2(mr\omega_B) = \frac{3}{2}mr\omega_B$

$\omega_B = \frac{2}{7}\frac{gt}{r}$

$\bar{v}_B = r\omega_B; \quad \bar{v}_B = \frac{2}{7}gt$

EQ(1): $\quad Qt = mr\omega_B; \quad Qt = mr\left(\frac{2}{7}\frac{gt}{r}\right)$

$Q = \frac{2}{7}mg = \frac{2}{7}W$

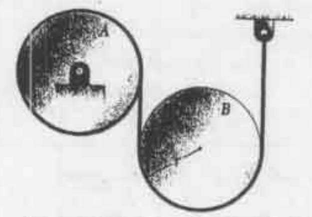
DATA: $W = 14\,lb, \ t = 3s$

(a) $\bar{v}_B = \frac{2}{7}gt = \frac{2}{7}(32.2\,ft/s^2)(3s) \qquad \bar{v}_B = 27.6\,ft/s\downarrow$ ◀

(b) $Q = \frac{2}{7}W = \frac{2}{7}(14\,lb) = 4\,lb$

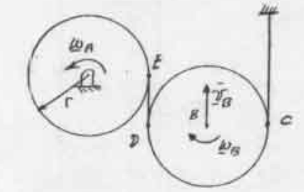
TENSION IN CONNECTING BELT = 4 lb ◀

17.73



GIVEN: TWO 14-lb CYLINDERS OF RADIUS 5 in. INITIALLY $\omega_A = 30\,rad/s\circlearrowright$

FIND: (a) TIME REQUIRED FOR ω_A TO BE REDUCED TO $\omega_A = 5\,rad/s\circlearrowright$
(b) TENSION IN BELT CONNECTING CYLINDERS



KINEMATICS: CYLINDER B

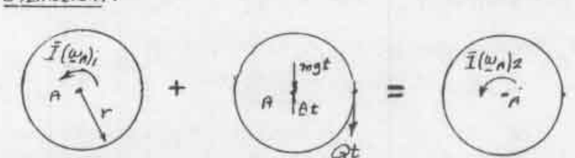
INSTANT. CENTER OF B IS AT C.

$\bar{v}_B = r\omega_B$

$v_D = v_E = 2r\omega_B$

CYLINDER A: $\quad \omega_A = \frac{v_E}{r} = 2\omega_B$

CYLINDER A:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

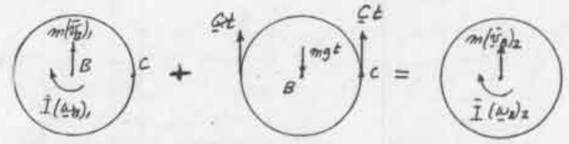
$+\circlearrowright$ MOMENTS ABOUT A: $\bar{I}(\omega_A)_1 - (Qt)r = \bar{I}(\omega_A)_2$

$(Qt)r = \frac{1}{2}mr^2\left[(\omega_A)_1 - (\omega_A)_2\right]$

$(Qt)r = \frac{1}{2}mr^2\left[2(\omega_B)_1 - 2(\omega_B)_2\right]$

$Qt = mr\left[(\omega_B)_1 - (\omega_B)_2\right] \qquad (1)$

CYLINDER B:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$+\circlearrowright$ MOMENTS ABOUT C:

$\bar{I}(\omega_B)_1 + m(\bar{v}_B)_1 r + Qt(2r) - (mgt)r = \bar{I}(\omega_B)_2 + m(\bar{v}_B)_2 r$

SUBSTITUTE $\bar{I} = \frac{1}{2}mr^2$; $(\bar{v}_B)_1 = r(\omega_B)_1$, AND $(\bar{v}_B)_2 = r(\omega_B)_2$

$Qt(2r) - (mgt)r = mr^2\left[(\omega_B)_2 - (\omega_B)_1\right] + \frac{1}{2}mr^2\left[(\omega_B)_2 - (\omega_B)_1\right]$

$2Qt - mgt = \frac{3}{2}mr\left[(\omega_B)_2 - (\omega_B)_1\right] \qquad (2)$

SUBSTITUTE FOR Qt FROM (1):

$2mr\left[(\omega_B)_1 - (\omega_B)_2\right] - mgt = \frac{3}{2}mr\left[(\omega_B)_2 - (\omega_B)_1\right]$

$t = \frac{7}{2}\frac{r}{g}\left[(\omega_B)_1 - (\omega_B)_2\right] \qquad (3)$

SUBSTITUTE FOR t FROM (3) INTO (1)

$Q\left\{\frac{7}{2}\frac{r}{g}\left[(\omega_B)_1 - (\omega_B)_2\right]\right\} = mr\left[(\omega_B)_1 - (\omega_B)_2\right]$

$Q = \frac{2}{7}mg = \frac{2}{7}W \qquad (4)$

DATA: $(\omega_A)_1 = 30\,rad/s \rightarrow (\omega_B)_1 = \frac{1}{2}(\omega_A)_1 = 15\,rad/s$

$(\omega_A)_2 = 5\,rad/s \rightarrow (\omega_B)_2 = \frac{1}{2}(\omega_A)_2 = 2.5\,rad/s$

$W = 14\,lb, \quad r = 5\,in. = 5/12\,ft$

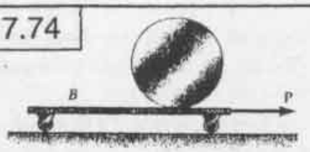
(a) EQ(3): $\quad t = \frac{7}{2}\cdot\frac{(5/12\,ft)}{32.2\,ft/s^2}\left[15\,rad/s - 2.5\,rad/s\right]$

$t = 0.5661\,s \qquad t = 0.566\,s$ ◀

(b) EQ(4): $\quad Q = \frac{2}{7}W = \frac{2}{7}(14\,lb) = 4\,lb$

TENSION IN CONNECTING BELT = 4 lb ◀

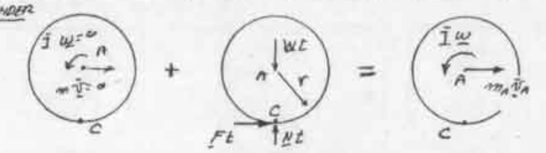
17.74



GIVEN:
CYLINDER: $m_A = 8\text{ kg},\ r = 240\text{ mm}$
CARRIAGE: $m_B = 3\text{ kg}$
SYSTEM AT REST WHEN $P = 10\text{ N}$ APPLIED FOR 1.25

FIND: (a) $\bar{v}_B$, (b) $\bar{v}_A$

CYLINDER



SYST MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↺ MOMENTS ABOUT C: $0 = \bar{I}\omega - m_A\bar{v}_A r$

$$\omega = \frac{m_A\bar{v}_A r}{\bar{I}} = \frac{m_A\bar{v}_A r}{\frac{1}{2}m_A r^2}\ ;\quad \omega = \frac{2\bar{v}_A}{r} \qquad (1)$$

$\xrightarrow{+}$ COMPONENTS: $Ft = m_A\bar{v}_A$

CARRIAGE:

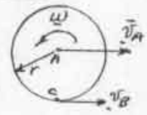
SYST MOMENTA$_1$ + SYST EXT IMP$_{1\to2}$ = SYST MOMENTA$_2$

$\xrightarrow{+}$ COMPONENTS: $Pt - Ft = m_B v_B$

$$Pt - m_A\bar{v}_A = m_B v_B$$

$$Pt = m_A\bar{v}_A + m_B v_B \qquad (2)$$

KINEMATICS: ASSUME ROLLING



$$v_B = \bar{v}_A + r\omega \qquad (3)$$

SUBSTITUTE FROM EQ (1)

$$v_B = \bar{v}_A + r\left(\frac{2\bar{v}_A}{r}\right);\quad v_B = 3\bar{v}_A \qquad (4)$$

EQ(2) $Pt = m_A\bar{v}_A + m_B(3\bar{v}_A)$

$$\bar{v}_A = \frac{Pt}{m_A + 3m_B} \qquad (5)$$

DATA: $m_A = 8\text{ kg},\quad m_B = 3\text{ kg}$
$P = 10\text{ N},\quad t = 1.25$

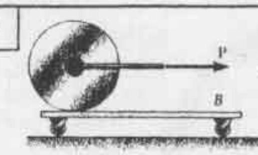
EQ(5) $\quad \bar{v}_A = \frac{(10\text{ N})(1.25)}{8\text{ kg} + 3(3\text{ kg})} = \frac{12}{17}\text{ m/s}$

$$\bar{v}_A = 0.706\text{ m/s} \rightarrow$$ ◀

EQ(4) $\quad v_B = 3v_A = 3\left(\frac{12}{17}\text{ m/s}\right) = \frac{36}{17}\text{ m/s}$

$$\bar{v}_B = 2.12\text{ m/s} \rightarrow$$ ◀

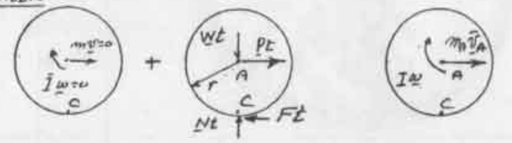
17.75



GIVEN:
CYLINDER: $m_A = 8\text{ kg},\ r = 240\text{ mm}$
CARRIAGE: $m_B = 3\text{ kg}$
SYSTEM AT REST WHEN $P = 10\text{ N}$ APPLIED FOR 1.25

FIND: (a) v_B, (b) $\bar{v}_A$

CYLINDER



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST MOMENTA$_2$

+↻ MOMENTS ABOUT A: $(Ft)r = \bar{I}\omega$

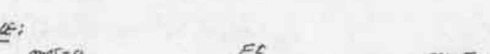
$$(Ft)r = \tfrac{1}{2}m_A r^2\omega$$

$$Ft = \tfrac{1}{2}m_A r\omega \qquad (1)$$

$\xrightarrow{+}$ COMPONENTS: $Pt - Ft = m_A\bar{v}_A \qquad (2)$

+↻ MOMENTS ABOUT C: $Pt = \bar{I}\omega + m_A\bar{v}_A r \qquad (3)$

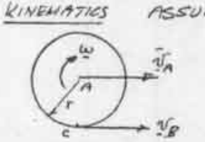
CARRIAGE:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

$\xrightarrow{+}$ COMPONENTS: $\quad Ft = m_B v_B$

KINEMATICS ASSUME ROLLING



$$v_B = \bar{v}_A - r\omega \qquad (4)$$

EQ(4): $\quad Ft = m_B(v_A - r\omega) \qquad (5)$

SUBSTITUTE EQ(1) → EQ(2): $\quad Pt - \frac{1}{2}m_A r\omega = m_A\bar{v}_A$

$$\bar{v}_A = \frac{Pt}{m_A} - \frac{1}{2}r\omega \qquad (6)$$

SUBSTITUTE EQ (1) → EQ(5):

$$\tfrac{1}{2}m_A r\omega = m_B(v_A - r\omega) \qquad (7)$$

SUBSTITUTE EQ(6) → EQ(7):

$$\tfrac{1}{2}m_A r\omega = m_B\left(\frac{Pt}{m_A} - \frac{1}{2}r\omega - r\omega\right)$$

$$\left(\tfrac{1}{2}m_A r + \tfrac{3}{2}m_B r\right)\omega = \frac{m_B}{m_A}Pt$$

$$\omega = \frac{2Pt}{r}\left(\frac{m_B}{m_A}\right)\frac{1}{m_A + 3m_B} \qquad (8)$$

DATA: $m_A = 8\text{ kg},\ m_B = 3\text{ kg}$
$P = 10\text{ N},\quad t = 1.25,\quad r = 0.24\text{ m}$

EQ(8): $\quad \omega = \frac{2(10\text{ N})(1.25)}{0.24\text{ m}} \cdot \frac{3\text{ kg}}{8\text{ kg}} \cdot \frac{1}{8\text{ kg} + 3(3\text{ kg})}$

$$\omega = \frac{37.5}{17}\text{ rad/s}$$

EQ(6): $\quad \bar{v}_A = \frac{Pt}{m_A} - \frac{1}{2}r\omega = \frac{(10\text{ N})(1.25)}{8\text{ kg}} - \frac{1}{2}(0.24\text{ m})\left(\frac{37.5}{17}\text{ rad/s}\right)$

$$\bar{v}_A = 1.5 - 0.2647 = 1.235\text{ m/s}$$

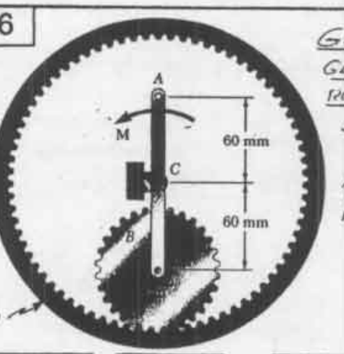
$$\bar{v}_A = 1.235\text{ m/s} \rightarrow$$ ◀

EQ(4): $\quad v_B = \bar{v}_A - r\omega = (1.235\text{ m/s}) - (0.24\text{ m})\left(\frac{37.5}{17}\text{ rad/s}\right)$

$$= 1.235 - 0.529 = 0.706\text{ m/s}$$

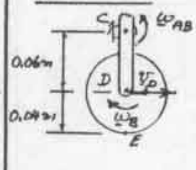
$$v_B = 0.706\text{ m/s} \rightarrow$$ ◀

17.76



GIVEN:
GEAR: $m_B = 1.8\,kg$, $\bar{k} = 32\,mm$
ROD: $m_{AD} = 2.5\,kg$
SYSTEM IS AT REST AT $t=0$
$M = 1.25\,N\cdot m$ IS APPLIED FOR $1.5\,s$
FIND: (a) ω_{AB}
(b) $\underline{v}_D$

KINEMATICS:



ROD ACD: $v_D = (0.06\,m)\,\omega_{AD}$ (1)

GEAR B: INST. CENTER AT E

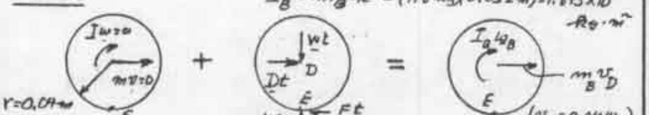
$v_D = (0.04\,m)\,\omega_B$

$v_D = v_D$: $0.06\,\omega_{AD} = 0.04\,\omega_B$

$\omega_B = 1.5\,\omega_{AD}$

GEAR B:

$\bar{I}_B = m_B \bar{k}^2 = (1.8\,kg)(0.032\,m)^2 = 1.843\times10^{-3}\,kg\cdot m^2$



SYST. MOMENTA$_1$ + SYST. EXT IMP$_{1\to2}$ = SYST. MOMENTA$_2$

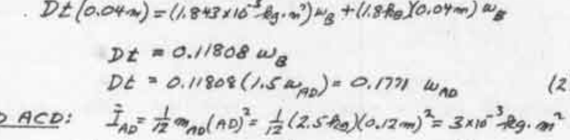
+) MOMENTS ABOUT E: $(Dt)r = \bar{I}_B\,\omega_B + m_B v_D r$

$Dt(0.04\,m) = (1.843\times10^{-3}\,kg\cdot m^2)\,\omega_B + (1.8\,kg)(0.04\,m)^2\omega_B$

$Dt = 0.11808\,\omega_B$

$Dt = 0.11808(1.5\,\omega_{AD}) = 0.1771\,\omega_{AD}$ (2)

ROD ACD: $\bar{I}_{AD} = \frac{1}{12}m_{AD}(AD)^2 = \frac{1}{12}(2.5\,kg)(0.12\,m)^2 = 3\times10^{-3}\,kg\cdot m^2$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ + SYST. MOMENTA$_2$

+) MOMENTS ABOUT C: $Mt - (Dt)(0.06\,m) = \bar{I}_{AD}\,\omega_{AD}$

$(1.25\,N\cdot m)(1.5\,s) - (Dt)(0.06\,m) = (3\times10^{-3}\,kg\cdot m^2)\,\omega_{AD}$

$1.875 - 0.06(Dt) = 3\times10^{-3}\,\omega_{AD}$

SUBSTITUTE FOR Dt FROM EQ (2)

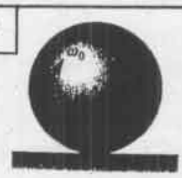
$1.875 - 0.06(0.1771\,\omega_{AD}) = 3\times10^{-3}\,\omega_{AD}$

$\underline{\omega}_{AD} = 137.6\,rad/s$ ↻ ◀

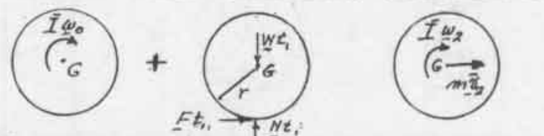
EQ.(1): $v_D = (0.06\,m)\,\omega_{AD} = (0.06\,m)(137.6\,rad/s)$

$\underline{v}_D = 8.26\,m/s \rightarrow$ ◀

17.77



GIVEN: SPHERE OF RADIUS r PLACED ON FLOOR (AT $t=0$) WITH $\underline{\bar{v}}=0$ AND $\underline{\omega}=\omega_0$ ↻.
COEF. OF KINETIC FRICTION = μ_k
FIND: (a) TIME t_1 WHEN ROLLING WITHOUT SLIDING STARTS
(b) $\underline{\bar{v}}$ AND $\underline{\omega}$ AT $t = t_1$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↑ y COMPONENTS: $Nt_1 - Wt_1 = 0$ $\quad N = W = mg$ (1)

$\xrightarrow{+}$ x COMPONENTS: $Ft_1 = m\bar{v}_2$ (2)

+) MOMENTS ABOUT G: $\bar{I}\omega_0 - Frt_1 = \bar{I}\omega_2$ (3)

SINCE $F = \mu_k N = \mu_k mg$, EQ(2) YIELDS

$\mu_k mg\,t_1 = m\bar{v}_2$ $\qquad \bar{v}_2 = \mu_k g t_1$ (4)

EQ(3): SINCE $\bar{I} = \frac{2}{5}mr^2$,

$\frac{2}{5}mr^2\omega_0 - (\mu_k mg)r\,t_1 = \frac{2}{5}mr^2\omega_2$

$\omega_2 = \omega_0 - \frac{5}{2}\frac{\mu_k g t_1}{r}$ (5)

SLIDING STOPS WHEN $\bar{v}_2 = r\omega$

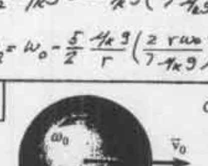
$\mu_k g t_1 = r\omega_0 - \frac{5}{2}\mu_k g t_1$

$\frac{7}{2}\mu_k g t_1 = r\omega_0$ $\qquad t_1 = \frac{2}{7}\frac{r\omega_0}{\mu_k g}$ ◀

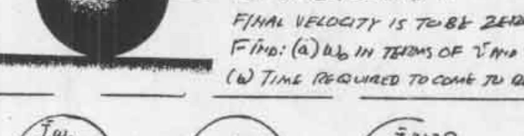
EQ(4): $\bar{v}_2 = \mu_k g\,t = \mu_k g\left(\frac{2}{7}\frac{r\omega_0}{\mu_k g}\right)$ $\qquad \underline{\bar{v}}_2 = \frac{2}{7}r\omega_0 \rightarrow$ ◀

EQ(5): $\omega_2 = \omega_0 - \frac{5}{2}\frac{\mu_k g}{r}\left(\frac{2\,r\omega_0}{7\,\mu_k g}\right)$ $\qquad \underline{\omega}_2 = \frac{2}{7}\,\omega_0$ ↻ ◀

17.78



GIVEN: SPHERE OF RADIUS r PLACED ON FLOOR WITH VELOCITIES SHOWN. IF FINAL VELOCITY IS TO BE ZERO,
FIND: (a) ω_0 IN TERMS OF $\bar{v}_0$ AND r.
(b) TIME REQUIRED TO COME TO REST



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ + SYST. MOMENTA$_2$

+↑ y COMPONENTS: $Nt - Wt = 0$ $\quad N = W = mg$ (1)

$\xrightarrow{+}$ x COMPONENTS: $m\bar{v}_0 - Ft = 0$ $\quad Ft = m\bar{v}_0$ (2)

+) MOMENTS ABOUT G: $\bar{I}\omega_0 - (Ft)r = 0$ (3)

SUBSTITUTE FOR Ft (from EQ 2) AND $\bar{I} = \frac{2}{5}mr^2$

EQ.(3): $\frac{2}{5}mr^2\omega_0 - (m\bar{v}_0)r = 0$

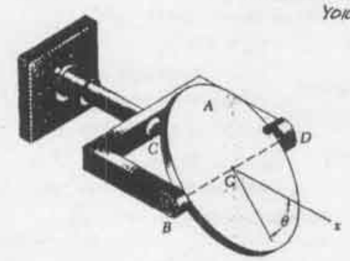
$\omega_0 = \frac{5}{2}\cdot\frac{\bar{v}_0}{r}$ ◀

EQ(2): $t = \frac{m\bar{v}_0}{F} = \frac{m\bar{v}_0}{\mu_k mg}$; $\qquad t = \frac{\bar{v}_0}{\mu_k g}$ ◀

17.79

GIVEN: DISK: $W_D = 2.5\,lb$, $r = 4\,in.$,
YOKE: $W_Y = 1.5\,lb$, $\bar{k} = 3\,in.$
WHEN $\theta = 0$, $\omega_x = 120\,rpm$

FIND: ω_x WHEN $\theta = 90°$



ROTATION ABOUT x AXIS:

SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT THE x AXIS. $\bar{I}_1\omega_1 = \bar{I}\omega_2$ (1)

$$\bar{I}_1 = \bar{I}_{Yoke} + \bar{I}_{DISK,\theta=0} = m_Y\bar{k}^2 + \frac{1}{4}m_D r^2$$

$$= \frac{1.5\,lb}{g}\left(\frac{3}{12}\,ft\right)^2 + \frac{1}{4}\,\frac{2.5\,lb}{g}\left(\frac{4}{12}\,in\right)^2 =$$

$$= \frac{0.09375}{g} + \frac{0.06944}{g} = 0.16319\,\frac{1}{g}$$

$$\bar{I}_2 = \bar{I}_{Yoke} + \bar{I}_{DISK,\theta=90°} = m_Y\bar{k}^2 + \frac{1}{2}m_D r^2$$

$$= \frac{1.5\,lb}{g}\left(\frac{3}{12}\,ft\right)^2 + \frac{1}{2}\,\frac{2.5\,lb}{g}\left(\frac{4}{12}\,in.\right) = 0.23264\,\frac{1}{g}$$

$\omega_1 = 120\,rpm$

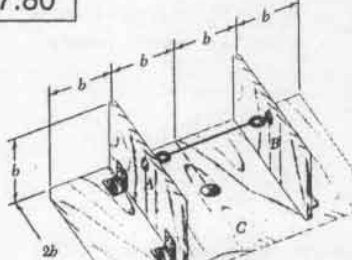
EQ(1): $0.16319\,\frac{1}{g}(120\,rpm) = 0.23264\,\frac{1}{g}\,\omega_2$

$\omega_2 = 84.17\,rpm$ $\qquad$ $\omega_2 = 84.2\,rpm$

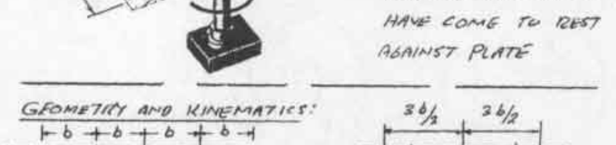
17.80

GIVEN: PANELS AND PLATE ARE MADE OF SAME MATERIAL AND ARE OF SAME THICKNESS. IN THE POSITION SHOWN ANGULAR VELOCITY $= \omega_0$

FIND: AFTER WIRE BREAKS ANGULAR VELOCITY WHEN PANELS HAVE COME TO REST AGAINST PLATE



GEOMETRY AND KINEMATICS:



PANELS IN UP POSITION $\qquad$ PANELS IN DOWN POSITION

$\bar{v}_0 = b\,\omega_0$ $\qquad$ $\bar{v}_1 = \frac{3}{2}\,b\,\omega_0$

LET ρ = MASS DENSITY, t = THICKNESS

PLATE: $m_{plate} = \rho t(2b)(4b) = 8\rho t b^2$

$$\bar{I}_{plate} = \frac{1}{12}(8\rho t b^2)\left[(2b)^2 + (4b)^2\right] = \frac{160}{12}\rho t b^4 = \frac{40}{3}\rho t b^4$$

(CONTINUED)

17.80 continued

EACH PANEL: $m_{panel} = \rho t(b)(2b) = 2\rho t b^2$

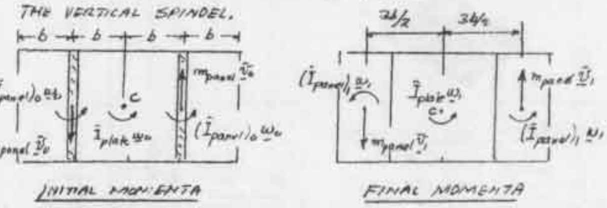
PANEL IN UP POSITION

$$(\bar{I}_{panel})_0 = \frac{1}{12}(2\rho t b^2)(2b)^2 = \frac{8}{12}\rho t b^4 = \frac{2}{3}\rho t b^4$$

PANEL IN DOWN POSITION

$$(\bar{I}_{panel})_1 = \frac{1}{12}(2\rho t b^2)\left[b^2 + (2b)^2\right] = \frac{10}{12}\rho t b^4 = \frac{5}{6}\rho t b^4$$

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT THE VERTICAL SPINDEL.



INITIAL MOMENTA $\qquad$ FINAL MOMENTA

+) MOMENTS ABOUT C:

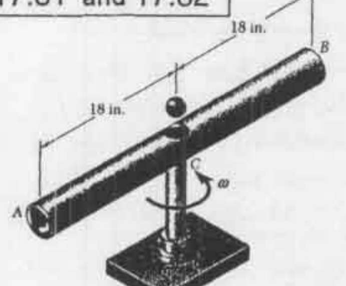
$$\bar{I}_{plate}\omega_0 + 2\left[(\bar{I}_{panel})_0\omega_0 + m_{panel}\bar{v}_0(b)\right] = \bar{I}_{plate}\omega_1 + 2\left[(\bar{I}_{panel})_1\omega_1 + m_{panel}\bar{v}_1\left(\frac{3b}{2}\right)\right]$$

$$\frac{40}{3}\rho t b^4\omega_0 + 2\left[\frac{2}{3}\rho t b^4\omega_0 + 2\rho t b^2(b\omega_0)b\right]$$
$$= \frac{40}{3}\rho t b^4\omega_0 + 2\left[\frac{5}{6}\rho t b^4\omega_1 + 2\rho t b^2\left(\frac{3}{2}b\omega_0\right)\left(\frac{3}{2}b\right)\right]$$

$$\left[\frac{40}{3} + \frac{4}{3} + 4\right]\rho t b^4\omega_0 = \left[\frac{40}{3} + \frac{10}{6} + 9\right]\rho t b^4\omega_1$$

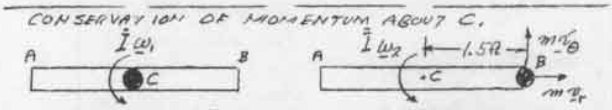
$\frac{56}{3}\omega_0 = 24\,\omega_1$; $\quad \omega_1 = \frac{56}{3(24)}\omega_0 \qquad \omega_1 = \frac{7}{9}\,\omega_0$

17.81 and 17.82



GIVEN: 4-lb TUBE AB INITIALLY $\omega = 8\,rad/s$
BALLS INTRODUCED TO TUBE
PROBLEM 17.82:
FIND: (a) ω AS A 0.8-lb BALL LEAVES TUBE
(b) ω AS A SECOND 0.8-lb BALL LEAVES TUBE.
PROBLEM 17.83:
FIND: ω AS A SINGLE 1.6-lb BALL LEAVES TUBE.

CONSERVATION OF MOMENTUM ABOUT C.



AS BALL IS INTRODUCED $\qquad$ AS BALL LEAVES TUBE

)MOMENTS ABOUT C: $\bar{I}\omega_1 = \bar{I}\omega_2 + m\,v_\theta(1.5\,ft)$ (1)

$v_\theta = (1.5\,ft)\omega_2 \qquad \bar{I} = \frac{1}{12}\,\frac{4\,lb}{g}(3\,ft)^2 = \frac{3}{g}$

PROBLEM 17.82: (a) FIRST 0.8-lb BALL, $\omega_1 = 8\,rad/s$

EQ(1): $\frac{3}{g}(8\,rad/s) = \frac{3}{g}\omega_2 + \frac{0.8\,lb}{g}(1.5\,\omega_2)(1.5\,ft)$

$24 = (3 + 1.8)\omega_2 \qquad \omega_2 = 5\,rad/s$

AS FIRST BALL LEAVES TUBE: $\omega = 5\,rad/s$

(b) SECOND 0.8-lb BALL. $\omega_1 = 5\,rad/s$

EQ(1): $\frac{3}{g}(5\,rad/s) = \frac{3}{g}\omega_2 + \frac{0.8\,lb}{g}(1.5\,\omega_2)(1.5\,ft)$

$15 = (3 + 1.8)\,\omega_2 \qquad \omega_2 = 3.125\,rad/s$

AS SECOND BALL LEAVES TUBE: $\omega = 3.125\,rad/s$

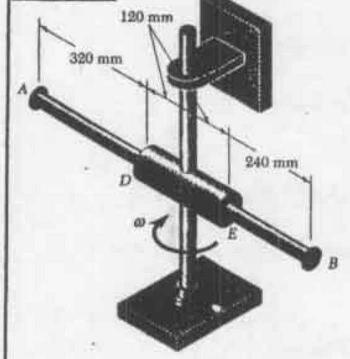
PROBLEM 17.83: A 1.6-lb BALL IS INTRODUCED, $\omega_1 = 8\,rad/s$

EQ(1) $\quad \frac{3}{g}(8\,rad/s) = \frac{3}{g}\omega_2 + \frac{1.6\,lb}{g}(1.5\,\omega_2)(1.5)$

$24 = (3 + 3.6)\omega_2 \qquad \omega_2 = 3.636\,rad/s$

AS 1.6-lb BALL LEAVES THE TUBE: $\omega = 3.64\,rad/s$

17.83



GIVEN: 3-kg ROD AB

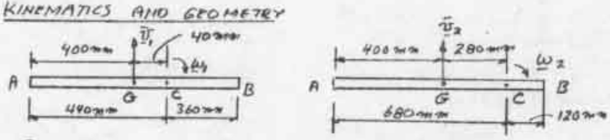
FOR CYLINDER DE: $\bar{I} = 0.025\ kg \cdot m^2$

IN POSITION SHOWN:

$\omega = 40\ rad/s$ AND END B OF ROD IS MOVING TOWARD E AT 75 mm/s.

FIND: ANGULAR VELOCITY OF ASSEMBLY AS END B STRIKES CYLINDER AT E.

KINEMATICS AND GEOMETRY



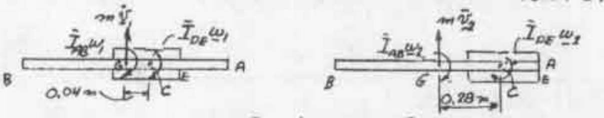
$\bar{v}_1 = (0.04\ m)\omega_1 = (0.04\ m)(40\ rad/s)$

$\bar{v}_1 = 1.6\ m/s$

$\bar{v}_2 = (0.28\ m)\omega_2$

INITIAL POSITION — FINAL POSITION

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT C.



+) MOMENTS ABOUT C: $\bar{I}_{AB} = \frac{1}{12}(3\ kg)(0.8\ m)^2 = 0.16\ kg \cdot m^2$

$\bar{I}_{AB}\omega_1 + m\bar{v}_1(0.04\ m) + \bar{I}_{DE}\omega_1 = \bar{I}_{AB}\omega_2 + m\bar{v}_2(0.28\ m) + \bar{I}_{DE}\omega_2$

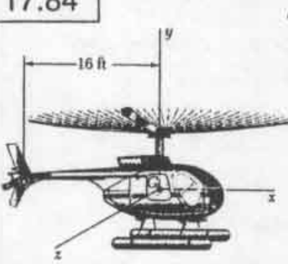
$(0.16\ kg \cdot m^2)(40\ rad/s) + (3\ kg)(1.6\ m/s)(0.04\ m) + (0.025\ kg \cdot m^2)(40\ rad/s)$

$= (0.16\ kg \cdot m^2)\omega_2 + (3\ kg)(0.28\omega_2)(0.28) + (0.025\ kg \cdot m^2)\omega_2$

$(6.4 + 0.192 + 1.00) = (0.16 + 0.2352 + 0.025)\omega_2$

$7.592 = 0.4202\,\omega_2$; $\omega_2 = 18.068\ rad/s$; $\omega_2 = 18.07\ rad/s$ ◀

17.84



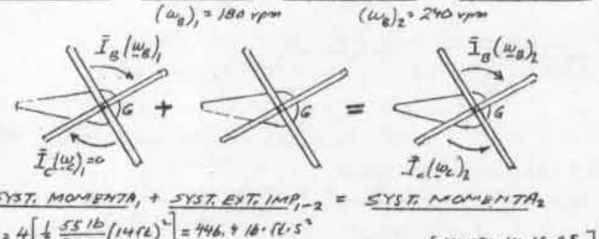
GIVEN: $\bar{I}_{CAB} = \bar{I}_C = 650\ lb \cdot ft \cdot s^2$

EACH BLADE WEIGHS 55 lb

INITIAL ANGULAR VELOCITY OF CAB = ZERO

FIND: ω_C AS ω_{BLADES} IS INCREASED FROM 180 rpm TO 240 rpm

$(\omega_B)_1 = 180$ rpm — $(\omega_B)_2 = 240$ rpm



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$\bar{I}_B = 4\left[\frac{1}{3}\frac{55\ lb}{32.2}(14\ ft)^2\right] = 446.4\ lb \cdot ft \cdot s^2$

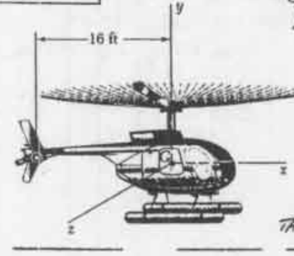
$(\omega_B)_2 - (\omega_C)_2 = 240$ rpm ; $(\omega_B)_2 = (\omega_C)_2 + 240$ rpm   [NOTE: ω IS OF BLADES RELATIVE TO CAB]

+) MOMENTS ABOUT G: $\bar{I}_B(\omega_B)_1 + 0 = \bar{I}_B(\omega_B)_2 + \bar{I}_C(\omega_C)_2$

$(446.4\ lb \cdot ft \cdot s^2)(180\ rpm) = (446.4\ lb \cdot ft \cdot s^2)\left[(\omega_C)_2 + 240\ rpm\right] + (650\ lb \cdot ft \cdot s^2)(\omega_C)_2$

$(\omega_C)_2 = \frac{26784}{1096.4}$ $(\omega_{CAB})_2 = 24.4$ rpm ◀

17.85



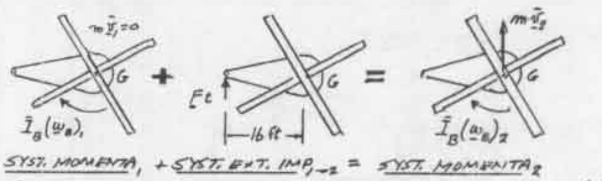
GIVEN: $\bar{I}_{CAB} = \bar{I}_c = 650\ lb \cdot ft \cdot s^2$

EACH BLADE WEIGHS 55 lb

$W_{CAB} = 1250$ lb

TAIL PROPELLER PREVENTS ROTATION OF CAB AS ω OF BLADES IS INCREASED FROM 180 rpm TO 240 rpm IN 12 s.

FIND: FORCE EXERTED BY TAIL PROPELLER AND FINAL $\bar{v}_C$.



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+) MOMENTS ABOUT G: $\bar{I}_B(\omega_B)_1 + Ft(16\ ft) = \bar{I}(\omega_B)_2$ (1)

+↑ COMPONENTS: $0 + Ft = m\bar{v}_2$ (2)

$\bar{I}_B = 4\left[\frac{1}{3}\frac{55\ lb}{32.2}(14\ ft)^2\right] = 446.4\ lb \cdot ft \cdot s^2$

$m = m_C + m_B = \frac{1}{32.2}\left[1250\ lb + 4(55\ lb)\right] = 45.65\ lb \cdot s^2/ft$

$(\omega_B)_1 = 180\ rpm\ \frac{2\pi}{60} = 18.85\ rad/s$; $(\omega_B)_2 = 240\ rpm\ \frac{2\pi}{60} = 25.13\ rad/s$

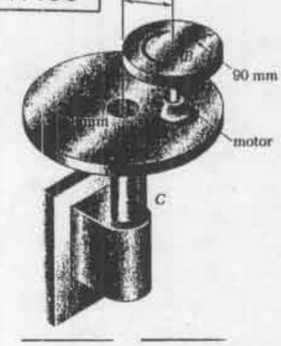
EQ(1): $(446.4\ lb \cdot ft \cdot s^2)(18.85\ rad/s) + (Ft)(16\ ft) = (446.4\ lb \cdot ft \cdot s^2)(25.13\ rad/s)$

$Ft = 175.3\ lb \cdot s$

EQ(2): $175.3\ lb \cdot s = (45.65\ lb \cdot s^2/ft)\bar{v}_2$; $\bar{v}_2 = 3.84\ ft/s$ ◀

FOR $t = 12\ s$: $Ft = F(12\ s) = 175.3\ lb \cdot s$; $F = 14.61$ lb ◀

17.86



GIVEN: $m_B = 4\ kg$

$\bar{I}_{AC} = 0.20\ kg \cdot m^2$

SYSTEM IS INITIALLY AT REST

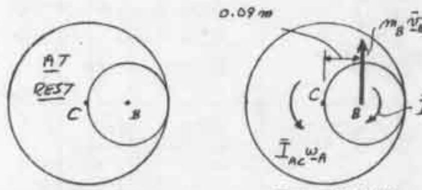
FIND: ω_B AND ω_A WHEN SPEED OF MOTOR REACHES 360 rpm

FOR DISK B

$\bar{I}_B = \frac{1}{2}(4\ kg)(0.09\ m)^2$

$\bar{I}_B = 16.2 \times 10^{-3}\ kg \cdot m^2$

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT SHAFT C



+) MOMENTS ABOUT C: $\bar{I}_{AC}\omega_A + m_B\bar{v}_B(0.09\ m) - \bar{I}_B\omega_B$

$(0.20\ kg \cdot m^2)\omega_A + (4\ kg)(0.09\omega_A)(0.09) - (16.2 \times 10^{-3}\ kg \cdot m^2)\omega_B$

$0.2324\,\omega_A - 0.0162\,\omega_B = 0$

$\omega_B = 14.346\,\omega_A$ (1)

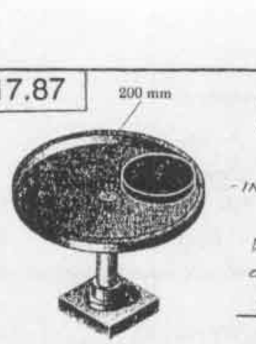
$\omega_{MOTOR} = \omega_A + \omega_B$

$360\ rpm = \omega_A + 14.346\,\omega_A$

$\omega_A = 23.46$ rpm $\omega_A = 23.5$ rpm ◀

EQ(1) $\omega_B = 14.346(23.46) = 336.55$ rpm $\omega_B = 337$ rpm ◀

17.87



GIVEN: FOR 200-mm RADIUS PLATFORM - RIM UNIT:

$m_A = 5\text{ kg}$, $\bar{k} = 175\text{ mm}$

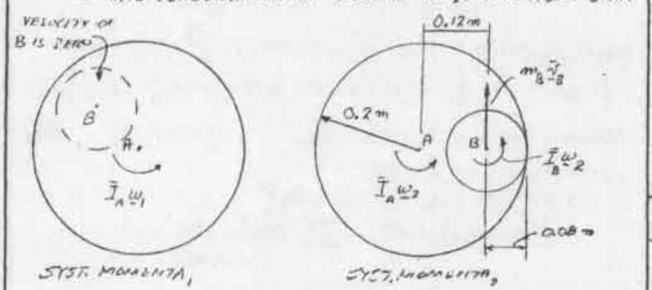
- INITIAL ANGULAR VELOCITY: $\omega_A = 50\text{ rpm}$

DISK: $m_B = 3\text{ kg}$, $r_B = 80\text{ mm}$

DISK PLACED, WITH NO VELOCITY, ON PLATFORM

FIND: FINAL ANGULAR VELOCITY

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT SHAFT



SYST. MOMENTA$_1$ SYST. MOMENTA$_2$

+↺ MOMENTS ABOUT A: $\bar{I}_A\omega_1 = \bar{I}_A\omega_2 + \bar{I}_B\omega_2 + m_B\bar{v}_B(0.12\text{ m})$ (1)

$\bar{I}_A = m_A\bar{k}^2 = (5\text{ kg})(0.175\text{ m})^2 = 0.153125\text{ kg}\cdot\text{m}^2$

$\bar{I}_B = \frac{1}{2}m_B r_B^2 = \frac{1}{2}(3\text{ kg})(0.08\text{ m})^2 = 9.6\times10^{-3}\text{ kg}\cdot\text{m}^2$

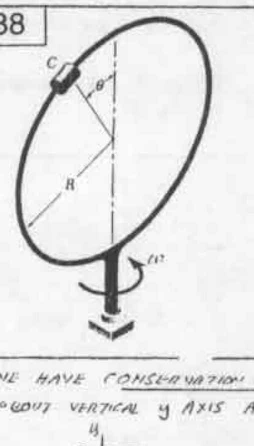
$\bar{v}_B = (0.12\text{ m})\omega_2$

EQ(1): $(0.153125\text{ kg}\cdot\text{m}^2)\omega_1 = (0.153125\text{ kg}\cdot\text{m}^2)\omega_2 + (9.6\times10^{-3}\text{ kg}\cdot\text{m}^2)\omega_2 + (3\text{ kg})(0.12\text{ m})^2\omega_2$

$0.153125\,\omega_1 = 0.20593\,\omega_2$

$\omega_2 = 0.7436\,\omega_1 = 0.7436(50\text{ rpm})$ $\quad\omega_2 = 37.2\text{ rpm}$ ◀

17.88



GIVEN: 2-kg COLLAR C

RING: $m_R = 3\text{ kg}$

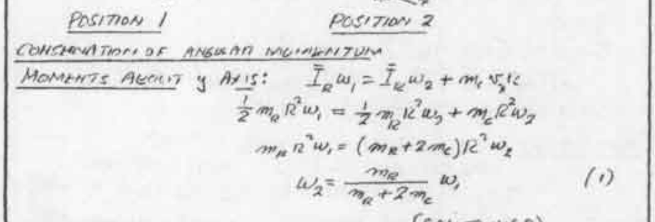
$R = 250\text{ mm}$

WHEN $\theta = 0$, $\omega_1 = 35\text{ rad/s}$ AND $v_C = 0$

FIND: (a) ω WHEN $\theta = 90°$

(b) VELOCITY OF COLLAR RELATIVE TO RING WHEN $\theta = 90°$

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT VERTICAL y AXIS AND CONSERVATION OF ENERGY



POSITION 1 POSITION 2

CONSERVATION OF ANGULAR MOMENTUM

MOMENTS ABOUT y AXIS: $\bar{I}_R\omega_1 = \bar{I}_R\omega_2 + m_c v_x R$

$\frac{1}{2}m_R R^2\omega_1 = \frac{1}{2}m_R R^2\omega_2 + m_c R^2\omega_2$

$m_R R^2\omega_1 = (m_R + 2m_c)R^2\omega_2$

$$\omega_2 = \frac{m_R}{m_R + 2m_c}\,\omega_1 \qquad (1)$$

(CONTINUED)

17.88 continued

$T_1 = \frac{1}{2}\bar{I}_R\omega_1^2 = \frac{1}{2}\left(\frac{1}{2}m_R R^2\right)\omega_1^2 = \frac{1}{4}m_R R^2\omega_1^2$

$V_1 = W_c R = m_c g R$

$T_2 = \frac{1}{2}\bar{I}_R\omega_2^2 + \frac{1}{2}m_c(v_x^2 + v_y^2) = \frac{1}{4}m_R R^2\omega_2^2 + \frac{1}{2}m_c R^2\omega_2^2 + \frac{1}{2}m_c v_y^2$

$V_2 = 0$

CONSERVATION OF ENERGY: $T_1 + V_1 = T_2 + V_2$

$$\frac{1}{4}m_R R^2\omega_1^2 + m_c g R = \left(\frac{1}{4}m_R + \frac{1}{2}m_c\right)R^2\omega_2^2 + \frac{1}{2}m_c v_y^2 \qquad (2)$$

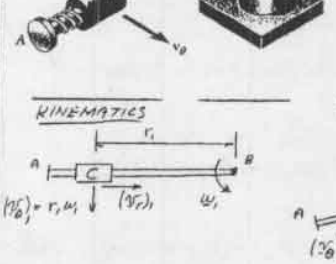
DATA: $m_c = 2\text{ kg}$, $m_R = 3\text{ kg}$, $R = 0.25\text{ m}$, $\omega_1 = 35\text{ rad/s}$

(a)
EQ(1) $\omega_2 = \frac{3\text{ kg}}{3\text{ kg} + 2(2\text{ kg})}(35\text{ rad/s})$ $\quad\omega_2 = 15\text{ rad/s}$ ◀

(b)
EQ(2): $\frac{1}{4}(3\text{ kg})(0.25\text{ m})^2(35\text{ rad/s})^2 + (2\text{ kg})(9.81\text{ m/s}^2)(0.25\text{ m})$
$= \left[\frac{1}{4}(3\text{ kg}) + \frac{1}{2}(2\text{ kg})\right](0.25\text{ m})^2(15\text{ rad/s})^2 + \frac{1}{2}(2\text{ kg})v_y^2$

$57.422 + 4.905 = 24.609 + v_y^2$; $v_y^2 = 37.716$; $\quad v_y = 6.14\text{ m/s}$ ◀

17.89

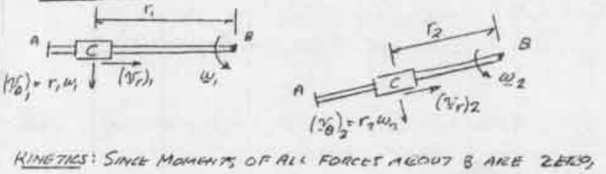


GIVEN: IN POSITION SHOWN $\omega_1 = 1.5\text{ rad/s}$, $(v_r)_1 = 1.5\text{ m/s}$

$m_c = 8\text{ kg}$, FOR ROD AND SPRING $I_B = 1.2\text{ kg}\cdot\text{m}^2$

FIND: (a) MINIMUM DISTANCE BETWEEN C AND B, (b) CORRESPONDING ANGULAR VELOCITY

KINEMATICS



KINETICS: SINCE MOMENTS OF ALL FORCES ABOUT B ARE ZERO, WE HAVE: $(H_B)_1 = (H_B)_2$: $I_B\omega_1 + m_c(v_\theta)_1 r_1 = I_B\omega_2 + m_c(v_\theta)_2 r_2$

$$(I_B + m_c r_1^2)\,\omega_1 = (I_B + m_c r_2^2)\,\omega_2 \qquad (1)$$

$$\left[1.2\text{ kg}\cdot\text{m}^2 + (8\text{ kg})(0.6\text{ m})^2\right](1.5\text{ rad/s}) = \left[1.2\text{ kg}\cdot\text{m}^2 + (8\text{ kg})r_2^2\right]\omega_2 \qquad (2)$$

CONSERVATION OF ENERGY SINCE $V_1 = V_2$, WE HAVE $T_1 = T_2$

$T_1 = \frac{1}{2}I_B\omega_1^2 + \frac{1}{2}m_c(v_\theta)_1^2 + \frac{1}{2}m_c(v_r)_1^2$
$= \frac{1}{2}(1.2\text{ kg}\cdot\text{m}^2)(1.5\text{ rad/s})^2 + \frac{1}{2}(8\text{ kg})(0.6\text{ m})^2(1.5\text{ rad/s})^2 + \frac{1}{2}(8\text{ kg})(1.5\text{ m/s})^2$

$T_1 = 13.59\text{ J}$

$T_2 = \frac{1}{2}I_B\omega_2^2 + \frac{1}{2}m_c(v_\theta)_2^2 + \frac{1}{2}m_c(v_r)_2^2$
$= \frac{1}{2}(1.2\text{ kg}\cdot\text{m}^2)\omega_2^2 + \frac{1}{2}(8\text{ kg})r_2^2\omega_2^2 + \frac{1}{2}(8\text{ kg})(v_r)_2^2$

$T_2 = 0.6\,\omega_2^2 + 4r_2^2\omega_2^2 + 4(v_r)_2^2$

$T_1 = T_2$: $\quad 13.59 = (0.6 + 4r_2^2)\omega_2^2 + 4(v_r)_2^2$ (3)

EQ(1): $\quad 6.12 = (1.2 + 8r_2^2)\omega_2$; $\quad \omega_2 = \frac{6.12}{1.2 + 8r_2^2} = \frac{3.06}{0.6 + 4r_2^2}$ (4)

EQ(3) $\quad 13.59 = (0.6 + 4r_2^2)\left[\frac{3.06}{0.6 + 4r_2^2}\right]^2 + 4(v_r)_2^2$

FOR r_{MINIMUM} WE HAVE $(v_r)_2 = 0$

$13.59 = \frac{(3.06)^2}{0.6 + 4r_2^2}$; $\quad 8.154 + 54.36\,r_2^2 = 9.364$

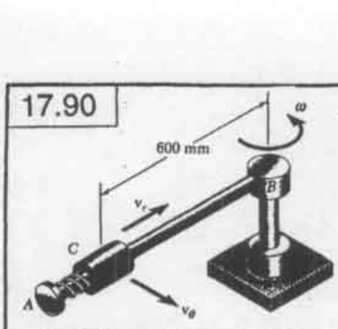
$r_2^2 = 22.25\times10^{-3}\text{ m}^2$

$r_2 = 0.1492\text{ m}$ $\quad r_2 = 149.2\text{ mm}$ ◀

EQ(4): $\omega_2 = \frac{3.06}{0.6 + 4r_2^2} = \frac{3.06}{0.6 + 4(22.25\times10^{-3})} = 4.441\text{ rad/s}$

$\omega_2 = 4.44\text{ rad/s}$ ◀

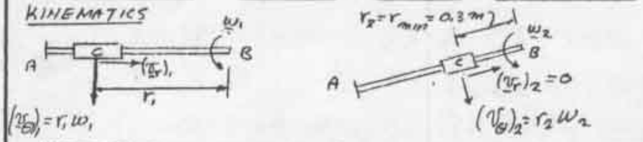
17.90



GIVEN: IN POSITION SHOWN
$\omega_1 = 1.5$ rad/s
$m_c = 8$ kg
FOR ROD + SPRING $I_B = 1.2$ kg·m²

FIND: v_r IF MINIMUM DISTANCE FROM COLLAR TO B IS TO BE 300 mm

KINEMATICS



$(v_\theta)_1 = r_1\omega_1$ $\qquad (v_\theta)_2 = r_2\omega_2$

KINETICS: SINCE MOMENTS OF ALL FORCES ABOUT B ARE ZERO, $(H_B)_1 = (H_B)_2$

$$I_B\omega_1 + m_c(v_\theta)_1 r_1 = I_B\omega_2 + m_c(v_\theta)_2 r_2$$

$$(I_B + m_c r_1^2)\omega_1 = (I_B + m_c r_2^2)\omega_2$$

DATA: $I_B = 1.2$ kg·m², $m_c = 8$ kg, $r_1 = 0.6$ m, $r_2 = r_{min} = 0.3$ m

$$\left[1.2\text{ kg}\cdot\text{m}^2 + (8\text{ kg})(0.6\text{ m})^2\right](1.5\text{ rad/s}) = \left[1.2\text{ kg}\cdot\text{m}^2 + (8\text{ kg})(0.3\text{ m})^2\right]\omega_2$$

$6.12 = 1.92\omega_2 \qquad \omega_2 = 3.1875$ rad/s

CONSERVATION OF ENERGY

SINCE $V_1 = V_2$, WE HAVE $T_1 = T_2$

$$T_1 = \tfrac{1}{2}I_B\omega_1^2 + \tfrac{1}{2}m_c(v_\theta)_1^2 + \tfrac{1}{2}m(v_r)_1^2$$

$$= \tfrac{1}{2}(1.2\text{ kg}\cdot\text{m}^2)(1.5\text{ rad/s})^2 + \tfrac{1}{2}(8\text{ kg})(0.6\text{ m})^2(1.5\text{ rad/s})^2 + \tfrac{1}{2}(8\text{ kg})(v_r)_1^2$$

$$T_1 = 4.59\text{ J} + 4(v_r)_1^2$$

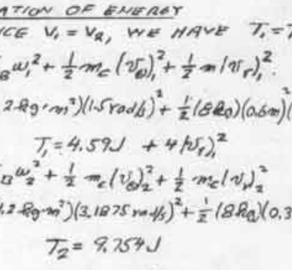
$$T_2 = \tfrac{1}{2}I_B\omega_2^2 + \tfrac{1}{2}m_c(v_\theta)_2^2 + \tfrac{1}{2}m_c(v_r)_2^2$$

$$= \tfrac{1}{2}(1.2\text{ kg}\cdot\text{m}^2)(3.1875\text{ rad/s})^2 + \tfrac{1}{2}(8\text{ kg})(0.3\text{ m})^2(3.1875\text{ rad/s})^2 + 0$$

$$T_2 = 9.754\text{ J}$$

$T_1 = T_2$: $\quad 4.59\text{ J} + 4(v_r)_1^2 = 9.754\text{ J}; \quad (v_r)_1 = 1.136$ m/s

17.91



GIVEN: $W_c = 6$ lb

MASS MOMENT OF INERTIA OF ROD AB AND SPRING ABOUT A,
$I_R = 0.35$ lb·ft·s²

SPRING: $k = 15$ lb/in. AND UNDEFORMED LENGTH = 10 in.

$\omega_1 = 12$ rad/s
$(v_r)_1 = 0$ OF COLLAR.
WHEN AC = 7.5 in.,

FIND: (a) ω_2
(b) $(v_r)_2$ OF COLLAR.

POTENTIAL ENERGY OF SPRING $\qquad$ UNDEFORMED LENGTH = 10 in.

POSITION 1: $\qquad$ POSITION 2:

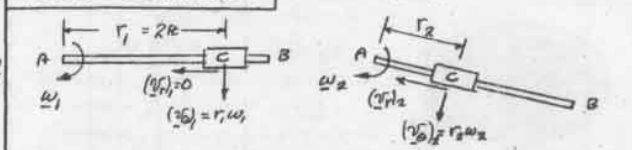
$\Delta = 26\text{ in.} - 10\text{ in.} = 16\text{ in.}$
$V_1 = \tfrac{1}{2}k\Delta^2 = \tfrac{1}{2}(15\text{ lb/in.})(16\text{ in})^2$
$= 1920$ in·lb
$V_1 = 160$ ft·lb

$\Delta = 12.5\text{ in.} - 10\text{ in.} = 2.5\text{ in}$
$V_2 = \tfrac{1}{2}k\Delta^2 = \tfrac{1}{2}(15\text{ lb/in.})(2.5\text{ in})^2$
$= 46.875$ in·lb
$V_2 = 3.91$ ft·lb

(CONTINUED)

17.91 continued

KINEMATICS:



KINETICS: SINCE MOMENTS OF ALL FORCES ABOUT SHAFT AT A ARE ZERO, $(H_A)_1 = (H_A)_2$

$$I_R\omega_1 + m_c(v_\theta)_1 r_1 = I_R\omega_2 + m_c(v_\theta)_2 r_2$$

$$(I_R + m_c r_1^2)\omega_1 = (I_R + m_c r_2^2)\omega_2$$

DATA: $I_R = 0.35$ lb·ft·s², $m_c = \frac{6\text{ lb}}{32.2}$, $r_1 = 2$ ft, $r_2 = \frac{7.5}{12}$ ft, $\omega_1 = 12$ rad/s

$$\left[0.35\text{ lb}\cdot\text{ft}\cdot\text{s}^2 + \frac{6\text{ lb}}{32.2}(2\text{ ft})^2\right](12\text{ rad/s}) = \left[0.35\text{ lb}\cdot\text{ft}\cdot\text{s}^2 + \frac{6\text{ lb}}{32.2}\left(\frac{7.5}{12}\text{ ft}\right)^2\right]\omega_2$$

$13.1441 = 0.42279\,\omega_2$; $\omega_2 = 31.089$ rad/s; $\qquad \omega_2 = 31.1$ rad/s

CONSERVATION OF ENERGY

$$T_1 = \tfrac{1}{2}I_R\omega_1^2 + \tfrac{1}{2}m_c(v_\theta)_1^2 + \tfrac{1}{2}m_c(v_r)_1^2$$

$$= \tfrac{1}{2}(0.35\text{ lb}\cdot\text{ft}\cdot\text{s}^2)(12\text{ rad/s})^2 + \tfrac{1}{2}\left(\frac{6\text{ lb}}{32.2}\right)(2\text{ ft})^2(12\text{ rad/s})^2 + 0$$

$$T_1 = 78.865\text{ ft}\cdot\text{lb}$$

$$T_2 = \tfrac{1}{2}I_R\omega_2^2 + \tfrac{1}{2}m(v_\theta)_2^2 + \tfrac{1}{2}m_c(v_r)_2^2$$

$$= \tfrac{1}{2}(0.35\text{ lb}\cdot\text{ft}\cdot\text{s}^2)(31.089\text{ rad/s})^2 + \tfrac{1}{2}\left(\frac{6\text{ lb}}{32.2}\right)\left(\frac{7.5}{12}\text{ ft}\right)^2(31.089\text{ rad/s})^2 + \tfrac{1}{2}\left(\frac{6\text{ lb}}{32.2}\right)(v_r)_2^2$$

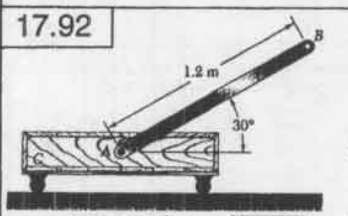
$$T_2 = 204.32 + 0.09317(v_r)_2^2$$

RECALL: $V_1 = 160$ ft·lb AND $V_2 = 3.91$ ft·lb

$T_1 + V_1 = T_2 + V_2$: $\quad 78.865 + 160 = 204.32 + 0.09317(v_r)_2^2 + 3.91$

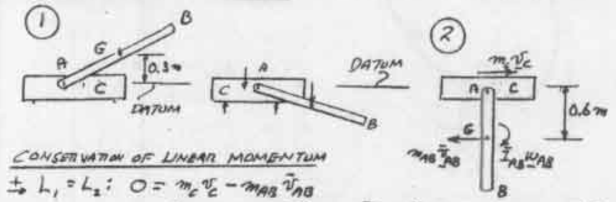
$30.638 = 0.09317(v_r)_2^2 \qquad (v_r)_2 = 18.13$ ft/s

17.92



GIVEN: $m_{AB} = 3$ kg
$m_c = 5$ kg
SYSTEM RELEASED FROM REST

FIND: (a) v_B AS AB IS VERTICAL
(b) v_c

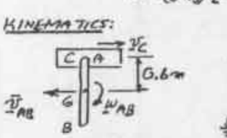


CONSERVATION OF LINEAR MOMENTUM

$\xrightarrow{+}$ $L_1 = L_2$: $\quad 0 = m_c v_c - m_{AB}\bar{v}_{AB}$

$0 = (5\text{ kg})v_c - (3\text{ kg})\bar{v}_{AB}$; $\quad \bar{v}_{AB} = \frac{5}{3}v_c \qquad (1)$

KINEMATICS:



$\xleftarrow{+}$ $\bar{v}_{AB} = -v_c + 0.6\,\omega_{AB}$

$\frac{5}{3}v_c = -v_c + 0.6\,\omega_{AB}$

$\frac{8}{3}v_c = 0.6\,\omega_{AB}$; $\quad v_c = 0.225\,\omega_{AB}$

EQ.(1): $\bar{v}_{AB} = \frac{5}{3}v_c = \frac{5}{3}(0.225\,\omega_{AB})$; $\quad \bar{v}_{AB} = 0.375\,\omega_{AB}$

CONSERVATION OF ENERGY

$V_1 = W_{AB}(0.3\text{ m}) = m_{AB}g(0.3\text{ m}) = (3\text{ kg})(9.81)(0.3\text{ m}) = 8.829$ J

$V_2 = -W_{AB}(0.6\text{ m}) = -m_{AB}g(0.6\text{ m}) = -(3\text{ kg})(9.81)(0.6\text{ m}) = -17.658$ J

$T_1 = 0$

$$T_2 = \tfrac{1}{2}m_c v_c^2 + \tfrac{1}{2}m_{AB}\bar{v}_{AB}^2 + \tfrac{1}{2}\bar{I}_{AB}\omega_{AB}^2$$

$$= \tfrac{1}{2}(5\text{ kg})(0.225\,\omega_{AB})^2 + \tfrac{1}{2}(3\text{ kg})(0.375\,\omega_{AB})^2 + \tfrac{1}{2}\left[\tfrac{1}{12}(3\text{ kg})(1.2\text{ m})^2\right]\omega_{AB}^2$$

$$T_2 = (0.1266 + 0.2109 + 0.1800)\omega_{AB}^2 = 0.5175\,\omega_{AB}^2$$

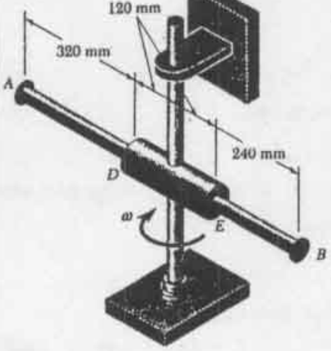
$T_1 + V_1 = T_2 + V_2$: $\quad 0 + 8.829 = 0.5175\,\omega_{AB}^2 - 17.658$

$26.487 = 0.5175\,\omega_{AB}^2$; $\quad \omega_{AB} = 7.154$ rad/s

$v_c = 0.225\,\omega_{AB} = 0.225(7.154) \qquad v_c = 1.610$ m/s →

$\overleftarrow{v_B} = \overrightarrow{v_c} + \overleftarrow{(AB)\omega_{AB}} = -1.610 + (1.2\text{ m})7.154\text{ rad/s} \qquad v_B = 6.975$ m/s ←

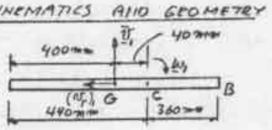
17.93

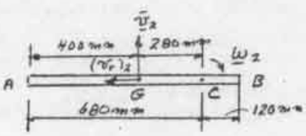


GIVEN: 3-kg ROD AB
FOR CYLINDER DE: $\bar{I} = 0.025\ kg\cdot m^2$
IN POSITION SHOWN
$\omega = 40$ rad/s AND
END B OF ROD IS MOVING
TOWARD E AT 75 mm/s

FIND: VELOCITY OF AB
RELATIVE TO DE AS
END B STRIKES END E
OF THE CYLINDER

KINEMATICS AND GEOMETRY



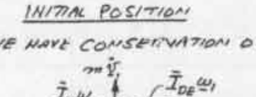


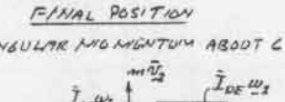
$\bar{v}_1 = (0.04\,m)\omega_1 = (0.04\,m)(40\ rad/s)$
$\bar{v}_1 = 1.6\ m/s$

$\bar{v}_2 = (0.28\,m)\omega_2$

INITIAL POSITION　　　　FINAL POSITION

WE HAVE CONSERVATION OF ANGULAR MOMENTUM ABOUT C.





+) MOMENTS ABOUT C: $\bar{I}_{AB} = \frac{1}{12}(3\,kg)(0.8\,m)^2 = 0.16\ kg\cdot m^2$

$$\bar{I}_{AB}\omega_1 + m\bar{v}_1(0.04\,m) + \bar{I}_{DE}\omega_1 = \bar{I}_{AB}\omega_2 + m\bar{v}_2(0.28\,m) + \bar{I}_{DE}\omega_2$$

$$(0.16\,kg\cdot m^2)(40\,rad/s) + (3\,kg)(1.6\,m/s)(0.04\,m) + (0.025\,kg\cdot m^2)(40\,rad/s)$$
$$= (0.16\,kg\cdot m^2)\omega_2 + (3\,kg)(0.28\,\omega_2)(0.28) + (0.025\,kg\cdot m^2)\omega_2$$

$$(6.4 + 0.192 + 1.00) = (0.16 + 0.2352 + 0.025)\omega_2$$

$7.592 = 0.4202\,\omega_2$;　$\omega_2 = 18.068$ rad/s:　$\omega_2 = 18.07$ rad/s

CONSERVATION OF ENERGY　$(v_r)_1 = 0.075$ m/s

$V_1 = V_2 = 0$

$$T_1 = \frac{1}{2}\bar{I}_{DE}\omega_1^2 + \frac{1}{2}\bar{I}_{AB}\omega_1^2 + \frac{1}{2}m_{AB}\bar{v}_1^2 + \frac{1}{2}m_{AB}(v_r)_1^2$$
$$= \frac{1}{2}(0.025\,kg\cdot m^2)(40\,rad/s)^2 + \frac{1}{2}(0.16\,kg\cdot m^2)(40\,rad/s)^2$$
$$+ \frac{1}{2}(3\,kg)(1.6\,m/s)^2 + \frac{1}{2}(3\,kg)(0.075\,m/s)^2$$

$T_1 = 20\,J + 128\,J + 3.84\,J + 0.008\,J = 151.85\,J$

$\bar{v}_2 = (0.28\,m)\omega_2 = (0.28\,m)(18.068\ rad/s) = 5.059\ m/s$

$$T_2 = \frac{1}{2}\bar{I}_{DE}\omega_2^2 + \frac{1}{2}\bar{I}_{AB}\omega_2^2 + \frac{1}{2}m_{AB}\bar{v}_2^2 + \frac{1}{2}m_{AB}(v_r)_2^2$$

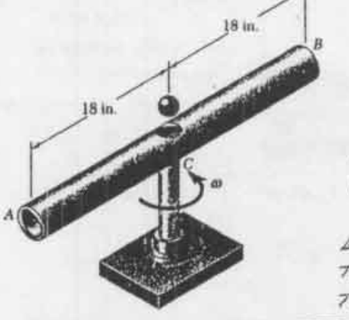
$$= \frac{1}{2}(0.025\,kg\cdot m^2)(18.068\,rad/s)^2 + \frac{1}{2}(0.16\,kg\cdot m^2)(18.068\,rad/s)^2$$
$$+ \frac{1}{2}(3\,kg)(5.059\,m/s)^2 + \frac{1}{2}(3\,kg)(v_r)_2^2$$

$T_2 = 4.081\,J + 26.116\,J + 38.391\,J + 1.5(v_r)_2^2$
$T_2 = 68.587\,J + 1.5(v_r)_2^2$

$T_1 + V_1 = T_2 + V_2$:　$151.85\,J + 0 = 68.587\,J + 1.5(v_r)_2^2$
$83.263 = 1.5(v_r)_2^2$

$(v_r)_2 = 7.45$ m/s ◀

17.94

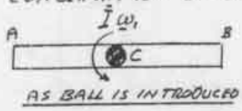


GIVEN: 4-lb TUBE AB
INITIALLY $\omega_1 = 8$ rad/s

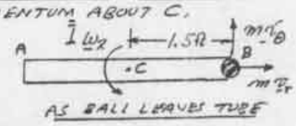
AN 0.8-lb BALL IS
INTRODUCED INTO TUBE
AND LEAVE TUBE AT B.
A SECOND 0.8-lb BALL
IS THEN PUT INTO TUBE

FIND: VELOCITY OF
EACH BALL RELATIVE TO
TUBE AS IT LEAVES THE
TUBE

CONSERVATION OF MOMENTUM ABOUT C.



AS BALL IS INTRODUCED



AS BALL LEAVES TUBE

) MOMENTS ABOUT C: $\bar{I}\omega_1 = \bar{I}\omega_2 + m\,v_\theta(1.5\,ft)$　　(1)

$v_\theta = (1.5\,ft)\omega_2$　　$\bar{I} = \frac{1}{12}\frac{4\,lb}{g}(3\,ft)^2 = \frac{3}{g}$

FIRST 0.8-lb BALL, $\omega_1 = 8$ rad/s

EQ(1): $\frac{3}{g}(8\,rad/s) = \frac{3}{g}\omega_2 + \frac{0.8\,lb}{g}(1.5\,\omega_2)(1.5\,ft)$

$24 = (3 + 1.8)\omega_2$　　$\omega_2 = 5$ rad/s

AS FIRST BALL LEAVES TUBE: $\omega = 5$ rad/s

SECOND 0.8-lb BALL.　$\omega_1 = 5$ rad/s

EQ(1): $\frac{3}{g}(5\,rad/s) = \frac{3}{g}\omega_2 + \frac{0.8\,lb}{g}(1.5\,\omega_2)(1.5\,ft)$

$15 = (3 + 1.8)\omega_2$　　$\omega_2 = 3.125$ rad/s

AS SECOND BALL LEAVES TUBE: $\omega = 3.125$ rad/s

CONSERVATION OF ENERGY

FIRST BALL:　$\omega_1 = 8$ rad/s,　$\omega_2 = 5$ rad/s

$V_1 = 0$,　$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}\left(\frac{3}{g}\right)(8\,rad/s)^2 = \frac{96}{g}$

$V_2 = 0$,　$T_2 = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\,v_\theta^2 + \frac{1}{2}m\,v_r^2$

$$= \frac{1}{2}\left(\frac{3}{g}\right)(5\,rad/s)^2 + \frac{1}{2}\left(\frac{0.8\,lb}{g}\right)(1.5)^2(5\,rad/s)^2 + \frac{1}{2}\left(\frac{0.8\,lb}{g}\right)v_r^2$$

$T_2 = \frac{37.5}{g} + \frac{22.5}{g} + \frac{0.4}{g}v_r^2$

$T_1 + V_1 = T_2 + V_2$:　$\frac{96}{g} + 0 = \frac{1}{g}(37.5 + 22.5) + \frac{0.4}{g}v_r^2 + 0$

$v_r^2 = 90$　　　$v_r = 9.49$ ft/s ◀

SECOND BALL　$\omega_1 = 5$ rad/s,　$\omega_2 = 3.125$ rad/s

$V_1 = 0$,　$T_1 = \frac{1}{2}\bar{I}\omega_1^2 = \frac{1}{2}\left(\frac{3}{g}\right)(5\,rad/s)^2 = \frac{37.5}{g}$

$V_2 = 0$,　$T_2 = \frac{1}{2}\bar{I}\omega_2 + \frac{1}{2}m\,v_\theta^2 + \frac{1}{2}m\,v_r^2$

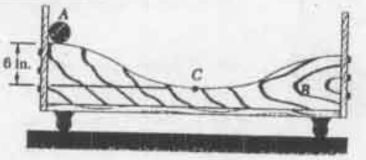
$$= \frac{1}{2}\left(\frac{3}{g}\right)(3.125\,rad/s)^2 + \frac{1}{2}\left(\frac{0.8}{g}\right)(1.5)^2(3.125\,rad/s)^2 + \frac{1}{2}\left(\frac{0.8\,lb}{g}\right)v_r^2$$

$$= \frac{14.684}{g} + \frac{8.789}{g} + \frac{0.4}{g}v_r^2$$

$T_1 + V_1 = T_2 + V_2$:　$\frac{37.5}{g} = \frac{14.684}{g} + \frac{8.789}{g} + \frac{0.4}{g}v_r^2$

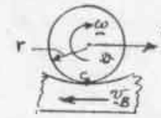
$v_r^2 = 35.156$　　　$v_r = 5.93$ ft/s ◀

17.95



GIVEN: $W_A = 6\,lb$
$W_B = 10\,lb$
CYLINDER RELEASED FROM REST.
FIND: v_B AS CYLINDER PASSES POINT C

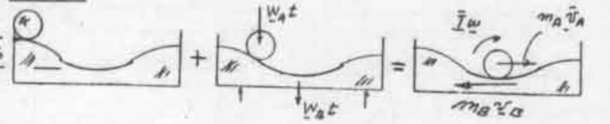
KINEMATICS (WHEN CYLINDER IS PASSING C)



$\pm\; v_B = v_C = r\omega - \bar{v}_A$

$\omega = \dfrac{\bar{v}_A + v_B}{r}$

KINETICS:



SYST OF MOMENTA$_1$ + SYST. EXT IMP$_{1\to2}$ = SYST MOMENTA$_2$

$\xrightarrow{+}$ x COMPONENTS: $m_A \bar{v}_A - m_B v_B = 0$

$\dfrac{6\,lb}{g}\bar{v}_A = \dfrac{10\,lb}{g} v_B$; $v_B = 0.6\,\bar{v}_A$

PRINCIPLE OF WORK-ENERGY

$U_{1\to2} = W_A(6\,in.) = (6\,lb)\left(\frac{6}{12}ft\right) = 3\,ft\cdot lb$; $T_1 = 0$

$T_2 = \frac{1}{2}m_A\bar{v}_A^2 + \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m_B v_B^2$

$v_B = 0.6\bar{v}_A$; $\omega = \dfrac{\bar{v}_A + v_B}{r} = \dfrac{\bar{v}_A + 0.6\bar{v}_A}{r} = \dfrac{1.6\bar{v}_A}{r}$

$T_2 = \frac{1}{2}\left(\frac{6\,lb}{g}\right)\bar{v}_A^2 + \frac{1}{2}\left[\frac{1}{2}\,\frac{6\,lb}{g}\,r^2\right]\left(\frac{1.6\bar{v}_A}{r}\right)^2 + \frac{1}{2}\,\frac{10\,lb}{g}(0.6\bar{v}_A)^2$

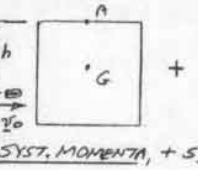
$= \frac{3}{g}\bar{v}_A^2 + \frac{3.84}{g}\bar{v}_A^2 + \frac{1.8}{g}\bar{v}_A^2 = \frac{12.48}{g}\bar{v}_A^2$

$T_1 + U_{1\to2} = T_2$: $0 + 3\,ft\cdot lb = \dfrac{8.64}{32.2}\bar{v}_A^2$

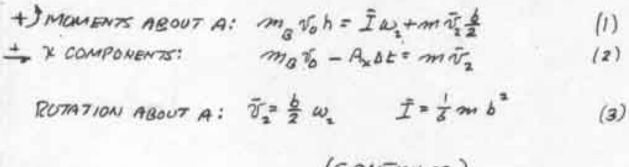
$\bar{v}_A^2 = 11.181$ $\qquad \bar{v}_A = 3.344\,ft/s \rightarrow$

$v_B = 0.6\bar{v}_A = 0.6(3.344)$ $\qquad \underline{v_B = 2.01\,ft/s \leftarrow}$ ◀

17.96 and 17.97



GIVEN: BULLET, $m_B = 45\,g$
$v_0 = 400\,m/s$
PLATE: $m = 9\,kg$, $b = 200\,mm$
PROB 17.96: FOR $h = 200\,mm$,
FIND: (a) $\bar{v}$ JUST AFTER IMPACT
(b) A_x IF $\Delta t = 2\,ms$
PROB 17.97: FIND
FIND: (a) h FOR $A_x = 0$
(b) $\bar{v}$ JUST AFTER IMPACT



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $m_B v_0 h = \bar{I}\omega_2 + m\bar{v}_2\frac{b}{2}$ (1)

$\xrightarrow{+}$ x COMPONENTS: $m_B v_0 - A_x\Delta t = m\bar{v}_2$ (2)

ROTATION ABOUT A: $\bar{v}_2 = \frac{b}{2}\omega_2$ $\qquad \bar{I} = \frac{1}{6}mb^2$ (3)

(CONTINUED)

17.96 and 17.97 continued

SUBSTITUTE FROM (3) INTO (1)

$m_B v_0 h = \frac{1}{6}mb^2\omega_2 + m\left(\frac{1}{2}b\omega_2\right)\frac{1}{2}b$

$m_B v_0 h = \frac{5}{12}mb^2\omega_2$ (4)

EQ(2): $A_x\Delta t = m_B v_0 - m\bar{v}_2$ (5)

DATA: $m_B = 0.045\,kg$, $v_0 = 400\,m/s$, $b = 0.2\,m$, $m = 9\,kg$, $\Delta t = 0.002\,s$

PROBLEM 17.96 FOR $h = 0.2\,m$

EQ(4) $(0.045\,kg)(400\,m/s)(0.2\,m) = \frac{5}{12}(9\,kg)(0.2\,m)^2\omega_2$

$\omega_2 = 24\,rad/s$

(a) $\bar{v}_2 = \frac{b}{2}\omega_2 = \frac{0.2\,m}{2}(24\,rad/s)$ $\qquad \underline{\bar{v}_2 = 2.4\,m/s \rightarrow}$ ◀

(b) EQ(5) $A_x(0.002\,s) = (0.045\,kg)(400\,m/s) - (9\,kg)(2.4\,m/s)$

$0.002A_x = 18 - 21.8$ $\qquad A_x = -1.8\,kN$ $\qquad \underline{A_x = 1.8\,kN \leftarrow}$ ◀

PROBLEM 17.97 FOR $A_x = 0$

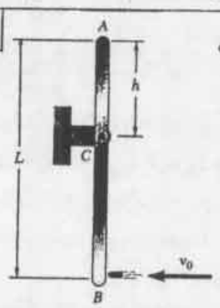
EQ(5) $m_B v_0 = m\bar{v}_2$; $mv_0 = m\left(\frac{b}{2}\omega_2\right)$; $\omega_2 = 2\frac{m_B}{m}\frac{v_0}{b}$

SUBSTITUTE INTO (4) $m_B v_0 h = \frac{5}{12}mb^2\left(2\frac{m_B}{m}\frac{v_0}{b}\right)$

(a) $h = \frac{5}{6}b = \frac{5}{6}(200\,mm)$ $\qquad \underline{h = 166.7\,mm}$ ◀

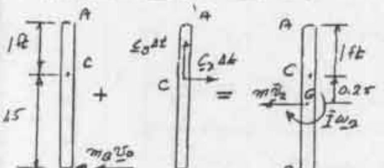
(b) $m_B v_0 = m\bar{v}_2$; $(0.045\,kg)(400\,m/s) = (9\,kg)\bar{v}_2$; $\underline{\bar{v}_2 = 2\,m/s \rightarrow}$ ◀

17.98



GIVEN:
BULLET: $W_B = 0.08\,lb$, $v_0 = 1800\,ft/s$
ROD: $W = 15\,lb$, $L = 30\,in.$
$h = 12\,in.$

FIND: (a) ω JUST AFTER IMPACT
(b) $\underline{C}$ FOR $\Delta t = 0.001\,s$



$\bar{I} = \frac{1}{12}mL^2$
$= \frac{1}{12}\left(\frac{15\,lb}{32.2}\right)\left(\frac{30}{12}ft\right)$
$\bar{I} = 0.24262\,lb\cdot ft\cdot s^2$

$\bar{v}_2 = (0.25\,ft)\omega_2$

+↻ MOMENTS ABOUT C: $m_B v_0(1\,ft) = \bar{I}\omega_2 + m\bar{v}_2(0.25\,ft)$

$\left(\frac{0.08\,lb}{32.2}\right)(1800\,ft/s)(1.5\,ft) = (0.24262\,lb\cdot ft\cdot s^2)\omega_2 + \left(\frac{15\,lb}{32.2}\right)(0.25)^2\omega_2$

$6.708 = (0.24262 + 0.02911)\omega_2$

(a) $\omega_2 = 24.686\,rad/s$ $\qquad \underline{\omega_2 = 24.7\,rad/s}$ ↺ ◀

(b) $\xrightarrow{+}$ x COMPONENTS:

$C_x\Delta t - m_B v_0 = -m\bar{v}_2$

$C_x\Delta t = m_B v_0 - m(0.25\,ft)\omega_2$

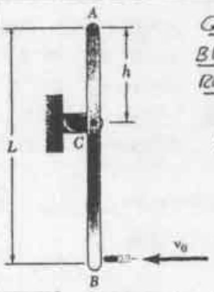
$= \left(\frac{0.08\,lb}{32.2}\right)(1800\,ft/s) - \left(\frac{15\,lb}{32.2}\right)(0.25\,ft)(24.686\,rad/s)$

$C_x\Delta t = 1.597\,lb\cdot s$

$\Delta t = 0.001\,s$

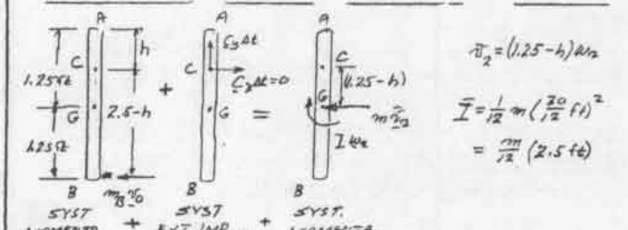
$C_x(0.001\,s) = 1.597\,lb\cdot s$ $\qquad \underline{C_x = 1597\,lb \rightarrow}$ ◀

17.99



GIVEN:
BULLET: $W_B = 0.08$ lb, $v_0 = 1800$ ft/s
ROD: $W = 15$ lb, $L = 30$ in.

FIND: (a) h FOR $C_x = 0$
(b) CORRESPONDING ω JUST AFTER IMPACT



$\bar{v}_2 = (1.25 - h)\omega_2$

$\bar{I} = \frac{1}{12} m\left(\frac{30}{12}\text{ ft}\right)^2 = \frac{m}{12}(2.5\text{ ft})^2$

SYST MOMENTA$_1$ + SYST EXT. IMP$_{1\to 2}$ = SYST. MOMENTA$_2$

$\xleftarrow{+}$ X COMPONENTS:

$m_B v_0 = m\bar{v}_2$

$m_B v_0 = m(1.25 - h)\omega_2 \qquad (1)$

+↺ MOMENTS ABOUT G: $m_B v_0 (1.25\text{ ft}) = \bar{I}\omega_2 \qquad (2)$

SUBSTITUTE FOR $m_B v_0$ FROM (1) INTO (2):

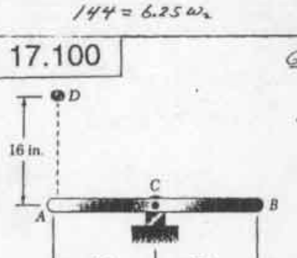
(a) $\left[\cancel{m}(1.25-h)\cancel{\omega_2}\right](1.25\text{ ft}) = \frac{\cancel{m}}{12}(2.5\text{ ft})^2\cancel{\omega_2}$

$(1.25 - h) = \frac{2.5^2}{12(1.25)} = 0.41667$; $\quad h = 0.8333\text{ ft} = 10\text{ in.}$

(b) EQ(1): $\frac{0.08\text{ lb}}{g}(1800\text{ ft/s}) = \frac{15\text{ lb}}{g}(1.25 - 0.8333)\omega_2$

$144 = 6.25\,\omega_2 \qquad \omega_2 = 23.0$ rad/s ↻

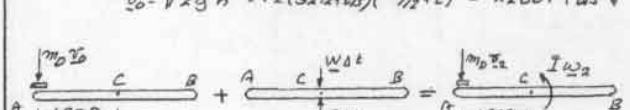
17.100



GIVEN: 0.6-lb MAGNET D
8-lb ROD AB
MAGNET RELEASED FROM POSITION SHOWN
FIND: ω AFTER IMPACT ($e = 0$)
v_D AFTER IMPACT ($e = 0$)

MAGNET STRIKES BAR WITH VELOCITY v_0

$v_0 = \sqrt{2gh} = \sqrt{2(32.2\text{ ft/s}^2)(16/12\text{ ft})} = 9.2664$ ft/s ↓



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to 2}$ = SYST. MOMENTA$_2$

$v_2 = (1.25\text{ ft})\omega_2$

(a)
+↺ MOMENTS ABOUT C: $m_D v_0 (1.25\text{ ft}) = \bar{I}\omega_2 + m_D v_2 (1.25\text{ ft})$

$\frac{0.6\text{ lb}}{g}(9.2664\text{ ft/s})(1.25\text{ ft}) = \frac{1}{12}\frac{8\text{ lb}}{g}(2.5\text{ ft})^2\omega_2 + \frac{0.6\text{ lb}}{g}(1.25\text{ ft})^2\omega_2$

$6.9498 = (4.1667 + 0.9375)\omega_2$

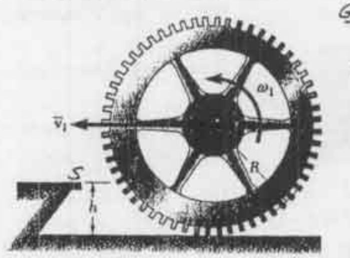
$6.9498 = 5.1047\,\omega_2$

$\omega_2 = 1.3616$ rad/s $\qquad \omega_2 = 1.362$ rad/s ↺

(b) $v_A = v_D = (1.25\text{ ft})\omega_2 = (1.25\text{ ft})(1.3616\text{ rad/s}) = 1.7020$ ft/s

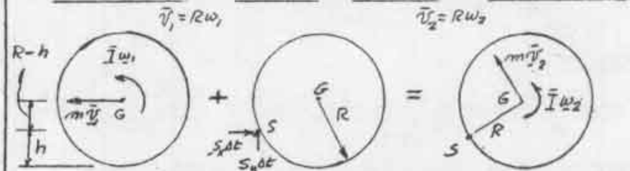
$v_A = 1.702$ ft/s ↓

17.101 and 17.102



GIVEN:
GEAR: $R = 150$ mm, $\bar{k} = 125$ mm
$\bar{v}_1 = 3$ m/s
GEAR ROLL AND HITS STEP, NO SLIPPING OCCURS BETWEEN STEP + GEAR
PROBLEM 17.101
FIND: ω_2 FOR $h = 75$ mm
PROBLEM 17.102
FIND: ω_2 FOR $h = 150$ mm

$\bar{v}_1 = R\omega_1 \qquad \bar{v}_2 = R\omega_2$



+↺ MOMENTS ABOUT S: $m\bar{v}_1(R-h) + \bar{I}\omega_1 = m\bar{v}_2 R + \bar{I}\omega_2$

$\cancel{m}(R\omega_1)(R-h) + \cancel{m}\bar{k}^2\omega_1 = \cancel{m}(R\omega_2)R + \cancel{m}\bar{k}^2\omega_2$

$\left[R(R-h) + \bar{k}^2\right]\omega_1 = (R^2 + \bar{k}^2)\omega_2$

$\omega_2 = \frac{R^2 + \bar{k}^2 - Rh}{R^2 + \bar{k}^2}\omega_1 \qquad \omega_2 = \left[1 - \frac{Rh}{R^2 + \bar{k}^2}\right]\omega_1 \qquad (1)$

DATA: $R = 150$ mm, $\bar{k} = 125$ mm, $\bar{v}_1 = 3$ m/s

$\omega_1 = \frac{\bar{v}_1}{R} = \frac{3\text{ m/s}}{0.150\text{ m}} = 20$ rad/s

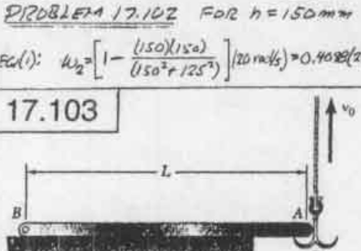
PROBLEM 17.101 FOR $h = 75$ mm

EQ(1): $\omega_2 = \left[1 - \frac{(150)(75)}{(150^2 + 125^2)}\right](20\text{ rad/s}) = 0.7049(20)$; $\quad \omega_2 = 14.10$ rad/s ↺

PROBLEM 17.102 FOR $h = 150$ mm

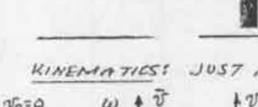
EQ(1): $\omega_2 = \left[1 - \frac{(150)(150)}{(150^2 + 125^2)}\right](20\text{ rad/s}) = 0.4098(20)$; $\quad \omega_2 = 8.20$ rad/s ↺

17.103



GIVEN: ROD OF MASS m
PLASTIC IMPACT BETWEEN HOOK C AND ROD AT A.
FIND: IMPULSE EXERTED ON ROD (a) AT A, (b) AT B.

KINEMATICS: JUST AFTER IMPACT, ROD ROTATES ABOUT B



$\omega = \frac{v_0}{L}, \quad \bar{v}_1 = \frac{1}{2}v_0$

KINETICS

SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to 2}$ = SYST. MOMENTA$_2$

+↺ MOMENTS ABOUT B: $(A\Delta t)L = m\bar{v}_2\frac{L}{2} + \bar{I}\omega_2$

$(A\Delta t)L = m\left(\frac{v_0}{2}\right)\frac{L}{2} + \frac{1}{12}mL^2\left(\frac{v_0}{L}\right)$

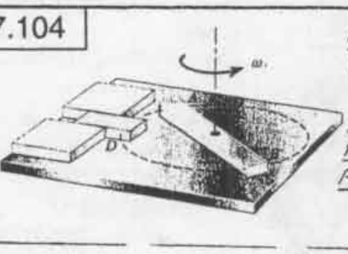
$A\Delta t = \left(\frac{1}{4} + \frac{1}{12}\right)m v_0 L \qquad A\Delta t = \frac{1}{3}m v_0$ ↑

+y COMPONENTS: $(A\Delta t) + (B\Delta t) = m\bar{v}_2$

$\frac{1}{3}m v_0 + (B\Delta t) = m\frac{v_0}{2}$

$B\Delta t = \frac{1}{6}m v_0 \qquad B\Delta t = \frac{1}{6}m v_0$ ↑

17.104

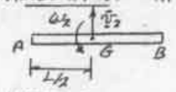


GIVEN: BAR AB OF MASS m AND LENGTH L.

IMPACT IS PERFECTLY PLASTIC

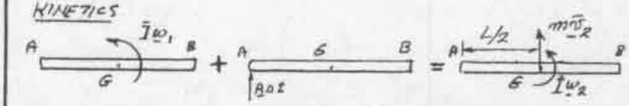
FIND: ω AND $\bar{v}$ JUST AFTER IMPACT

KINEMATICS: AFTER IMPACT $v_A = 0$ $(e = 0)$



$$\bar{v}_2 = \frac{L}{2}\omega_2 \uparrow \qquad (1)$$

KINETICS



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

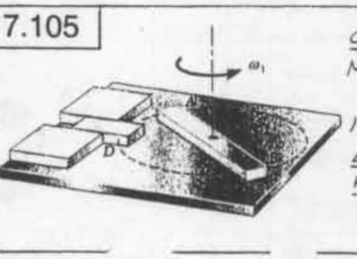
+↻ MOMENTS ABOUT A: $\bar{I}\omega_1 = m\bar{v}_2\frac{L}{2} + \bar{I}\omega_2$

$$\frac{1}{12}mL^2\omega_1 = m\left(\frac{L}{2}\right)^2\omega_2 + \frac{1}{12}mL^2\omega_2$$

$$\frac{\omega_1}{12} = \frac{\omega_2}{3} \qquad \omega_2 = \frac{1}{4}\omega_1 ↻ \blacktriangleleft$$

EQ(1): $\bar{v}_2 = \frac{L}{2}\omega_2 = \frac{L}{2}\left(\frac{1}{4}\omega_1\right)$ $\qquad \bar{v}_2 = \frac{1}{8}L\omega_1 \uparrow \blacktriangleleft$

17.105



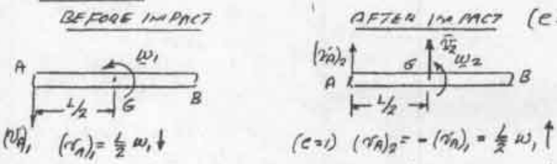
GIVEN: ROD AB OF MASS m AND LENGTH L

IMPACT IS PERFECTLY ELASTIC.

FIND: ω AND $\bar{v}$ JUST AFTER IMPACT

KINEMATICS

BEFORE IMPACT $\qquad$ AFTER IMPACT $(e = 1)$



$(v_A)_1 = \frac{L}{2}\omega_1 \downarrow$ $\qquad (e=1)$ $(v_A)_2 = -(v_A)_1 = \frac{L}{2}\omega_1 \uparrow$

$$+\uparrow \bar{v}_2 = (v_A)_2 + \frac{L}{2}\omega_2 = \frac{L}{2}\omega_1 + \frac{L}{2}\omega_2$$

$$\bar{v}_2 = \frac{L}{2}(\omega_1 + \omega_2) \uparrow \qquad (1)$$

KINETICS

SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $\bar{I}\omega_1 = m\bar{v}_2\frac{L}{2} + \bar{I}\omega_2$

$$\frac{1}{12}mL^2\omega_1 = m\frac{L}{2}(\omega_1+\omega_2)\frac{L}{2} + \frac{1}{12}mL^2\omega^2$$

$$\left(\frac{1}{12} - \frac{1}{4}\right)\omega_1 = \left(\frac{1}{4} + \frac{1}{12}\right)\omega_2$$

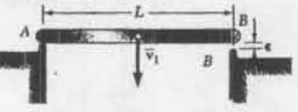
$$-\frac{1}{6}\omega_1 = \frac{1}{3}\omega_2 \qquad \omega_2 = -\frac{1}{2}\omega_1$$

$$\omega_2 = \frac{1}{2}\omega_1 ↺ \blacktriangleleft$$

EQ(1) $\bar{v}_2 = \frac{L}{2}(\omega_1 + \omega_2) = \frac{L}{2}\left(\omega_1 - \frac{1}{2}\omega_1\right)$

$$\bar{v}_2 = \frac{1}{4}\omega_1 L \uparrow \blacktriangleleft$$

17.106

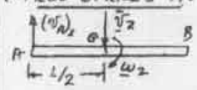


GIVEN: ROD STRIKES WITH $\bar{v}_1 \downarrow$ AND $\omega_1 = 0$

PERFECTLY ELASTIC IMPACT $(e=1)$

FIND: $\bar{v}$ AND ω AFTER ROD STRIKES (a) A, (b) B, (c) AGAIN A.

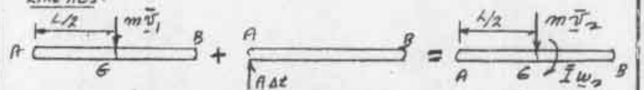
(a) ROD STRIKES A: $(v_A)_1 = \bar{v}_1 \downarrow$; SINCE $e=1$, $(v_A)_2 = \bar{v}_1 \uparrow$



$$+\downarrow \bar{v}_2 = -(v_A)_2 + \frac{L}{2}\omega_2$$

$$\bar{v}_2 = \left(\frac{L}{2}\omega_2 - \bar{v}_1\right)\downarrow$$

KINETICS:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $m\bar{v}_1\frac{L}{2} = m\bar{v}_2\frac{L}{2} + \bar{I}\omega_2$

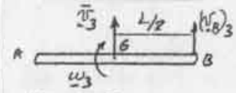
$$m\bar{v}_1\frac{L}{2} = m\left(\frac{L}{2}\omega_2 - \bar{v}_1\right)\frac{L}{2} + \frac{1}{12}mL^2\omega_2$$

$$v_1 L = \frac{1}{3}L^2\omega_2 \qquad \omega_2 = 3\frac{\bar{v}_1}{L} ↻ \blacktriangleleft$$

$$\bar{v}_2 = \frac{L}{2}\omega_2 - \bar{v}_1 = \frac{L}{2}\left(3\frac{\bar{v}_1}{L}\right) - \bar{v}_1 \qquad \bar{v}_2 = \frac{1}{2}\bar{v}_1 \downarrow \blacktriangleleft$$

$+\downarrow (v_B)_2 = -(v_A)_2 + L\omega_2 = -\bar{v}_1 + L\left(3\frac{\bar{v}_1}{L}\right)$; $(v_B)_2 = 2\bar{v}_1 \downarrow$

(b) ROD STRIKES B: SINCE $e=1$, $(v_B)_3 = -(v_B)_2 = 2\bar{v}_1 \uparrow$



$$+\uparrow \bar{v}_3 = (v_B)_3 + \frac{L}{2}\omega_3$$

$$\bar{v}_3 = 2\bar{v}_1 + \frac{L}{2}\omega_3 \uparrow$$

KINETICS:

+↻ MOMENTS ABOUT B: $\bar{I}\omega_2 - m\bar{v}_2\frac{L}{2} = \bar{I}\omega_3 + m\bar{v}_3\frac{L}{2}$

$$\frac{1}{12}mL^2\left(3\frac{\bar{v}_1}{L}\right) - m\left(\frac{\bar{v}_1}{2}\right)\frac{L}{2} = \frac{1}{12}mL^2\omega_3 + m\left(2\bar{v}_1 + \frac{L}{2}\omega_3\right)\frac{L}{2}$$

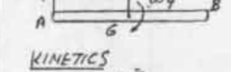
$$0 = \frac{1}{3}mL^2\omega_3 + m\bar{v}_1 L$$

$$\omega_3 = -3\frac{\bar{v}_1}{L} \qquad \omega_3 = 3\frac{\bar{v}_1}{L} ↺ \blacktriangleleft$$

$$\bar{v}_3 = 2\bar{v}_1 + \frac{L}{2}\omega_3 = 2\bar{v}_1 + \frac{L}{2}\left(-3\frac{\bar{v}_1}{L}\right) = \frac{1}{2}\bar{v}_1 \qquad \bar{v}_3 = \frac{1}{2}\bar{v}_1 \uparrow \blacktriangleleft$$

$+\downarrow (v_A)_3 = -(v_B)_3 + L\omega_3 = -2\bar{v}_1 + L\left(\frac{3\bar{v}_1}{L}\right) = \bar{v}_1 \downarrow$

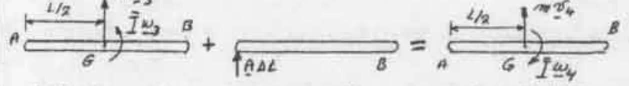
(c) ROD AGAIN STRIKES A: SINCE $e=1$, $(v_A)_4 = -(v_A)_3 = \bar{v}_1 \uparrow$



$$+\uparrow \bar{v}_4 = (v_A)_4 - \frac{L}{2}\omega_4$$

$$\bar{v}_4 = \bar{v}_1 - \frac{L}{2}\omega_4 \uparrow$$

KINETICS



SYST. MOMENTA$_3$ + SYST. EXT. IMP$_{3\to4}$ = SYST. MOMENTA$_4$

+↻ MOMENTS ABOUT A: $\bar{I}\omega_3 + m\bar{v}_3\frac{L}{2} = m\bar{v}_4\frac{L}{2} - \bar{I}\omega_4$

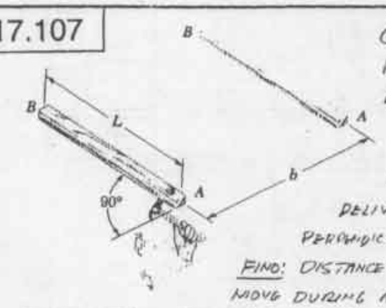
$$\frac{1}{12}mL^2\left(3\frac{\bar{v}_1}{L}\right) + m\left(\frac{1}{2}\bar{v}_1\right)\frac{L}{2} = m\left(\bar{v}_1 - \frac{L}{2}\omega_4\right)\frac{L}{2} - \frac{1}{12}mL^2\omega_4$$

$$\left(\frac{1}{4} + \frac{1}{4} - \frac{1}{2}\right)L\bar{v}_1 = \left(-\frac{1}{4} - \frac{1}{12}\right)L^2\omega_4$$

$$0 = -\frac{1}{3}L^2\omega_4 \qquad \omega_4 = 0 \blacktriangleleft$$

$$\bar{v}_4 = \bar{v}_1 - \frac{L}{2}\omega_4 = \bar{v}_1 - 0 \qquad \bar{v}_4 = \bar{v}_1 \uparrow \blacktriangleleft$$

17.107

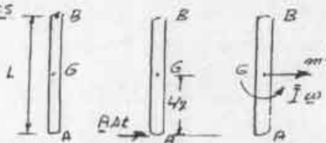


GIVEN: ROD AB IS AT REST ON A FRICTIONLESS HORIZONTAL TABLE. IT IS STRUCK BY A HAMMER THAT DELIVERS AN IMPULSE AT A PERPENDICULAR TO THE ROD

FIND: DISTANCE b THAT ROD WILL MOVE DURING A COMPLETE REVOLUTION

DENOTE BY $A\Delta t$ IMPULSE DELIVERED BY HAMMER

KINETICS



$\bar{I} = \frac{1}{12} mL^2$

SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

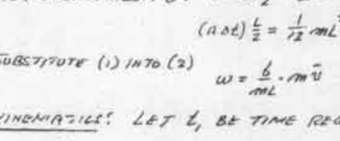
$\xrightarrow{+}$ x COMPONENTS: $A\Delta t = m\bar{v}$ (1)

$+\circlearrowleft$ MOMENTS ABOUT G: $(A\Delta t)\frac{L}{2} = \bar{I}\omega$

$(A\Delta t)\frac{L}{2} = \frac{1}{12} mL^2 \omega \qquad \omega = \frac{6}{mL}(A\Delta t)$ (2)

SUBSTITUTE (1) INTO (2) $\quad \omega = \frac{6}{mL}\cdot m\bar{v} \qquad \omega = \frac{6}{L}\bar{v}$ (3)

KINEMATICS: LET t_1 BE TIME REQUIRED FOR ONE REVOLUTION



$\theta = \omega t$

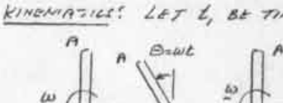
$2\pi = \omega t_1$

$2\pi = \left(\frac{6}{L}\bar{v} t_1\right)$

$\bar{v} t_1 = \frac{2\pi}{6} L$

$b = \bar{v} t_1 = \frac{2\pi}{6} L \qquad b = \frac{\pi}{3} L \rightarrow$

17.108

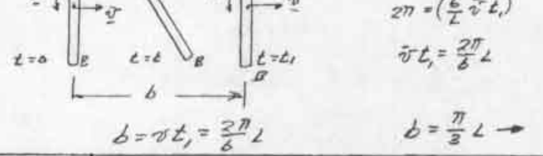


GIVEN: SPHERE ROLLS AND HITS HORIZONTAL SURFACE. AFTER SLIPPING IT STARTS ROLLING AGAIN

FIND: $\bar{v}_2$ AND ω_2 AS IT ROLLS TO THE LEFT

POSITION 2, SPHERE HAS RESUMED ROLLING, $\bar{v}_2 = r\omega_2$

$\bar{v}_1 = r\omega_1$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

$+\circlearrowleft$ MOMENTS ABOUT A: $\bar{I}\omega_1 + (m\bar{v}_1\cos\beta)r = \bar{I}\omega_2 + m\bar{v}_2 r$

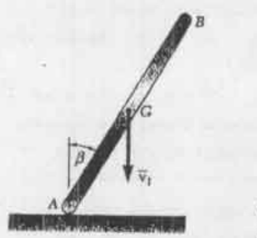
$\frac{2}{5}mr^2\omega_1 + (mr\omega_1\cos\beta)r = \frac{2}{5}mr^2\omega_2 + m(r\omega_2)r$

$\left(\frac{2}{5} + \cos\beta\right)\omega_1 = \frac{7}{5}\omega_2$

$\omega_2 = \frac{1}{7}(2 + 5\cos\beta)\omega_1 \circlearrowleft$

$\bar{v}_2 = r\omega_2 = \frac{1}{7}(2+5\cos\beta)r\omega_1 \qquad \bar{v}_2 = \frac{1}{7}(2+5\cos\beta)\bar{v}_1 \leftarrow$

17.109 and 17.110

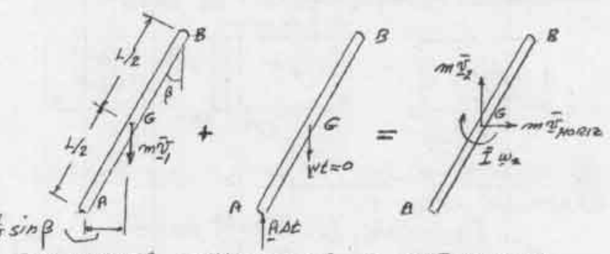


GIVEN: ROD AB STRIKES FRICTIONLESS SURFACE WITH THE VELOCITY SHOWN.

DERIVE AN EXPRESSION FOR ω IMMEDIATELY AFTER IMPACT.

PROBLEM 17.109 ASSUME PERFECTLY ELASTIC IMPACT, $(e = 1)$

PROBLEM 17.110 ASSUME PERFECTLY PLASTIC IMPACT $(e = 0)$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

$\xrightarrow{+}$ x COMPONENTS: $0 = m\bar{v}_{HORIZ} \qquad \bar{v}_{HORIZ} = 0$

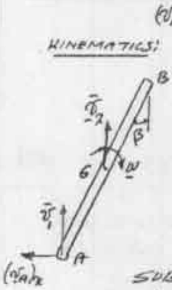
$+\circlearrowleft$ MOMENTS ABOUT A:

$m\bar{v}_1\left(\frac{L}{2}\sin\beta\right) = \bar{I}\omega_2 - m\bar{v}_2\left(\frac{L}{2}\sin\beta\right)$ (1)

PROBLEM 17.109: ELASTIC IMPACT AT A $(e=1)$

$(v_A)_1 = \bar{v}_1\downarrow \quad \therefore \left[(v_A)_2\right]_y = \bar{v}_1\uparrow$

KINEMATICS:



$(v_A)_2 = \bar{v}_G + \bar{v}_{A/G}$

$\left[\bar{v}_1\uparrow\right] + \left[(v_A)_x\leftarrow\right] = \left[\bar{v}_2\uparrow\right] + \left[\frac{L}{2}\omega_2 \measuredangle\beta\right]$

$+\uparrow y$ COMPONENTS:

$\bar{v}_1 = \bar{v}_2 + \frac{L}{2}\omega\sin\beta$

$\bar{v}_2 = \bar{v}_1 - \frac{L}{2}\omega_2\sin\beta$

SUBSTITUTE INTO (1):

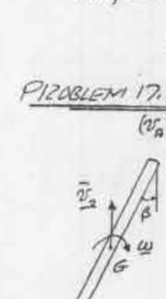
$m\bar{v}_1\frac{L}{2}\sin\beta = \frac{1}{12}mL^2\omega_2 - m\left(\bar{v}_1 - \frac{L}{2}\omega_2\sin\beta\right)\left(\frac{L}{2}\sin\beta\right)$

$m\bar{v}_1 L\sin\beta = mL^2\left(\frac{1}{12} + \frac{1}{4}\sin^2\beta\right)\omega_2$

$\omega_2 = \frac{\bar{v}_1}{L}\cdot\frac{12\sin\beta}{3\sin^2\beta + 1} \circlearrowright$

PROBLEM 17.110: PLASTIC IMPACT $(e=0)$

$(v_A)_1 = \bar{v}_1\downarrow \quad \therefore \left[(v_A)_2\right]_y = 0$



$(v_A)_2 = \bar{v}_G + \bar{v}_{A/G}$

$\left[(v_A)_x\leftarrow\right] = \left[\bar{v}_2\uparrow\right] + \left[\frac{L}{2}\omega_2 \measuredangle\beta\right]$

$+y$ COMPONENTS:

$0 = \bar{v}_2 + \frac{L}{2}\omega_2\sin\beta$

$\bar{v}_2 = -\frac{L}{2}\omega_2\sin\beta$

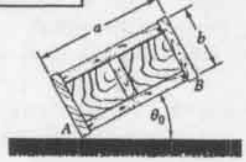
SUBSTITUTE INTO (1)

$m\bar{v}_1\frac{L}{2}\sin\beta = \frac{1}{12}mL^2\omega_2 - m\left(-\frac{L}{2}\omega_2\sin\beta\right)\frac{L}{2}\sin\beta$

$m\bar{v}_1\frac{L}{2}\sin\beta = mL^2\left(\frac{1}{12} + \frac{1}{4}\sin^2\beta\right)\omega_2$

$\omega_2 = \frac{\bar{v}_1}{L}\cdot\frac{6\sin\beta}{3\sin^2\beta + 1} \circlearrowright$

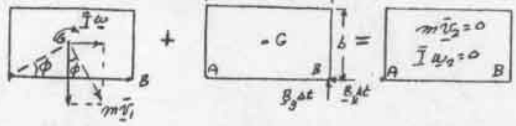
17.111



GIVEN: UNIFORM CRATE IS RELEASED FROM REST. IMPACT AT B IS PERFECTLY PLASTIC.

FIND: SMALLEST VALUE OF $\frac{a}{b}$ FOR WHICH CORNER A REMAINS IN CONTACT WITH FLOOR

WE CONSIDER THE LIMITING CASE WHEN THE CRATE IS JUST READY TO ROTATE ABOUT B. AT THAT INSTANT THE VELOCITIES MUST BE ZERO AND THE REACTION AT CORNER A MUST BE ZERO.



SYST MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↺ MOMENTS ABOUT B

$$\bar I\omega_1 + (m\bar v_1)_x\frac{b}{2} - (m\bar v_1)_y\frac{a}{2} + 0 = 0 \qquad (1)$$

NOTE: $\sin\phi = \frac{b}{\sqrt{a^2+b^2}}$, $\cos\phi = \frac{a}{\sqrt{a^2+b^2}}$

$$\bar v_1 = (AG)\omega_1 = \frac{1}{2}\sqrt{a^2+b^2}\,\omega_1$$

THUS: $(m\bar v_1)_x = (m\bar v_1)\sin\phi = \frac{m}{2}\sqrt{a^2+b^2}\,\omega_1\frac{b}{\sqrt{a^2+b^2}} = \frac{1}{2}mb\omega_1$

ALSO, $(m\bar v_1)_y = (m\bar v_1)\cos\phi = \frac{1}{2}ma\omega_1$

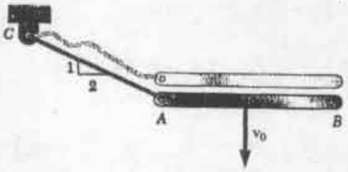
$$\bar I = \frac{1}{12}m(a^2+b^2)$$

EQ(1): $\frac{1}{12}m(a^2+b^2)\omega_1 + \frac{1}{2}(mb\omega_1)\frac{b}{2} - \frac{1}{2}(ma\omega_1)\frac{a}{2} = 0$

$$\frac{1}{3}mb^2\omega_1 - \frac{1}{6}ma^2\omega_1 = 0$$

$$\frac{b^2}{a^2} = 2 \qquad \frac{b}{a} = \frac{1}{\sqrt{2}}$$

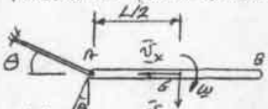
17.112



GIVEN: ROD OF LENGTH L. ASSUMING PERFECTLY PLASTIC IMPACT,

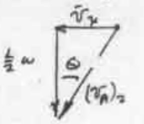
FIND: ω AND $\bar v$ JUST AFTER CORD BECOMES TAUT.

KINEMATICS (JUST AFTER IMPACT) LET $\theta = \tan^{-1}\frac{1}{2}$



$$(v_A)_2 = \bar v_G + v_{A/G}$$

$$\left[(v_A)_2 \measuredangle\theta\right] = \left[\bar v_x \leftarrow\right] + \left[\bar v_y - \frac{L}{2}\omega \downarrow\right]$$

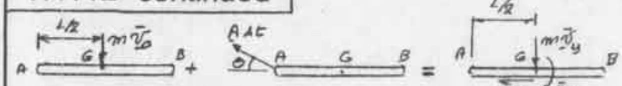


$$\tan\theta = \frac{\bar v_x}{\bar v_y - \frac{L}{2}\omega}$$

$$\bar v_x = \left(\bar v_y - \frac{L}{2}\omega\right)\tan\theta \qquad (1)$$

(CONTINUED)

17.112 continued



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↺ MOMENTS ABOUT A

$$m\bar v_0\frac{L}{2} = \bar I\omega + m\bar v_y\frac{L}{2}$$

$$m\bar v_0\frac{L}{2} = \frac{1}{12}mL^2\omega + m\bar v_y\frac{L}{2}$$

$$\bar v_0 = \frac{1}{6}L\omega + \bar v_y \qquad (2)$$

+ COMPONENTS:

$$m\bar v_0\cos\theta = m\bar v_x\sin\theta + m\bar v_y\cos\theta$$

$$\bar v_0 = \bar v_x\tan\theta + \bar v_y \qquad (3)$$

(1)→(3) $\bar v_0 = \left(\bar v_y - \frac{L}{2}\omega\right)\tan^2\theta + \bar v_y$

$$\bar v_0 = \bar v_y(1+\tan^2\theta) - \frac{L}{2}\omega\tan^2\theta$$

$$\omega = \frac{2}{L}\left(\bar v_y\frac{1+\tan^2\theta}{\tan^2\theta} - \frac{\bar v_0}{\tan^2\theta}\right) \qquad (4)$$

(4)→(2) $\bar v_0 = \frac{L}{6}\cdot\frac{2}{L}\left(\bar v_y\frac{1+\tan^2\theta}{\tan^2\theta} - \frac{\bar v_0}{\tan^2\theta}\right) + \bar v_y$

$$\bar v_0 = \bar v_y\left(1+\frac{1}{3}\frac{1+\tan^2\theta}{\tan^2\theta}\right) - \frac{1}{3}\frac{\bar v_0}{\tan^2\theta}$$

$$3\bar v_0\tan^2\theta = \bar v_y(1+4\tan^2\theta) - \bar v_0$$

$$\bar v_y = \frac{1+3\tan^2\theta}{1+4\tan^2\theta}\bar v_0 \qquad (5)$$

(5)→(2) $\bar v_0 = \frac{L}{6}\cdot\omega + \frac{1+3\tan^2\theta}{1+4\tan^2\theta}\bar v_0$

$$\omega = \frac{6}{L}\left[1 - \frac{1+3\tan^2\theta}{1+4\tan^2\theta}\right]\bar v_0 = \frac{6}{L}\left[\frac{1+4\tan^2\theta - 1 - 3\tan^2\theta}{1+4\tan^2\theta}\right]$$

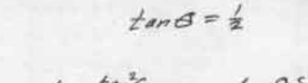
$$\omega = \frac{6}{L}\frac{\tan^2\theta}{1+4\tan^2\theta}\bar v_0 \qquad (6)$$

(6) AND (5)→(1) $\bar v_x = \left(\bar v_y - \frac{L}{2}\omega\right)\tan\theta$

$$\bar v_x = \left[\frac{1+3\tan^2\theta}{1+4\tan^2\theta}\bar v_0 - \frac{L}{2}\cdot\frac{6}{L}\cdot\frac{\tan^2\theta}{1+4\tan^2\theta}\bar v_0\right]\tan\theta$$

$$\bar v_x = \frac{\tan\theta}{1+4\tan^2\theta}\bar v_0 \qquad (7)$$

DATA:



$$\tan\theta = \frac{1}{2}$$

EQ(6): $\omega = \frac{6}{L}\frac{\tan^2\theta}{1+4\tan^2\theta}v_0 = \frac{6}{L}\frac{0.5^2}{1+4(0.5)^2}v_0 = \frac{1.5}{2}\frac{v_0}{L}$

$$\omega = \frac{3}{4}\frac{v_0}{L}$$

EQ(7): $\bar v_x = \frac{\tan\theta}{1+4\tan\theta}\bar v_0 = \frac{0.5}{1+4(0.5)^2}v_0 = \frac{0.5}{2}v_0$

$$\bar v_x = \frac{1}{4}v_0 \leftarrow$$

EQ(5) $\bar v_y = \frac{1+3\tan^2\theta}{1+4\tan^2\theta}v_0 = \frac{1+3(0.5)^2}{1+4(0.5)^2}v_0 = \frac{1.75}{2}v_0$

$$\bar v_y = \frac{7}{8}v_0 \downarrow$$

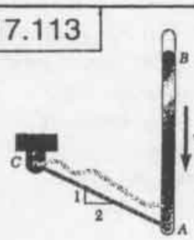
CHECK: EQ(1) $\bar v_x = \left(\bar v_y - \frac{L}{2}\omega\right)\tan\theta$

$$= \left(\frac{7}{8}\bar v_0 - \frac{L}{2}\cdot\frac{3}{4}\frac{v_0}{L}\right)(0.5)$$

$$= \left(\frac{7-3}{8}v_0\right)0.5$$

$$\bar v_x = \frac{1}{4}v_0 \checkmark$$

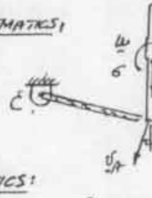
17.113



GIVEN: ROD AB OF LENGTH L. ASSUMING PERFECTLY PLASTIC IMPACT

FIND: ω AND $\bar v$ IMMEDIATELY AFTER CORD BECOMES TAUT.

KINEMATICS:

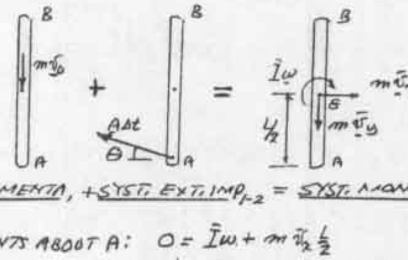


$\bar v = v_A \angle\theta + \frac{L}{2}\omega \rightarrow$

$\bar v_x = (\frac{L}{2}\omega - v_A\sin\theta)\rightarrow$

$\bar v_y = v_A\cos\theta\downarrow$

KINETICS:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↺ MOMENTS ABOUT A: $0=\bar I\omega + m\bar v_x\frac{L}{2}$

$$0=\frac{1}{12}mL^2\omega + m\left(\frac{L}{2}\omega - v_A\sin\theta\right)\frac{L}{2}$$

$$0=\frac{1}{3}\cancel{m}L^2\omega - \cancel{m}v_A\frac{L}{2}\sin\theta$$

$$\omega=\frac{3}{2}\frac{v_A}{L}\sin\theta \qquad (1)$$

+↗ COMPONENTS: $\cancel{m}v_0\cos\theta = \cancel{m}\bar v_y\cos\theta - \cancel{m}\bar v_x\sin\theta$

$$v_0\cos\theta = v_A\cos^2\theta - \left(\frac{L}{2}\omega - v_A\sin\theta\right)\sin\theta$$

$$v_0\cos\theta = v_A(\cos^2\theta+\sin^2\theta) - \frac{L}{2}\omega\sin\theta$$

$$v_A = v_0\cos\theta + \frac{L}{2}\omega\sin\theta \qquad (2)$$

(2)→(1) $\omega = \frac{3}{2L}\left(v_0\cos\theta + \frac{L}{2}\omega\sin\theta\right)\sin\theta$

$$\omega = \frac{3v_0}{2L}\cos\theta\sin\theta + \frac{3}{4}\omega\sin^2\theta$$

$$\omega = \frac{3}{2}\frac{v_0}{L}\,\frac{\cos\theta\sin\theta}{1-\frac{3}{4}\sin^2\theta}$$

For $\theta=\tan^{-1}0.5$, $\cos\theta=\frac{2}{\sqrt5}$ AND $\sin\theta=\frac{1}{\sqrt5}$

$$\omega = \frac{3}{2}\frac{v_0}{L}\cdot\frac{(2/\sqrt5)(1/\sqrt5)}{1-\frac{3}{4}(\frac{1}{\sqrt5})^2} = \frac{3}{5}\cdot\frac{1}{0.85}\frac{v_0}{L}$$

$\omega=0.7059\frac{v_0}{L}$ $\qquad \omega=0.706\frac{v_0}{L}$ ↻ ◀

EQ(2) $v_A = v_0\cos\theta + \frac{L}{2}\omega\sin\theta$

$$= v_0\frac{2}{\sqrt5} + \frac{L}{2}\left(0.7059\frac{v_0}{L}\right)\frac{1}{\sqrt5}$$

$$=(0.8944+0.1578)v_0$$

$$v_A = 1.0522\,v_0$$

$$\bar v_x = \frac{L}{2}\omega - v_A\sin\theta = \frac{L}{2}\left(0.7059\frac{v_0}{L}\right) - (1.0522v_0)\frac{1}{\sqrt5}$$

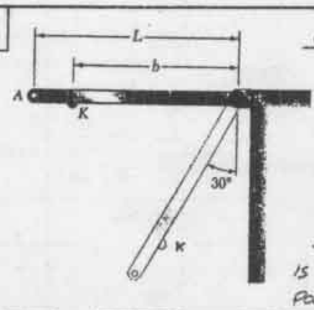
$$=(0.35295-0.47059)v_0 = -0.11764v_0$$

$\bar v_x = 0.1176\,v_0 \leftarrow$ ◀

$$\bar v_y = v_A\cos\theta = (1.0522v_0)\frac{2}{\sqrt5} = 0.9411\,v_0$$

$\bar v_y = 0.941\,v_0\downarrow$ ◀

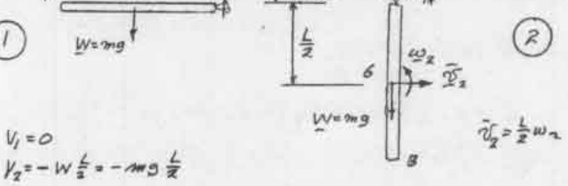
17.114



GIVEN: ROD IS RELEASED FROM POSITION SHOWN AND REBOUNDS TO 30° WITH THE VERTICAL.

FIND: (a) COEF. OF RESTITUTION, (b) SHOW THAT REBOUND IS INDEPENDENT OF POSITION OF KNOB K

CONSERVATION OF ENERGY: BEFORE IMPACT



$V_1=0$

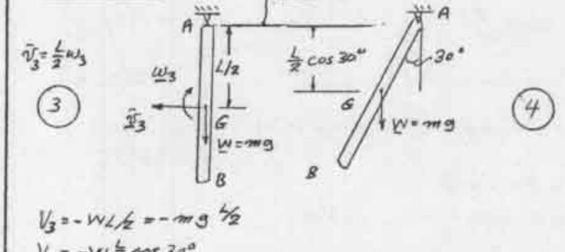
$V_2=-W\frac{L}{2}=-mg\frac{L}{2}$

$T_1=0$

$$T_2=\frac12\bar I\omega_2^2+\frac12m\bar v_2^2=\frac12\left(\frac{1}{12}mL^2\right)\omega_2^2+\frac12m\left(\frac{L}{2}\omega_2\right)^2=\frac16mL^2\omega_2^2$$

$T_1+V_1=T_2+V_2$: $\quad 0=\frac16mL^2\omega_2^2=-mg\frac{L}{2}$; $\quad \omega_2^2=3\frac{g}{L}$.

AFTER IMPACT:



$V_3=-WL/2=-mg\,L/2$

$V_4=-W\frac{L}{2}\cos30°$

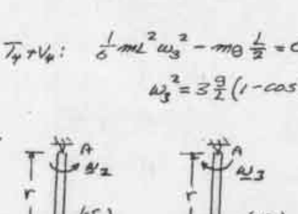
$$T_3=\frac12\bar I\omega_3^2+\frac12m\bar v_3^2=\frac12\left(\frac{1}{12}mL^2\right)\omega_3^2+\frac12m\left(\frac{L}{2}\omega_3\right)^2=\frac16mL^2\omega_3^2$$

$T_4=0$

$T_3+V_3=T_4+V_4$: $\quad \frac16mL^2\omega_3^2-mg\frac{L}{2}=0-mg\frac{L}{2}\cos30°$

$$\omega_3^2=3\frac{g}{L}(1-\cos30°).$$

IMPACT



$(v_K)_2=r\omega_2=r\sqrt{3\frac{g}{L}}$ $\qquad (v_K)_3=r\omega_3=r\sqrt{3\frac{g}{L}(1-\cos30°)}$

COEFFICIENT OF RESTITUTION

$$e=\frac{(v_K)_3}{(v_K)_2}=\frac{r\sqrt{3\frac{g}{L}(1-\cos30°)}}{r\sqrt{3\frac{g}{L}}} \qquad e=\sqrt{(1-\cos30°)}$$

$$e=\sqrt{(1-\cos30°)}=\sqrt{1-0.86603}$$

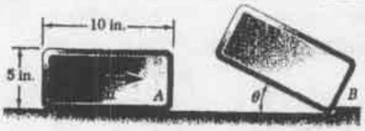
$e=0.366$ ◀

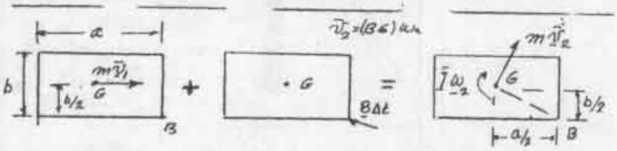
WE NOTE THAT RESULT IS INDEPENDENT OF THE POSITION OF THE KNOB.

17.115

GIVEN: UNIFORM BLOCK STRIKES OBSTRUCTION AT B.

FIND: $\bar{v}_1$ FOR WHICH MAXIMUM VALUE OF $\theta = 30°$





SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to 2}$ = SYST. MOMENTA$_2$

+) MOMENTS ABOUT B: $m\bar{v}_1 \frac{b}{2} = \bar{I}\omega_2 + m\bar{v}_2(BG)$

$$m\bar{v}_1 \frac{b}{2} = \bar{I}\omega_2 + m(BG)^2\omega_2$$

$$BG^2 = (a/2)^2 + (b/2)^2 = \frac{1}{4}(a^2+b^2)$$

$$\bar{I} = \frac{1}{12}m(a^2+b^2)$$

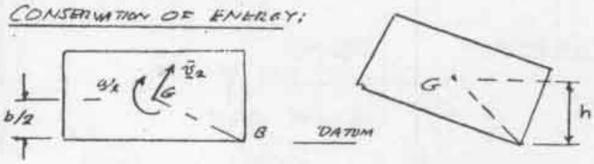
$$\cancel{m}\bar{v}_1 \frac{b}{2} = \frac{1}{12}\cancel{m}(a^2+b^2)\omega_2 + \cancel{m}\frac{1}{4}(a^2+b^2)\omega_2$$

$$\bar{v}_1 \frac{b}{2} = \frac{1}{3}(a^2+b^2)\omega_2 \qquad \omega_2 = \frac{3}{2}\frac{b}{a^2+b^2}\bar{v}_1$$

DATA: $a = \frac{10}{12}$ ft $\quad b = \frac{5}{12}$ ft

$$\omega_2 = \frac{3}{2}\frac{5/12}{\left[\left(\frac{10}{12}\right)^2 + \left(\frac{5}{12}\right)^2\right]}\bar{v}_1 \qquad \omega_2 = 0.720\,\bar{v}_1 \qquad (1)$$

CONSERVATION OF ENERGY:



$$T_1 = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\bar{v}_2^2 = \frac{1}{2}\frac{1}{12}m(a^2+b^2)\omega_2^2 + \frac{1}{2}m\frac{1}{4}(a^2+b^2)\omega_2^2$$

$$T_1 = \frac{1}{6}m(a^2+b^2)\omega_2^2$$

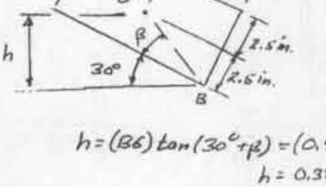
$V_1 = W\frac{b}{2} = mg\frac{b}{2}$

$V_2 = Wh = mgh \qquad T_2 = 0$

$T_1 + V_1 = T_2 + V_2$: $\quad \frac{1}{6}\cancel{m}(a^2+b^2)\omega_2^2 + \cancel{m}g\frac{b}{2} = \cancel{m}g\,h$

$$\omega_2^2 = \frac{6(h - b/2)}{(a^2+b^2)}g \qquad (2)$$

For $\theta_{max} = 30°$



$$\tan\beta = \frac{2.5\text{ in}}{5\text{ in.}}$$

$$\beta = 26.565°$$

$$BG = \sqrt{2.5^2 + 5^2} = 5.5902\text{ in}$$

$$BG = 0.46585\text{ ft}$$

$$h = (BG)\tan(30° + \beta) = (0.46585\text{ ft})\sin(30° + 26.565°)$$

$$h = 0.38876\text{ ft}$$

EQ(2): $\quad \omega_2^2 = \frac{6(h - b/2)}{g(a^2+b^2)} = \frac{6\left(0.38876 - \frac{2.5}{12}\right)}{\left[\left(\frac{10}{12}\right)^2 + \left(\frac{5}{12}\right)^2\right]}32.2 = 40.156$

$$\omega_2 = 6.337\text{ rad/s}$$

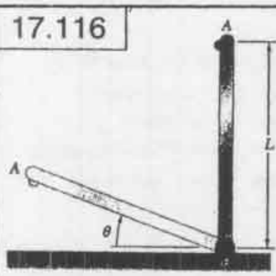
EQ(1): $\quad \omega_2 = 0.720\,\bar{v}_1$; $\quad 6.337 = 0.720\,\bar{v}_1$

$$\bar{v}_1 = 8.80\text{ ft/s}$$

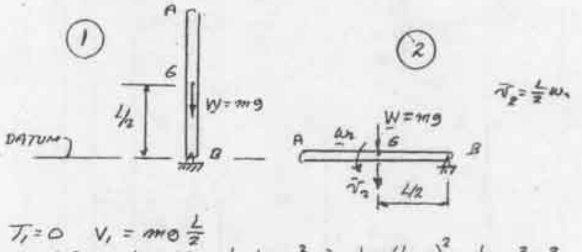
17.116

GIVEN: ROD AB IS GIVEN SLIGHT NUDGE AND ROTATES COUNTERCLOCKWISE HITS SURFACE AND REBOUNDS $e = 0.40$

FIND: MAXIMUM θ OF REBOUND.



CONSERVATION OF ENERGY:



$T_1 = 0 \quad V_1 = mg\frac{L}{2}$

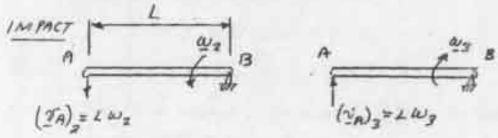
$$T_2 = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\bar{v}_2^2 = \frac{1}{2}\cdot\frac{1}{12}mL^2\omega_2^2 + \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2 = \frac{1}{6}mL^2\omega_2^2$$

$V_2 = 0$

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + mg\frac{L}{2} = \frac{1}{6}mL^2\omega_2^2 + 0$

$$\omega_2^2 = 3\frac{g}{L}$$

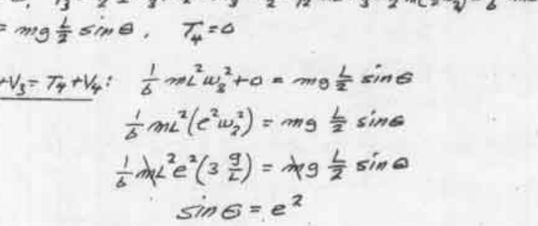
IMPACT



$$e = \frac{(v_A)_3}{(v_A)_2} = \frac{L\omega_3}{L\omega_2}: \qquad \omega_3 = e\,\omega_2$$

$$\text{OR;} \qquad \omega_3^2 = e^2\omega_2^2$$

CONSERVATION OF ENERGY



$$V_3 = 0, \quad T_3 = \frac{1}{2}\bar{I}\omega_3^2 + \frac{1}{2}m\bar{v}_3^2 = \frac{1}{2}\cdot\frac{1}{12}mL^2\omega_3^2 + \frac{1}{2}m\left(\frac{L}{2}\omega_3\right)^2 = \frac{1}{6}mL^2\omega_3^2$$

$V_4 = mg\frac{L}{2}\sin\theta, \quad T_4 = 0$

$T_3 + V_3 = T_4 + V_4$: $\quad \frac{1}{6}mL^2\omega_3^2 + 0 = mg\frac{L}{2}\sin\theta$

$$\frac{1}{6}mL^2(e^2\omega_2^2) = mg\frac{L}{2}\sin\theta$$

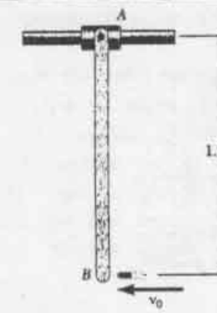
$$\frac{1}{6}\cancel{m}L^2e^2\left(3\frac{g}{L}\right) = \cancel{m}g\frac{L}{2}\sin\theta$$

$$\sin\theta = e^2$$

For $e = 0.40$ $\qquad \sin\theta = (0.40)^2 = 0.16$

$$\theta = 9.21°$$

17.117 and 17.118

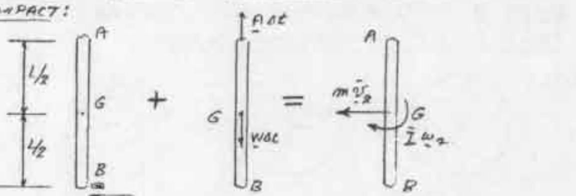


GIVEN: 30-g BULLET FIRED INTO THE 8-kg BEAM. COLLAR A SLIDES FREELY.

PROBLEM 17.117
FIND: MAXIMUM ANGLE OF ROTATION OF BEAM FOR $\bar{v}_0 = 350$ m/s ←

PROBLEM 17.118:
FIND: $\bar{v}_0$ FOR WHICH MAXIMUM ANGLE OF ROTATION OF BEAM IS 90°

IMPACT:

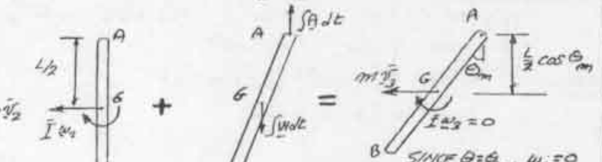


SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

$\stackrel{+}{\leftarrow}$ COMPONENTS: $m_B v_0 = m\bar{v}_2$ (1)

+↻ MOMENTS ABOUT G: $m_B v_0 \frac{L}{2} = \bar{I}\omega_2$; $\omega_2 = \frac{m_B v_0 L}{2\bar{I}}$ (2)

SUBSTITUTE FOR $m_B v_0$: $m\bar{v}_2 \frac{L}{2} = \frac{1}{12} mL^2 \omega_2$ $\quad \bar{v}_2 = \frac{L}{6}\omega_2$



SYST. MOMENTA$_2$ + SYST. EXT. IMP$_{2\to3}$ = SYST. MOMENTA$_3$

$\stackrel{+}{\leftarrow}$ COMPONENTS: $m\bar{v}_2 = m\bar{v}_3$ $\quad \bar{v}_2 = \bar{v}_3$

CONSERVATION OF ENERGY: CHOOSE DATUM AT A.

$$T_2 = \tfrac{1}{2}m\bar{v}_2^2 + \tfrac{1}{2}\bar{I}\omega_2^2 = \tfrac{1}{2}m\left(\tfrac{L}{6}\omega_2\right)^2 + \tfrac{1}{2}\left(\tfrac{1}{12}mL^2\right)\omega_2^2 = \tfrac{1}{18}mL^2\omega_2^2$$

$$V_2 = -W\tfrac{L}{2} = -mg\tfrac{L}{2}$$

$$T_3 = \tfrac{1}{2}m\bar{v}_3^2 + \tfrac{1}{2}\bar{I}\omega_3^2 = \tfrac{1}{2}m\bar{v}_2^2 + 0 = \tfrac{1}{2}m\left(\tfrac{L}{6}\omega_2\right)^2 = \tfrac{1}{72}mL^2\omega_2^2$$

$$V_3 = -W\tfrac{L}{2}\cos\theta_m = -mg\tfrac{L}{2}\cos\theta_m$$

$T_2 + V_2 = T_3 + V_3$: $\quad \frac{1}{18}\cancel{m}L^2\omega_2^2 - \cancel{m}g\frac{L}{2} = \frac{1}{72}\cancel{m}L^2\omega_2^2 - \cancel{m}g\frac{L}{2}\cos\theta_m$

$$\left(\tfrac{1}{18} - \tfrac{1}{72}\right)L^2\omega_2^2 = g\tfrac{L}{2}(1 - \cos\theta_m)$$

SUBSTITUTE FOR ω_2 FROM EQ(2):

$$\frac{1}{24}L^2\left(\frac{m_B v_0 L}{2\bar{I}}\right)^2 = g\frac{L}{2}(1 - \cos\theta_m)$$

$$\frac{L\, m_B^2 v_0^2 L^2}{48g\left(\frac{1}{12}mL^2\right)^2} = (1 - \cos\theta) \; ; \quad \cos\theta_m = 1 - \frac{3m_B^2 v_0^2}{gL\, m^2} \quad (3)$$

DATA: $L = 1.2$ m, $m_B = 0.03$ kg, $m = 8$ kg

PROBLEM 17.117: FOR $\bar{v}_0 = 350$ m/s ←

EQ(3): $\cos\theta_m = 1 - \dfrac{3(0.03\text{ kg})^2(350\text{ m/s})^2}{(9.81\text{ m/s}^2)(1.2\text{ m})(8\text{ kg})^2} = 1 - 0.4391$

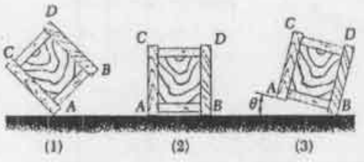
$\cos\theta_m = 0.5609$ $\qquad \theta_m = 55.9°$ ◀

PROBLEM 17.118: FOR $\theta_m = 90°$, $\cos\theta_m = 0$

EQ(3): $0 = 1 - \dfrac{3(0.03\text{ kg})^2 v_0^2}{(9.81\text{ m/s}^2)(1.2\text{ m})(8\text{ kg})^2}$; $\quad 1 - 3.5837\times10^{-6}\, v_0^2 = 0$

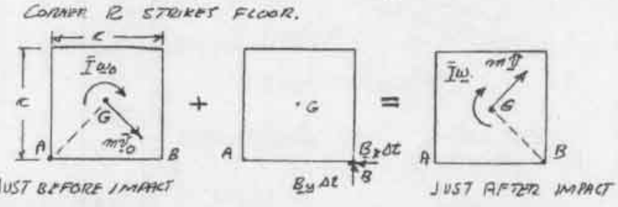
$v_0^2 = 279.04\times10^3$ $\qquad \bar{v}_0 = 528$ m/s ← ◀

17.119



GIVEN: UNIFORM CRATE RELEASED FROM POSITION 1. NO SLIPPING
FIND: (a) ω JUST AFTER IMPACT, (b) ENERGY LOST IN IMPACT, (c) MAXIMUM ANGLE θ.

DENOTE BY $\underline{\omega_0}$ ANGULAR VELOCITY ABOUT A JUST BEFORE CORNER B STRIKES FLOOR.



JUST BEFORE IMPACT ROTATION ABOUT A $\qquad$ JUST AFTER IMPACT ROTATION ABOUT B

SYST. MOMENTA$_0$ + SYST. EXT. IMP$_{0\to1}$ = SYST. MOMENTA$_1$

+↻ MOMENTS ABOUT B:

$$\bar{I}\omega_0 + 0 = \bar{I}\omega + m\bar{v}(BG) \quad (1)$$

$AG = BG = \sqrt{2}\,\frac{c}{2} = \frac{c}{\sqrt{2}}$ $\qquad \bar{I} = \frac{1}{6}mc^2$

$\bar{v}_0 = (AG)\omega_0 = \frac{c}{\sqrt{2}}\omega_0$ $\qquad \bar{v} = (BG)\omega = \frac{c}{\sqrt{2}}\omega$

EQ(1): $\frac{1}{6}mc^2\omega_0 = \frac{1}{6}mc^2\omega + m\left(\frac{c}{\sqrt{2}}\omega\right)\frac{c}{\sqrt{2}}$

(a) $\frac{1}{6}mc^2\omega_0 = \frac{2}{3}mc^2\omega$ $\qquad \underline{\omega} = \frac{1}{4}\omega_0$ ↻ ◀

(b) KINETIC ENERGY LOST:

SEE EQ. 17.10 page 1049. FOR ROTATION ABOUT A OR B

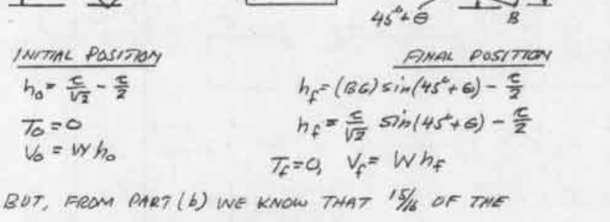
$T_0 = \frac{1}{2}I_A\omega_0^2$ $\qquad$ NOTE: $I_A = I_B$

$T_1 = \frac{1}{2}I_B\omega^2$

$$\frac{\Delta T}{T_0} = \frac{T_0 - T_1}{T_0} = \frac{\frac{1}{2}I_A\omega_0^2 - \frac{1}{2}I_B\omega^2}{\frac{1}{2}I_A\omega_0^2} = \frac{\omega_0^2 - \omega^2}{\omega_0^2} = 1 - \left(\frac{\omega}{\omega_0}\right)^2$$

$$\text{ENERGY LOST} = 1 - \left(\frac{\omega}{\omega_0}\right)^2 = 1 - \left(\frac{1}{4}\right)^2 = \frac{15}{16} \; ◀$$

(c)



INITIAL POSITION

$h_0 = \frac{c}{\sqrt{2}} - \frac{c}{2}$

$T_0 = 0$

$V_0 = W h_0$

FINAL POSITION

$h_f = (BG)\sin(45° + \theta) - \frac{c}{2}$

$h_f = \frac{c}{\sqrt{2}}\sin(45° + \theta) - \frac{c}{2}$

$T_f = 0$, $V_f = W h_f$

BUT, FROM PART (b) WE KNOW THAT 15/16 OF THE ENERGY IS LOST, THUS

$V_f = \frac{1}{16}V_0$; $\quad W h_f = \frac{1}{16}W h_0$

$$h_f = \frac{1}{16}h_0$$

$$\frac{c}{\sqrt{2}}\sin(45° + \theta) - \frac{c}{2} = \frac{1}{16}\left(\frac{c}{\sqrt{2}} - \frac{c}{2}\right)$$

$$\frac{1}{\sqrt{2}}\sin(45° + \theta) = \frac{1}{16}\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right) + \frac{1}{2}$$

$$\sin(45° + \theta) = \frac{2 + 15\sqrt{2}}{32} = 0.72541$$

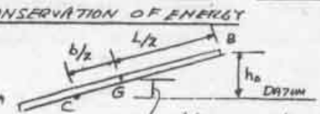
$$45° + \theta = 46.503°$$

$$\theta = 1.50° \; ◀$$

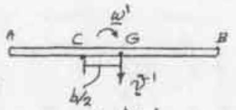
17.120

GIVEN: ROD OF LENGTH $L = 30$ in. $b = 5$ in. ROD IS RELEASED WHEN $h_0 = 4$ in.
FIND: (a) h_1 AFTER FIRST IMPACT. (b) h_2 AFTER SECOND IMPACT.

CONSERVATION OF ENERGY



$$a_0 = h_0 \frac{b/2}{L/2 + b/2} = h_0 \frac{b}{L+b}$$



$$v' = \frac{b}{2}\omega'$$

POSITION "O"

$$V_0 = mg a_0; \quad T_0 = 0$$

POSITION "PRIME"

$$V' = 0; \quad T' = \frac{1}{2}\bar{I}\omega'^2 + \frac{1}{2}m\bar{v}'^2$$

$$T' = \frac{1}{2}\left(\frac{1}{12}mL^2\right)\omega'^2 + \frac{1}{2}m\left(\frac{b}{2}\omega'\right)^2$$

$$T' = \frac{1}{24}m\omega'^2(L^2 + 3b^2)$$

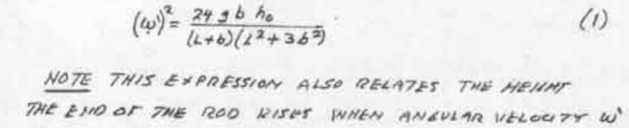
$$T_0 + V_0 = T' + V': \quad 0 + mga_0 = \frac{1}{24}m\omega'^2(L^2+3b^2)$$

$$\frac{1}{24}\, g h_0 \frac{b}{L+b} = \omega'^2(L^2+3b^2)$$

$$(\omega')^2 = \frac{24 g b h_0}{(L+b)(L^2+3b^2)} \qquad (1)$$

NOTE THIS EXPRESSION ALSO RELATES THE HEIGHT THE END OF THE ROD RISES WHEN ANGULAR VELOCITY ω' OCCURS WHEN ROD IS HORIZONTAL

IMPACT



SYST MOMENTA POSITION "PRIME" + SYST. EXT. IMP. = SYST. MOMENTA POSITION "DOUBLE PRIME"

$$v' = \frac{b}{2}\omega' \qquad v'' = \frac{b}{2}\omega''$$

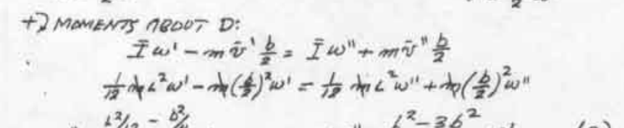
+↻ MOMENTS ABOUT D:

$$\bar{I}\omega' - m\bar{v}'\frac{b}{2} = \bar{I}\omega'' + m\bar{v}''\frac{b}{2}$$

$$\frac{1}{12}mL^2\omega' - m\left(\frac{b}{2}\right)^2\omega' = \frac{1}{12}mL^2\omega'' + m\left(\frac{b}{2}\right)^2\omega''$$

$$\omega'' = \frac{L^2/12 - b^2/4}{L^2/12 + b^2/4}\omega' \qquad \omega'' = \frac{L^2 - 3b^2}{L^2+3b^2}\omega' \qquad (2)$$

FIRST IMPACT:



EQ(1):

$$(\omega')^2 = \frac{24gb}{(L+b)}\cdot\frac{h_0}{(L^2+3b^2)} \qquad (\omega'')^2 = \frac{24gb}{(L+b)}\cdot\frac{h_1}{(L^2+3b^2)}$$

SQUARE EQ(2):

$$(\omega'')^2 = \frac{(L^2-3b^2)^2}{(L^2+3b^2)^2}(\omega')^2$$

$$\left[\frac{24gb}{(L+b)}\cdot\frac{h_1}{(L^2+3b^2)}\right] = \frac{(L^2-3b^2)^2}{(L^2+3b^2)^2}\cdot\left[\frac{24gb}{(L+b)}\cdot\frac{h_0}{(L^2+3b^2)}\right]$$

$$h_1 = \left[\frac{L^2-3b^2}{L^2+3b^2}\right]^2 h_0 \qquad (3)$$

SECOND IMPACT: $h_0 \rightarrow h_1$, $h_1 \rightarrow h_2$

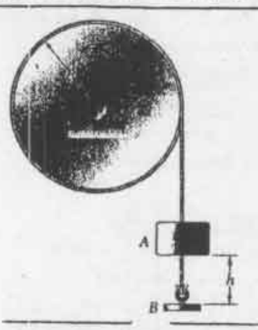
$$h_2 = \left[\frac{L^2-3b^2}{L^2+3b^2}\right]^4 h_0$$

DATA: $h_0 = 4$ in., $L = 30$ in., $d = 6$ in. $\quad \frac{L^2-3b^2}{L^2+3b^2} = \frac{30^2-3(5)^2}{30^2+3(5)^2} = 0.84615$

EQ(3): $h_1 = (0.84615)^2(4\text{ in.}) = 2.8639$ in. $\qquad h_1 = 2.86$ in ◀

EQ(4): $h_2 = (0.84615)^4(4\text{ in.}) = 2.0505$ in. $\qquad h_2 = 2.05$ in. ◀

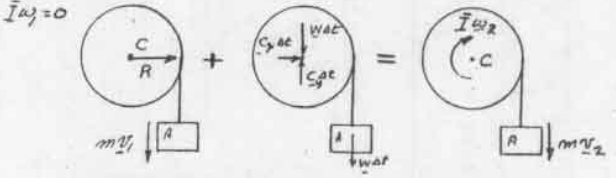
17.121 and 17.122



GIVEN: 3-lb COLLAR A DROPS $h = 15$ in.
8-lb DISK OF RADIUS $R = 9$ in.
IMMEDIATELY AFTER IMPACT
FIND: (a) v_A, (b) ω
PROBLEM 17.121 ASSUME PERFECTLY PLASTIC IMPACT.
PROBLEM 17.122 ASSUME PERFECTLY ELASTIC IMPACT.

COLLAR A FALLS A DISTANCE h: $\quad v_1 = \sqrt{2gh}$

PRINCIPLE OF IMPULSE-MOMENTUM



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C:

$$m v_1 R = \bar{I}\omega_2 + m v_2 R \qquad (1)$$

PROBLEM 17.121

PLASTIC IMPACT $\quad e = 0 \quad v_2 = R\omega_2; \; \omega_2 = \frac{v_2}{R}$

M = MASS OF DISK; $\bar{I} = \frac{1}{2}MR^2$

EQ.(1):

$$m v_1 R = \frac{1}{2}MR^2\left(\frac{v_2}{R}\right) + m v_2 R$$

$$m v_1 = \frac{1}{2}M v_2 + m v_2$$

$$v_2 = \frac{2m}{2m+M}v_1 \qquad (3)$$

DATA: $m = \frac{3\text{ lb}}{g}$; $M = \frac{8\text{ lb}}{g}$; $h = 15$ in.

$$v_1 = \sqrt{2gh} = \sqrt{2(32.2\text{ ft/s}^2)(\tfrac{15}{12}\text{ ft})} = 8.972\text{ ft/s}$$

EQ(3) $\quad v_2 = \frac{2(3/g)}{2(3/g)+\frac{8}{g}}(8.972\text{ ft/s}) = 3.845$ ft/s $\qquad v_2 = 3.85$ ft/s ↓ ◀

$\omega_2 = \frac{v_2}{R} = \frac{3.845\text{ ft/s}}{(\frac{9}{12}\text{ ft})} = 5.127\ \frac{\text{rad}}{\text{s}} \qquad \omega_2 = 5.13$ rad/s ↻ ◀

PROBLEM 17.122:

ELASTIC IMPACT $\quad e = 1 \quad (v_B)_2 - (v_A)_2 = (v_A)_1 - (v_B)_1$

$(v_B)_1 = 0$; $(v_B)_2 = R\omega_2$:

$$R\omega_2 - v_2 = v_1; \quad \omega_2 = (v_1 + v_2)/R \qquad (2)$$

EQ(1):

$$m v_1 R = \frac{1}{2}MR^2\left(\frac{v_1+v_2}{R}\right) + m v_2 R$$

$$m v_1 R = \frac{1}{2}MRv_1 + \frac{1}{2}MRv_2 + m v_2 R$$

$$v_2 = \frac{2m - M}{2m+M}v_1 \qquad (4)$$

DATA: $m = 3\text{ lb}/g$; $M = 8\text{ lb}/g$; $h = 15$ in., $v_1 = 8.972$ ft/s

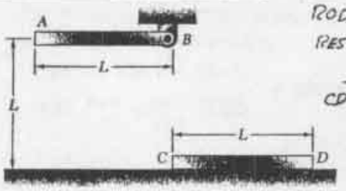
EQ.(4) $\quad v_2 = \frac{2(3/g) - 8/g}{2(3/g)+8/g}(8.972\text{ ft/s}) = -1.2817$ ft/s

$v_2 = 1.282$ ft/s ↑ ◀

EQ(2): $\quad \omega_2 = \frac{v_1 + v_2}{R} = \frac{8.972\text{ ft/s} - 1.282\text{ ft/s}}{(\frac{9}{12}\text{ ft})} = 10.254$ rad/s

$\omega_2 = 10.25$ rad/s ↻ ◀

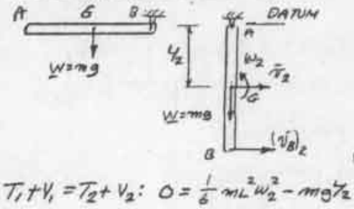
17.123 and 17.124



GIVEN: IDENTICAL RODS AB + CD
ROD AB IS RELEASED FROM REST IN POSITION SHOWN.
FIND: VELOCITY OF CD JUST AFTER IMPACT
PROBLEM 17.123: FOR COEF. OF RESTITUTION EQUAL TO $e = 0.50$.
PROBLEM 17.125: FOR $e = 1$

ROD AB SWINGS TO VERTICAL POSITION

CONSERVATION OF ENERGY



$T_1 = V_1 = 0$

$T_2 = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\bar{v}_2^2$

$= \frac{1}{2}(\frac{1}{12}mL^2)\omega_2^2 + \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2$

$T_2 = \frac{1}{6}mL^2\omega_2^2$

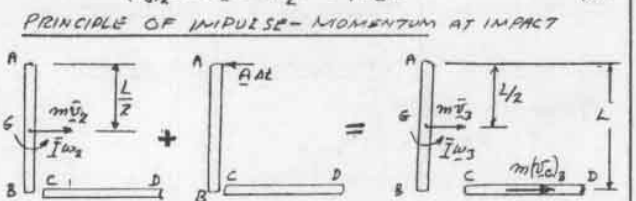
$V_2 = -mg\frac{L}{2}$

$T_1 + V_1 = T_2 + V_2: \quad 0 = \frac{1}{6}mL^2\omega_2^2 - mg\frac{L}{2}$

$\omega_2^2 = \frac{3g}{L} \qquad \omega_2 = \sqrt{\frac{3g}{L}} \qquad (1)$

$(v_B)_2 = L\omega_2 = L\sqrt{\frac{3g}{L}} = \sqrt{3gL} \qquad (2)$

PRINCIPLE OF IMPULSE-MOMENTUM AT IMPACT



SYST. MOMENTA$_2$ + SYST. EXT. IMP$_{2-3}$ = SYST. MOMENTA$_3$

+↻ MOMENTS ABOUT A:

$\bar{I}\omega_2 + m\bar{v}_2\frac{L}{2} = \bar{I}\omega_3 + m\bar{v}_3\frac{L}{2} + m(v_C)_3 L$

$\frac{1}{12}mL^2\omega_2 + m\left(\frac{L}{2}\right)^2\omega_2 = \frac{1}{12}mL^2\omega_3 + m\left(\frac{L}{2}\right)^2\omega_3 + m(v_C)_3 L$

$\frac{1}{3}L\omega_2 - \frac{1}{3}L\omega_3 = +(v_C)_3\ ; \qquad \omega_2 - \omega_3 = \frac{3(v_C)_3}{L} \qquad (3)$

IMPACT: $(v_B)_2 e = (v_C)_3 - L\omega_3\ ; \quad (v_C)_3 = (v_B)_2 e + L\omega_3 \quad (4)$

EQ(3): $\omega_2 - \omega_3 = \frac{3}{L}\left((v_B)_2 e + L\omega_3\right); \quad \omega_2 - \omega_3 = \frac{3}{L}(v_B)_2 e + 3\omega_3$

$\omega_2 - 4\omega_3 = \frac{3}{L}(v_B)_2 e$

SUBSTITUTE FROM (1) AND (2)

$\sqrt{\frac{3g}{L}} - 4\omega_3 = \frac{3}{L}\sqrt{3gL}\ e$

$4\omega_3 = \sqrt{\frac{3g}{L}} - \frac{3}{L}\sqrt{3gL}\ e\ ; \quad 4\omega_3 = \frac{1}{L}\sqrt{3gL} - \frac{3}{L}\sqrt{3gL}\ e$

$\omega_3 = \frac{1}{4L}(1-3e)\sqrt{3gL}$

EQ.(4): $(v_C)_3 = (v_B)_2 e + L\omega_3 = (\sqrt{3gL})e + L\left[\frac{1}{4L}(1-3e)\sqrt{3gL}\right]$

$(v_C)_3 = \sqrt{3gL}\left(e + \frac{1}{4} - \frac{3}{4}e\right) = \sqrt{3gL}\ \frac{1}{4}(1+e)$

$(v_C)_3 = \frac{1}{4}(1+e)\sqrt{3gL} \qquad (5)$

PROBLEM 17.123: FOR $e = 0.5$

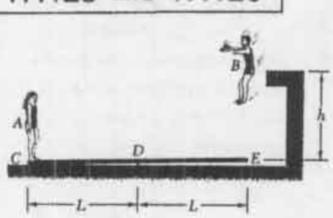
EQ(5): $(v_C)_3 = \frac{1}{4}(1+0.5)\sqrt{3gL} = \frac{3}{8}\sqrt{3gL}$

$v_{CD} = \frac{3}{8}\sqrt{3gL}$ →

PROBLEM 17.124: FOR $e = 1$

EQ(5): $(v_C)_3 = \frac{1}{4}(1+1)\sqrt{3gL}$ $\qquad v_{CD} = \frac{1}{2}\sqrt{3gL}$ →

17.125 and 17.126

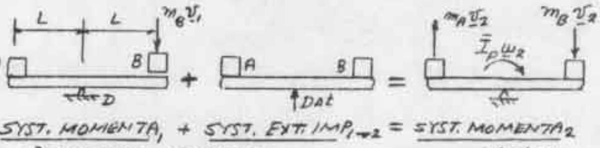


GIVEN: GYMNAST A IS AT REST. GYMNAST B JUMPS ON TO PLANK AT E.
$h = 2.5\ m$
MASS OF PLANK: $m_p = 15\ kg$
ASSUMING PERFECTLY PLASTIC IMPACT,
FIND: HEIGHT THAT GYMNAST RISE

PROBLEM 17.125: USE $m_A = 55\ kg$ AND $m_B = 70\ kg$
PROBLEM 17.126: USE $m_A = 70\ kg$ AND $m_B = 55\ kg$

VELOCITY OF B AS IT STRIKES E: $v_1 = \sqrt{2gh}$

PRINCIPLE OF IMPULSE-MOMENTUM



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT D: $\qquad v_2 = L\omega_2$

$m_B v_1 L = \bar{I}_D\omega_2 + m_A v_2 L + m_B v_2 L$

$m_B v_1 L = \frac{1}{12}m_p(2L)^2\omega_2 + (m_A + m_B)(L\omega_2)L$

$m_B v_1 L = \frac{1}{3}m_p L^2\omega_2 + (m_A + m_B)L^2\omega_2$

$\omega_2 = \frac{m_B}{\frac{1}{3}m_p + m_A + m_B}\ \frac{v_1}{L}$

$v_2 = L\omega_2 = \frac{3m_B}{m_p + 3m_A + 3m_B}\ v_1 \qquad (1)$

FOR $h = 2.5m \qquad v_1 = \sqrt{2gh} = \sqrt{2(9.81 m/s^2)(2.5m)} = 7.0036\ m/s$

PROBLEM 17.125

$m_p = 15\ kg \qquad m_A = 55\ kg \qquad m_B = 70\ kg$

EQ(1): $v_2 = \frac{3(70)}{15 + 3(55) + 3(70)}(7.0036\ m/s) = \frac{210}{390}(7.0036)$

$v_2 = 3.771\ m/s$ ↑

$v_2 = \sqrt{2gh_2} \qquad h_2 = \frac{v_2^2}{2g} = \frac{(3.771\ m/s)^2}{2(9.81\ m/s^2)} = 0.725\ m$

GYMNAST A RISES 725 mm

PROBLEM 17.126

$m_p = 15\ kg \qquad m_A = 70\ kg \qquad m_B = 55\ kg$

EQ.(1): $v_2 = \frac{3(55)}{15 + 3(70) + 3(55)}(7.0036\ m/s) = \frac{165}{390}(7.0036)$

$v_2 = 2.963\ m/s$ ↑

$v_2 = \sqrt{2gh_2} \qquad h_2 = \frac{v_2^2}{2g} = \frac{(2.963\ m/s)^2}{2(9.81\ m/s^2)} = 0.447\ m$

GYMNAST A RISES 447 mm

17.127

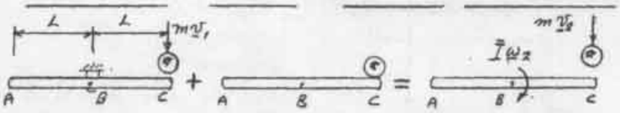
GIVEN: SPHERE $m = 800\,g$, $\bar{v}_1 = 3\ m/s \downarrow$

ROD: $L = 750\,mm$, $M = 2.4\,kg$

COEF. OF RESTITUTION $e = 0.5$

JUST AFTER IMPACT

FIND: (a) $\underline{\omega}$, (b) $\bar{v}_D$.

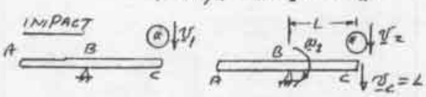


SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↺ MOMENTS ABOUT B:

$$m\bar{v}_1 L = \bar{I}\omega_2 + m\bar{v}_2 L \qquad (1)$$

IMPACT



$$v_1 e = v_C - v_2\ ; \quad v_1 e = L\omega_2 - v_2\ ; \quad v_2 = L\omega_2 - v_1 e \qquad (2)$$

EQ(1): $m v_1 L = \frac{1}{12}M(2L)^2\omega_2 + m(L\omega_2 - v_1 e)L$

$$m v_1 = \tfrac{1}{3}ML\,\omega_2 + mL\omega_2 - m v_1 e$$

$$m(1+e)\frac{v_1}{L} = \left(\tfrac{1}{3}M + m\right)\omega_2 \qquad (3)$$

DATA: $m = 0.8\,kg$, $M = 2.4\,kg$, $v_1 = 3\,m/s$, $e = 0.5$, $L = 0.75\,m$

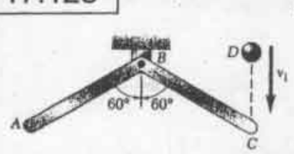
EQ(3): $(0.8\,kg)(1+0.5)\frac{3\,m/s}{0.75\,m} = \left[\tfrac{1}{3}(2.4\,kg) + 0.8\,kg\right]\omega_2$

$4.8 = 1.6\,\omega_2$ $\qquad \omega_2 = 3\,rad/s \qquad \underline{\omega}_2 = 3\,rad/s$ ↺ ◀

EQ(2): $v_2 = L\omega_2 - v_1 e = (0.75\,m)(3\,rad/s) - (3\,m/s)(0.5)$

$v_2 = 2.25 - 1.5 \qquad (\bar{v}_D)_2 = \bar{v}_2 = 0.75\,m/s \downarrow$ ◀

17.128



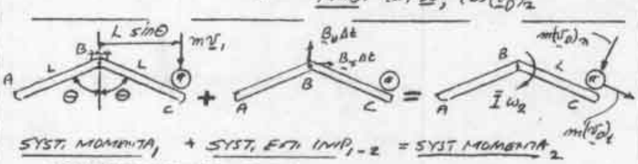
GIVEN: SPHERE: $m = 800\,g$, $v_1 = 3\,m/s \downarrow$

ROD: $L = 750\,mm$, $M = 2.4\,kg$

COEF. OF RESTITUTION $e = 0.5$

JUST AFTER IMPACT

FIND: (a) $\underline{\omega}$, (b) $(\bar{v}_D)_2$

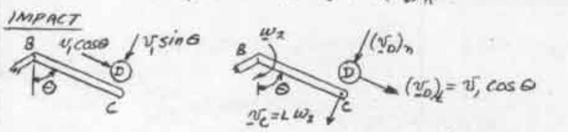


SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↺ MOMENTS ABOUT B:

$$m v_1 L \sin\theta = \bar{I}\omega_2 + m(v_D)_n L \qquad (1)$$

IMPACT



$$(v_1 \sin\theta)e = L\omega_2 - (v_D)_n\ ; \qquad (v_D)_n = L\omega_2 - (v_1\sin\theta)e \qquad (2)$$

EQ(1): $m v_1 L \sin\theta = \frac{1}{12}M(2L)^2\omega_2 + m\left[L\omega_2 - (v_1\sin\theta)e\right]L$

$$m v_1 \sin\theta = \tfrac{1}{3}ML\omega_2 + mL\omega_2 - m(v_1\sin\theta)e$$

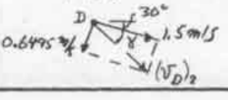
$$m(1+e)\frac{v_1}{L}\sin\theta = \left(\tfrac{1}{3}M + m\right)\omega_2$$

$$(0.8\,kg)(1+0.5)\frac{3\,m/s}{0.75\,m}\sin 60^\circ = \left[\tfrac{1}{3}(2.4\,kg) + 0.8\,kg\right]\omega_2$$

$4.157 = 1.6\,\omega_2 \qquad \omega_2 = 2.598\,rad/s \qquad \underline{\omega}_2 = 2.60\,\frac{rad}{s}$ ↺ ◀

EQ(2): $(v_D)_n = (0.75\,m)(2.598\,rad/s) - (3\,m/s)\sin 60^\circ(0.5)$; $(v_D)_n = 0.6495\,m/s$

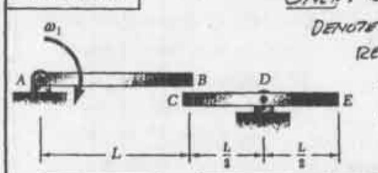
$(v_D)_n = 0.6495\,m/s \measuredangle 30^\circ \qquad (v_D)_t = (3\,m/s)\cos 60^\circ = 1.5\,m/s \measuredangle 30^\circ$



$(v_D)_2 = 1.635\,m/s$, $\gamma = 23.4^\circ$

$(\underline{v}_D)_2 = 1.635\,m/s \measuredangle 53.4^\circ$ ◀

17.129



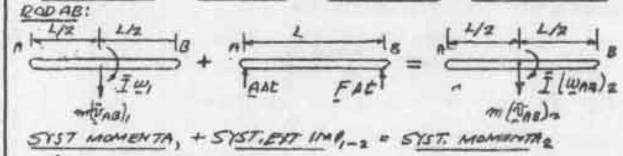
GIVEN: RODS OF MASS m

DENOTE BY e THE COEF. OF RESTITUTION.

JUST AFTER IMPACT,

FIND: $\underline{\omega}_{AB}$ AND $\underline{\omega}_{CE}$

ROD AB:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

+↺ MOMENTS ABOUT A:

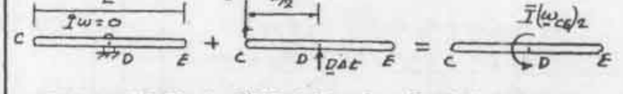
$$\bar{I}\omega_1 + m(\bar{v}_{AB})_1\frac{L}{2} - (F\Delta t)L = \bar{I}(\omega_{AB})_2 + m(\bar{v}_{AB})_2\frac{L}{2}$$

$$\frac{1}{12}mL^2\omega_1 + m\left(\omega_1\frac{L}{2}\right)\frac{L}{2} - (F\Delta t)L = \frac{1}{12}mL^2(\omega_{AB})_2 + m\frac{L}{2}(\omega_{AB})_2\frac{L}{2}$$

$$\frac{1}{3}mL^2\omega_1 - (F\Delta t)L = \frac{1}{3}mL^2(\omega_{AB})_2$$

$$F\Delta t = \frac{1}{3}mL^2\left[\omega_1 - (\omega_{AB})_2\right] \qquad (1)$$

ROD CE:



SYST MOMENTA$_1$ + SYST EXT IMP$_{1\to 2}$ = SYST MOMENTA$_2$

+↻ MOMENTS ABOUT D:

$$(F\Delta t)\frac{L}{2} = \bar{I}(\omega_{CE})_2$$

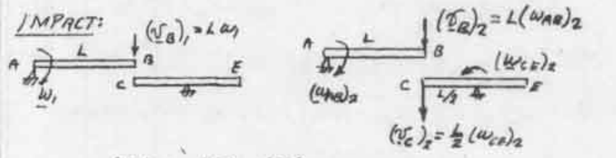
$$(F\Delta t)\frac{L}{2} = \frac{1}{12}mL^2(\omega_{CE})_2$$

SUBSTITUTE FOR $(F\Delta t)$ FROM (1)

$$\frac{1}{3}mL^2\left[\omega_1 - (\omega_{AB})_2\right]\frac{L}{2} = \frac{1}{12}mL^2(\omega_{CE})_2$$

$$\omega_1 - (\omega_{AB})_2 = \frac{1}{2}(\omega_{CE})_2 \qquad (2)$$

IMPACT:



$$(v_B)_1 e = (v_C)_2 - (v_B)_2$$

$$L\omega_1 e = \frac{L}{2}(\omega_{CE})_2 - L(\omega_{AB})_2$$

$$(\omega_{AB})_2 = \frac{1}{2}(\omega_{CE})_2 - \omega_1 e \qquad (3)$$

EQ(2): $\omega_1 - \left[\frac{1}{2}(\omega_{CE})_2 - \omega_1 e\right] = \frac{1}{2}(\omega_{CE})_2$

$$\omega_1(1+e) = (\omega_{CE})_2$$

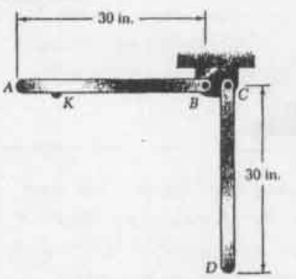
$(\underline{\omega}_{CE})_2 = \omega_1(1+e)$ ↻ ◀

EQ(3): $(\omega_{AB})_2 = \frac{1}{2}\omega_1(1+e) - \omega_1 e$

$$= \frac{1}{2}\omega_1 + \frac{1}{2}\omega_1 e - \omega_1 e$$

$(\underline{\omega}_{AB})_2 = \frac{1}{2}\omega_1(1-e)$ ↻ ◀

17.130



GIVEN: $W_{AB} = 5\,lb$

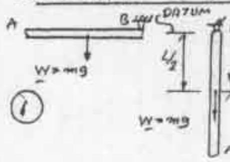
$W_{CD} = 3\,lb$

COEF. OF RESTITUTION $e = 0.8$

SYSTEM IS RELEASED FROM REST IN POSITION SHOWN

FIND: MAXIMUM ANGLE θ_m THROUGH WHICH ROD CD WILL ROTATE AFTER THE IMPACT.

CONSERVATION OF ENERGY:



$T_1 = V_1 = 0$

$V_2 = -mg\,\frac{L}{2}$

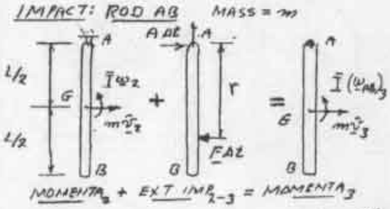
$T_2 = \frac{1}{2}\bar{I}\omega_2^2 + \frac{1}{2}m\bar{v}_2^2$

$= \frac{1}{2}\,\frac{1}{12}mL^2\omega_2^2 + \frac{1}{2}m\left(\frac{L}{2}\omega_2\right)^2$

$T_2 = \frac{1}{6}mL^2\omega_2^2$

$T_1 + V_1 = T_2 + V_2$: $\quad 0 = \frac{1}{6}mL^2\omega_2^2 - mL\frac{L}{2}$; $\quad \omega_2^2 = 3\frac{g}{L} \qquad (1)$

IMPACT: ROD AB $\quad$ MASS $= m$



MOMENTA$_2$ + EXT IMP$_{2-3}$ = MOMENTA$_3$

+) MOMENTS ABOUT A

$\bar{I}\omega_2 + m\bar{v}_2\frac{L}{2} - (F\Delta t)r$

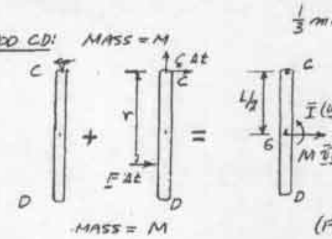
$= \bar{I}(\omega_{AB})_3 + m\bar{v}_3\frac{L}{2}$

$\frac{1}{12}mL^2\omega_2 + m\left(\frac{L}{2}\right)^2\omega_2 - (F\Delta t)r$

$= \frac{1}{12}mL^2(\omega_{AB})_3 + m\left(\frac{L}{2}\right)^2(\omega_{AB})_3$

$\frac{1}{3}mL^2\left[\omega_2 - (\omega_{AB})_3\right] = (F\Delta t)r \qquad (2)$

ROD CD: $\quad$ MASS $= M$



+) MOMENTS ABOUT C

$0 + (F\Delta t)r$

$= \bar{I}(\omega_{CD})_3 + M\bar{v}_3\frac{L}{2}$

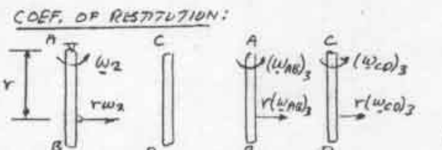
$(F\Delta t)r = \frac{M}{12}L^2(\omega_{CD})_3 + M\left(\frac{L}{2}\right)^2(\omega_{CD})_3$

$(F\Delta t)r = \frac{1}{3}ML^2(\omega_{CD})_3 \qquad (3)$

EQUATE $(F\Delta t)r$ FROM (2) AND (3)

$\frac{1}{3}mL^2\left[\omega_2 - (\omega_{AB})_3\right] = \frac{1}{3}ML^2(\omega_{CD})_3 \qquad (4)$

COEF. OF RESTITUTION:



IMPACT VELOCITIES

$(r\,\omega_2)e = r(\omega_{CD})_3 - r(\omega_{AB})_3$; $\quad (\omega_{AB})_3 = (\omega_{CD})_3 - e\,\omega_2$

SUBSTITUTE INTO (4)

$m\left\{\omega_2 - \left[(\omega_{CD})_3 - e\,\omega_2\right]\right\} = M(\omega_{CD})_3$

$\omega_2 - (\omega_{CD})_3 + e\,\omega_2 = \frac{M}{m}(\omega_{CD})_3$

$(\omega_{CD})_3 = \frac{1+e}{1+\frac{M}{m}}\,\omega_2 \qquad (5)$

(CONTINUED)

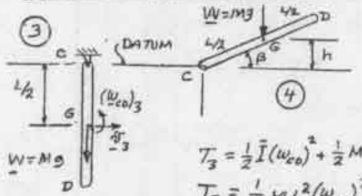
17.130 continued

DATA: $m = (5\,lb)/g \quad M = (3\,lb)/g \quad e = 0.8 \quad L = 2.5\,ft$

EQ(1): $\omega_2^2 = \frac{3g}{L} = \frac{3(32.2\,ft/s^2)}{2.5\,ft} \qquad \omega_2 = 6.216\,rad/s$

EQ(5) $(\omega_{CD})_3 = \frac{1+e}{1+\frac{M}{m}}\,\omega_2 = \frac{1+0.8}{1+\frac{3}{5}}\,6.216\,rad/s = 6.993\,rad/s$

CONSERVATION OF ENERGY $\quad$ ROD CD: MASS $= M$



$T_3 = \frac{1}{2}\bar{I}(\omega_{CD})_3^2 + \frac{1}{2}M\bar{v}^2 = \frac{1}{2}\frac{M}{12}(\omega_{CD})_3^2 + \frac{1}{2}M\left(\frac{L}{2}\right)^2(\omega_{CD})_3^2$

$T_3 = \frac{1}{6}ML^2(\omega_{CD})_3^2$

$V_3 = -Mg\,\frac{L}{2}$

$V_4 = +Mg\,h \qquad T_4 = 0$

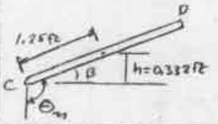
$T_3 + V_3 = T_4 + V_4$

$\frac{1}{6}ML^2(\omega_{CD})_3^2 - Mg\frac{L}{2} = Mg\,h$

$h + \frac{L}{2} = \frac{L^2}{6g}(\omega_{CD})_3^2$

$h + \frac{2.5\,ft}{2} = \frac{(2.5\,ft)^2}{6(32.2\,ft/s^2)}(6.993\,rad/s)^2$

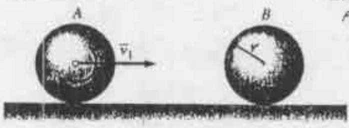
$h + 1.25\,ft = 1.582\,ft \qquad h = 0.332\,ft$



$\beta = 15.4°$

$\theta_m = 90° + 15.4° \qquad \theta_m = 105.4°$ ◀

17.131



GIVEN: SPHERE A ROLLS AND STRIKES SPHERE B. ASSUME PERFECTLY ELASTIC IMPACT AND DENOTE COEF. OF KINETIC FRICTION BY μ_k.

FIND: JUST AFTER IMPACT, (a) ω AND $\bar{v}$ OF EACH SPHERE. (b) FINAL VELOCITY OF EACH SPHERE.

(a) IMMEDIATELY AFTER IMPACT

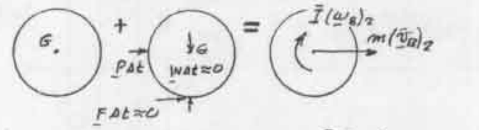
SPHERE A $\quad$ WAS ROLLING $\bar{v}_1 = r\omega_1$

+) MOMENTS ABOUT G

$\bar{I}\omega_1 = \bar{I}(\omega_A)_2$

$(\omega_A)_2 = \omega_1$ ◀

SPHERE B:



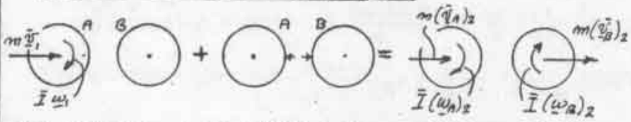
+) MOMENTS ABOUT G: $\quad \bar{I}(\omega_B)_2 = 0$

$(\omega_B)_2 = 0$ ◀

(CONTINUED)

17.131 continued

CONSIDER BOTH SPHERES AS A SYSTEM



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

$\xrightarrow{+}$ COMPONENTS: $\quad m\bar{v}_1 = m(\bar{v}_A)_2 + m(\bar{v}_B)_2$

$$\bar{v}_1 = (\bar{v}_A)_2 + (\bar{v}_B)_2 \qquad (1)$$

RELATIVE VELOCITIES $\quad (e=1)$

$$(\bar{v}_B)_2 - (\bar{v}_A)_2 = e\bar{v}_1 = \bar{v}_1 \qquad \bar{v}_1 = (\bar{v}_B)_2 - (\bar{v}_A)_2 \qquad (2)$$

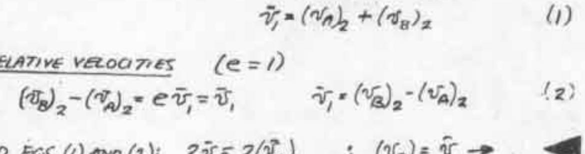
ADD EQS. (1) AND (2): $2\bar{v}_1 = 2(\bar{v}_B)_2$; $(\bar{v}_B)_2 = \bar{v}_1 \rightarrow$ ◀

SUBTRACT EQ(2) FROM EQ(1): $0 = 2(\bar{v}_A)_2$; $(\bar{v}_A)_2 = 0$ ◀

(b) MOTION AFTER SPHERES START ROLLING UNIFORMLY

NOTE: TIME INTERVAL IS NOT SMALL AND IMPULSES OF FRICTION FORCES MUST BE INCLUDED

SPHERE A:



SYST. MOMENTA$_2$ + SYST. EXT. IMP$_{2\to3}$ = SYST. MOMENTA$_3$

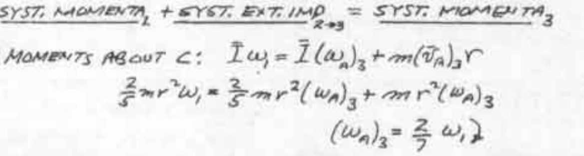
+↻ MOMENTS ABOUT C: $\quad \bar{I}\omega_1 = \bar{I}(\omega_A)_3 + m(\bar{v}_A)_3 r$

$$\tfrac{2}{5}mr^2\omega_1 = \tfrac{2}{5}mr^2(\omega_A)_3 + mr^2(\omega_A)_3$$

$$(\omega_A)_3 = \tfrac{2}{7}\,\omega_1 \circlearrowright$$

$(\bar{v}_A)_3 = r(\omega_A)_3 = \frac{2}{7}r\omega_1 = \frac{2}{7}\bar{v}_1 \qquad (\bar{v}_A)_3 = \frac{2}{7}\bar{v}_1 \rightarrow$ ◀

SPHERE B:



SYST MOMENTA$_2$ + SYST EXT IMP$_{2-3}$ = SYST. MOMENTA$_3$

+↻ MOMENTS ABOUT D:

$$m\bar{v}_1 r = \bar{I}(\omega_B)_3 + m(\bar{v}_B)_3 r$$

$$m\bar{v}_1 r = \tfrac{2}{5}mr^2(\omega_B)_3 + mr^2(\omega_B)_3$$

$$(\omega_B)_3 = \tfrac{5}{7}\tfrac{\bar{v}_1}{r} \qquad \text{BUT, } \omega_1 = \tfrac{\bar{v}_1}{r}$$

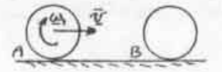
$$(\omega_B)_3 = \tfrac{5}{7}\omega_1 \circlearrowright$$

$(\bar{v}_B)_3 = r(\omega_B)_3 = r\left(\frac{5}{7}\frac{\bar{v}_1}{r}\right) \qquad (\bar{v}_B)_3 = \frac{5}{7}\bar{v}_1 \rightarrow$ ◀

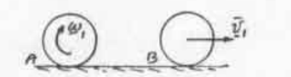
SUMMARY

INITIAL MOTION

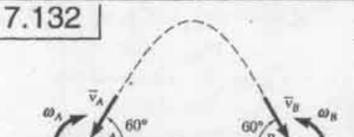
JUST AFTER IMPACT



FINAL (UNIFORM) MOTION



17.132



GIVEN: BALL BOUNCES AS SHOWN. $\omega_A = \omega_0$, $\bar{v}_A = \bar{v}_0$

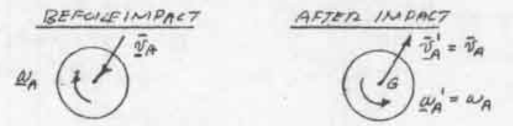
FIND: ω_0 IN TERMS OF $\bar{v}_0$ AND r.

SINCE THE LINEAR AND ANGULAR VELOCITIES ARE CHANGED DURING A SHORT INTERVAL Δt, BOTH THE NORMAL AND FRICTION FORCES ARE IMPULSIVE

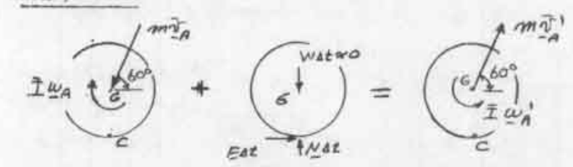
WE ASSUME THAT NO SLIPPING OCCURS.

FOR THE VELOCITY OF THE BALL TO BE REVERSED AT EACH IMPACT, WE MUST HAVE AT POINT A.

BEFORE IMPACT $\qquad$ AFTER IMPACT



IMPACT AT A:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT C:

$$\bar{I}\omega_A - (m\bar{v}_A\cos 60^\circ)r = -\bar{I}\omega_A' + (m\bar{v}_A'\cos 60^\circ)r$$

SUBSTITUTE: $\quad \omega_A' = \omega_A = \omega_0$

$$\bar{v}_A' = \bar{v}_A = \bar{v}_0$$

$$2\bar{I}\omega_0 = 2(m\bar{v}_0\cos 60^\circ)r$$

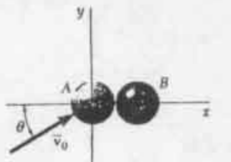
$$\omega_0 = \frac{(m\bar{v}_0\cos 60^\circ)r}{\bar{I}}$$

$$\omega_0 = \frac{m\bar{v}_0(\frac{1}{2})r}{\frac{2}{5}mr^2}$$

$$\omega_0 = \frac{5}{4}\frac{\bar{v}_0}{r}$$ ◀

17.133 and 17.134

GIVEN: BALL A IS ROLLING WITHOUT SLIPPING WHEN IT HITS BALL B. COEF. OF KINETIC FRICTION IS μ_k. ASSUMING PERFECTLY ELASTIC IMPACT,



PROBLEM 17.133
FIND: (a) $\bar{v}$ AND ω OF EACH BALL, (b) $\bar{v}_B$ AFTER IT STARTS ROLLING

PROBLEM 17.134: FIND EQUATION OF PATH OF BALL A

PROBLEM 17.133

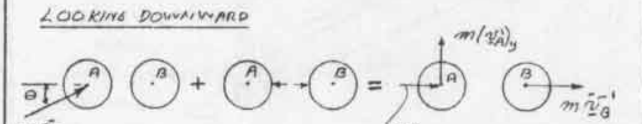
(a) MOTION IMMEDIATELY AFTER IMPACT

FRICTION FORCES ARE NON IMPULSIVE, THUS ANGULAR MOMENTUM (AND THUS ω) OF EACH BALL IS UNCHANGED. WE HAVE $\quad \underline{\omega}_B' = 0$

AND SINCE BALL A WAS ROLLING:

$\underline{\omega}_A' = \frac{\bar{v}_0}{r} \measuredangle \theta \qquad \underline{\omega}_A' = \frac{\bar{v}_0}{r}\left(-\sin\theta\, \underline{i} + \cos\theta\, \underline{j}\right)$

LOOKING DOWNWARD



SYST. MOMENTA$_0$ + SYST. EXT. IMP$_{0\to1}$ = SYST. MOMENTA$_1$

$\xrightarrow{+}$ COMPONENTS: $\quad m\bar{v}_0\cos\theta = m(v_A')_x + m\bar{v}_B' \qquad (1)$

$+\uparrow$ COMPONENTS OF BALL A: $\quad m\bar{v}_0\sin\theta = m(\bar{v}_A')_y$

$$(\bar{v}_A')_y = \bar{v}_0\sin\theta \qquad (2)$$

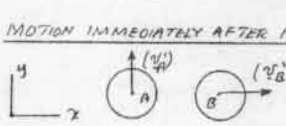
FOR ELASTIC IMPACT $e = 1$

$$v_B' - (v_A')_x = \bar{v}_0\cos\theta \qquad (3)$$

SOLVING SIMULTANEOUSLY EQS(1) AND (2)

$$(v_A')_x = 0 \qquad (v_B') = \bar{v}_0\cos\theta \qquad (4)$$

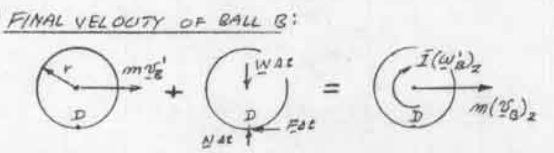
MOTION IMMEDIATELY AFTER IMPACT:



$\underline{v}_A' = \bar{v}_0\sin\theta\, \underline{j}$

$\underline{v}_B' = \bar{v}_0\cos\theta\, \underline{i}$

(b) FINAL VELOCITY OF BALL B:



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$+\circlearrowright$ MOMENTS ABOUT D: $\quad m v_B' r = \bar{I}(\omega_B)_2 + m(v_B)_2 r \qquad (5)$

WE RECALL: $v_B' = \bar{v}_0\cos\theta$ AND $\bar{I} = \frac{2}{5}mr^2$

BALL ROLLS: $\qquad (v_B)_2 = r(\omega_B)_2$

EQ(5): $\quad mr\bar{v}_0\cos\theta = \frac{2}{5}mr^2(\omega_B)_2 + mr(\omega_B)_2 r$

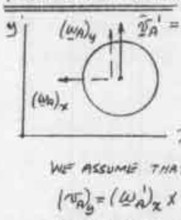
$(\omega_B)_2 = \frac{5}{7}\frac{\bar{v}_0}{r}\cos\theta \qquad (\bar{v}_B)_2 = r(\omega_B)_2 \rightarrow$

$$(\underline{v}_B)_2 = \frac{5}{7}\bar{v}_0\cos\theta\, \underline{i}$$

(CONTINUED)

17.133 and 17.134 continued

PROBLEM 17.134 $\quad$ LOOKING DOWNWARD ON BALL A



$\underline{\omega}_A' = \frac{\bar{v}_0}{r}\left(-\sin\theta\, \underline{i} + \cos\theta\, \underline{j}\right)$

$\underline{\bar{v}}_A' = \bar{v}_0\sin\theta\, \underline{j}$

WE ASSUME THAT BALL A ROLLS WITHOUT SLIPPING IN y DIRECTION

$(v_A)_y = (\omega_A')_x \times r\underline{k} = -\left(\frac{\bar{v}_0}{r}\sin\theta\, \underline{i}\right)\times r\underline{k} = +\, v_0\sin\theta\, \underline{j}$

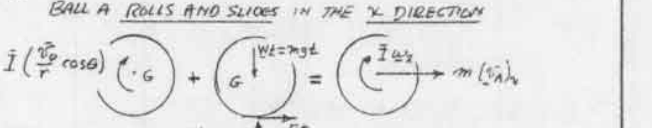
ASSUMPTION IS CORRECT

THE y COMPONENT OF VELOCITY IS CONSTANT AND THUS THE y COORDINATE AT ANY TIME t IS

$$y = (v_A')t = (\bar{v}_0\sin\theta)t \qquad (1)$$

μ_k = COEF. OF KINETIC FRICTION BETWEEN BALLS AND TABLE

BALL A ROLLS AND SLIDES IN THE x DIRECTION



$\xrightarrow{+}$ x COMPONENTS: $\quad 0 + \mu_k mg\, t = m(\bar{v}_A)_x$

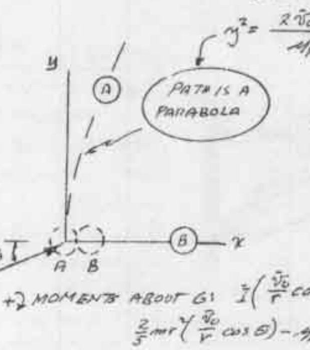
$$m(\bar{v}_A)_x = \mu_k g\, t$$

$(a_A)_x = \mu_k g = \text{CONSTANT} \qquad x = \frac{1}{2}at^2 \qquad x = \frac{1}{2}\mu_k g t^2 \qquad (2)$

ELIMINATE t BETWEEN EQS (1) AND (2)

$t = y/(v_0\sin\theta)$; $\quad x = \frac{1}{2}\mu_k g\left(\frac{y^2}{v_0^2\sin^2\theta}\right)$;

$$y^2 = \frac{2\bar{v}_0^2\sin^2\theta}{\mu_k g}\, x$$



SLIPPING ENDS AND UNIFORM MOTION BEGINS WHEN

$$\omega_x = \frac{(v_A)_x}{r} = \frac{\mu_k g t_f}{r}$$

$+\circlearrowright$ MOMENTS ABOUT G: $\quad \bar{I}\left(\frac{\bar{v}_0}{r}\cos\theta\right) - (\mu_k mgt) = \bar{I}\omega_2$

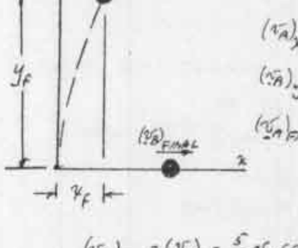
$$\frac{2}{5}mr^2\left(\frac{\bar{v}_0}{r}\cos\theta\right) - \mu_k mgt = \frac{2}{5}mr^2\left(\frac{\mu_k g t_f}{r}\right)$$

$$\frac{2}{5}\bar{v}_0\cos\theta = \frac{7}{5}\mu_k g t_f$$

ROLLING WITHOUT SLIDING BEGINS WHEN $\quad t_f = \frac{2}{7}\cdot\frac{\bar{v}_0\cos\theta}{\mu_k g}$

EQ.(2): $\quad x_f = \frac{1}{2}\mu_k g t_f^2 = \frac{2}{49}\frac{\bar{v}_0^2\cos^2\theta}{\mu_k g}$

EQ(1): $\quad y_f = \bar{v}_0\sin\theta\, t_f = \frac{2}{7}\frac{\bar{v}_0^2\sin\theta\cos\theta}{\mu_k g}$



FINAL VELOCITIES (UNIFORM MOTION)

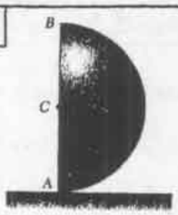
$(v_A)_x = \mu_k g t_f = \frac{2}{7}\bar{v}_0\cos\theta$

$(v_A)_y = \bar{v}_0\sin\theta$

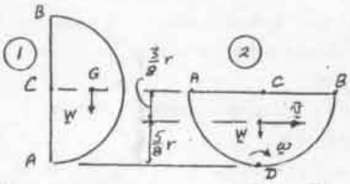
$(\underline{v}_A)_{FINAL} = \left(\frac{2}{7}\bar{v}_0\cos\theta\right)\underline{i} + (\bar{v}_0\sin\theta)\underline{j}$

$(\underline{v}_B)_{FINAL} = (v_B)_2 = \frac{5}{7}v_0\cos\theta\, \underline{i}$

17.135



GIVEN: UNIFORM HEMISPHERE IS RELEASED FROM REST AND ROLLS WITHOUT SLIDING. AFTER HEMISPHERE ROLLS THROUGH 90°; FIND: (a) $\underline{\omega}$, (b) NORMAL REACTION.



$\bar{v} = \frac{5}{8} r\omega$

$\bar{I} = \frac{2}{5} mr^2 - m\left(\frac{3}{8}r\right)^2$

$\bar{I} = \left(\frac{2}{5} - \frac{9}{64}\right) mr^2$

(a) WORK-ENERGY: $U_{1\to2} = W\left(\frac{3}{8}r\right) = \frac{3}{8} mgr$

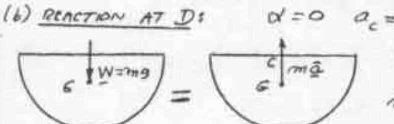
$T_1 = 0, \quad T_2 = \frac{1}{2}\bar{I}\omega^2 + \frac{1}{2}m\bar{v}^2 = \frac{1}{2}\left(\frac{2}{5} - \frac{9}{64}\right)mr^2\omega^2 + \frac{1}{2}m\left(\frac{5}{8}r\right)^2\omega^2$

$T_2 = \frac{13}{40} mr^2\omega^2$

$T_1 + U_{1-2} = T_2: \quad 0 + \frac{3}{8} mgr = \frac{13}{40} mr^2\omega^2$

$\omega^2 = \frac{15}{13}\,\frac{g}{r} \qquad \underline{\omega} = 1.074\sqrt{g/r}$ ◄

(b) REACTION AT $\underline{D}$: $\quad \alpha = 0 \quad a_C = 0$



$\bar{a} = a_C + a_{G/C} = \left(\frac{3}{8}r\right)\omega^2 \uparrow$

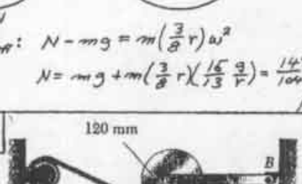
$m\bar{a} = m\left(\frac{3}{8}r\right)\omega^2 \uparrow$

$+\uparrow \Sigma F = \Sigma F_{eff}: \quad N - mg = m\left(\frac{3}{8}r\right)\omega^2$

$N = mg + m\left(\frac{3}{8}r\right)\left(\frac{15}{13}\,\frac{g}{r}\right) = \frac{149}{104} mg$

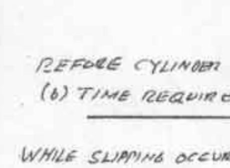
$N = 1.433 mg \uparrow$ ◄

17.136



GIVEN: $\mu_k = 0.15$, $v = 25$ m/s ↘ CYLINDER IS AT REST WHEN PLACED ON BELT. FIND: (a) NUMBER OF REVOLUTIONS BEFORE CYLINDER REACHES CONSTANT VELOCITY. (b) TIME REQUIRED TO REACH CONSTANT VELOCITY.

WHILE SLIPPING OCCURS:



$+\uparrow \Sigma F_y = 0$

$N\cos\beta - \mu_k N \sin\beta - mg = 0$

$N = \dfrac{mg}{\cos\beta - \mu_k \sin\beta} \qquad (1)$

SLIPPING OCCURS UNTIL: $\omega = \frac{v}{r}$

WORK-ENERGY: $M_F = Fr$ = MOMENT OF $\underline{F}$ ABOUT A.

$U_{1-2} = M_F\theta = Fr\theta = \mu_k N r\theta$

$T_1 = 0; \quad T_2 = \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{4}mv^2$

$T_1 + U_{1-2} = T_2: \quad 0 + \mu_k N r\theta = \frac{1}{4} mv^2$

$\theta = \frac{1}{4}\frac{mv^2}{\mu_k r}\frac{1}{N} = \frac{1}{4}\frac{mv^2}{\mu_k r}\cdot\frac{\cos\beta - \mu_k\sin\beta}{mg}$

$\theta = \frac{1}{4}\frac{v^2}{\mu_k rg}\cdot(\cos\beta - \mu_k\sin\beta) \qquad (2)$

(CONTINUED)

17.136 continued

PRINCIPLE OF IMPULSE-MOMENTUM

$\omega = \frac{v}{r}$

SYST. MOMENTA$_1$ + SYST. EXT. IMP.$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT A: $\quad Ftr = \bar{I}\omega$

$\mu_k N t r = \frac{1}{2}mr^2\left(\frac{v}{r}\right)$

SUBSTITUTE FOR N:

$\mu_k\left(\dfrac{mg}{\cos\beta - \mu_k\sin\beta}\right)tr = \frac{1}{2}mrv$

$t = \frac{1}{2}\frac{v}{\mu_k g}(\cos\beta - \mu_k\sin\beta) \qquad (3)$

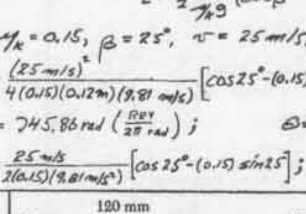
DATA: $\mu_k = 0.15$, $\beta = 25°$, $v = 25$ m/s, $r = 0.12$ m

EQ(2): $\theta = \dfrac{(25\text{ m/s})^2}{4(0.15)(0.12\text{ m})(9.81\text{ m/s}^2)}\left[\cos 25° - (0.15)\sin 25°\right]$

$\theta = 745.86$ rad $\left(\frac{\text{REV}}{2\pi\text{ rad}}\right)$; $\qquad \theta = 118.7$ revolutions ◄

EQ(3): $t = \dfrac{25\text{ m/s}}{2(0.15)(9.81\text{ m/s}^2)}\left[\cos 25° - (0.15)\sin 25°\right]; \quad t = 7.16$ s ◄

17.137



GIVEN: $\mu_k = 0.15$, $v = 25$ m/s ↗ CYLINDER AT REST PLACED ON BELT. UNTIL MOTION BECOMES UNIFORM FIND: (a) NUMBER OF REVOLUTIONS REQUIRED. (b) TIME INTERVAL REQUIRED

WHILE SLIPPING OCCURS:

$+\uparrow \Sigma F_y = 0: \quad N\cos\beta + \mu_k N \sin\beta - mg = 0$

$N = \dfrac{mg}{\cos\beta + \mu_k \sin\beta} \qquad (1)$

FOR CYLINDER SLIPPING OCCURS UNTIL $\omega = \frac{v}{r}$

WORK-ENERGY: $M_F = Fr$ = MOMENT OF $\underline{F}$ ABOUT A.

$U_{1\to2} = M_F\theta = Fr\theta = \mu_k N r\theta$

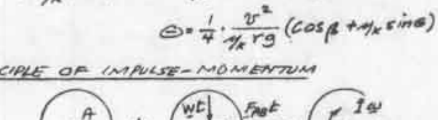
$T_1 = 0; \quad T_2 = \frac{1}{2}\bar{I}\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{4}mv^2$

$T_1 + U_{1\to2} = T_2: \quad 0 + \mu_k N r\theta = \frac{1}{4}mv^2$

$\theta = \frac{1}{4}\frac{mv^2}{\mu_k r}\cdot\frac{1}{N} = \frac{1}{4}\frac{mv^2}{\mu_k r}\cdot\frac{\cos\beta + \mu_k\sin\theta}{mg}$

$\theta = \frac{1}{4}\cdot\frac{v^2}{\mu_k rg}(\cos\beta + \mu_k\sin\theta) \qquad (2)$

PRINCIPLE OF IMPULSE-MOMENTUM



+↻ MOMENTS ABOUT A: $\quad Ftr = \bar{I}\omega$

$\mu_k N t r = \frac{1}{2}mr^2\left(\frac{v}{r}\right)$

SUBSTITUTE FOR N:

$\mu_k\left(\dfrac{mg}{\cos\beta + \mu_k\sin\beta}\right)tr = \frac{1}{2}mrv$

$t = \frac{1}{2}\cdot\frac{v}{\mu_k g}\cdot(\cos\beta + \mu_k\sin\beta) \qquad (3)$

DATA: $\mu_k = 0.15$, $\beta = 25°$, $v = 25$ m/s, $r = 0.12$ m

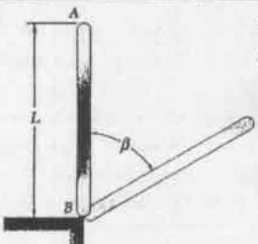
EQ(2): $\theta = \dfrac{(25\text{ m/s})^2}{4(0.15)(0.12\text{ m})(9.81\text{ m/s}^2)}\left[\cos 25° + (0.15)\sin 25°\right]$

$\theta = 858.05$ rad $\left(\frac{\text{rev}}{2\pi\text{ rad}}\right) \qquad \theta = 136.6$ revolutions ◄

EQ(3): $t = \dfrac{25\text{ m/s}}{2(0.15)(9.81\text{ m/s}^2)}\left[\cos 25° + (0.15)\sin 25°\right]$

$t = 8.24$ s ◄

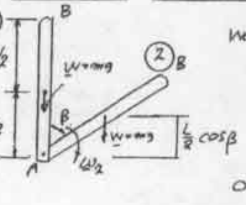
17.138



GIVEN: ROD AB IS GIVEN A SLIGHT MOTION CLOCKWISE

FIND: (a) ANGLE β WHEN ROD LOSES CONTACT WITH CORNER

(b) CORRESPONDING $\underline{v}_A$



WORK-ENERGY:

$U_{1-2} = W\left(\frac{L}{2} - \frac{L}{2}\cos\beta\right) = mg\frac{L}{2}(1-\cos\beta)$

$T_1 = 0$

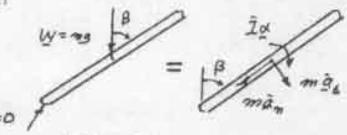
$T_2 = \frac{1}{2}I_A\omega_2^2 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_2^2 = \frac{1}{6}mL^2\omega_2^2$

$T_1 + U_{1-2} = T_2$

$0 + mg\frac{L}{2}(1-\cos\beta) = \frac{1}{6}mL^2\omega_2^2$

$\omega_2^2 = \frac{3g}{L}(1-\cos\beta) \qquad (1)$

(a)



$a_n = \frac{L}{2}\omega_2^2$

$a_n = \frac{3}{2}g(1-\cos\beta)$

$+\swarrow \Sigma F = \Sigma F_{eff}$: $\quad mg\cos\beta = ma_n = m\frac{3}{2}g(1-\cos\beta)$

$\cos\beta = \frac{3}{2} - \frac{3}{2}\cos\beta$

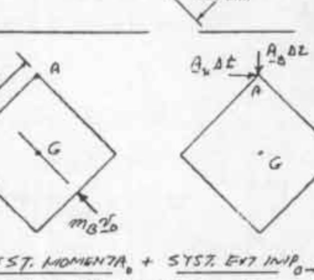
$2.5\cos\beta = 1.5; \quad \cos\beta = 0.6; \quad \beta = 53.1°$ ◀

(b) WHEN $\cos\beta = 0.6$

EQ(1) $\omega_2^2 = \frac{3g}{L}(1-0.6) = 1.2\frac{g}{L}; \quad \omega_2 = \sqrt{1.2\frac{g}{L}}$

$v_A = L\omega_2 = L\sqrt{1.2\frac{g}{L}} \qquad \underline{v}_A = \sqrt{1.2gL} \measuredangle 53.1°$ ◀

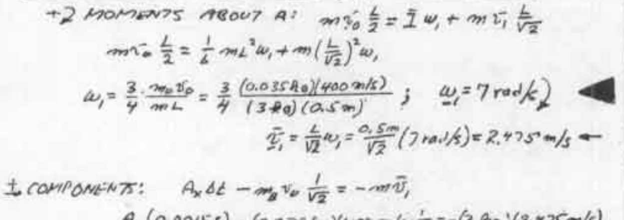
17.139



GIVEN: 35-g BULLET FIRED WITH $v_0 = 400$ m/s BECOMES EMBEDDED IN $\Delta t = 1.5$ ms. MASS OF PLATE = 3 kg

FIND: (a) ω JUST AFTER BULLET BECOMES EMBEDDED.

(b) IMPULSIVE REACTION AT A.



$\bar{v}_1 = \frac{L}{\sqrt{2}}\omega_1$

$\bar{I} = \frac{1}{6}mL^2$

SYST. MOMENTA$_0$ + SYST. EXT IMP$_{0-1}$ = SYST. MOMENTA$_1$

$+\circlearrowright$ MOMENTS ABOUT A: $\quad m v_0\frac{L}{2} = \bar{I}\omega_1 + m\bar{v}_1\frac{L}{\sqrt{2}}$

$m v_0\frac{L}{2} = \frac{1}{6}mL^2\omega_1 + m\left(\frac{L}{\sqrt{2}}\right)^2\omega_1$

$\omega_1 = \frac{3}{4}\cdot\frac{m_B v_0}{mL} = \frac{3}{4}\frac{(0.035\text{ kg})(400\text{ m/s})}{(3\text{ kg})(0.5\text{ m})}; \quad \underline{\omega}_1 = 7\text{ rad/s} \circlearrowright$ ◀

$\bar{v}_1 = \frac{L}{\sqrt{2}}\omega_1 = \frac{0.5\text{ m}}{\sqrt{2}}(7\text{ rad/s}) = 2.475\text{ m/s} \leftarrow$

$\pm$ COMPONENTS: $\quad A_x\Delta t - m_B v_0\frac{1}{\sqrt{2}} = -m\bar{v}_1$

$A_x(0.0015\text{ s}) - (0.035\text{ kg})(400\text{ m/s})\frac{1}{\sqrt{2}} = -(3\text{ kg})(2.475\text{ m/s})$

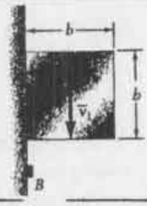
$A_x(0.0015\text{ s}) - 9.899 = -7.425; \quad \underline{A}_x = 1650\text{ N} \rightarrow$

$+\downarrow$ COMPONENTS: $\quad A_y\Delta t - m_B v_0 = 0$

$A_y(0.0015\text{ s}) - (0.035\text{ kg})(400\text{ m/s})\frac{1}{\sqrt{2}}; \quad A_y = 6600\text{ N}\downarrow$

$\underline{A} = 6.80\text{ kN} \measuredangle 76.0°$ ◀

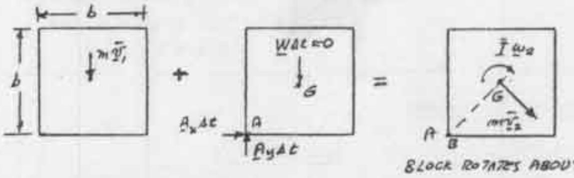
17.140



GIVEN: UNIFORM BLOCK

JUST AFTER A PERFECTLY PLASTIC IMPACT AT B

FIND: (a) $\underline{\omega}$, (b) $\underline{\bar{v}}$.



BLOCK ROTATES ABOUT B

SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$+\circlearrowright$ MOMENTS ABOUT A: $\quad m\bar{v}_1\frac{b}{2} = \bar{I}\omega_2 + m\bar{v}_2(AG)$

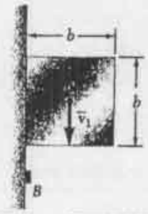
$(AG) = b/\sqrt{2} \qquad \bar{v}_2 = (AG)\omega_2 = \frac{b}{\sqrt{2}}\omega_2 \qquad \bar{I} = \frac{1}{6}mb^2$

$m\bar{v}_1\frac{b}{2} = \frac{1}{6}mb^2\omega_2 + m\left(\frac{b}{\sqrt{2}}\right)^2\omega_2$

$\frac{1}{2}\bar{v}_1 = \frac{2}{3}b\,\omega_2; \qquad \underline{\omega}_2 = \frac{3}{4}\frac{\bar{v}_1}{b} \circlearrowright$ ◀

$\bar{v}_2 = (AG)\omega_2 = \frac{b}{\sqrt{2}}\cdot\frac{3}{4}\frac{\bar{v}_1}{b}; \qquad \underline{\bar{v}}_2 = \frac{3\sqrt{2}}{8}\bar{v}_1 \measuredangle 45°$ ◀

17.141

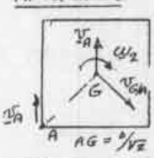


GIVEN: UNIFORM BLOCK

JUST AFTER A PERFECTLY ELASTIC IMPACT AT B

FIND: (a) ω, (b) $\bar{v}$.

KINEMATICS:



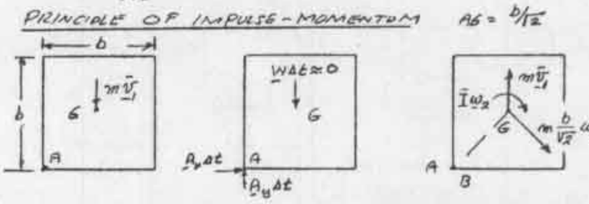
AFTER ELASTIC IMPACT ($e = 1$)

$v_A = \bar{v}_1\uparrow$

$\bar{v}_2 = v_A + v_{G/A}$

$\bar{v}_2 = [\bar{v}_1\uparrow] + \left[\frac{b}{\sqrt{2}}\omega_2 \measuredangle 45°\right] \qquad (1)$

PRINCIPLE OF IMPULSE-MOMENTUM $\qquad AG = b/\sqrt{2}$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$+\circlearrowright$ MOMENTS ABOUT A:

$m\bar{v}_1\frac{b}{2} = \bar{I}\omega_2 - m\bar{v}_1\frac{b}{2} + m\frac{b}{\sqrt{2}}\omega_2\frac{b}{\sqrt{2}}$

$\bar{I} = \frac{1}{6}mb^2 \qquad (AG) = \frac{b}{\sqrt{2}}$

$m\bar{v}_1\frac{b}{2} = \frac{1}{6}mb^2\omega_2 - m\bar{v}_1\frac{b}{2} + m\left(\frac{b}{\sqrt{2}}\right)^2\omega_2$

$m\bar{v}_1 b = \frac{2}{3}b^2\omega_2 \qquad \underline{\omega}_2 = \frac{3}{2}\frac{\bar{v}_1}{b} \circlearrowright$ ◀

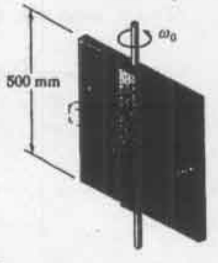
EQ(1) $\quad \bar{v}_2 = [\bar{v}_1\uparrow] + \left[\frac{b}{\sqrt{2}}\cdot\frac{3}{2}\frac{\bar{v}_1}{b} \measuredangle 45°\right]$

$= [\bar{v}_1\uparrow] + \left[\frac{3}{2\sqrt{2}}\bar{v}_1\sin 45°\downarrow\right] + \left[\frac{3}{2\sqrt{2}}\bar{v}_1\cos 45°\rightarrow\right]$

$= [\bar{v}_1\uparrow] + \left[\frac{3}{4}\bar{v}_1\downarrow\right] + \left[\frac{3}{4}\bar{v}_1\rightarrow\right]$

$\underline{\bar{v}}_2 = 0.791\bar{v}_1 \measuredangle 18.4°$ ◀

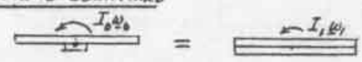
17.142



GIVEN: 3-kg BAR AB
4-kg PLATE
$\omega_0 = 120$ rpm

AFTER BAR SWUNG TO HORIZONTAL,
FIND: (a) $\underline{\omega}$, (b) ENERGY LOST DURING PLASTIC IMPACT AT C

(a) LOOKING DOWNWARD



CONSERVATION OF ANGULAR MOMENTUM ABOUT SHAFT

$$I_0\omega_0 = I_1\omega_1 \qquad (1)$$

$I_0 = I_{PLATE} = \frac{1}{12}(4\,kg)L^2$

$I_1 = I_{PLATE} + I_{BAR} = \frac{1}{12}(4\,kg)L^2 + \frac{1}{12}(3\,kg)L^2 = \frac{1}{12}(7\,kg)L^2$

EQ(1): $\frac{1}{12}(4\,kg)L^2(120\,rpm) = \frac{1}{12}(7\,kg)L^2\omega_1$

$\omega_1 = \frac{4}{7}(120\,rpm) \qquad \omega_1 = 68.6\,rpm$ ◀

(b) ENERGY: (WE MUST USE rad/s)

$\omega_0 = 120\,rpm\left(\frac{2\pi}{60}\right) = 4\pi\,rad/s = 12.566\,rad/s$

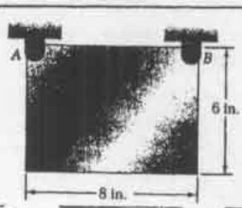
$\omega_1 = \frac{4}{7}\omega_0 = \frac{4}{7}(4\pi\,rad/s) = 7.181\,rad/s$

$T_0 = \frac{1}{2}I_0\omega_0^2 = \frac{1}{2}\left[\frac{1}{12}(4\,kg)(0.5\,m)^2\right](12.566\,rad/s)^2 = 6.580\,J$

$T_1 = \frac{1}{2}I\omega_1^2 = \frac{1}{2}\left[\frac{1}{12}(7\,kg)(0.5\,m)^2\right](7.181\,rad/s)^2 = 3.760\,J$

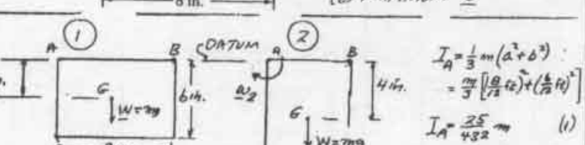
ENERGY LOST = 6.580 J − 3.760 J = 2.82 J ◀

17.143



GIVEN: PIN B IS REMOVED AND PLATE SWINGS ABOUT A
FIND: (a) $\underline{\omega}$ AFTER 90° ROTATION,
(b) MAXIMUM $\underline{\omega}$

(a)



$I_A = \frac{1}{3}m(a^2+b^2) = \frac{m}{3}\left[\left(\frac{8}{12}ft\right)^2 + \left(\frac{6}{12}ft\right)^2\right]$

$I_A = \frac{25}{432}m \qquad (1)$

SEE EQ 17.10 page 1049

$T = \frac{1}{2}I_A\omega^2$

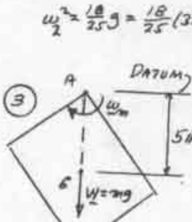
$U_{1-2} = W(4\,in. - 3\,in.) = mg\left(\frac{1}{12}ft\right)$

$T_1 = 0: \quad T_2 = \frac{1}{2}I_A\omega_2^2 = \frac{1}{2}\frac{25}{432}m\omega^2$

$T_1 + U_{1-2} = T_2$: $\cancel{m}g\left(\frac{1}{12}\right) = \frac{1}{2}\frac{25}{432}\cancel{m}\omega_2^2 \qquad \omega_2^2 = \frac{18}{25}g$

$\omega_2^2 = \frac{18}{25}g = \frac{18}{25}(32.2) = 23.184 \qquad \underline{\omega}_2 = 4.81\,rad/s$ ◀

(b)



$AG = \sqrt{(3\,in)^2 + (4\,in)^2} = 5\,in.$

EQ(1): $I_A = \frac{25}{432}m$

$U_{1-3} = W(5\,in. - 3\,in.) = mg\left(\frac{2}{12}ft\right)$

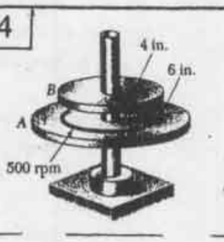
$T_1 = 0$

$T_3 = \frac{1}{2}I_A\omega_3^2 = \frac{1}{2}\frac{25}{432}\omega_3^2$

$T_1 + U_{1-3} = T_3 \qquad \cancel{m}g\left(\frac{2}{12}\right) = \frac{1}{2}\frac{25}{432}\cancel{m}\omega_3^2$

$\omega_3^2 = \frac{36}{25}g = \frac{36}{25}(32.2) = 46.368 \qquad \underline{\omega}_3 = 6.81\,rad/s$ ◀

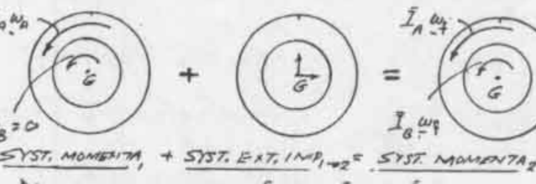
17.144



GIVEN: DISK OF SAME THICKNESS AND SAME MATERIAL
DISK B IS AT REST WHEN IT IS DROPPED ON DISK A
KNOWING $W_A = 18\,lb$,
FIND: (a) FINAL $\underline{\omega}$ OF DISKS
(b) CHANGE IN KINETIC ENERGY

(a)

FOR A DISK: $m = \rho V = \rho\pi r^2 t$; $\bar{I} = \frac{1}{2}mr^2 = \frac{1}{2}(\rho\pi r^2 t)r^2 = \frac{1}{2}\rho\pi t r^4$



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↺ MOMENTS ABOUT G: $\bar{I}_A\omega_A = \bar{I}_A\omega_f + \bar{I}_B\omega_f$

$$\frac{1}{2}\rho\pi t r_A^4\omega_A = \left(\frac{1}{2}\rho\pi t r_A^4 + \frac{1}{2}\rho\pi t r_B^4\right)\omega_f$$

$$\omega_f = \frac{r_A^4}{r_A^4 + r_B^4}\omega_A$$

$\omega_f = \frac{(6\,in.)^4}{(6\,in.)^4 + (4\,in.)^4}(500\,rpm) = 417.526\,rpm$

$\omega_f = 418\,rpm$ ◀

(b) ENERGY: $\left.\begin{array}{l} W_A = m_A g = \rho g\pi t r_A^2 \\ W_B = m_B g = \rho g\pi t r_B^2 \end{array}\right\} \quad \frac{W_A}{W_B} = \frac{r_A^2}{r_B^2}$

$W_A = 18\,lb$; $W_B = \left(\frac{r_B}{r_A}\right)^2 W_A = \left(\frac{4\,in.}{6\,in.}\right)^2(18\,lb) = 8\,lb$

INITIAL KINETIC ENERGY $\quad \omega_A = 500\,rpm\left(\frac{2\pi}{60}\right) = 52.36\,rad/s$

$T_1 = \frac{1}{2}\bar{I}_A\omega_A^2 = \frac{1}{2}\left[\frac{1}{2}\frac{18\,lb}{32.2}\left(\frac{6}{12}ft\right)^2\right](52.36\,rad/s)^2 = 95.784\,ft\cdot lb$

$\omega_f = 417.526\,rpm\left(\frac{2\pi}{60}\right) = 43.723\,rad/s$

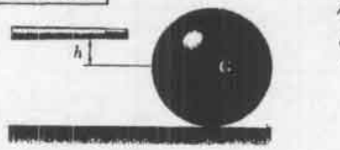
$T_2 = \frac{1}{2}(\bar{I}_A + \bar{I}_B)\omega_f^2 = \frac{1}{2}\left[\frac{1}{2}\frac{18\,lb}{32.2}\left(\frac{6}{12}ft\right)^2 + \frac{1}{2}\frac{8\,lb}{32.2}\left(\frac{4}{12}ft\right)^2\right](43.723\,rad/s)^2$

$T_2 = 79.985\,ft\cdot lb$

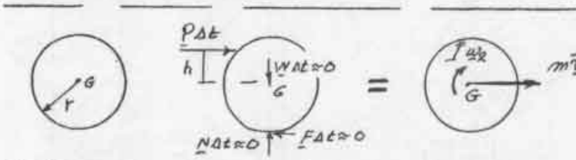
ENERGY LOSS: $\Delta T = T_2 - T_1 = 79.985\,ft\cdot lb - 95.784\,ft\cdot lb$

$\Delta T = -15.80\,ft\cdot lb$ ◀

17.145



FIND: DISTANCE h IF BALL IS TO START ROLLING WITHOUT SLIDING



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to2}$ = SYST. MOMENTA$_2$

+↻ MOMENTS ABOUT G: $(P\Delta t)h = \bar{I}\omega_2 \qquad (1)$

$\xrightarrow{+}$ COMPONENTS: $\quad P\Delta t = m\bar{v}_2 \qquad (2)$

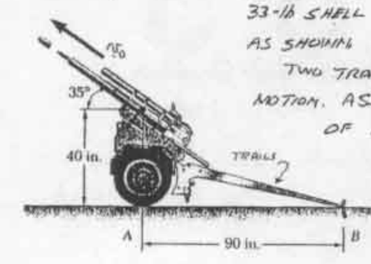
DIVIDE EQ(1) BY EQ(2) MEMBER BY MEMBER

$$h = \frac{\bar{I}}{m}\cdot\frac{\omega_2}{\bar{v}_2}$$

FOR ROLLING $\bar{v} = r\omega_2$

$$h = \frac{\frac{2}{5}mr^2}{m}\cdot\frac{\omega_2}{r\omega_2} \qquad h = \frac{2}{5}r$$ ◀

17.146



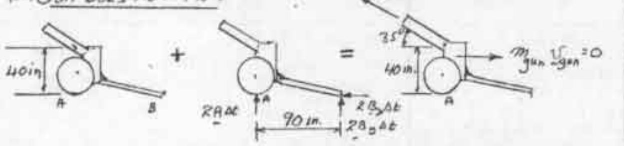
GIVEN: 4980-lb GUN FIRES 33-lb SHELL WITH $v_0 = 1450$ ft/s AS SHOWN

TWO TRAILS PREVENT HORIZONTAL MOTION. ASSUMING RECOIL TIME OF $\Delta t = 0.6$ s,

FIND: (a) IMPULSIVE REACTIONS AT EACH WHEEL AND EACH TRAIL

(b) v_{GUN} IF TRAILS DO NOT EXERT ANY HORIZ. FORCE

(a) GUN DOES NOT MOVE



SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1-2}$ = SYST. MOMENTA$_2$

$\xrightarrow{+}$ COMPONENTS: $\quad 2B_x \Delta t = m_s v_0 \cos 35°$

$$2B_x(0.6\,s) = \left(\frac{33\,lb}{32.2}\right)(1450\,ft/s)\cos 35°$$

$$B_x = 1014.4\,lb \qquad B_x = 1014\,lb \leftarrow$$

$+\circlearrowleft$ MOMENTS ABOUT A: $\quad 2B_y \Delta t (90\,in.) = (m_s v_0 \cos 35°)(40\,in.)$

$$2B_y(0.6\,s)(90\,in) = \left(\frac{33\,lb}{32.2}\right)(1450\,ft/s)\cos 35°(40\,in)$$

$$B_y = 450.8\,lb \qquad B_y = 451\,lb \uparrow$$

$+\uparrow$ COMPONENTS: $\quad 2A\Delta t + 2B_y \Delta t = m_s v_0 \sin 35°$

$$2(A + B_y)(0.6\,s) = \left(\frac{33\,lb}{32.2}\right)(1450\,lb)\sin 35°$$

$$A + B_y = 710.3\,lb$$

$$A + 450.8\,lb = 710.3\,lb \qquad A = 259\,lb \uparrow$$

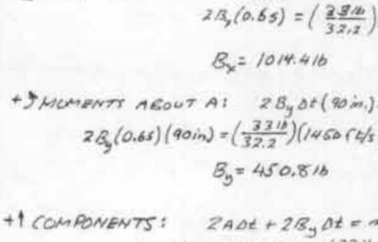
(b) TRAILS NOT EMBEDDED $\quad B_x = 0, \quad v_{GUN} \neq 0$

$\xrightarrow{+}$ COMPONENTS: $\quad 0 + 0 = -m_s v_0 \sin 35° + m_{gun} v_{gun}$

$$0 = -\left(\frac{33\,lb}{g}\right)(1450\,ft/s)\cos 35° + \left(\frac{4980\,lb}{g}\right)v_{gun}$$

$$v_{gun} = 7.871\,ft/s \qquad v_{gun} = 7.87\,ft/s \rightarrow$$

17.C1



GIVEN: 3-kg ROD AB

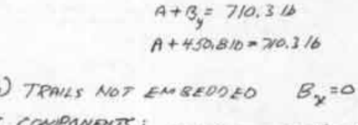
5-kg CART C

ROD RELEASED FROM REST WHEN $\theta = 30°$.

FIND: v_C AND v_B FOR $\theta = 30°$ TO $-90°$ USING 10° DECREMENTS.

ALSO, FIND θ FOR MAXIMUM v_C TO LEFT AND CORRESPONDING v_C

KINEMATICS $\quad AB = L$



$$\bar{v} = \vec{v}_C + \tfrac{1}{2}\omega \angle \theta$$

$$\bar{v}_x = \left[v_C + \tfrac{1}{2}\omega \sin\theta\right] \rightarrow$$

$$\bar{v}_y = \left[\tfrac{L}{2}\omega \cos\theta\right] \downarrow$$

$$v_B = \vec{v}_C + L\omega \angle \theta$$

$$(v_B)_x = \left[v_C + L\omega \sin\theta\right] \rightarrow \qquad (1)$$

$$(v_B)_y = \left[L\omega \cos\theta\right] \downarrow \qquad (2)$$

(CONTINUED)

17.C1 continued

POSITION (1) $\qquad$ POSITION (2) $\quad \theta = \theta_2$

SYST. MOMENTA$_1$ + SYST. EXT. IMP$_{1\to 2}$ = SYST. MOMENTA$_2$

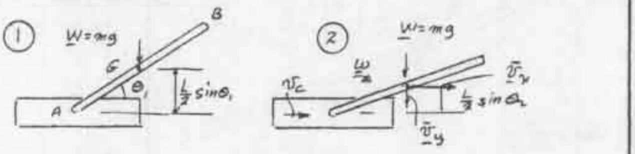
$\xrightarrow{+}$ $\quad 0 + 0 = m_C v_C + m_{AB}\bar{v}_x$

$$0 = m_C v_C + m_{AB}\left(v_C + \tfrac{L}{2}\omega \sin\theta_2\right)$$

$$(m_C + m_{AB})v_C = -m_{AB}\tfrac{L}{2}\omega \sin\theta_2$$

$$v_C = -\left[\left(\frac{m_{AB}}{m_C + m_{AB}}\right)\frac{L}{2}\sin\theta_2\right]\omega = \left[\text{COEFF } (1)\right]\omega \qquad (3)$$

CONSERVATION OF ENERGY



$$V_1 = mg\tfrac{L}{2}\sin\theta_1 \qquad V_2 = mg\tfrac{L}{2}\sin\theta_2 \qquad \bar{I} = \tfrac{1}{12}m_{AB}L^2$$

$$T_1 = 0 \qquad T_2 = \tfrac{1}{2}m_C v_C^2 + \tfrac{1}{2}\bar{I}\omega^2 + \tfrac{1}{2}m_{AB}\bar{v}^2$$

$$\bar{v}^2 = \bar{v}_x^2 + \bar{v}_y^2 = \left(v_C + \tfrac{L}{2}\omega\sin\theta\right)^2 + \left(\tfrac{L}{2}\omega\cos\theta\right)^2$$

$$\bar{v}^2 = v_C^2 + L\omega\sin\theta + \tfrac{1}{4}L^2\omega^2$$

$$= \left[(\text{COEFF } (1))^2 + (\text{COEFF } (1))L\sin\theta + \tfrac{L^2}{4}\right]\omega^2$$

$$\bar{v}^2 = \left[\text{COEFF } (2)\right]\omega^2 \qquad (4)$$

$$T_2 = \tfrac{1}{2}\left\{m_C(\text{COEFF } (1)) + \bar{I} + m_{AB}(\text{COEFF } (2))\right\}\omega^2$$

$$T_2 = \tfrac{1}{2}\left[(\text{COEFF } (3))\right]\omega^2 \qquad (5)$$

$T_1 + V_1 = T_2 + V_2$: $\quad 0 + m_{AB}g\tfrac{L}{2}\sin\theta_1 = \tfrac{1}{2}\left[\text{COEFF } (3)\right]\omega^2 + m_{AB}\tfrac{L}{2}\sin\theta_2$

$$\omega^2 = \frac{m_{AB}gL(\sin\theta_1 - \sin\theta)}{\left[\text{COEFF } (3)\right]} \qquad (6)$$

OUTLINE OF PROGRAM:

ENTER DATA: $L = 1.2$ m, $m_C = 5$ kg, $m_{AB} = 3$ kg, $\theta_1 = 30°$.

PROGRAM IN SEQUENCE EQS. (3), (4), AND (5) WHICH CONTAIN THE THREE COEFFICIENTS, THEN PROGRAM EQS. (1) AND (2) THAT INVOLVE $(v_B)_x$ AND $(v_B)_y$

EVALUATE AND PRINT

$$\theta,\ \omega,\ \bar{v},\ v_C,\ (v_B)_x,\ (v_B)_y$$

```
Linear velocities positive to the right and up
Omega positive clockwise
```

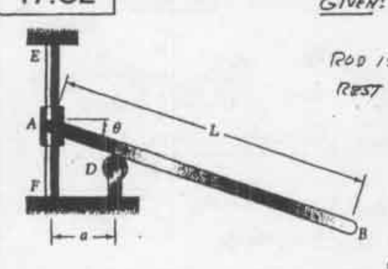
theta deg.	omega rad/s	vAB=$\bar{v}$ m/s	vC m/s	vBx m/s	vBy m/s
30.00	0.000	0.000	0.000	0.000	0.000
20.00	2.002	1.157	-0.154	0.667	-2.257
10.00	2.841	1.689	-0.111	0.481	-3.358
0.00	3.502	2.101	0.000	0.000	-4.202
-10.00	4.082	2.427	0.159	-0.691	-4.824
-20.00	4.621	2.672	0.356	-1.541	-5.211
-30.00	5.136	2.837	0.578	-2.504	-5.338
-40.00	5.631	2.923	0.814	-3.529	-5.177
-50.00	6.098	2.933	1.051	-4.555	-4.704
-60.00	6.516	2.881	1.270	-5.502	-3.910
-70.00	6.854	2.795	1.449	-6.279	-2.813
-80.00	7.076	2.715	1.568	-6.795	-1.475
-90.00	7.154	2.683	1.610	-6.975	-0.000

Find maximum velocity of cart to the left.

19.70	2.0315	1.1760	-0.15408479
19.69	2.0325	1.1766	-0.15408483
19.68	2.0335	1.1772	-0.15408479

$(v_C)_{max}$ TO LEFT = 0.1541 m/s WHEN $\theta = 19.7°$

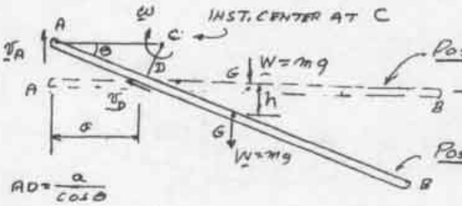
17.C2



GIVEN: $L = 800$ mm, $m = 5$ kg, $a = 200$ mm

ROD IS RELEASED FROM REST WHEN $\theta = 0$

FIND: ω AND v_A FOR VALUES OF θ FROM 0 TO 50° USING 5° INCREMENTS. ALSO FIND ω_{max} AND CORRESPONDING VALUE OF θ.



$AD = \dfrac{a}{\cos\theta}$

$DG = AG - AD = \dfrac{L}{2} - \dfrac{a}{\cos\theta}$; $h = (DG)\sin\theta$

$CD = (AD)\tan\theta = a\dfrac{\tan\theta}{\cos\theta}$; $AC = \dfrac{AD}{\cos\theta} = \dfrac{a}{\cos^2\theta}$

MASS MOMENT OF INERTIA ABOUT INST. CENTER

$$I_c = \bar{I} + m\left[(CD)^2 + (DG)^2\right]$$

$$I_c = \frac{1}{12}mL^2 + m\left[a^2\frac{\tan^2\theta}{\cos^2\theta} - \left(\frac{L}{2}\sin\theta - a\tan\theta\right)^2\right] \quad (1)$$

CONSERVATION OF ENERGY

$T_1 = 0$, $V_1 = 0$,

$V_2 = -Wh = -mgh$

$T_2 = \frac{1}{2}I_c\omega^2$ (SEE EQ 17.10 page 1049)

$T_1 + V_1 = T_2 + V_2$ $\quad 0 + 0 = \frac{1}{2}I_c\omega^2 - mgh$

$$\omega^2 = \frac{2mgh}{I_c} \qquad \omega = \sqrt{\frac{2mgh}{I_c}} \quad (2)$$

VELOCITY OF A: $\quad v_A = (AC)\omega$

OUTLINE OF PROGRAM

PROGRAM IN SEQUENCE, AD, DG, h, CD, AC, I_c, ω, v_A.
EVALUATE AND PRINT θ, h, ω, AND v_A FOR VALUES OF θ FROM 0 TO 50° AT 5° INTERVALS.

```
L = 800 mm        a = 200 mm        m = 5 kg
```

theta deg	h mm	omega rad / s	vA m/s
0.000	0.000	0.000	0.000
5.000	17.365	1.911	0.385
10.000	34.194	2.680	0.553
15.000	49.938	3.235	0.693
20.000	64.014	3.648	0.826
25.000	75.786	3.934	0.958
30.000	84.530	4.079	1.088
35.000	89.389	4.051	1.208
40.000	89.295	3.811	1.299
45.000	82.843	3.325	1.330
50.000	68.067	2.592	1.255

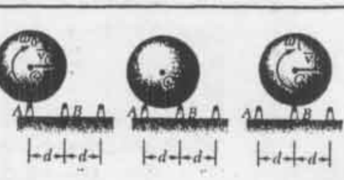
+++

Find theta for max omega

theta deg	h mm	$omega_{MAX}$ rad / s
31.810	86.788	4.0907731056
31.820	86.799	4.0907735825
31.830	86.810	4.0907735825
31.840	86.821	4.0907731056

+++

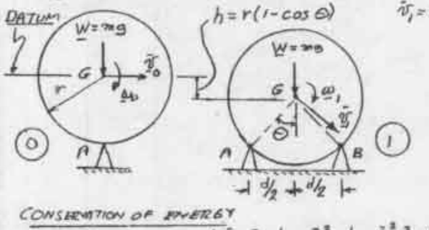
17.C3



GIVEN: 10-in. RADIUS SPHERE ROLLS OVER BARS WITHOUT SLIPPING. KNOWING THAT $\omega_0 = 1.5$ rad/s ↻ AND ASSUMING PERFECTLY PLASTIC IMPACTS, FOR $d = 1$ in. TO 6 in. USING 0.5-in. INCREMENTS,

FIND: (a) ω_1 AS G PASSES DIRECTLY ABOVE B
(b) NUMBER OF BARS THE SPHERE WILL ROLL OVER AFTER LEAVING BAR A

$\bar{v}_0 = r\omega_0$ $\quad \theta = \sin^{-1}\frac{d}{2r}$ $\quad h = r(1-\cos\theta)$ $\quad \bar{v}_1 = r\omega_1$ $\quad \bar{I} = m\bar{k}^2$ $\quad \bar{k}^2 = \frac{2}{5}$



CONSERVATION OF ENERGY

$V_0 = 0$; $\quad T_0 = \frac{1}{2}\bar{I}\omega_0^2 + \frac{1}{2}m\bar{v}_0^2 = \frac{1}{2}m\bar{k}^2\omega_0^2 + \frac{1}{2}mr^2\omega_0^2 = \frac{1}{2}m(\bar{k}^2 + r^2)\omega_0^2$

$V_1 = -mgh$; $\quad T_1 = \frac{1}{2}m(\bar{k}^2 + r^2)\omega_1^2$

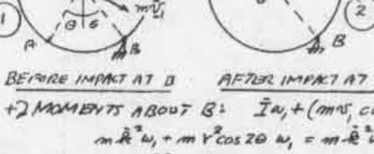
$T_0 + V_0 = T_1 + V_1$: $\quad \frac{1}{2}m(\bar{k}^2 + r^2)\omega_0^2 = \frac{1}{2}m(\bar{k}^2 + r^2)\omega_1^2 - mgh$

$$\omega_1^2 = \omega_0^2 + \frac{2gh}{\bar{k}^2 + r^2} \quad (1)$$

AFTER IMPACT AT B: SPHERE ROTATES ABOUT B

CONSERVATION OF ANG. MOMENTUM ABOUT B

$\bar{v}_1 = r\omega_1$
$\bar{v}_2 = r\omega_2$



BEFORE IMPACT AT B $\qquad$ AFTER IMPACT AT B

+↺ MOMENTS ABOUT B: $\quad \bar{I}\omega_1 + (m\bar{v}_1\cos 2\theta)r = \bar{I}\omega_2 + m\bar{v}_2 r$

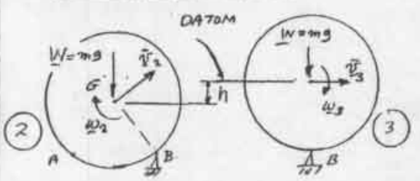
$m\bar{k}^2\omega_1 + mr^2\cos 2\theta\,\omega_1 = m\bar{k}^2\omega_2 + mr^2\omega_2$

$$\omega_2 = \frac{\bar{k}^2 + r^2\cos 2\theta}{\bar{k}^2 + r^2}\omega_1 \quad (2)$$

SPHERE ROTATES ABOUT B UNTIL G IS ABOVE B

CONSERVATION OF ENERGY

$\bar{v}_2 = r\omega_2$
$\bar{v}_3 = r\omega_3$



$V_2 = -mgh$; $\quad T_2 = \frac{1}{2}m(\bar{k}^2 + r^2)\omega_2^2$

$V_3 = 0$; $\quad T_3 = \frac{1}{2}m(\bar{k}^2 + r^2)\omega_3^2$

$T_2 + V_2 = T_3 + V_3$:

$\frac{1}{2}m(\bar{k}^2 + r^2)\omega_2 - mgh = \frac{1}{2}m(\bar{k}^2 + r^2)\omega_3^2$

$$\omega_3^2 = \omega_2^2 - \frac{2gh}{\bar{k}^2 + r^2} \quad (3)$$

(CONTINUED)

17.C3 continued

WE HAVE FOUND:
$$\omega_1^2 = \omega_0^2 + \frac{2gh}{\bar{k}^2+r^2} \qquad (1)$$
$$\omega_2 = \frac{\bar{k}^2 + r^2\cos 2\theta}{\bar{k}^2+r^2}\,\omega_1 \qquad (2)$$
$$\omega_3^2 = \omega_2^2 - \frac{2gh}{\bar{k}^2+r^2} \qquad (3)$$

ω_3 IS ANGULAR VELOCITY OF SPHERE AS G PASSES OVER B. (THIS IS SHOWN AS ω_1 IN PROBLEM FIGURE)

AS SPHERE ROLLS FROM THE $(n-1)^{th}$ BAR TO THE n^{th} BAR.

OUTLINE OF PROGRAM

ENTER DATA: $r = \frac{10}{12}$ ft, $\omega_0 = 1.5$ rad/s, $\bar{k}^2 = 0.4$

FOR $d = \frac{1}{12}$ ft TO $\frac{6}{12}$ ft; INCREMENT $\frac{0.5}{12}$ ft
- $\omega_n = \omega_0$
- FOR $n = 1$ TO 1000; INCREMENT = 1
  - $\theta = \sin^{-1}(d/2r)$
  - $h = r(1-\cos\theta)$
  - $\omega_1 = \left\{\omega_m^2 + 2gh/(\bar{k}^2+r^2)\right\}^{1/2}$
  - $\omega_2 = \left\{(\bar{k}^2 + r^2\cos 2\theta)/(\bar{k}^2+r^2)\right\}\omega_1$
  - $\omega_3 = \left\{\omega_1^2 - 2gh/(\bar{k}^2+r^2)\right\}^{1/2}$
  - IF $n=1$ PRINT ω_3 (G IS ABOVE B)
  - IF $\omega_3 < 0 \rightarrow$ STOP.
  - (n IS NUMBER OF BARS ROLLED OVER)
- NEXT

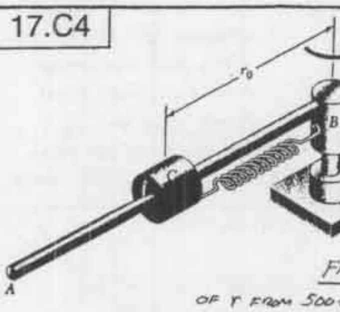
NEXT

r = 10.000 in. omega0 = 1.500 rad/s

Distance between bars in.	omega when G is over B rad/s	Number of bars sphere rolls over
1.0	1.494	491
1.5	1.487	169
2.0	1.476	76
2.5	1.460	40
3.0	1.438	23
3.5	1.409	14
4.0	1.370	9
4.5	1.319	6
5.0	1.252	4
5.5	1.164	3
6.0	1.047	2

NOTE: FOR $d = 7$ in. SPHERE FAILS TO REACH A POSITION WITH G ABOVE B

17.C4



GIVEN: $m_C = 2.5$ kg

SPRING: $k = 750$ N/m

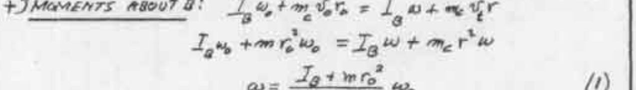
UNSTRETCHED LENGTH: $r_0 = 500$ mm

ROD AND HUB: $I_B = 0.3$ kg·m^2

INITIALLY: $r_0 = 500$ mm, $\omega_0 = 10$ rad/s

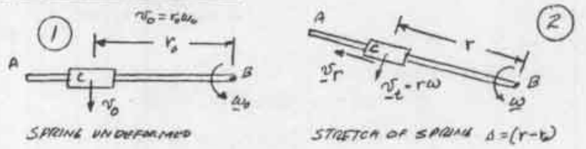
FIND: ω AND $v_{C/B}$ FOR VALUES OF r FROM 500 mm TO 700 mm AT 25-mm INCREMENTS. ALSO FIND r_{max}.

CONSERVATION OF ANGULAR MOMENTUM ABOUT B



+↺ MOMENTS ABOUT B: $I_B\omega_0 + m_C v_0 r_0 = I_B\omega + m_C v_t r$
$$I_B\omega_0 + m r_0^2\omega_0 = I_B\omega + m_C r^2\omega$$
$$\omega = \frac{I_B + m r_0^2}{I_B + m r^2}\,\omega_0 \qquad (1)$$

CONSERVATION OF ENERGY



$T_1 = \frac{1}{2}I_B\omega_0^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}I_B\omega_0^2 + \frac{1}{2}mr_0^2\omega_0^2 = \frac{1}{2}(I_B + mr_0^2)\omega_0^2 \qquad V_1 = 0$

$T_2 = \frac{1}{2}I_B\omega^2 + \frac{1}{2}mv_t^2 + \frac{1}{2}mv_r^2 + \frac{1}{2}k\Delta^2$

$\quad = \frac{1}{2}I_B\omega^2 + \frac{1}{2}mr^2\omega^2 + \frac{1}{2}mv_r^2$

$T_2 = \frac{1}{2}(I_B + mr^2)\omega^2 + \frac{1}{2}mv_r^2 \qquad V_2 = \frac{1}{2}k(r-r_0)^2$

$T_1 + V_1 = T_2 + V_2$: $\frac{1}{2}(I_B + mr_0^2)\omega_0^2 = \frac{1}{2}(I_B + mr^2)\omega^2 + \frac{1}{2}mv_r^2 + \frac{1}{2}k(r-r_0)^2$

$$v_r = \left\{\frac{1}{m}\left[(I_B + mr_0^2)\omega_0^2 - (I_B + mr^2)\omega^2 - k(r-r_0)^2\right]\right\}^{1/2} \qquad (2)$$

OUTLINE OF PROGRAM:

ENTER DATA: $m = 2.5$ kg, $I_B = 0.3$ kg·m^2, $r_0 = 0.5$ m, $k = 750$ N/m AND $\omega_0 = 10$ rad/s

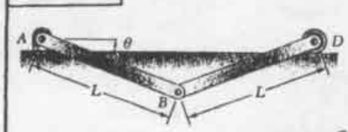
PROGRAM EQ(1) AND THEN EQ(2). EVALUATE AND PRINT ω AND v_t FOR VALUES OF r FROM 0.5 m TO 0.7 m AT 0.025 m INCREMENTS. THEN SEEK r_{max} WHERE $v_t = 0$

r mm	omega rad/s	v radial m/s
500.00	10.000	0.000
525.00	9.352	1.486
550.00	8.757	1.962
575.00	8.211	2.221
600.00	7.708	2.341
625.00	7.246	2.346
650.00	6.820	2.239
675.00	6.428	2.007
700.00	6.066	1.599

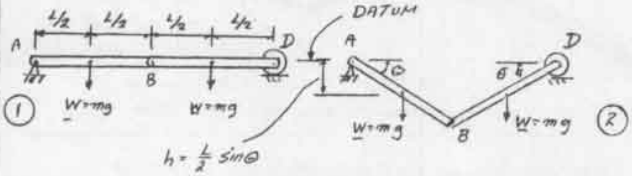
Find r maximum (where vr = 0)

r mm	omega rad/s	[v radial]^2
731.75	5.645	0.0014211
731.76	5.645	0.0004968
731.77	5.645	-0.0004275
731.78	5.645	-0.0013555

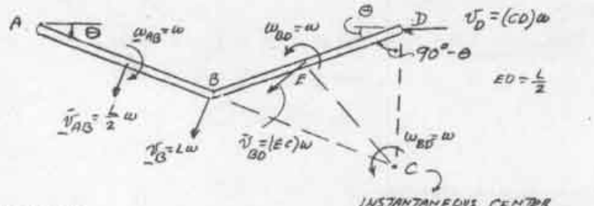
17.C5



GIVEN: $L = 30$ in.
BARS ARE RELEASED FROM REST WHEN $\theta = 0$.
FIND: ω_{AB} AND v_D FOR VALUES OF θ FROM 0 TO 90° USING 10° INCREMENTS



KINEMATICS OF POSITION 2:



$AB = BC = L$

IN ΔACD: $CD = 2L \sin\theta$ (1)

IN ΔCED: (LAW OF COSINES)

$$(EC)^2 = (CD)^2 + (L/2)^2 - 2(CD)(L/2)\cos(90° - \theta)$$

$$EC = \left[(CD)^2 + (L/2)^2 - 2(CD)(L/2)\cos(90° - \theta)\right]^{1/2} \qquad (2)$$

CONSERVATION OF ENERGY

$$V_1 = 0 \qquad T_1 = 0$$

$$V_2 = -2mg\left(\tfrac{L}{2}\sin\theta\right) = -mgL\sin\theta$$

$$T_2 = \tfrac{1}{2}m\bar{v}_{AB}^2 + \tfrac{1}{2}\bar{I}\omega_{AB}^2 + \tfrac{1}{2}m\bar{v}_{BD}^2 + \tfrac{1}{2}\bar{I}\omega_{BD}^2$$

$$= \tfrac{1}{2}m\left(\tfrac{L}{2}\omega\right)^2 + \tfrac{1}{2}\left(\tfrac{1}{12}mL^2\right)\omega^2 + \tfrac{1}{2}m(EC)^2\omega^2 + \tfrac{1}{2}\left(\tfrac{1}{12}mL^2\right)\omega^2$$

$$T_2 = \left[\tfrac{1}{8} + \tfrac{1}{24} + \tfrac{1}{2}\left(\tfrac{EC}{L}\right)^2 + \tfrac{1}{24}\right]mL^2\omega^2$$

$$T_2 = \tfrac{1}{24}\left[5 + 12\left(\tfrac{EC}{L}\right)^2\right]mL^2\omega^2$$

$T_1 + V_1 = T_2 + V_2$

$$0 + 0 = -mgL\sin\theta + \tfrac{1}{24}\left[5 + 12\left(\tfrac{EC}{L}\right)^2\right]mL^2\omega^2$$

$$\omega = \left[\frac{24g}{L}\cdot\frac{\sin\theta}{5 + \left(\frac{EC}{L}\right)^2}\right]^{1/2} \qquad (3)$$

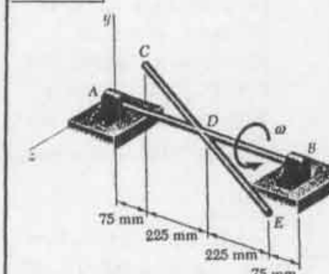
VELOCITY OF D: $v_D = (CD)\,\omega$ (4)

OUTLINE OF PROGRAM:

ENTER $L = 30$ in. $= 2.5$ ft, $g = 32.2$ ft/s²
PROGRAM, IN SEQUENCE, EQS. (1), (2), (3), AND (4)
EVALUATE AND PRINT ω AND v_D FOR VALUES OF θ FROM 0 TO 90° USING 10° INCREMENTS.

theta deg.	omega rad/s	vD ft/s
0	0.0000	0.0000
10	2.4806	2.1537
20	3.1277	5.3487
30	3.3226	8.3066
40	3.3302	10.7031
50	3.2746	12.5423
60	3.2088	13.8945
70	3.1544	14.8210
80	3.1198	15.3622
90	3.1081	15.5403

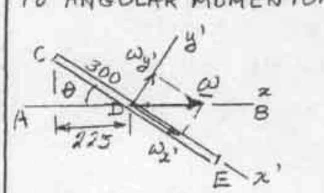
18.1



GIVEN:
TWO UNIFORM RODS AB AND CE ARE WELDED AT MIDPOINTS D.
MASS OF EACH ROD = 1.5 kg
LENGTH = 600 mm
ASSEMBLY HAS CONSTANT ANG. VEL $\omega = 12$ rad/s.

FIND:
ANG. MOMENTUM $\underline{H}_D$.

SINCE ROD AB HAS MOM. OF INERTIA ≈ 0 ABOUT AXIS OF ROTATION, ONLY ROD CE CONTRIBUTES TO ANGULAR MOMENTUM.



SINCE CD = 300 mm, $\cos\theta = \frac{225}{300}$ $\theta = 41.41°$

USING THE PRINCIPAL CENTROIDAL AXES $x'y'z$, WE HAVE

$\omega_{x'} = \omega\cos\theta$ $\bar{I}_{x'} = 0$

$\omega_{y'} = \omega\sin\theta$ $\bar{I}_{y'} = \frac{1}{12}m\ell^2$

$\omega_z = 0$ $\bar{I}_z = \frac{1}{12}m\ell^2$

EQS. (18.10):

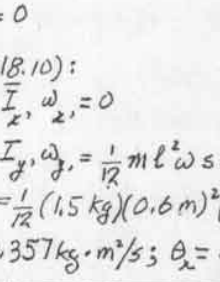
$H_{x'} = \bar{I}_{x'}\omega_{x'} = 0$ $H_z = \bar{I}_z\omega_z = 0$

$H_{y'} = \bar{I}_{y'}\omega_{y'} = \frac{1}{12}m\ell^2\omega\sin\theta$

$= \frac{1}{12}(1.5\text{ kg})(0.6\text{ m})^2(12\text{ rad/s})\sin 41.41° = 0.357$

$\underline{H}_D = 0.357\text{ kg}\cdot\text{m}^2/\text{s}$; $\theta_x = 48.6°$, $\theta_y = 41.4°$, $\theta_z = 90°$ ◄

18.2



GIVEN:
THIN, HOMOGENEOUS DISK OF MASS m AND RADIUS r SPINS AT CONSTANT RATE ω_1.
FORK-ENDED ROD SPINS AT CONSTANT RATE ω_2.

FIND:
ANGULAR MOMENTUM $\underline{H}_G$ OF DISK.

SINCE THE x, y, z AXES ARE PRINCIPAL CENTROIDAL AXES, WE CAN USE EQS. (18.10) WITH

$\bar{I}_x = \bar{I}_y = \frac{1}{4}mr^2$, $\bar{I}_z = \frac{1}{2}mr^2$

$\omega_x = 0$, $\omega_y = \omega_2$, $\omega_z = \omega_1$

AND WRITE

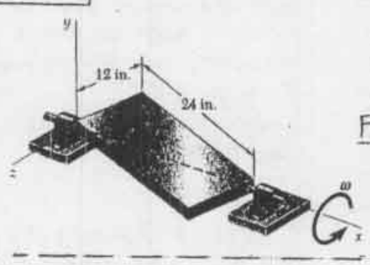
$H_x = \bar{I}_x\omega_x = 0$

$H_y = \bar{I}_y\omega_y = \frac{1}{4}mr^2\omega_2$

$H_z = \bar{I}_z\omega_z = \frac{1}{2}mr^2\omega_1$

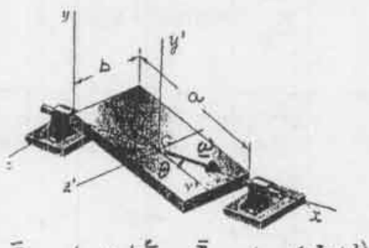
$\underline{H}_G = \frac{1}{4}mr^2(\omega_2\underline{j} + 2\omega_1\underline{k})$ ◄

18.3



GIVEN:
RECTANGULAR PLATE SHOWN WEIGHS 18 lb AND ROTATES WITH CONSTANT $\omega = 5$ rad/s

FIND:
ANGULAR MOMENTUM $\underline{H}_G$ ABOUT MASS CENTER G.



WE USE THE PRINCIPAL CENTROIDAL AXES $Gx'y'z'$

WE HAVE

$\omega_{x'} = \omega\cos\theta$

$\omega_{y'} = 0$

$\omega_{z'} = -\omega\sin\theta$

$\bar{I}_{x'} = \frac{1}{12}mb^2$, $\bar{I}_{y'} = \frac{1}{12}m(a^2+b^2)$, $\bar{I}_{z'} = \frac{1}{12}ma^2$

USING EQS. (18.10):

$H_{x'} = \bar{I}_{x'}\omega_{x'} = \frac{1}{12}mb^2\omega\cos\theta$

$H_{y'} = \bar{I}_{y'}\omega_{y'} = 0$

$H_{z'} = \bar{I}_{z'}\omega_{z'} = -\frac{1}{12}ma^2\omega\sin\theta$

WE HAVE

$\underline{H}_G = H_{x'}\underline{i}' + H_{y'}\underline{j}' + H_{z'}\underline{k}'$

WHERE $\underline{i}', \underline{j}', \underline{k}'$ ARE THE UNIT VECTORS ALONG THE x', y', z' AXES.

$\underline{H}_G = \frac{1}{12}mb^2\omega\cos\theta\,\underline{i}' - \frac{1}{12}ma^2\omega\sin\theta\,\underline{k}'$

TO RETURN TO THE ORIGINAL x, y, z AXES, WE NOTE THAT

$\underline{i}' = \underline{i}\cos\theta + \underline{k}\sin\theta$ $\underline{k}' = -\underline{i}\sin\theta + \underline{k}\cos\theta$

THEREFORE

$\underline{H}_G = \frac{1}{12}mb^2\omega(\cos^2\theta\,\underline{i} + \cos\theta\sin\theta\,\underline{k}) +$
$+ \frac{1}{12}ma^2\omega(\sin^2\theta\,\underline{i} - \sin\theta\cos\theta\,\underline{k})$

$\underline{H}_G = \frac{1}{12}m\omega[(a^2\sin^2\theta + b^2\cos^2\theta)\underline{i} - (a^2-b^2)\sin\theta\cos\theta\,\underline{k}]$

GIVEN DATA:

$m = (18\text{ lb})/(32.2\text{ ft/s}^2) = 0.55901\text{ lb}\cdot\text{s}^2/\text{ft}$

$a = 24\text{ in.} = 2\text{ ft}$ $b = 12\text{ in.} = 1\text{ ft}$

$\tan\theta = \frac{b}{a} = 0.5$ $\theta = 26.565°$

THUS:

$\underline{H}_G = \frac{1}{12}(0.55901\text{ lb}\cdot\text{s}^2/\text{ft})\omega[(4\sin^2 26.565° + \cos^2 26.565°)\underline{i} - (4-1)\sin 26.565°\cos 26.565°\,\underline{k}](\text{ft}^2)$

$\underline{H}_G = (0.046584\text{ lb}\cdot\text{s}^2/\text{ft})\omega(1.600\,\underline{i} - 1.200\,\underline{k})(\text{ft}^2)$

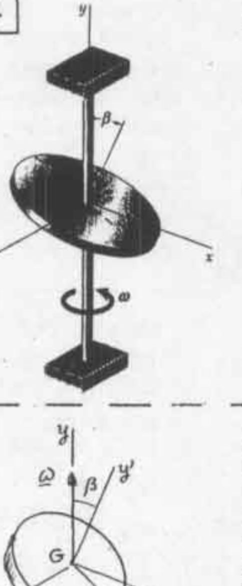
$\underline{H}_G = [(0.074534\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\underline{i} - (0.055901\text{ lb}\cdot\text{ft}\cdot\text{s}^2)\underline{k}]\omega$ (1)

LETTING $\omega = 5$ rad/s,

$\underline{H}_G = (0.3727\text{ lb}\cdot\text{ft}\cdot\text{s})\underline{i} - (0.2795\text{ lb}\cdot\text{ft}\cdot\text{s})\underline{k}$ (2)

$\underline{H}_G = (0.373\text{ lb}\cdot\text{ft}\cdot\text{s})\underline{i} - (0.280\text{ lb}\cdot\text{ft}\cdot\text{s})\underline{k}$ ◄

18.4



GIVEN:
HOMOGENEOUS DISK OF MASS m AND RADIUS r MOUNTED ON SHAFT AB WITH $\beta = 25°$. SHAFT ROTATES WITH CONSTANT $\underline{\omega}$.

FIND:
ANGLE θ FORMED BY AB AND ANG. MOMENTUM $\underline{H}_G$ OF DISK ABOUT G.

WE USE THE PRINCIPAL CENTROIDAL AXES $Gx'y'z'$.
WE HAVE:

$\bar{I}_{x'} = \bar{I}_{z'} = \frac{1}{4} m r^2$

$\bar{I}_{y'} = \frac{1}{2} m r^2$

$\omega_{x'} = -\omega \sin\beta$

$\omega_{y'} = \omega \cos\beta$

$\omega_{z'} = 0$

USING EQS. (18.10):

$H_{x'} = \bar{I}_{x'} \omega_{x'} = -\frac{1}{4} m r^2 \omega \sin\beta$

$H_{y'} = \bar{I}_{y'} \omega_{y'} = \frac{1}{2} m r^2 \omega \cos\beta$

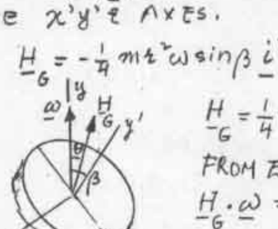
$H_{z'} = \bar{I}_{z'} \omega_{z'} = 0$

WE HAVE

$\underline{H}_G = H_{x'} \underline{i}' + H_{y'} \underline{j}' + H_{z'} \underline{k}$

WHERE $\underline{i}', \underline{j}', \underline{k}$ ARE THE UNIT VECTORS ALONG THE $x'y'z$ AXES.

$$\underline{H}_G = -\frac{1}{4} m r^2 \omega \sin\beta\, \underline{i}' + \frac{1}{2} m r^2 \omega \cos\beta\, \underline{j}' \qquad (1)$$



$\underline{H}_G = \frac{1}{4} m r^2 \omega \left(-\sin\beta\, \underline{i}' + 2\cos\beta\, \underline{j}'\right)$

FROM EQ. (3.24) WE HAVE

$\underline{H}_G \cdot \underline{\omega} = |\underline{H}_G| \omega \cos\theta$

$$\cos\theta = \frac{\underline{H}_G \cdot \underline{\omega}}{|\underline{H}_G| \omega} \qquad (2)$$

BUT $\underline{H}_G \cdot \underline{\omega} = \frac{1}{4} m r^2 \omega \left(-\sin\beta\, \underline{i}' + 2\cos\beta\, \underline{j}'\right) \cdot \omega \underline{j}$

OR, OBSERVING THAT $\underline{i}' \cdot \underline{j} = -\sin\beta$ AND $\underline{j}' \cdot \underline{j} = \cos\beta$,

$$\underline{H}_G \cdot \underline{\omega} = \frac{1}{4} m r^2 \omega^2 (\sin^2\beta + 2\cos^2\beta) = \frac{1}{4} m r^2 \omega^2 (1 + \cos^2\beta) \qquad (3)$$

ALSO

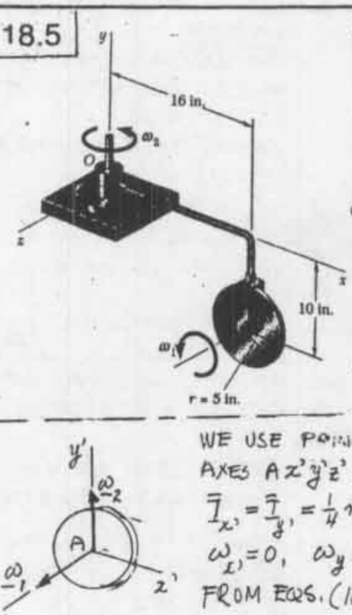
$$|\underline{H}_G| \omega = \frac{1}{4} m r^2 \omega^2 \sqrt{\sin^2\beta + 4\cos^2\beta} = \frac{1}{4} m r^2 \omega^2 \sqrt{1 + 3\cos^2\beta} \qquad (4)$$

SUBSTITUTING FROM (3) AND (4) INTO (2),

$$\cos\theta = \frac{1 + \cos^2\beta}{\sqrt{1 + 3\cos^2\beta}}$$

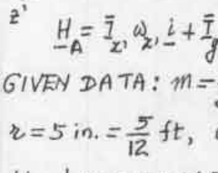
FOR $\beta = 25°$, $\cos\theta = 0.9786$ $\qquad \theta = 11.88°$ ◀

18.5



GIVEN:
HOMOGENEOUS DISK OF WEIGHT $W = 8$ lb ROTATES AT CONSTANT RATE $\omega_1 = 12$ rad/s. ARM OA ROTATES AT CONSTANT RATE $\omega_2 = 4$ rad/s

FIND:
ANGULAR MOMENTUM $\underline{H}_A$ OF DISK ABOUT ITS CENTER A.



WE USE PRINCIPAL CENTROIDAL AXES $Ax'y'z'$. WE HAVE

$\bar{I}_{x'} = \bar{I}_{y'} = \frac{1}{4} m r^2 \qquad \bar{I}_{z'} = \frac{1}{2} m r^2$

$\omega_{x'} = 0, \quad \omega_{y'} = \omega_2, \quad \omega_{z'} = \omega_1$

FROM EQS. (18.10):

$$\underline{H}_A = \bar{I}_{x'} \omega_{x'} \underline{i} + \bar{I}_{y'} \omega_{y'} \underline{j} + \bar{I}_{z'} \omega_{z'} \underline{k} = \frac{1}{4} m r^2 (\omega_2 \underline{j} + 2\omega_1 \underline{k})$$

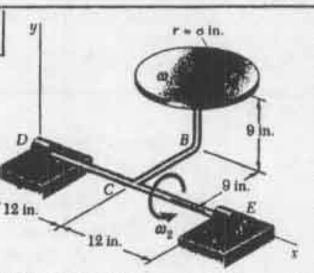
GIVEN DATA: $m = \frac{W}{g} = \frac{8 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.24845$ lb·s²/ft

$r = 5$ in. $= \frac{5}{12}$ ft, $\omega_1 = 12$ rad/s, $\omega_2 = 4$ rad/s

$$\underline{H}_A = \frac{1}{4}(0.24845)\left(\frac{5}{12}\right)^2 \left[4\underline{j} + 2(12)\underline{k}\right]$$
$$= (0.043133 \text{ lb·ft·s})\underline{j} + (0.25880 \text{ lb·ft·s})\underline{k}$$

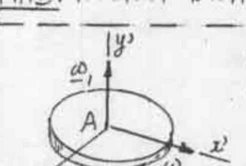
$\underline{H}_A = (0.0431 \text{ lb·ft·s})\underline{j} + (0.259 \text{ lb·ft·s})\underline{k}$ ◀

18.6



GIVEN:
HOMOGENEOUS DISK OF WEIGHT $W = 6$ lb ROTATES AT CONSTANT RATE $\omega_1 = 16$ rad/s. SHAFT DCE ROTATES AT CONSTANT RATE $\omega_2 = 8$ rad/s.

FIND: ANG. MOMENTUM $\underline{H}_A$ OF DISK ABOUT ITS CENTER A.



WE USE PRINCIPAL CENTROIDAL AXES $Ax'y'z'$. WE HAVE

$\bar{I}_{x'} = \bar{I}_{z'} = \frac{1}{4} m r^2, \quad \bar{I}_{y'} = \frac{1}{2} m r^2$

$\omega_{x'} = \omega_2, \quad \omega_{y'} = \omega_1, \quad \omega_{z'} = 0$

FROM EQS. (18.10):

$$\underline{H}_A = \bar{I}_{x'} \omega_{x'} \underline{i} + \bar{I}_{y'} \omega_{y'} \underline{j} + \bar{I}_{z'} \omega_{z'} \underline{k} = \frac{1}{4} m r^2 (\omega_2 \underline{i} + 2\omega_1 \underline{j})$$

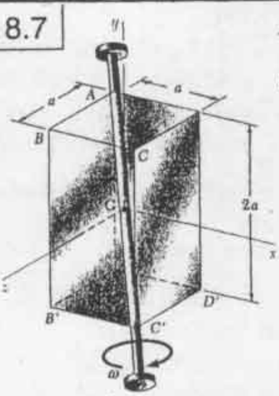
GIVEN DATA: $m = \frac{W}{g} = \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.186335$ lb·s²/ft

$r = 8$ in. $= \frac{2}{3}$ ft, $\omega_1 = 16$ rad/s, $\omega_2 = 8$ rad/s

$$\underline{H}_A = \frac{1}{4}(0.186335)\left(\frac{2}{3}\right)^2 \left[8\underline{i} + 2(16)\underline{j}\right]$$
$$= (0.16563 \text{ lb·ft·s})\underline{i} + (0.66252 \text{ lb·ft·s})\underline{j}$$

$\underline{H}_A = (0.1656 \text{ lb·ft·s})\underline{i} + (0.663 \text{ lb·ft·s})\underline{j}$ ◀

18.7



GIVEN:
SOLID RECTANGULAR PARALLELEPIPED SHOWN, OF MASS m, IT ROTATES ABOUT ITS DIAGONAL AC' AT CONSTANT RATE ω.

FIND:
(a) MAGNITUDE OF ANGULAR MOMENTUM $\underline{H}_G$.
(b) ANGLE THAT $\underline{H}_G$ FORMS WITH AC'.

WE DENOTE BY $\bar{I}_x$, $\bar{I}_y$, $\bar{I}_z$ THE PRINCIPAL CENTROIDAL MOMENTS OF INERTIA. WE HAVE

$$\underline{\omega} = \omega\,\frac{-a\underline{i}+2a\underline{j}-a\underline{k}}{a\sqrt{6}} = \frac{\omega}{\sqrt{6}}(-\underline{i}+2\underline{j}-\underline{k}) \qquad (1)$$

$$\underline{H}_G = \bar{I}_x\omega_x\underline{i}+\bar{I}_y\omega_y\underline{j}+\bar{I}_z\omega_z\underline{k} = \frac{\omega}{\sqrt{6}}(-\bar{I}_x\underline{i}+2\bar{I}_y\underline{j}-\bar{I}_z\underline{k}) \qquad (2)$$

COMPUTATION OF THE MOMENTS OF INERTIA:

$$\bar{I}_x = \bar{I}_z = \frac{1}{12}m(a^2+4a^2) = \frac{5}{12}ma^2$$

$$\bar{I}_y = \frac{1}{12}m(a^2+a^2) = \frac{1}{6}ma^2$$

SUBSTITUTE INTO (2):

$$\underline{H}_G = \frac{\omega}{\sqrt{6}}\left(-\frac{5}{12}ma^2\underline{i}+\frac{2}{6}ma^2\underline{j}-\frac{5}{12}ma^2\underline{k}\right)$$

$$\underline{H}_G = \frac{ma^2\omega}{12\sqrt{6}}(-5\underline{i}+4\underline{j}-5\underline{k}) \qquad (3)$$

(a) $$|\underline{H}_G| = \frac{ma^2\omega}{12\sqrt{6}}\sqrt{25+16+25} = ma^2\omega\frac{\sqrt{11}}{12} \qquad (4)$$

$$|\underline{H}_G| = 0.276\,ma^2\omega$$ ◄

(b) FROM EQ. (3.24) WE HAVE

$$\underline{H}_G\cdot\underline{\omega} = |\underline{H}_G|\,\omega\cos\theta$$

$$\cos\theta = \frac{\underline{H}_G\cdot\underline{\omega}}{|\underline{H}_G|\,\omega} \qquad (5)$$

RECALLING (1) AND (3):

$$\underline{H}_G\cdot\underline{\omega} = \frac{ma^2\omega}{12\sqrt{6}}(-5\underline{i}+4\underline{j}-5\underline{k})\cdot\frac{\omega}{\sqrt{6}}(-\underline{i}+2\underline{j}-\underline{k})$$

$$= \frac{ma^2\omega^2}{72}(5+8+5) = \frac{1}{4}ma^2\omega^2 \qquad (6)$$

RECALLING (4): $|\underline{H}_G|\,\omega = \frac{\sqrt{11}}{12}ma^2\omega^2 \qquad (7)$

SUBSTITUTING FROM (6) AND (7) INTO (5):

$$\cos\theta = \frac{1/4}{\sqrt{11}/12} = \frac{3}{\sqrt{11}} = 0.90453$$

$$\theta = 25.239^\circ$$

$$\theta = 25.2^\circ$$ ◄

18.8

GIVEN: SOLID PARALLELEPIPED OF PROB. 18.7 IS REPLACED BY HOLLOW ONE MADE OF 6 THIN METAL PLATES.

FIND: (a) MAGNITUDE OF ANG. MOMENTUM $\underline{H}_G$.
(b) ANGLE THAT $\underline{H}_G$ FORMS WITH AC'.

WE DENOTE BY $\bar{I}_x$, $\bar{I}_y$, $\bar{I}_z$ THE PRINCIPAL CENTROIDAL MOMENTS OF INERTIA. WE HAVE

$$\underline{\omega} = \omega\,\frac{-a\underline{i}+2a\underline{j}-a\underline{k}}{a\sqrt{6}} = \frac{\omega}{\sqrt{6}}(-\underline{i}+2\underline{j}-\underline{k}) \qquad (1)$$

$$\underline{H}_G = \bar{I}_x\omega_x\underline{i}+\bar{I}_y\omega_y\underline{j}+\bar{I}_z\omega_z\underline{k} = \frac{\omega}{\sqrt{6}}(-\bar{I}_x\underline{i}+2\bar{I}_y\underline{j}-\bar{I}_z\underline{k}) \qquad (2)$$

COMPUTATION OF MOMENTS OF INERTIA:

EACH OF THE TWO SQUARE PLATES HAS A MASS EQUAL TO $m/10$ AND EACH OF THE RECTANGULAR PLATES HAS A MASS EQUAL TO $m/5$. USING THE PARALLEL-AXIS THEOREM WHEN NEEDED, WE OBTAIN:

	$\bar{I}_x$	$\bar{I}_y$	$\bar{I}_z$
SQUARE PLATES	$\frac{2m}{10}\left(\frac{a^2}{12}+a^2\right) = \frac{13}{60}ma^2$	$\frac{2m}{10}\frac{a^2}{6} = \frac{ma^2}{30}$	$\frac{13}{60}ma^2$
RECTANG. PLATES // yz PLANE	$\frac{2m}{5}\frac{a^2+4a^2}{12} = \frac{1}{6}ma^2$	$\frac{2m}{5}\left[\frac{a^2}{12}+\left(\frac{a}{2}\right)^2\right] = \frac{2}{15}ma^2$	$\frac{2m}{5}\left[\frac{a^2}{3}+\left(\frac{a}{2}\right)^2\right] = \frac{7}{30}ma^2$
REC. PL. // xy PLANE	$\frac{7}{30}ma^2$	$\frac{2}{15}ma^2$	$\frac{1}{6}ma^2$
SUMS	$\frac{37}{60}ma^2$	$\frac{9}{30}ma^2$	$\frac{37}{60}ma^2$

SUBSTITUTE THE VALUES OBTAINED FOR $\bar{I}_x$, $\bar{I}_y$, $\bar{I}_z$ INTO (2):

$$\underline{H}_G = \frac{ma^2\omega}{60\sqrt{6}}(-37\underline{i}+36\underline{j}-37\underline{k}) \qquad (3)$$

(a) $$|\underline{H}_G| = \frac{ma^2\omega}{60\sqrt{6}}\sqrt{(37)^2+(36)^2+(37)^2} = \frac{ma^2\omega}{60\sqrt{6}}\sqrt{4034} \qquad (4)$$

$$|\underline{H}_G| = 0.432157\,ma^2\omega, \quad |\underline{H}_G| = 0.432\,ma^2\omega$$ ◄

(b) WE RECALL EQ. (5) IN SOLUTION OF PROB. 18.7:

$$\cos\theta = \frac{\underline{H}_G\cdot\underline{\omega}}{|\underline{H}_G|\,\omega} \qquad (5)$$

RECALLING (1) AND (3) ABOVE:

$$\underline{H}_G\cdot\underline{\omega} = \frac{ma^2\omega}{60\sqrt{6}}(-37\underline{i}+36\underline{j}-37\underline{k})\cdot\frac{\omega}{\sqrt{6}}(-\underline{i}+2\underline{j}-\underline{k})$$

$$= \frac{ma^2\omega^2}{360}(37+72+37) = \frac{146}{360}ma^2\omega^2 \qquad (6)$$

RECALLING (4) ABOVE:

$$|\underline{H}_G|\,\omega = \frac{\sqrt{4034}}{60\sqrt{6}}ma^2\omega^2 \qquad (7)$$

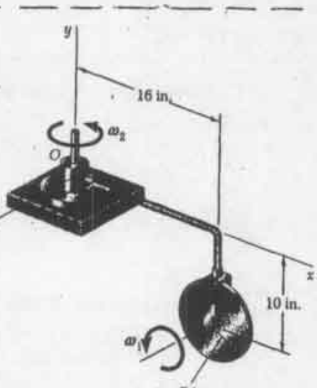
SUBSTITUTING FROM (6) AND (7) INTO (5):

$$\cos\theta = \frac{146}{360}\,\frac{60\sqrt{6}}{\sqrt{4034}} = \frac{146}{\sqrt{6\times4034}} = 0.93845$$

$$\theta = 20.208^\circ$$

$$\theta = 20.2^\circ$$ ◄

18.9 GIVEN: DISK OF PROB. 18.5 WITH $W = 8\,lb$, $\omega_1 = 12\,rad/s$, AND $\omega_2 = 4\,rad/s$.
FIND: ANGULAR MOMENTUM $\underline{H}_O$ ABOUT POINT O.



WE USE EQ. (18.11):

$$\underline{H}_O = \bar{\underline{r}} \times m\bar{\underline{v}} + \underline{H}_G \qquad (1)$$

WHERE

$$\bar{\underline{r}} = \underline{r}_A = \left(\tfrac{16}{12}\,ft\right)\underline{i} - \left(\tfrac{10}{12}\,ft\right)\underline{j}$$

$$\bar{\underline{r}} = \underline{r}_A = \left(\tfrac{4}{3}\,ft\right)\underline{i} - \left(\tfrac{5}{6}\,ft\right)\underline{j}$$

$$m = \frac{W}{g} = \frac{8\,lb}{32.2\,ft/s^2} = 0.24845\ lb\cdot s^2/ft$$

$$\bar{\underline{v}} = \underline{v}_A = \underline{\omega}_2 \times \underline{r}_A = (4\,rad/s)\underline{j} \times \left(\tfrac{4}{3}\underline{i} - \tfrac{5}{6}\underline{j}\right)$$

$$\bar{\underline{v}} = -\left(\tfrac{16}{3}\,ft/s\right)\underline{k}$$

FROM THE SOLUTION OF PROB. 18.5, WE RECALL THAT

$$\underline{H}_G = \underline{H}_A = (0.0431\,lb\cdot ft\cdot s)\underline{j} + (0.259\,lb\cdot ft\cdot s)\underline{k}$$

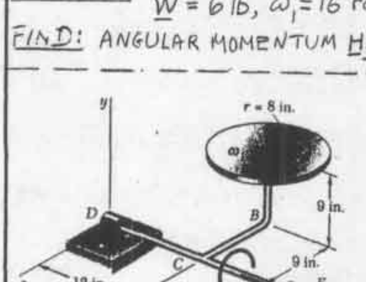
SUBSTITUTING INTO (1):

$$\underline{H}_O = \left(\tfrac{4}{3}\underline{i} - \tfrac{5}{6}\underline{j}\right) \times 0.24845\left(-\tfrac{16}{3}\underline{k}\right) + 0.0431\underline{j} + 0.259\underline{k}$$

$$= 1.7668\,\underline{j} + 1.1042\,\underline{i} + 0.0431\,\underline{j} + 0.259\,\underline{k}$$

$$\underline{H}_O = (1.104\,lb\cdot ft\cdot s)\underline{i} + (1.810\,lb\cdot ft\cdot s)\underline{j} + (0.259\,lb\cdot ft\cdot s)\underline{k}$$

18.10 GIVEN: DISK OF PROB. 18.6 WITH $W = 6\,lb$, $\omega_1 = 16\,rad/s$, AND $\omega_2 = 8\,rad/s$.
FIND: ANGULAR MOMENTUM $\underline{H}_D$ ABOUT POINT D.



WE USE EQ. (18.11) WITH RESPECT TO D:

$$\underline{H}_D = \bar{\underline{r}} \times m\bar{\underline{v}} + \underline{H}_G \qquad (1)$$

WHERE

$$\bar{\underline{r}} = \underline{r}_A = (1\,ft)\underline{i} + (0.75\,ft)\underline{j} - (0.75\,ft)\underline{k}$$

$$m = \frac{W}{g} = \frac{6\,lb}{32.2\,ft/s^2} = 0.186335\ lb\cdot s^2/ft$$

$$\bar{\underline{v}} = \underline{v}_A = \underline{\omega}_2 \times \underline{r}_A = (8\,rad/s)\underline{i} \times (\underline{i} + 0.75\underline{j} - 0.75\underline{k})$$

$$\bar{\underline{v}} = (6\,ft/s)\underline{j} + (6\,ft/s)\underline{k}$$

FROM THE SOLUTION OF PROB. 18.6, WE RECALL THAT

$$\underline{H}_G = \underline{H}_A = (0.1656\,lb\cdot ft\cdot s)\underline{i} + (0.663\,lb\cdot ft\cdot s)\underline{j}$$

SUBSTITUTING INTO (1):

$$\underline{H}_D = (\underline{i} + 0.75\underline{j} - 0.75\underline{k}) \times 0.186335(6\underline{j} + 6\underline{k}) + 0.1656\,\underline{i} + 0.663\,\underline{j}$$

$$= 1.1180\,\underline{k} - 1.1180\,\underline{j} + 0.8385\,\underline{i} + 0.8385\,\underline{i} + 0.1656\,\underline{i} + 0.663\,\underline{j}$$

$$\underline{H}_D = (1.843\,lb\cdot ft\cdot s)\underline{i} - (0.455\,lb\cdot ft\cdot s)\underline{j} + (1.118\,lb\cdot ft\cdot s)\underline{k}$$

18.11

GIVEN: PROJECTILE WITH $m = 30\,kg$, $\bar{k}_x = 60\,mm$, $\bar{k}_y = 250\,mm$; ANGLE $\theta = -5°$, ANG. MOM. $\underline{H}_G = (320\,g\cdot m^2/s)\underline{i} - (9\,g\cdot m^2/s)\underline{j}$.
RESOLVE $\underline{\omega}$ INTO COMPONENTS
(a) ALONG Gx (RATE OF SPIN)
(b) ALONG GD (RATE OF PRECESSION)

BECAUSE OF AXISYMMETRY OF PROJECTILE, THE x AND y AXES ARE PRINCIPAL CENTROIDAL AXES.

$$\bar{I}_x = m\bar{k}_x^2 = (30\,kg)(0.060\,m)^2 = 0.108\ kg\cdot m^2$$

$$\bar{I}_y = m\bar{k}_y^2 = (30\,kg)(0.250\,m)^2 = 1.875\ kg\cdot m^2$$

GIVEN: $H_x = 0.320\,kg\cdot m^2/s$, $H_y = -0.009\,kg\cdot m^2/s$

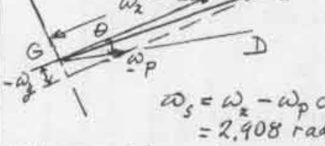
FROM EQS. (18.10):

$$\omega_x = \frac{H_x}{\bar{I}_x} = \frac{0.320\,kg\cdot m^2/s}{0.108\,kg\cdot m^2} = 2.9630\ rad/s$$

$$\omega_y = \frac{H_y}{\bar{I}_y} = \frac{-0.009\,kg\cdot m^2/s}{1.875\,kg\cdot m^2} = -0.00480\ rad/s$$

THUS: $\underline{\omega} = (2.9630\,rad/s)\underline{i} - (0.00480\,rad/s)\underline{j}$

WE MUST NOW RESOLVE ω INTO OBLIQUE COMPONENTS ALONG Gx AND GD.



WE NOTE THAT

$$-\omega_y = \omega_p \sin\theta$$

$$\omega_p = \frac{-\omega_y}{\sin\theta} = \frac{+0.00480}{\sin 5°}$$

$$\omega_p = 0.055074\ rad/s$$

$$\omega_s = \omega_x - \omega_p\cos\theta = 2.9630 - 0.055074\cos 5° = 2.908\ rad/s$$

ANSWERS: (a) $\omega_s = 2.91\,rad/s$. (b) $\omega_p = 0.0551\,rad/s$

18.12 GIVEN: PROJECTILE OF PROB. 18.11. ADDITIONAL DATA: $\bar{v} = 650\,m/s$.
FIND: ANG. MOM. $\underline{H}_A$. (RESOLVE INTO x, y, z COMP.)

RESOLVE $\bar{\underline{v}}$ INTO RECTANG. COMP. ALONG x AND y AXES.

$$\bar{\underline{v}} = (650\,m/s)(\cos 5°\,\underline{i} - \sin 5°\,\underline{j}) = (647.53\,m/s)\underline{i} - (56.65\,m/s)\underline{j}$$

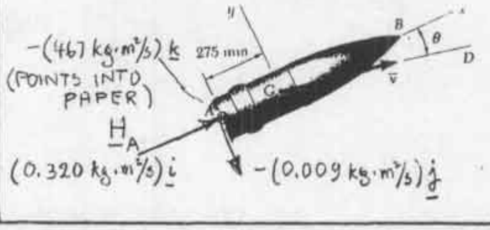
USING EQ. (18.11) AND RECALLING DATA FROM PROB. 18.11,

$$\underline{H}_A = \bar{\underline{r}} \times m\bar{\underline{v}} + \underline{H}_G$$

$$= (0.275\,m)\underline{i} \times (30\,kg)[(647.58\,m/s)\underline{i} - (56.65\,m/s)\underline{j}] + (0.320\,kg\cdot m^2/s)\underline{i} - (0.009\,kg\cdot m^2/s)\underline{j}$$

$$= -(467.37\,kg\cdot m^2/s)\underline{k} + (0.320\,kg\cdot m^2/s)\underline{i} - (0.009\,kg\cdot m^2/s)\underline{j}$$

$$\underline{H}_A = (0.320\,kg\cdot m^2/s)\underline{i} - (0.009\,kg\cdot m^2/s)\underline{j} - (467\,kg\cdot m^2/s)\underline{k}$$



18.13

(a) Show that the angular momentum $\mathbf{H}_B$ of a rigid body about point B can be obtained by adding to the angular momentum $\mathbf{H}_A$ of that body about point A the vector product of the vector $\mathbf{r}_{A/B}$ drawn from B to A and the linear momentum $m\bar{\mathbf{v}}$ of the body:

$$\mathbf{H}_B = \mathbf{H}_A + \mathbf{r}_{A/B} \times m\bar{\mathbf{v}}$$

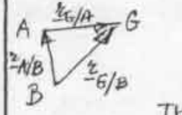
(b) Further show that when a rigid body rotates about a fixed axis, its angular momentum is the same about any two points A and B located on the fixed axis ($\mathbf{H}_A = \mathbf{H}_B$) if, and only if, the mass center G of the body is located on the fixed axis.

(a) USING EQ. (18.11) TO DETERMINE $\underline{H}_A$ AND THEN $\underline{H}_B$:

$$\underline{H}_A = \underline{r}_{G/A} \times m\bar{\underline{v}} + \underline{H}_G \qquad (1)$$

$$\underline{H}_B = \underline{r}_{G/B} \times m\bar{\underline{v}} + \bar{H}_G \qquad (2)$$

SUBTRACTING (1) FROM (2):



$$\underline{H}_B - \underline{H}_A = (\underline{r}_{G/B} - \underline{r}_{G/A}) m\bar{\underline{v}}$$

BUT $\underline{r}_{G/B} = \underline{r}_{A/B} + \underline{r}_{G/A}$

THUS: $\underline{H}_B = \underline{H}_A + \underline{r}_{A/B} \times m\bar{\underline{v}}$ (3)

(Q.E.D.)

(b) IT FOLLOWS FROM EQ. (3) THAT $\underline{H}_A = \underline{H}_B$ IF, AND ONLY IF,

$$\underline{r}_{A/B} \times m\bar{\underline{v}} = 0 \qquad (4)$$

BUT, DENOTING BY $\underline{\lambda}_{A/B}$ THE UNIT VECTOR ALONG THE FIXED AXIS, WE HAVE $\bar{\underline{v}} = \omega\underline{\lambda}_{A/B} \times \underline{r}_{G/A}$; EQ (4) YIELDS

$$\underline{r}_{A/B} \times m(\omega\underline{\lambda}_{A/B} \times \underline{r}_{G/A}) \qquad (5)$$

WE NOTE THAT $\underline{r}_{A/B}$ IS PERPENDICULAR TO $\omega\underline{\lambda}_{A/B} \times \underline{r}_{G/A}$ AND, THUS, NOT PARALLEL TO IT. THEREFORE, THIS SECOND VECTOR MUST BE ZERO, WHICH WILL OCCUR IF $\underline{r}_{G/A}$ IS PARALLEL TO $\underline{\lambda}_{AB}$, THAT IS, <u>IF, AND ONLY IF, G IS LOCATED ON AB.</u>

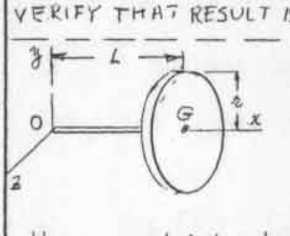
(Q.E.D)

18.14

<u>GIVEN:</u> DISK OF SAMPLE PROB. 18.2 AND ANSWERS TO PART d OF THAT PROBLEM:

$$m\bar{\underline{v}} = mr\omega_1\underline{k}, \quad \underline{H}_G = \tfrac{1}{2}mr^2\omega_1\left(\underline{i} - \tfrac{r}{2L}\underline{j}\right)$$

<u>FIND:</u> ANG. MOMENTUM $\underline{H}_O$ USING EQ. (18.11), AND VERIFY THAT RESULT IS SAME AS IN PART b. OF S.P. 18.2.



EQ. (18.11):

$$\underline{H}_O = \bar{\underline{r}} \times m\bar{\underline{v}} + \underline{H}_G$$

$$= L\underline{i} \times mr\omega_1\underline{k} + \tfrac{1}{2}mr^2\omega_1\left(\underline{i} - \tfrac{r}{2L}\underline{j}\right)$$

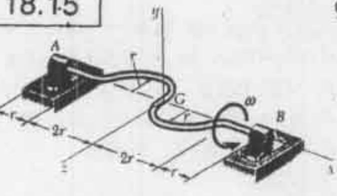
$$\underline{H}_O = -mrL\omega_1\underline{j} + \tfrac{1}{2}mr^2\omega_1\underline{i} - \tfrac{1}{4}mr^2\omega_1\tfrac{r}{L}\underline{j}$$

$$= \tfrac{1}{2}mr^2\omega_1\underline{i} - mL^2\tfrac{r}{L}\omega_1\underline{j} - \tfrac{1}{4}mr^2\tfrac{r}{L}\omega_1\underline{j}$$

$$\underline{H}_O = \tfrac{1}{2}mr^2\omega_1\underline{i} - m\left(L^2 + \tfrac{1}{4}r^2\right)(r\omega_1/L)\underline{j}$$

WHICH IS THE ANSWER OBTAINED IN PART b OF SAMPLE PROB. 18.2.

18.15



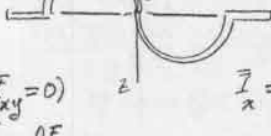
<u>GIVEN:</u>
SHAFT OF MASS m, MADE OF ROD OF UNIFORM CROSS SECTION ROTATES WITH CONSTANT ANG. VEL. $\underline{\omega}$.

<u>FIND:</u>
(a) ANG. MOM. $\underline{H}_G$,
(b) ANGLE FORMED BY $\underline{H}_G$ AND AXIS AB.

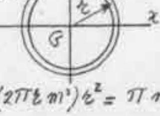
<u>MASS PER UNIT LENGTH</u> $= m' = \dfrac{m}{r + \pi r + \pi r + r} = \dfrac{m}{2(\pi+1)r}$

<u>MOMENTS AND PRODUCTS OF INERTIA</u> $\bar{I}_x$, $\bar{I}_{xy}$, AND $\bar{I}_{zx}$

$\bar{I}_x$ OF



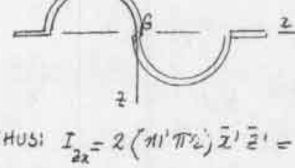
$= I_x$ OF



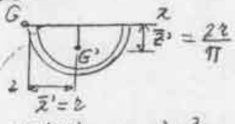
$(\bar{I}_{xy} = 0)$

$$\bar{I}_x = \tfrac{1}{2}I_G = \tfrac{1}{2}(2\pi r m')r^2 = \pi m' r^3$$

$\bar{I}_{zx}$ OF



$= 2\,\bar{I}_{zx}$ OF



THUS: $\bar{I}_{zx} = 2(m'\pi r)\bar{z}'\bar{x}' = 2m'\pi r(r)\left(\frac{2r}{\pi}\right) = 4m'r^3$

(a) <u>ANGULAR MOMENTUM $\underline{H}_G$</u>

WE USE EQS. (18.7). SINCE $\omega_x = \omega$, $\omega_y = \omega_z = 0$, WE HAVE

$$H_x = \bar{I}_x\omega = \pi m' r^3\omega \qquad (1)$$

$$H_y = -\bar{I}_{xy}\omega = 0$$

$$H_z = -\bar{I}_{zx}\omega = -4m'r^3\omega \qquad (2)$$

THUS:

$$\underline{H}_G = H_x\underline{i} + H_z\underline{k} = m'r^3\omega(\pi\underline{i} - 4\underline{k})$$

OR, RECALLING THE EXPRESSION OBTAINED FOR m':

$$\underline{H}_G = \frac{mr^3\omega}{2(\pi+1)r}(\pi\underline{i} - 4\underline{k}) =$$

$$= mr^2\omega\left[\frac{\pi}{2(\pi+1)}\underline{i} - \frac{2}{\pi+1}\underline{k}\right]$$

$$\underline{H}_G = mr^2\omega(0.379\,\underline{i} - 0.483\,\underline{k})$$

(b) <u>ANGLE FORMED BY $\underline{H}_G$ AND AXIS AB.</u>

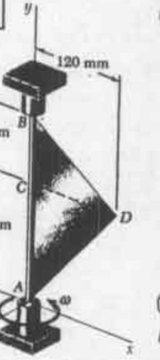
DENOTING BY θ THAT ANGLE, WE HAVE

$$\tan\theta = \frac{|H_z|}{H_x}$$

AND, RECALLING (1) AND (2):

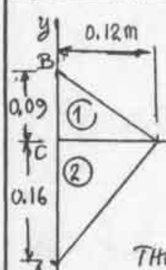
$$\tan\theta = \frac{4m'r^3\omega}{\pi m' r^3\omega} = \frac{4}{\pi} \qquad \theta = 51.9°$$

18.16



GIVEN:
TRIANGULAR PLATE SHOWN HAS MASS $m = 7.5\text{ kg}$ AND IS WELDED TO SHAFT AB. PLATE ROTATES AT CONSTANT RATE $\omega = 12\text{ rad/s}$.

FIND:
(a) ANG. MOMENTUM $\underline{H}_C$
(b) ANG. MOMENTUM $\underline{H}_A$
(FIND $\bar{\underline{v}}$ AND USE PROPERTY INDICATED IN PROB. 18.13a.)



(a) WE DIVIDE PLATE INTO TWO RIGHT TRIANGLES AND COMPUTE THEIR PRODUCTS OF INERTIA.

$m_1 = \frac{9}{25}(7.5\text{ kg}) = 2.7\text{ kg}$

$m_2 = \frac{16}{25}(7.5\text{ kg}) = 4.8\text{ kg}$

FROM SAMPLE PROB. 9.6, WE RECALL THAT $I_{xy,\,AREA} = \frac{1}{24} b^2 h^2$

THEREFORE $I_{xy,\,MASS} = \frac{m}{\frac{1}{2}bh}\left(\frac{1}{24} b^2 h^2\right) = \frac{mbh}{12}$

TRIANGLE 1: $(I_{x'y})_1 = \frac{1}{12}(2.7\text{ kg})(0.12\text{ m})(0.09\text{ m}) = 2.43\times 10^{-3}\text{ kg}\cdot\text{m}^2$

TRIANGLE 2: $(I_{x'y})_2 = \frac{1}{12}(4.8\text{ kg})(0.12\text{ m})(-0.16\text{ m}) = -7.68\times 10^{-3}\text{ kg}\cdot\text{m}^2$

THUS, FOR THE PLATE, $I_{x'y} = (2.43 - 7.68)10^{-3} = -5.25\times 10^{-3}\text{ kg}\cdot\text{m}^2$

$I_{x'y} = -5.25\text{ g}\cdot\text{m}^2$

WE NOTE THAT $I_{yz'} = 0$.

MOMENT OF INERTIA $\underline{I}_y$ OF ENTIRE PLATE:

$I_{y,\,AREA} = \frac{1}{12} b h^3$, $I_{y,\,MASS} = \frac{m}{\frac{1}{2}bh}\left(\frac{1}{12}bh^3\right) = \frac{1}{6} m h^2$

$I_y = \frac{1}{6}(7.5\text{ kg})(0.12\text{ m})^2 = 0.018\text{ kg}\cdot\text{m}^2 = 18\text{ g}\cdot\text{m}^2$

ANGULAR MOMENTUM $\underline{H}_C$

WE USE EQS. (18.13) TO OBTAIN THE COMPONENTS $H_{x'}, H_y, H_{z'}$ OF $\underline{H}_C$

$H_{x'} = -I_{x'y}\,\omega = -(-5.25\text{ g}\cdot\text{m}^2)(12\text{ rad/s}) = +63.0\text{ g}\cdot\text{m}^2/\text{s}$

$H_y = I_y\,\omega = (18\text{ g}\cdot\text{m}^2)(12\text{ rad/s}) = 216\text{ g}\cdot\text{m}^2/\text{s}$

$H_{z'} = -I_{yz'}\,\omega = 0$

THEREFORE $\underline{H}_C = (63.0\text{ g}\cdot\text{m}^2/\text{s})\underline{i} + (216\text{ g}\cdot\text{m}^2/\text{s})\underline{j}$ ◀

(b) ANGULAR MOMENTUM $\underline{H}_A$.

WE APPLY THE EQUATION GIVEN IN PART a OF PROB. 18.13 TO POINTS A AND C.

$$\underline{H}_A = \underline{H}_C + \underline{r}_{C/A} \times m\bar{\underline{v}} \qquad (1)$$

WHERE $\underline{r}_{C/A} = (0.16\text{ m})\underline{j}$. NOTING THAT THE DISTANCE FROM THE AXIS OF ROTATION AB TO THE MASS CENTER G OF THE PLATE IS $\bar{r} = \frac{1}{3}(0.12\text{ m}) = 0.04\text{ m}$, WE HAVE

$m\bar{\underline{v}} = m(\underline{\omega}\times\bar{\underline{r}}) = (7.5\text{ kg})(12\text{ rad/s})\underline{j}\times(0.04\text{ m})\underline{i}$

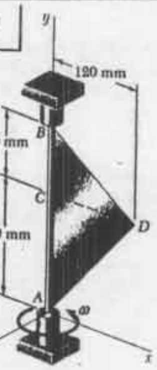
$= -(3.60\text{ kg}\cdot\text{m/s})\underline{k} = -(3600\text{ g}\cdot\text{m/s})\underline{k}$

$\underline{r}_{C/A}\times m\bar{\underline{v}} = (0.16\text{ m})\underline{j}\times(-3600\text{ g}\cdot\text{m/s})\underline{k} = -(576\text{ g}\cdot\text{m}^2/\text{s})\underline{i}$

SUBSTITUTING FOR $\underline{H}_C$ AND $\underline{r}_{C/A}\times m\bar{\underline{v}}$ INTO (1):

$\underline{H}_A = -(513\text{ g}\cdot\text{m}^2/\text{s})\underline{i} + (216\text{ g}\cdot\text{m}^2/\text{s})\underline{j}$ ◀

18.17



GIVEN
TRIANGULAR PLATE SHOWN HAS MASS $m = 7.5\text{ kg}$ AND IS WELDED TO SHAFT AB. PLATE ROTATES AT CONSTANT RATE $\omega = 12\text{ rad/s}$.

FIND:
(a) ANG. MOMENTUM $\underline{H}_C$
(b) ANG. MOMENTUM $\underline{H}_B$
(FIND $\bar{\underline{v}}$ AND USE PROPERTY INDICATED IN PROB. 18.13a.)

(a) SEE PART a OF SOLUTION OF PROB. 18.16. WE FIND

ANG. MOMENTUM $\underline{H}_C = (63.0\text{ g}\cdot\text{m}^2/\text{s})\underline{i} + (216\text{ g}\cdot\text{m}^2/\text{s})\underline{j}$ ◀

(b) ANG. MOMENTUM $\underline{H}_B$.

WE APPLY THE EQUATION GIVEN IN PART a OF PROB. 18.13 TO POINTS B AND C.

$$\underline{H}_B = \underline{H}_C + \underline{r}_{C/B}\times m\bar{\underline{v}} \qquad (1)$$

WHERE $\underline{r}_{C/B} = -(0.09\text{ m})\underline{j}$.

NOTING THAT THE DISTANCE FROM THE AXIS OF ROTATION AB TO THE MASS CENTER G OF THE PLATE IS

$\bar{r} = \frac{1}{3}(0.12\text{ m}) = 0.04\text{ m}$

WE HAVE

$m\bar{\underline{v}} = m(\underline{\omega}\times\bar{\underline{r}}) = (7.5\text{ kg})(12\text{ rad/s})\underline{j}\times(0.04\text{ m})\underline{i}$

$= -(3.60\text{ kg}\cdot\text{m/s})\underline{k} = -(3600\text{ g}\cdot\text{m/s})\underline{k}$

AND

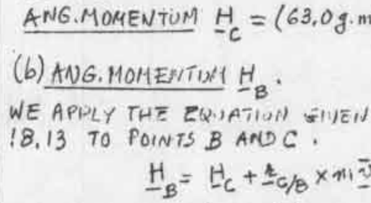
$\underline{r}_{C/B}\times m\bar{\underline{v}} = -(0.09\text{ m})\underline{j}\times(-3600\text{ g}\cdot\text{m/s})\underline{k}$

$= +(324\text{ g}\cdot\text{m}^2/\text{s})\underline{i}$

SUBSTITUTING FOR $\underline{H}_C$ AND $\underline{r}_{C/B}\times m\bar{\underline{v}}$ INTO (1):

$\underline{H}_B = (387\text{ g}\cdot\text{m}^2/\text{s})\underline{i} + (216\text{ g}\cdot\text{m}^2/\text{s})\underline{j}$ ◀

18.18



GIVEN:
SHAFT OF PROB. 18.15

FIND:
ANG. MOM. OF SHAFT
(a) ABOUT A
(b) ABOUT B

WE FIRST DETERMINE $\underline{H}_G$. SEE SOLUTION OF PROB. 18.15. WE FOUND

$$\underline{H}_G = m r^2\omega(0.379\,\underline{i} - 0.483\,\underline{k})$$

FROM EQ. (18.11) WE HAVE

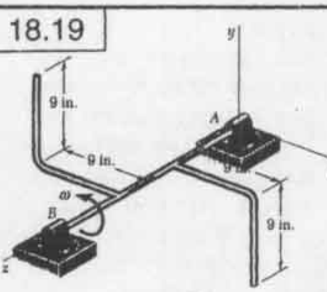
$\underline{H}_A = \underline{r}_{G/A}\times m\bar{\underline{v}} + \underline{H}_G \qquad \underline{H}_B = \underline{r}_{G/B}\times m\bar{\underline{v}} + \underline{H}_G$

BUT $\bar{\underline{v}} = 0$ SINCE G IS LOCATED ON AXIS AB. THUS:

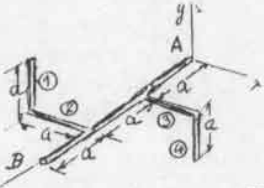
(a) AND (b): $\underline{H}_A = \underline{H}_B = \underline{H}_G = m r^2\omega(0.379\,\underline{i} - 0.483\,\underline{k})$ ◀

NOTE. THE RESULT OBTAINED VERIFIES THE PROPERTY INDICATED IN PROB. 18.13b, NAMELY, THAT IF THE MASS CENTER G OF A BODY ROTATING ABOUT A FIXED AXIS IS LOCATED ON THAT AXIS, THE ANGULAR MOMENTUM IS THE SAME ABOUT ANY TWO POINTS OF THAT AXIS.

18.19



GIVEN:
TWO L-SHAPED ARMS, EACH WEIGHING 5 lb, ARE WELDED AT THE ONE-THIRD POINTS OF THE 27-in. SHAFT AB. ASSEMBLY ROTATES AT CONSTANT 360-rpm RATE.
FIND: (a) $\underline{H}_A$
(b) ANGLE FORMED BY $\underline{H}_A$ AND AB.



MOMENTS AND PRODUCTS OF INERTIA

FOR EACH NUMBERED ELEMENT:
$a = 9\text{ in.} = 0.75\text{ ft}.$
$m = \frac{1}{2}(5\text{ lb})/g = 2.5/g$

FOR 1 AND 4: $I_z = \bar{I} + md^2 = \frac{1}{12}ma^2 + m\left(\frac{a^2}{4}+a^2\right) = \frac{4}{3}ma^2$

FOR 2 AND 3: $I_z = \frac{1}{3}ma^2$

FOR ASSEMBLY: $I_z = 2\left(\frac{4}{3}ma^2 + \frac{1}{3}ma^2\right) \qquad I_z = \frac{10}{3}ma^2$

PRODUCTS OF INERTIA OF ASSEMBLY:

$$I_{xz} = (I_{xz})_1 + (I_{xz})_2 + (I_{xz})_3 + (I_{xz})_4$$
$$= m(-a)(2a) + m\left(-\frac{a}{2}\right)(2a) + m\left(\frac{a}{2}\right)a + m(a)a = -\frac{3}{2}ma^2$$
$$I_{yz} = m\left(\frac{a}{2}\right)(2a) + 0 + 0 + m\left(-\frac{a}{2}\right)a = \frac{1}{2}ma^2$$

(a) ANGULAR MOMENTUM ABOUT A

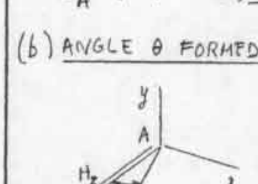
WE USE EQS. (18.13) TO OBTAIN THE COMPONENTS OF $\underline{H}_A$.
WE HAVE $\omega_z = \omega = 360\text{ rpm} = 6(2\pi) = 12\pi\text{ rad/s},\ \omega_x = \omega_y = 0.$

$$H_x = -I_{xz}\omega_z = +\frac{3}{2}ma^2(12\pi) = 18ma^2\pi$$
$$H_y = -I_{yz}\omega_z = -\frac{1}{2}ma^2(12\pi) = -6ma^2\pi$$
$$H_z = I_z\omega_z = \frac{10}{3}ma^2(12\pi) = 40ma^2\pi$$

THUS: $\underline{H}_A = H_x\underline{i} + H_y\underline{j} + H_z\underline{k} = 2ma^2\pi(9\underline{i} - 3\underline{j} + 20\underline{k})$
$$= 2\left(\frac{2.5}{32.2}\text{ lb}\cdot\text{s}^2/\text{ft}\right)(0.75\text{ ft})^2(\pi\text{ rad/s})(9\underline{i} - 3\underline{j} + 20\underline{k})$$
$$= (0.2744\text{ lb}\cdot\text{ft}\cdot\text{s})(9\underline{i} - 3\underline{j} + 20\underline{k})$$

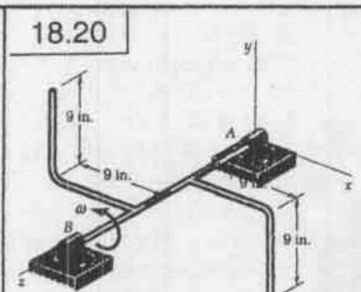
$$\underline{H}_A = (2.47\text{ lb}\cdot\text{ft}\cdot\text{s})\underline{i} - (0.823\text{ lb}\cdot\text{ft}\cdot\text{s})\underline{j} + (5.49\text{ lb}\cdot\text{ft}\cdot\text{s})\underline{k}$$ ◀

(b) ANGLE θ FORMED BY $\underline{H}_A$ AND AB

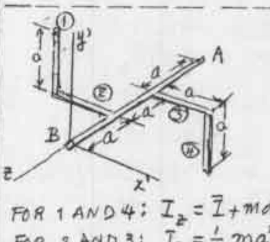


WE NOTE THAT $\theta = \theta_z$
AND RECALL THAT
$H_z = |\underline{H}_A|\cos\theta_z$
THUS:
$$\cos\theta = \frac{H_z}{|H_A|} = \frac{2ma^2\pi(20)}{2ma^2\pi\sqrt{9^2+3^2+20^2}}$$
$$= \frac{20}{\sqrt{490}} = 0.90351$$
$$\theta = 25.4°$$ ◀

18.20



GIVEN:
TWO L-SHAPED ARMS, EACH WEIGHING 5 lb, ARE WELDED AT THE ONE-THIRD POINTS OF THE 27-in. SHAFT AB. ASSEMBLY ROTATES AT CONSTANT 360-rpm RATE.
FIND: (a) $\underline{H}_B$
(b) ANGLE FORMED BY $\underline{H}_B$ AND BA.



WE WILL USE AXES x', y', z WITH ORIGIN AT B.

MOMENTS AND PRODUCTS OF INERTIA

FOR EACH NUMBERED ELEMENT:
$a = 9\text{ in.} = 0.75\text{ ft}$
$m = \frac{1}{2}(5\text{ lb})/g = 2.5/g$

FOR 1 AND 4: $I_z = \bar{I} + md^2 = \frac{1}{12}ma^2 + m\left(\frac{a^2}{4}+a^2\right) = \frac{4}{3}ma^2$

FOR 2 AND 3: $I_z = \frac{1}{3}ma^2$

FOR ASSEMBLY: $I_z = 2\left(\frac{4}{3}ma^2 + \frac{1}{3}ma^2\right) \qquad I_z = \frac{10}{3}ma^2$

PRODUCTS OF INERTIA OF ASSEMBLY:

$$I_{x'z} = (I_{x'z})_1 + (I_{x'z})_2 + (I_{x'z})_3 + (I_{x'z})_4$$
$$= m(-a)(-a) + m\left(-\frac{a}{2}\right)(-a) + m\left(\frac{a}{2}\right)(-2a) + m(a)(-2a) = -\frac{3}{2}ma^2$$
$$I_{y'z} = m\left(\frac{a}{2}\right)(-a) + 0 + 0 + m\left(-\frac{a}{2}\right)(-2a) = \frac{1}{2}ma^2$$

(a) ANGULAR MOMENTUM ABOUT B

WE USE EQS. (18.13) TO OBTAIN THE COMPONENTS OF $\underline{H}_B$.
WE HAVE $\omega_z = \omega = 360\text{ rpm} = 6(2\pi) = 12\pi\text{ rad/s},\ \omega_{x'} = \omega_{y'} = 0.$

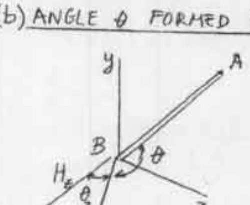
$$H_{x'} = -I_{x'z}\omega_z = +\frac{3}{2}ma^2(12\pi) = 18ma^2\pi$$
$$H_{y'} = -I_{y'z}\omega_z = -\frac{1}{2}ma^2(12\pi) = -6ma^2\pi$$
$$H_z = I_z\omega_z = \frac{10}{3}ma^2(12\pi) = 40ma^2\pi$$

THUS: $\underline{H}_B = H_{x'}\underline{i} + H_{y'}\underline{j} + H_z\underline{k} = 2ma^2\pi(9\underline{i} - 3\underline{j} + 20\underline{k})$
$$= 2\left(\frac{2.5}{32.2}\text{ lb}\cdot\text{s}^2/\text{ft}\right)(0.75\text{ ft})^2(\pi\text{ rad/s})(9\underline{i} - 3\underline{j} + 20\underline{k})$$
$$= (0.2744\text{ lb}\cdot\text{ft}\cdot\text{s})(9\underline{i} - 3\underline{j} + 20\underline{k})$$

$$\underline{H}_B = (2.47\text{ lb}\cdot\text{ft}\cdot\text{s})\underline{i} - (0.823\text{ lb}\cdot\text{ft}\cdot\text{s})\underline{j} + (5.49\text{ lb}\cdot\text{ft}\cdot\text{s})\underline{k}$$ ◀

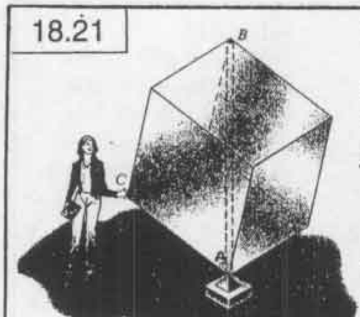
NOTE. THIS IS THE SAME ANSWER THAT WAS OBTAINED FOR $\underline{H}_A$ IN PROB. 18.19. THIS COULD HAVE BEEN ANTICIPATED, SINCE THE MASS CENTER G OF THE ASSEMBLY LIES ON THE FIXED AXIS AB (CF. PROB. 18.13 b).

(b) ANGLE θ FORMED BY $\underline{H}_B$ AND BA



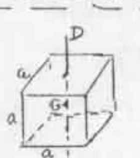
WE NOTE THAT $\theta = \pi - \theta_z$
AND RECALL THAT
$H_z = |\underline{H}_B|\cos\theta_z$
THUS:
$$\cos\theta = \cos(\pi - \theta_z) = -\cos\theta_z$$
$$= -\frac{H_z}{|H_B|} = -\frac{2ma^2\pi(20)}{2ma^2\pi\sqrt{9^2+3^2+20^2}}$$
$$= -\frac{20}{\sqrt{490}} = -0.90351$$
$$\theta = 154.6°$$ ◀

18.21



GIVEN:
HOLLOW CUBE CONSISTS OF SIX 5×5 ft ALUMINUM SHEETS AND CAN ROTATE ABOUT VERTICAL DIAGONAL AB. STUDENT PUSHES CORNER C FOR 1.2 s IN DIRECTION PERPENDICULAR TO PLANE ABC WITH FORCE OF 12.5 lb, CAUSING CUBE TO COMPLETE 1 REV IN 5 s.
FIND: WEIGHT OF CUBE.

HINT: PERP. DISTANCE FROM C TO AB IS $a\sqrt{2/3}$, WHERE a IS SIDE OF CUBE.



FOR CUBE, $I_{AB} = I_{DD'}$ SINCE THE ELLIPSOID OF INERTIA AT G IS A SPHERE (SEC. 9.17).
FOR THE TWO HORIZONTAL FACES
$$(I_{DD'})_H = 2\left(\frac{m}{6}\right)\left(\frac{a^2}{6}\right) = \frac{ma^2}{18}$$
WHERE m = MASS OF CUBE

FOR THE FOUR VERTICAL FACES
$$(I_{DD'})_V = 4\left(\frac{m}{6}\right)\left[\frac{a^2}{12} + \left(\frac{a}{2}\right)^2\right] = \frac{2ma^2}{9}$$

FOR THE WHOLE CUBE:
$$I_{AB} = I_{DD'} = (I_{DD'})_H + (I_{DD'})_V = \frac{ma^2}{18} + \frac{2ma^2}{9} = \frac{5}{18}ma^2$$

IMPULSE-MOMENTUM PRINCIPLE

ANG. IMPULSE ABOUT AB = FINAL ANG. MOMENTUM ABOUT AB
$$(F\Delta t)a\sqrt{2/3} = \frac{5}{18}ma^2\omega \qquad (1)$$

GIVEN DATA: $F = 12.5$ lb, $\Delta t = 1.2$ s, $a = 5$ ft, $\omega = \frac{2\pi \text{ rad}}{5 \text{ s}}$

SUBSTITUTE DATA AND $m = W/g$ INTO (1):
$$(12.5 \text{ lb})(1.2 \text{ s})(5 \text{ ft})\sqrt{\frac{2}{3}} = \frac{5}{18}\,\frac{W}{32.2 \text{ ft/s}^2}(5 \text{ ft})^2\left(\frac{2\pi \text{ rad}}{5 \text{ s}}\right)$$

SOLVING FOR W: $W = 225.96$ lb $\qquad W = 226$ lb

18.22

GIVEN: ALUMINUM CUBE OF PROB. 18.21 IS REPLACED BY CUBE CONSISTING OF SIX PLYWOOD SHEETS, WEIGHING 20 lb EACH. STUDENT PUSHES CORNER C AS IN PROB. 18.21 (FOR 1.2 s WITH 12.5-lb FORCE).
FIND: TIME REQUIRED FOR CUBE TO COMPLETE 1 REV.

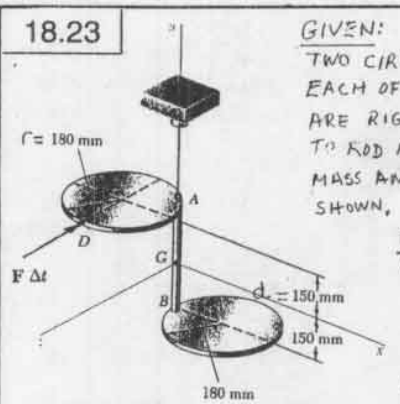
SEE SOLUTION OF PROB. 18.21 FOR DERIVATION OF
$$(F\Delta t)a\sqrt{2/3} = \frac{5}{18}ma^2\omega \qquad (1)$$

GIVEN DATA: $F = 12.5$ lb, $\Delta t = 1.2$ s, $a = 5$ ft
$$m = \frac{W}{g} = \frac{6(20 \text{ lb})}{32.2 \text{ ft/s}^2} = 3.727 \text{ lb}\cdot\text{s}^2/\text{ft}$$

SUBSTITUTE DATA INTO (1):
$$(12.5 \text{ lb})(1.2 \text{ s})(5 \text{ ft})\sqrt{\frac{2}{3}} = \frac{5}{18}(3.727 \text{ lb}\cdot\text{s}^2/\text{ft})(5 \text{ ft})^2\omega$$

SOLVING FOR ω: $\omega = 2.366 \text{ s}^{-1}$
$$\tau = \frac{2\pi}{\omega} = \frac{2\pi}{2.366 \text{ s}^{-1}} = 2.6556 \text{ s}$$
$$\tau = 2.66 \text{ s}$$

18.23



GIVEN:
TWO CIRCULAR PLATES, EACH OF MASS $m = 4$ kg, ARE RIGIDLY CONNECTED TO ROD AB OF NEGLIGIBLE MASS AND SUSPENDED AS SHOWN. AN IMPULSE $F\Delta t = -(2.4 \text{ N}\cdot\text{s})\mathbf{k}$ IS APPLIED AT D.
FIND:
(a) VELOCITY $\bar{v}$ OF MASS CENTER G.
(b) ANGULAR VELOCITY ω OF ASSEMBLY.

COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

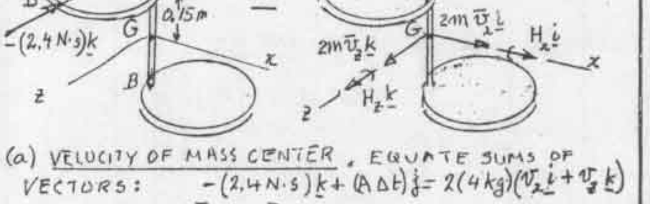
FOR UPPER PLATE:
$$I_x = \bar{I}_{x'} + md^2 = m\left(\tfrac{1}{4}r^2 + d^2\right) = (4 \text{ kg})\left[\tfrac{1}{4}(0.18 \text{ m})^2 + (0.15 \text{ m})^2\right] = 0.1224 \text{ kg}\cdot\text{m}^2$$
$$I_y = \bar{I}_{y'} + mr^2 = \tfrac{1}{2}mr^2 + mr^2 = \tfrac{3}{2}mr^2 = \tfrac{3}{2}(4 \text{ kg})(0.18 \text{ m})^2 = 0.1944 \text{ kg}\cdot\text{m}^2$$
$$I_z = \bar{I}_{z'} + m(r^2 + d^2) = \tfrac{1}{4}mr^2 + m(r^2+d^2) = m\left(\tfrac{5}{4}r^2 + d^2\right) = (4 \text{ kg})\left[\tfrac{5}{4}(0.18 \text{ m})^2 + (0.15 \text{ m})^2\right] = 0.252 \text{ kg}\cdot\text{m}^2$$
$$I_{xy} = m(-r)(d) = -mrd = -(4 \text{ kg})(0.18 \text{ m})(0.15 \text{ m})$$
$$I_{xy} = -0.108 \text{ kg}\cdot\text{m}^2, \quad I_{yz} = 0, \quad I_{zx} = 0$$

FOR LOWER PLATE: WE OBTAIN THE SAME RESULTS

THUS, FOR ASSEMBLY, WE DOUBLE RESULTS FOR UPPER PLATE:
$$\bar{I}_x = 0.2448 \text{ kg}\cdot\text{m}^2, \ \bar{I}_y = 0.3888 \text{ kg}\cdot\text{m}^2, \ \bar{I}_z = 0.504 \text{ kg}\cdot\text{m}^2$$
$$\bar{I}_{xy} = -0.216 \text{ kg}\cdot\text{m}^2, \ \bar{I}_{yz} = 0, \ \bar{I}_{zx} = 0 \qquad (1)$$

IMPULSE-MOMENTUM PRINCIPLE

WE NOTE THAT THE IMPULSIVE FORCES ARE F AND, POSSIBLY, THE FORCE AT A. ALSO, FROM CONSTRAINTS, $\bar{v}_y = 0$



(a) VELOCITY OF MASS CENTER. EQUATE SUMS OF VECTORS: $-(2.4 \text{ N}\cdot\text{s})\mathbf{k} + (A\Delta t)\mathbf{j} = 2(4 \text{ kg})(\bar{v}_x\mathbf{i} + \bar{v}_z\mathbf{k})$

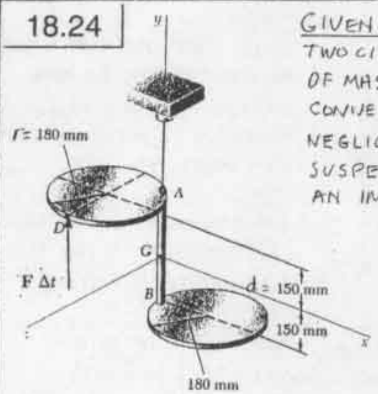
THUS: $A\Delta t = 0$, $\bar{v}_x = 0$, $\bar{v}_z = -0.3$ m/s
$$\bar{\mathbf{v}} = -(0.300 \text{ m/s})\mathbf{k}$$

(b) ANGULAR VELOCITY. EQUATE SUMS OF MOMENTS ABOUT G:
$$[(-0.18 \text{ m})\mathbf{i} + (0.15 \text{ m})\mathbf{j}] \times (-2.4 \text{ N}\cdot\text{s})\mathbf{k} = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$$
$$-(0.432 \text{ kg}\cdot\text{m}^2)\mathbf{j} - (0.360 \text{ kg}\cdot\text{m}^2)\mathbf{i} = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$$
$$H_x = -0.360, \quad H_y = -0.432, \quad H_z = 0 \qquad (2)$$

SUBSTITUTE FROM (1) AND (2) INTO EQS. (18.7):
$$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z: \quad -0.360 = +0.2448\,\omega_x + 0.216\,\omega_y \qquad (3)$$
$$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z: \quad -0.432 = +0.216\,\omega_x + 0.3888\,\omega_y \qquad (4)$$
$$H_z = -\bar{I}_{xz}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z: \quad 0 = \omega_z \qquad (5)$$

SOLVE (3) AND (4): $\omega_x = -0.96154$, $\omega_y = -0.57692$
$$\boldsymbol{\omega} = -(0.962 \text{ rad/s})\mathbf{i} - (0.577 \text{ rad/s})\mathbf{j}$$

18.24



GIVEN:
TWO CIRCULAR PLATES, EACH OF MASS $m = 4\,kg$, ARE RIGIDLY CONNECTED TO ROD AB OF NEGLIGIBLE MASS AND SUSPENDED AS SHOWN. AN IMPULSE

$$\underline{F}\Delta t = (2.4\,N\cdot s)\underline{j}$$

IS APPLIED AT D.

FIND:
(a) VELOCITY $\bar{\underline{v}}$ OF MASS CENTER G,
(b) ANGULAR VELOCITY $\underline{\omega}$ OF ASSEMBLY.

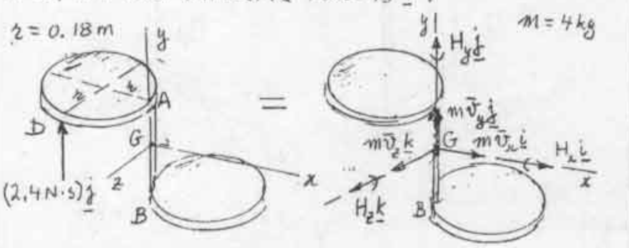
COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

SEE SOLUTION OF PROB. 18.23 WHERE WE FOUND

$$\bar{I}_x = 0.2448\,kg\cdot m^2,\ \bar{I}_y = 0.3888\,kg\cdot m^2,\ \bar{I}_z = 0.504\,kg\cdot m^2$$
$$\bar{I}_{xy} = -0.216\,kg\cdot m^2,\ \bar{I}_{yz} = 0,\ \bar{I}_{zx} = 0 \qquad (1)$$

IMPULSE-MOMENTUM PRINCIPLE

WE NOTE THAT THE CORD AT A WILL BECOME SLACK THUS, THE ONLY IMPULSIVE FORCE IS $\underline{F}$.



(a) VELOCITY OF MASS CENTER

EQUATE SUMS OF VECTORS:

$$(2.4\,N\cdot s)\underline{j} = 2(4\,kg)(v_x\underline{i} + v_y\underline{j} + v_z\underline{k})$$

THUS: $v_x = 0$, $v_y = 0.300\,m/s$, $v_z = 0$

$$\bar{\underline{v}} = (0.300\,m/s)\underline{j}$$

(b) ANGULAR VELOCITY

EQUATE SUMS OF MOMENTS ABOUT G:

$$[(-0.18\,m)\underline{i} + (0.18\,m)\underline{k}] \times (2.4\,N\cdot s)\underline{j} = H_x\underline{i} + H_y\underline{j} + H_z\underline{k}$$
$$(0.432\,kg\cdot m^2)(-\underline{k} - \underline{i}) = H_x\underline{i} + H_y\underline{j} + H_z\underline{k}$$

THUS: $H_x = -0.432\,kg\cdot m^2$, $H_y = 0$, $H_z = -0.432\,kg\cdot m^2$ (2)

SUBSTITUTE FROM (1) AND (2) INTO EQS. (18.7):

$$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z:\quad -0.432 = +0.2448\omega_x + 0.216\omega_y \qquad (3)$$
$$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z:\quad 0 = +0.216\omega_x + 0.3888\omega_y \qquad (4)$$
$$H_z = -\bar{I}_{xz}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z:\quad -0.432 = +0.504\,\omega_z \qquad (5)$$

SOLVING (3) AND (4) SIMULTANEOUSLY,

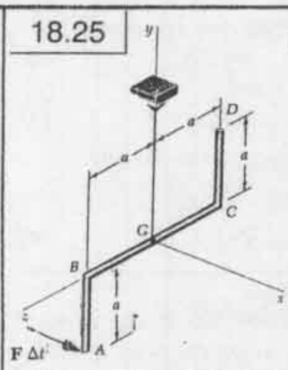
$$\omega_x = -3.4616\,rad/s,\quad \omega_y = 1.9231\,rad/s$$

SOLVING (5) FOR ω_z: $\omega_z = -0.8571\,rad/s$

THUS:

$$\underline{\omega} = -(3.46\,rad/s)\underline{i} + (1.923\,rad/s)\underline{j} - (0.857\,rad/s)\underline{k}$$

18.25



GIVEN:
UNIFORM BENT ROD OF MASS m IS SUSPENDED AS SHOWN. ROD IS HIT AT A WITH IMPULSE $\underline{F}\Delta t$ IN DIRECTION PERPENDICULAR TO PLANE CONTAINING ROD.

FIND:
IMMEDIATELY AFTER IMPACT
(a) VELOCITY $\bar{\underline{v}}$ OF MASS CENTER
(b) ANGULAR VELOCITY $\underline{\omega}$ OF ROD.

COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

PORTION BC: $(I_x)_{BC} = (I_y)_{BC} = \frac{1}{12}(\frac{m}{2})(2a)^2 = \frac{1}{6}ma^2$, $(I_z)_{BC} = 0$

$(I_{xy})_{BC} = (I_{yz})_{BC} = (I_{zx})_{BC} = 0$

PORTIONS AB AND CD:

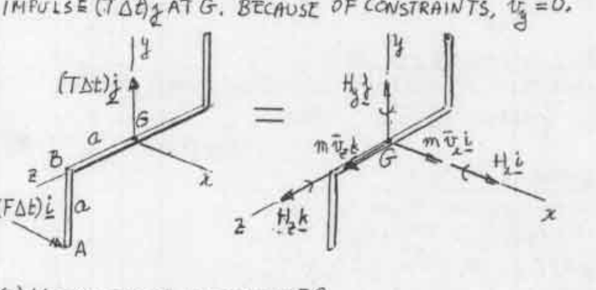
$$(I_x)_{AB} = (I_x)_{CD} = \bar{I} + (\tfrac{m}{4})d^2 = \tfrac{1}{12}(\tfrac{m}{4})a^2 + \tfrac{m}{4}(a^2 + \tfrac{a^2}{4}) = \tfrac{1}{3}ma^2$$
$$(I_y)_{AB} = (I_y)_{CD} = \tfrac{m}{4}a^2,\quad (I_z)_{AB} = (I_z)_{CD} = \tfrac{1}{3}\tfrac{m}{4}a^2 = \tfrac{1}{12}ma^2$$
$$(I_{xy})_{AB} = (I_{xy})_{CD} = 0,\ (I_{zx})_{AB} = (I_{zx})_{CD} = 0,\ (I_{yz})_{AB} = (I_{yz})_{CD} = -\tfrac{m}{4}(a)(\tfrac{a}{2}) = -\tfrac{ma^2}{8}$$

THE MOMENTS AND PRODUCTS OF INERTIA OF THE ROD ARE OBTAINED BY ADDING THE ABOVE VALUES:

$$\bar{I}_x = \tfrac{1}{6}ma^2 + \tfrac{1}{3}ma^2 + \tfrac{1}{3}ma^2 = \tfrac{5}{6}ma^2$$
$$\bar{I}_y = \tfrac{1}{6}ma^2 + \tfrac{1}{4}ma^2 + \tfrac{1}{4}ma^2 = \tfrac{2}{3}ma^2 \qquad (1)$$
$$\bar{I}_z = 0 + \tfrac{1}{12}ma^2 + \tfrac{1}{12}ma^2 = \tfrac{1}{6}ma^2$$
$$\bar{I}_{xy} = 0,\ \bar{I}_{yz} = 0 - \tfrac{1}{8}ma^2 - \tfrac{1}{8}ma^2 = -\tfrac{1}{4}ma^2,\ \bar{I}_{zx} = 0$$

IMPULSE-MOMENTUM PRINCIPLE

THE IMPULSES CONSIST OF $\underline{F}\Delta t = (F\Delta t)\underline{i}$ AND, POSSIBLY, AN IMPULSE $(T\Delta t)\underline{j}$ AT G. BECAUSE OF CONSTRAINTS, $\bar{v}_y = 0$.



(a) VELOCITY OF MASS CENTER

EQUATE SUMS OF VECTORS: $(F\Delta t)\underline{i} + (T\Delta t)\underline{j} = m\bar{v}_x\underline{i} + m\bar{v}_z\underline{k}$

THUS: $\bar{v}_x = (F\Delta t)/m$, $\bar{v}_z = 0$, $T\Delta t = 0$

$$\bar{\underline{v}} = (F\Delta t/m)\underline{i}$$

(b) ANGULAR VELOCITY

EQUATE MOMENTS ABOUT G:

$$(-a\underline{j} + a\underline{k}) \times (F\Delta t)\underline{i} = H_x\underline{i} + H_y\underline{j} + H_z\underline{k}$$
$$aF\Delta t(\underline{k} + \underline{j}) = H_x\underline{i} + H_y\underline{j} + H_z\underline{k}$$

THUS: $H_x = 0$, $H_y = aF\Delta t$, $H_z = aF\Delta t$ (2)

SUBSTITUTE FROM (1) AND (2) INTO EQS. (18.7):

$$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z:\quad 0 = \tfrac{5}{6}ma^2\omega_x \qquad \omega_x = 0 \qquad (3)$$
$$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z:\quad aF\Delta t = \tfrac{2}{3}ma^2\omega_y + \tfrac{1}{4}ma^2\omega_z \qquad (4)$$
$$H_z = -\bar{I}_{xz}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z:\quad aF\Delta t = \tfrac{1}{4}ma^2\omega_y + \tfrac{1}{6}ma^2\omega_z \qquad (5)$$

(CONTINUED)

18.25 continued

WE REPEAT THE FOLLOWING EQS.:

$$\omega_x = 0 \qquad (3)$$

$$aF\Delta t = \tfrac{2}{3} ma^2\omega_y + \tfrac{1}{4} ma^2\omega_z \qquad (4)$$

$$aF\Delta t = \tfrac{1}{4} ma^2\omega_y + \tfrac{1}{6} ma^2\omega_z \qquad (5)$$

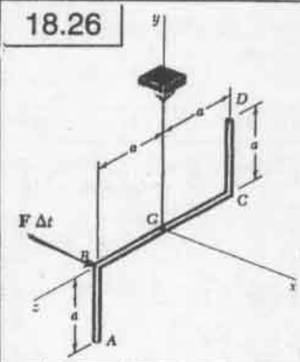
SOLVING (4) AND (5) SIMULTANEOUSLY, WE OBTAIN

$$\omega_y = -\frac{12}{7}\frac{F\Delta t}{ma} \qquad \omega_z = \frac{60}{7}\frac{F\Delta t}{ma}$$

THUS:

$$\underline{\omega} = (12\,F\Delta t/7ma)(-\underline{j} + 5\underline{k})$$

18.26



GIVEN:
UNIFORM BENT ROD OF MASS m IS SUSPENDED AS SHOWN. ROD IS HIT AT B WITH IMPULSE $F\Delta t$ IN DIRECTION PERPENDICULAR TO PLANE CONTAINING ROD.

FIND:
IMMEDIATELY AFTER IMPACT
(a) VELOCITY OF MASS CENTER
(b) ANGULAR VELOCITY $\underline{\omega}$ OF ROD.

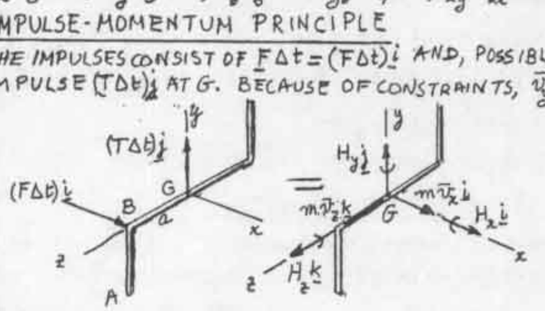
MOMENTS AND PRODUCTS OF INERTIA

SEE SOLUTION OF PROB. 18.25. WE OBTAINED

$$\bar{I}_x = \tfrac{5}{6} ma^2,\ \bar{I}_y = \tfrac{2}{3} ma^2,\ \bar{I}_z = \tfrac{1}{6} ma^2,\ \bar{I}_{yz} = -\tfrac{1}{4} ma^2,\ \bar{I}_{xy} = \bar{I}_{zx} = 0 \qquad (1)$$

IMPULSE-MOMENTUM PRINCIPLE

THE IMPULSES CONSIST OF $\underline{F}\Delta t = (F\Delta t)\underline{i}$ AND, POSSIBLY, AN IMPULSE $(T\Delta t)\underline{j}$ AT G. BECAUSE OF CONSTRAINTS, $\bar{v}_y = 0$.



(a) VELOCITY OF MASS CENTER

EQUATE SUMS OF VECTORS: $(F\Delta t)\underline{i} + (T\Delta t)\underline{j} = m\bar{v}_x\underline{i} + m\bar{v}_z\underline{k}$

THUS: $\bar{v}_x = (F\Delta t)/m,\ \bar{v}_z = 0,\ T\Delta t = 0$

$$\bar{\underline{v}} = (F\Delta t/m)\,\underline{i}$$

(b) ANGULAR VELOCITY

EQUATE MOMENTS ABOUT G:

$$a\underline{k} \times (F\Delta t)\underline{i} = H_x\underline{i} + H_y\underline{j} + H_z\underline{k}$$

$$(aF\Delta t)\,\underline{j} = H_x\underline{i} + H_y\underline{j} + H_z\underline{k}$$

THUS: $H_x = 0,\ H_y = aF\Delta t,\ H_z = 0 \qquad (2)$

SUBSTITUTE FROM (1) AND (2) INTO EQS. (18.7):

$$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{zx}\omega_z: \quad 0 = \tfrac{5}{6} ma^2\omega_x \quad \omega_x = 0 \qquad (3)$$

$$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z: \quad aF\Delta t = \tfrac{2}{3} ma^2\omega_y - \tfrac{1}{4} ma^2\omega_z \qquad (4)$$

$$H_z = -\bar{I}_{zx}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z: \quad 0 = -\tfrac{1}{4} ma^2\omega_y + \tfrac{1}{6} ma^2\omega_z \qquad (5)$$

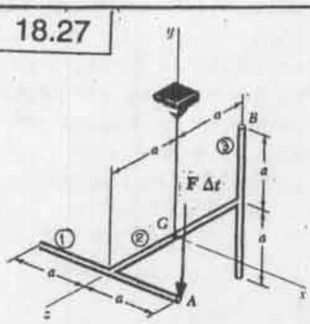
SOLVING (4) AND (5) SIMULTANEOUSLY, WE OBTAIN

$$\omega_y = \frac{24}{7}\frac{F\Delta t}{ma} \qquad \omega_z = -\frac{36}{7}\frac{F\Delta t}{ma}$$

THUS:

$$\underline{\omega} = (12\,F\Delta t/7ma)(2\underline{j} - 3\underline{k})$$

18.27



GIVEN:
THREE RODS, EACH OF MASS m AND LENGTH $2a$ ARE WELDED TO FORM ASSEMBLY. ASSEMBLY IS HIT VERTICALLY AT A AS SHOWN.

FIND:
IMMEDIATELY AFTER IMPACT
(a) VELOCITY OF MASS CENTER
(b) ANGULAR VELOCITY $\underline{\omega}$ OF ASSEMBLY.

COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

$$\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3 = ma^2 + \tfrac{1}{3} ma^2 + m\left(a^2 + \tfrac{a^2}{3}\right) = \tfrac{8}{3} ma^2$$

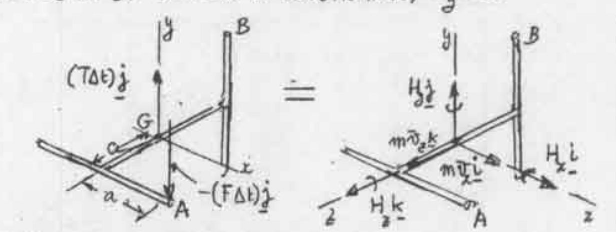
$$\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3 = m\left(a^2 + \tfrac{a^2}{3}\right) + \tfrac{1}{3} ma^2 + ma^2 = \tfrac{8}{3} ma^2 \qquad (1)$$

$$\bar{I}_z = (I_z)_1 + (I_z)_2 + (I_z)_3 = \tfrac{1}{3} ma^2 + 0 + \tfrac{1}{3} ma^2 = \tfrac{2}{3} ma^2$$

$$\bar{I}_{xy} = 0,\ \bar{I}_{yz} = 0,\ \bar{I}_{zx} = 0$$

IMPULSE-MOMENTUM PRINCIPLE

THE IMPULSES CONSIST OF $-(F\Delta t)\underline{j}$ APPLIED AT A AND $(T\Delta t)\underline{j}$ APPLIED AT G. BECAUSE OF CONSTRAINTS, $\bar{v}_y = 0$.



(a) VELOCITY OF MASS CENTER

EQUATE SUMS OF VECTORS: $(T\Delta t)\underline{j} - (F\Delta t)\underline{j} = m\bar{v}_x\underline{i} + m\bar{v}_z\underline{k}$

THUS: $T\Delta t = F\Delta t,\ \bar{v}_x = 0,\ \bar{v}_z = 0$. SINCE $\bar{v}_y = 0$ FROM ABOVE,

$$\bar{\underline{v}} = 0$$

(b) ANGULAR VELOCITY

EQUATE MOMENTS ABOUT G:

$$(a\underline{i} + a\underline{k}) \times (-F\Delta t)\underline{j} = H_x\underline{i} + H_y\underline{j} + H_z\underline{k}$$

$$-(aF\Delta t)\underline{k} + (aF\Delta t)\underline{i} = H_x\underline{i} + H_y\underline{j} + H_z\underline{k}$$

THUS: $H_x = aF\Delta t,\ H_y = 0,\ H_z = -aF\Delta t \qquad (2)$

SINCE THE THREE PRODUCTS OF INERTIA ARE ZERO, THE x, y, AND z AXES ARE PRINCIPAL CENTROIDAL AXES AND WE CAN USE EQS. (18.10). SUBSTITUTING FROM (1) AND (2) INTO THESE EQUATIONS, WE HAVE

$$H_x = \bar{I}_x\omega_x: \ aF\Delta t = \tfrac{8}{3} ma^2\omega_x \qquad \omega_x = 3F\Delta t/8ma \qquad (3)$$

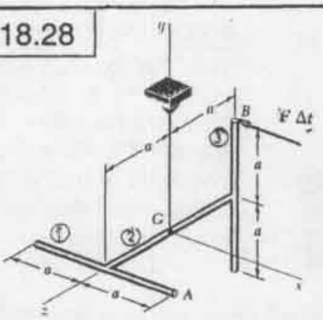
$$H_y = \bar{I}_y\omega_y: \ 0 = \tfrac{8}{3} ma^2\omega_y \qquad \omega_y = 0 \qquad (4)$$

$$H_z = \bar{I}_z\omega_z: \ -aF\Delta t = \tfrac{2}{3} ma^2 \qquad \omega_z = -3F\Delta t/2ma \qquad (5)$$

THEREFORE:

$$\underline{\omega} = (3F\Delta t/8ma)(\underline{i} - 4\underline{k})$$

18.28



GIVEN: THREE RODS, EACH OF MASS m AND LENGTH $2a$ ARE WELDED TO FORM ASSEMBLY, WHICH IS HIT AT B IN DIRECTION OPPOSITE TO x AXIS.

FIND: IMMEDIATELY AFTER IMPACT (a) VELOCITY OF MASS CENTER, (b) ANGULAR VELOCITY $\underline{\omega}$.

COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

$$\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3 = ma^2 + \tfrac{1}{3}ma^2 + m\left(a^2 + \tfrac{a^2}{3}\right) = \tfrac{8}{3}ma^2$$

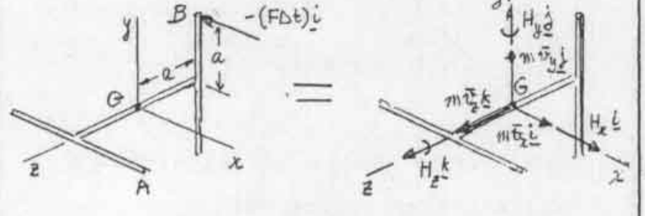
$$\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3 = m\left(a^2 + \tfrac{a^2}{3}\right) + \tfrac{1}{3}ma^2 + ma^2 = \tfrac{8}{3}ma^2 \qquad (1)$$

$$\bar{I}_z = (I_z)_1 + (I_z)_2 + (I_z)_3 = \tfrac{1}{3}ma^2 + 0 + \tfrac{1}{3}ma^2 = \tfrac{2}{3}ma^2$$

$$\bar{I}_{xy} = \bar{I}_{yz} = \bar{I}_{zx} = 0$$

IMPULSE-MOMENTUM PRINCIPLE

THE ONLY IMPULSE IS $\underline{F}\Delta t = -(F\Delta t)\underline{i}$.



(a) VELOCITY OF MASS CENTER

EQUATE SUMS OF VECTORS:

$$-(F\Delta t)\underline{i} = m\bar{v}_x\underline{i} + m\bar{v}_y\underline{j} + m\bar{v}_z\underline{k}$$

THUS: $\bar{v}_x = -F\Delta t/m$, $\bar{v}_y = 0$, $\bar{v}_z = 0$

$$\bar{\underline{v}} = -(F\Delta t/m)\underline{i}$$

(b) ANGULAR VELOCITY

EQUATE MOMENTS ABOUT G:

$$(a\underline{j} - a\underline{k}) \times (-F\Delta t)\underline{i} = H_x\underline{i} + H_y\underline{j} + H_z\underline{k}$$

$$(aF\Delta t)\underline{k} + (aF\Delta t)\underline{j} = H_x\underline{i} + H_y\underline{j} + H_z\underline{k}$$

THUS: $H_x = 0$, $H_y = aF\Delta t$, $H_z = aF\Delta t \qquad (2)$

SINCE THE THREE PRODUCTS OF INERTIA ARE ZERO, THE x, y, AND z AXES ARE PRINCIPAL CENTROIDAL AXES AND WE CAN USE EQS. (18.10). SUBSTITUTING FROM (1) AND (2) INTO THESE EQUATIONS, WE HAVE

$H_x = \bar{I}_x\omega_x$: $\quad 0 = \tfrac{8}{3}ma^2\omega_x \quad \omega_x = 0 \qquad (3)$

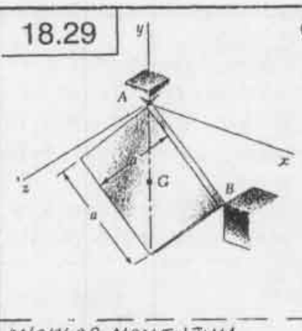
$H_y = \bar{I}_y\omega_y$: $\quad aF\Delta t = \tfrac{8}{3}ma^2\omega_y \quad \omega_y = 3F\Delta t/8ma \qquad (4)$

$H_z = \bar{I}_z\omega_z$: $\quad aF\Delta t = \tfrac{2}{3}ma^2\omega_z \quad \omega_z = 3F\Delta t/2ma \qquad (5)$

THEREFORE:

$$\underline{\omega} = (3F\Delta t/8ma)(\underline{j} + 4\underline{k})$$

18.29



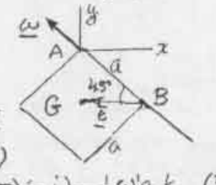
GIVEN: SQUARE PLATE OF MASS m SUPPORTED BY BALL AND SOCKET WITH ANGULAR VELOCITY $\omega_0\underline{j}$ WHEN IT STRIKES OBSTRUCTION AT B IN xy PLANE ($e = 0$).

FIND: IMMEDIATELY AFTER IMPACT (a) ANG. VELOCITY OF PLATE, (b) VELOCITY OF G.

ANGULAR MOMENTUM

BECAUSE OF SYMMETRY OF SQUARE PLATE, $\bar{I}$ IS THE SAME ABOUT ANY AXIS THROUGH G WITHIN xy PLANE. (CF. SEC. 9.17), $\bar{I} = \frac{1}{12}ma^2$. IT FOLLOWS THAT $\underline{H}_G = \frac{1}{12}ma^2\underline{\omega}$ (1) FOR ANY $\underline{\omega}$.

VELOCITIES AFTER IMPACT



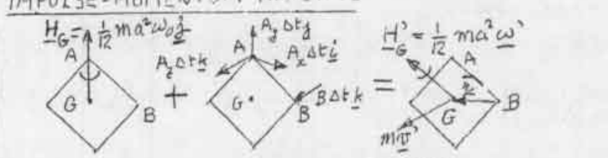
SINCE $e = 0$, CORNER B REMAINS IN CONTACT WITH OBSTRUCTION AND PLATE ROTATES ABOUT AB

$$\underline{\lambda}_{BA} = -\cos 45°\,\underline{i} + \sin 45°\,\underline{j} = (-\underline{i} + \underline{j})/\sqrt{2}$$

$$\underline{\omega}' = \omega'\underline{\lambda}_{BA} = \omega'(-\underline{i} + \underline{j})/\sqrt{2} \qquad (2)$$

$$\bar{\underline{v}}' = \underline{\omega}' \times \underline{r} = [\omega'(-\underline{i} + \underline{j})/\sqrt{2}] \times (a/\sqrt{2})(-\underline{i}) = \tfrac{1}{2}\omega' a\underline{k} \qquad (3)$$

IMPULSE-MOMENTUM PRINCIPLE



EQUATING MOMENTS ABOUT LINE BA:

$$H_G\cos 45° + 0 = H'_G + \underline{\lambda}_{BA}\cdot(\underline{r} \times m\bar{\underline{v}}')$$

RECALLING (1), (2), (3), AND VALUE OF $\underline{\lambda}_{BA}$,

$$\tfrac{1}{12}ma^2\omega_0\cos 45° = \tfrac{1}{12}ma^2\omega' + [(-\underline{i} + \underline{j})/\sqrt{2}]\cdot\left[-\tfrac{a}{\sqrt{2}}\underline{i} \times \tfrac{1}{2}m\omega' a\underline{k}\right]$$

$$\frac{\omega_0}{12\sqrt{2}} = \omega'\left(\frac{1}{12} + \frac{1}{4}\right) \qquad \omega' = \frac{1}{4\sqrt{2}}\omega_0 \qquad (4)$$

(a) ANGULAR VELOCITY

FROM (2) AND (4): $\underline{\omega}' = \frac{-\underline{i} + \underline{j}}{\sqrt{2}}\frac{\omega_0}{4\sqrt{2}} \qquad \underline{\omega}' = \frac{1}{8}\omega_0(-\underline{i} + \underline{j})$

(b) VELOCITY OF G. FROM (3) AND (4):

$\bar{\underline{v}}' = \frac{1}{2}\cdot\frac{1}{4\sqrt{2}}\omega_0 a\underline{k} = 0.08839\,\omega_0 a\underline{k} \qquad \bar{\underline{v}}' = 0.0884\,\omega_0 a\underline{k}$

18.30

GIVEN: IMPACT DESCRIBED IN PROB. 18.29.

FIND: IMPULSE ON PLATE AT (a) B, (b) A.

SEE SOLUTION OF PROB. 18.29 FOR IMPULSE-MOMENTUM DIAGRAM AND DETERMINATION OF $\underline{\omega}'$ AND $\bar{\underline{v}}'$.

(a) EQUATING MOMENTS ABOUT A:

$$\tfrac{1}{12}ma^2\omega_0\underline{j} + \tfrac{a}{\sqrt{2}}(\underline{i} - \underline{j}) \times B\Delta t\,\underline{k} = \tfrac{1}{12}ma^2\underline{\omega}' - \tfrac{a}{\sqrt{2}}\underline{j} \times m\bar{\underline{v}}'$$

SUBSTITUTING FOR $\underline{\omega}'$ AND $\bar{\underline{v}}'$ AND PERFORMING PRODUCTS

$$\tfrac{1}{12}ma^2\omega_0\underline{j} - \tfrac{a}{\sqrt{2}}B\Delta t(\underline{j} + \underline{i}) = \tfrac{1}{12}ma^2\tfrac{1}{8}\omega_0(-\underline{i} + \underline{j}) - \tfrac{a}{\sqrt{2}}\underline{j} \times m\tfrac{\omega_0 a}{8\sqrt{2}}\underline{k}$$

$$= ma^2\omega_0\left(-\tfrac{1}{96}\underline{i} + \tfrac{1}{96}\underline{j} - \tfrac{1}{16}\underline{i}\right)$$

EQUATING THE COEFF. OF $\underline{i}$:

$-\frac{a}{\sqrt{2}}B\Delta t = -\frac{7}{96}ma^2\omega_0 \qquad B\Delta t = 0.10312\,m\omega_0 a$

$\underline{B}\Delta t = 0.1031\,m\omega_0 a\underline{k}$

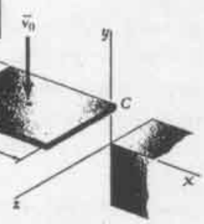
(b) EQUATING SUMS OF VECTORS:

$$\underline{A}\Delta t + \underline{B}\Delta t = m\bar{\underline{v}}'$$

$$\underline{A}\Delta t = m\bar{\underline{v}}' - \underline{B}\Delta t = m(0.08839\,\omega_0 a\underline{k}) - 0.10312\,m\omega_0 a\underline{k}$$

$$\underline{A}\Delta t = -0.01473\,m\omega_0 a\underline{k}$$

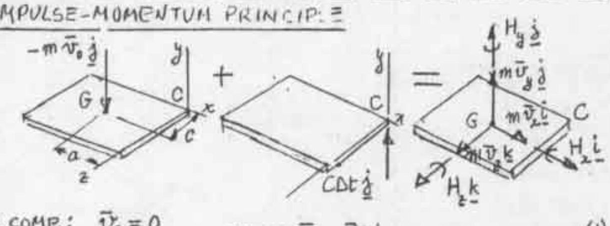
18.31



GIVEN:
RECTANGULAR PLATE OF MASS m FALLING WITH $\bar{v}_0$ AND NO ANG. VELOCITY STRIKES OBSTRUCTION ($e=0$).

FIND:
ANG. VELOCITY OF PLATE IMMEDIATELY AFTER IMPACT.

IMPULSE-MOMENTUM PRINCIPLE



x COMP.: $\bar{v}_x = 0$
z COMP.: $\bar{v}_z = 0$ THUS: $\underline{\bar{v}} = \bar{v}_y\underline{j}$ (1)

y COM.: $-m\bar{v}_0 + C\Delta t = m\bar{v}_y$ $\quad C\Delta t = m(\bar{v}_0 + \bar{v}_y)$ (2)

EQUATING MOMENTS ABOUT C:

$$(-a\underline{i}+c\underline{k})\times(-m\bar{v}_0\underline{j}) = (-a\underline{i}+c\underline{k})\times m\bar{v}_y\underline{j} + H_x\underline{i}+H_y\underline{j}+H_z\underline{k}$$

$$m\bar{v}_0(a\underline{k}+c\underline{i}) = -m\bar{v}_y(a\underline{k}+c\underline{i}) + H_x\underline{i}+H_y\underline{j}+H_z\underline{k} \quad (3)$$

SINCE $e=0$, PLATE ROTATES ABOUT C IMMEDIATELY AFTER IMPACT

$$\underline{\bar{v}} = \underline{\omega}\times\underline{r}_{G/C}: \quad \bar{v}_y\underline{j} = \begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ \omega_x & \omega_y & \omega_z\\ -a & 0 & c\end{vmatrix} = \omega_y c\underline{i} - (\omega_x c+\omega_z a)\underline{j} + \omega_y a\underline{k}$$

EQUATE COEFF. OF UNIT VECTORS: $\omega_y = 0$, $\bar{v}_y = -(\omega_x c+\omega_z a)$ (4)

USE EQS. (18.10):

$$H_x = \bar{I}_x\omega_x = \tfrac{1}{12}m(2c)^2\omega_x = \tfrac{1}{3}mc^2\omega_x$$
$$H_y = \bar{I}_y\omega_y = 0 \quad \text{[BECAUSE OF (4)]} \qquad (5)$$
$$H_z = \bar{I}_z\omega_z = \tfrac{1}{12}m(2a)^2\omega_z = \tfrac{1}{3}ma^2\omega_z$$

SUBSTITUTE FROM (4) AND (5) INTO (3):

$$m\bar{v}_0(a\underline{k}+c\underline{i}) = +m(\omega_x c+\omega_z a)(a\underline{k}+c\underline{i}) + \tfrac{1}{3}mc^2\omega_x\underline{i} + \tfrac{1}{3}ma^2\omega_z\underline{k}$$
$$= \left(\tfrac{4}{3}mc^2\omega_x + mac\omega_z\right)\underline{i} + \left(\tfrac{4}{3}ma^2\omega_z + mac\omega_x\right)\underline{k}$$

DIVIDE BY m AND EQUATE COEFF. OF UNIT VECTORS:

$$\tfrac{4}{3}c^2\omega_x + ac\,\omega_z = \bar{v}_0 c \qquad (6)$$
$$ac\,\omega_x + \tfrac{4}{3}a^2\omega_z = \bar{v}_0 a \qquad (7)$$

SOLVE (6) AND (7) SIMULTANEOUSLY:

$$\omega_x = 3\bar{v}_0/7c, \quad \omega_z = 3\bar{v}_0/7a \qquad \underline{\omega} = \tfrac{3}{7}\bar{v}_0\left(\tfrac{1}{c}\underline{i} + \tfrac{1}{a}\underline{k}\right)$$

18.32

GIVEN: IMPACT DESCRIBED IN PROB. 18.31

FIND:
(a) VELOCITY OF G IMMEDIATELY AFTER IMPACT,
(b) IMPULSE ON PLATE DURING IMPACT.

(a) FROM SOLUTION OF PROB. 18.31:

EQS. (1) AND (4): $\underline{\bar{v}} = \bar{v}_y\underline{j} = -(\omega_x c + \omega_z a)\underline{j}$

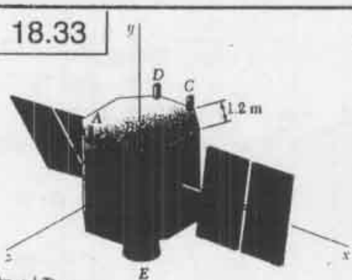
FROM ANSWER TO PROB. 18.31: $\omega_x = \frac{3\bar{v}_0}{7c}$, $\omega_z = \frac{3\bar{v}_0}{7a}$

THUS: $\underline{\bar{v}} = -\left(\frac{3\bar{v}_0}{7} + \frac{3\bar{v}_0}{7}\right)\underline{j}$ $\qquad \underline{\bar{v}} = -\frac{6}{7}\bar{v}_0\underline{j}$

(b) FROM IMPULSE-MOMENTUM DIAGRAM OF PROB. 18.31; EQUATING SUMS OF VECTORS:

$$-m\bar{v}_0\underline{j} + C\Delta t\,\underline{j} = m\underline{\bar{v}}$$
$$C\Delta t\,\underline{j} = m\underline{\bar{v}} + m\bar{v}_0\underline{j} = -\tfrac{6}{7}m\bar{v}_0\underline{j} + m\bar{v}_0\underline{j} = \tfrac{1}{7}m\bar{v}_0\underline{j}$$
$$\underline{C}\Delta t = \tfrac{1}{7}m\bar{v}_0\underline{j}$$

18.33



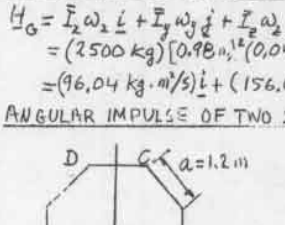
GIVEN: PROBE WITH $m = 2500$ kg, $k_x = 0.98$ m, $k_y = 1.06$ m, $k_z = 1.02$ m; 500-N MAIN THRUSTER E; 20-N THRUSTERS A, B, C, D CAN EXPEL FUEL IN y DIR. PROBE HAS ANG. VELOCITY $\underline{\omega} = (0.040 \text{ rad/s})\underline{i} + (0.060 \text{ rad/s})\underline{k}$

FIND:
(a) WHICH TWO THRUSTERS SHOULD BE USED TO REDUCE $\underline{\omega}$ TO ZERO,
(b) OPERATING TIME OF THESE THRUSTERS,
(c) HOW LONG SHOULD E BE ACTIVATED IF $\underline{\bar{v}}$ IS TO BE UNCHANGED

INITIAL ANGULAR MOMENTUM

$$\underline{H}_G = \bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j} + \bar{I}_z\omega_z\underline{k} = m(k_x^2\omega_x\underline{i} + k_y^2\omega_y\underline{j} + k_z^2\omega_z\underline{k})$$
$$= (2500 \text{ kg})[(0.98\text{ m})^2(0.04 \text{ rad/s})\underline{i} + 0 + (1.02\text{ m})^2(0.06 \text{ rad/s})\underline{k}]$$
$$= (96.04 \text{ kg}\cdot\text{m}^2/\text{s})\underline{i} + (156.06 \text{ kg}\cdot\text{m}^2/\text{s})\underline{k} \qquad (1)$$

ANGULAR IMPULSE OF TWO 20-N THRUSTERS



LET US ASSUME THAT THRUSTERS A AND B WILL BE USED.
FROM GEOMETRY OF OCTOGON,
$x_B = \frac{1}{2}a = \frac{1}{2}(1.2 \text{ m}) = 0.6$ m
$z_B = \frac{1}{2}a + a\sin 45° = 1.2071a = 1.44853$ m
$x_A = -x_B \quad z_A = z_B$

$$\text{ANG. IMPULSE ABOUT } G = \underline{r}_A\times(-F\Delta t_A)\underline{j} + \underline{r}_B\times(-F\Delta t_B)\underline{j}$$
$$= (-x_B\underline{i} + z_B\underline{k})\times(-F\Delta t_A)\underline{j} + (x_B\underline{i} + z_B\underline{k})\times(-F\Delta t_B)\underline{j}$$
$$= x_B(F\Delta t_A - F\Delta t_B)\underline{k} + z_B(F\Delta t_A + F\Delta t_B)\underline{i}$$
$$= (0.6\text{ m})(F\Delta t_A - F\Delta t_B)\underline{k} + (1.44853\text{ m})(F\Delta t_A + F\Delta t_B)\underline{i} \qquad (2)$$

IMPULSE-MOMENTUM PRINCIPLE

SINCE THE FINAL ANGULAR VELOCITY AND, THUS, THE FINAL ANGULAR MOMENTUM MUST BE ZERO, THE SUM OF (1) AND (2) MUST BE ZERO. EQUATING THE COEFF. OF $\underline{i}$ AND $\underline{k}$ TO ZERO:

$$(1.44853 \text{ m})(F\Delta t_A + F\Delta t_B) + 96.04 \text{ kg}\cdot\text{m}^2/\text{s} = 0$$
$$(0.6\text{ m})(F\Delta t_A - F\Delta t_B) + 156.06 \text{ kg}\cdot\text{m}^2/\text{s} = 0$$

OR $\quad F\Delta t_A + F\Delta t_B = -66.302$ N·s (3)
$\qquad F\Delta t_A - F\Delta t_B = -260.1$ N·s (4)

SOLVING (3) AND (4) SIMULTANEOUSLY:

$F\Delta t_A = -163.20$ N·s $\qquad F\Delta t_B = 96.90$ N·s

THE FACT THAT $F\Delta t_A < 0$ INDICATES THAT THE DIAGONALLY OPPOSITE THRUSTER SHOULD BE USED INSTEAD OF A THUS

(a) THRUSTERS B AND C

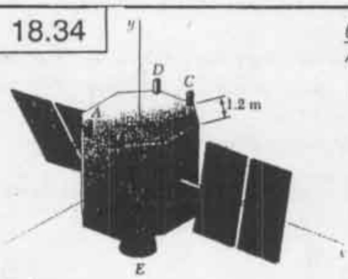
(b) $F\Delta t_B = 96.90$ N·s, $\Delta t_B = \frac{96.90 \text{ N·s}}{20 \text{ N}} = 4.84$ s

$F\Delta t_C = 163.20$ N·s, $\Delta t_C = \frac{163.20 \text{ N·s}}{20 \text{ N}} = 8.16$ s

(c) IF THE VELOCITY $\underline{\bar{v}}$ OF THE MASS CENTER IS TO BE UNCHANGED, THE RESULTANT OF THE LINEAR IMPULSES MUST BE ZERO.

$$-(F\Delta t_B)\underline{j} - (F\Delta t_C)\underline{j} + (500 \text{ N})\Delta t_E\underline{j} = 0$$
$$-96.90 \text{ N·s} - 163.20 \text{ N·s} + (500 \text{ N})\Delta t_E = 0$$
$$\Delta t_E = \frac{260.1 \text{ N·s}}{500 \text{ N}} = 0.5202 \text{ s} \qquad \Delta t_E = 0.520 \text{ s}$$

18.34



GIVEN: PROBE WITH $m = 2500\,kg$, $k_x = 0.98\,m$, $k_y = 1.06\,m$, $k_z = 1.02\,m$; 500-N MAIN THRUSTER E; 20-N THRUSTERS A, B, C, D CAN EXPEL FUEL IN y DIR. PROBE HAS ANG. VELOCITY $\underline{\omega} = (0.060\,rad/s)\underline{i} - (0.040\,rad/s)\underline{k}$

FIND:
(a) WHICH TWO THRUSTERS SHOULD BE USED TO REDUCE $\underline{\omega}$ TO ZERO,
(b) OPERATING TIME OF THESE THRUSTERS,
(c) HOW LONG SHOULD E BE ACTIVATED IF $\bar{\underline{v}}$ IS TO BE UNCHANGED

INITIAL ANGULAR MOMENTUM

$$\underline{H}_G = \bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j} + \bar{I}_z\omega_z\underline{k} = m(k_x^2\omega_x\underline{i} + k_y^2\omega_y\underline{j} + k_z^2\omega_z\underline{k})$$
$$= (2500\,kg)[(0.98\,m)^2(0.06\,rad/s)\underline{i} + 0 + (1.02\,m)^2(-0.04\,rad/s)\underline{k}]$$
$$= (144.06\,kg\cdot m^2/s)\underline{i} - (104.04\,kg\cdot m^2/s)\underline{k} \qquad (1)$$

ANGULAR IMPULSE OF TWO 20-N THRUSTERS

SEE SOLUTION OF PROB. 18.33. ASSUMING THAT THRUSTERS A AND B ARE USED, WE FOUND ANG. IMPULSE ABOUT G

$$= (0.6\,m)(F\Delta t_A - F\Delta t_B)\underline{k} + (1.44853\,m)(F\Delta t_A + F\Delta t_B)\underline{i} \qquad (2)$$

IMPULSE-MOMENTUM PRINCIPLE

SINCE THE FINAL ANG. VELOCITY AND, THUS, THE FINAL ANG. MOMENTUM MUST BE ZERO, THE SUM OF (1) AND (2) MUST BE ZERO. EQUATING THE COEFF. OF $\underline{i}$ AND $\underline{k}$ TO ZERO,

$$(1.44853\,m)(F\Delta t_A + F\Delta t_B) + 144.06\,kg\cdot m^2/s = 0$$
$$(0.6\,m)(F\Delta t_A - F\Delta t_B) - 104.04\,kg\cdot m^2/s = 0$$

OR
$$F\Delta t_A + F\Delta t_B = -99.453\,N\cdot s \qquad (3)$$
$$F\Delta t_A - F\Delta t_B = 173.40\,N\cdot s \qquad (4)$$

SOLVING (3) AND (4) SIMULTANEOUSLY:

$$F\Delta t_A = 36.974\,N\cdot s \qquad F\Delta t_B = -136.43\,N\cdot s$$

THE FACT THAT $F\Delta t_B < 0$ INDICATES THAT THE THRUSTER D, WHICH IS DIAGONALLY OPPOSITE TO B SHOULD BE USED INSTEAD OF B. THUS:

(a) THRUSTERS A AND D ◀

(b) $F\Delta t_A = 36.974\,N\cdot s$, $\Delta t_A = \dfrac{36.974\,N\cdot s}{20\,N} = 1.8487\,s$

$F\Delta t_D = 136.43\,N\cdot s$, $\Delta t_D = \dfrac{136.43\,N\cdot s}{20\,N} = 6.821\,s$

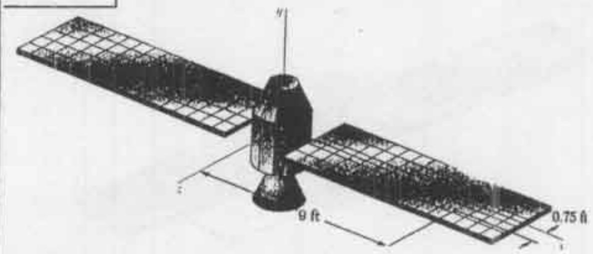
$\Delta t_A = 1.849\,s$; $\Delta t_D = 6.82\,s$ ◀

(c) IF THE VELOCITY $\bar{v}$ OF THE MASS CENTER IS TO BE UNCHANGED, THE RESULTANT OF THE LINEAR IMPULSES MUST BE ZERO.

$$-(F\Delta t_A)\underline{j} - (F\Delta t_D)\underline{j} + (500\,N)\Delta t_E\,\underline{j} = 0$$
$$-36.974\,N\cdot s - 136.43\,N\cdot s + (500\,N)\Delta t_E = 0$$
$$\Delta t_E = \frac{173.40\,N\cdot s}{500\,N} = 0.34685$$

$\Delta t_E = 0.347\,s$ ◀

18.35



GIVEN:
PROBE WITH PRINCIPAL CENTROIDAL AXES x, y, z, AND $W = 3000\,lb$, $k_x = 1.375\,ft$, $k_y = 1.425\,ft$, $k_z = 1.250\,ft$. PROBE HAS NO ANG. VELOCITY WHEN STRUCK AT A BY 5-OZ METEORITE WITH VELOCITY RELATIVE TO PROBE

$$\underline{v}_0 = (2400\,ft/s)\underline{i} - (3000\,ft/s)\underline{j} + (3200\,ft/s)\underline{k}$$

METEORITE EMERGES ON OTHER SIDE OF PANEL MOVING IN SAME DIRECTION WITH SPEED REDUCED BY 20%
FIND: FINAL ANGULAR VELOCITY OF PROBE.

ANGULAR MOMENTUM OF METEORITE ABOUT G.

$$(\underline{H}_G)_M = \underline{r}_A \times m_M\underline{v}_0$$
$$= [(9\,ft)\underline{i} + (0.75\,ft)\underline{k}] \times \frac{(5/16)\,lb}{32.2\,ft/s^2}[(2400\,ft/s)\underline{i} - 3000\underline{j} + 3200\underline{k}]$$
$$= (9.705\times10^{-3}\,lb\cdot s^2/ft)(-27\underline{k} - 28.8\underline{j} + 1.8\underline{j} + 2.25\underline{i})(10^3\,ft^2/s)$$
$$= (9.705\,lb\cdot ft\cdot s)(2.25\underline{i} - 27\underline{j} - 27\underline{k})$$
$$(\underline{H}_G)_M = (21.836\,lb\cdot ft\cdot s)(\underline{i} - 12\underline{j} - 12\underline{k}) \qquad (1)$$

FINAL ANGULAR MOMENTUM OF PROBE

$$(\underline{H}_G)_P = \bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j} + \bar{I}_z\omega_z\underline{k} = m(k_x^2\omega_x\underline{i} + k_y^2\omega_y\underline{j} + k_z^2\omega_z\underline{k})$$
$$= \frac{3000\,lb}{32.2\,ft/s^2}[(1.375\,ft)^2\omega_x\underline{i} + (1.425\,ft)^2\omega_y\underline{j} + (1.250\,ft)^2\omega_z\underline{k}]$$
$$= (176.15\,lb\cdot ft\cdot s^2)\omega_x\underline{i} + (189.19\,lb\cdot ft\cdot s^2)\omega_y\underline{j} + (145.57\,lb\cdot ft\cdot s^2)\omega_z\underline{k} \qquad (2)$$

WE EXPRESS THAT $(H_G)_P = 0.20(H_G)_M$
RECALLING (1) AND (2):

$$176.15\omega_x\underline{i} + 189.19\omega_y\underline{j} + 145.57\omega_z\underline{k} =$$
$$= 0.20(21.836)(\underline{i} - 12\underline{j} - 12\underline{k})$$
$$= 4.3672(\underline{i} - 12\underline{j} - 12\underline{k})$$

EQUATING THE COEFF. OF THE UNIT VECTORS:

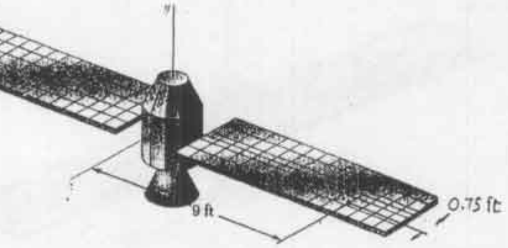
$176.15\,\omega_x = 4.367 \qquad \omega_x = 0.02479\,rad/s$

$189.19\,\omega_y = -52.406 \qquad \omega_y = -0.2770\,rad/s$

$145.57\,\omega_z = -52.406 \qquad \omega_z = -0.3600\,rad/s$

$$\underline{\omega} = (0.0248\,rad/s)\underline{i} - (0.277\,rad/s)\underline{j} - (0.360\,rad/s)\underline{k}$$ ◀

18.36



GIVEN:
PROBE WITH PRINCIPAL CENTROIDAL AXES x, y, z, AND $W = 3000$ lb, $k_x = 1.375$ ft, $k_y = 1.425$ ft, $k_z = 1.250$ ft.
PROBE HAS NO ANGULAR VELOCITY WHEN STRUCK AT A BY 5-OZ METEORITE WHICH EMERGES ON OTHER SIDE OF PANEL MOVING IN SAME DIRECTION WITH SPEED REDUCED BY 25%
FINAL ANGULAR VELOCITY OF PROBE IS

$$\underline{\omega} = (0.05 \text{ rad/s})\underline{i} - (0.12 \text{ rad/s})\underline{j} + \omega_z \underline{k}$$

AND x COMPONENT OF CHANGE IN $\overline{v}$ OF PROBE IS $\Delta \overline{v}_x = -0.675$ in./s.

FIND: (a) ω_z,
(b) RELATIVE VELOCITY $\underline{v}_0$ OF METEORITE WITH WHICH IT STRIKES PANEL.

CONSERVATION OF LINEAR MOMENTUM IN x DIRECTION
SINCE 25% OF LINEAR MOM. OF METEORITE IS TRANSFERRED TO PROBE:

$$0.25 \frac{(5/16)\text{ lb}}{g}(v_0)_x = \frac{3000 \text{ lb}}{g}\Delta \overline{v}_x$$

$$(v_0)_x = 38.4 \times 10^3 \Delta \overline{v}_x = 38.4 \times 10^3(-0.675 \text{ in./s}) = 25.92 \times 10^3 \text{ in./s}$$

$$(v_0)_x = -2160 \text{ ft/s} \quad \triangleleft$$

CONSERVATION OF ANGULAR MOMENTUM ABOUT G
INITIAL ANG. MOM. OF METEORITE:

$$(\underline{H}_G)_M = \underline{r}_A \times m_M \underline{v}_0 = [(9 \text{ ft})\underline{i} + (0.75 \text{ ft})\underline{k}] \times \frac{(5/16)\text{ lb}}{g}[(v_0)_x\underline{i} + (v_0)_y\underline{j} + (v_0)_z\underline{k}]$$

RECALLING THAT $(v_0)_x = -2160$ ft/s AND USING DETERMINANT

$$(H_G)_M = \frac{(5/16)\text{ lb}}{32.2 \text{ ft/s}^2}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 9 \text{ ft} & 0 & 0.75 \text{ ft} \\ -2160 \text{ ft/s} & (v_0)_y & (v_0)_z \end{vmatrix}$$

$$(H_G)_M = \frac{(5/16)\text{ lb}}{32.2 \text{ ft/s}^2}\left[-0.75(v_0)_y\underline{i} - (1620 + 9(v_0)_z)\underline{j} + 9(v_0)_y\underline{k}\right] \quad (1)$$

FINAL ANG. MOM. OF PROBE:

$$(\underline{H}_G)_P = \overline{I}_x\omega_x\underline{i} + \overline{I}_y\omega_y\underline{j} + \overline{I}_z\omega_z\underline{k} = m(k_x^2\omega_x\underline{i} + k_y^2\omega_y\underline{j} + k_z^2\omega_z\underline{k})$$

$$= \frac{3000 \text{ lb}}{32.2 \text{ ft/s}^2}\left[(1.375 \text{ ft})^2(0.05 \text{ rad/s})\underline{i} - (1.425 \text{ ft})^2(0.12 \text{ rad/s})\underline{j} + (1.250 \text{ ft})^2\omega_z\underline{k}\right] \quad (2)$$

SINCE 25% OF ANGULAR MOM. OF METEORITE IS TRANSFERRED TO PROBE, $(\underline{H}_G)_P = 0.25(\underline{H}_G)_M$ OR, RECALLING (1) AND (2):

$$3000\left[(1.375)^2(0.05)\underline{i} - (1.425)^2(0.12)\underline{j} + (1.250)^2\omega_z\underline{k}\right]$$
$$= 0.25(5/16)\left[-0.75(v_0)_y\underline{i} - (1620 + 9(v_0)_z)\underline{j} + 9(v_0)_y\underline{k}\right]$$

EQUATE THE COEFF. OF UNIT VECTORS:

(i) $203.59 = -0.058594(v_0)_y \qquad (v_0)_y = -4840$ ft/s $\triangleleft$

(j) $-731.03 = -126.56 - 0.70313(v_0)_z \qquad (v_0)_z = 859.7$ ft/s $\triangleleft$

(k) $4687.5\,\omega_z = 0.70313(-4840) \qquad \omega_z = -0.726$ rad/s $\triangleleft$

ANSWERS:

(a) $\omega_z = -0.726$ rad/s ◀

(b) $\underline{v}_0 = -(2160 \text{ ft/s})\underline{i} - (4840 \text{ ft/s})\underline{j} + (860 \text{ ft/s})\underline{k}$ ◀

18.37

GIVEN:
RIGID BODY WITH FIXED POINT O, ANG. VELOCITY $\underline{\omega}$, ANGULAR MOMENTUM $\underline{H}_0$, AND KINETIC ENERGY T.
SHOW THAT: (a) $\underline{H}_0 \cdot \underline{\omega} = 2T$,
(b) $\theta < 90°$, WHERE θ IS ANGLE BETWEEN $\underline{\omega}$ AND $\underline{H}_0$

(a) USING PRINCIPAL AXES AS COORDINATE AXES, WE WRITE

$$\underline{H}_0 \cdot \underline{\omega} = (H_x\underline{i} + H_y\underline{j} + H_z\underline{k}) \cdot (\omega_x\underline{i} + \omega_y\underline{j} + \omega_z\underline{k})$$
$$= H_x\omega_x + H_y\omega_y + H_z\omega_z \quad (1)$$

SINCE x, y, z ARE PRINCIPAL AXES,

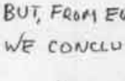
$$H_x = I_x\omega_x \qquad H_y = I_y\omega_y \qquad H_z = I_z\omega_z$$

SUBSTITUTE INTO (1):

$$\underline{H}_0 \cdot \underline{\omega} = I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 \quad (2)$$

BUT, FROM EQ. (18.20), $T = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2)$
WE CONCLUDE THAT

$$\underline{H}_0 \cdot \underline{\omega} = 2T \qquad (\text{Q.E.D.})$$ ◀



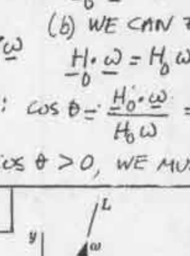
(b) WE CAN EXPRESS THE SCALAR PRODUCT AS

$$\underline{H}_0 \cdot \underline{\omega} = H_0\,\omega\cos\theta$$

THUS: $\cos\theta = \dfrac{\underline{H}_0 \cdot \underline{\omega}}{H_0\,\omega} = \dfrac{2T}{H_0\,\omega} > 0$, SINCE $T > 0$

SINCE $\cos\theta > 0$, WE MUST HAVE $\theta < 90°$ (Q.E.D.) ◀

18.38



GIVEN:
RIGID BODY WITH FIXED POINT O;
$\underline{\omega}$ = INSTANTANEOUS ANG. VELOCITY;
I_{OL} = MOMENT OF INERTIA OF BODY ABOUT LINE OF ACTION OL OF $\underline{\omega}$.
SHOW THAT $T = \frac{1}{2}I_{OL}\,\omega^2$
(a) USING EQS. (9.46) AND (18.19),
(b) CONSIDERING T AS THE SUM OF THE K.E. OF PARTICLES P_i.

(a) EQ. (18.19):

$$T = \tfrac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 - 2I_{xy}\omega_x\omega_y - 2I_{yz}\omega_y\omega_z - 2I_{zx}\omega_z\omega_x)$$

LET
$$\omega_x = \omega\cos\theta_x = \omega\lambda_x$$
$$\omega_y = \omega\cos\theta_y = \omega\lambda_y$$
$$\omega_z = \omega\cos\theta_z = \omega\lambda_z$$

SUBSTITUTE INTO EQ. (18.19):

$$T = \tfrac{1}{2}(I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x)\omega^2$$

BUT, BY EQ. (9.46) OF SEC. 9.16, EXPRESSION IN PARENTHESES IS I_{OL}. THUS:

$$T = \tfrac{1}{2}I_{OL}\,\omega^2 \qquad (\text{Q.E.D.})$$ ◀

(b) EACH PARTICLE P_i DESCRIBES A CIRCLE OF RADIUS ρ_i CENTERED ON OL WITH A SPEED $v_i = \rho_i\,\omega$ THEREFORE

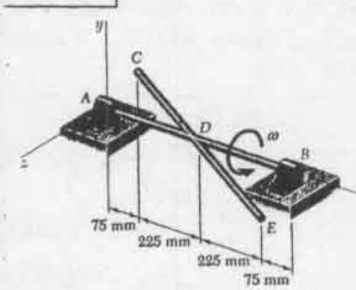
$$T = \tfrac{1}{2}\sum_i(\Delta m_i)v_i^2 = \tfrac{1}{2}\sum_i(\Delta m_i)\rho_i^2\omega^2$$
$$= \tfrac{1}{2}\Big(\sum_i \rho_i^2\Delta m_i\Big)\omega^2$$

BUT $\sum_i \rho_i^2\Delta m_i = I_{OL}$

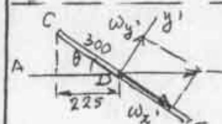
THEREFORE:

$$T = \tfrac{1}{2}I_{OL}\,\omega^2 \qquad (\text{Q.E.D.})$$ ◀

18.39



GIVEN:
ASSEMBLY OF PROB. 18.1.
FOR EACH ROD:
$m = 1.5\text{ kg}$
LENGTH = 600 mm
ASSEMBLY ROTATES WITH $\omega = 12\text{ rad/s}$.
FIND:
KINETIC ENERGY OF ASSEMBLY.



USING PRINCIPAL AXES $x'y'z$:

$\cos\theta = \frac{225}{300}$ $\quad \theta = 41.41°$

$\omega_{x'} = \omega\cos\theta \quad \omega_{y'} = \omega\sin\theta \quad \omega_z = 0$

$\bar{I}_{x'} = 0, \quad \bar{I}_{y'} = \frac{1}{12}m\ell^2, \quad I_z = \frac{1}{12}m\ell^2$

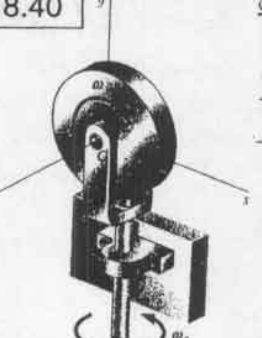
EQ. (18.20): $T = \frac{1}{2}(\bar{I}_{x'}\omega_{x'}^2 + \bar{I}_{y'}\omega_{y'}^2 + \bar{I}_z\omega_z^2)$

$T = \frac{1}{2}(0 + \frac{1}{12}m\ell^2\omega^2\sin^2\theta + 0)$

$= \frac{1}{24}(1.5\text{ kg})(0.6\text{ m})^2(12\text{ rad/s})^2\sin^2 41.41°$

$T = 1.417\text{ J}$ ◀

18.40



GIVEN:
DISK OF PROB. 18.2 OF MASS m AND RADIUS r ROTATING AS SHOWN.
FIND:
KINETIC ENERGY OF DISK.

EQ. (18.20):

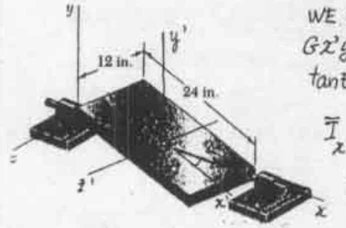
$T = \frac{1}{2}(\bar{I}_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2)$

$= \frac{1}{2}(0 + \frac{1}{4}mr^2\omega_2^2 + \frac{1}{2}mr^2\omega_1^2)$

$T = \frac{1}{8}mr^2(\omega_2^2 + 2\omega_1^2)$ ◀

18.41

GIVEN: 18-lb RECTANGULAR PLATE OF PROB. 18.3 ROTATING WITH $\omega = 5\text{ rad/s}$ ABOUT x AXIS.
FIND: KINETIC ENERGY OF PLATE



WE USE PRINC. CENTROIDAL AXES $Gx'y'z'$ WITH

$\tan\theta = \frac{12\text{ in.}}{24\text{ in.}} = 0.5 \quad \theta = 26.565°$

$\bar{I}_{x'} = \frac{1}{12}\frac{18\text{ lb}}{g}(1\text{ ft})^2 = \frac{1.5}{g}$

$\bar{I}_{z'} = \frac{1}{12}\frac{18\text{ lb}}{g}(2\text{ ft})^2 = \frac{6}{g}$

EQ. (18.20): $T = \frac{1}{2}(\bar{I}_{x'}\omega_{x'}^2 + \bar{I}_{y'}\omega_{y'}^2 + \bar{I}_{z'}\omega_{z'}^2)$

$T = \frac{1}{2}\left[\frac{1.5}{g}(5\text{ rad/s})^2\cos^2 26.565° + 0 + \frac{6}{g}(5\text{ rad/s})^2\sin^2 26.565°\right]$

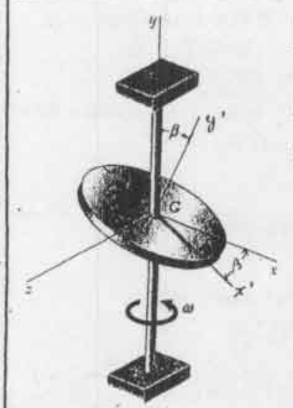
$= \frac{1}{2}\,\frac{1.5}{32.2}(5)^2(\cos^2 26.565° + 4\sin^2 26.565°)$

$= (0.58230\text{ ft·lb})(0.8 + 4\times 0.2) = 0.9317\text{ ft·lb}$

$T = 0.932\text{ ft·lb}$ ◀

18.42

GIVEN: DISK OF PROB. 18.4. WITH $\beta = 25°$.
FIND: KINETIC ENERGY OF DISK.



WE RESOLVE $\underline{\omega} = \omega\underline{j}$ ALONG THE PRINCIPAL CENTROIDAL AXES $Gx'y'z$:

$\omega_{x'} = -\omega\sin\beta, \quad \omega_{y'} = \omega\cos\beta, \quad \omega_z = 0$

EQ (18.10):

$T = \frac{1}{2}(\bar{I}_{x'}\omega_{x'}^2 + \bar{I}_{y'}\omega_{y'}^2 + \bar{I}_z\cdot 0)$

$= \frac{1}{2}(\frac{1}{4}mr^2\omega^2\sin^2\beta + \frac{1}{2}mr^2\omega^2\cos^2\beta)$

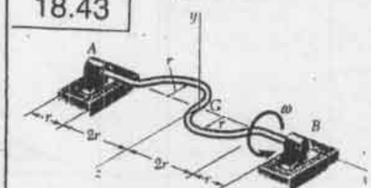
$= \frac{1}{8}mr^2\omega^2(\sin^2\beta + 2\cos^2\beta)$

$= \frac{1}{8}mr^2\omega^2(1 + \cos^2\beta)$

$= \frac{1}{8}mr^2\omega^2(1 + \cos^2 25°)$

$T = 0.228\,mr^2\omega^2$ ◀

18.43



GIVEN:
SHAFT OF PROB. 18.15 OF MASS m, ROTATING WITH ANG. VEL. $\underline{\omega}$.
FIND:
KINETIC ENERGY OF SHAFT.

MASS PER UNIT LENGTH $= m' = \frac{m}{2r + 2\pi r} = \frac{m}{2(\pi+1)r}$

SINCE $\omega_y = \omega_z = 0$, EQ. (18.19) REDUCES TO $T = \frac{1}{2}I_x\omega^2$.
BUT I_x OF BOTH SEMICIRCULAR PORTIONS OF ROD IS SAME AS OF FULL CIRCULAR ROD; THAT IS,

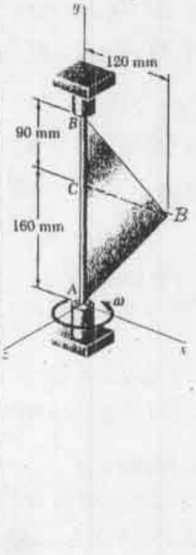
$I_x = \frac{1}{2}(2\pi r m')r^2 = \pi r^3 m' = \pi r^3\frac{m}{2(\pi+1)r} = \frac{\pi r^2}{2(\pi+1)}m$

THEREFORE, $T = \frac{1}{2}\frac{\pi r^2}{2(\pi+1)}m\omega^2 = \frac{\pi}{4(\pi+1)}mr^2\omega^2$

$T = 0.1896\,mr^2\omega^2$ ◀

18.44

GIVEN: TRIANGULAR PLATE OF PROB. 18.16 OF MASS $m = 7.5\text{ kg}$ WITH ANG. VEL. $\omega = 12\text{ rad/s}$
FIND: KINETIC ENERGY OF PLATE



SINCE $\omega_x = \omega_z = 0$, EQ. (18.19) REDUCES TO

$$T = \frac{1}{2}I_y\omega^2 \qquad (1)$$

BUT $I_{y,\text{AREA}} = \frac{1}{12}bh^3$

AND $I_{y,\text{MASS}} = \frac{m}{\frac{1}{2}bh}(\frac{1}{12}bh^3) = \frac{1}{6}mh^2$

WHERE $m = 7.5\text{ kg}$, $h = CB = (0.12\text{ m})$

THUS $I_{y,\text{MASS}} = \frac{1}{6}(7.5\text{ kg})(0.12\text{ m})^2$

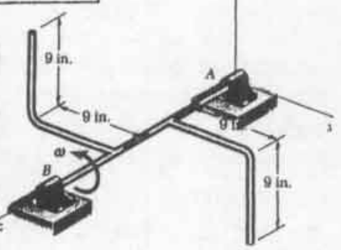
$= 18.00\times 10^{-3}\text{ kg·m}^2$

SUBSTITUTING THIS VALUE FOR I_y AND 12 rad/s FOR ω INTO (1), WE HAVE

$T = \frac{1}{2}(18.00\times 10^{-3}\text{ kg·m}^2)(12\text{ rad/s})^2$

$T = 1.296\text{ J}$ ◀

18.45



GIVEN: ASSEMBLY OF PROB. 18.19 WHICH ROTATES AT 360 rpm. EACH L-SHAPED ARM WEIGHS 5 lb.

FIND: KINETIC ENERGY OF ASSEMBLY.

SINCE $\omega_x = \omega_y = 0$, EQ. (18.19) REDUCES TO

$$T = \frac{1}{2} I_z \omega^2$$

FOR ONE ARM (OF MASS m): $\frac{a}{2}$, $\frac{a}{2}$, G_1, d, a

$$I_z = (\bar{I}_z)_1 + \frac{m}{2} d^2 + (\bar{I}_z)_2$$

$$= \frac{1}{12}\frac{m}{2}a^2 + \frac{m}{2}\left(a^2 + \frac{a^2}{4}\right) + \frac{1}{3}\frac{m}{2}a^2 = \frac{5}{6} ma^2$$

FOR BOTH ARMS: $I_z = \frac{5}{3} ma^2 = \frac{5}{3} \frac{5\ \text{lb}}{32.2\ \text{ft/s}^2}\left(\frac{3}{4}\ \text{ft}\right)^2$

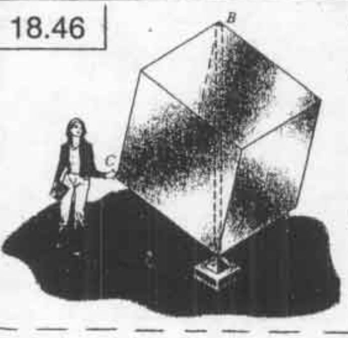
$$= 0.14557\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

AND $\omega = 360 \frac{\text{rev}}{\text{min}} = 360 \frac{2\pi\ \text{rad}}{60\ \text{s}} = 12\pi\ \text{rad/s}$

THUS: $T = \frac{1}{2} I_z \omega^2 = \frac{1}{2}(0.14557\ \text{lb}\cdot\text{ft}\cdot\text{s}^2)(12\pi\ \text{rad/s})^2$

$$T = 103.5\ \text{ft}\cdot\text{lb}$$

18.46



GIVEN: HOLLOW 5×5 ft ALUMINUM CUBE OF PROB. 18.21. STUDENT PUSHES CORNER C FOR 1.25 IN DIRECTION PERPENDICULAR TO PLANE ABC WITH 12.5-lb FORCE, CAUSING CUBE TO COMPLETE 1 REV IN 5 s.

FIND: KINETIC ENERGY IMPARTED TO CUBE.

DIRECT COMPUTATION OF K.E.

WE HAVE $\omega = (2\pi\ \text{rad})/5\ \text{s} = 1.2566\ \text{rad/s}$

WE RECALL FROM PROB. 18.21 THAT AB IS A PRINCIPAL AXIS AND THAT $I_{AB} = \frac{5}{18} ma^2$ THUS, EQ. (18.19) YIELDS

$$T = \frac{1}{2} I_{AB} \omega^2 = \frac{1}{2}\frac{5}{18} ma^2\omega^2 = \frac{5}{36} m (5\ \text{ft})^2 (1.2566\ \text{rad/s})^2$$

BUT WE FOUND IN PROB. 18.21 THAT $W = 226$ lb

THUS: $T = \frac{5}{36}\frac{226\ \text{lb}}{32.2\ \text{ft/s}^2}(5\ \text{ft})^2(1.2566\ \text{rad/s})^2 = 38.48\ \text{ft}\cdot\text{lb}$

$$T = 38.5\ \text{ft}\cdot\text{lb}$$

ALTERNATIVE SOLUTION

WE NOTE THAT THE K.E. IMPARTED TO THE CUBE IS EQUAL TO THE WORK $U_{1\to2}$ DONE BY THE STUDENT!

$$T = U_{1\to2} = F\Delta s$$

WHERE $F = 12.5$ lb AND $\Delta s = \frac{1}{2} v \Delta t = \frac{1}{2}\omega r \Delta t$

RECALLING THAT THE RADIUS r OF THE CIRCLE DESCRIBED BY C IS (SEE HINT IN PROB. 18.21)

$$r = a\sqrt{2/3} = (5\ \text{ft})\sqrt{2/3} = 4.0825\ \text{ft}$$

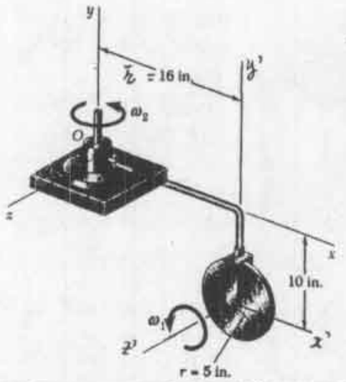
WE HAVE $\Delta s = \frac{1}{2}(1.2566\ \text{rad/s})(4.0825\ \text{ft})(1.2\ \text{s}) = 3.078\ \text{ft}$

AND $T = (12.5\ \text{lb})(3.078\ \text{ft}) = 38.48\ \text{ft}\cdot\text{lb}$, $T = 38.5\ \text{ft}\cdot\text{lb}$

18.47

GIVEN: DISK OF PROB. 18.5 WITH WEIGHT $W = 8$ lb, AND ANGULAR VELOCITIES $\omega_1 = 12$ rad/s AND $\omega_2 = 4$ rad/s.

FIND: KINETIC ENERGY OF DISK.



EQ. (18.17):

$$T = \frac{1}{2} m\bar{v}^2 + \frac{1}{2}(\bar{I}_{x'}\omega_{x'}^2 + \bar{I}_{y'}\omega_{y'}^2 + \bar{I}_{z'}\omega_{z'}^2)$$

$$= \frac{1}{2} m\omega_2^2 \bar{r}^2 + \frac{1}{2}\left(0 + \frac{1}{4} mr^2\omega_2^2 + \frac{1}{2} mr^2\omega_1^2\right)$$

$$= \frac{1}{2}\frac{8\ \text{lb}}{32.2\ \text{ft/s}^2}\left[(4\ \text{rad/s})^2\left(\frac{16}{12}\ \text{ft}\right)^2 + \frac{1}{4}\left(\frac{5}{12}\ \text{ft}\right)^2(4\ \text{rad/s})^2 + \frac{1}{2}\left(\frac{5}{12}\right)^2(12)^2\right]$$

$$= (0.12422)[28.444 + 0.6944 + 12.5]$$

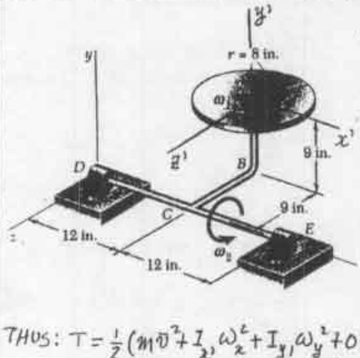
$$= 5.1724\ \text{ft}\cdot\text{lb}$$

$$T = 5.17\ \text{ft}\cdot\text{lb}$$

18.48

GIVEN: DISK OF PROB. 18.6 WITH WEIGHT $W = 6$ lb AND ANGULAR VELOCITIES $\omega_1 = 16$ rad/s AND $\omega_2 = 8$ rad/s.

FIND: KINETIC ENERGY OF DISK.



EQ. (18.17):

$$T = \frac{1}{2} m\bar{v}^2 + \frac{1}{2}(\bar{I}_{x'}\omega_{x'}^2 + \bar{I}_{y'}\omega_{y'}^2 + \bar{I}_{z'}\omega_{z'}^2)$$

WHERE $\bar{v}^2 = \omega_2^2 (AC)^2$

WITH $(AC)^2 = (AB)^2 + (BC)^2$

$(AC)^2 = 2\left(\frac{9}{12}\ \text{ft}\right)^2 = 1.125$

$\bar{I}_{x'} = \frac{1}{4} mr^2 = \frac{1}{4} m\left(\frac{8}{12}\ \text{ft}\right)^2 = 0.1111\, m$

$\bar{I}_{y'} = \frac{1}{2} mr^2 = 0.2222\, m$

THUS: $T = \frac{1}{2}(m\bar{v}^2 + I_{x'}\omega_{x'}^2 + I_{y'}\omega_{y'}^2 + 0)$

$$= \frac{1}{2} m [1.125\omega_2^2 + 0.1111\omega_2^2 + 0.2222\omega_1^2]$$

$$= \frac{1}{2}\frac{6\ \text{lb}}{32.2\ \text{ft/s}^2}[1.236(8\ \text{rad/s})^2 + 0.2222(16\ \text{rad/s})^2]$$

$$T = 12.67\ \text{ft}\cdot\text{lb}$$

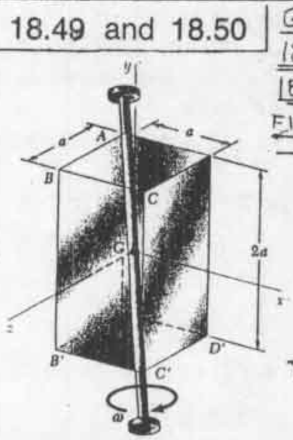
18.49 and 18.50

GIVEN: PARALLELEPIPED OF

18.49: PROB. 18.7 (SOLID)

18.50: PROB. 18.8 (HOLLOW)

FIND: KINETIC ENERGY



SINCE G IS FIXED AND x, y, z ARE PRINCIPAL AXES, USE (18.20):

$$T = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2)$$

WITH $\omega_x = -\frac{a}{\sqrt{a^2+a^2+4a^2}}\omega$

$\omega_x = -\frac{\omega}{\sqrt{6}}$, $\omega_y = \frac{2\omega}{\sqrt{6}}$, $\omega_z = -\frac{\omega}{\sqrt{6}}$

THUS: $T = \frac{1}{12}(I_x + 4I_y + I_z)\omega^2$ (1)

18.49 WE HAVE $I_x = I_z = \frac{1}{12} m[a^2 + (2a)^2] = \frac{5}{12} ma^2$, $I_y = \frac{1}{6} ma^2$

SUBSTITUTE IN (1):

$$T = \frac{1}{12} ma^2\left(\frac{5}{12} + \frac{4}{6} + \frac{5}{12}\right)\omega^2 = \frac{1}{12} ma^2\left(\frac{3}{2}\omega^2\right) \qquad T = \frac{1}{8} ma^2\omega^2$$

(CONTINUED)

18.49 and 18.50 continued

WE RECALL FROM THE PREVIOUS PAGE

$$T=\frac{1}{12}(I_x+4I_y+I_z)\omega^2 \qquad (1)$$

18.50: SEE SOLUTION OF PROB. 18.8 FOR THE DETERMINATION OF THE PRINCIPAL MOMENTS OF INERTIA:

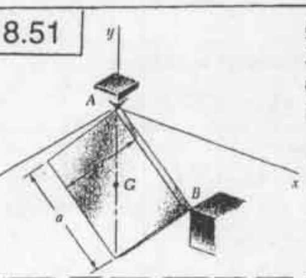
$$I_x=\frac{37}{60}ma^2 \qquad I_y=\frac{9}{30}ma^2 \qquad I_z=\frac{37}{60}ma^2$$

SUBSTITUTE IN EQ.(1):

$$T=\frac{1}{12}ma^2\left(\frac{37}{60}+\frac{4\times 9}{30}+\frac{37}{60}\right)\omega^2=\frac{146}{720}ma^2\omega^2$$

$$T=0.203\,ma^2\omega^2$$

18.51



GIVEN: SQUARE PLATE OF PROB. 18.29 OF MASS m WITH ANG. VEL $\omega_0\underline{j}$ STRIKES B WITH $e=0$

FIND: KINETIC ENERGY LOST IN IMPACT.

WE RECALL FROM PROB. 18.29 THAT $\bar{I}=\frac{1}{12}ma^2$ ABOUT ANY AXIS THROUGH G IN THE PLANE OF THE PLATE.

KINETIC ENERGY BEFORE IMPACT

$$T_0=\frac{1}{2}\bar{I}\omega_0^2=\frac{1}{2}\left(\frac{1}{12}ma^2\right)\omega_0^2=\frac{1}{24}ma^2\omega_0^2$$

KINETIC ENERGY AFTER IMPACT

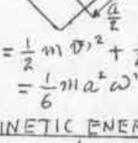


PLATE ROTATES ABOUT AB.
WE FOUND IN PROB. 18.29 THAT $\omega'=\frac{1}{4\sqrt{2}}\omega_0$ AND $\bar{v}'=\omega'(a/2)$

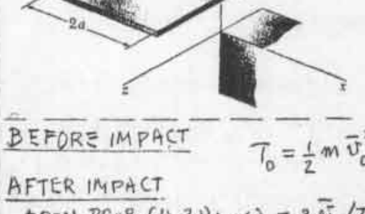
THEREFORE, FROM EQ. (18.17).

$$T=\frac{1}{2}m\bar{v}'^2+\frac{1}{2}\bar{I}\omega'^2=\frac{1}{2}m\omega'^2\left(\frac{a}{2}\right)^2+\frac{1}{2}\left(\frac{1}{12}ma^2\right)\omega'^2$$
$$=\frac{1}{6}ma^2\omega'^2=\frac{1}{6}ma^2\left(\frac{\omega_0}{4\sqrt{2}}\right)^2=\frac{1}{192}ma^2\omega_0^2$$

KINETIC ENERGY LOST

$$=\frac{1}{24}ma^2\omega_0^2-\frac{1}{192}ma^2\omega_0^2=\frac{7}{192}ma^2\omega_0^2$$

18.52



GIVEN: RECTANGULAR PLATE OF PROBS. 18.31 AND 18.32 OF MASS m FALLING WITH VELOCITY $\bar{v}_0$ AND $\omega_0=0$ HITS OBSTRUCTION ($e=0$)

FIND: KINETIC ENERGY LOST IN IMPACT.

BEFORE IMPACT $\qquad T_0=\frac{1}{2}m\bar{v}_0^2$

AFTER IMPACT

FROM PROB. (18.31): $\omega_x=3\bar{v}_0/7c,\ \omega_y=0,\ \omega_z=3\bar{v}_0/7a$

FROM PROB. (18.32): $\quad \bar{\underline{v}}=-(6\bar{v}_0/7)\underline{j}$

EQ. (18.17):

$$T=\frac{1}{2}m\bar{v}^2+\frac{1}{2}\left(\bar{I}_{x'}\omega_x^2+\bar{I}_{y'}\omega_y^2+\bar{I}_{z'}\omega_z^2\right)$$
$$=\frac{1}{2}m\left(\frac{6}{7}\bar{v}_0\right)^2+\frac{1}{2}\left[\frac{1}{3}mc^2\left(\frac{3\bar{v}_0}{7c}\right)^2+0+\frac{1}{3}ma^2\left(\frac{3\bar{v}_0}{7a}\right)^2\right]$$
$$=\frac{1}{2}m\bar{v}_0^2\left(\frac{1}{7}\right)^2[36+3+3]=\frac{1}{2}m\bar{v}_0^2\frac{42}{49}=\frac{1}{2}\frac{6}{7}m\bar{v}_0^2$$

KINETIC ENERGY LOST

$$T_0-T=\frac{1}{2}m\bar{v}_0^2\left(1-\frac{6}{7}\right) \qquad T_0-T=\frac{1}{14}m\bar{v}_0^2$$

18.53

GIVEN: SPACE PROBE OF PROB. 18.35, WITH $W=3000$ lb, $k_x=1.375$ ft, $k_y=1.425$ ft, $k_z=1.250$ ft.

FIND: KINETIC ENERGY OF PROBE IN ITS MOTION ABOUT ITS MASS CENTER AFTER ITS COLLISION WITH METEORITE.

SEE SOLUTION OF PROB. 18.35 FOR DETERMINATION OF

$$\omega_x=0.0248\text{ rad/s},\ \omega_y=-0.277\text{ rad/s},\ \omega_z=-0.360\text{ rad/s}$$

IN MOTION ABOUT G, G IS A FIXED POINT AND THE x,y,z AXES ARE PRINCIPAL AXES. WE USE EQ (18.20):

$$T'=\frac{1}{2}(I_x\omega_x^2+I_y\omega_y^2+I_z\omega_z^2)=\frac{1}{2}m(k_x^2\omega_x^2+k_y^2\omega_y^2+k_z^2\omega_z^2)$$
$$=\frac{1}{2}\frac{3000\text{ lb}}{32.2\text{ ft/s}^2}\left[(1.375\text{ ft}\times 0.0248\text{ rad/s})^2+(1.425\text{ ft}\times 0.277\text{ rad/s})^2+(1.250\text{ ft}\times 0.360\text{ rad/s})^2\right]$$
$$=\frac{1}{2}\frac{3000\text{ lb}}{32.2\text{ ft/s}^2}(0.3595\text{ ft}^2/\text{s}^2)=16.747\text{ ft}\cdot\text{lb}$$

$$T'=16.75\text{ ft}\cdot\text{lb}$$

18.54

GIVEN: SPACE PROBE OF PROB. 18.36, WITH $W=3000$ lb, $k_x=1.375$ ft, $k_y=1.425$ ft, $k_z=1.250$ ft.

FIND: KINETIC ENERGY OF PROBE IN ITS MOTION ABOUT ITS MASS CENTER AFTER ITS COLLISION WITH METEORITE.

SEE STATEMENT AND SOLUTION OF PROB. 18.36 FOR THE VALUES OF $\omega_x,\omega_y,\omega_z$ AFTER COLLISION:

$$\omega_x=0.05\text{ rad/s},\ \omega_y=-0.12\text{ rad/s},\ \omega_z=-0.726\text{ rad/s}$$

IN MOTION ABOUT G, G IS A FIXED POINT AND THE x,y,z AXES ARE PRINCIPAL AXES. WE USE EQ. (18.20):

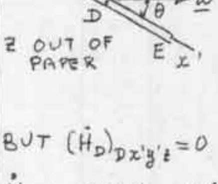
$$T'=\frac{1}{2}(I_x\omega_x^2+I_y\omega_y^2+I_z\omega_z^2)=\frac{1}{2}m(k_x^2\omega_x^2+k_y^2\omega_y^2+k_z^2\omega_z^2)$$
$$=\frac{1}{2}\frac{3000\text{ lb}}{32.2\text{ ft/s}^2}\left[(1.375\text{ ft}\times 0.05\text{ rad/s})^2+(1.425\text{ ft}\times 0.12\text{ rad/s})^2+(1.250\text{ ft}\times 0.726\text{ rad/s})^2\right]$$
$$=\frac{1}{2}\frac{3000\text{ lb}}{32.2\text{ ft/s}^2}(0.8575\text{ ft}^2/\text{s}^2)=39.946\text{ ft}\cdot\text{lb}$$

$$T'=39.9\text{ ft}\cdot\text{lb}$$

18.55

GIVEN: ASSEMBLY OF PROB 18.1
FOR EACH ROD: $m=1.5$ kg, $\ell=600$ mm
ASSEMBLY ROTATES WITH $\omega=12$ rad/s.

FIND: RATE OF CHANGE $\dot{\underline{H}}_D$ OF ANG. MOMENTUM $\underline{H}_D$



FROM PROB. 18.1: $\theta=41.41°$

USING PRINCIPAL AXES x',y',z:

$$\underline{\omega}=\omega(\cos\theta\,\underline{i}'+\sin\theta\,\underline{j}')$$
$$\underline{H}_D=\frac{1}{12}m\ell^2\omega\sin\theta\,\underline{j}'$$

EQ.(18.22) YIELDS

$$\dot{\underline{H}}_D=(\dot{\underline{H}}_D)_{Dx'y'z}+\underline{\Omega}\times\underline{H}_D$$

BUT $(\dot{H}_D)_{Dx'y'z}=0$ AND $\underline{\Omega}=\underline{\omega}$. THUS:

$$\dot{\underline{H}}_D=\underline{\omega}\times\underline{H}_D=\omega(\cos\theta\,\underline{i}'+\sin\theta\,\underline{j}')\times\frac{1}{12}m\ell^2\omega\sin\theta\,\underline{j}'$$
$$=\frac{1}{12}m\ell^2\omega^2\sin\theta\cos\theta\,\underline{k}=\frac{1}{24}m\ell^2\omega^2\sin 2\theta\,\underline{k}$$

WITH GIVEN DATA,

$$\dot{\underline{H}}_D=\frac{1}{24}(1.5\text{ kg})(0.6\text{ m})^2(12\text{ rad/s})^2\sin 82.82°\,\underline{k}$$

$$\dot{\underline{H}}_D=(3.21\text{ N}\cdot\text{m})\underline{k}$$

18.56

GIVEN: DISK OF PROB. 18.2.
FIND: RATE OF CHANGE $\dot{\underline{H}}_G$ OF $\underline{H}_G$.

FROM PROB. 18.2:

$$\underline{\omega} = \omega_2 \underline{j} + \omega_1 \underline{k}$$

$$\underline{H}_G = \tfrac{1}{4} m r^2 (\omega_2 \underline{j} + 2\omega_1 \underline{k})$$

WE NOTE THAT THE ANGULAR VELOCITY OF THE FRAME $Gxyz$ IS

$$\underline{\Omega} = \omega_2 \underline{j}$$

EQ. (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = 0 + \underline{\Omega} \times \underline{H}_G$

THUS: $\dot{\underline{H}}_G = \omega_2 \underline{j} \times \tfrac{1}{4} m r^2 (\omega_2 \underline{j} + 2\omega_1 \underline{k})$

$$\dot{\underline{H}}_G = \tfrac{1}{2} m r^2 \omega_1 \omega_2 \underline{i}$$

18.57

GIVEN: PLATE OF PROB. 18.3 WEIGHING 18 lb, WHICH ROTATES WITH $\omega = 5$ rad/s.
FIND: RATE OF CHANGE $\dot{\underline{H}}_G$ OF $\underline{H}_G$.

WE HAVE $\underline{\omega} = (5 \text{ rad/s}) \underline{i}$

SEE SOLUTION OF PROB. 18.3 FOR THE DERIVATION OF EQ. (2):

$$\underline{H}_G = (0.3727 \text{ lb·ft·s}) \underline{i} - (0.2795 \text{ lb·ft·s}) \underline{k}$$

EQ. (18.22):

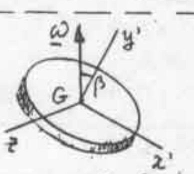
$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{xyz} + \underline{\Omega} \times \underline{H}_G = 0 + \underline{\omega} \times \underline{H}_G$$

THUS: $\dot{\underline{H}}_G = (5 \text{ rad/s}) \underline{i} \times [0.3727 \text{ lb·ft·s}) \underline{i} - (0.2795 \text{ lb·ft·s}) \underline{k}]$

$$\dot{\underline{H}}_G = (1.398 \text{ lb·ft}) \underline{j}$$

18.58

GIVEN: DISK AND SHAFT OF PROB. 18.4.
FIND: RATE OF CHANGE $\dot{\underline{H}}_G$ OF $\underline{H}_G$.



USING THE PRINCIPAL AXES $Gx'y'z$, WE FOUND IN PROB. 18.4 THAT

$$\underline{\omega} = \omega(-\sin\beta \, \underline{i}' + \cos\beta \, \underline{j}')$$

$$\underline{H}_G = \tfrac{1}{4} m r^2 \omega (-\sin\beta \, \underline{i}' + 2\cos\beta \, \underline{j}')$$

EQ. (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gx'y'z} + \underline{\Omega} \times \underline{H}_G = 0 + \underline{\omega} \times \underline{H}_G$

$$\dot{\underline{H}}_G = \omega(-\sin\beta \, \underline{i}' + \cos\beta \, \underline{j}') \times \tfrac{1}{4} m r^2 \omega (-\sin\beta \, \underline{i}' + 2\cos\beta \, \underline{j}')$$

$$= \tfrac{1}{4} m r^2 \omega^2 (-2\sin\beta\cos\beta \, \underline{k} + \cos\beta \sin\beta \, \underline{k})$$

$$= -\tfrac{1}{8} m r^2 \omega^2 \sin 2\beta \, \underline{k} = -\tfrac{1}{8} m r^2 \omega^2 \sin 50°$$

$$\dot{\underline{H}}_G = -0.0958 \, m r^2 \omega^2 \underline{k}$$

18.59

GIVEN: DISK OF PROB. 18.5 WEIGHING 8 lb, WITH $\omega_1 = 12$ rad/s AND $\omega_2 = 4$ rad/s.
FIND: RATE OF CHANGE $\dot{\underline{H}}_A$ OF $\underline{H}_A$.

USING PRINCIPAL CENTROIDAL AXES $Ax'y'z'$:

$$\underline{\omega} = \omega_2 \underline{j} + \omega_1 \underline{k} \qquad \underline{\Omega} = \omega_2 \underline{j}$$

$$\underline{H}_A = \bar{I}_{x'} \omega_{x'} \underline{i} + \bar{I}_{y'} \omega_{y'} \underline{j} + \bar{I}_{z'} \omega_{z'} \underline{k} = \tfrac{1}{4} m r^2 \omega_2 \underline{j} + \tfrac{1}{2} m r^2 \omega_1 \underline{k}$$

EQ. (18.22): $\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Ax'y'z'} + \underline{\Omega} \times \underline{H}_A = 0 + \omega_2 \underline{j} \times \underline{H}_A$

$$\dot{\underline{H}}_A = \omega_2 \underline{j} \times (\tfrac{1}{4} m r^2 \omega_2 \underline{j} + \tfrac{1}{2} m r^2 \omega_1 \underline{k}) = \tfrac{1}{2} m r^2 \omega_1 \omega_2 \underline{i}$$

WITH GIVEN DATA: $\dot{\underline{H}}_A = \frac{1}{2} \frac{8 \text{ lb}}{32.2 \text{ ft/s}^2} (\frac{5}{12} \text{ ft})^2 (12 \text{ rad/s})(4 \text{ rad/s}) \underline{i}$

$$\dot{\underline{H}}_A = (1.035 \text{ lb·ft}) \underline{i}$$

18.60

GIVEN: DISK OF PROB. 18.6 WEIGHING 6 lb WITH $\omega_1 = 16$ rad/s AND $\omega_2 = 8$ rad/s.
FIND: RATE OF CHANGE $\dot{\underline{H}}_A$ OF $\underline{H}_A$.

USING PRINCIPAL CENTROIDAL AXES $Ax'y'z'$:

$$\underline{\omega} = \omega_2 \underline{i} + \omega_1 \underline{j} \qquad \underline{\Omega} = \omega_2 \underline{i}$$

$$\underline{H}_A = \bar{I}_{x'} \omega_{x'} \underline{i} + \bar{I}_{y'} \omega_{y'} \underline{j} + \bar{I}_{z'} \omega_{z'} \underline{k} = \tfrac{1}{4} m r^2 \omega_2 \underline{i} + \tfrac{1}{2} m r^2 \omega_1 \underline{j}$$

EQ. (18.22):

$$\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Ax'y'z'} + \underline{\Omega} \times \underline{H}_A = 0 + \omega_2 \underline{i} \times \underline{H}_A$$

$$= \omega_2 \underline{i} \times (\tfrac{1}{4} m r^2 \omega_2 \underline{i} + \tfrac{1}{2} m r^2 \omega_1 \underline{j}) = \tfrac{1}{2} m r^2 \omega_1 \omega_2 \underline{k}$$

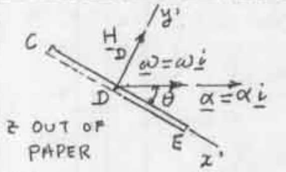
WITH GIVEN DATA:

$$\dot{\underline{H}}_A = \frac{1}{2} \frac{6 \text{ lb}}{32.2 \text{ ft/s}^2} (\frac{8}{12} \text{ ft})^2 (16 \text{ rad/s})(8 \text{ rad/s}) \underline{k}$$

$$\dot{\underline{H}}_A = (5.30 \text{ lb·ft}) \underline{k}$$

18.61

GIVEN: ASSEMBLY OF PROB. 18.1.
FOR EACH ROD: $m = 1.5$ kg, $\ell = 600$ mm
AT INSTANT CONSIDERED, $\underline{\omega} = (12 \text{ rad/s}) \underline{i}$, $\underline{\alpha} = (96 \text{ rad/s}^2) \underline{i}$.
FIND: RATE OF CHANGE $\dot{\underline{H}}_D$ OF $\underline{H}_D$.



FROM PROB. 18.1: $\theta = 41.41°$
USING PRINCIPAL AXES x', y', z:

$$\underline{\omega} = \omega(\cos\theta \, \underline{i}' + \sin\theta \, \underline{j}')$$

$$\underline{\alpha} = \alpha(\cos\theta \, \underline{i}' + \sin\theta \, \underline{j}')$$

$$\underline{H}_D = \tfrac{1}{12} m \ell^2 \omega \sin\theta \, \underline{j}'$$

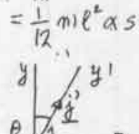
$$(\dot{\underline{H}}_D)_{Dx'y'z} = \tfrac{1}{12} m \ell^2 \dot{\omega} \sin\theta \, \underline{j}' = \tfrac{1}{12} m \ell^2 \alpha \sin\theta \, \underline{j}'$$

APPLY EQ. (18.22), OBSERVING THAT $\underline{\Omega} = \underline{\omega}$:

$$\dot{\underline{H}}_D = (\dot{\underline{H}}_D)_{Dx'y'z} + \underline{\Omega} \times \underline{H}_D = (\dot{\underline{H}}_D)_{Dx'y'z} + \underline{\omega} \times \underline{H}_D$$

$$= \tfrac{1}{12} m \ell^2 \alpha \sin\theta \, \underline{j}' + \omega(\cos\theta \, \underline{i}' + \sin\theta \, \underline{j}') \times \tfrac{1}{12} m \ell^2 \omega \sin\theta \, \underline{j}'$$

$$= \tfrac{1}{12} m \ell^2 \alpha \sin\theta \, \underline{j}' + \tfrac{1}{12} m \ell^2 \omega^2 \cos\theta \sin\theta \, \underline{k}$$



BUT $\underline{j}' = \sin\theta \, \underline{i} + \cos\theta \, \underline{j}$

THUS:

$$\dot{\underline{H}}_D = \tfrac{1}{12} m \ell^2 \alpha \sin\theta (\sin\theta \, \underline{i} + \cos\theta \, \underline{j}) + \tfrac{1}{12} m \ell^2 \omega^2 \cos\theta \sin\theta \, \underline{k}$$

$$\dot{\underline{H}}_D = \tfrac{1}{12} m \ell^2 \sin\theta (\alpha \sin\theta \, \underline{i} + \alpha \cos\theta \, \underline{j} + \omega^2 \cos\theta \, \underline{k}) \quad (1)$$

WITH GIVEN DATA:
$m = 1.5$ kg, $\ell = 0.6$ m, $\omega = 12$ rad/s, $\alpha = 96$ rad/s², $\theta = 41.41°$.

$$\dot{\underline{H}}_D = \tfrac{1}{12}(1.5 \text{ kg})(0.6 \text{ m})^2 \sin 41.41° [(96 \text{ rad/s}^2) \sin 41.41° \, \underline{i} + (96 \text{ rad/s}^2) \cos 41.41° \, \underline{j} + (12 \text{ rad/s})^2 \cos 41.41° \, \underline{k}]$$

$$\dot{\underline{H}}_D = (1.890 \text{ N·m}) \underline{i} + (2.14 \text{ N·m}) \underline{j} + (3.21 \text{ N·m}) \underline{k}$$

18.62

GIVEN: ASSEMBLY OF PROB. 18.1.
FOR EACH ROD: $m = 1.5$ kg, $\ell = 600$ mm.
AT INSTANT CONSIDERED, $\underline{\omega} = (12 \text{ rad/s})$, $\underline{\alpha} = -(96 \text{ rad/s}^2) \underline{i}$.
FIND: RATE OF CHANGE $\dot{\underline{H}}_D$ OF $\underline{H}_D$.

SUBSTITUTE GIVEN DATA INTO EQ. (1) OF PROB. 18.61.

$$\dot{\underline{H}}_D = \tfrac{1}{12} m \ell^2 \sin\theta (\alpha \sin\theta \, \underline{i} + \alpha \cos\theta \, \underline{j} + \omega^2 \cos\theta \, \underline{k}) \quad (1)$$

$$\dot{\underline{H}}_D = \tfrac{1}{12}(1.5 \text{ kg})(0.6 \text{ m})^2 \sin 41.41° [-(96 \text{ rad/s}^2) \sin 41.41° \, \underline{i} + (-96 \text{ rad/s}^2) \cos 41.41° \, \underline{j} + (12 \text{ rad/s})^2 \cos 41.41° \, \underline{k}]$$

$$\dot{\underline{H}}_D = -(1.890 \text{ N·m}) \underline{i} - (2.14 \text{ N·m}) \underline{j} + (3.21 \text{ N·m}) \underline{k}$$

18.63 GIVEN: AT INSTANT CONSIDERED, 18-lb PLATE OF PROB. 18.3 HAS $\underline{\omega} = (5\ \text{rad/s})\underline{i}$ AND $\underline{\alpha} = -(20\ \text{rad/s}^2)\underline{i}$.
FIND: RATE OF CHANGE $\dot{\underline{H}}_G$ OF $\underline{H}_G$

SEE SOLUTION OF PROB. 18.3 FOR THE DERIVATION OF EQ.(1):

$$\underline{H}_G = [(0.074534\ \text{lb·ft·s}^2)\underline{i} - (0.055901\ \text{lb·ft·s}^2)\underline{k}]\omega \quad (1)$$

SINCE $\dot{\omega} = \alpha$, WE HAVE

$$(\dot{\underline{H}}_G)_{xyz} = (0.074534\,\underline{i} - 0.055901\,\underline{k})\alpha$$

SINCE $\Omega = \omega$, EQ.(18.22) YIELDS

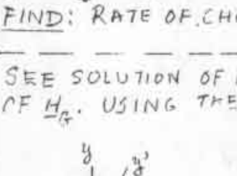
$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{xyz} + \underline{\omega}\times\underline{H}_G$$
$$= (0.074534\,\underline{i} - 0.055901\,\underline{k})\alpha + \omega\,\underline{i}\times(0.074534\,\underline{i} - 0.055901\,\underline{k})\omega$$
$$= 0.074534\,\alpha\,\underline{i} - 0.055901\,\alpha\,\underline{k} + 0.055901\,\omega^2\,\underline{j}$$

LETTING $\alpha = -20\ \text{rad/s}^2$ AND $\omega = 5\ \text{rad/s}$,

$$\dot{\underline{H}}_G = 0.074534(-20)\,\underline{i} + 0.055901(5)^2\,\underline{j} - 0.055901(-20)\,\underline{k}$$
$$\dot{\underline{H}}_G = -(1.491\ \text{lb·ft})\underline{i} + (1.398\ \text{lb·ft})\underline{j} + (1.118\ \text{lb·ft})\underline{k}$$

18.64 GIVEN: AT INSTANT CONSIDERED, SHAFT OF PROB. 18.4 HAS ANGULAR VELOCITY $\underline{\omega} = \omega\,\underline{j}$ AND ANGULAR ACCELERATION $\underline{\alpha} = \alpha\,\underline{j}$
FIND: RATE OF CHANGE $\dot{\underline{H}}_G$ OF $\underline{H}_G$

SEE SOLUTION OF PROB. 18.4 FOR THE DETERMINATION OF $\underline{H}_G$. USING THE PRINCIPAL CENTROIDAL AXES $Gx'y'z$, WE OBTAINED EQ.(1):



$$\underline{H}_G = \tfrac{1}{4}mr^2\omega(-\sin\beta\,\underline{i}' + 2\cos\beta\,\underline{j}')$$

TO REVERT TO THE ORIGINAL AXES $Gxyz$, WE OBSERVE THAT

$$\underline{i}' = \underline{i}\cos\beta - \underline{j}\sin\beta$$
$$\underline{j}' = \underline{i}\sin\beta + \underline{j}\cos\beta$$

SUBSTITUTING INTO (1):

$$\underline{H}_G = \tfrac{1}{4}mr^2\omega\,[-\sin\beta(\underline{i}\cos\beta - \underline{j}\sin\beta) + 2\cos\beta(\underline{i}\sin\beta + \underline{j}\cos\beta)]$$
$$= \tfrac{1}{4}mr^2\omega\,[\sin\beta\cos\beta\,\underline{i} + (1+\cos^2\beta)\,\underline{j}]$$

SINCE $\dot{\omega} = \alpha$

$$(\dot{\underline{H}}_G)_{Gxyz} = \tfrac{1}{4}mr^2\alpha\,[\sin\beta\cos\beta\,\underline{i} + (1+\cos^2\beta)\underline{j}]$$

WE USE EQ.(18.22) WITH $\underline{\Omega} = \underline{\omega} = \omega\,\underline{j}$:

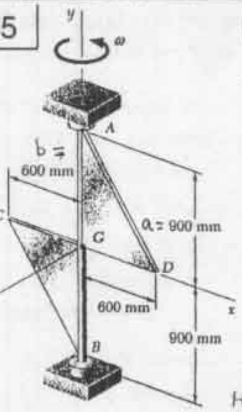
$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = \tfrac{1}{4}mr^2\alpha\,[\sin\beta\cos\beta\,\underline{i} + (1+\cos^2\beta)\underline{j}]$$
$$+\ \omega\underline{j}\times\tfrac{1}{4}mr^2\omega[\sin\beta\cos\beta\,\underline{i} + (1+\cos^2\beta)\,\underline{j}]$$

$$\dot{\underline{H}}_G = \tfrac{1}{4}mr^2\alpha\,[\sin\beta\cos\beta\,\underline{i} + (1+\cos^2\beta)\underline{j}] - \tfrac{1}{4}mr^2\omega^2\sin\beta\cos\beta\,\underline{k}$$

LETTING $\beta = 25°$:

$$\dot{\underline{H}}_G = \tfrac{1}{4}mr^2\alpha\,(0.38302\,\underline{i} + 1.8214\,\underline{j}) - \tfrac{1}{4}mr^2\omega^2(0.38302)\underline{k}$$
$$\dot{\underline{H}}_G = mr^2(0.0958\,\alpha\,\underline{i} + 0.455\,\alpha\,\underline{j} - 0.0958\,\omega^2\,\underline{k})$$

18.65



GIVEN:
ASSEMBLY CONSISTING OF TWO TRIANGULAR PLATES, EACH OF MASS $m = 5\ \text{kg}$, WELDED TO VERTICAL SHAFT. ASSEMBLY ROTATES WITH CONSTANT $\omega = 8\ \text{rad/s}$.
FIND:
DYNAMIC REACTIONS AT A AND B.

SINCE $\underline{\omega} = \omega\underline{j}$, EQS. (18.7) YIELD

$$H_x = -I_{xy}\omega,\ H_y = I_y\omega,\ H_z = -I_{yz}\omega \quad (1)$$

MOMENTS AND PRODUCTS OF INERTIA:

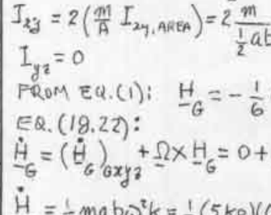
$$I_y = 2\left(\tfrac{m}{A}I_{y,\text{AREA}}\right) = 2\,\tfrac{m}{\frac{1}{2}ab}\left(\tfrac{1}{12}ab^3\right) = \tfrac{1}{3}mb^2 \quad \text{[cf. front cover]}$$
$$I_{xy} = 2\left(\tfrac{m}{A}I_{xy,\text{AREA}}\right) = 2\,\tfrac{m}{\frac{1}{2}ab}\left(\tfrac{1}{24}a^2b^2\right) = \tfrac{1}{6}mab \quad \text{[cf. Sample Prob. 9.6]}$$
$$I_{yz} = 0$$

FROM EQ.(1): $\underline{H}_G = -\tfrac{1}{6}mab\omega\,\underline{i} + \tfrac{1}{3}mb^2\omega\,\underline{j} \quad (2)$

EQ. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + \underline{\omega}\times\underline{H}_G = \omega\underline{j}\times m\omega(-\tfrac{1}{6}ab\,\underline{i} + \tfrac{1}{3}b^2\underline{j})$$
$$\dot{\underline{H}}_G = \tfrac{1}{6}mab\omega^2\underline{k} = \tfrac{1}{6}(5\ \text{kg})(0.9\ \text{m})(0.6\ \text{m})(8\ \text{rad/s})^2 = (28.8\ \text{N·m})\underline{k}$$

EQUATIONS OF MOTION: WE EQUATE THE SYSTEMS OF EXTERNAL AND EFFECTIVE FORCES.

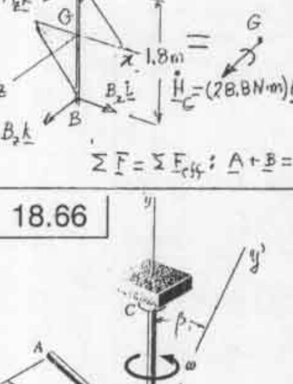


$$\Sigma\underline{M}_B = \Sigma(\underline{M}_B)_{\text{eff}}:$$
$$(1.8\ \text{m})\underline{j}\times(A_x\underline{i} + A_z\underline{k}) = (28.8\ \text{N·m})\underline{k}$$
$$-1.8A_x\underline{k} + 1.8A_z\underline{i} = 28.8\,\underline{k}$$
$$A_x = -16\ \text{N},\quad A_z = 0$$

$\Sigma\underline{F} = \Sigma\underline{F}_{\text{eff}}:\ \underline{A} + \underline{B} = 0$

$$\underline{A} = -(16.00\ \text{N})\underline{i}$$
$$\underline{B} = (16.00\ \text{N})\underline{i}$$

18.66



GIVEN:
ROD AB OF MASS m IS WELDED TO SHAFT CD, OF LENGTH 2b, WHICH ROTATES AT CONSTANT RATE ω.
FIND:
DYNAMIC REACTIONS AT C AND D.

USING THE PRINCIPAL AXES $Gx'y'z$:

$$\bar{I}_{x'} = 0,\ \bar{I}_{y'} = \bar{I}_z = \tfrac{1}{3}mb^2$$
$$\omega_{x'} = -\omega\sin\beta,\ \omega_{y'} = \omega\cos\beta,\ \omega_z = 0$$
$$\underline{H}_G = \bar{I}_{x'}\omega_{x'}\underline{i}' + \bar{I}_{y'}\omega_{y'}\underline{j}' + \bar{I}_z\omega_z\underline{k}$$
$$\underline{H}_G = \tfrac{1}{3}mb^2\omega\cos\beta\,\underline{j}'$$

OR, SINCE $\underline{j}' = \underline{i}\sin\beta + \underline{j}\cos\beta$: $\underline{H}_G = \tfrac{1}{3}mb^2\omega\cos\beta(\sin\beta\,\underline{i} + \cos\beta\,\underline{j}) \quad (1)$

EQ.(18.22):

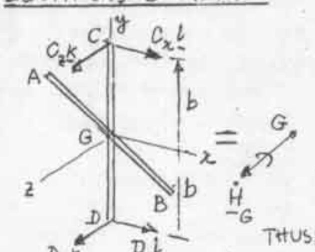
$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + \underline{\omega}\times\underline{H}_G$$
$$= \omega\underline{j}\times\tfrac{1}{3}mb^2\omega\cos\beta(\sin\beta\,\underline{i} + \cos\beta\,\underline{j})$$
$$= -\tfrac{1}{3}mb^2\omega^2\sin\beta\cos\beta\,\underline{k}$$

(CONTINUED)

18.66 continued

WE RECALL FROM PREVIOUS PAGE: $\dot{\underline{H}}_G = -\frac{1}{3} m b^2 \omega^2 \sin\beta \cos\beta\, \underline{k}$

EQUATIONS OF MOTION



WE EQUATE THE SYSTEMS OF EXTERNAL AND EFFECTIVE FORCES

$\Sigma \underline{M}_D = \Sigma (\underline{M}_D)_{eff}$:

$2b\underline{j} \times (C_x \underline{i} + C_z \underline{k}) = \dot{\underline{H}}_G$

$-2bC_x \underline{k} + 2bC_z \underline{i} = -\frac{1}{3} m b^2 \omega^2 \sin\beta\cos\beta\, \underline{k}$

THUS: $C_x = \frac{1}{6} m b \omega^2 \sin\beta \cos\beta$, $C_z = 0$

$\underline{C} = \frac{1}{6} m b \omega^2 \sin\beta \cos\beta\, \underline{i}$ ◀

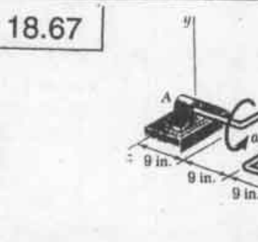
$\Sigma \underline{F} = \Sigma \underline{F}_{eff}$: $\underline{C} + \underline{D} = 0$ $\underline{D} = -\frac{1}{6} m b \omega^2 \sin\beta \cos\beta\, \underline{i}$ ◀

18.67

GIVEN: 16-lb SHAFT WITH UNIFORM CROSS SECTION ROTATES AT CONSTANT RATE $\omega = 12$ rad/s.

FIND: DYNAMIC REACTIONS AT A AND B

MOMENTS AND PRODUCTS OF INERTIA



WE DENOTE BY a THE LENGTH OF AN ELEMENT OF ROD AND BY m ITS MASS. USING THE CENTROIDAL AXES $Gxy'z'$:

$\bar{I}_x = 2ma^2 + 4\left(\frac{1}{3} m a^2\right) = \frac{10}{3} m a^2$

$\bar{I}_{xy'} = 0$

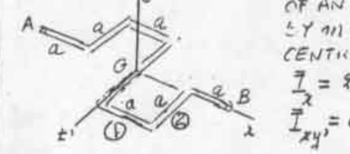
$\bar{I}_{xz'} = 2m\bar{x}_1\bar{z}_1' + 2m\bar{x}_2\bar{z}_2' = 2m\left(\frac{a}{2}\right)a + 2ma\left(\frac{a}{2}\right) = 2ma^2$

$$\underline{H}_G = \bar{I}_x \omega \underline{i} - \bar{I}_{xy'} \omega \underline{j} - \bar{I}_{xz'} \omega \underline{k} = \frac{10}{3} m a^2 \omega \underline{i} - 2ma^2\omega \underline{k} \quad (1)$$

EQ. (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxy'z'} + \underline{\Omega} \times \underline{H}_G = 0 + \omega \underline{i} \times \underline{H}_G$

$\dot{\underline{H}}_G = \omega \underline{i} \times \left(\frac{10}{3} m a^2 \omega \underline{i} - 2ma^2\omega \underline{k}\right) = 2ma^2\omega^2 \underline{j}$

EQUATIONS OF MOTION



$\Sigma \underline{M}_A = \Sigma (\underline{M}_A)_{eff}$:

$4a\underline{i} \times (B_y \underline{j} + B_z \underline{k}) = \dot{\underline{H}}_G$

$4aB_y \underline{k} - 4aB_z \underline{j} = 2ma^2\omega^2 \underline{j}$

THUS: $B_y = 0$, $B_z = -\frac{1}{2} m a \omega^2$

$\underline{B} = -\frac{1}{2} m a \omega^2 \underline{k}$

$\Sigma \underline{F} = \Sigma \underline{F}_{eff}$: $\underline{A} + \underline{B} = 0$ $\underline{A} = -\underline{B} = \frac{1}{2} m a \omega^2 \underline{k}$

DATA: $m = \frac{1}{8}\frac{W}{g} = \frac{1}{8}\frac{16\text{ lb}}{32.2\text{ ft/s}^2} = 0.062112$ lb·s²/ft

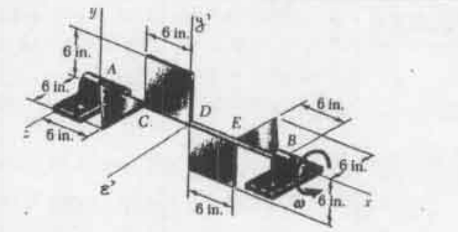
$a = 9\text{ in.} = 0.75$ ft $\omega = 12$ rad/s

THUS:

$A = \frac{1}{2}(0.062112\text{ lb·s}^2/\text{ft})(0.75\text{ ft})(12\text{ rad/s})^2 = 3.354$ lb

$\underline{A} = (3.35\text{ lb})\underline{k}$; $\underline{B} = -(3.35\text{ lb})\underline{k}$ ◀

18.68



GIVEN:

ASSEMBLY WEIGHS 2.7 lb AND ROTATES AT CONSTANT RATE $\omega = 240$ rpm

FIND: DYNAMIC REACTIONS AT A AND B

COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA

WE USE THE CENTROIDAL AXES $Dxy'z'$.

FOR EACH SQUARE: $m = \frac{1}{3}\frac{2.7\text{ lb}}{g} = \frac{0.9}{g}$

$I_x = \frac{1}{3} m a^2 = \frac{1}{3}\frac{0.9}{g}\left(\frac{1}{2}\text{ ft}\right)^2 = 0.075/g$

$I_{xy'} = m\left(\frac{a}{2}\right)\left(-\frac{a}{2}\right) = -\frac{1}{4}\frac{0.9}{g}\left(\frac{1}{2}\text{ ft}\right)^2 = -0.05625/g$, $\bar{I}_{xz} = 0$

FOR EACH TRIANGLE: $m = \frac{1}{6}\frac{2.7\text{ lb}}{g} = \frac{0.45}{g}$

$I_{x,\text{MASS}} = I_{x,\text{AREA}}\frac{m}{A} = \frac{1}{12}a^4 \frac{m}{\frac{1}{2}a^2} = \frac{1}{6} m a^2 = \frac{1}{6}\frac{0.45}{g}\left(\frac{1}{2}\right)^2 = \frac{0.01875}{g}$

$I_{xz',\text{AREA}} = A\bar{x}'\bar{z}' + \bar{I}_{x''z''} = \frac{1}{2}a^2\left(\frac{4}{3}a\right)\left(-\frac{a}{3}\right) + \frac{1}{72}a^4 = -\frac{15}{72}a^4$ [CF. SAMPLE PROB. 9.6]

$I_{xz',\text{MASS}} = I_{xz',\text{AREA}}\frac{m}{A} = -\frac{15}{72}a^4\frac{m}{\frac{1}{2}a^2} = -\frac{5}{12} m a^2 = -\frac{5}{12}\frac{0.45}{g}\left(\frac{1}{2}\text{ ft}\right)^2$

$= -0.046875/g$ $I_{xy'} = 0$

FOR ASSEMBLY:

$I_x = (2 \times 0.075 + 2 \times 0.01875)/g = 0.1875/g$

$I_{xy'} = 2(-0.05625)/g = -0.1125/g$

$I_{xz'} = 2(-0.046875)/g = -0.09375/g$

ANGULAR MOMENTUM $\underline{H}_D$

$\underline{H}_D = I_x \omega \underline{i} - I_{xy'}\omega \underline{j} - I_{xz'}\omega \underline{k}$

$$\underline{H}_D = (0.1875\underline{i} + 0.1125\underline{j} + 0.09375\underline{k})(\omega/g) \quad (1)$$

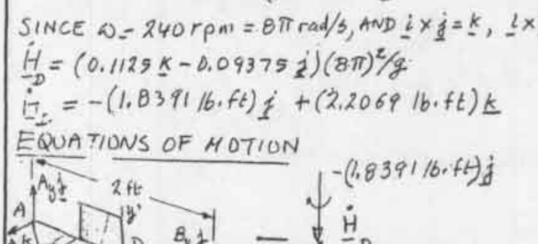
EQ. (18.22): $\dot{\underline{H}}_D = (\dot{\underline{H}}_D)_{Dxy'z'} + \underline{\Omega} \times \underline{H}_D = 0 + \omega\underline{i} \times \underline{H}_G$

SINCE $\omega = 240$ rpm $= 8\pi$ rad/s, AND $\underline{i} \times \underline{j} = \underline{k}$, $\underline{i} \times \underline{k} = -\underline{j}$,

$\dot{\underline{H}}_D = (0.1125\underline{k} - 0.09375\underline{j})(8\pi)^2/g$

$\dot{\underline{H}}_D = -(1.8391\text{ lb·ft})\underline{j} + (2.2069\text{ lb·ft})\underline{k}$

EQUATIONS OF MOTION



$\Sigma \underline{M}_A = \Sigma (\underline{M}_A)_{eff}$: $(2\text{ ft})\underline{i} \times (B_y\underline{j} + B_z\underline{k}) = -1.8391\underline{j} + 2.2069\underline{k}$

$2B_y\underline{k} - 2B_z\underline{j} = -1.8391\underline{j} + 2.2069\underline{k}$

THUS: $B_y = \frac{1}{2}(2.2069) = 1.1034$ lb

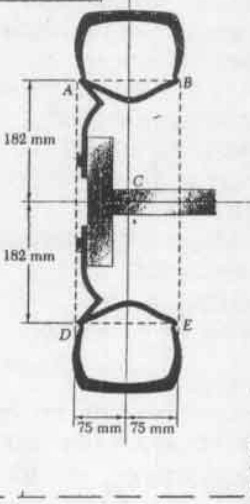
$B_z = \frac{1}{2}(1.8391) = 0.9196$ lb

$\underline{B} = (1.103\text{ lb})\underline{j} + (0.920\text{ lb})\underline{k}$ ◀

$\Sigma \underline{F} = \Sigma \underline{F}_{eff}$: $\underline{A} + \underline{B} = 0$

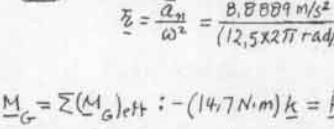
$\underline{A} = -\underline{B}$ $\underline{A} = -(1.103\text{ lb})\underline{j} - (0.920\text{ lb})\underline{k}$ ◀

18.69



GIVEN:
18-kg WHEEL IS ATTACHED TO BALANCING MACHINE.
WHEN MACHINE SPINS AT THE RATE OF 12.5 rev/s, WHEEL IS FOUND TO EXERT ON MACHINE A FORCE-COUPLE SYSTEM CONSISTING OF
$\underline{F} = (160\text{ N})\underline{j}$ APPLIED AT C
AND $\underline{M}_C = (14.7\text{ N·m})\underline{k}$.
FIND:
(a) DISTANCE $\bar{z}$ FROM x AXIS TO G, AND I_{xy} AND I_{zx},
(b) THE TWO CORRECTIVE MASSES REQUIRED TO BALANCE THE WHEEL AND AT WHICH OF POINTS A, B, C, D THEY SHOULD BE PLACED.

(a) THE FORCES EXERTED ON THE WHEEL MUST BE EQUIVALENT TO THE EFFECTIVE FORCES.



$\Sigma \underline{F} = \Sigma \underline{F}_{eff}$:
$-(160\text{ N})\underline{j} = -(18\text{ kg})\bar{a}_n\underline{j}$
$\bar{a}_n = 8.8889\text{ m/s}^2$
BUT $\bar{a}_n = \bar{z}\omega^2$

$$\bar{z} = \frac{\bar{a}_n}{\omega^2} = \frac{8.8889\text{ m/s}^2}{(12.5\times 2\pi\text{ rad/s})^2} = 1.4410\times 10^{-3}\text{ m}$$

$$\bar{z} = 1.441\text{ mm}$$

$\Sigma \underline{M}_G = \Sigma(\underline{M}_G)_{eff}: -(14.7\text{ N·m})\underline{k} = \dot{\underline{H}}_G \quad (1)$

BUT $\underline{H}_G = I_x\omega\underline{i} - I_{xy}\omega\underline{j} - I_{xz}\omega\underline{k}$

AND $\dot{\underline{H}}_G = \underline{\omega}\times\underline{H}_G = \omega\underline{i}\times(I_x\omega\underline{i} - I_{xy}\omega\underline{j} - I_{xz}\omega\underline{k})$

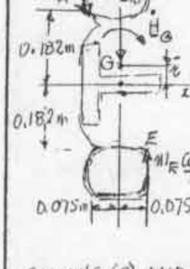
$$\dot{\underline{H}}_G = -I_{xy}\omega^2\underline{k} + I_{xz}\omega^2\underline{j}$$

SUBSTITUTE IN (1): $-14.7\underline{k} = -I_{xy}\omega^2\underline{k} + I_{xz}\omega^2\underline{j}$

THUS:
$$I_{xy} = \frac{14.7\text{ N·m}}{(12.5\times 2\pi\text{ rad/s})^2} = 2.3831\times 10^{-3}\text{ kg·m}^2 \quad \text{AND } I_{xz} = 0$$

$$I_{xy} = 2.38\text{ g·m}^2, \quad I_{xz} = 0$$

(b) WITH CORRECTIVE MASSES THE FORCES EXERTED ON THE WHEEL ARE EQUIVALENT TO ZERO. FOR THE EFFECTIVE FORCES TO ALSO BE EQUIVALENT TO ZERO, THE MASSES MUST BE PLACED AT A AND E:



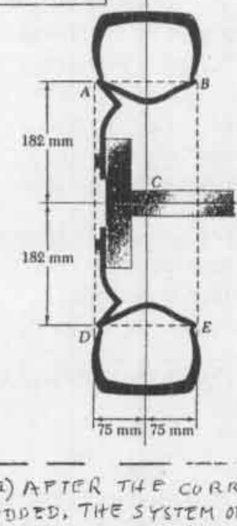
$+\uparrow\Sigma(F_y)_{eff} = 0: m_E a_E - m_A a_A - m\bar{a} = 0$
$(m_E - m_A)(0.182\text{ m})\omega^2 - (18\text{ kg})\bar{z}\omega^2$
$(m_E - m_A)(0.182\text{ m}) = (18\text{ kg})(1.441\times 10^{-3}\text{ m})$
$m_E - m_A = 0.14252\text{ kg} \quad (2)$

$+\circlearrowleft\Sigma(M_C)_{eff} = 0$:
$(m_E a_E + m_A a_A)(0.075\text{ m}) - \dot{H}_G = 0$
$(m_E + m_A)(0.182)\omega^2(0.075) - 14.7 = 0$
$(m_E + m_A)(0.182)(12.5\times 2\pi)^2(0.075) = 14.7$
$m_E + m_A = 0.17458\text{ kg} \quad (3)$

SOLVING (2) AND (3) SIMULTANEOUSLY:
$$m_A = 16.034\times 10^{-3}\text{ kg}, \quad m_E = 158.55\times 10^{-3}\text{ kg}$$

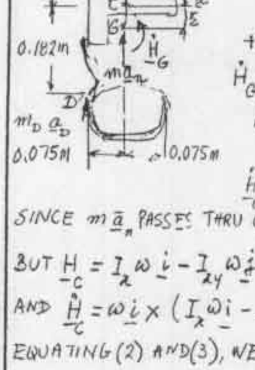
AT A AND E; $m_A = 16.03\text{ g}$, $m_E = 158.6\text{ g}$

18.70



GIVEN:
18-kg WHEEL IS ATTACHED TO BALANCING MACHINE AND SPINS AT THE RATE OF 15 rev/s. MECHANIC FINDS THAT A 170-g MASS PLACED AT B AND A 56-g MASS PLACED AT D ARE NEEDED TO BALANCE WHEEL.
FIND:
BEFORE THE CORRECTIVE MASSES HAVE BEEN ATTACHED:
(a) DISTANCE $\bar{z}$ FROM x AXIS TO G, AND I_{xy} AND I_{zx},
(b) THE FORCE-COUPLE SYSTEM AT C EQUIVALENT TO THE FORCES EXERTED BY THE WHEEL ON THE MACHINE.

(a) AFTER THE CORRECTIVE MASSES HAVE BEEN ADDED, THE SYSTEM OF THE EXTERNAL FORCES IS ZERO THEREFORE, THE SYSTEM OF THE EFFECTIVE FORCES MUST ALSO BE EQUIVALENT TO ZERO. SINCE THE LARGER OF THE TWO MASSES IS PLACED ABOVE THE x AXIS, THE MASS CENTER G OF THE UNBALANCED WHEEL MUST BE BELOW THAT AXIS.



$+\uparrow\Sigma(F_y)_{eff} = 0: m\bar{a}_n - m_B a_B + m_D a_D = 0$
$(18\text{ kg})\bar{z}\omega^2 - (0.170\text{ kg})(0.182\text{ m})\omega^2 +$
$+ (0.056\text{ kg})(0.182\text{ m})\omega^2 = 0$
$18\bar{z} = (0.170)(0.182) - (0.056)(0.182)$
$\bar{z} = 1.1527\times 10^{-3}\text{ m} \quad \bar{z} = 1.153\text{ mm}$

$+\circlearrowleft\Sigma(M_G)_{eff} = 0$:
$\dot{H}_G - m_B a_B(0.075\text{ m}) - m_D a_D(0.075\text{ m}) = 0$
$\dot{H}_G = m_B r_B\omega^2(0.075) + m_D r_D\omega^2(0.075)$
$= (0.170 + 0.056)(0.182)(0.075)\omega^2$
$\dot{\underline{H}}_G = 3.0849\times 10^{-3}\omega^2\underline{k} \quad (1)$

SINCE $m\bar{a}_n$ PASSES THRU C, $\dot{\underline{H}}_C = \dot{\underline{H}}_G = 3.0849\times 10^{-3}\omega^2\underline{k} \quad (2)$

BUT $\underline{H}_C = I_x\omega\underline{i} - I_{xy}\omega\underline{j} - I_{xz}\omega\underline{k}$

AND $\dot{\underline{H}}_C = \omega\underline{i}\times(I_x\omega\underline{i} - I_{xy}\omega\underline{j} - I_{xz}\omega\underline{k}) = -I_{xy}\omega^2\underline{k} + I_{xz}\omega^2\underline{j} \quad (3)$

EQUATING (2) AND (3), WE HAVE $-I_{xy} = 3.0849\times 10^{-3}$, $I_{xz} = 0$

$$I_{xy} = -3.08\text{ g·m}^2, \quad I_{xz} = 0$$

(b) THE FORCE-COUPLE SYSTEM EXERTED ON THE WHEEL BEFORE THE CORRECTIVE MASSES HAVE BEEN ATTACHED IS EQUAL TO THE EFFECTIVE FORCES:

$\underline{F} = m\bar{\underline{a}}_n = m\bar{z}\omega^2\underline{j} = (18\text{ kg})(1.1527\times 10^{-3}\text{ m})(15\times 2\pi\text{ rad/s})^2\underline{j} = (184.3\text{ N})\underline{j}$

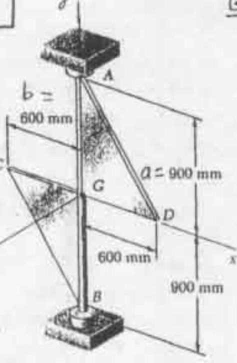
$\underline{M}_C = \underline{M}_G = \dot{\underline{H}}_G = 3.0849\times 10^{-3}(15\times 2\pi\text{ rad/s})^2\underline{k} = (27.4\text{ N·m})\underline{k}$

THE FORCE-COUPLE SYSTEM EXERTED BY THE WHEEL ON THE MACHINE BEFORE THE CORRECTIVE MASSES HAVE BEEN ATTACHED

$$\underline{F}' = -\underline{F} = -(184.3\text{ N})\underline{j},$$

$$\underline{M}'_C = -\underline{M}_C = -(27.4\text{ N·m})\underline{k}$$

18.71



GIVEN: ASSEMBLY OF PROB. 18.65 CONSISTING OF TWO TRIANGULAR PLATES, EACH OF MASS $m = 5$ kg, IS AT REST WHEN A COUPLE OF MOMENT $\underline{M}_0 = (36\text{ N}\cdot\text{m})\underline{j}$ IS APPLIED TO SHAFT AB.

FIND: (a) ANGULAR ACCELERATION OF ASSEMBLY, (b) INITIAL DYNAMIC REACTIONS AT A AND B.

SEE SOLUTION OF PROB. 18.65 FOR DERIVATION OF EQ. (2):

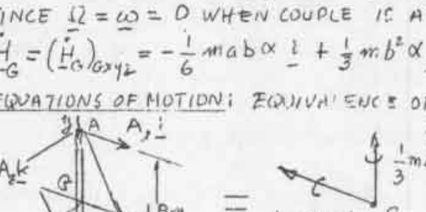
$$\underline{H}_G = -\tfrac{1}{6} mab\omega\,\underline{i} + \tfrac{1}{3} mb^2\omega\,\underline{j} \qquad (2)$$

EQ (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = (\dot{\underline{H}}_G)_{Gxyz} + 0$

SINCE $\underline{\Omega} = \underline{\omega} = 0$ WHEN COUPLE IS APPLIED. THUS

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} = -\tfrac{1}{6} mab\alpha\,\underline{i} + \tfrac{1}{3} mb^2\alpha\,\underline{j} \qquad (3)$$

EQUATIONS OF MOTION: EQUIVALENCE OF APPLIED AND EFFECTIVE FORCES.



$\Sigma\,\underline{M}_B = \Sigma(\underline{M}_B)_{eff}$:

$$(1.8\text{ m})\underline{j}\times(A_x\underline{i} + A_z\underline{k}) + M_0\underline{j} = -\tfrac{1}{6} mab\alpha\,\underline{i} + \tfrac{1}{3} mb^2\alpha\,\underline{j}$$

$$-1.8A_x\underline{k} + 1.8A_z\underline{i} + M_0\underline{j} = -\tfrac{1}{6} mab\alpha\,\underline{i} + \tfrac{1}{3} mb^2\alpha\,\underline{j}$$

EQUATING THE COEFF. OF $\underline{i}, \underline{j}, \underline{k}$:

($\underline{i}$) $(1.8\text{ m})A_z = -\tfrac{1}{6} mab\alpha$ (4)

($\underline{j}$) $M_0 = \tfrac{1}{3} mb^2\alpha$ (5)

($\underline{k}$) $A_x = 0$ (6)

(a) ANGULAR ACCELERATION

SUBSTITUTING GIVEN DATA IN (5):

$36\text{ N}\cdot\text{m} = \tfrac{1}{3}(5\text{ kg})(0.6\text{ m})^2\alpha$

$\alpha = 60.0\text{ rad/s}^2$ ◀

(b) INITIAL DYNAMIC REACTIONS

EQ. (4): $(1.8\text{ m})A_z = -\tfrac{1}{6}(5\text{ kg})(0.9\text{ m})(0.6\text{ m})(60\text{ rad/s}^2)$

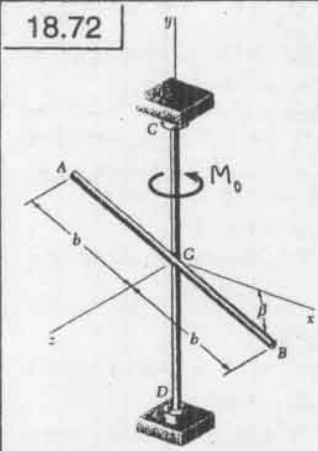
$A_z = -15.00$ N

RECALLING EQ (6), $A_x = 0$, $\underline{A} = -(15.00\text{ N})\underline{k}$ ◀

$\Sigma\underline{F} = \Sigma(\underline{F})_{eff}$:

$\underline{A} + \underline{B} = 0$, $\underline{B} = -\underline{A}$. $\underline{B} = (15.00\text{ N})\underline{k}$ ◀

18.72



GIVEN: ASSEMBLY OF PROB. 18.66, CONSISTING OF ROD OF MASS m WELDED TO SHAFT CD OF LENGTH $2b$. ASSEMBLY IS AT REST WHEN COUPLE OF MOMENT $\underline{M}_0 = M_0\underline{j}$ IS APPLIED TO SHAFT CD

FIND: (a) ANGULAR ACCELERATION OF ASSEMBLY, (b) INITIAL DYNAMIC REACTIONS AT C AND D.

SEE SOLUTION OF PROB. 18.66 FOR DERIVATION OF EQ. (1):

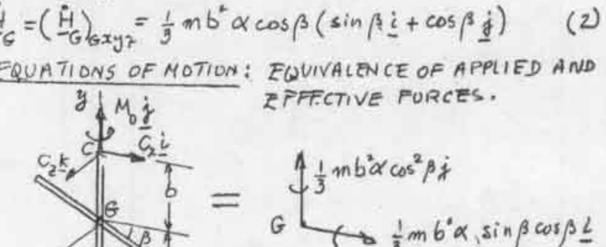
$$\underline{H}_G = \tfrac{1}{3} mb^2\omega\cos\beta(\sin\beta\,\underline{i} + \cos\beta\,\underline{j}) \qquad (1)$$

EQ. (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = (\dot{\underline{H}}_G)_{Gxyz} + 0$

SINCE $\underline{\Omega} = \underline{\omega} = 0$ WHEN COUPLE IS APPLIED. THUS

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} = \tfrac{1}{3} mb^2\alpha\cos\beta(\sin\beta\,\underline{i} + \cos\beta\,\underline{j}) \qquad (2)$$

EQUATIONS OF MOTION: EQUIVALENCE OF APPLIED AND EFFECTIVE FORCES.



$\Sigma\underline{M}_D = \Sigma(\underline{M}_D)_{eff}$:

$$2b\underline{j}\times(C_x\underline{i} + C_z\underline{k}) + M_0\underline{j} = \tfrac{1}{3} mb^2\alpha\sin\beta\cos\beta\,\underline{i} + \tfrac{1}{3} mb^2\alpha\cos^2\beta\,\underline{j}$$

$$-2bC_x\underline{k} + 2bC_z\underline{i} + M_0\underline{j} = \tfrac{1}{3} mb^2\alpha\sin\beta\cos\beta\,\underline{i} + \tfrac{1}{3} mb^2\alpha\cos^2\beta\,\underline{j}$$

EQUATING THE COEFF. OF $\underline{i}, \underline{j}, \underline{k}$:

($\underline{i}$) $2bC_z = \tfrac{1}{3} mb^2\alpha\sin\beta\cos\beta$ (3)

($\underline{j}$) $M_0 = \tfrac{1}{3} mb^2\alpha\cos^2\beta$ (4)

($\underline{k}$) $C_x = 0$ (5)

(a) ANGULAR ACCELERATION

FROM EQ. (4): $\alpha = 3M_0/mb^2\cos^2\beta$ ◀

(b) INITIAL DYNAMIC REACTIONS

FROM EQ (3):

$C_z = \tfrac{1}{6} mb\alpha\sin\beta\cos\beta = \tfrac{1}{6} mb\sin\beta\cos\beta(3M_0/mb^2\cos^2\beta)$

$C_z = (M_0/2b)\tan\beta$

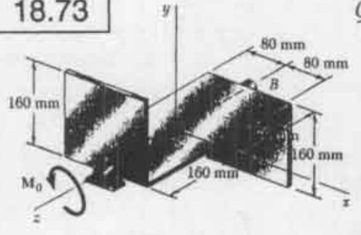
RECALLING EQ. (5), $C_x = 0$,

$\underline{C} = (M_0/2b)\tan\beta\,\underline{k}$ ◀

$\Sigma\underline{F} = \Sigma(\underline{F})_{eff}$:

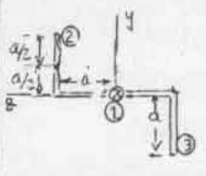
$\underline{C} + \underline{D} = 0$, $\underline{D} = -\underline{C}$ $\underline{D} = -(M_0/2b)\tan\beta\,\underline{k}$ ◀

18.73



GIVEN:
2.4-kg COMPONENT SHOWN IS AT REST WHEN COUPLE $\underline{M}_0 = (0.8\ \text{N·m})\underline{k}$ IS APPLIED TO IT.
FIND:
(a) ANG. ACCELERATION
(b) DYNAMIC REACTIONS AT A AND B IMMEDIATELY AFTER COUPLE IS APPLIED

COMPUTATION OF MOMENTS AND PRODUCTS OF INERTIA



TOTAL MASS = $m = 2.4$ kg, $a = 160$ mm

PORTION 1:
$I_z = \frac{1}{12}\left(\frac{m}{2}\right)a^2 = \frac{1}{24}ma^2, \quad I_{yz} = I_{zx} = 0$

PORTIONS 2 AND 3:
$I_z = 2\left(\frac{m}{4}\right)\left[\frac{a^2}{6} + \left(\frac{a}{2}\right)^2\right] = \frac{5}{24}ma^2$
$I_{yz} = 2\left(\frac{m}{4}\right)\left(\frac{a}{2}\right)a = \frac{1}{4}ma^2, \quad I_{xz} = 0$

COMPONENT:
$I_z = \frac{1}{24}ma^2 + \frac{5}{24}ma^2 = \frac{1}{4}ma^2, \quad I_{yz} = \frac{1}{4}ma^2, \quad I_{xz} = 0$

ANGULAR MOMENTUM:
$\underline{H}_G = -I_{xz}\omega\underline{i} - I_{yz}\omega\underline{j} + I_z\omega\underline{k} = 0 - \frac{1}{4}ma^2\omega\underline{j} + \frac{1}{4}ma^2\omega\underline{k}$

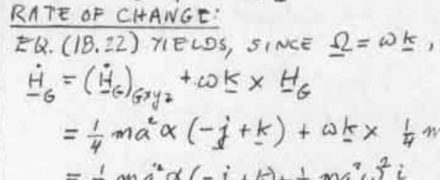
$$\underline{H}_G = \frac{1}{4}ma^2\omega(-\underline{j} + \underline{k}) \qquad (1)$$

RATE OF CHANGE:
EQ. (18.22) YIELDS, SINCE $\underline{\Omega} = \omega\underline{k}$,

$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \omega\underline{k} \times \underline{H}_G$
$= \frac{1}{4}ma^2\alpha(-\underline{j} + \underline{k}) + \omega\underline{k} \times \frac{1}{4}ma^2\omega(-\underline{j} + \underline{k})$
$= \frac{1}{4}ma^2\alpha(-\underline{j} + \underline{k}) + \frac{1}{4}ma^2\omega^2\underline{i}$

$$\dot{\underline{H}}_G = \frac{1}{4}ma^2(\omega^2\underline{i} - \alpha\underline{j} + \alpha\underline{k}) \qquad (2)$$

EQUATIONS OF MOTION



$\Sigma\underline{M}_B = \Sigma(\underline{M}_B)_{eff}$: $2a\underline{k} \times (A_x\underline{i} + A_y\underline{j}) + M_0\underline{k} = \dot{\underline{H}}_G$

$$2aA_x\underline{j} - 2aA_y\underline{i} + M_0\underline{k} = \frac{1}{4}ma^2\omega^2\underline{i} - \frac{1}{4}ma^2\alpha\underline{j} + \frac{1}{4}ma^2\alpha\underline{k} \qquad (3)$$

(a) ANG. ACCELERATION
EQUATE COEFF. OF $\underline{k}$ IN (3):

$M_0 = \frac{1}{4}ma^2\alpha \qquad \alpha = \frac{4M_0}{ma^2} = \frac{4(0.8\ \text{N·m})}{(2.4\ \text{kg})(0.16\ \text{m})^2} = 52.083\ \text{rad/s}^2 \qquad (4)$

$\alpha = 52.1\ \text{rad/s}^2$ ◀

(b) DYNAMIC REACTIONS
EQUATE COEFF. OF $\underline{j}$ IN (3):

$2aA_x = -\frac{1}{4}ma^2\alpha = -\frac{1}{4}ma^2\frac{4M_0}{ma^2} = -M_0$

$$A_x = -\frac{M_0}{2a} = -\frac{0.8\ \text{N·m}}{2(0.16\ \text{m})} = -2.50\ \text{N} \qquad (5)$$

EQUATE COEFF. OF $\underline{i}$ IN (3):

$-2aA_y = \frac{1}{4}ma^2\omega^2 \qquad A_y = -\frac{1}{8}ma\omega^2 \qquad (6)$

SINCE $\omega = 0$, $A_y = 0$; THUS: $\underline{A} = -(2.50\ \text{N})\underline{i}$ ◀

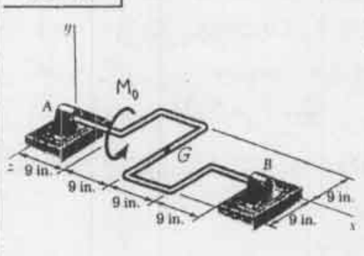
$\Sigma\underline{F} = \Sigma(\underline{F})_{eff}$: $\underline{A} + \underline{B} = 0$, $\underline{B} = -\underline{A}$, $\underline{B} = (2.50\ \text{N})\underline{i}$ ◀

18.74

GIVEN: COMPONENT OF PROB. 18.73.
FIND: DYNAMIC REACTIONS AT A AND B AFTER ONE FULL REVOLUTION

SEE SOLUTION OF PROB. 18.73 FOR DERIVATION OF EQS. (4), (5), AND (6).
FROM EQ. (4), $\alpha = 52.083\ \text{rad/s}^2$.
FOR ONE FULL REVOLUTION, $\theta = 2\pi$ rad
FROM EQS. (15.16):
$\omega^2 = 2\alpha\theta = 2(52.083\ \text{rad/s}^2)(2\pi\ \text{rad}) = 654.49\ \text{rad}^2/\text{s}^2$
EQ. (5): $A_x = -2.50$ N
EQ. (6): $A_y = -\frac{1}{8}(2.4\ \text{kg})(0.16\ \text{m})(654.49\ \text{rad}^2/\text{s}^2) = -31.4$ N
THEREFORE:
$\underline{A} = -(2.50\ \text{N})\underline{i} - (31.4\ \text{N})\underline{j}$; $\underline{B} = -\underline{A} = (2.50\ \text{N})\underline{i} + (31.4\ \text{N})\underline{j}$ ◀

18.75



GIVEN:
16-lb SHAFT OF PROB. 18.67 IS AT REST WHEN A COUPLE $\underline{M}_0$ IS APPLIED TO IT, CAUSING ANGULAR ACCEL. $\underline{\alpha} = (20\ \text{rad/s}^2)\underline{i}$.
FIND:
(a) COUPLE $\underline{M}_0$,
(b) DYNAMIC REACTIONS AT A AND B IMMEDIATELY AFTER $\underline{M}_0$ IS APPLIED.

SEE SOLUTION OF PROB. 18.67 FOR DERIVATION OF

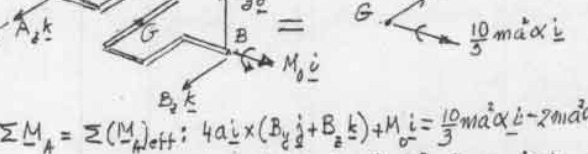
$$\underline{H}_G = \frac{10}{3}ma^2\omega\underline{i} - 2ma^2\omega\underline{k} \qquad (1)$$

EQ. (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = (\dot{\underline{H}}_G)_{Gxyz} + 0$

SINCE $\dot{\omega} = \alpha$: $\quad \dot{\underline{H}}_G = \frac{10}{3}ma^2\alpha\underline{i} - 2ma^2\alpha\underline{k} \qquad (2)$

WHERE $a = 9\ \text{in.} = 0.75$ ft AND $m = \frac{1}{8}(16\ \text{lb}) = 2$ lb

EQUATIONS OF MOTION



$\Sigma\underline{M}_A = \Sigma(\underline{M}_A)_{eff}$: $4a\underline{i} \times (B_y\underline{j} + B_z\underline{k}) + M_0\underline{i} = \frac{10}{3}ma^2\alpha\underline{i} - 2ma^2\alpha\underline{k}$

$$4aB_y\underline{k} - 4aB_z\underline{j} + M_0\underline{i} = \frac{10}{3}ma^2\alpha\underline{i} - 2ma^2\alpha\underline{k} \qquad (3)$$

(a) COUPLE $\underline{M}_0$
EQUATE COEFF. OF $\underline{i}$ IN EQ. (3):

$M_0 = \frac{10}{3}ma^2\alpha = \frac{10}{3}\frac{2\ \text{lb}}{32.2\ \text{ft/s}^2}(0.75\ \text{ft})^2(20\ \text{rad/s}^2) = 2.329$ lb·ft

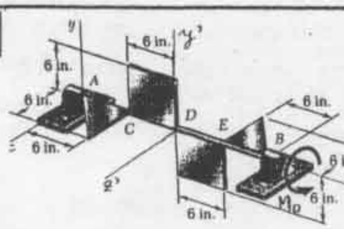
$\underline{M}_0 = (2.33\ \text{lb·ft})\underline{i}$ ◀

(b) DYNAMIC REACTIONS AT $t = 0$
EQUATE COEFF. OF $\underline{j}$ IN EQ. (3): $\quad B_z = 0$
EQUATE COEFF. OF $\underline{k}$ IN EQ. (3):
$4aB_y = -2ma^2\alpha$
$B_y = -\frac{1}{2}ma\alpha = -\frac{1}{2}\frac{2\ \text{lb}}{32.2\ \text{ft/s}^2}(0.75\ \text{ft})(20\ \text{rad/s}^2) = -0.466$ lb
THEREFORE: $\underline{B} = -(0.466\ \text{lb})\underline{j}$

$\Sigma\underline{F} = \Sigma(\underline{F}_{eff})$: $\underline{A} + \underline{B} = 0 \qquad \underline{A} = -\underline{B} = (0.466\ \text{lb})\underline{j}$

THUS: $\quad \underline{A} = (0.466\ \text{lb})\underline{j}$; $\underline{B} = -(0.466\ \text{lb})\underline{j}$ ◀

18.76



GIVEN:
THE 2.7-lb ASSEMBLY OF PROB. 18.68 IS AT REST WHEN A COUPLE $\underline{M}_0$ IS APPLIED TO AXLE AB, CAUSING AN ANGULAR ACCELERATION $\underline{\alpha} = (150 \text{ rad/s}^2)\underline{i}$.

FIND: (a) THE COUPLE $\underline{M}_0$.
(b) THE DYNAMIC REACTIONS AT A AND B IMMEDIATELY AFTER $\underline{M}_0$ IS APPLIED.

SEE SOLUTION OF PROB. 18.68 FOR DERIVATION OF EQ.(1):

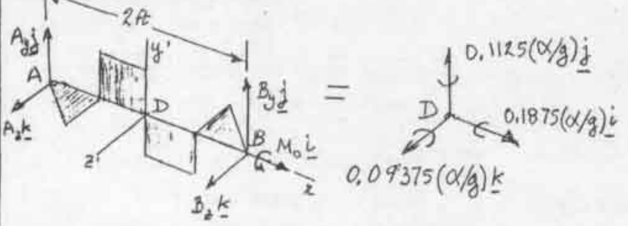
$$\underline{H}_D = (0.1875\,\underline{i} + 0.1125\,\underline{j} + 0.09375\,\underline{k})(\omega/g) \qquad (1)$$

WHERE THE NUMERICAL VALUES ARE EXPRESSED IN lb·ft²

EQ. (18.22): $\dot{\underline{H}}_D = (\dot{\underline{H}})_{Dx'y'z'} + \underline{\Omega}\times\underline{H}_D = (\dot{\underline{H}})_{Dx'y'z'} + 0$

SINCE $\dot{\omega}=\alpha$: $\dot{\underline{H}}_D = (0.1875\,\underline{i} + 0.1125\,\underline{j} + 0.09375)(\alpha/g)$ (2)

EQUATIONS OF MOTION



$\Sigma\underline{M}_A = \Sigma(\underline{M}_A)_{eff}$:

$$(2\text{ ft})\underline{i}\times(B_y\underline{j}+B_z\underline{k}) + M_0\underline{i} = 0.1875(\alpha/g)\underline{i} + 0.1125(\alpha/g)\underline{j} + 0.09375(\alpha/g)\underline{k}$$

$$(2\text{ft})B_y\underline{k} - (2\text{ft})B_z\underline{j} + M_0\underline{i} = 0.1875(\alpha/g)\underline{i} + 0.1125(\alpha/g)\underline{j} + 0.09375(\alpha/g)\underline{k} \qquad (3)$$

(a) COUPLE $\underline{M}_0$

EQUATE COEFF. OF $\underline{i}$ IN EQ.(3):

$$M_0 = 0.1875(\alpha/g) = (0.1875\text{ lb·ft}^2)\frac{150\text{ rad/s}^2}{32.2\text{ ft/s}^2} = 0.873\text{ lb·ft}$$

$$\underline{M}_0 = (0.873\text{ lb·ft})\underline{i}$$

(b) DYNAMIC REACTIONS AT $t=0$.

EQUATE COEFF. OF $\underline{k}$ IN EQ.(3):

$$(2\text{ ft})B_y = 0.09375(\alpha/g) = (0.09375\text{ lb·ft}^2)\frac{150\text{ rad/s}^2}{32.2\text{ ft/s}^2} = 0.43672\text{ lb·ft}$$

$$B_y = 0.218\text{ lb}$$

EQUATE COEFF. OF $\underline{j}$ IN EQ.(3):

$$-(2\text{ft})B_z = 0.1125(\alpha/g) = (0.1125\text{ lb·ft}^2)\frac{150\text{ rad/s}^2}{32.2\text{ ft/s}^2} = 0.52407\text{ lb·ft}$$

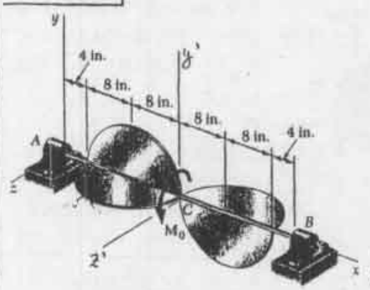
$$B_z = -0.262\text{ lb}$$

THUS: $\underline{B} = (0.218\text{ lb})\underline{j} - (0.262\text{ lb})\underline{k}$

$\Sigma\underline{F} = \Sigma(\underline{F})_{eff}$: $\underline{A}+\underline{B}=0$, $\underline{A} = -\underline{B}$

$$\underline{A} = -(0.218\text{ lb})\underline{j} + (0.262\text{ lb})\underline{k}$$

18.77



GIVEN:
ASSEMBLY WEIGHS 12 lb AND CONSISTS OF 4 SEMICIRCULAR PLATES. ASSEMBLY IS AT REST AT $t=0$ WHEN COUPLE $\underline{M}_0$ IS APPLIED FOR ONE FULL REVOLUTION WHICH LASTS 2 s.

FIND: (a) THE COUPLE $\underline{M}_0$
(b) THE DYNAMIC REACTIONS AT A AND B AT $t=0$

MASS OF ASSEMBLY $= m = \dfrac{12\text{ lb}}{32.2\text{ ft/s}^2} = 0.37267\text{ lb·s}^2/\text{ft}$

RADIUS OF SEMICIRCULAR PLATES $= r = 8\text{ in.} = \frac{2}{3}\text{ ft}$

MOMENTS AND PRODUCTS OF INERTIA

FOR ASSEMBLY: $I_x = 2\left(\frac{m}{2}\right)\frac{r^2}{4} = \frac{1}{4}mr^2$

FOR EACH VERTICAL PLATE: $I_{xy'} = \frac{m}{4}\bar{x}\bar{y} = -\frac{m}{4}r\left(\frac{4r}{3\pi}\right)$

$I_{xy'} = -\dfrac{mr^2}{3\pi}$ $\quad I_{zz'} = 0$

FOR EACH HORIZONTAL PLATE:

$I_{xy'} = 0$ $\quad I_{xz'} = -\dfrac{mr^2}{3\pi}$

FOR ASSEMBLY: $I_{xy'} = I_{xz'} = 2\left(-\dfrac{mr^2}{3\pi}\right) = -\dfrac{2mr^2}{3\pi}$

ANGULAR MOMENTUM

FROM EQS. (18.13) WITH $\omega_x = \omega$, $\omega_{y'} = \omega_{z'} = 0$:

$$\underline{H}_C = I_x\omega\underline{i} - I_{xy'}\omega\underline{j} - I_{xz'}\omega\underline{k} = mr^2\omega\left(\tfrac{1}{4}\underline{i} + \tfrac{2}{3\pi}\underline{j} + \tfrac{2}{3\pi}\underline{k}\right) \quad (1)$$

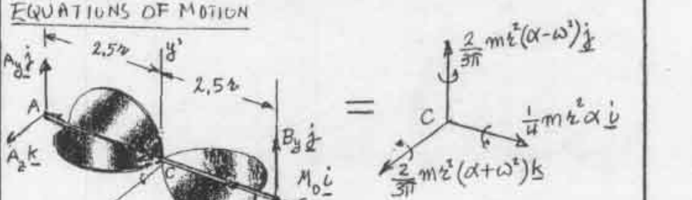
EQ. (18.22): $\dot{\underline{H}}_C = (\dot{\underline{H}}_C)_{Cxy'z'} + \underline{\Omega}\times\underline{H}_C$

$$\dot{\underline{H}}_C = mr^2\alpha\left(\tfrac{1}{4}\underline{i} + \tfrac{2}{3\pi}\underline{j} + \tfrac{2}{3\pi}\underline{k}\right) + \omega\underline{i}\times mr^2\omega\left(\tfrac{1}{4}\underline{i} + \tfrac{2}{3\pi}\underline{j} + \tfrac{2}{3\pi}\underline{k}\right)$$

$$= mr^2\alpha\left(\tfrac{1}{4}\underline{i} + \tfrac{2}{3\pi}\underline{j} + \tfrac{2}{3\pi}\underline{k}\right) + \tfrac{2}{3\pi}mr^2\omega^2(\underline{k}-\underline{j})$$

$$\dot{\underline{H}}_C = \tfrac{1}{4}mr^2\alpha\underline{i} + \tfrac{2}{3\pi}mr^2(\alpha-\omega^2)\underline{j} + \tfrac{2}{3\pi}mr^2(\alpha+\omega^2)\underline{k} \quad (2)$$

EQUATIONS OF MOTION



$\Sigma\underline{M}_A = \Sigma(\underline{M}_A)_{eff}$: $5r\underline{i}\times(B_y\underline{j}+B_z\underline{k}) + M_0\underline{i} = \dot{\underline{H}}_C$

$$5rB_y\underline{k} - 5rB_z\underline{j} + M_0\underline{i} = \tfrac{1}{4}mr^2\alpha\underline{i} + \tfrac{2}{3\pi}mr^2(\alpha-\omega^2)\underline{j} + \tfrac{2}{3\pi}mr^2(\alpha+\omega^2)\underline{k} \quad (3)$$

(a) COUPLE $\underline{M}_0$

EQUATE COEFF. OF $\underline{i}$: $M_0 = \frac{1}{4}mr^2\alpha$

SINCE ASSEMBLY ROTATES THROUGH $\theta = 2\pi$ rad IN 2 s:

$\theta = \frac{1}{2}\alpha t^2$, $\alpha = 2\theta/t^2 = 4\pi/4 = \pi\text{ rad/s}^2$. THUS:

$$M_0 = \tfrac{1}{4}(0.37267\text{ lb·s}^2/\text{ft})\left(\tfrac{2}{3}\text{ft}\right)^2(\pi\text{ rad/s}^2) = 0.1301\text{ lb·ft}$$

$$\underline{M}_0 = (0.1301\text{ lb·ft})\underline{i}$$

(b) DYNAMIC REACTIONS AT $t=0$

EQUATING THE COEFF. OF $\underline{j}$ AND $\underline{k}$ IN (3) AND SETTING $\omega=0$ AND $\alpha = \pi\text{ rad/s}^2$:

($\underline{j}$) $-5rB_z = \frac{2}{3\pi}mr^2(\pi\text{ rad/s}^2)$, $B_z = -\frac{2}{15}(0.37267)\left(\frac{2}{3}\right) = -0.0331$ lb

($\underline{k}$) $5rB_y = \frac{2}{3\pi}mr^2(\pi\text{ rad/s}^2)$. $\quad B_y = +0.0331$ lb

THUS: $\underline{B} = (0.0331\text{ lb})\underline{j} - (0.0331\text{ lb})\underline{k}$

$\Sigma\underline{F} = \Sigma\underline{F}_{eff}$: $\underline{A}+\underline{B}=0$, $\underline{A} = -(0.0331\text{ lb})\underline{j} + (0.0331\text{ lb})\underline{k}$

18.78 GIVEN: ASSEMBLY OF PROB. 18.77

FIND: DYNAMIC REACTIONS AT A AND B AT $t = 2s$.

SEE SOLUTION OF PROB. 18.77 FOR DERIVATION OF EQ.(3):

$$5r B_y \underline{k} - 5r B_z \underline{j} + M_0 \underline{i} = \frac{1}{4} m r^2 \alpha \underline{i} + \frac{2}{3\pi} m r^2 (\alpha - \omega^2) \underline{j} + \frac{2}{3\pi} m r^2 (\alpha + \omega^2) \underline{k} \quad (3)$$

WHERE $m = 0.37267\ \text{lb}\cdot\text{s}^2/\text{ft}$ AND $r = \frac{2}{3}$ ft

SINCE ASSEMBLY ROTATES THROUGH $\theta = 2\pi$ rad in 2 s:

$\theta = \frac{1}{2}\alpha t^2, \quad \alpha = 2\theta/t^2 = 4\pi/4 = \pi\ \text{rad/s}^2$

AT $t = 2\,s$: $\omega = \alpha t = (\pi\ \text{rad/s}^2)(2\,s) = 2\pi$ rad/s

EQUATING THE COEFF. OF $\underline{j}$ AND $\underline{k}$ IN EQ.(3) AND SUBSTITUTING THE ABOVE VALUES:

($\underline{j}$) $-5r B_z = \frac{2}{3\pi} m r^2 (\pi - 4\pi^2) \qquad B_z = -\frac{2}{15} m r (1 - 4\pi)$

$B_z = -\frac{2}{15}(0.37267)(\frac{2}{3})(1 - 4\pi) = +0.383$ lb

($\underline{k}$) $5r B_y = \frac{2}{3\pi} m r^2 (\pi + 4\pi^2) \qquad B_y = \frac{2}{15} m r (1 + 4\pi)$

$B_y = \frac{2}{15}(0.37267)(\frac{2}{3})(1 + 4\pi) = 0.449$ lb

THUS: $\underline{B} = (0.449\ \text{lb})\underline{j} + (0.383\ \text{lb})\underline{k}$ ◀

$\Sigma \underline{F} = \Sigma (\underline{F})_{eff}$: $\underline{A} + \underline{B} = 0$

$A = -\underline{B} \qquad \underline{A} = -(0.449\ \text{lb})\underline{j} - (0.383\ \text{lb})\underline{k}$ ◀

18.79 GIVEN:

FLYWHEEL RIGIDLY ATTACHED TO CRANKSHAFT OF AUTOMOBILE ENGINE IS EQUIVALENT TO 400-mm-DIAM, 15-mm-THICK STEEL PLATE (DENSITY = 7860 kg/m³). AUTOMOBILE TRAVELS ON UNBANKED CURVE OF 200-m RADIUS AT 90 km/h WITH FLYWHEEL ROTATING AT 2700 rpm.

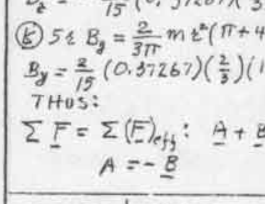
FIND:

MAGNITUDE OF COUPLE EXERTED BY FLYWHEEL ON CRANKSHAFT, ASSUMING AUTOMOBILE TO HAVE

(a) REAR-WHEEL DRIVE WITH ENGINE MOUNTED LONGITUDINALLY,

(b) FRONT-WHEEL DRIVE WITH ENGINE MOUNTED TRANSVERSELY.

(a) REAR-WHEEL DRIVE (LONGITUDINAL MOUNTING)



ASSUME SENSES SHOWN FOR $\underline{\omega}_x, \underline{\omega}_y, \bar{v}$.

$\bar{v} = 90$ km/h $= 25$ m/s

$\omega_x = 2700\ \text{rpm}\left(\frac{2\pi\ \text{rad}}{60\,s}\right) = 282.74$ rad/s

$\omega_y = \frac{\bar{v}}{\rho} = \frac{25\ \text{m/s}}{200\ \text{m}} = 0.125$ rad/s

$\bar{I}_x = \frac{1}{2} m r^2 = \frac{1}{2}(\rho \pi r^2 t) r^2$

$\bar{I}_x = \frac{1}{2}\rho\pi r^4 t = \frac{1}{2}(7860\ \text{kg/m}^3)\pi(0.2\ \text{m})^4(0.015\ \text{m}) = 0.29632\ \text{kg}\cdot\text{m}^2$

ANGULAR MOMENTUM ABOUT G:

$\underline{H}_G = \bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j}$

EQ. (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = 0 + \omega_y \underline{j} \times (\bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j})$

$\dot{\underline{H}}_G = -\bar{I}_x \omega_x \omega_y \underline{k} = -(0.29632\ \text{kg}\cdot\text{m}^2)(282.74\ \text{rad/s})(0.125\ \text{rad/s})\underline{k}$

$\dot{\underline{H}}_G = -(10.47\ \text{N}\cdot\text{m})\underline{k}$

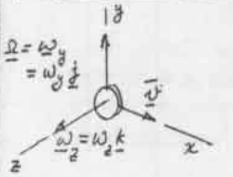
THE COUPLE EXERTED ON THE FLYWHEEL, THEREFORE, MUST BE $\underline{M} = \dot{\underline{H}}_G = -(10.47\ \text{N}\cdot\text{m})\underline{k}$, AND THE COUPLE EXERTED BY THE FLYWHEEL IS $-\underline{M} = (10.47\ \text{N}\cdot\text{m})\underline{k}$

ANSWER: $10.47\ \text{N}\cdot\text{m}$ ◀

(CONTINUED)

18.79 continued

(b) FRONT-WHEEL DRIVE (TRANSVERSE MOUNTING)



WE ASSUME THE SAME DIRECTION OF MOTION OF THE CAR AS IN PART a

REFERRING TO THE NUMERICAL VALUES FOUND IN PART a:

$\omega_z = 282.74$ rad/s

$\omega_y = 0.125$ rad/s

$\bar{I}_z = 0.29632\ \text{kg}\cdot\text{m}^2$

ANGULAR MOMENTUM ABOUT G:

$\underline{H}_G = \bar{I}_y \omega_y \underline{j} + \bar{I}_z \omega_z \underline{k}$

EQ. (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = 0 + \omega_y \underline{j} \times (\bar{I}_y \omega_y \underline{j} + \bar{I}_z \omega_z \underline{k})$

$\dot{\underline{H}}_G = \bar{I}_z \omega_y \omega_z \underline{i} = (0.29632\ \text{kg}\cdot\text{m}^2)(0.125\ \text{rad/s})(282.74\ \text{rad/s})\underline{i}$

$\dot{\underline{H}}_G = (10.47\ \text{N}\cdot\text{m})\underline{i}$

THE COUPLE EXERTED ON THE FLYWHEEL, THEREFORE, MUST BE $\underline{M} = \dot{\underline{H}}_G = (10.47\ \text{N}\cdot\text{m})\underline{i}$, AND THE COUPLE EXERTED BY THE FLYWHEEL IS $\underline{M} = -(10.47\ \text{N}\cdot\text{m})\underline{i}$

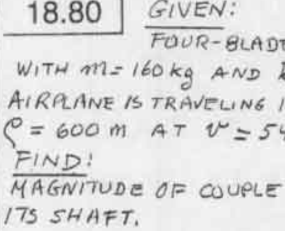
ANSWER: $10.47\ \text{N}\cdot\text{m}$ ◀

18.80 GIVEN:

FOUR-BLADED AIRPLANE PROPELLER WITH $m = 160$ kg AND $\bar{k} = 800$ mm ROTATES AT 1600 rpm. AIRPLANE IS TRAVELING IN CIRCULAR PATH WITH $\rho = 600$ m AT $v = 540$ km/h.

FIND:

MAGNITUDE OF COUPLE EXERTED BY PROPELLER ON ITS SHAFT.



WE ASSUME SENSES SHOWN FOR $\underline{\omega}_x$, $\underline{\Omega}_y$, AND $\bar{v}$

$\bar{v} = 540$ km/h $= 150$ m/s

$\omega_x = 1600\ \text{rpm}\left(\frac{2\pi\ \text{rad}}{60\,s}\right) = 167.55$ rad/s

$\omega_y = \frac{\bar{v}}{\rho} = \frac{150\ \text{m/s}}{600\ \text{m}} = 0.25$ rad/s

$\bar{I}_x = m\bar{k}^2 = (160\ \text{kg})(0.8\ \text{m})^2 = 102.4\ \text{kg}\cdot\text{m}^2$

ANGULAR MOMENTUM ABOUT G:

$\underline{H}_G = \bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j}$

EQ (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = 0 + \omega_y \underline{j} \times (\bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j})$

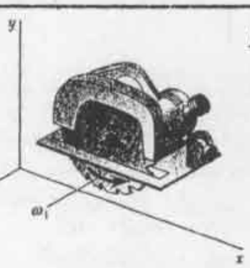
$\dot{\underline{H}}_G = -\bar{I}_x \omega_x \omega_y \underline{k} = -(102.4\ \text{kg}\cdot\text{m}^2)(167.55\ \text{rad/s})(0.25\ \text{rad/s})\underline{k}$

$\dot{\underline{H}}_G = -(4289\ \text{N}\cdot\text{m})\underline{k} = -(4.29\ \text{kN}\cdot\text{m})\underline{k}$

THE COUPLE EXERTED ON THE PROPELLER, THEREFORE, MUST BE $\underline{M} = \dot{\underline{H}}_G = -(4.29\ \text{kN}\cdot\text{m})\underline{k}$, AND THE COUPLE EXERTED BY THE PROPELLER ON ITS SHAFT IS $-\underline{M} = (4.29\ \text{kN}\cdot\text{m})\underline{k}$.

ANSWER: $4.29\ \text{kN}\cdot\text{m}$ ◀

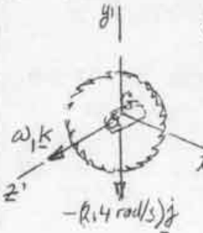
18.81



GIVEN:
FOR BLADE AND ROTOR OF MOTOR OF PORTABLE SAW:
$W = 2.5\ lb$, $\bar{k} = 1.5\ in.$
BLADE ROTATES AS SHOWN AT RATE
$\omega_1 = 1500\ rpm$

FIND: COUPLE $\underline{M}$ THAT WORKER MUST EXERT ON HANDLE TO ROTATE SAW WITH CONSTANT $\underline{\omega}_2 = -(2.4\ rad/s)\underline{j}$.

USING AXES CENTERED AT MASS CENTER G OF BLADE AND ROTOR AND ROTATING WITH CASING:



$$\omega_z = \omega_1 = 1500\ rpm\left(\frac{2\pi\ rad}{60\ s}\right) = 50\pi\ rad/s$$

$$\Omega = \omega_y = \omega_2 = -2.4\ rad/s$$

$$\bar{I}_{z'} = \frac{W}{g}\bar{k}^2 = \frac{2.5\ lb}{32.2\ ft/s^2}\left(\frac{1.5}{12}\ ft\right)^2 = 1.2131 \times 10^{-3}\ lb\cdot ft\cdot s^2$$

ANGULAR MOMENTUM ABOUT G:

$$\underline{H}_G = \bar{I}_{y'}\omega_y\underline{j} + \bar{I}_{z'}\omega_z\underline{k}$$

EQ. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gx'y'z'} + \underline{\Omega}\times\underline{H}_G = 0 + \omega_y\underline{j}\times(I_{y'}\omega_y\underline{j} + \bar{I}_{z'}\omega_z\underline{k})$$

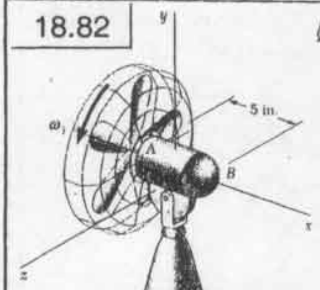
$$= \bar{I}_{z'}\omega_y\omega_z\underline{i} = (1.2131\times10^{-3}\ lb\cdot ft\cdot s^2)(-2.4\ rad/s)(50\pi\ rad/s)\underline{i}$$

$$\dot{\underline{H}}_G = -(0.457\ lb\cdot ft)\underline{i}$$

THE COUPLE THAT THE WORKER MUST APPLY IS

$\underline{M} = \dot{\underline{H}}_G$ $\qquad$ $\underline{M} = -(0.457\ lb\cdot ft)\underline{i}$ ◄

18.82



GIVEN:
FOR BLADE AND ROTOR OF MOTOR OF OSCILLATING FAN:
$W = 8\ oz$, $\bar{k} = 3\ in.$
BEARING SUPPORTS AT A AND B ARE 5 in. APART.
BLADE ROTATES AT RATE $\omega_1 = 1800\ rpm$.
FIND:
DYNAMIC REACTIONS AT A AND B WHEN MOTOR CASING HAS ANG. VEL. $\underline{\omega}_2 = (0.6\ rad/s)\underline{j}$.

ANGULAR MOMENTUM ABOUT MASS CENTER:

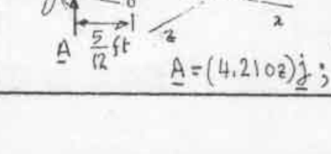
$$\underline{H}_G = \bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j} = \bar{I}_x\omega_1\underline{i} + \bar{I}_y\omega_2\underline{j}$$

EQ. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + \omega_2\underline{j}\times(\bar{I}_x\omega_1\underline{i} + \bar{I}_y\omega_2\underline{j})$$

$$\dot{\underline{H}}_G = -\bar{I}_x\omega_1\omega_2\underline{k} = -\frac{(8/16)\ lb}{32.2\ ft/s^2}\left(\frac{3}{12}\ ft\right)^2(1800\ rpm)\frac{2\pi\ rad}{60\ s}(0.6\ rad/s)\underline{k}$$

$$= -(0.10976\ lb\cdot ft)\underline{k}$$



THE REACTIONS AT A AND B FORM A COUPLE EQUIVALENT TO $\dot{\underline{H}}_G$:

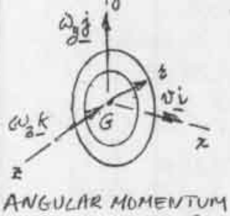
$A(5/12\ ft) = 0.10976\ lb\cdot ft$

$A = 0.26343\ lb = 4.21\ oz$

$\underline{A} = (4.21\ oz)\underline{j}$; $\underline{B} = -(4.21\ oz)\underline{j}$ ◄

18.83

GIVEN:
AUTOMOBILE TRAVELS AROUND UNBANKED CURVE WITH $\rho = 150\ m$ AT SPEED $v = 95\ km/h$.
FOR EACH WHEEL: $m = 22\ kg$, DIAM. $= 575\ mm$, $\bar{k} = 225\ mm$.
TRANSVERSE DISTANCE BETWEEN WHEELS $= 1.5\ m$.
FIND: ADDITIONAL REACTION ΔR EXERTED BY GROUND ON EACH OUTSIDE WHEEL DUE TO MOTION OF CAR.



FOR EACH WHEEL:

$$v = 95\ km/h = 26.389\ m/s$$

$$\omega_y = \frac{v}{\rho} = \frac{26.389\ m/s}{150\ m} = 0.17593\ rad/s$$

$$\omega_z = -\frac{v}{r} = -\frac{26.389\ m/s}{(0.575\ m)/2} = -91.787\ rad/s$$

$$\bar{I}_z = m\bar{k}^2 = (22\ kg)(0.225\ m)^2 = 1.1138\ kg\cdot m^2$$

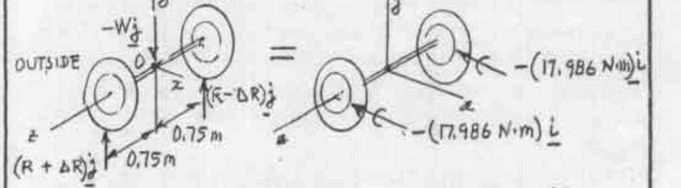
ANGULAR MOMENTUM OF EACH WHEEL:

$$\underline{H}_G = \bar{I}_y\omega_y\underline{j} + \bar{I}_z\omega_z\underline{k}$$

$$\text{EQ.}(18.22):\ \dot{\underline{H}}_G = (\dot{\underline{H}})_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + \omega_y\underline{j}\times(\bar{I}_y\omega_y\underline{j} + \bar{I}_z\omega_z\underline{k})$$

$$\dot{\underline{H}}_G = \bar{I}_z\omega_y\omega_z\underline{i} = (1.1138\ kg\cdot m^2)(0.17593\ rad/s)(-91.787\ rad/s)\underline{i} = -(17.986\ N\cdot m)\underline{i}$$

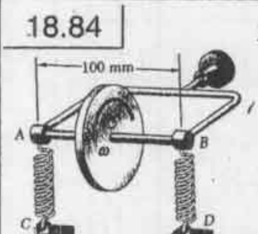
EQUATIONS OF MOTION FOR TWO WHEELS ON SAME AXLE:



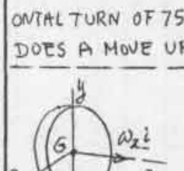
$$\Sigma\underline{M}_O = \Sigma(\underline{M}_O)_{eff}:\ -2(\Delta R)(0.75\ m)\underline{i} = -2(17.986\ N\cdot m)\underline{i}$$

$\Delta R = 23.98\ N$ $\qquad$ $\Delta R = 24.0\ N\uparrow$ ◄

18.84



GIVEN:
TYPE OF AIRCRAFT TURN INDICATOR.
UNIFORM DISK: $m = 200\ g$, $r = 40\ mm$
SPINS AT RATE OF 10 000 rpm.
EACH SPRING HAS 500-N/m CONSTANT.
SPRINGS EXERT EQUAL FORCES ON YOKE AB IN STRAIGHT FLIGHT PATH.
FIND:
ANGLE OF ROTATION OF YOKE IN HORIZONTAL TURN OF 750-m RADIUS TO THE RIGHT WITH $v = 800\ km/h$.
DOES A MOVE UP OR DOWN?



$$\omega_x = 10\ 000\ rpm = 1047.2\ rad/s$$

$$v = 800\ km/h = 222.2\ m/s$$

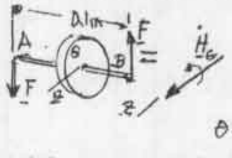
$$\omega_y = -\frac{v}{\rho} = -\frac{222.2\ m/s}{750\ m} = -0.2963\ rad/s$$

$$\underline{H}_G = \bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j}$$

$$\text{EQ.}(18.22):\ \dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + \omega_y\underline{j}\times(\bar{I}_x\omega_x\underline{i} + \bar{I}_y\omega_y\underline{j})$$

$$\dot{\underline{H}}_G = -\bar{I}_x\omega_x\omega_y\underline{k} = -\tfrac{1}{2}(0.2\ kg)(0.04\ m)^2(1047.2\ rad/s)(-0.2963\ rad/s)\underline{k}$$

$$= +(0.049645\ N\cdot m)\underline{k}$$



WE HAVE

$$(0.1\ m)F = \dot{H}_G = 0.049645\ N\cdot m$$

$$F = 0.49645\ N$$

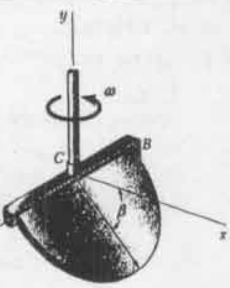
$$F = kx:\ x = \frac{0.49645\ N}{500\ N/m} = 0.9929\ mm$$

$$\theta = \frac{x}{GA} = \frac{0.9929\ mm}{50\ mm} = 0.01986\ rad = 1.14^\circ$$ ◄

SINCE SPRING AT A PULLS DOWN, A IS MOVING UP ◄

18.85 and 18.86

GIVEN:
SEMICIRCULAR PLATE WITH $r = 120$ mm IS HINGED TO CLEVIS WHICH ROTATES WITH CONSTANT ω.

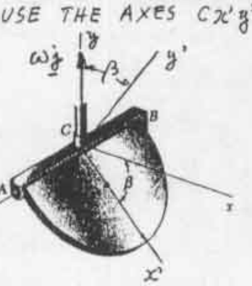


PROBLEM 18.85:
FIND:
(a) β WHEN $\omega = 15$ rad/s,
(b) LARGEST ω FOR WHICH PLATE REMAINS VERTICAL ($\beta = 90°$)

PROBLEM 18.86:
FIND ω FOR WHICH $\beta = 50°$.

MOMENTS AND PRODUCTS OF INERTIA

WE USE THE AXES $Cx'y'z$ SHOWN.



WE NOTE THAT $I_{x'}$ AND $I_{y'}$ ARE HALF THOSE FOR A CIRCULAR PLATE, AND SO IS THE MASS m. THUS

$$I_{x'} = \frac{1}{4} m r^2$$
$$I_{y'} = \frac{1}{2} m r^2$$

BECAUSE OF SYMMETRY, ALL PRODUCTS OF INERTIA ARE EQUAL TO ZERO:

$$I_{x'y'} = I_{y'z} = I_{zx'} = 0$$

ANGULAR MOMENTUM ABOUT C

$$\underline{H}_C = I_{x'}\omega_{x'}\,\underline{i}' + I_{y'}\omega_{y'}\,\underline{j}'$$
$$= \frac{1}{4}mr^2(-\omega\sin\beta)\underline{i}' + \frac{1}{2}mr^2(\omega\cos\beta)\underline{j}'$$
$$= \frac{1}{4}mr^2\omega\,(-\sin\beta\,\underline{i}' + 2\cos\beta\,\underline{j}')$$

SINCE C IS A FIXED POINT, WE CAN USE EQ. (18.28):

$$\Sigma \underline{M}_C = (\dot{\underline{H}}_C)_{Cx'y'z} + \underline{\Omega}\times\underline{H}_C = 0 + \omega\underline{j}\times\underline{H}_C$$

OR, SINCE $\underline{j} = -\underline{i}'\sin\beta + \underline{j}'\cos\beta$:

$$\Sigma\underline{M}_C = \omega(-\underline{i}'\sin\beta + \underline{j}'\cos\beta)\times\frac{1}{4}mr^2\omega(-\sin\beta\,\underline{i}' + 2\cos\beta\,\underline{j}')$$
$$= \frac{1}{4}mr^2\omega^2(-2\sin\beta\cos\beta\,\underline{k} + \cos\beta\sin\beta\,\underline{k})$$
$$\Sigma\underline{M}_C = -\frac{1}{4}mr^2\omega^2\sin\beta\cos\beta\,\underline{k} \qquad (1)$$

BUT
$$\Sigma\underline{M}_C = -mg\,\bar{x}'\cos\beta\,\underline{k}$$
$$= -mg\frac{4r}{3\pi}\cos\beta\,\underline{k} \qquad (2)$$

EQUATING (1) AND (2):

$$\frac{1}{4}mr^2\omega^2\sin\beta\cos\beta = \frac{4mgr}{3\pi}\cos\beta$$

$$\omega^2\sin\beta = \frac{16}{3\pi}\frac{g}{r} = \frac{16}{3\pi}\frac{9.81\text{ m/s}^2}{0.12\text{ m}} \qquad \omega^2\sin\beta = 138.78\text{ s}^{-2} \quad (3)$$

PROBLEM 18.85

(a) LET $\omega = 15$ rad/s IN (3): $\sin\beta = \frac{138.78}{(15)^2} = 0.61681$

$$\beta = 38.1° \;◀$$

(b) LET $\beta = 90°$ IN (3): $\omega^2 = 138.78\text{ s}^{-2}$, $\omega = 11.78$ rad/s ◀

PROBLEM 18.86

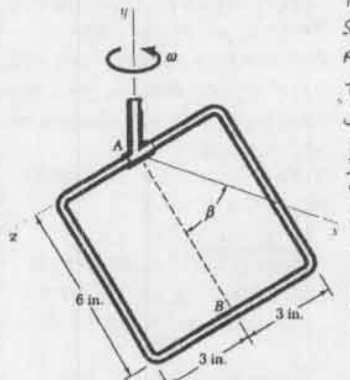
LET $\beta = 50°$ IN EQ. (3):

$$\omega^2 = \frac{138.78\text{ s}^{-2}}{\sin 50°} = 181.17\text{ s}^{-2}$$

$$\omega = 13.46\text{ rad/s} \;◀$$

18.87 and 18.88

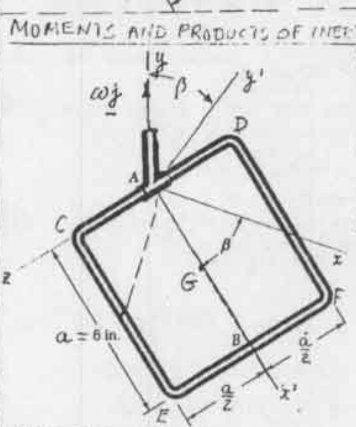
GIVEN:
ROD BENT TO FORM 6-in. SQUARE FRAME WHICH IS ATTACHED BY COLLAR AT A TO SHAFT ROTATING WITH CONSTANT ω.



PROBLEM 18.87:
FIND:
(a) β WHEN $\omega = 9.8$ rad/s,
(b) LARGEST ω FOR WHICH $\beta = 90°$.

PROBLEM 18.88:
FIND ω FOR WHICH $\beta = 48°$.

MOMENTS AND PRODUCTS OF INERTIA. (MASS OF FRAME $= m$)



WE USE THE AXES $Ax'y'z$ SHOWN

FOR CD:
$$I_{x'} = I_{y'} = \frac{1}{12}\frac{m}{4}a^2 = \frac{1}{48}ma^2$$

FOR EF: $I_{x'} = \frac{1}{48}ma^2$
$$I_{y'} = \frac{1}{48}ma^2 + \frac{m}{4}a^2 = \frac{13}{48}ma^2$$

FOR CE OR DF:
$$I_{x'} = \frac{m}{4}\left(\frac{a}{2}\right)^2 = \frac{1}{16}ma^2$$
$$I_{y'} = \frac{1}{48}ma^2 + \frac{m}{4}\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]$$
$$= \left(\frac{1}{48} + \frac{1}{8}\right)ma^2 = \frac{7}{48}ma^2$$

FOR ENTIRE FRAME:

$$I_{x'} = \left[\frac{1}{48} + \frac{1}{48} + 2\left(\frac{1}{16}\right)\right]ma^2 = \frac{1}{6}ma^2;\quad I_{y'} = \left[\frac{1}{48} + \frac{13}{48} + 2\left(\frac{7}{48}\right)\right]ma^2 = \frac{7}{12}ma^2$$

BECAUSE OF SYMMETRY: $I_{x'y'} = I_{y'z} = I_{zx'} = 0$

ANGULAR MOMENTUM ABOUT A

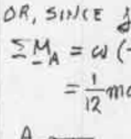
$$\underline{H}_A = I_{x'}\omega_{x'}\,\underline{i}' + I_{y'}\omega_{y'}\,\underline{j}' = \frac{1}{6}ma^2(-\omega\sin\beta)\underline{i}' + \frac{7}{12}ma^2(\omega\cos\beta)\underline{j}'$$

SINCE A IS FIXED, WE USE EQ. (18.28):

$$\Sigma\underline{M}_A = (\dot{\underline{H}}_A)_{Ax'y'z} + \underline{\Omega}\times\underline{H}_A = 0 + \omega\underline{j}\times\underline{H}_A$$

OR, SINCE $\underline{j} = -\underline{i}'\sin\beta + \underline{j}'\cos\beta$:

$$\Sigma\underline{M}_A = \omega(-\underline{i}'\sin\beta + \underline{j}'\cos\beta)\times\frac{1}{12}ma^2\omega(-2\sin\beta\,\underline{i}' + 7\cos\beta\,\underline{j}')$$
$$= \frac{1}{12}ma^2\omega^2(-7\sin\beta\cos\beta\,\underline{k} + 2\cos\beta\sin\beta\,\underline{k})$$
$$\Sigma\underline{M}_A = -\frac{5}{12}ma^2\omega^2\sin\beta\cos\beta\,\underline{k} \qquad (1)$$



BUT $\Sigma M_A = -mg\left(\frac{a}{2}\right)\cos\beta\,\underline{k} \qquad (2)$

EQUATING (1) AND (2):

$$\frac{5}{12}ma^2\omega^2\sin\beta\cos\beta = -\frac{1}{2}mga\cos\beta$$

$$\omega^2\sin\beta = \frac{6}{5}\frac{g}{a} = \frac{6}{5}\frac{32.2\text{ ft/s}^2}{(6/12)\text{ ft}} \qquad \omega^2\sin\beta = 77.28\text{ s}^{-2} \quad (3)$$

PROBLEM 18.87

(a) LET $\omega = 9.8$ rad/s IN (3): $\sin\beta = \frac{77.28}{(9.8)^2} = 0.80466$

$$\beta = 53.6° \;◀$$

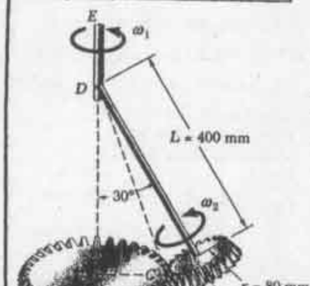
(b) LET $\beta = 90°$ IN (3):
$$\omega^2 = 77.28\text{ s}^{-2} \qquad \omega = 8.79\text{ rad/s} \;◀$$

PROBLEM 18.88

LET $\beta = 48°$ IN EQ. (3):

$$\omega^2 = \frac{77.28\text{ s}^{-2}}{\sin 48°} = 103.99\text{ s}^{-2} \qquad \omega = 10.20\text{ rad/s} \;◀$$

18.89 and 18.90

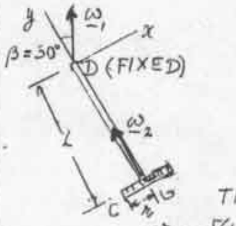


GIVEN:
950-g GEAR A CONSTRAINED TO ROLL ON FIXED GEAR B, BUT FREE TO ROTATE ABOUT AD. AXLE AD CONNECTED BY CLEVIS TO SHAFT DE WHICH ROTATES WITH CONSTANT ω_1.
(GEAR A CAN BE CONSIDERED AS THIN DISK.)

PROBLEM 18.89:
FIND LARGEST ALLOWABLE ω, IF GEAR A IS NOT TO LOSE CONTACT WITH GEAR B.

PROBLEM 18.90:
FIND FORCE $\underline{F}$ EXERTED BY GEAR B ON GEAR A WHEN $\omega_1 = 4$ rad/s. ($\underline{F}$ IS $\perp$ CD.)

ANGULAR VELOCITY OF GEAR A



WE USE THE AXES $Gxyz$ SHOWN AND EXPRESS THAT $\underline{v}_C = 0$:

$\underline{v}_C = \underline{\omega} \times \vec{DC} = 0$

WHERE

$\underline{\omega} = (\omega_1 \cos\beta + \omega_2)\underline{j} + \omega_1 \sin\beta\, \underline{i}$ (1)

$\vec{DC} = -(L\underline{j} + r\underline{i})$

THUS:

$v_C = -[(\omega_1 \cos\beta + \omega_2)\underline{j} + \omega_1 \sin\beta\, \underline{i}] \times (L\underline{j} + r\underline{i}) = 0$

$(\omega_1 \cos\beta + \omega_2) r\, \underline{k} - (\omega_1 \sin\beta) L\, \underline{k} = 0$

THUS: $\omega_1 \cos\beta + \omega_2 = (\omega_1 \sin\beta)(L/r)$

SUBSTITUTE INTO (1): $\underline{\omega} = \omega_1 \sin\beta (\underline{i} + \frac{L}{r}\underline{j})$ (2)

ANGULAR MOMENTUM ABOUT D

$\underline{H}_D = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} = m(L^2 + \frac{r^2}{4})\omega_1 \sin\beta\, \underline{i} + m\frac{r^2}{2}\omega_1 \frac{L}{r}\sin\beta\, \underline{j}$

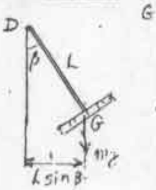
$\underline{H}_D = m\omega_1 \sin\beta [(L^2 + \frac{r^2}{4})\underline{i} + \frac{1}{2} r L\, \underline{j}]$ (3)

SINCE D IS A FIXED POINT, WE USE EQ. (18.28):

$\Sigma \underline{M}_D = (\dot{\underline{H}}_D)_{Dxyz} + \underline{\Omega} \times \underline{H}_D = 0 + (\omega_1 \sin\beta\, \underline{i} + \omega_1 \cos\beta\, \underline{j}) \times \underline{H}_D$

$= m\omega_1^2 \sin\beta [\frac{1}{2} r L \sin\beta - (L^2 + \frac{r^2}{4})\cos\beta]\, \underline{k}$ (4)

PROB. 18.89: WHEN FORCE EXERTED BY GEAR B ON GEAR A BECOMES ZERO:



$\Sigma \underline{M}_D = -mgL\sin\beta\, \underline{k}$ (5)

EQUATING (4) AND (5):

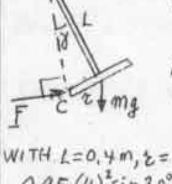
$m\omega_1^2 \sin\beta [\frac{1}{2} r L \sin\beta - (L^2 + \frac{r^2}{4})\cos\beta] = -mgL\sin\beta$

$\omega_1^2 [(L^2 + \frac{r^2}{4})\cos\beta - \frac{1}{2} r L \sin\beta] = gL$

WITH $L = 0.4$ m, $r = 0.08$ m, $\beta = 30°$, $g = 9.81$ m/s^2

$0.13195\,\omega_1^2 = 3.924$ $\quad\omega_1 = 5.45$ rad/s ◀

PROB. 18.90: MOMENT OF $\underline{F}$ ABOUT D $= +F\sqrt{L^2 + r^2}\,\underline{k}$



THUS, EQ. (5) ABOVE MUST BE REPLACED BY

$\Sigma \underline{M}_D = (F\sqrt{L^2 + r^2} - mgL\sin\beta)\underline{k}$ (6)

EQUATING (4) AND (6):

$m\omega_1^2 \sin\beta [\frac{1}{2} r L \sin\beta - (L^2 + \frac{r^2}{4})\cos\beta]$
$= F\sqrt{L^2 + r^2} - mgL\sin\beta$

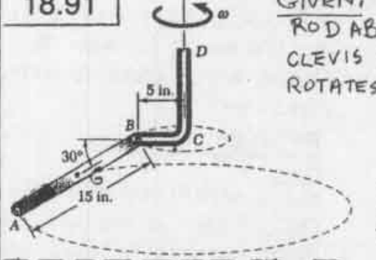
WITH $L = 0.4$ m, $r = 0.08$ m, $\beta = 30°$, $g = 9.81$ m/s^2, $m = 0.95$ kg, $\omega = 4$ rad/s:

$0.95(4)^2 \sin 30°(-0.13195) = F\sqrt{0.1664} - (0.95)(9.81)(0.4)\sin 30°$

$0.40792\,F = 1.8639 - 1.0028 = 0.8611$ $\quad F = 2.11$ N

$\tan\gamma = \frac{r}{L} = 0.2$, $\gamma = 11.31°$, $\beta - \gamma = 18.7°$ $\quad \underline{F} = 2.11\text{ N} \measuredangle 18.7°$ ◀

18.91

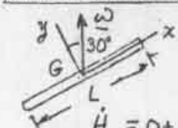


GIVEN:
ROD AB IS ATTACHED BY A CLEVIS TO ARM BCD WHICH ROTATES WITH CONSTANT $\underline{\omega}$.

FIND:
MAGNITUDE OF $\underline{\omega}$

LET $L = 15$ in. $= 1.25$ ft
THEN: $BC = 5$ in. $= L/3$

ANGULAR MOMENTUM ABOUT G



$\underline{\omega} = \omega \sin 30°\, \underline{i} + \omega \cos 30°\, \underline{j}$

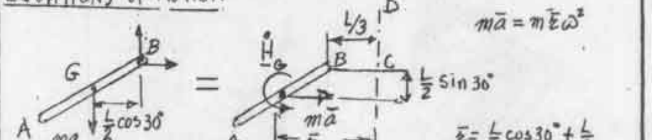
$\underline{H}_G = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} = 0 + \frac{1}{12} mL^2 \omega \cos 30°\, \underline{j}$

EQ (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G$

$\dot{\underline{H}}_G = 0 + \underline{\omega} \times \underline{H}_G = (\omega \sin 30°\, \underline{i} + \omega \cos 30°\, \underline{j}) \times \frac{1}{12} mL^2 \omega \cos 30°\, \underline{j}$

$\dot{\underline{H}}_G = \frac{1}{12} mL^2 \omega^2 \sin 30° \cos 30°\, \underline{k}$

EQUATIONS OF MOTION



$m\bar{a} = m\bar{r}\omega^2$

$\bar{r} = \frac{L}{2}\cos 30° + \frac{L}{3}$

$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$: $mg(\frac{L}{2}\cos 30°) = \dot{H}_G + (m\bar{a})(\frac{L}{2}\sin 30°)$

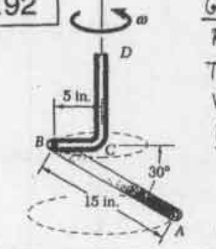
$\frac{1}{2} mgL\cos 30° = \frac{1}{12} mL^2 \omega^2 \sin 30° \cos 30° + m(\frac{L}{2}\cos 30° + \frac{L}{3})\omega^2 (\frac{L}{2}\sin 30°)$

$\frac{1}{2}\frac{g}{L}\cos 30° = (\frac{1}{3}\sin 30° \cos 30° + \frac{1}{6}\sin 30°)\omega^2$

$\frac{1}{2}\frac{32.2\text{ ft/s}^2}{1.25\text{ ft}}\cos 30° = 0.22767\,\omega^2$, $\quad\omega^2 = 48.994$

$\omega = 7.00$ rad/s ◀

18.92



GIVEN:
ROD AB IS ATTACHED BY A CLEVIS TO ARM BCD WHICH ROTATES WITH CONSTANT $\underline{\omega}$.

FIND:
MAGNITUDE OF $\underline{\omega}$

LET $L = 15$ in. $= 1.25$ ft
THEN: $BC = 5$ in. $= L/3$

ANGULAR MOMENTUM ABOUT G:

$\underline{\omega} = -\omega \sin 30°\, \underline{i} + \omega \cos 30°\, \underline{j}$

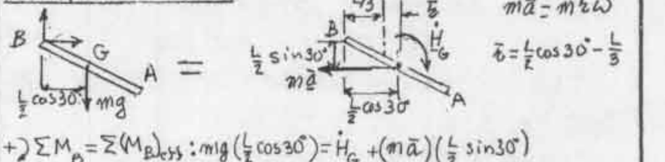
$\underline{H}_G = I_x \omega_x \underline{i} + I_y \omega_y \underline{j} = 0 + \frac{1}{12} mL^2 \omega \cos 30°\, \underline{j}$

EQ. (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = 0 + \underline{\omega} \times \underline{H}_G$

$\dot{\underline{H}}_G = (-\omega \sin 30°\, \underline{i} + \omega \cos 30°\, \underline{j}) \times \frac{1}{12} mL^2 \omega \cos 30°\, \underline{j}$

$\dot{\underline{H}}_G = -\frac{1}{12} mL^2 \omega^2 \sin 30° \cos 30°\, \underline{k}$

EQUATIONS OF MOTION



$m\bar{a} = m\bar{r}\omega^2$

$\bar{r} = \frac{L}{2}\cos 30° - \frac{L}{3}$

$+\circlearrowright \Sigma M_B = \Sigma (M_B)_{eff}$: $mg(\frac{L}{2}\cos 30°) = \dot{H}_G + (m\bar{a})(\frac{L}{2}\sin 30°)$

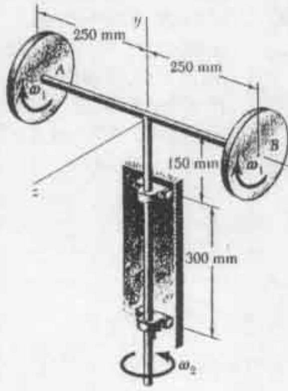
$\frac{1}{2} mgL\cos 30° = \frac{1}{12} mL^2 \omega^2 \sin 30° \cos 30° + m(\frac{L}{2}\cos 30° - \frac{L}{3})\omega^2 (\frac{L}{2}\sin 30°)$

$\frac{1}{2}\frac{g}{L}\cos 30° = (\frac{1}{3}\sin 30° \cos 30° - \frac{1}{6}\sin 30°)\omega^2$

$\frac{1}{2}\frac{32.2\text{ ft/s}^2}{1.25\text{ ft}}\cos 30° = 0.061004\,\omega^2$, $\quad\omega^2 = 182.83$

$\omega = 13.52$ rad/s ◀

18.93 and 18.94



GIVEN:
FOR EACH DISK:
$m = 5\ kg,\ r = 100\ mm$
$\omega_1 = 1500\ rpm$

PROB 18.93:
FOR $\omega_2 = 45\ rpm$ FIND DYNAMIC REACTIONS AT C AND IF
(a) BOTH DISKS ROTATE AS SHOWN
(b) DIRECTION OF SPIN OF B IS REVERSED

PROB. 18.94:
FIND MAX. ALLOWABLE ω_2 IF DYNAMIC REACTIONS AT C AND D ARE NOT TO EXCEED 250 N EACH.

ANGULAR MOMENTUM OF EACH DISK ABOUT ITS MASS CENTER

$$\underline{H}_G = \bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j} = -\tfrac{1}{2} m r^2 \omega_1 \underline{i} + \tfrac{1}{4} m r^2 \omega_2 \underline{j}$$

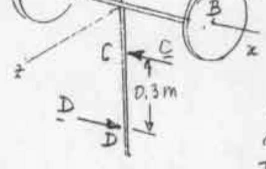
$$\underline{H}_G = \tfrac{1}{4} m r^2 (-2\omega_1 \underline{i} + \omega_2 \underline{j}) \qquad (1)$$

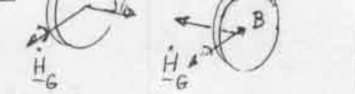
EQ. (18.22):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G = 0 + \omega_2 \underline{j} \times \tfrac{1}{4} m r^2 (-2\omega_1 \underline{i} + \omega_2 \underline{j})$$

$$\dot{\underline{H}}_G = +\tfrac{1}{2} m r^2 \omega_1 \omega_2 \underline{k} \qquad (2)$$

EQUATIONS OF MOTION





SINCE $m\bar{a}_A$ AND $m\bar{a}_B$ CANCEL OUT, EFFECTIVE FORCES REDUCE TO COUPLE $2\dot{\underline{H}}_G = m r^2 \omega_1 \omega_2 \underline{k}$

IT FOLLOWS THAT THE REACTIONS FORM AN EQUIVALENT COUPLE WITH

$$-\underline{C} = \underline{D} = (m r^2 \omega_1 \omega_2 / 0.3\ m)\ \underline{i} \qquad (3)$$

PROBLEM 18.93

(a) WITH $m = 5\ kg$, $r = 0.1\ m$, $\omega_1 = 1500\ rpm = 50\pi\ rad/s$, AND $\omega_2 = 45\ rpm = 1.5\pi\ rad/s$, EQ. (3) YIELDS

$$C = D = (5\ kg)(0.1\ m)^2 (50\pi\ rad/s)(1.5\pi\ rad/s)/(0.3\ m) = 123.37\ N$$

$$\underline{C} = -(123.4\ N)\ \underline{i}\ ;\ \underline{D} = (123.4\ N)\ \underline{i}$$

(b) WITH DIRECTION OF SPIN OF B REVERSED, ITS ANGULAR MOMENTUM WILL ALSO BE REVERSED AND THE EFFECTIVE FORCES (AND, THUS, THE APPLIED FORCES) REDUCE TO ZERO:

$$\underline{C} = \underline{D} = 0$$

PROBLEM 18.94

MAKING $C = D = 250\ N$ IN EQ. (3) YIELDS

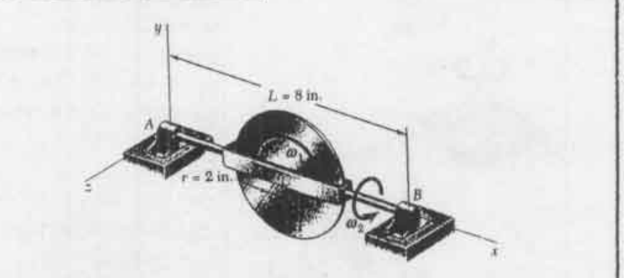
$$\frac{m r^2 \omega_1 \omega_2}{0.3\ m} = 250\ N$$

WITH $m = 5\ kg$, $r = 0.1\ m$, $\omega_1 = 1500\ rpm = 50\pi\ rad/s$ WE HAVE

$$\omega_2 = \frac{(250\ N)(0.3\ m)}{(5\ kg)(0.1\ m)^2 (50\pi\ rad/s)} = 9.5493\ rad/s$$

$$\omega_2 = 91.2\ rpm$$

18.95 and 18.96

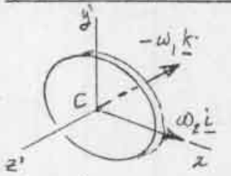


GIVEN: 10-oz DISK SPINS AT RATE $\omega_1 = 750\ rpm$

PROBLEM 18.95:
FOR $\omega_2 = 6\ rad/s$ FIND THE DYNAMIC REACTIONS AT A AND B.

PROBLEM 18.96:
FIND MAX. ALLOWABLE ω_2 IF DYNAMIC REACTIONS AT A AND B ARE NOT TO EXCEED 0.25 lb EACH.

ANGULAR MOMENTUM ABOUT C



$$\underline{H}_C = \bar{I}_x \omega_x \underline{i} + \bar{I}_z \omega_z \underline{k}$$

$$= \tfrac{1}{4} m r^2 \omega_2 \underline{i} - \tfrac{1}{2} m r^2 \omega_1 \underline{k}$$

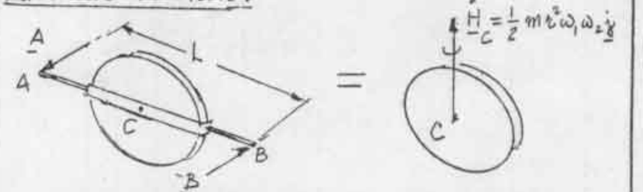
$$\underline{H}_C = \tfrac{1}{4} m r^2 (\omega_2 \underline{i} - 2\omega_1 \underline{k}) \qquad (1)$$

EQ. (18.22):

$$\dot{\underline{H}}_C = (\dot{\underline{H}}_C)_{Cxy'z'} + \underline{\Omega} \times \underline{H}_C = 0 + \omega_2 \underline{i} \times \tfrac{1}{4} m r^2 (\omega_2 \underline{i} - 2\omega_1 \underline{k})$$

$$\dot{\underline{H}}_C = \tfrac{1}{2} m r^2 \omega_1 \omega_2 \underline{j} \qquad (2)$$

EQUATIONS OF MOTION



$$\Sigma \underline{M}_A = \Sigma (\underline{M}_A)_{eff}:\quad BL = \tfrac{1}{2} m r^2 \omega_1 \omega_2 \qquad A = B = \frac{m r^2 \omega_1 \omega_2}{2L} \quad (3)$$

PROBLEM 18.95

LETTING $m = \frac{W}{g} = \frac{(10/16)\ lb}{32.2\ ft/s^2} = 0.01941\ lb \cdot s^2/ft$, $r = \frac{1}{6}\ ft$, $L = \frac{2}{3}\ ft$

$\omega_1 = 750\ rpm = 25\pi\ rad/s$, $\omega_2 = 6\ rad/s$ IN EQ. (3):

$$A = B = \frac{(0.01941\ lb \cdot s^2/ft)(\frac{1}{6}\ ft)^2 (25\pi\ rad/s)(6\ rad/s)}{2(\frac{2}{3}\ ft)} = 0.1906\ lb$$

$$\underline{A} = (0.1906\ lb)\ \underline{k}\ ;\ \underline{B} = -(0.1906\ lb)\ \underline{k}$$

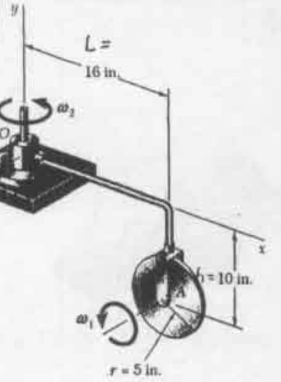
PROBLEM 18.96

LETTING $A = B = 0.25\ lb$, $m = 0.01941\ lb \cdot s^2/ft$, $r = \frac{1}{6}\ ft$, $L = \frac{2}{3}\ ft$ AND $\omega_1 = 750\ rpm = 25\pi\ rad/s$ IN EQ. (3) AND SOLVING FOR ω_2:

$$\omega_2 = \frac{2(\frac{2}{3}\ ft)(0.25\ lb)}{(0.01941\ lb \cdot s^2/ft)(\frac{1}{6}\ ft)^2 (25\pi\ rad/s)} = 7.872\ rad/s$$

$$\omega_2 = 7.87\ rad/s$$

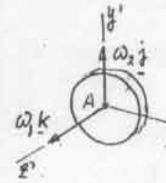
18.97



GIVEN:
DISK OF WEIGHT $W = 8\ lb$ ROTATES AT CONSTANT $\omega_1 = 12\ rad/s$. ARM OA ROTATES AT CONSTANT $\omega_2 = 4\ rad/s$.

FIND:
FORCE-COUPLE SYSTEM REPRESENTING DYNAMIC REACTION AT SUPPORT O.

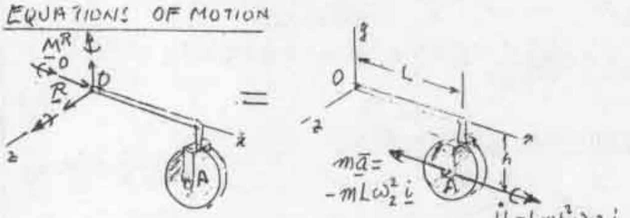
ANGULAR MOMENTUM ABOUT A



$$\underline{H}_A = \bar{I}_{y'}\omega_y \underline{j} + \bar{I}_{z'}\omega_z \underline{k}$$
$$= \tfrac{1}{4}mr^2\omega_2 \underline{j} + \tfrac{1}{2}mr^2\omega_1 \underline{k}$$
$$\underline{H}_A = \tfrac{1}{4}mr^2(\omega_2 \underline{j} + 2\omega_1 \underline{k}) \qquad (1)$$

EQ. (18.22):
$$\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Ax'y'z'} + \underline{\Omega}\times\underline{H}_A = 0 + \omega_2 \underline{j}\times \underline{H}_A$$
$$\dot{\underline{H}}_A = \omega_2 \underline{j} \times \tfrac{1}{4}mr^2(\omega_2 \underline{j} + 2\omega_1 \underline{k})$$
$$\dot{\underline{H}}_A = \tfrac{1}{2}mr^2\omega_1\omega_2 \underline{i} \qquad (2)$$

EQUATIONS OF MOTION



$$\Sigma \underline{F} = \Sigma(\underline{F})_{eff}: \quad \underline{R} = -mL\omega_2^2 \underline{i} \qquad (3)$$
$$\Sigma \underline{M}_O = \Sigma(\underline{M}_O)_{eff}:$$
$$\underline{M}^R_O = \dot{\underline{H}}_A + (L\underline{i} - h\underline{j})\times(-mL\omega_2^2 \underline{i})$$
$$= \tfrac{1}{2}mr^2\omega_1\omega_2 \underline{i} - mhL\omega_2^2 \underline{k} \qquad (4)$$

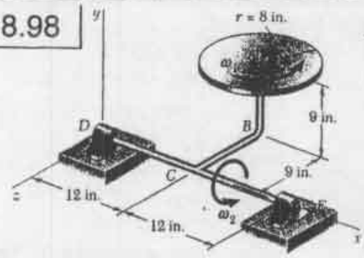
WITH GIVEN DATA:
$$m = \frac{W}{g} = \frac{8\ lb}{32.2\ ft/s^2} = 0.24845\ lb\cdot s^2/ft,\ L = \tfrac{4}{3}\ ft,\ h = \tfrac{5}{6}\ ft$$
$$\omega_1 = 12\ rad/s,\ \omega_2 = 4\ rad/s,\ r = \tfrac{5}{12}\ ft.$$

EQ. (3): $\underline{R} = -(0.24845\ lb\cdot s^2/ft)(\tfrac{4}{3}\ ft)(4\ rad/s)^2 \underline{i}$
$$= -(5.300\ lb)\underline{i}$$

EQ. (4): $\underline{M}^R_O = \tfrac{1}{2}(0.24845\ lb\cdot s^2/ft)(\tfrac{5}{12}\ ft)^2(12\ rad/s)(4\ rad/s)\underline{i} - (0.24845\ lb\cdot s^2/ft)(\tfrac{5}{6}\ ft)(\tfrac{4}{3}\ ft)(4\ rad/s)^2 \underline{k}$
$$= (1.0352\ lb\cdot ft)\underline{i} - (4.417\ lb\cdot ft)\underline{k}$$

FORCE-COUPLE AT O:
$$\underline{R} = -(5.30\ lb)\underline{i}\ ;\ \underline{M}^R_O = (1.035\ lb\cdot ft)\underline{i} - (4.42\ lb\cdot ft)\underline{k}$$

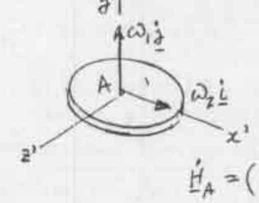
18.98



GIVEN
DISK OF WEIGHT $W = 6\ lb$ ROTATES AT CONSTANT $\omega_1 = 16\ rad/s$. ARM ABC IS WELDED TO SHAFT DCE WHICH ROTATES AT CONSTANT $\omega_2 = 8\ rad/s$.

FIND: DYNAMIC REACTIONS AT D AND E.

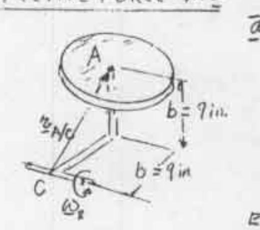
ANGULAR MOMENTUM ABOUT A



$$\underline{H}_A = \bar{I}_{x'}\omega_x \underline{i} + \bar{I}_{y'}\omega_y \underline{j}$$
$$= \tfrac{1}{4}mr^2\omega_2 \underline{i} + \tfrac{1}{2}mr^2\omega_1 \underline{j}$$
$$\underline{H}_A = \tfrac{1}{4}mr^2(\omega_2 \underline{i} + 2\omega_1 \underline{j}) \qquad (1)$$

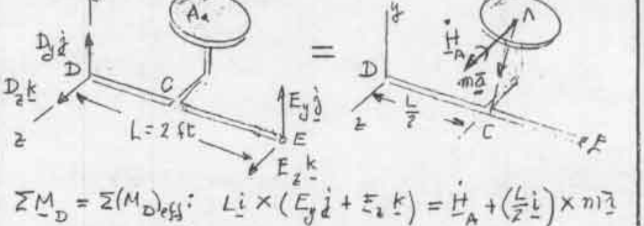
EQ. (18.22):
$$\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Ax'y'z'} + \underline{\Omega}\times\underline{H}_A = 0 + \omega_2 \underline{i}\times\underline{H}_A$$
$$\dot{\underline{H}}_A = \omega_2 \underline{i} \times \tfrac{1}{4}mr^2(\omega_2 \underline{i} + 2\omega_1 \underline{j})$$
$$\dot{\underline{H}}_A = \tfrac{1}{2}mr^2\omega_1\omega_2 \underline{k} \qquad (2)$$

EFFECTIVE FORCE $m\bar{\underline{a}}$



$$\bar{\underline{a}} = -\underline{r}_{A/C}\,\omega_2^2$$
$$= -(b\underline{j} - b\underline{k})\omega_2^2$$
$$m\bar{\underline{a}} = mb\omega_2^2(-\underline{j} + \underline{k}) \qquad (3)$$

EQUATIONS OF MOTION
APPLIED FORCES ARE EQUIVALENT TO EFFECTIVE FORCES



$$\Sigma \underline{M}_D = \Sigma(\underline{M}_D)_{eff}: \quad L\underline{i}\times(E_y\underline{j} + E_z\underline{k}) = \dot{\underline{H}}_A + (\tfrac{L}{2}\underline{i})\times m\bar{\underline{a}}$$

RECALLING EQS. (2) AND (3):
$$L\underline{i}\times(E_y\underline{j} + E_z\underline{k}) = \tfrac{1}{2}mr^2\omega_1\omega_2\underline{k} + \tfrac{L}{2}\underline{i}\times mb\omega_2^2(-\underline{j} + \underline{k})$$
$$LE_y\underline{k} - LE_z\underline{j} = \tfrac{1}{2}mr^2\omega_1\omega_2\underline{k} - \tfrac{1}{2}mbL\omega_2^2\underline{k} - \tfrac{1}{2}mbL\omega_2^2\underline{j}$$

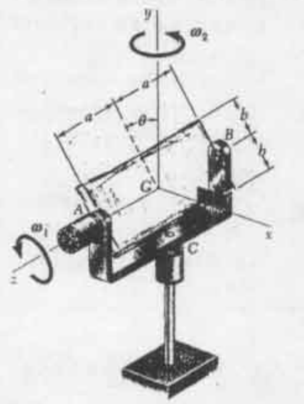
EQUATING COEFF. OF UNIT VECTORS:
$$E_y = \tfrac{1}{2}m\left[(r^2/L)\,\omega_1\omega_2 - b\omega_2^2\right] \qquad E_z = \tfrac{1}{2}mb\omega_2^2 \qquad (4)$$

WITH GIVEN DATA: $m = \frac{6\ lb}{32.2\ ft/s^2} = 0.18634\ lb\cdot s^2/ft$, $r = \tfrac{2}{3}\ ft$, $L = 2\ ft$, $b = 0.75\ ft$, $\omega_1 = 16\ rad/s$, $\omega_2 = 8\ rad/s$:
$$E_y = -1.822\ lb,\quad E_z = 4.472\ lb$$
$$\underline{E} = -(1.822\ lb)\underline{j} + (4.47\ lb)\underline{k}$$

$$\Sigma \underline{F} = \Sigma \underline{F}_{eff}: \quad \underline{D} + \underline{E} = m\bar{\underline{a}}$$

RECALLING (3) AND GIVEN DATA:
$$\underline{D} = m\bar{\underline{a}} - \underline{E} = mb\omega_2^2(-\underline{j} + \underline{k}) - \underline{E}$$
$$= (0.18634)(0.75)(8)^2(-\underline{j} + \underline{k}) + (1.822\ lb)\underline{j} - (4.472\ lb)\underline{k}$$
$$= (1.822 - 8.944)\underline{j} + (8.944 - 4.472)\underline{k}$$
$$\underline{D} = -(7.12\ lb)\underline{j} + (4.47\ lb)\underline{k}$$

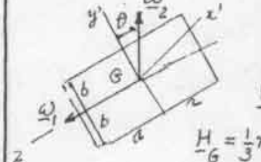
18.99 and 18.100



GIVEN:
ADVERTISING PANEL
$m = 48\,kg$, $2a = 2.4\,m$, $2b = 1.6\,m$.
MOTOR AT A KEEPS PANEL ROTATING ABOUT AB AT CONSTANT RATE ω_1.
MOTOR AT C KEEPS FRAME ROTATING AT CONSTANT ω_2.
PANEL COMPLETES FULL REVOLUTION IN 6 s.
FRAME COMPLETES FULL REVOLUTION IN 12 s.

PROBLEM 18.99:
EXPRESS DYNAMIC REACTION AT D AS FUNCTION OF θ.

PROBLEM 18.100:
SHOW THAT (a) DYNAMIC REACTION AT D IS INDEPENDENT OF LENGTH $2a$,
(b) AT ANY INSTANT $M_1/M_2 = \omega_2/2\omega_1$, WHERE M_1 AND M_2 ARE THE MAGNITUDES OF THE COUPLES EXERTED BY THE MOTORS AT A AND C, RESPECTIVELY.



USING AXES $Gx'y'z$ WITH x' PERPENDICULAR TO PANEL:
$\omega_{x'} = \omega_2 \sin\theta$, $\omega_{y'} = \omega_2\cos\theta$, $\omega_z = \omega_1$

$\underline{H}_G = \bar{I}_{x'}\omega_{x'}\underline{i}' + \bar{I}_{y'}\omega_{y'}\underline{j}' + \bar{I}_z\omega_z\underline{k}$

$\underline{H}_G = \frac{1}{3}m(a^2+b^2)\omega_2\sin\theta\,\underline{i}' + \frac{1}{3}ma^2\omega_2\cos\theta\,\underline{j}' + \frac{1}{3}mb^2\omega_1\underline{k}$ (1)

TO REVERT TO THE ORIGINAL FRAME $Gxyz$, WE NOTE THAT

$\underline{i}' = \cos\theta\,\underline{i} + \sin\theta\,\underline{j}$
$\underline{j}' = -\sin\theta\,\underline{i} + \cos\theta\,\underline{j}$

SUBSTITUTE IN (1):

$\underline{H}_G = \frac{1}{3}m(a^2+b^2)\omega_2\sin\theta(\cos\theta\,\underline{i} + \sin\theta\,\underline{j}) + \frac{1}{3}ma^2\omega_2\cos\theta(-\sin\theta\,\underline{i} + \cos\theta\,\underline{j}) + \frac{1}{3}mb^2\omega_1\underline{k}$

$\underline{H}_G = \frac{1}{3}m\left[b^2\omega_2\sin\theta\cos\theta\,\underline{i} + (a^2+b^2\sin^2\theta)\omega_2\underline{j} + b^2\omega_1\underline{k}\right]$ (2)

EQ.(18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = (\dot{\underline{H}}_G)_{Gxyz} + \omega_2\underline{j}\times\underline{H}_G$ (3)

THE FIRST TERM IS OBTAINED BY DIFFERENTIATING $\underline{H}_G$ WITH RESPECT TO t, ASSUMING FRAME $Gxyz$ TO BE FIXED:

$(\dot{\underline{H}}_G)_{Gxyz} = \frac{1}{3}m\left[b^2\omega_2(\cos^2\theta - \sin^2\theta)\dot\theta\,\underline{i} + 2b^2\omega_2\sin\theta\cos\theta\,\dot\theta\,\underline{j}\right]$

OBSERVING THAT $\dot\theta = \omega_1$ AND SUBSTITUTING INTO (3):

$\dot{\underline{H}}_G = \frac{1}{3}mb^2\omega_1\omega_2\left[(\cos^2\theta - \sin^2\theta)\underline{i} + 2\sin\theta\cos\theta\,\underline{j}\right] + \omega_2\underline{j}\times\frac{1}{3}mb^2(\omega_2\sin\theta\cos\theta\,\underline{i} + \omega_1\underline{k})$

$\dot{\underline{H}}_G = \frac{1}{3}mb^2\omega_1\omega_2\left[(\cos^2\theta - \sin^2\theta)\underline{i} + 2\sin\theta\cos\theta\,\underline{j}\right] + \frac{1}{3}mb^2(\omega_1\omega_2\underline{i} - \omega_2^2\sin\theta\cos\theta\,\underline{k})$

$\dot{\underline{H}}_G = \frac{1}{3}mb^2(2\omega_1\omega_2\cos^2\theta\,\underline{i} + \omega_1\omega_2\sin2\theta\,\underline{j} - \frac{1}{2}\omega_2^2\sin2\theta\,\underline{k})$ (4)

THE REACTION AT D MUST BE EQUAL TO COUPLE $\dot{\underline{H}}_G$

PROBLEM 18.99. WITH GIVEN DATA:

$\underline{M}_D = \dot{\underline{H}}_G = \frac{1}{3}(48\,kg)(0.8\,m)^2\left[2\left(\frac{2\pi}{6}\right)\left(\frac{2\pi}{12}\right)\cos^2\theta\,\underline{i} + \left(\frac{2\pi}{6}\right)\left(\frac{2\pi}{12}\right)\sin2\theta\,\underline{j} - \frac{1}{2}\left(\frac{2\pi}{12}\right)^2\sin2\theta\,\underline{k}\right]$

$\underline{M}_D = (11.23\,N\cdot m)\cos^2\theta\,\underline{i} + (5.61\,N\cdot m)\sin2\theta\,\underline{j} - (1.404\,N\cdot m)\sin2\theta\,\underline{k}$ ◄

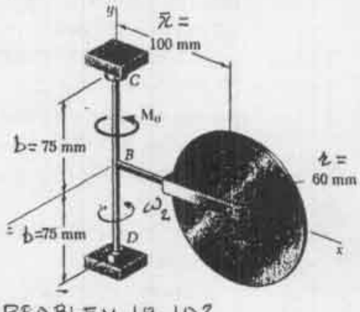
PROBLEM 18.100

(a) EQ.(4) DOES NOT CONTAIN a. ◄

(b) FROM (4): $M_1 = \frac{1}{6}mb^2\omega_2^2\sin2\theta$, $M_2 = \frac{1}{3}mb^2\omega_1\omega_2\sin2\theta$

THUS: $M_1/M_2 = \omega_2/2\omega_1$ ◄

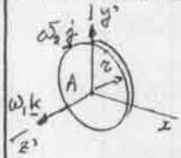
18.101 and 18.102



PROBLEM 18.101:
GIVEN:
3-Kg DISK SPINS AT CONSTANT $\omega_1 = 60\,rad/s$;
ARM AB AND SHAFT ARE AT REST WHEN $\underline{M}_0 = (0.40\,N.m)\,\underline{j}$ IS APPLIED FOR 2 s.
FIND:
DYNAMIC REACTIONS AT C AND D AFTER $\underline{M}_0$ IS REMOVED.

PROBLEM 18.102
GIVEN: 3-kg DISK SPINS AT CONSTANT $\omega_1 = 60\,rad/s$.
ARM AB AND SHAFT ARE AT REST WHEN $\underline{M}_0$ IS APPLIED FOR 3 s, WITH ANG. VELOCITY OF SHAFT REACHING 18 rad/s.
FIND: (a) $\underline{M}_0$, (b) DYNAMIC REACTIONS AT C AND D AFTER $\underline{M}_0$ IS REMOVED.

ANGULAR MOMENTUM ABOUT A



$\underline{H}_A = \bar{I}_{y'}\omega_{y'}\underline{j} + \bar{I}_{z'}\omega_{z'}\underline{k} = \frac{1}{4}mr^2\omega_2\underline{j} + \frac{1}{2}mr^2\omega_1\underline{k}$

$\underline{H}_A = \frac{1}{4}mr^2(\omega_2\underline{j} + 2\omega_1\underline{k})$ (1)

EQ.(18.22): $\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Axy'z'} + \underline{\Omega}\times\underline{H}_A$

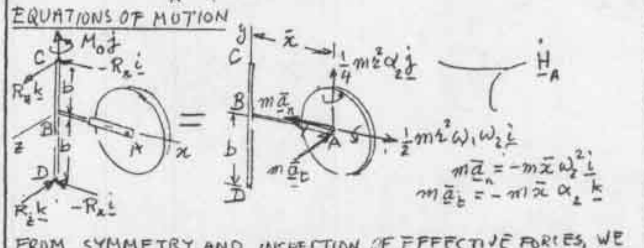
SINCE DISK HAS AN ANG. ACCEL. $\alpha_2\underline{j} = \dot\omega_2\underline{j}$, WE HAVE

$(\dot{\underline{H}}_A)_{Axy'z'} = \frac{1}{4}mr^2\dot\omega_2\underline{j} = \frac{1}{4}mr^2\alpha_2\underline{j}$. ALSO, $\underline{\Omega} = \omega_2\underline{j}$

THUS: $\dot{\underline{H}}_A = \frac{1}{4}mr^2\alpha_2\underline{j} + \omega_2\underline{j}\times\frac{1}{4}mr^2(\omega_2\underline{j} + 2\omega_1\underline{k})$

$\dot{\underline{H}}_A = \frac{1}{2}mr^2\omega_1\omega_2\underline{i} + \frac{1}{4}mr^2\alpha_2\underline{j}$ (2)

EQUATIONS OF MOTION



FROM SYMMETRY AND INSPECTION OF EFFECTIVE FORCES, WE FIND THAT THE COMPONENTS OF THE REACTIONS AT C AND D ARE EQUAL IN MAGNITUDE AND DIRECTED AS SHOWN.

$\Sigma M_y = \Sigma(M_y)_{eff}$: $M_0 = \frac{1}{4}mr^2\alpha_2 + \bar{x}(m\bar{x}\alpha_2) = m(\frac{1}{4}r^2 + \bar{x}^2)\alpha_2$

$M_0 = (3\,kg)\left[\frac{1}{4}(0.06\,m)^2 + (0.1\,m)^2\right]\alpha_2$ $M_0 = 0.0327\alpha_2$ (3)

$\Sigma F_x = \Sigma(F_x)_{eff}$: $2R_x = m\bar{x}\omega_2^2 = (3\,kg)(0.1\,m)\omega_2^2$, $R_x = 0.15\omega_2^2$ (4)

$\Sigma M_x = \Sigma(M_x)_{eff}$: $2bR_z = \frac{1}{2}mr^2\omega_1\omega_2$

$R_z = (mr^2/4b)\omega_1\omega_2 = \frac{(3\,kg)(0.06\,m)^2(60\,rad/s)}{4(0.075\,m)}\omega_2$, $R_z = 2.16\omega_2$ (5)

PROBLEM 18.101

LET $M_0 = 0.40\,N\cdot m$ IN (3): $\alpha_2 = \frac{0.40}{0.0327} = 12.232\,rad/s^2$

FOR $t = 2\,s$: $\omega_2 = \alpha_2 t = (12.232\,rad/s^2)(2\,s) = 24.464\,rad/s$

EQS.(4) AND (5): $R_x = 0.15\omega_2^2 = 89.8\,N$; $R_z = 2.16\omega_2 = 52.8\,N$

$\underline{C} = -(89.8\,N)\underline{i} + (52.8\,N)\underline{k}$; $\underline{D} = -(89.8\,N)\underline{i} - (52.8\,N)\underline{k}$ ◄

PROBLEM 18.102

$\omega_2 = \alpha_2 t$: $18\,rad/s = \alpha_2(3\,s)$, $\alpha_2 = 6\,rad/s^2$

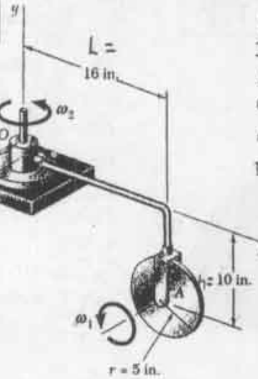
(a) EQ.(3): $M_0 = 0.0327(6) = 0.1962\,N\cdot m$ $\underline{M}_0 = (0.1962\,N\cdot m)\underline{j}$ ◄

(b) EQ.(4): $R_x = 0.15(18\,rad/s)^2 = 48.6\,N$

EQ.(5): $R_z = 2.16(18\,rad/s) = 38.88\,N$

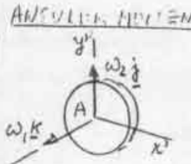
$\underline{C} = -(48.6\,N)\underline{i} + (38.9\,N)\underline{k}$; $\underline{D} = -(48.6\,N)\underline{i} - (38.9\,N)\underline{k}$ ◄

18.103



GIVEN:
DISK OF WEIGHT $W = 8$ lb.
AT INSTANT SHOWN $\omega_1 = 12$ rad/s AND DECREASES AT RATE OF 4 rad/s² DUE TO BEARING FRICTION.
ARM OA ROTATES AT CONSTANT $\omega_2 = 4$ rad/s.
FIND:
FORCE-COUPLE SYSTEM REPRESENTING DYNAMIC REACTION AT SUPPORT O.

ANGULAR MOMENTUM ABOUT A



$$\underline{H}_A = \bar{I}_{y'}\omega_y \underline{j} + \bar{I}_{z'}\omega_z \underline{k} = \tfrac{1}{4} m r^2 \omega_2 \underline{j} + \tfrac{1}{2} m r^2 \omega_1 \underline{k}$$

$$\underline{H}_A = \tfrac{1}{4} m r^2 (\omega_2 \underline{j} + 2\omega_1 \underline{k}) \qquad (1)$$

EQ. (18.22): $\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Ax'y'z'} + \underline{\Omega} \times \underline{H}_A$

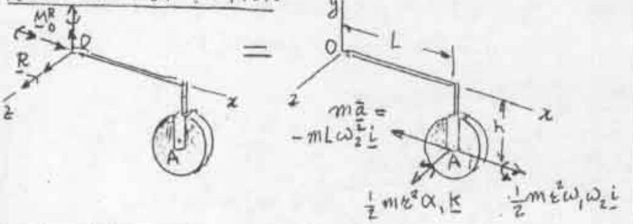
WHERE THE FIRST TERM IS OBTAINED BY DIFFERENTIATING $\underline{H}_A$ ASSUMING THE FRAME $Ax'y'z'$ TO BE FIXED:

$$(\dot{\underline{H}}_A)_{Ax'y'z'} = \tfrac{1}{2} m r^2 \dot{\omega}_1 \underline{k} = \tfrac{1}{2} m r^2 \alpha_1 \underline{k} \qquad \text{WITH } \alpha_1 = -4 \text{ rad/s}^2$$

THUS: $\dot{\underline{H}}_A = \tfrac{1}{2} m r^2 \alpha_1 \underline{k} + \omega_2 \underline{j} \times \tfrac{1}{4} m r^2 (\omega_2 \underline{j} + 2\omega_1 \underline{k})$

$$\dot{\underline{H}}_A = \tfrac{1}{2} m r^2 (\omega_1 \omega_2 \underline{i} + \alpha_1 \underline{k}) \qquad (2)$$

EQUATIONS OF MOTION



$\Sigma \underline{F} = \Sigma(\underline{F})_{eff}$: $\underline{R} = -mL\omega_2^2 \underline{i} \qquad (3)$

$\Sigma \underline{M}_O = \Sigma(\underline{M}_O)_{eff}$: $\underline{M}_O^R = \dot{\underline{H}}_A + (L\underline{i} - h\underline{j}) \times (-mL\omega_2^2 \underline{i})$

$$\underline{M}_O^R = \tfrac{1}{2} m r^2 \omega_1 \omega_2 \underline{i} + \tfrac{1}{2} m r^2 \alpha_1 \underline{k} - mhL\omega_2^2 \underline{k}$$

$$\underline{M}_O^R = \tfrac{1}{2} m r^2 \omega_1 \omega_2 \underline{i} + m\left(\tfrac{1}{2} r^2 \alpha_1 - hL\omega_2^2\right)\underline{k} \qquad (4)$$

WITH GIVEN DATA:

$m = \frac{W}{g} = \frac{8 \text{ lb}}{32.2 \text{ ft/s}^2} = 0.24845$ lb·s²/ft, $L = \frac{4}{3}$ ft, $h = \frac{5}{6}$ ft, $r = \frac{5}{12}$ ft

$\omega_1 = 12$ rad/s, $\alpha_1 = -4$ rad/s², $\omega_2 = 4$ rad/s

EQ. (3): $\underline{R} = -(0.24845 \text{ lb·s}^2/\text{ft})(\tfrac{4}{3} \text{ ft})(4 \text{ rad/s})^2 \underline{i}$
$= -(5.300 \text{ lb})\underline{i}$

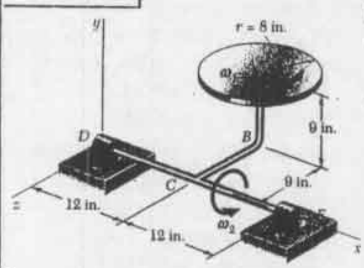
EQ. (4): $\underline{M}_O^R = \tfrac{1}{2}(0.24845 \text{ lb·s}^2/\text{ft})(\tfrac{5}{12}\text{ ft})^2(12 \text{ rad/s})(4 \text{ rad/s}) +$
$+ (0.24845 \text{ lb·s}^2/\text{ft})\left[\tfrac{1}{2}(\tfrac{5}{12}\text{ ft})^2(-4 \text{ rad/s}^2) - (\tfrac{5}{6}\text{ ft})(\tfrac{4}{3}\text{ ft})(4 \text{ rad/s})^2\right]\underline{k}$
$= (1.0352 \text{ lb·ft})\underline{i} - 0.24845(0.34722 + 17.778)\underline{k}$

$\underline{M}_O^R = (1.0352 \text{ lb·ft})\underline{i} - (4.503 \text{ lb·ft})\underline{k}$

FORCE-COUPLE AT O:

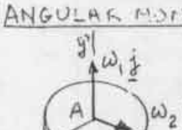
$\underline{R} = -(5.30 \text{ lb})\underline{i}$; $\underline{M}_O^R = (1.035 \text{ lb·ft})\underline{i} - (4.50 \text{ lb·ft})\underline{k}$ ◀

18.104



GIVEN:
DISK OF WEIGHT $W = 6$ lb ROTATES WITH CONSTANT $\underline{\omega}_1 = (16 \text{ rad/s})\underline{j}$.
AT INSTANT SHOWN, SHAFT DCE HAS $\underline{\omega}_2 = (8 \text{ rad/s})\underline{i}$ AND $\underline{\alpha}_2 = (6 \text{ rad/s}^2)\underline{i}$.
FIND:
(a) COUPLE APPLIED TO SHAFT
(b) DYNAMIC REACTIONS AT D AND E.

ANGULAR MOMENTUM ABOUT A



$$\underline{H}_A = \bar{I}_{x'}\omega_x \underline{i} + \bar{I}_{y'}\omega_y \underline{j} = \tfrac{1}{4} m r^2 \omega_2 \underline{i} + \tfrac{1}{2} m r^2 \omega_1 \underline{j}$$

$$\underline{H}_A = \tfrac{1}{4} m r^2 (\omega_2 \underline{i} + 2\omega_1 \underline{j}) \qquad (1)$$

EQ. (18.22): $\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Ax'y'z'} + \underline{\Omega} \times \underline{H}_A$

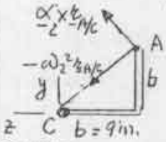
WHERE THE FIRST TERM IS OBTAINED BY DIFFERENTIATING $\underline{H}_A$ ASSUMING THE FRAME $Dx'y'z'$ TO BE FIXED:

$$(\dot{\underline{H}}_A)_{Ax'y'z'} = \tfrac{1}{4} m r^2 \dot{\omega}_2 \underline{i} = \tfrac{1}{4} m r^2 \alpha_2 \underline{i}$$

THUS: $\dot{\underline{H}}_A = \tfrac{1}{4} m r^2 \alpha_2 \underline{i} + \omega_2 \underline{i} \times \tfrac{1}{4} m r^2 (\omega_2 \underline{i} + 2\omega_1 \underline{j})$

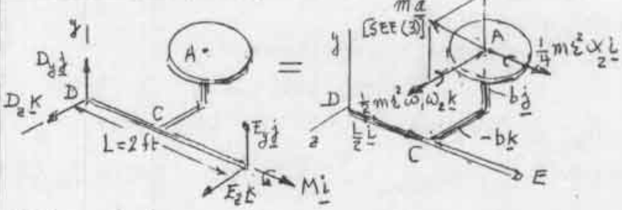
$$\dot{\underline{H}}_A = \tfrac{1}{4} m r^2 \alpha_2 \underline{i} + \tfrac{1}{2} m r^2 \omega_1 \omega_2 \underline{k} \qquad (2)$$

ACCELERATION OF MASS CENTER



$$\bar{\underline{a}} = \underline{\alpha}_2 \times \underline{r}_{A/C} - \omega_2^2 \underline{r}_{A/C}$$
$$= \alpha_2 \underline{i} \times (b\underline{j} - b\underline{k}) - \omega_2^2 (b\underline{j} - b\underline{k})$$
$$\bar{\underline{a}} = b(\alpha_2 - \omega_2^2)\underline{j} + b(\alpha_2 + \omega_2^2)\underline{k} \qquad (3)$$

EQUATIONS OF MOTION



$\Sigma \underline{M}_D = \Sigma(\underline{M}_D)_{eff}$:

$L\underline{i} \times (E_y \underline{j} + E_z \underline{k}) + M\underline{i} = (\tfrac{L}{2}\underline{i} + b\underline{j} - b\underline{k}) \times mb\left[(\alpha_2 - \omega_2^2)\underline{j} + (\alpha_2 + \omega_2^2)\underline{k}\right] + \dot{\underline{H}}_A$

$LE_y \underline{k} - LE_z \underline{j} + M\underline{i} = \tfrac{1}{2} mbL\left[(\alpha_2 - \omega_2^2)\underline{k} - (\alpha_2 + \omega_2^2)\underline{j}\right] + 2mb^2 \alpha_2 \underline{i} +$
$+ \tfrac{1}{4} m r^2 \alpha_2 \underline{i} + \tfrac{1}{2} m r^2 \omega_1 \omega_2 \underline{k}$

EQUATING COEFF. OF UNIT VECTORS:

($\underline{i}$) $M = m(2b^2 + \tfrac{1}{4} r^2)\alpha_2 \qquad (4)$

($\underline{j}$) $-LE_z = -\tfrac{1}{2} mbL(\alpha_2 + \omega_2^2)$, $\quad E_z = \tfrac{1}{2} mb(\alpha_2 + \omega_2^2) \qquad (5)$

($\underline{k}$) $LE_y = \tfrac{1}{2} m\left[bL(\alpha_2 - \omega_2^2) + r^2 \omega_1 \omega_2\right]$
$E_y = \tfrac{1}{2} m\left[b(\alpha_2 - \omega_2^2) + \tfrac{r^2}{L}\omega_1 \omega_2\right] \qquad (6)$

$\Sigma F_y = \Sigma(F_y)_{eff}$: $D_y + E_y = ma_y \qquad D_y = mb(\alpha_2 - \omega_2^2) - E_y$
$D_y = \tfrac{1}{2} m\left[b(\alpha_2 - \omega_2^2) - \tfrac{r^2}{L}\omega_1 \omega_2\right] \qquad (7)$

$\Sigma F_z = \Sigma(F_z)_{eff}$: $D_z + E_z = ma_z \qquad D_z = mb(\alpha_2 + \omega_2^2) - E_z$
$D_z = \tfrac{1}{2} mb(\alpha_2 + \omega_2^2)$

WITH GIVEN DATA:

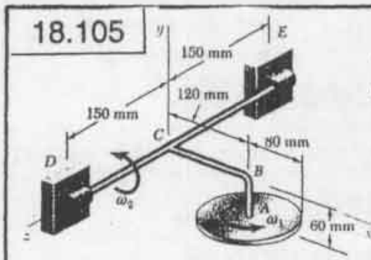
$m = 6/32.2 = 0.18634$, $L = 2$ ft, $b = \frac{3}{4}$ ft, $r = \frac{2}{3}$ ft, $\omega_1 = 16$, $\omega_2 = 8$, $\alpha_2 = 6$

(a) $M = 1.382$ lb·ft $\qquad \underline{M} = (1.382 \text{ lb·ft})\underline{i}$ ◀

(b) $D_y = -6.70$ lb, $E_y = -1.403$ lb, $D_z = E_z = 4.89$ lb

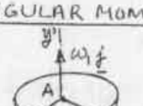
$\underline{D} = -(6.70 \text{ lb})\underline{j} + (4.89 \text{ lb})\underline{k}$; $\underline{E} = -(1.403 \text{ lb})\underline{j} + (4.89 \text{ lb})\underline{k}$ ◀

18.105



GIVEN:
2.5-kg DISK ROTATES WITH $\underline{\omega}_1 = \omega_1 \underline{j}$, $\underline{\alpha}_1 = -(15\,\text{rad/s}^2)\underline{j}$.
SHAFT DCE ROTATES WITH CONSTANT $\underline{\omega}_2 = (12\,\text{rad/s})\underline{k}$.
FIND:
DYNAMIC REACTIONS AT D AND E WHEN ω_1 HAS DECREASED TO 50 rad/s.

ANGULAR MOMENTUM ABOUT A



$\underline{H}_A = \bar{I}_{y'}\omega_{y'}\underline{j} + \bar{I}_{z'}\omega_{z'}\underline{k} = \frac{1}{2}mr^2\omega_1\underline{j} + \frac{1}{4}mr^2\omega_2\underline{k}$

EQ. (18.22): $\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Ax'y'z'} + \underline{\Omega} \times \underline{H}_A$

THE FIRST TERM IS THE RATE OF CHANGE OF $\underline{H}_A$ WITH RESPECT TO THE ROTATING FRAME $Ax'y'z'$.

$(\dot{\underline{H}}_A)_{Ax'y'z'} = \frac{1}{2}mr^2\dot{\omega}_1\underline{j} = \frac{1}{2}mr^2\alpha_1\underline{j}$; ALSO: $\underline{\Omega} = \omega_2\underline{k}$

THUS: $\dot{\underline{H}}_A = \frac{1}{2}mr^2\alpha_1\underline{j} + \omega_2\underline{k} \times (\frac{1}{2}mr^2\omega_1\underline{j} + \frac{1}{4}mr^2\omega_2\underline{k})$

$\dot{\underline{H}}_A = \frac{1}{2}mr^2\alpha_1\underline{j} - \frac{1}{2}mr^2\omega_1\omega_2\underline{i} = \frac{1}{2}mr^2(-\omega_1\omega_2\underline{i} + \alpha_1\underline{j})$

$= \frac{1}{2}(2.5\,\text{kg})(0.08\,\text{m})^2[-(50\,\text{rad/s})(12\,\text{rad/s})\underline{i} + (-15\,\text{rad/s}^2)\underline{j}]$

$$\dot{\underline{H}}_A = -(4.8\,\text{N}\cdot\text{m})\underline{i} - (0.120\,\text{N}\cdot\text{m})\underline{j} \qquad (1)$$

ACCELERATION OF MASS CENTER

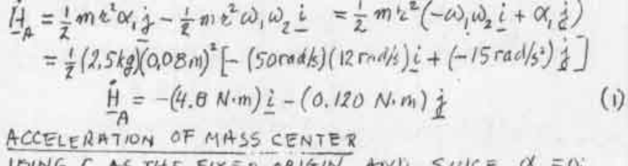
USING C AS THE FIXED ORIGIN, AND SINCE $\alpha_2 = 0$:

$\bar{\underline{a}} = -\underline{r}_{A/C}\,\omega_2^2 = -[(0.12\,\text{m})\underline{i} - (0.06\,\text{m})\underline{j}](12\,\text{rad/s})^2$

$\bar{\underline{a}} = -(17.28\,\text{m/s}^2)\underline{i} + (8.64\,\text{m/s}^2)\underline{j}$

THUS: $m\bar{\underline{a}} = (2.5\,\text{kg})\bar{\underline{a}}$ $\qquad m\bar{\underline{a}} = -(43.2\,\text{N})\underline{i} + (21.6\,\text{N})\underline{j} \quad (2)$

EQUATIONS OF MOTION



$\Sigma\underline{M}_D = \Sigma(\underline{M}_D)_{eff}$:

$-(0.3\,\text{m})\underline{k} \times (E_x\underline{i} + E_y\underline{j}) = -(4.8\,\text{N}\cdot\text{m})\underline{i} - (0.12\,\text{N}\cdot\text{m})\underline{j} + \underline{r}_{A/D} \times m\bar{\underline{a}}$

$-0.3E_x\underline{j} + 0.3E_y\underline{i} = -4.8\underline{i} - 0.12\underline{j} + (-0.15\underline{k} + 0.12\underline{i} - 0.06\underline{j}) \times (-43.2\underline{i} + 21.6\underline{j})$

$-0.3E_x\underline{j} + 0.3E_y\underline{i} = -4.8\underline{i} - 0.12\underline{j} + 6.48\underline{j} + 3.24\underline{i} + \cancel{2.592\underline{k}} - \cancel{2.592\underline{k}}$

$-0.3E_x\underline{j} + 0.3E_y\underline{i} = -1.56\underline{i} + 6.36\underline{j}$

EQUATING THE COEFF. OF THE UNIT VECTORS:

$-0.3E_x = 6.36 \qquad E_x = -21.2\,\text{N}$

$0.3E_y = -1.56 \qquad E_y = -5.20\,\text{N}$

$\Sigma F_x = \Sigma(F_x)_{eff}$: $D_x - 21.2\,\text{N} = -43.2\,\text{N} \qquad D_x = -22.0\,\text{N}$

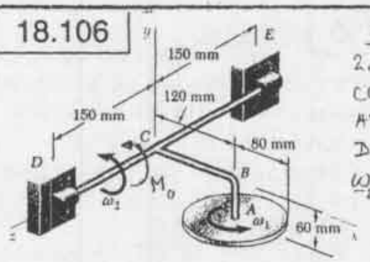
$\Sigma F_y = \Sigma(F_y)_{eff}$: $D_y - 5.20\,\text{N} = 21.6\,\text{N} \qquad D_y = 26.8\,\text{N}$

ANSWER

$\underline{D} = -(22.0\,\text{N})\underline{i} + (26.8\,\text{N})\underline{j}$; $\underline{E} = -(21.2\,\text{N})\underline{i} - (5.20\,\text{N})\underline{j}$

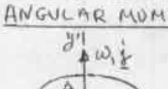
(ANSWER GIVEN WITH RESPECT TO ROTATING Cxyz AXES)

18.106



GIVEN:
2.5-kg DISK ROTATES WITH CONSTANT $\underline{\omega}_1 = (50\,\text{rad/s})\underline{j}$.
AT INSTANT SHOWN, SHAFT DCE ROTATES WITH $\underline{\omega}_2 = (12\,\text{rad/s})\underline{k}$, $\underline{\alpha}_2 = (8\,\text{rad/s}^2)\underline{k}$.
FIND:
(a) COUPLE $\underline{M}_0$ APPLIED TO THE SHAFT,
(b) DYNAMIC REACTIONS AT D AND E

ANGULAR MOMENTUM ABOUT A



$\underline{H}_A = \bar{I}_{y'}\omega_{y'}\underline{j} + \bar{I}_{z'}\omega_{z'}\underline{k} = \frac{1}{2}mr^2\omega_1\underline{j} + \frac{1}{4}mr^2\omega_2\underline{k}$

EQ.(18.22): $\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{Ax'y'z'} + \underline{\Omega} \times \underline{H}_A$

THE FIRST TERM IS THE RATE OF CHANGE OF $\underline{H}_A$ WITH RESPECT TO THE FRAME $Ax'y'z'$ WHICH ROTATES AT $\underline{\Omega} = \omega_2\underline{k}$.

$(\dot{\underline{H}}_A)_{Ax'y'z'} = \frac{1}{4}mr^2\dot{\omega}_2\underline{k} = \frac{1}{4}mr^2\alpha_2\underline{k}$

THUS: $\dot{\underline{H}}_A = \frac{1}{4}mr^2\alpha_2\underline{k} + \omega_2\underline{k} \times (\frac{1}{2}mr^2\omega_1\underline{j} + \frac{1}{4}mr^2\omega_2\underline{k})$

$\dot{\underline{H}}_A = \frac{1}{4}mr^2\alpha_2\underline{k} - \frac{1}{2}mr^2\omega_1\omega_2\underline{i} = \frac{1}{4}mr^2(-2\omega_1\omega_2\underline{i} + \alpha_2\underline{k})$

$= \frac{1}{4}(2.5\,\text{kg})(0.08\,\text{m})^2[-2(50\,\text{rad/s})(12\,\text{rad/s})\underline{i} + (8\,\text{rad/s}^2)\underline{k}]$

$$\dot{\underline{H}}_A = -(4.8\,\text{N}\cdot\text{m})\underline{i} + (0.032\,\text{N}\cdot\text{m})\underline{k} \qquad (1)$$

ACCELERATION OF MASS CENTER

USING C AS THE FIXED ORIGIN:

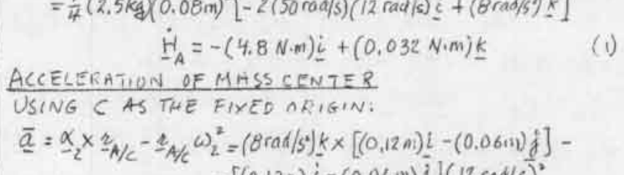
$\bar{\underline{a}} = \underline{\alpha}_2 \times \underline{r}_{A/C} - \underline{r}_{A/C}\,\omega_2^2 = (8\,\text{rad/s}^2)\underline{k} \times [(0.12\,\text{m})\underline{i} - (0.06\,\text{m})\underline{j}] - [(0.12\,\text{m})\underline{i} - (0.06\,\text{m})\underline{j}](12\,\text{rad/s})^2$

$= (0.96\,\text{m/s}^2)\underline{j} + (0.48\,\text{m/s}^2)\underline{i} - (17.28\,\text{m/s}^2)\underline{i} + (8.64\,\text{m/s}^2)\underline{j}$

$\bar{\underline{a}} = -(16.8\,\text{m/s}^2)\underline{i} + (9.6\,\text{m/s}^2)\underline{j}$

THUS: $m\bar{\underline{a}} = (2.5\,\text{kg})\bar{\underline{a}}$ $\qquad m\bar{\underline{a}} = -(42.0\,\text{N})\underline{i} + (24.0\,\text{N})\underline{j} \quad (2)$

EQUATIONS OF MOTION



$\Sigma\underline{M}_D = \Sigma(\underline{M}_D)_{eff}$:

$-(0.3\,\text{m})\underline{k} \times (E_x\underline{i} + E_y\underline{j}) + M_0\underline{k} = -(4.8\,\text{N}\cdot\text{m})\underline{i} + (0.032\,\text{N}\cdot\text{m})\underline{k} + \underline{r}_{A/D} \times m\bar{\underline{a}}$

$-0.3E_x\underline{j} + 0.3E_y\underline{i} + M_0\underline{k} = -4.8\underline{i} + 0.032\underline{k} + (-0.15\underline{k} + 0.12\underline{i} - 0.06\underline{j}) \times (-42\underline{i} + 24\underline{j})$

$-0.3E_x\underline{j} + 0.3E_y\underline{i} + M_0\underline{k} = -4.8\underline{i} + 0.032\underline{k} + 6.30\underline{j} + 3.60\underline{i} + 2.88\underline{k} - 2.52\underline{k}$

EQUATING THE COEFF. OF UNIT VECTORS:

(a) ($\underline{k}$) $M_0 = 0.032 + 2.88 - 2.52 = 0.392\,\text{N}\cdot\text{m}$

$\underline{M}_0 = (0.392\,\text{N}\cdot\text{m})\underline{k}$

(b) ($\underline{j}$) $-0.3E_x = 6.30 \qquad E_x = -21.0\,\text{N}$

($\underline{i}$) $0.3E_y = -4.8 + 3.6 = -1.20 \qquad E_y = -4.00\,\text{N}$

$\Sigma F_x = \Sigma(F_x)_{eff}$: $D_x - 21.0\,\text{N} = -42.0\,\text{N} \qquad D_x = -21.0\,\text{N}$

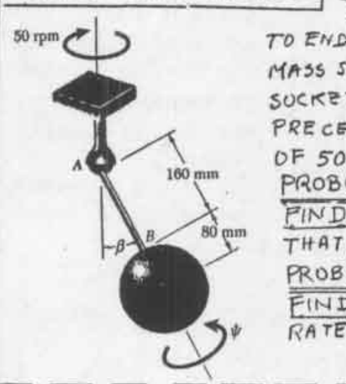
$\Sigma F_y = \Sigma(F_y)_{eff}$: $D_y - 4.00\,\text{N} = 24.0\,\text{N} \qquad D_y = 28.0\,\text{N}$

$\underline{D} = -(21.0\,\text{N})\underline{i} + (28.0\,\text{N})\underline{j}$; $\underline{E} = -(21.0\,\text{N})\underline{i} - (4.00\,\text{N})\underline{j}$

(ANSWER GIVEN WITH RESPECT TO ROTATING Cxyz AXES)

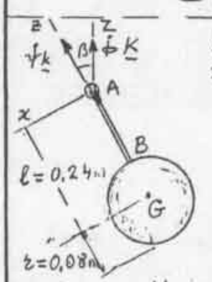
18.107 and 18.108

GIVEN:
SOLID SPHERE WELDED TO END OF ROD AB OF NEGLIGIBLE MASS SUPPORTED BY BALL AND SOCKET AT A. SPHERE PRECESSES AT CONSTANT RATE OF 50 rpm AS SHOWN.



PROBLEM 18.107:
FIND RATE OF SPIN $\dot\psi$, KNOWING THAT $\beta = 25°$.

PROBLEM 18.108:
FIND β, KNOWING THAT RATE OF SPIN IS $\dot\psi = 800$ rpm.



ANGULAR VELOCITIES:

SPHERE: $\underline{\omega} = \dot\phi \underline{K} + \dot\psi \underline{k}$

$$\underline{\omega} = -\dot\phi \sin\beta\, \underline{i} + (\dot\psi + \dot\phi\cos\beta)\underline{k}$$

FRAME Axyz: $\underline{\Omega} = \dot\phi \underline{K}$

$$\underline{\Omega} = -\dot\phi \sin\beta\, \underline{i} + \dot\phi\cos\beta\, \underline{k}$$

ANGULAR MOMENTUM ABOUT A

$$\underline{H}_A = \bar I_x \omega_x \underline{i} + \bar I_z \omega_z \underline{k}$$

$$\underline{H}_A = -m\left(\tfrac{2}{5}c^2 + l^2\right)\dot\phi \sin\beta\, \underline{i} + \tfrac{2}{5} m c^2 (\dot\psi + \dot\phi\cos\beta)\underline{k}$$

SINCE A IS FIXED, WE USE EQ. (18.28):

$$\Sigma \underline{M}_A = (\dot{\underline{H}}_A)_{Axyz} + \underline{\Omega}\times\underline{H}_A = 0 + (-\dot\phi \sin\beta\,\underline{i} + \dot\phi\cos\beta)\underline{k} \times \underline{H}_A$$

$$= (-\dot\phi\sin\beta\,\underline{i} + \dot\phi\cos\beta\,\underline{k}) \times m\left[-\left(\tfrac{2}{5}c^2 + l^2\right)\dot\phi\sin\beta\,\underline{i} + \tfrac{2}{5}c^2(\dot\psi + \dot\phi\cos\beta)\underline{k}\right]$$

$$= m\dot\phi\sin\beta\left[\tfrac{2}{5}c^2(\dot\psi + \dot\phi\cos\beta) - \left(\tfrac{2}{5}c^2 + l^2\right)\dot\phi\cos\beta\right]\underline{j} \qquad (1)$$

BUT $\Sigma \underline{M}_A = -l\underline{k} \times (-mg\underline{K}) = -mgl\sin\beta\,\underline{j}$ (2)

EQUATING (1) AND (2):

$$m\dot\phi\sin\beta\left[\tfrac{2}{5}c^2(\dot\psi + \dot\phi\cos\beta) - \left(\tfrac{2}{5}c^2 + l^2\right)\dot\phi\cos\beta\right] = -mgl\sin\beta$$

$$\tfrac{2}{5}c^2(\dot\psi + \dot\phi\cos\beta) = \left(\tfrac{2}{5}c^2 + l^2\right)\dot\phi\cos\beta - \frac{gl}{\dot\phi} \qquad (3)$$

GIVEN DATA: (NOTE THAT $\dot\phi$ IS NEGATIVE)
$c = 0.08$ m, $l = 0.24$ m, $g = 9.81$ m/s²; $\dot\phi = -50$ rpm $= -5.236$ rad/s

$$2.56\times10^{-3}(\dot\psi - 5.236\cos\beta) = 60.16\times10^{-3}(-5.236\cos\beta) + 449.7\times10^{-3}$$

$$\dot\psi = -117.81\cos\beta + 175.65 \qquad (4)$$

PROBLEM 18.107
WITH $\beta = 25°$, EQ (4) YIELDS

$$\dot\psi = -117.81\cos 25° + 175.65 = +68.875 \text{ rad/s}$$
$$= 657.7 \text{ rpm}$$

$\dot\psi = 658$ rpm ◄

PROBLEM 18.108

WITH $\dot\psi = 800$ rpm $= 83.776$ rad/s, EQ. (4) READS

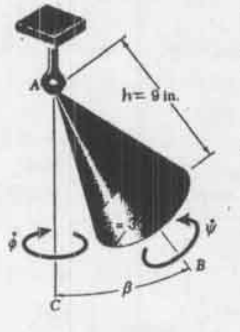
$$83.776 = -117.81\cos\beta + 175.65$$

$\cos\beta = 0.77985$ $\quad \beta = 38.753°$

$\beta = 38.8°$ ◄

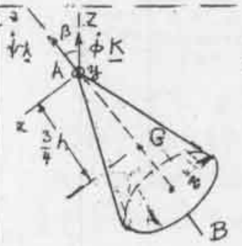
18.109 and 18.110

CONE SUPPORTED BY BALL AND SOCKET AT A.



PROBLEM 18.109:
GIVEN:
PRECESSES AS SHOWN AT CONSTANT RATE OF 40 rpm WITH $\beta = 40°$
FIND:
RATE OF SPIN $\dot\psi$

PROBLEM 18.110
GIVEN:
$\dot\psi = 3000$ rpm, $\beta = 60°$
FIND:
TWO POSSIBLE VALUES OF $\dot\phi$



ANGULAR VELOCITIES

CONE: $\underline{\omega} = \dot\phi \underline{K} + \dot\psi \underline{k}$

$$\underline{\omega} = -\dot\phi\sin\beta\,\underline{i} + (\dot\psi + \dot\phi\cos\beta)\underline{k}$$

FRAME Axyz: $\underline{\Omega} = \dot\phi \underline{K}$

$$\underline{\Omega} = -\dot\phi\sin\beta\,\underline{i} + \dot\phi\cos\beta\,\underline{k}$$

ANGULAR MOMENTUM ABOUT A

$$\underline{H}_A = I_x \omega_x \underline{i} + I_z \omega_z \underline{k}$$

$$\underline{H}_A = -\tfrac{3}{5}m\left(\tfrac{r^2}{4} + h^2\right)\dot\phi\sin\beta\,\underline{i} + \tfrac{3}{10} m r^2(\dot\psi + \dot\phi\cos\beta)\underline{k}$$

SINCE A IS FIXED, WE USE EQ. (18.28):

$$\Sigma\underline{M}_A = (\dot{\underline{H}}_A)_{Axyz} + \underline{\Omega}\times\underline{H}_A = 0 + (-\dot\phi\sin\beta\,\underline{i} + \dot\phi\cos\beta\,\underline{k}) \times \underline{H}_A$$

$$= (-\dot\phi\sin\beta\,\underline{i} + \dot\phi\cos\beta\,\underline{k}) \times \left[-\tfrac{3}{5}m\left(\tfrac{r^2}{4} + h^2\right)\dot\phi\sin\beta\,\underline{i} + \tfrac{3}{10}mr^2(\dot\psi + \dot\phi\cos\beta)\underline{k}\right]$$

$$= \tfrac{3}{5}m\dot\phi\sin\beta\left[\tfrac{1}{2}r^2(\dot\psi + \dot\phi\cos\beta) - \left(\tfrac{r^2}{4} + h^2\right)\dot\phi\cos\beta\right]\underline{j} \qquad (1)$$

BUT $\Sigma\underline{M}_A = -\frac{3}{4}h\underline{k} \times (-mg\underline{K}) = -\frac{3}{4}mgh\sin\beta\,\underline{j}$ (2)

EQUATING (1) AND (2):

$$\tfrac{3}{5}m\dot\phi\sin\beta\left[\tfrac{1}{2}r^2(\dot\psi + \dot\phi\cos\beta) - \left(\tfrac{r^2}{4} + h^2\right)\dot\phi\cos\beta\right] = -\tfrac{3}{4}mgh\sin\beta$$

$$\tfrac{1}{2}r^2(\dot\psi + \dot\phi\cos\beta) - \left(\tfrac{r^2}{4} + h^2\right)\dot\phi\cos\beta = -\frac{5}{4}\frac{gh}{\dot\phi}$$

$$\tfrac{1}{2}r^2\dot\psi - \left(h^2 - \tfrac{r^2}{4}\right)\dot\phi\cos\beta = -\frac{5}{4}\frac{gh}{\dot\phi}$$

WITH $r = \frac{1}{4}$ ft, $h = \frac{3}{4}$ ft, $g = 32.2$ ft/s², AND MULTIPLYING BY 32,

$$\dot\psi - 17.5\,\dot\phi\cos\beta = -966/\dot\phi \qquad (3)$$

PROBLEM 18.109
LETTING $\dot\phi = -40$ rpm $= -4.1888$ rad/s, $\beta = 40°$ IN (3),

$$\dot\psi - 17.5(-4.1888)\cos 40° = -966/(-4.1888)$$

$$\dot\psi = -56.154 + 230.616 = 174.46 \text{ rad/s} = 1666.0 \text{ rpm}$$

$\dot\psi = 1666$ rpm ◄

PROBLEM 18.110
LETTING $\dot\psi = 3000$ rpm $= 314.16$ rad/s, $\beta = 60°$ IN (3),

$$314.16 - 17.5\,\dot\phi\cos 60° = -966/\dot\phi$$
$$8.75\dot\phi^2 - 314.16\dot\phi - 966 = 0$$
$$\dot\phi^2 - 35.904\dot\phi - 110.4 = 0$$
$$\dot\phi = \tfrac{1}{2}\left(35.904 \pm \sqrt{(35.904)^2 + 4(110.4)}\right) = \tfrac{1}{2}(35.904 \pm 41.602)$$

$\dot\phi = +38.753$ rad/s $= +370$ rpm

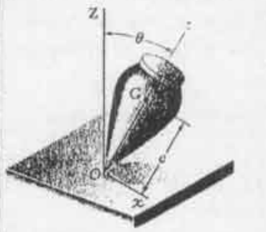
$\dot{\underline{\phi}} = (370 \text{ rpm})\underline{K}$ ◄
(SENSE OPPOSITE TO SENSE SHOWN)

OR $\dot\phi = -2.849$ rad/s $= -27.2$ rpm

$\dot{\underline{\phi}} = -(27.2 \text{ rpm})\underline{K}$ ◄
(SAME SENSE AS SHOWN)

18.111 and 18.112

TOP SUPPORTED AT FIXED POINT O.



PROBLEM 18.111:

GIVEN:

$m = 85\,g$, $k_z = 21\,mm$, $k_x = 45\,mm$

$c = 37.5\,mm$, $\theta = 30°$,

RATE OF SPIN ABOUT z AXIS $=$ $\dot\psi = 1800$ rpm.

FIND:

TWO POSSIBLE RATES $\dot\phi$ OF STEADY PRECESSION.

PROBLEM 18.112

GIVEN: $I_z = I$, $I_x = I'$, ω_z = RECTANGULAR COMPONENT OF $\underline{\omega}$ ALONG z AXIS

(a) SHOW THAT $(I\omega_z - I'\dot\phi\cos\theta)\dot\phi = Wc$

(b) SHOW THAT $I\dot\psi\dot\phi \approx Wc$ IF $\dot\psi >> \dot\phi$

(c) FIND PERCENT ERROR WHEN EXPRESSION UNDER b IS USED TO APPROXIMATE THE SLOWER $\dot\phi$ OF PROB. 18.111.

WE RECALL FROM PAGE 1150 THE FOLLOWING EQS.

$$\underline{\omega} = -\dot\phi\sin\theta\,\underline{i} + \omega_z\underline{k} \qquad (18.40)$$

$$\underline{H}_O = -I'\dot\phi\sin\theta\,\underline{i} + I\omega_z\underline{k} \qquad (18.41)$$

$$\underline{\Omega} = -\dot\phi\sin\theta\,\underline{i} + \dot\phi\cos\theta\,\underline{k} \qquad (18.42)$$

SINCE O IS A FIXED POINT, WE USE EQ. (18.28):

$$\Sigma\underline{M}_O = (\dot{\underline{H}}_O)_{Oxyz} + \underline{\Omega}\times\underline{H}_O = 0 + \underline{\Omega}\times H_O$$

$$= (-\dot\phi\sin\theta\,\underline{i} + \dot\phi\cos\theta\,\underline{k})\times(-I'\dot\phi\sin\theta\,\underline{i} + I\omega_z\underline{k})$$

$$= (I\omega_z\dot\phi\sin\theta - I'\dot\phi^2\cos\theta\sin\theta)\underline{j}$$

$$= (I\omega_z - I'\dot\phi\cos\theta)\dot\phi\sin\theta\,\underline{j} \qquad (1)$$

WHERE $\underline{j}$ IS $\perp$ PLANE OZZ AND POINTS AWAY

BUT $\Sigma\underline{M}_O = c\underline{k}\times(-W\underline{K}) = Wc\sin\theta\,\underline{j} \qquad (2)$

EQUATING EQS. (1) AND (2):

$$(I\omega_z - I'\dot\phi\cos\theta)\dot\phi = Wc \qquad \triangleleft \quad (3)$$

PROBLEM 18.111

SINCE $I = mk_z^2$, $I' = mk_x^2$, $W = mg$, EQ (3) YIELDS

$$(k_z^2\omega_z - k_x^2\dot\phi\cos\theta)\dot\phi = gc$$

WHERE $\omega_z = \dot\psi + \dot\phi\cos\theta$

WITH GIVEN DATA AND $\dot\psi = 1800\,rpm = 60\pi$ rad/s :

$$[(0.021)^2(60\pi + \dot\phi\cos30°) - (0.045)^2\dot\phi\cos30°]\dot\phi = 9.81(0.0375)$$

$$[(0.045)^2 - (0.021)^2]\cos30°\dot\phi^2 - (0.021)^2 60\pi\,\dot\phi + 9.81(0.0375) = 0$$

$$\dot\phi^2 - 60.597\,\dot\phi + 268.17 = 0$$

SOLVING: $\dot\phi = 30.299 \pm 25.492$

$\dot\phi = 55.79$ rad/s AND $\dot\phi = 4.807$ rad/s

ANSWER: 533 rpm AND 45.9 rpm ◀

PROBLEM 18.112

(a) SEE DERIVATION OF EQ. (3) ABOVE ◀

(b) FOR $\dot\psi >> \dot\phi$, $\omega_z \approx \dot\psi$, AND EQ. (3) REDUCES TO

$$(I\dot\psi - I'\dot\phi\cos\theta)\dot\phi = Wc$$

AND, WITH $\dot\psi >> \dot\phi$, TO

$$I\dot\psi\dot\phi = Wc \qquad (Q.E.D.) \quad ◀$$

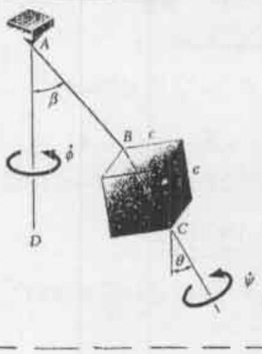
(c) WITH DATA OF PROB. 18.111, ABOVE EQUATION YIELDS

$$\dot\phi = \frac{Wc}{I\dot\psi} = \frac{mgc}{mk_z^2\dot\psi} = \frac{9.81(0.0375)}{(0.021)^2(60\pi\,rad/s)} = 4.4255\,rad/s = 42.26\,rpm$$

$$\%\,ERROR = 100\,\frac{42.26 - 45.90}{45.90} = -7.9\,\% \quad ◀$$

18.113 and 18.114

SOLID CUBE ATTACHED TO CORD AB



PROBLEM 18.113:

GIVEN:

$c = 80\,mm$, $\beta = 30°$

$\dot\psi = 40$ rad/s, $\dot\phi = 5$ rad/s.

FIND: θ

PROBLEM 18.114:

GIVEN:

$c = 120\,mm$, $AB = 240\,mm$

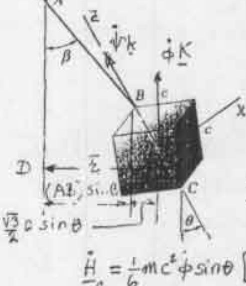
$\theta = 25°$, $\beta = 40°$

FIND:

(a) $\dot\psi$, (b) $\dot\phi$

WE RECALL FROM SEC. 9.17 THAT, SINCE THE 3 PRINCIPAL MOMENTS OF INERTIA OF A CUBE ARE EQUAL, ITS MOMENT OF INERTIA ABOUT ANY LINE THROUGH G IS ALSO $\bar I = \frac{1}{6}mc^2$

USING Gxyz AXES WITH z ALONG CB, x IN ABD PLANE AND y $\perp$ ABD AND POINTING AWAY, WE HAVE



CUBE: $\underline{\omega} = \dot\phi\sin\theta\,\underline{i} + (\dot\psi + \dot\phi\cos\theta)\underline{k}$

$$\underline{H}_G = \tfrac{1}{6}mc^2[\dot\phi\sin\theta\,\underline{i} + (\dot\psi + \dot\phi\cos\theta)\underline{k}]$$

FRAME Gxyz:

$$\underline{\Omega} = \dot\phi\sin\theta\,\underline{i} + \dot\phi\cos\theta\,\underline{k}$$

EQ. (18.22):

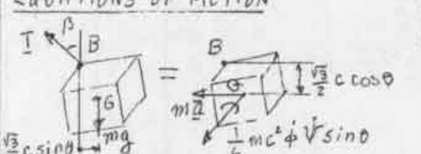
$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G$$

$$\dot{\underline{H}}_G = 0 + (\dot\phi\sin\theta\,\underline{i} + \dot\phi\cos\theta\,\underline{k})\times \tfrac{1}{6}mc^2[\dot\phi\sin\theta\,\underline{i} + (\dot\psi + \dot\phi\cos\theta)\underline{k}]$$

$$\dot{\underline{H}}_G = \tfrac{1}{6}mc^2\dot\phi\sin\theta\,[-(\dot\psi + \dot\phi\cos\theta)\underline{j} + \dot\phi\cos\theta\,\underline{j}]$$

$$\dot{\underline{H}}_G = -\tfrac{1}{6}mc^2\dot\phi\dot\psi\sin\theta\,\underline{j} \qquad (1)$$

EQUATIONS OF MOTION



$\Sigma\underline{F} = \Sigma(\underline{F})_{eff}$:

HORIZ. COMP.: $T\sin\beta = m\bar a \qquad (2)$

VERTICAL COMP.: $T\cos\beta - mg = 0$

$T\cos\beta = mg \qquad (3)$

DIVIDE (2) BY (3): $\tan\beta = \frac{\bar a}{g}$ $\qquad \bar a = g\tan\beta \qquad (4)$

$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$:

$$-mg\tfrac{\sqrt3}{2}c\sin\theta = \tfrac{1}{6}mc^2\dot\phi\dot\psi\sin\theta - (mg\tan\beta)\tfrac{\sqrt3}{2}c\cos\theta$$

DIVIDE BY $mg\frac{\sqrt3}{2}c\cos\theta$ AND SOLVE FOR $\tan\theta$:

$$\tan\theta = \frac{\tan\beta}{1 + (c\dot\phi\dot\psi/3\sqrt3 g)} \qquad (5)$$

PROBLEM 18.113

LETTING $\beta = 30°$, $c = 0.08\,m$, $\dot\phi = 5$ rad/s, $\dot\psi = 40$ rad/s, $g = 9.81\,m/s^2$

IN (5): $\tan\theta = 0.43942 \qquad \theta = 23.7°$ ◀

PROBLEM 18.114

$\bar a = \bar r\dot\phi^2$ RECALLING (4): $\dot\phi^2 = \frac{\bar a}{\bar r} = \frac{g\tan\beta}{(AB)\sin\beta + \frac{\sqrt3}{2}c\sin\theta}$

LETTING $\beta = 40°$, $\theta = 25°$, $AB = 0.24\,m$, $c = 0.12\,m$, $g = 9.81\,m/s^2$:

$\dot\phi = 6.4447$ rad/s $\qquad$ (b) $\dot\phi = 6.44$ rad/s ◀

SOLVING (5) FOR $c\dot\phi\dot\psi$,

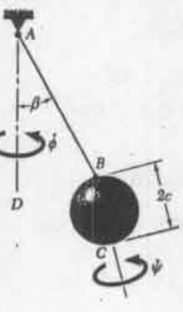
$$c\dot\phi\dot\psi = 3\sqrt3 g\left(\frac{\tan\beta}{\tan\theta} - 1\right) = 3\sqrt3(9.81)\left(\frac{\tan40°}{\tan25°} - 1\right) = 40.752$$

$$\dot\psi = \frac{40.752}{(0.12)(6.4447)} = 52.694\,rad/s$$

(a) $\dot\psi = 52.7$ rad/s ◀

18.115 and 18.116

SOLID SPHERE ATTACHED TO CORD AB.



PROBLEM 18.115:

GIVEN:

$c = 3\text{ in.},\ \beta = 40°,\ \dot\phi = 6\text{ rad/s}$

FIND:

ANGLE θ, KNOWING THAT

(a) $\dot\psi = 0$, (b) $\dot\psi = 50$ rad/s, (c) $\dot\psi = -50$ rad/s.

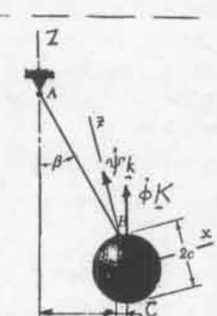
PROBLEM 18.116:

GIVEN:

$c = 3\text{ in.},\ AB = 15\text{ in.},\ \theta = 20°,\ \beta = 35°$

FIND:

(a) $\dot\psi$, (b) $\dot\phi$



USING Gxyz AXES WITH z ALONG CB, x IN ABD PLANE AND y ⊥ ABD AND POINTING AWAY:

SPHERE: $\underline{\omega} = \dot\phi\sin\theta\,\underline{i} + (\dot\psi + \dot\phi\cos\theta)\,\underline{k}$

$\underline{H}_G = \frac{2}{5}mc^2[\dot\phi\sin\theta\,\underline{i} + (\dot\psi + \dot\phi\cos\theta)\underline{k}]$

FRAME Gxyz:

$\underline{\Omega} = \dot\phi\sin\theta\,\underline{i} + \dot\phi\cos\theta\,\underline{k}$

EQ. (18.22):

$\dot{\underline{H}}_G = (\dot{H}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G$

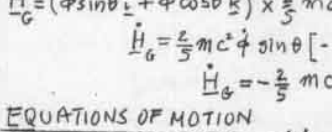
$= 0 + (\dot\phi\sin\theta\,\underline{i} + \dot\phi\cos\theta\,\underline{k})\times\underline{H}_G$

$\dot{\underline{H}}_G = (\dot\phi\sin\theta\,\underline{i} + \dot\phi\cos\theta\,\underline{k})\times\frac{2}{5}mc^2[\dot\phi\sin\theta\,\underline{i} + (\dot\psi + \dot\phi\cos\theta)\underline{k}]$

$\dot{\underline{H}}_G = \frac{2}{5}mc^2\dot\phi\sin\theta[-(\dot\psi + \dot\phi\cos\theta)\underline{j} + \dot\phi\cos\theta\,\underline{j}]$

$$\dot{\underline{H}}_G = -\frac{2}{5}mc^2\dot\phi\dot\psi\sin\theta\,\underline{j} \qquad (1)$$

EQUATIONS OF MOTION



$\Sigma\underline{F} = \Sigma(\underline{F})_{eff}$:

HORIZ. COMP.:

$T\sin\beta = m\bar{a} \qquad (2)$

VERTICAL COMP

$T\cos\beta - W = 0$

$T\cos\beta = mg \qquad (3)$

DIVIDE (2) BY (3): $\tan\beta = \frac{\bar{a}}{g} \qquad \bar{a} = g\tan\beta \qquad (4)$

$+\circlearrowleft \Sigma M_B = \Sigma(M_B)_{eff}$:

$-mg\,c\sin\theta = \frac{2}{5}mc^2\dot\phi\dot\psi\sin\theta - (mg\tan\beta)c\cos\theta$

DIVIDE BY $mgc\cos\theta$ AND SOLVE FOR $\tan\theta$:

$$\tan\theta = \frac{\tan\beta}{1 + (2c\dot\phi\dot\psi/5g)} \qquad (5)$$

PROBLEM 18.115

LETTING $\beta = 40°$, $c = \frac{1}{4}$ ft, $\dot\phi = 6$ rad/s, $g = 32.2$ ft/s² IN (5):

$\tan\theta = \tan 40°/(1 + 0.018634\,\dot\psi)$

(a) FOR $\dot\psi = 0$: $\tan\theta = \tan 40°$ $\quad\theta = 40.0°$ ◀

(b) FOR $\dot\psi = 50$ rad/s: $\tan\theta = 0.43438$ $\quad\theta = 23.5°$ ◀

(c) FOR $\dot\psi = -50$ rad/s: $\tan\theta = 12.285$ $\quad\theta = 85.3°$ ◀

PROBLEM 18.116

$\bar{a} = \bar{r}\dot\phi^2$ RECALLING (4): $\dot\phi^2 = \frac{\bar{a}}{\bar{r}} = \frac{g\tan\beta}{(AB)\sin\beta + c\sin\theta}$

WITH $\beta = 35°$, $\theta = 20°$, $AB = 1.25$ ft, $c = 0.25$ ft, $g = 32.2$ ft/s²:

$\dot\phi = 5.3006$ rad/s $\qquad$ (b) $\dot\phi = 5.30$ rad/s ◀

SOLVING (5) FOR $c\dot\phi\dot\psi$,

$c\dot\phi\dot\psi = 2.5g\left(\frac{\tan\beta}{\tan\theta} - 1\right) = 2.5(32.2)\left(\frac{\tan 35°}{\tan 20°} - 1\right) = 74.366$

$\dot\psi = \frac{74.366}{(0.25)(5.3006)} = 56.12$ rad/s $\qquad$ (a) $\dot\psi = 56.1$ rad/s ◀

18.117

(PRECESSION OF THE EQUINOXES)

GIVEN:

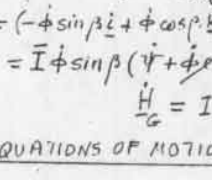
RATE OF PRECESSION OF EARTH ABOUT GA = 1 rev IN 25 800 yr

FOR EARTH: $\rho_{ave} = 5.51$

$r_{ave} = 6370$ km, $I = \frac{2}{5}mr_{ave}^2$

FIND:

AVERAGE VALUE OF COUPLE $\underline{M}$ DUE TO GRAVITATIONAL ATTRACTION OF SUN, MOON, AND PLANETS.



WE USE Gxyz AXES (WITH y POINTING AWAY).

TOTAL ANG. VEL. OF EARTH

$\underline{\omega} = -\dot\phi\sin\beta\,\underline{i} + (\dot\psi + \dot\phi\cos\beta)\underline{k}$

FRAME Gxyz:

$\underline{\Omega} = -\dot\phi\sin\beta\,\underline{i} + \dot\phi\cos\beta\,\underline{k}$

$\underline{H}_G = -\bar{I}\dot\phi\sin\beta\,\underline{i} + \bar{I}(\dot\psi + \dot\phi\cos\beta)\underline{k}$

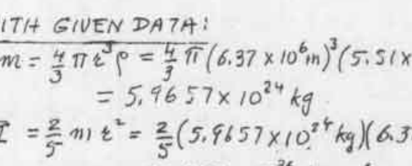
EQ. (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}})_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + (-\dot\phi\sin\beta\,\underline{i} + \dot\phi\cos\beta\,\underline{k})\times\underline{H}_G$

$\dot{\underline{H}}_G = (-\dot\phi\sin\beta\,\underline{i} + \dot\phi\cos\beta\,\underline{k})\times[-\bar{I}\dot\phi\sin\beta\,\underline{i} + \bar{I}(\dot\psi + \dot\phi\cos\beta\,\underline{k})]$

$= \bar{I}\dot\phi\sin\beta(\dot\psi + \cancel{\dot\phi\cos\beta} - \cancel{\dot\phi\cos\beta})\underline{j}$

$$\dot{\underline{H}}_G = I\dot\phi\dot\psi\sin\beta\,\underline{j}$$

EQUATIONS OF MOTION



$\Sigma\underline{M}_G = \Sigma(\underline{M}_G)_{eff}$: $\qquad \underline{M} = \dot{\underline{H}}_G = I\dot\phi\dot\psi\sin\beta$

(NOTE SENSE OF $\underline{M}$)

WITH GIVEN DATA:

$m = \frac{4}{3}\pi r^3\rho = \frac{4}{3}\pi(6.37\times10^6\text{ m})^3(5.51\times10^3\text{ kg/m}^3)$

$= 5.9657\times10^{24}$ kg

$\bar{I} = \frac{2}{5}mr^2 = \frac{2}{5}(5.9657\times10^{24}\text{ kg})(6.37\times10^6\text{ m})^2$

$= 96.827\times10^{36}$ kg·m²

$\dot\phi = \frac{2\pi\text{ rad}}{(25\,800\text{ yr})(365.24\text{ day/yr})(24\text{ h/day})(3600\text{ s/h})} = 7.717\times10^{-12}$ rad/s

$\dot\psi = \frac{2\pi\text{ rad}}{(23.93\text{ h})(3600\text{ s/h})} = 72.935\times10^{-6}$ rad/s, $\beta = 23.45°$

$M = \dot{H}_G = I\dot\phi\dot\psi\sin\beta$

$= (96.827\times10^{36}\text{ kg·m}^2)(7.717\times10^{-12}\text{ s}^{-1})(72.935\times10^{-6}\text{ s}^{-1})\sin 23.45°$

$= 21.69\times10^{21}$ N·m

$$M = 21.7\times10^{21}\text{ N·m}$$ ◀

18.118

GIVEN:

PROJECTILE WITH $m = 20$ kg

$\bar{k}_z = 50$ mm, $\bar{k}_x = 200$ mm

$\bar{v} = 600$ m/s (HORIZONTAL),

DRAG $= \underline{D} = 120$ N (HORIZONTAL)

$\beta = 3°$, $c = 150$ mm

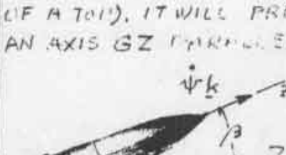
$\dot{\psi} = 6000$ rpm

FIND:

(a) APPROXIMATE VALUE OF RATE OF PRECESSION,

(b) EXACT VALUES OF TWO POSSIBLE RATES OF PRECESSION

SINCE THE DRAG $\underline{D}$ IS A FORCE CONSTANT IN MAGNITUDE AND DIRECTION (LIKE THE WEIGHT OF A TOP), IT WILL PRECESS, LIKE A TOP, ABOUT AN AXIS GZ PARALLEL TO THAT FORCE.



USING THE AXES Gxyz, WITH y POINTING AWAY FROM PROJECTILE:

$$\underline{\omega} = \dot{\phi}\sin\beta\,\underline{i} + (\dot{\psi} + \dot{\phi}\cos\beta)\underline{k}$$

$$\underline{H}_G = \bar{I}_x\,\dot{\phi}\sin\beta\,\underline{i} + \bar{I}_z(\dot{\psi} + \dot{\phi}\cos\beta)\underline{k}$$

FRAME Gxyz: $\underline{\Omega} = \dot{\phi}\sin\beta\,\underline{i} + \dot{\phi}\cos\beta\,\underline{k}$

EQ. (18.22): $\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = 0 + \underline{\Omega}\times\underline{H}_G$

$$\dot{\underline{H}}_G = (\dot{\phi}\sin\beta\,\underline{i} + \dot{\phi}\cos\beta\,\underline{k})\times\left[\bar{I}_x\dot{\phi}\sin\beta\,\underline{i} + \bar{I}_z(\dot{\psi} + \dot{\phi}\cos\beta)\underline{k}\right]$$

$$= \dot{\phi}\sin\beta\left[-\bar{I}_z(\dot{\psi} + \dot{\phi}\cos\beta) + \bar{I}_x\dot{\phi}\cos\beta\right]\underline{j}$$

THUS: $\Sigma(\underline{M}_G)_{eff} = \dot{\underline{H}}_G = \dot{\phi}\sin\beta\left[(\bar{I}_x - \bar{I}_z)\dot{\phi}\cos\beta - \bar{I}_z\dot{\psi}\right]\underline{j}$ (1)

ON THE OTHER HAND,

$\Sigma\underline{M}_G = c\underline{k}\times(-D\underline{K}) = -cD\sin\beta\,\underline{j}$ (2)

$\Sigma\underline{M}_G = \Sigma(\underline{M}_G)_{eff}$: $-cD = \dot{\phi}\left[(\bar{I}_x - \bar{I}_z)\dot{\phi}\cos\beta - \bar{I}_z\dot{\psi}\right]$ (3)

(a) APPROXIMATE VALUE OF $\dot{\phi}$

SINCE $\dot{\psi} \gg \dot{\phi}$, WE MAY NEGLECT THE FIRST TERM IN THE BRACKET IN (3). WE OBTAIN

$$\bar{I}_z\dot{\phi}\dot{\psi} = cD \qquad (4)$$

WITH GIVEN DATA: $\bar{I}_z = m\bar{k}_z^2 = (20\text{ kg})(0.05\text{ m})^2 = 0.05\text{ kg}\cdot\text{m}^2$

$c = 0.15$ m, $D = 120$ N, $\dot{\psi} = 6000$ rpm $= 200\pi$ rad/s:

$0.05\,\dot{\phi}(200\pi) = (0.15)(120)$ $\qquad \dot{\phi} = 0.5730$ rad/s

$\dot{\phi} = 5.47$ rpm

(b) EXACT VALUES OF $\dot{\phi}$

USING EQ. (3) WITH THE ABOVE DATA AND WITH $\beta = 3°$ AND $\bar{I}_x = m\bar{k}_x^2 = (20\text{ kg})(0.2\text{ m})^2 = 0.8\text{ kg}\cdot\text{m}^2$:

$$-(0.15\text{ m})(120\text{ N}) = \dot{\phi}\left[(0.8 - 0.05)\dot{\phi}\cos 3° - 0.05(200\pi)\right]$$

$$0.74897\,\dot{\phi}^2 - 31.416\,\dot{\phi} + 18 = 0$$

$$\dot{\phi}^2 - 41.945\,\dot{\phi} + 24.033 = 0$$

$$\dot{\phi} = \tfrac{1}{2}\left(41.945 \pm \sqrt{(41.945)^2 - 4(24.033)}\right)$$

$$= \tfrac{1}{2}(41.945 \pm 40.783)\text{ rad/s}$$

$\dot{\phi} = 41.364$ rad/s n. $\qquad \dot{\phi} = 0.58101$ rad/s

$\dot{\phi} = 395$ rpm AND $\dot{\phi} = 5.55$ rpm

18.119

GIVEN:

AXISYMMETRICAL BODY UNDER NO FORCE

I = MOMENT OF INERTIA ABOUT AXIS OF SYMMETRY

I' = — — — — TRANSVERSE AXIS THRU G.

H_G = ANG. MOM. ABOUT G.

SHOW THAT:

$$\dot{\phi} = \frac{H_G}{I'} \quad \text{AND} \quad \dot{\psi} = \frac{H_G\cos\theta\,(I' - I)}{II'}$$

FROM EQ. (18.40), PAGE 1146:

$$\omega_x = -\dot{\phi}\sin\theta \qquad (1)$$

FROM THE FIRST OF EQS. (18.47), PAGE 1147:

$$\omega_x = -\frac{H_G\sin\theta}{I'} \qquad (2)$$

EQUATING THE R.H. MEMBERS OF (1) AND (2):

$$-\dot{\phi}\sin\theta = -\frac{H_G\sin\theta}{I'} \qquad \dot{\phi} = \frac{H_G}{I'} \qquad (Q.E.D.)(3)$$

FROM FIG. 18.21: $\dot{\psi} = \omega_z - \dot{\phi}\cos\theta$ (4)

FROM EQS. (18.48): $\omega_z = \dfrac{H_G\cos\theta}{I}$ (5)

FROM EQ. (3) ABOVE: $\dot{\phi}\cos\theta = \dfrac{H_G\cos\theta}{I'}$ (6)

SUBSTITUTE FROM (5) AND (6) INTO (4):

$$\dot{\psi} = H_G\cos\theta\left(\frac{1}{I} - \frac{1}{I'}\right)$$

$$\dot{\psi} = \frac{H_G\cos\theta\,(I' - I)}{II'} \qquad (Q.E.D)$$

18.120

GIVEN:

AXISYMMETRICAL BODY UNDER NO FORCE

I = MOMENT OF INERTIA ABOUT AXIS OF SYMMETRY

I' = — — — — TRANSVERSE AXIS THRU G

θ = ANGLE BETWEEN AXES OF PRECESSION & SPIN

ω_z = COMPONENT OF $\underline{\omega}$ ALONG AXIS OF SYMMETRY

SHOW THAT:

(a) $\qquad \dot{\phi} = \dfrac{I\omega_z}{I'\cos\theta}$

(b) EQ. (18.44) IS SATISFIED

(a) SEE SOLUTION OF PROB. 18.119 FOR DERIVATION OF EQ. (3): $\qquad \dot{\phi} = \dfrac{H_G}{I'}$ (3)

FROM EQS. (18.48): $H_G = \dfrac{I\omega_z}{\cos\theta}$

SUBSTITUTE FOR H_G IN (3):

$$\dot{\phi} = \frac{I\omega_z}{I'\cos\theta} \qquad (Q.E.D.)$$

(b) FROM RELATION JUST OBTAINED, WE HAVE

$$I\omega_z - I'\dot{\phi}\cos\theta = 0$$

WHICH SHOWS THAT, FOR AN AXISYMMETRICAL BODY UNDER NO FORCE, THE R.H. MEMBER OF

$$\Sigma\underline{M}_O = (I\omega_z - I'\dot{\phi}\cos\theta)\dot{\phi}\sin\theta\,\underline{j} \qquad (18.44)$$

IS EQUAL TO ZERO. BUT, SINCE THERE IS NO FORCE, WE ALSO HAVE $\Sigma\underline{M}_O = 0$ AND EQ. (18.44) IS SATISFIED. (Q.E.D)

18.121 GIVEN:

AXISYMMETRICAL BODY UNDER NO FORCE

I = MOMENT OF INERTIA ABOUT AXIS OF SYMMETRY.

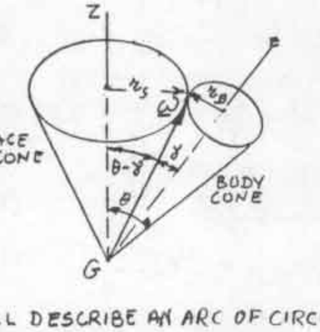
I' = – – – – – TRANSVERSE AXIS THRU G

ω_z = COMPONENT OF $\underline{\omega}$ ALONG AXIS OF SYMMETRY.

SHOW THAT:

ANGULAR VELOCITY $\underline{\omega}$ IS OBSERVED FROM THE BODY TO ROTATE ABOUT THE AXIS OF SYMMETRY AT THE RATE

$$n = \frac{I'-I}{I'}\omega_z$$



ASSUMING DIRECT PRECESSION ($I' > I$), WE CONSIDER THE SPACE AND BODY CONES. THE PLANE ZGz ROTATES ABOUT THE Z AXIS AT THE RATE $\dot\phi$; SO WILL THE VECTOR $\underline{\omega}$ CONTAINED IN THAT PLANE. THUS, THE TIP OF $\underline{\omega}$ WILL DESCRIBE AN ARC OF CIRCLE OF LENGTH $r_s\dot\phi\,\Delta t$ IN THE TIME Δt. BUT, ACCORDING TO THE DEFINITION OF n, THE VECTOR $\underline{\omega}$ IS OBSERVED TO ROTATE AT THE RATE n WITH RESPECT TO THE BODY. THUS THE TIP OF $\underline{\omega}$ WILL DESCRIBE AN ARC OF CIRCLE OF LENGTH $r_B\, n\,\Delta t$ IN THE TIME Δt. SINCE THE BODY CONE ROLLS ON THE SPACE CONE, WE HAVE

$$r_s\dot\phi\,\Delta t = r_B\, n\,\Delta t \qquad (1)$$

BUT, FROM THE SKETCH ABOVE,

$$r_s = \omega\sin(\theta-\gamma) \quad \text{AND} \quad r_B = \omega\sin\gamma$$

SUBSTITUTING INTO (1):

$$\dot\phi\sin(\theta-\gamma) = n\sin\gamma$$

$$n = \dot\phi\,\frac{\sin(\theta-\gamma)}{\sin\gamma} \qquad (2)$$

WE RECALL THE RELATION DERIVED IN PROB 18.120:

$$\dot\phi = \frac{I\omega_z}{I'\cos\theta}$$

SUBSTITUTING INTO (2) AND EXPANDING $\sin(\theta-\gamma)$:

$$n = \frac{I\omega_z}{I'\cos\theta}\;\frac{\sin\theta\cos\gamma-\sin\gamma\cos\theta}{\sin\gamma}$$

$$= \frac{I\omega_z}{I'}\left(\frac{\tan\theta}{\tan\gamma}-1\right)$$

RECALLING FROM EQ. (18.49) THAT $\frac{\tan\theta}{\tan\gamma} = \frac{I'}{I}$, WE HAVE

$$n = \frac{I}{I'}\left(\frac{I'}{I}-1\right)\omega_z$$

$$n = \frac{I'-I}{I'}\omega_z \quad (Q.E.D.)$$

NOTE. FOR $I > I'$ (RETROGRADE PRECESSION), WE WOULD FIND

$$n = \frac{I-I'}{I'}\omega_z$$

18.122 GIVEN:

AXISYMMETRICAL BODY UNDER NO FORCE AND IN RETROGRADE PRECESSION ($I > I'$).

SHOW THAT:

(a) RATE OF RETROGRADE PRECESSION CANNOT BE LESS THAN TWICE THE RATE OF SPIN: $|\dot\phi| \ge 2|\dot\psi|$,

(b) THE AXIS OF SYMMETRY IN FIG. 18.24 CAN NEVER LIE WITHIN THE SPACE CONE.

(a) WE RECALL THE RELATION DERIVED IN PROB. 18.120:

$$\dot\phi = \frac{I\omega_z}{I'\cos\theta} \qquad (1)$$

OR

$$I'\dot\phi\cos\theta = I\omega_z$$

SUBSTITUTING $\omega_z = \dot\psi + \dot\phi\cos\theta$, WE HAVE

$$I'\dot\phi\cos\theta = I(\dot\psi + \dot\phi\cos\theta)$$

SOLVING FOR $\dot\phi$,

$$\dot\phi = -\frac{I}{I-I'}\,\frac{\dot\psi}{\cos\theta} \quad \text{OR} \quad \dot\phi = -\frac{\sec\theta}{1-(I'/I)}\dot\psi \qquad (2)$$

FOR RETROGRADE PRECESSION, $I'/I < 1$

ON THE OTHER HAND, THE SMALLEST POSSIBLE VALUE OF I'/I IS 1/2 (WHICH CORRESPONDS TO THE CASE OF A FLAT DISK OR ANNULUS).

THUS:

$$\frac{1}{2} \le \frac{I'}{I} < 1 \quad \text{OR} \quad \frac{1}{2} \ge 1-\frac{I'}{I} > 0$$

$$\text{OR} \quad \frac{1}{1-(I/I')} \ge 2$$

RECALLING THAT $\sec\theta \ge 1$, WE MUST HAVE FROM (2)

$$|\dot\phi| \ge 2|\dot\psi| \quad (Q.E.D.)$$

(b) WE RECALL EQ. (18.49):

$$\tan\gamma = \frac{I}{I'}\tan\theta$$

SINCE $\frac{I'}{I} \ge \frac{1}{2}$ AS SHOWN ABOVE, $\frac{I}{I'} \le 2$ AND

$$\tan\gamma \le 2\tan\theta \qquad (3)$$

WE WRITE THE TRIGONOMETRIC IDENTITY

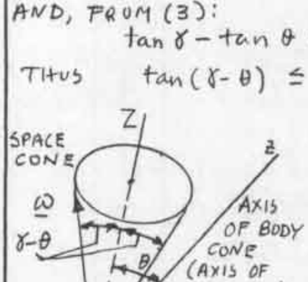
$$\tan(\gamma-\theta) = \frac{\tan\gamma-\tan\theta}{1+\tan\gamma\tan\theta}$$

SINCE $\gamma < \frac{\pi}{2}$ AND $\theta < \frac{\pi}{2}$, WE HAVE $1+\tan\gamma\tan\theta \ge 1$

AND, FROM (3):

$$\tan\gamma - \tan\theta \le \tan\theta$$

THUS $\quad \tan(\gamma-\theta) \le \frac{\tan\theta}{1}$



$$\tan(\gamma-\theta) \le \tan\theta$$

$$\gamma-\theta \le \theta$$

THE z AXIS CANNOT LIE WITHIN THE SPACE CONE (SEE SKETCH)

(Q.E.D.)

18.123 (FREE PRECESSION OF THE EARTH)

GIVEN:

I = MOM. OF INERTIA OF EARTH ABOUT AXIS OF SYMMETRY.
I' = " " " " " — TRANSVERSE AXIS

$$I' = 0.9967\,I$$

RELATION DERIVED IN PROB. 18.121:

$$n = \frac{I - I'}{I'}\,\omega_z \qquad (\text{FOR } I > I')$$

WHERE ω_z = COMPONENT OF $\underline{\omega}$ OF EARTH ALONG AXIS OF SYMMETRY, AND n = RATE AT WHICH $\underline{\omega}$ IS OBSERVED FROM THE EARTH TO ROTATE ABOUT ITS AXIS OF SYMMETRY.

FIND:

PERIOD OF PRECESSION OF NORTH POLE.

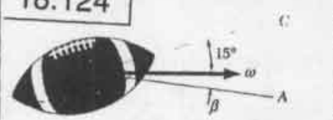
PERIOD OF PRECESSION

$$= \frac{2\pi}{n} = \frac{I'}{I - I'}\,\frac{2\pi}{\omega_z} = \frac{I'}{I - I'}\,(1\ \text{day})$$

BUT $\dfrac{I'}{I - I'} = \dfrac{0.9967\,I}{0.0033\,I} = 302$

THUS: PERIOD OF PRECESSION = 302 days

18.124



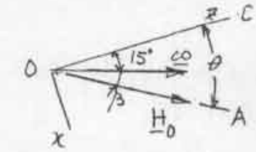
GIVEN:

FOOTBALL KICKED WITH HORIZONTAL ANG. VEL. $\underline{\omega}$ OF MAGNITUDE 200 rpm. RATIO OF AXIAL AND TRANSVERSE MOMENTS OF INERTIA IS $I/I' = 1/3$.

FIND:

(a) ANGLE β BETWEEN $\underline{\omega}$ AND PRECESSION AXIS OA.
(b) RATES OF PRECESSION AND SPIN.

(a) USING REFERENCE FRAME Oxyz WITH y POINTING AWAY



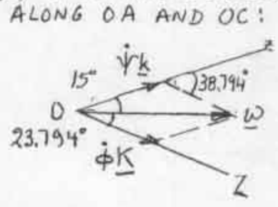
$$\omega_x = \omega \sin 15°$$
$$\omega_y = 0$$
$$\omega_z = \omega \cos 15°$$
$$H_x = I'\omega_x = I'\omega \sin 15°$$
$$H_y = I'\omega_y = 0$$
$$H_z = I\omega_z = I\omega \cos 15°$$

$$\tan\theta = \frac{H_x}{H_z} = \frac{I'\omega \sin 15°}{I\omega \cos 15°} = \frac{1}{I/I'}\tan 15° = 3\tan 15°$$

$$\tan\theta = 0.80385 \qquad \theta = 38.794°$$

$$\beta = \theta - 15° = 38.794° - 15° = 23.794° \qquad \beta = 23.8°$$

(b) USING THE OBLIQUE COMPONENTS OF $\underline{\omega}$ ALONG OA AND OC:



LAW OF SINES:

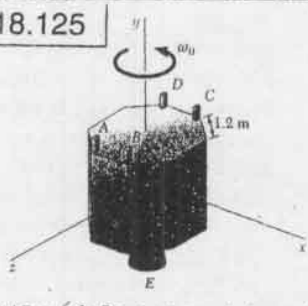
$$\frac{\omega}{\sin 38.794°} = \frac{\dot\phi}{\sin 15°} = \frac{\dot\psi}{\sin 23.794°}$$

SETTING ω = 200 rpm, WE FIND

RATE OF PRECESSION = $\dot\phi$ = 82.6 rpm

RATE OF SPIN = $\dot\psi$ = 128.8 rpm

18.125



GIVEN:

2500-kg SATELLITE, 2.4-m HIGH WITH OCTAGONAL BASE. $k_x = k_z = 0.90$ m, $k_y = 0.98$ m. SATELLITE SPINNING AT RATE OF 36 rev/h ABOUT Gy WHEN 20-N THRUSTERS AT A AND B ARE ACTIVATED FOR 2 s, EXPELLING FUEL IN POSITIVE y DIRECTION

FIND: (a) PRECESSION AXIS, (b) $\dot\phi$, (c) $\dot\psi$.

INITIAL ANG. VELOCITY:

$$\underline{\omega}_0 = \left(36\,\frac{\text{rev}}{\text{h}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\underline{j} = (0.062832\ \text{rad/s})\,\underline{j}$$

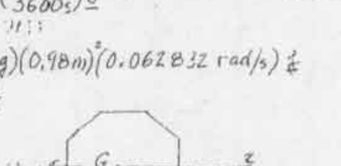
INITIAL ANG. MOMENTUM:

$$(\underline{H}_G)_0 = mk_y^2\,\underline{\omega}_0 = (2500\ \text{kg})(0.98\ \text{m})^2(0.062832\ \text{rad/s})\,\underline{j}$$
$$= (150.86\ \text{kg}\cdot\text{m}^2/\text{s})\,\underline{j}$$

ANG. IMPULSE:

$$\underline{M}_G\Delta t = (1.4485\ \text{m})\underline{k} \times$$
$$\times 2(-20\ \text{N})\underline{j}(2\ \text{s})$$

$$\underline{M}_G\Delta t = (115.88\ \text{kg}\cdot\text{m}^2/\text{s})\,\underline{i}$$



PRINCIPLE OF IMPULSE AND MOMENTUM

FINAL MOMENTUM:

$$\underline{H}_G = (\underline{H}_G)_0 + \underline{M}_G\Delta t = (150.86\ \text{kg}\cdot\text{m}^2/\text{s})\underline{j} + (115.88\ \text{kg}\cdot\text{m}^2/\text{s})\underline{i}$$

$$\underline{H}_G = (115.88\ \text{kg}\cdot\text{m}^2/\text{s})\underline{i} + (150.86\ \text{kg}\cdot\text{m}^2/\text{s})\underline{j} \qquad (1)$$

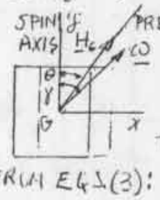
WE RECALL THAT

$$\underline{H}_G = I_x\omega_x\underline{i} + I_y\omega_y\underline{j} + I_z\omega_z\underline{k}$$

$$\underline{H}_G = (2500\ \text{kg})(0.90\ \text{m})^2\omega_x\underline{i} + (2500\ \text{kg})(0.98\ \text{m})^2\omega_y\underline{j} + I_z\omega_z\underline{k} \qquad (2)$$

EQUATING THE COEFF. OF $\underline{i}, \underline{j}, \underline{k}$ IN (1) AND (2):

$$2025\,\omega_x = 115.88 \qquad \omega_x = 57.225\times10^{-3}\ \text{rad/s}$$
$$2401\,\omega_y = 150.86 \qquad \omega_y = 62.832\times10^{-3}\ \text{rad/s} \qquad (3)$$
$$I_z\omega_z = 0 \qquad \omega_z = 0$$



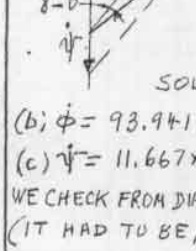
(a) FROM EQ. (1):

$$\tan\theta = \frac{H_x}{H_y} = \frac{115.88}{150.86} \qquad \theta = 37.529°$$

THUS: $\theta_x = 52.5°$, $\theta_y = 37.5°$, $\theta_z = 90°$

FROM EQS.(3): $\tan\gamma = \dfrac{\omega_x}{\omega_y} = \dfrac{57.225}{62.832} \qquad \gamma = 42.326°$

$$\omega = \sqrt{\omega_x^2 + \omega_y^2} = 84.986\times10^{-3}\ \text{rad/s}$$



LAW OF SINES:

$$\frac{\omega}{\sin\theta} = \frac{\dot\phi}{\sin\gamma} = \frac{\dot\psi}{\sin(\gamma - \theta)}$$

$$\frac{84.986\times10^{-3}}{\sin 37.529°} = \frac{\dot\phi}{\sin 42.326°} = \frac{\dot\psi}{\sin 4.797}$$

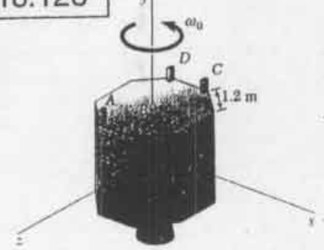
SOLVING FOR $\dot\phi$ AND $\dot\psi$:

(b) $\dot\phi = 93.941\times10^{-3}$ rad/s $\qquad \dot\phi = 53.8$ rev/h

(c) $\dot\psi = 11.667\times10^{-3}$ rad/s $\qquad \dot\psi = 6.68$ rev/h

WE CHECK FROM DIAGRAM THAT PRECESSION IS RETROGRADE. (IT HAD TO BE, SINCE $k_y > k_x$ AND, THUS, $I > I'$.)

18.126



GIVEN:
2500-kg SATELLITE, 2.4-m HIGH WITH OCTOGONAL BASE. $k_x = k_z = 0.90\text{ m}$, $k_y = 0.98\text{ m}$. SATELLITE SPINNING AT RATE OF 36 rev/h ABOUT Gy WHEN 20-N THRUSTERS AT A AND D ARE ACTIVATED FOR 2 s, EXPELLING FUEL IN POSITIVE y DIRECTION

FIND: (a) PRECESSION AXIS, (b) $\dot{\phi}$, (c) $\dot{\psi}$.

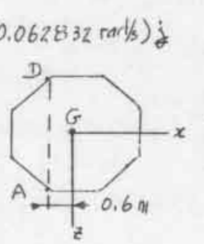
INITIAL ANG. VELOCITY:

$$\underline{\omega}_0 = \left(36\frac{\text{rev}}{h}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1h}{3600\text{ s}}\right)\underline{j} = (0.062832\text{ rad/s})\underline{j}$$

INITIAL ANG. MOMENTUM:

$$(\underline{H}_G)_0 = m k_y^2 \underline{\omega}_0 = (2500\text{ kg})(0.98\text{ m})^2(0.062832\text{ rad/s})\underline{j}$$
$$= (150.86\text{ kg}\cdot\text{m}^2/\text{s})\underline{j}$$

ANG. IMPULSE:



$$\underline{M}_G\Delta t = -(0.6\text{m})\underline{i}\times 2(-20\text{N})\underline{j}(2\text{s})$$
$$\underline{M}_G\Delta t = (48.0\text{ kg}\cdot\text{m}^2/\text{s})\underline{k}$$

PRINCIPLE OF IMPULSE AND MOMENTUM

FINAL MOMENTUM:

$$\underline{H}_G = (\underline{H}_G)_0 + \underline{M}_G\Delta t = (150.86\text{ kg}\cdot\text{m}^2/\text{s})\underline{j} + (48.0\text{ kg}\cdot\text{m}^2/\text{s})\underline{k} \quad (1)$$

WE RECALL THAT

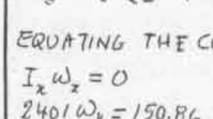
$$\underline{H}_G = I_x\omega_x\underline{i} + I_y\omega_y\underline{j} + I_z\omega_z\underline{k}$$
$$\underline{H}_G = I_x\omega_x\underline{i} + (2500\text{ kg})(0.98\text{m})^2\omega_y\underline{j} + (2500\text{ kg})(0.90\text{m})^2\omega_z\underline{k} \quad (2)$$

EQUATING THE COEFF. OF $\underline{i}, \underline{j}, \underline{k}$ IN (1) AND (2):

$I_x\omega_x = 0$ $\qquad \omega_x = 0$

$2401\,\omega_y = 150.86$ $\qquad \omega_y = 62.832\times10^{-3}\text{ rad/s} \quad (3)$

$2025\,\omega_z = 48.0$ $\qquad \omega_z = 23.704\times10^{-3}\text{ rad/s}$



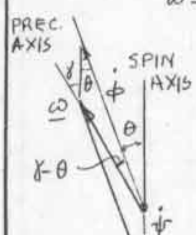
(a) FROM EQ.(1):

$$\tan\theta = \frac{H_z}{H_y} = \frac{48.0}{150.86} \qquad \theta = 17.650°$$

THUS: $\theta_x = 90°$, $\theta_y = 17.65°$, $\theta_z = 72.35°$ ◀

FROM EQS.(3): $\tan\gamma = \dfrac{\omega_z}{\omega_y} = \dfrac{23.704}{62.832}$ $\qquad \gamma = 20.669°$

$$\omega = \sqrt{\omega_y^2 + \omega_z^2} = 67.155\times10^{-3}\text{ rad/s}$$



LAW OF SINES:

$$\frac{\omega}{\sin\theta} = \frac{\dot{\phi}}{\sin\gamma} = \frac{\dot{\psi}}{\sin(\gamma-\theta)}$$

$$\frac{67.155\times10^{-3}}{\sin 17.650°} = \frac{\dot{\phi}}{\sin 20.669°} = \frac{\dot{\psi}}{\sin 3.019°}$$

SOLVING FOR $\dot{\phi}$ AND $\dot{\psi}$:

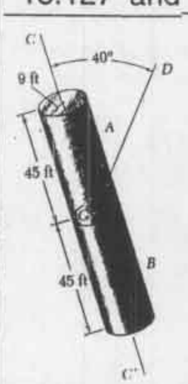
(b) $\dot{\phi} = 78.177\times10^{-3}\text{ rad/s}$ $\qquad \dot{\phi} = 44.8\text{ rev/h}$ ◀

(c) $\dot{\psi} = 11.665\times10^{-3}\text{ rad/s}$ $\qquad \dot{\psi} = 6.68\text{ rev/h}$ ◀

WE CHECK FROM DIAGRAM THAT PRECESSION IS RETROGRADE

(IT HAD TO BE, SINCE $k_y > k_z$ AND, THUS, $I > I'$.)

18.127 and 18.128



GIVEN:
SPACE STATION CONSISTS OF TWO SECTIONS A AND B OF THE SAME WEIGHT WHICH ARE RIGIDLY CONNECTED. EACH SECTION IS DYNAMICALLY EQUIVALENT TO A HOMOGENEOUS CYLINDER. STATION IS PRECESSING ABOUT GD AT THE CONSTANT RATE OF 2 rev/h.

PROBLEM 18.127:
FIND THE RATE OF SPIN OF THE STATION ABOUT CC'.

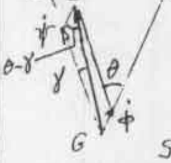
PROBLEM 18.128:
IF CONNECTION IS SEVERED BETWEEN A AND B, FIND FOR SECTION A:
(a) THE ANGLE BETWEEN CC' AND THE PRECESSION AXIS, (b) $\dot{\phi}$, (c) $\dot{\psi}$.

FOR ENTIRE STATION: $\qquad \theta = 40°$

$$I = \tfrac{1}{2}ma^2 \qquad I' = \tfrac{1}{12}m(3a^2 + L^2) \qquad \frac{I}{I'} = \frac{6a^2}{3a^2+L^2}$$

$$\text{EQ.(18.49)}: \tan\gamma = \frac{I}{I'}\tan\theta = \frac{6(9)^2}{3(9)^2+(90)^2}\tan 40°$$
$$= 58.252\times10^{-3}\tan 40°, \quad \gamma = 2.7984°$$

PROBLEM 18.127



LAW OF SINES:

$$\frac{\omega}{\sin\theta} = \frac{\dot{\phi}}{\sin\gamma} = \frac{\dot{\psi}}{\sin(\theta-\gamma)}$$

WITH $\dot{\phi} = 2$ rev/h:

$$\frac{\omega}{\sin 40°} = \frac{2\text{ rev/h}}{\sin 2.7984°} = \frac{\dot{\psi}}{\sin 37.202°}$$

SOLVING FOR ω AND $\dot{\psi}$:

$\omega = 26.332$ rev/h $\qquad \dot{\psi} = 24.8$ rev/h ◀

PROBLEM 18.128

FOR SECTION A:

(a) ANGLE BETWEEN SPIN AXIS AND $\underline{\omega}$ IS STILL $\gamma = 2.7984°$

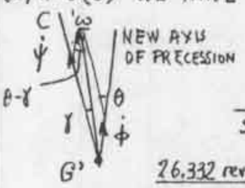
$$\text{NOW: } \frac{I}{I'} = \frac{6a^2}{3a^2+L^2} = \frac{6(9)^2}{3(9)^2+(45)^2} = 0.21429$$

$$\text{EQ.(18.49)}: \tan\gamma = \frac{I}{I'}\tan\theta \qquad \tan\gamma = 0.21429\tan\theta$$

$$\tan\theta = \frac{\tan\gamma}{0.21429} = \frac{\tan 2.7984°}{0.21429} = 0.22811$$

$\theta = 12.850°$ $\qquad \theta = 12.85°$ ◀

(b) AND (c) WE HAVE $\omega = 26.332$ rev/h, $\gamma = 2.7984°$, AND $\theta = 12.850°$



LAW OF SINES:

$$\frac{\omega}{\sin\theta} = \frac{\dot{\phi}}{\sin\gamma} = \frac{\dot{\psi}}{\sin(\theta-\gamma)}$$

$$\frac{26.332\text{ rev/h}}{\sin 12.850°} = \frac{\dot{\phi}}{\sin 2.7984°} = \frac{\dot{\psi}}{\sin 10.052°}$$

SOLVING FOR $\dot{\phi}$ AND $\dot{\psi}$:

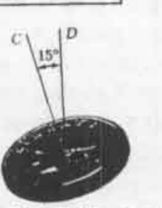
(b) $\dot{\phi} = 5.781$ rev/h $\qquad \dot{\phi} = 5.78$ rev/h ◀

(c) $\dot{\psi} = 20.665$ rev/h $\qquad \dot{\psi} = 20.7$ rev/h ◀

18.129

GIVEN:

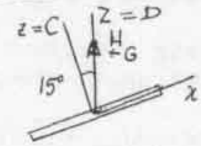
COIN SPINS AT THE RATE OF 600 rpm ABOUT AXIS GC PERPENDICULAR TO COIN AND PRECESSES ABOUT VERTICAL DIRECTION GD.



FIND:

(a) ANGLE BETWEEN $\underline{\omega}$ AND GD,

(b) RATE OF PRECESSION ABOUT GD.



IT FOLLOWS FROM THE ABOVE STATEMENT THAT $\underline{H}_G$ IS DIRECTED AS SHOWN AND THAT THE ANGLE BETWEEN THE AXES OF SPIN AND PRECESSION IS $\theta = 15°$

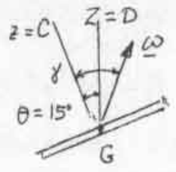
FOR DISK:

$$I = I_z = \frac{1}{2} m r^2 \qquad I' = I_x = \frac{1}{4} m r^2$$

EQ. (18.49):

$$\tan\gamma = \frac{I}{I'} \tan\theta = 2 \tan 15° \qquad \gamma = 28.187°$$

(a) ANGLE BETWEEN $\underline{\omega}$ AND GD



THE ANGLE γ WE HAVE FOUND IS THE ANGLE BETWEEN $\underline{\omega}$ AND GC. THE ANGLE BETWEEN $\underline{\omega}$ AND GD IS

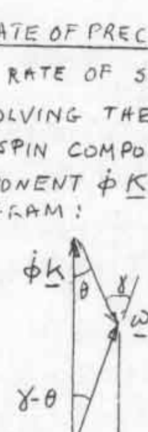
$$\gamma - \theta = 28.187° - 15° = 13.187°$$

$$\gamma - \theta = 13.19° \quad ◀$$

(b) RATE OF PRECESSION

THE RATE OF SPIN IS $\dot{\psi} = 600$ rpm

RESOLVING THE ANGULAR VELOCITY $\underline{\omega}$ INTO ITS SPIN COMPONENT $\dot{\psi}\underline{k}$ AND ITS PRECESSION COMPONENT $\dot{\phi}\underline{K}$, WE DRAW THE FOLLOWING DIAGRAM:



LAW OF SINES:

$$\frac{\dot{\phi}}{\sin\gamma} = \frac{\dot{\psi}}{\sin(\gamma-\theta)}$$

$$\dot{\phi} = \dot{\psi} \frac{\sin\gamma}{\sin(\gamma-\theta)} = (600\text{rpm}) \frac{\sin 28.187°}{\sin 13.187°}$$

$$\dot{\phi} = 1242 \text{ rpm} \quad ◀$$

WE NOTE FROM DIAGRAM THAT THE PRECESSION IS RETROGRADE

THIS COULD HAVE BEEN ANTICIPATED, SINCE $I/I' = 2 > 1$.

18.130

SOLVE SAMPLE PROB. 18.6, ASSUMING THAT THE METEORITE STRIKES THE SATELLITE AT C WITH $\underline{v}_0 = (2000 \text{ m/s})\underline{i}$.

(a) ANGULAR VELOCITY AFTER IMPACT

FROM SAMPLE PROB. 18.6:

$$I = I_z = \frac{1}{2} ma^2 \qquad I' = I_x = I_y = \frac{5}{4} ma^2$$

ANG. MOMENTUM AFTER IMPACT:

$$\underline{H}_G = \underline{r}_C \times m_0 \underline{v}_0 + I\omega_0 \underline{k}$$
$$= (-a\underline{j} - a\underline{k}) \times m_0 v_0 \underline{i} + I\omega_0 \underline{k}$$
$$\underline{H}_G = -m_0 v_0 a \underline{j} + (I\omega_0 + m_0 v_0 a)\underline{k} \qquad (1)$$

BUT $\underline{H}_G = I'\omega_x \underline{i} + I'\omega_y \underline{j} + I\omega_z \underline{k}$ (2)

EQUATING THE COEFF. OF THE UNIT VECTORS IN (1) AND (2)

$$\omega_x = 0 \qquad \omega_y = -\frac{m_0 v_0 a}{I'} = -\frac{4}{5}\frac{m_0 v_0}{ma}$$

$$\omega_z = \omega_0 + \frac{m_0 v_0 a}{I} = \omega_0 + 2\frac{m_0 v_0}{ma}$$

GIVEN DATA: $\omega_0 = 60$ rpm $= 6.283$ rad/s

$\frac{m_0}{m} = 0.001 \qquad a = 0.8$ m $\qquad v_0 = 2000$ m/s

WE FIND

$$\omega_x = 0 \qquad \omega_y = -2 \text{ rad/s} \qquad \omega_z = 11.283 \text{ rad/s}$$

$$\omega = \sqrt{\omega_y^2 + \omega_z^2} = 11.459 \text{ rad/s} \qquad \omega = 109.4 \text{ rpm} \quad ◀$$

$$\cos\gamma_x = 0 \qquad \cos\gamma_y = \frac{\omega_y}{\omega} = -0.17453 \qquad \cos\gamma_z = \frac{\omega_z}{\omega} = 0.98464$$

$$\gamma_x = 90°, \quad \gamma_y = 100.05°, \quad \gamma_z = 10.05° \quad ◀$$

(b) PRECESSION AXIS

SINCE IT IS DIRECTED ALONG $\underline{H}_G$, WE USE EQ (1) AND WRITE

$$H_x = 0, \quad H_y = -m_0 v_0 a = -\frac{m}{1000}(2000)(0.8) = -(1.6)m$$

$$H_z = I\omega_0 + m_0 v_0 a = \frac{1}{2} ma^2\omega_0 + m_0 v_0 a$$
$$= \frac{1}{2} m(0.8)^2(6.283) + (1.6)m = (3.6106)m$$

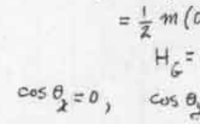
$$H_G = \sqrt{H_y^2 + H_z^2} = (3.9492)m$$

$$\cos\theta_x = 0, \quad \cos\theta_y = \frac{H_y}{H_G} = -0.40515, \quad \cos\theta_z = \frac{H_z}{H_G} = 0.91425$$

DIRECTION OF PRECESSION AXIS IS

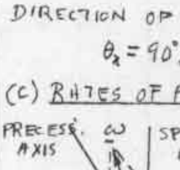
$$\theta_x = 90°, \quad \theta_y = 113.9°, \quad \theta_z = 23.9° \quad ◀$$

(c) RATES OF PRECESSION AND SPIN



WE HAVE

$$\theta = \theta_z = 23.9°$$
$$\gamma = \gamma_z = 10.05°$$
$$\theta - \gamma = 13.85°$$



LAW OF SINES:

$$\frac{\omega}{\sin\theta} = \frac{\dot{\phi}}{\sin\gamma} = \frac{\dot{\psi}}{\sin(\theta-\gamma)}$$

$$\frac{109.4\text{rpm}}{\sin 23.9°} = \frac{\dot{\phi}}{\sin 10.05°} = \frac{\dot{\psi}}{\sin 13.85°}$$

SOLVING FOR $\dot{\phi}$ AND $\dot{\psi}$

RATE OF PRECESSION $= \dot{\phi} = 47.1$ rpm ◀

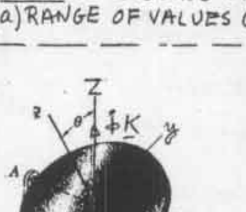
RATE OF SPIN $= \dot{\psi} = 64.6$ rpm ◀

18.131 and 18.132

GIVEN:
DISK OF MASS m IS FREE TO ROTATE ABOUT AB.
FORK-ENDED SHAFT OF NEGLIGIBLE MASS IS FREE TO ROTATE IN BEARING C.

PROBLEM 18.131:
INITIALLY, $\theta_0 = 90°$, $\dot\theta_0 = 0$, $\dot\phi_0 = 8$ rad/s.
IF DISK SLIGHTLY DISTURBED FIND IN ENSUING MOTION
(a) MINIMUM VALUE OF $\dot\phi$.
(b) MAXIMUM VALUE OF $\dot\theta$.

PROBLEM 18.132:
INITIALLY $\theta_0 = 30°$, $\dot\theta_0 = 0$, $\dot\phi_0 = 8$ rad/s.
FIND IN ENSUING MOTION:
(a) RANGE OF VALUES OF θ, (b) MINIMUM $\dot\phi$, (c) MAXIMUM $\dot\theta$.



USING THE AXES $Gxyz$:

$$\underline{\omega} = \dot\theta\,\underline{i} + \dot\phi\sin\theta\,\underline{j} + \dot\phi\cos\theta\,\underline{k}$$

CONSERVATION OF ANGULAR MOMENTUM:
SINCE DISK IS FREE TO ROTATE ABOUT THE Z AXIS, WE HAVE

$$H_Z = \text{constant} \qquad (1)$$

BUT $H_Z = H_y \sin\theta + H_z\cos\theta$

$$H_Z = I_y\omega_y\sin\theta + I_z\omega_z\cos\theta = \tfrac{1}{4}ma^2\dot\phi\sin^2\theta + \tfrac{1}{2}ma^2\dot\phi\cos^2\theta$$
$$= \tfrac{1}{4}ma^2\dot\phi(\sin^2\theta + 2\cos^2\theta) = \tfrac{1}{4}ma^2\dot\phi\,(1+\cos^2\theta)$$

USING THE INITIAL CONDITIONS, EQ. (1) YIELDS

$$\dot\phi\,(1+\cos^2\theta) = \dot\phi_0(1+\cos^2\theta_0) \qquad (2)$$

CONSERVATION OF ENERGY
SINCE NO WORK IS DONE, WE HAVE $T = \text{constant}$ (3)
WHERE

$$T = \tfrac{1}{2}\left(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2\right)$$
$$T = \tfrac{1}{2}\left(\tfrac{1}{4}ma^2\dot\theta^2 + \tfrac{1}{4}ma^2\dot\phi^2\sin^2\theta + \tfrac{1}{2}ma^2\dot\phi^2\cos^2\theta\right)$$
$$= \tfrac{1}{8}ma^2\left[\dot\theta^2 + \dot\phi^2(\sin^2\theta + 2\cos^2\theta)\right] = \tfrac{1}{8}ma^2\left[\dot\theta^2 + \dot\phi^2(1+\cos^2\theta)\right]$$

USING THE INITIAL CONDITIONS, INCLUDING $\dot\theta_0 = 0$, EQ. (3) YIELDS

$$\dot\theta^2 + \dot\phi^2(1+\cos^2\theta) = \dot\phi_0^2(1+\cos^2\theta_0) \qquad (4)$$

PROBLEM 18.131

(a) WITH $\theta_0 = 90°$ AND $\dot\phi_0 = 8$ rad/s, EQ. (2) YIELDS $\dot\phi = \dfrac{8}{1+\cos^2\theta}$

$\dot\phi$ IS MINIMUM FOR $\theta = 0$: $\quad \dot\phi_{min} = 4.00$ rad/s ◄

(b) EQ. (4) YIELDS $\dot\theta^2 = 64 - \dot\phi^2(1+\cos^2\theta) = 64\left(1 - \dfrac{1}{1+\cos^2\theta}\right)$

$\dot\theta^2$ IS LARGEST FOR $\theta = 0$:

$\dot\theta^2_{max} = 64\left(1 - \tfrac{1}{2}\right) = 32 \qquad \dot\theta_{max} = 5.66$ rad/s ◄

PROBLEM 18.132

(a) WITH $\theta_0 = 30°$, $\dot\phi_0 = 8$ rad/s IN (2)

$$\dot\phi\,(1+\cos^2\theta) = 14 \qquad \dot\phi = 14/(1+\cos^2\theta) \qquad (5)$$

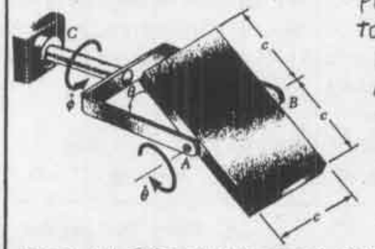
SUBSTITUTE IN (4) AND SOLVE FOR $\dot\theta^2$: $\dot\theta^2 = 112 - \dfrac{196}{1+\cos^2\theta}$ (6)

SINCE $\dot\theta^2 \geq 0$, WE MUST HAVE $1+\cos^2\theta \geq \frac{196}{112}$; $-30° \leq \theta \leq 30°$ ◄

(b) FROM (5), $\dot\phi$ IS MINIMUM FOR $\theta = 0$: $\quad \dot\phi_{min} = 7.00$ rad/s ◄

(c) FROM (6), $\dot\theta$ IS MAXIMUM FOR $\theta = 0$: $\quad \dot\theta_{max} = 3.74$ rad/s ◄

18.133 and 18.134

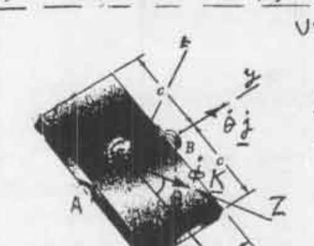


GIVEN:
PLATE OF MASS m IS FREE TO ROTATE ABOUT AB.
FORK-ENDED SHAFT OF NEGLIGIBLE MASS IS FREE TO ROTATE IN BEARING C.

PROBLEM 18.133:
INITIALLY $\theta_0 = 30°$, $\dot\theta_0 = 0$, $\dot\phi_0 = 6$ rad/s.

FIND IN ENSUING MOTION (a) RANGE OF VALUES OF θ (b) MINIMUM VALUE OF $\dot\phi$, (c) MAXIMUM VALUE OF $\dot\theta$.

PROBLEM 18.134:
INITIALLY $\theta_0 = 0$, $\dot\theta = 0$, $\dot\phi_0 = 6$ rad/s. IF PLATE IS SLIGHTLY DISTURBED, FIND IN ENSUING MOTION
(a) MINIMUM VALUE OF $\dot\phi$, (b) MAXIMUM VALUE OF $\dot\theta$.



USING THE AXES $Gxyz$

$$\underline{\omega} = \dot\phi\cos\theta\,\underline{i} + \dot\theta\,\underline{j} + \dot\phi\sin\theta\,\underline{k}$$

CONSERVATION OF ANGULAR MOMENTUM
SINCE PLATE IS FREE TO ROTATE ABOUT Z AXIS,

$$H_Z = \text{constant} \qquad (1)$$

BUT $H_Z = H_x\cos\theta + H_z\sin\theta$

$$H_Z = I_x\omega_x\cos\theta + I_z\omega_z\sin\theta = \tfrac{1}{12}mc^2\dot\phi\cos^2\theta + \tfrac{5}{12}mc^2\dot\phi\sin^2\theta$$
$$= \tfrac{1}{12}mc^2\dot\phi(\cos^2\theta + 5\sin^2\theta) = \tfrac{1}{12}mc^2\dot\phi\,(1+4\sin^2\theta)$$

USING THE INITIAL CONDITIONS, EQ (1) YIELDS

$$\dot\phi(1+4\sin^2\theta) = \dot\phi_0(1+4\sin^2\theta_0) \qquad (2)$$

CONSERVATION OF ENERGY
SINCE NO WORK IS DONE, WE HAVE $T = \text{constant}$ (3)
WHERE $\quad T = \tfrac{1}{2}\left(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2\right)$

$$T = \tfrac{1}{2}\left(\tfrac{1}{12}mc^2\dot\phi^2\cos^2\theta + \tfrac{1}{3}mc^2\dot\theta^2 + \tfrac{5}{12}mc^2\dot\phi^2\sin^2\theta\right)$$
$$= \tfrac{1}{24}mc^2\left[4\dot\theta^2 + \dot\phi^2(\cos^2\theta + 5\sin^2\theta)\right] = \tfrac{1}{24}mc^2\left[4\dot\theta^2 + \dot\phi^2(1+4\sin^2\theta)\right]$$

USING THE INITIAL CONDITIONS, INCLUDING $\dot\theta_0 = 0$, EQ. (3) YIELDS

$$4\dot\theta^2 + \dot\phi^2(1+4\sin^2\theta) = \dot\phi_0^2(1+4\sin^2\theta_0) \qquad (4)$$

PROBLEM 18.133

(a) WITH $\theta_0 = 30°$ AND $\dot\phi_0 = 6$ rad/s IN (2) AND (4):

$$\dot\phi(1+4\sin^2\theta) = 12 \qquad 4\dot\theta^2 + \dot\phi^2(1+4\sin^2\theta) = 72 \qquad (2', 4')$$

ELIMINATING $\dot\phi$ AND SOLVING FOR $\dot\theta^2$: $\dot\theta^2 = 18 - \dfrac{36}{1+4\sin^2\theta}$ (5)

FOR $\dot\theta^2 \geq 0$: $1+4\sin^2\theta \geq 2$, $\sin^2\theta \geq \frac{1}{4}$, $\quad 30° \leq \theta \leq 150°$ ◄

(b) FROM (2'), $\dot\phi$ IS MIN. FOR $\theta = 90°$: $\quad \dot\phi_{min} = 2.40$ rad/s ◄

(c) FROM (5), $\dot\theta$ IS MAX. FOR $\theta = 90°$: $\quad \dot\theta_{max} = 3.29$ rad/s ◄

PROBLEM 18.134

(a) WITH $\theta_0 = 0$, $\dot\phi_0 = 6$ rad/s, EQ. (2) YIELDS $\dot\phi = \dfrac{6}{1+4\sin^2\theta}$

$\dot\phi$ IS MINIMUM FOR $\theta = 90°$: $\quad \dot\phi_{min} = 1.200$ rad/s ◄

(b) EQ. (4) YIELDS: $4\dot\theta^2 = 36 - \dot\phi^2(1+4\sin^2\theta) = 36\left(1 - \dfrac{1}{1+4\sin^2\theta}\right)$

$\dot\theta^2$ IS LARGEST FOR $\theta = 90°$:

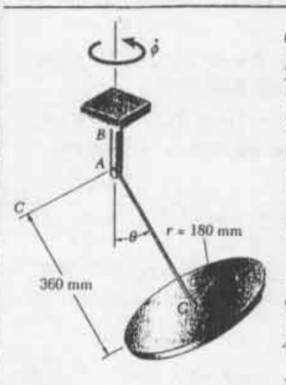
$4\dot\theta^2_{max} = 36\left(1 - \tfrac{1}{5}\right) \qquad \dot\theta^2_{max} = 7.20 \qquad \dot\theta_{max} = 2.68$ rad/s ◄

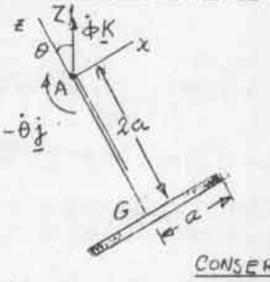
18.135 and 18.136

GIVEN:
DISK WELDED TO ROD AG OF NEGLIGIBLE MASS CONNECTED BY CLEVIS TO SHAFT AB. ROD AND DISK FREE TO ROTATE ABOUT AC; SHAFT FREE TO ROTATE ABOUT VERTICAL AXIS. INITIALLY, $\theta_0 = 90°$, $\dot\theta_0 = 0$.

PROBLEM 18.135:
KNOWING THAT $\dot\phi_{max} = 2\dot\phi_0$
FIND:
(a) θ_{min}, (b) $\dot\phi_0$

PROBLEM 18.136:
KNOWING THAT $\theta_{min} = 30°$,
FIND: (a) $\dot\phi_0$, (b) $\dot\phi_{max}$.





USING AXES Axyz, WITH y POINTING INTO PAPER,

$\underline{\omega} = \dot\phi \sin\theta\, \underline{i} - \dot\theta\, \underline{j} + \dot\phi \cos\theta\, \underline{k}$

$I_x = I_y = \frac{17}{4} ma^2,\quad I_z = \frac{1}{2} ma^2$

$H_x = I_x \omega_x = \frac{17}{4} ma^2 \dot\phi \sin\theta$

$H_z = I_z \omega_z = \frac{1}{2} ma^2 \dot\phi \cos\theta$

CONSERVATION OF ANG. MOM. ABOUT Z

SINCE ONLY FORCES ARE REACTION AT A AND $\underline{W} = -mg\underline{K}$, WE HAVE $\Sigma M_Z = 0$ AND H_Z = constant. THUS,

$H_Z = H_x \sin\theta + H_z \cos\theta = \frac{1}{4} ma^2 \dot\phi (17\sin^2\theta + 2\cos^2\theta)$

$H_Z = \frac{1}{4} ma^2 \dot\phi (2 + 15\sin^2\theta) = \text{constant} \quad (1)$

USING THE INITIAL CONDITIONS, EQ. (1) YIELDS

$\dot\phi(2 + 15\sin^2\theta) = 17\dot\phi_0 \quad (2)$

CONSERVATION OF ENERGY

$T = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2) = \frac{1}{2}\frac{ma^2}{4}(17\dot\phi^2\sin^2\theta + 17\dot\theta^2 + 2\dot\phi^2\cos^2\theta)$

$T = \frac{1}{8}ma^2[(2 + 15\sin^2\theta)\dot\phi^2 + 17\dot\theta^2] \qquad V = -2mga\cos\theta$

USING THE INITIAL CONDITIONS, WE WRITE T+V = const.

$(2 + 15\sin^2\theta)\dot\phi^2 + 17\dot\theta^2 - 16\frac{g}{a}\cos\theta = 17\dot\phi_0^2 \quad (3)$

PROBLEM 18.135

(a) LET $\dot\phi = \dot\phi_{max} = 2\dot\phi_0$ IN (2): $2\dot\phi_0(2 + 15\sin^2\theta) = 17\dot\phi_0$

$2 + 15\sin^2\theta = 8.5,\quad \sin\theta = \sqrt{0.4333},\quad \theta = 41.169°$

$\theta_{min} = 41.2°$

(b) LET $\theta = 41.169°$, $\dot\theta = 0$, $\dot\phi = 2\dot\phi_0$ IN (3):

$(2 + 15\sin^2 41.169°)(4\dot\phi_0^2) - 16\frac{g}{a}\cos 41.169° = 17\dot\phi_0^2$

$17\dot\phi_0^2 = 12.044\left(\frac{9.81}{0.18}\right) \qquad \dot\phi_0^2 = 38.613 \qquad \dot\phi_0 = 6.21 \text{ rad/s}$

PROBLEM 18.136

(a) LET $\theta = 30°$ IN (2): $\dot\phi(2 + 3.75) = 17\dot\phi_0$, $\dot\phi = \frac{17}{5.75}\dot\phi_0 \quad (4)$

LET $\theta = 30°$, $\dot\theta = 0$ IN (3):

$(2 + 3.75)\dot\phi^2 - 16\left(\frac{9.81}{0.18}\right)\cos 30° = 17\dot\phi_0^2$

SUBSTITUTE FOR $\dot\phi$ FROM (4):

$5.75\left(\frac{17}{5.75}\right)^2 \dot\phi_0^2 - 872\cos 30° = 17\dot\phi_0^2$

$17\left(\frac{17}{5.75} - 1\right)\dot\phi_0^2 = 872\cos 30° \qquad \dot\phi_0 = 4.7649 \text{ rad/s}$

$\dot\phi_0 = 4.76 \text{ rad/s}$

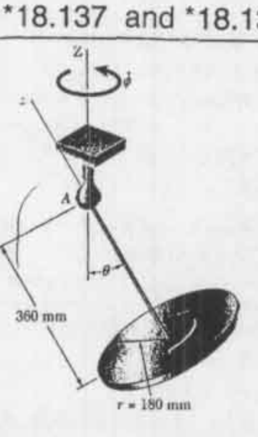
(b) FROM (4): $\dot\phi_{max} = \frac{17}{5.75}(4.7649) \qquad \dot\phi_{max} = 14.09 \text{ rad/s}$

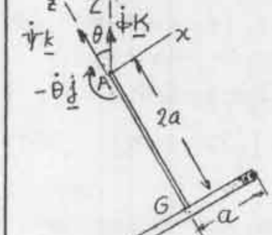
*18.137 and *18.138

GIVEN:
DISK WELDED TO ROD AG OF NEGLIGIBLE MASS SUPPORTED BY BALL AND SOCKET AT A.
INITIALLY, $\theta_0 = 90°$, $\dot\phi_0 = \dot\theta_0 = 0$.

PROBLEM 18.137:
KNOWING THAT $\dot\psi_0 = 50$ rad/s,
FIND: (a) θ_{min}.
(b) $\dot\phi$ AND $\dot\psi$ FOR $\theta = \theta_{min}$

PROBLEM 18.138:
KNOWING THAT $\theta_{min} = 30°$.
FIND: (a) $\dot\psi_0$,
(b) $\dot\phi$ AND $\dot\psi$ FOR $\theta = \theta_{min}$





USING AXES Axyz, WITH y AXIS POINTING INTO PAPER.

$\underline{\omega} = \dot\phi \sin\theta\, \underline{i} - \dot\theta\, \underline{j} + (\dot\psi + \dot\phi\cos\theta)\, \underline{k}$

$I_x = I_y = \frac{17}{4} ma^2,\quad I_z = \frac{1}{2} ma^2$

$\underline{H}_A = \frac{ma^2}{4}[17\dot\phi\sin\theta\, \underline{i} - 17\dot\theta\, \underline{j} + 2(\dot\psi + \dot\phi\cos\theta)\underline{k}]$

CONSERVATION OF ANG. MOMENTUM

SINCE THE ONLY EXTERNAL FORCES ARE THE REACTION AT A AND THE WEIGHT $\underline{W} = -mg\underline{K}$ AT G, WE HAVE $\Sigma M_Z = 0$, $\Sigma M_z = 0$. SINCE Z IS PART OF A NEWTONIAN FRAME, IT FOLLOWS THAT H_Z = CONST.; BECAUSE OF THE AXISYMMETRY OF THE DISK, IT ALSO FOLLOWS THAT H_z = CONST. (SEE PROB. 18.139). USING THE INITIAL CONDITIONS WE WRITE

$H_z = \text{const.}: \qquad \dot\psi + \dot\phi\cos\theta = \dot\psi_0 \quad (1)$

NOTING THAT $H_Z = \underline{H}_A \cdot \underline{K} = \frac{ma^2}{4}[17\dot\phi\sin^2\theta + 2(\dot\psi + \dot\phi\cos\theta)\cos\theta]$

AND SUBSTITUTING FROM (1) FOR THE INSIDE PARENTHESIS,

$H_Z = \text{const.}: \qquad 17\dot\phi\sin^2\theta + 2\dot\psi_0\cos\theta = 0 \quad (2)$

CONSERVATION OF ENERGY

$T = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2) = \frac{1}{2}\frac{ma^2}{4}[17\dot\phi^2\sin^2\theta + 17\dot\theta^2 + 2(\dot\psi + \dot\phi\cos\theta)^2]$

$T = \frac{1}{8}ma^2(17\dot\phi^2\sin^2\theta + 17\dot\theta^2 + 2\dot\psi_0^2) \qquad V = -2mga\cos\theta$

$T + V = \text{const.}: \quad 17\dot\phi^2\sin^2\theta + 17\dot\theta^2 + \cancel{2\dot\psi_0^2} - 16\frac{g}{a}\cos\theta = \cancel{2\dot\psi_0^2}$

$\dot\phi^2\sin^2\theta + \dot\theta^2 = \frac{16}{17}\frac{g}{a}\cos\theta \quad (3)$

PROBLEM 18.137

(a) FROM (2): $\dot\phi = -\frac{2}{17}\dot\psi_0\frac{\cos\theta}{\sin^2\theta} = -\frac{2}{17}(50 \text{ rad/s})\frac{\cos\theta}{\sin^2\theta}$

CARRY INTO (3) AND LET $\dot\theta = 0$ FOR $\theta = \theta_{min}$:

$\left(\frac{100}{17}\right)^2\frac{\cos^2\theta}{\sin^2\theta} = \frac{16}{17}\frac{9.81 \text{ m/s}^2}{0.18 \text{ m}} \qquad \frac{10\times10^3}{16\times17}\frac{0.18}{9.81}\cos\theta = 1 - \cos^2\theta$

$\cos^2\theta + 0.67458\cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2}(-0.67458 \pm 2.1107) = 0.71806 \qquad \theta = 44.105°$

OR -1.3926 (IMPOSSIBLE) $\qquad \theta = 44.1°$

(b) SUBSTITUTING $\dot\psi_0 = 50$ rad/s AND $\theta = 44.105°$ IN (2) AND (1)

EQ.(2): $\dot\phi = -\frac{2}{17}(50)\frac{\cos 44.105°}{\sin^2 44.105°} \qquad \dot\phi = -8.72 \text{ rad/s}$

EQ.(1): $\dot\psi = 50 - (-8.72)\cos 44.105° \qquad \dot\psi = 56.3 \text{ rad/s}$

PROBLEM 18.138

LET $\theta = 30°$, $\dot\theta = 0$ IN (3): $\dot\phi^2 = \frac{16}{17}\frac{9.81 \text{ m/s}^2}{0.18 \text{ m}}\frac{\cos 30°}{\sin^2 30°}$, $\dot\phi = \pm 13.330$

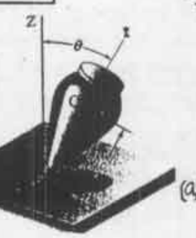
FROM (2), WE NOTE THAT $\dot\phi < 0$ FOR $\dot\psi_0 > 0$. THUS: $\dot\phi = -13.33$ rad/s

EQ.(2): $\dot\psi_0 = -\frac{17}{2}(-13.33)\frac{\sin^2 30°}{\cos 30°} = 32.708$, $\dot\psi_0 = 32.7$ rad/s

EQ.(1): $\dot\psi = 32.708 - (-13.33)\cos 30° \qquad \dot\psi = 44.3$ rad/s

*18.139

GIVEN:
TOP WITH FIXED POINT O
ϕ, θ, ψ = EULERIAN ANGLES
I = MOM. OF INERTIA ABOUT z AXIS
I' = — — — TRANSVERSE AXIS THROUGH O.

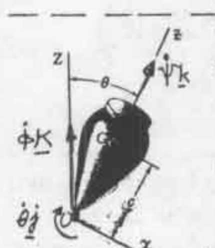


SHOW THAT:

(a) $I'\dot\phi\sin^2\theta + I(\dot\psi + \dot\phi\cos\theta)\cos\theta = \alpha$ (1)

$I(\dot\psi + \dot\phi\cos\theta) = \beta$ (2)

(b) ω_z = CONST. AND $\dot\phi$ = FUNCTION OF θ



WE USE FRAME $Oxyz$ WITH y AXIS POINTING INTO PAPER

ANG. VELOCITY OF TOP:

$$\underline{\omega} = -\dot\phi\sin\theta\,\underline{i} + \dot\theta\,\underline{j} + (\dot\psi + \dot\phi\cos\theta)\underline{k} \quad (A)$$

ANG. VELOCITY OF FRAME:

$$\underline{\Omega} = -\dot\phi\sin\theta\,\underline{i} + \dot\theta\,\underline{j} + \dot\phi\cos\theta\,\underline{k} \quad (B)$$

ANG. MOMENTUM ABOUT O:

$$\underline{H}_O = I_x\omega_x\underline{i} + I_y\omega_y\underline{j} + I_z\omega_z\underline{k}$$

$$\underline{H}_O = -I'\dot\phi\sin\theta\,\underline{i} + I'\dot\theta\,\underline{j} + I(\dot\psi + \dot\phi\cos\theta)\underline{k} \quad (C)$$

(a) WE RECALL $\quad \Sigma\underline{M}_O = \dot{\underline{H}}_O \quad (18.27)$

SINCE THE ONLY EXTERNAL FORCES ARE THE REACTION AT O AND THE WEIGHT $\underline{W} = -mg\underline{K}$ AT G, WE HAVE $\Sigma M_Z = 0$ AND FROM (18.27) $\dot H_Z = 0$. SINCE THE Z AXIS IS PART OF A NEWTONIAN FRAME OF REFERENCE, IT FOLLOWS THAT H_Z = constant. BUT $H_Z = \underline{H}_O \cdot \underline{K}$.

SUBSTITUTING FOR $\underline{H}_O$ FROM (C) AND NOTING THAT $\underline{i}\cdot\underline{K} = -\sin\theta$, $\underline{j}\cdot\underline{K} = 0$, $\underline{k}\cdot\underline{K} = \cos\theta$, WE HAVE

$$H_Z = \underline{H}_O\cdot\underline{K} = -I'\dot\phi\sin\theta(-\sin\theta) + 0 + I(\dot\psi + \dot\phi\cos\theta)\cos\theta$$

THUS: $\quad I'\dot\phi\sin^2\theta + I(\dot\psi + \dot\phi\cos\theta)\cos\theta = \alpha \quad (1)$ ◀

WHERE α IS A CONSTANT.

WE OBSERVE THAT WE ALSO HAVE $\Sigma M_z = 0$, BUT WE CANNOT CONCLUDE THAT H_z = const., SINCE THE z AXIS IS NOT PART OF A NEWTONIAN FRAME OF REFERENCE.

USING EQ. (18.28), WE WRITE

$$\Sigma\underline{M}_O = (\dot{\underline{H}}_O)_{Oxyz} + \underline{\Omega}\times\underline{H}_O \quad (18.28)$$

SUBSTITUTING FROM (B) AND (C) INTO (18.28),

$$\Sigma\underline{M}_O = -I'\frac{d}{dt}(\dot\phi\sin\theta)\underline{i} + I'\ddot\theta\,\underline{j} + I\frac{d}{dt}(\dot\psi + \dot\phi\cos\theta)\underline{k} +$$
$$(-\dot\phi\sin\theta\,\underline{i} + \dot\theta\,\underline{j} + \dot\phi\cos\theta\,\underline{k})\times[-I'\dot\phi\sin\theta\,\underline{i} + I'\dot\theta\,\underline{j} + I(\dot\psi + \dot\phi\cos\theta)\underline{k}]$$

CONSIDERING ONLY THE COEFFICIENTS OF $\underline{k}$, WE OBTAIN

$$\Sigma M_z = I\frac{d}{dt}(\dot\psi + \dot\phi\cos\theta) - I'\dot\phi\dot\theta\sin\theta + I'\dot\phi\dot\theta\sin\theta = 0$$

BUT THE SECOND AND THIRD TERMS CANCEL OUT, DUE TO <u>THE AXISYMMETRY OF THE TOP</u>. THUS

$$\Sigma M_z = I\frac{d}{dt}(\dot\psi + \dot\phi\cos\theta) = 0$$

AND $\quad I(\dot\psi + \dot\phi\cos\theta) = \beta \quad (2)$ ◀

WHERE β IS A CONSTANT

(b) FROM EQ. (A) WE HAVE $\omega_z = \dot\psi + \dot\phi\cos\theta$ AND, IN VIEW OF (2):

$$\omega_z = \beta/I = \text{constant} \quad (3)$$ ◀

SUBSTITUTING FOR $I(\dot\psi + \dot\phi\cos\theta)$ FROM (2) INTO (1):

$$I'\dot\phi\sin^2\theta + \beta\cos\theta = \alpha \quad (4)$$

$$\dot\phi = \frac{\alpha - \beta\cos\theta}{I'\sin^2\theta} \quad \text{(FUNCTION OF } \theta) \quad (5)$$ ◀

*18.140

GIVEN:
TOP OF PROB. 18.139

SHOW THAT:

(a) A THIRD EQUATION OF MOTION CAN BE OBTAINED FROM THE PRINCIPLE OF CONSERVATION OF ENERGY.

(b) BY ELIMINATING $\dot\phi$ AND $\dot\psi$ FROM THAT EQUATION AND EQS (1) AND (2) OF PROB. 18.140 AN EQUATION $\dot\theta^2 = f(\theta)$ CAN BE OBTAINED, WHERE

$$f(\theta) = \frac{1}{I'}\left(2E - \frac{\beta^2}{I} - 2mgc\cos\theta\right) - \left(\frac{\alpha - \beta\cos\theta}{I'\sin\theta}\right)^2 \quad (1)$$

(c) BY INTRODUCING THE VARIABLE $x = \cos\theta$, THE MAX. AND MIN. VALUES OF θ CAN BE OBTAINED BY SOLVING THE CUBIC EQUATION

$$\left(2E - \frac{\beta^2}{I} - 2mgcx\right)(1 - x^2) - \frac{1}{I'}(\alpha - \beta x)^2 = 0 \quad (2)$$

(a) CONSERVATION OF ENERGY.

$$T = \tfrac{1}{2}(I'\omega_x^2 + I'\omega_y^2 + I\omega_z^2)$$

REFERRING TO EQ. (A) OF PROB. 18.139:

$$T = \tfrac{1}{2}[I'\dot\phi^2\sin^2\theta + I'\dot\theta^2 + I(\dot\psi + \dot\phi\cos\theta)^2], \quad V = mgc\cos\theta$$

$T + V = E$:

$$\tfrac{1}{2}[I'\dot\phi^2\sin^2\theta + I'\dot\theta^2 + I(\dot\psi + \dot\phi\cos\theta)^2] + mgc\cos\theta = E \quad (6)$$ ◀

(b) SUBSTITUTING IN (6) FOR $\dot\phi$ FROM EQ. (5) OF PROB. 18.139 AND FOR $(\dot\psi + \dot\phi\cos\theta)$ FROM EQ. (2) OF PROB. 18.139, AND MULTIPLYING BY 2:

$$I'\left(\frac{\alpha - \beta\cos\theta}{I'\sin^2\theta}\right)^2\sin^2\theta + I'\dot\theta^2 + I\left(\frac{\beta}{I}\right)^2 + 2mgc\cos\theta = 2E$$

$$\frac{(\alpha - \beta\cos\theta)^2}{I'\sin^2\theta} + I'\dot\theta^2 + \frac{\beta^2}{I} + 2mgc\cos\theta = 2E$$

SOLVING FOR $\dot\theta^2$, WE OBTAIN

$$\dot\theta^2 = f(\theta)$$

WHERE

$$f(\theta) = \frac{1}{I'}\left(2E - \frac{\beta^2}{I} - 2mgc\cos\theta\right) - \left(\frac{\alpha - \beta\cos\theta}{I'\sin\theta}\right)^2$$ ◀

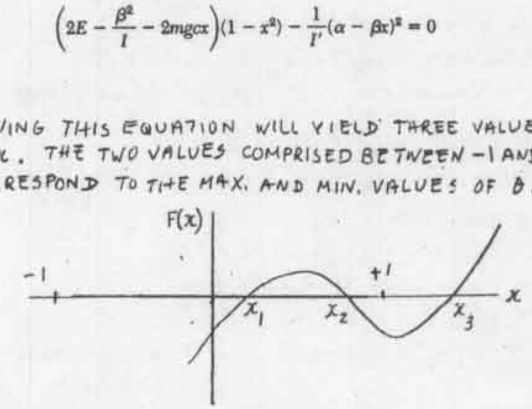
(c) SETTING $\cos\theta = x$,

$$f(\theta) = \frac{1}{I'}\left(2E - \frac{\beta^2}{I} - 2mgcx\right) - \frac{(\alpha - \beta x)^2}{I'^2(1 - x^2)}$$

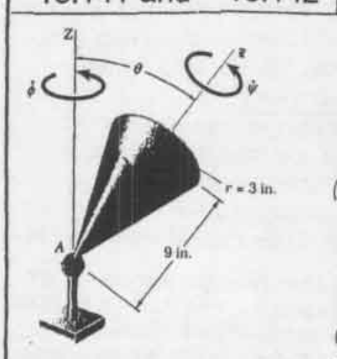
LETTING $f(\theta) = 0$ AND MULTIPLYING BY $I'(1 - x^2)$, WE OBTAIN THE CUBIC EQUATION $F(x) = 0$:

$$\left(2E - \frac{\beta^2}{I} - 2mgcx\right)(1 - x^2) - \frac{1}{I'}(\alpha - \beta x)^2 = 0$$ ◀

SOLVING THIS EQUATION WILL YIELD THREE VALUES OF x. THE TWO VALUES COMPRISED BETWEEN -1 AND $+1$ CORRESPOND TO THE MAX. AND MIN. VALUES OF θ.



* 18.141 and * 18.142



GIVEN: SOLID CONE. INITIALLY, $\theta_0 = 30°$, $\dot\theta_0 = 0$, $\dot\psi_0 = 300$ rad/s. USING EQ. (2) OF PROB. 18.140 AND

PROBLEM 18.141:
KNOWING THAT $\dot\phi_0 = 20$ rad/s
FIND: (a) θ_{max}, (b) CORRESPONDING $\dot\psi$ AND $\dot\phi$

PROBLEM 18.142:
KNOWING THAT $\dot\phi_0 = -4$ rad/s
FIND: (a) θ_{max}, (b) CORRESPONDING $\dot\psi$ AND $\dot\phi$
(c) VALUE OF θ FOR WHICH SENSE OF $\dot\phi$ IS REVERSED

WE FIRST DETERMINE THE FOLLOWING CONSTANTS:

$I = \frac{3}{10} m r^2 = \frac{3}{10} m (0.25\ ft)^2 = (18.75 \times 10^{-3}\ ft^2) m$

$I' = \frac{3}{5} m\left(\frac{1}{4}r^2 + h^2\right) = \frac{3}{5}\left[\frac{1}{4}(0.25\ ft)^2 + (0.75\ ft)^2\right] m$
$= (346.875 \times 10^{-3}\ ft^2)\ m$

$c = AG = \frac{3}{4} h = \frac{3}{4}(0.75\ ft) = 562.5 \times 10^{-3}\ ft$

NEXT, WE DETERMINE THE CONSTANTS β, α, AND E FROM EQS. (2) AND (4) OF THE SOLUTION OF PROB. 18.139 AND FROM EQ. (6) OF THE SOLUTION OF PROB. 18.140, USING THE APPROPRIATE INITIAL CONDITIONS.

PROBLEM 18.141

$\beta = I(\dot\psi_0 + \dot\phi_0 \cos\theta_0) = (18.75 \times 10^{-3}) m (300 + 20 \cos 30°)$
$= (5.94976) m$

$\alpha = I' \dot\phi_0 \sin^2\theta_0 + \beta \cos\theta_0 = (346.875 \times 10^{-3}) m (20) \sin^2 30° + (5.94976) m \cos 30° = (6.88702)\ m$

$E = \frac{1}{2}\left[I' \dot\phi_0^2 \sin^2\theta_0 + I' \dot\theta_0^2 + I(\dot\psi_0 + \dot\phi_0 \cos\theta_0)^2\right] + mgc\cos\theta_0$
$= \frac{1}{2}\left[(346.875 \times 10^{-3}) m (20)^2 \sin^2 30° + 0 + \frac{(5.94976\ m)^2}{18.75 \times 10^{-3}\ m}\right] + m(32.2)(562.5 \times 10^{-3}) \cos 30° = (977.020)\ m$

SUBSTITUTE IN EQ. (2) OF PROB. 18.140:

$$\left(2E - \frac{\beta^2}{I} - 2mgcx\right)(1 - x^2) - \frac{1}{I'}(\alpha - \beta x)^2 = 0$$

$$(66.0593 - 36.225x)(1 - x^2) - 2.88288(6.88702 - 5.94976x)^2 = 0$$

(a) SOLVING: $x = 0.743151$ $\theta_{max} = 42.0°$ ◀

(b) EQ. (5) OF PROB. 18.139:

$\dot\phi = \frac{\alpha - \beta\cos\theta}{I' \sin^2\theta} = \frac{6.88702 - 5.94976 \cos 42.0°}{(346.875 \times 10^{-3}) \sin^2 42.0°} = 15.8748$ rad/s

FROM EQ. (2): $\dot\psi = \frac{\beta}{I} - \dot\phi\cos\theta = \frac{5.94976}{18.75 \times 10^{-3}} - (15.8748) \cos 42.0°$

$\dot\psi = 306$ rad/s ; $\dot\phi = 15.87$ rad/s ◀

PROBLEM 18.142

$\beta = I(\dot\psi_0 + \dot\phi_0 \cos\theta_0) = (18.75 \times 10^{-3}) m (300 - 4 \cos 30°) = (5.56005) m$

$\alpha = I' \dot\phi_0 \sin^2\theta_0 + \beta\cos\theta_0 = (346.875 \times 10^{-3}) m (-4) \sin^2 30° + (5.56005) m \cos 30° = (4.46827) m$

$E = \frac{1}{2}\left[I' \dot\phi_0^2 \sin^2\theta_0 + I' \dot\theta_0^2 + I(\dot\psi_0 + \dot\phi_0\cos\theta_0)^2\right] + mgc\cos\theta_0$
$= \frac{1}{2}\left[(346.875 \times 10^{-3}) m (-4)^2 \sin^2 30° + 0 + \frac{(5.56005\ m)^2}{18.75 \times 10^{-3}\ m}\right] + m(32.2)(562.5 \times 10^{-3}) \cos 30° = 840.76\ m$

SUBSTITUTE IN EQ. (2) OF PROB. 18.140:

$$\left(2E - \frac{\beta^2}{I} - 2mgcx\right)(1 - x^2) - \frac{1}{I'}(\alpha - \beta x)^2 = 0$$

$$(32.765 - 36.225x)(1 - x^2) - 2.88288(4.46827 - 5.56005x)^2 = 0$$

(a) SOLVING: $x = 0.37166$, $\theta_{max} = 68.18°$, $\theta_{max} = 68.2°$ ◀

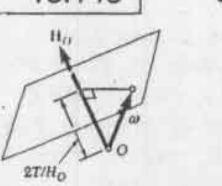
(b) EQ. (5): $\dot\phi = \frac{\alpha - \beta\cos\theta}{I' \sin^2\theta} = \frac{4.46827 - 5.56005 \cos 68.18°}{(346.875 \times 10^{-3}) \sin^2 68.18°} = 8.0335$ rad/s

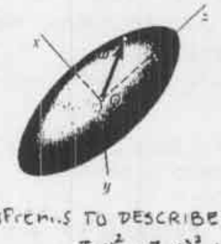
EQ. (2): $\dot\psi = (\beta/I) - \dot\phi\cos\theta = 296.536 - 8.0335 \cos 68.18° = 293.55$ rad/s

$\dot\psi = 294$ rad/s ; $\dot\phi = 8.03$ rad/s ◀

(c) $\dot\phi$ REVERSES FOR $\alpha - \beta\cos\theta = 0$, $\cos\theta = \frac{4.46827}{5.56005}$, $\theta = 36.5°$ ◀

* 18.143





GIVEN:
RIGID BODY OF ARBITRARY SHAPE SUPPORTED AT ITS MASS CENTER O AND SUBJECTED TO NO FORCE (EXCEPT AT SUPPORT O).

SHOW THAT:
(a) $\underline{H}_O$ = constant (IN MAGNITUDE & DIR^N)
T = constant
PROJ. OF $\underline{\omega}$ ALONG $\underline{H}_O$ = constant
(b) TIP OF $\underline{\omega}$ DESCRIBES CURVE ON FIXED PLANE (THE "INVARIABLE PLANE") PERP. TO $\underline{H}_O$ AND AT DISTANCE $2T/H_O$ FROM O.
(c) WITH RESPECT TO PRINCIPAL AXES $Oxyz$ ATTACHED TO BODY, $\underline{\omega}$ APPEARS TO DESCRIBE A CURVE ON ELLIPSOID OF EQUATION

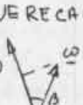
$$I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 = 2T \quad \text{(POINSOT ELLIPSOID)}$$

(a) FROM EQ. (18.27): $\Sigma \underline{M}_O = \dot{\underline{H}}_O$

SINCE $\Sigma \underline{M}_O = 0$: $\underline{H}_O$ = constant (1) ◀

$T + V$ = const.; SINCE V = const, T = constant (2) ◀

WE RECALL FROM PROB. 18.37 THAT $\underline{H}_O \cdot \underline{\omega} = 2T$



BUT $\underline{H}_O \cdot \underline{\omega} = H_O\,\omega\cos\beta$

THUS $H_O\,\omega\cos\beta = 2T$

PROJ. OF $\underline{\omega}$ ON $\underline{H}_O = \omega\cos\beta = \frac{2T}{H_O}$ = const (3) ◀

(b) IT FOLLOWS FROM (3) THAT THE TIP OF $\underline{\omega}$ MUST REMAIN IN A PLANE $\perp$ $\underline{H}_O$ AT A DISTANCE $2T/H_O$ FROM O.

(c) FROM EQ. (18.20):

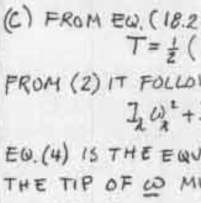
$$T = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2)$$

FROM (2) IT FOLLOWS THAT

$$I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 = 2T = \text{const.} \quad (4)$$ ◀

EQ. (4) IS THE EQUATION OF AN ELLIPSOID ON WHICH THE TIP OF $\underline{\omega}$ MUST LIE. THIS IS <u>POINSOT ELLIPSOID</u>. COMPARING EQ. (4) WITH EQ. (9.49) OF SEC. 9.17, WE NOTE THAT POINSOT ELLIPSOID HAS THE <u>SAME SHAPE</u> AS THE <u>ELLIPSOID OF INERTIA</u> OF THE BODY, BUT A DIFFERENT SIZE.

* 18.144



GIVEN:
POINSOT ELLIPSOID AND INVARIABLE PLANE DEFINED IN PROB. 18.143.

SHOW THAT:
(a) THE ELLIPSOID IS TANGENT TO THE PLANE.
(b) AS THE BODY MOVES THE POINSOT ELLIPSOID ROLLS ON THE INVARIABLE PLANE.

(a) AT THE TIP OF $\underline{\omega}$ THE DIRECTION OF THE NORMAL TO THE ELLIPSOID IS THAT OF <u>grad</u> $F(\omega_x, \omega_y, \omega_z)$, WHERE F DENOTES THE LEFT-HAND MEMBER OF EQ. (4) OF PROB. 18.143. FROM SEC. 13.7:

$$\text{grad } F = \frac{\partial F}{\partial\omega_x}\underline{i} + \frac{\partial F}{\partial\omega_y}\underline{j} + \frac{\partial F}{\partial\omega_z}\underline{k}$$
$$= 2I_x\omega_x\underline{i} + 2I_y\omega_y\underline{j} + 2I_z\omega_z\underline{k}$$
$$= 2(I_x\omega_x\underline{i} + I_y\omega_y\underline{j} + I_z\omega_z\underline{k}) = 2\underline{H}_O$$

THUS, THE NORMAL TO POINSOT ELLIPSOID IS PARALLEL TO $\underline{H}_O$. IT FOLLOWS THAT POINSOT ELLIPSOID IS TANGENT TO THE INVARIABLE PLANE ◀

(CONTINUED)

* 18.144 continued

(b) THE POINSOT ELLIPSOID IS PART OF THE BODY WHOSE MOTION IS BEING ANALYZED, AND ITS POINT OF CONTACT WITH THE INVARIABLE PLANE IS THE TIP OF THE VECTOR $\underline{\omega}$. SINCE $\underline{\omega}$ DEFINES THE INSTANTANEOUS AXIS OF ROTATION, THE POINT OF CONTACT HAS ZERO VELOCITY. THUS, THE POINSOT ELLIPSOID ROLLS ON THE INVARIABLE PLANE ◄ (WITH ITS CENTER O REMAINING FIXED).

* 18.145

GIVEN: AXISYMMETRICAL RIGID BODY SUPPORTED AT ITS MASS CENTER O AND SUBJECTED TO NO FORCE (EXCEPT AT SUPPORT O).

USING THE RESULTS OBTAINED IN PROBS. 18.143-144, SHOW THAT THE POINSOT ELLIPSOID IS AN ELLIPSOID OF REVOLUTION AND THE SPACE AND BODY CONES ARE BOTH CIRCULAR AND TANGENT TO EACH OTHER.

FURTHER SHOW THAT

(a) THE TWO CONES ARE TANGENT EXTERNALLY AND THE PRECESSION IS DIRECT WHEN $I < I'$,

WHERE I = MOM. OF INERTIA ABOUT AXIS OF SYMMETRY

I' = — — — — TRANSVERSE AXIS,

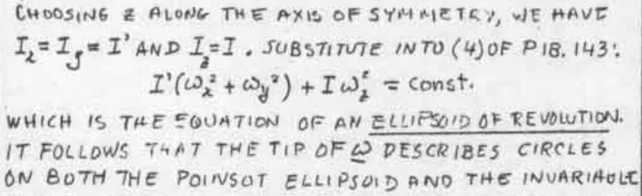
(b) THE SPACE CONE IS INSIDE THE BODY CONE AND THE PRECESSION IS RETROGRADE WHEN $I > I'$.

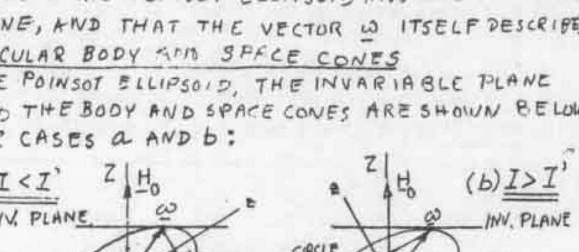
CHOOSING z ALONG THE AXIS OF SYMMETRY, WE HAVE $I_x = I_y = I'$ AND $I_z = I$. SUBSTITUTE INTO (4) OF P18.143:

$$I'(\omega_x^2 + \omega_y^2) + I\omega_z^2 = \text{const.}$$

WHICH IS THE EQUATION OF AN ELLIPSOID OF REVOLUTION. IT FOLLOWS THAT THE TIP OF $\underline{\omega}$ DESCRIBES CIRCLES ON BOTH THE POINSOT ELLIPSOID AND THE INVARIABLE PLANE, AND THAT THE VECTOR $\underline{\omega}$ ITSELF DESCRIBES CIRCULAR BODY AND SPACE CONES.

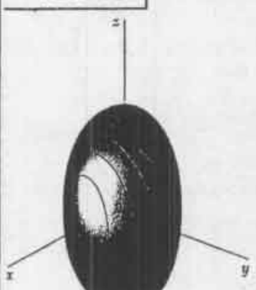
THE POINSOT ELLIPSOID, THE INVARIABLE PLANE AND THE BODY AND SPACE CONES ARE SHOWN BELOW FOR CASES a AND b:





DIRECT PRECESSION　　　RETROGRADE PRECESSION

* 18.146



GIVEN: RIGID BODY OF ARBITRARY SHAPE AND ITS POINSOT ELLIPSOID (CF. PROBS. 18.143 AND 18.144.

SHOW THAT:

(a) CURVE DESCRIBED BY TIP OF $\underline{\omega}$ ON POINSOT ELLIPSOID IS DEFINED BY

$$I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2 = 2T = \text{constant} \quad (1)$$

$$I_x^2\omega_x^2 + I_y^2\omega_y^2 + I_z^2\omega_z^2 = H_O^2 = \text{constant} \quad (2)$$

AND CAN THUS BE OBTAINED BY INTERSECTING THE POINSOT ELLIPSOID WITH THE ELLIPSOID DEFINED BY (2).

(b) ASSUMING $I_x > I_y > I_z$, THE CURVES (CALLED POLHODES) OBTAINED FOR VARIOUS VALUES OF H_O HAVE THE SHAPES INDICATED IN FIGURE

(c) THE BODY CAN ROTATE ABOUT A FIXED AXIS ONLY IF THAT AXIS COINCIDES WITH ONE OF THE PRINCIPAL AXES, THIS MOTION BEING STABLE IF THE AXIS IS THE MAJOR OR MINOR AXIS OF THE POINSOT ELLIPSOID (x OR z AXIS) AND UNSTABLE IF IT IS THE INTERMEDIATE AXIS (y AXIS).

(a) EQ. (1) IN STATEMENT EXPRESSES CONSERVATION OF ENERGY; THIS IS EQ. (4) OF PROB. 18.143.

WE NOW EXPRESS THAT THE MAGNITUDE OF $\underline{H}_O$ IS CONSTANT:

$$H_O^2 = H_x^2 + H_y^2 + H_z^2 = I_x^2\omega_x^2 + I_y^2\omega_y^2 + I_z^2\omega_z^2 = \text{const.}$$

WHICH IS EQ (2) IN STATEMENT. SINCE THE COORDINATES $\omega_x, \omega_y, \omega_z$ OF THE TIP OF $\underline{\omega}$ MUST SATISFY BOTH EQS. (1) AND (2), THE CURVE DESCRIBED BY THE TIP OF $\underline{\omega}$ IS THE INTERSECTION OF THE TWO ELLIPSOIDS.

(b) WE NOW WRITE THE EQUATIONS OF THE TWO ELLIPSOIDS IN THE STANDARD FORM

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

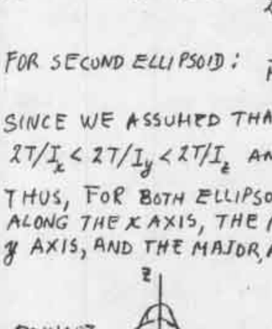
WHERE a, b, c ARE THE SEMI-AXES OF THE ELLIPSOID. WE HAVE

FOR POINSOT ELLIPSOID: $$\frac{\omega_x^2}{2T/I_x} + \frac{\omega_y^2}{2T/I_y} + \frac{\omega_z^2}{2T/I_z} = 1 \quad (3)$$

FOR SECOND ELLIPSOID: $$\frac{\omega_x^2}{H_O^2/I_x^2} + \frac{\omega_y^2}{H_O^2/I_y^2} + \frac{\omega_z^2}{H_O^2/I_z^2} = 1 \quad (4)$$

SINCE WE ASSUMED THAT $I_x > I_y > I_z$, WE HAVE $2T/I_x < 2T/I_y < 2T/I_z$ AND $H_O^2/I_x^2 < H_O^2/I_y^2 < H_O^2/I_z^2$

THUS, FOR BOTH ELLIPSOIDS, THE MINOR AXIS IS DIRECTED ALONG THE x AXIS, THE INTERMEDIATE AXIS ALONG THE y AXIS, AND THE MAJOR AXIS ALONG THE z AXIS.

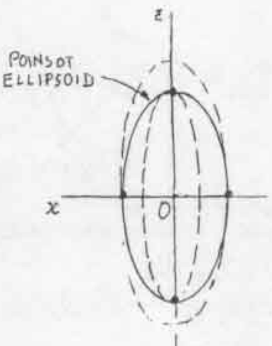


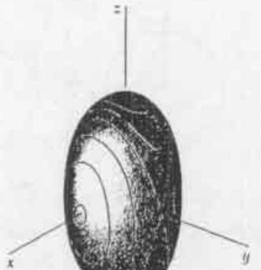
HOWEVER, BECAUSE THE RATIO OF THE MAJOR TO THE MINOR SEMIAXIS IS $\sqrt{I_x/I_z}$ FOR THE POINSOT ELLIPSOID AND I_x/I_z FOR THE SECOND ELLIPSOID, THE SHAPE OF THE LATTER WILL BE MORE "PRONOUNCED".

(CONTINUED)

*18.146 continued

THE LARGEST ELLIPSOID OF THE SECOND TYPE TO BE IN CONTACT WITH THE POINSOT ELLIPSOID WILL BE OUTSIDE THAT ELLIPSOID AND TOUCH IT AT ITS POINTS OF INTERSECTION WITH THE x AXIS, AND THE SMALLEST WILL BE INSIDE THE POINSOT ELLIPSOID AND TOUCH IT AT ITS POINTS OF INTERSECTION WITH THE z AXIS (SEE LEFT-HAND SKETCH). ALL ELLIPSOIDS OF THE SECOND TYPE COMPRISED BETWEEN THESE TWO WILL INTERSECT THE POINSOT ELLIPSOID ALONG THE POLHODES AS SHOWN IN THE RIGHT-HAND FIGURE.





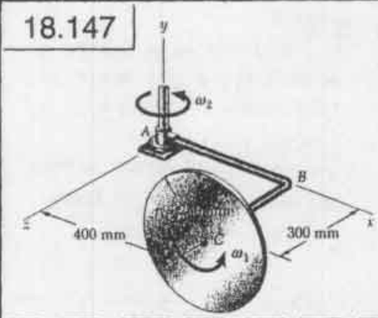
NOTE THAT THE ELLIPSOID OF THE SECOND TYPE WHICH HAS THE SAME INTERMEDIATE AXIS AS THE POINSOT ELLIPSOID INTERSECTS THAT ELLIPSOID ALONG TWO ELLIPSES WHOSE PLANES CONTAIN THE y AXIS. THESE CURVES ARE NOT POLHODES, SINCE THE TIP OF ω WILL NOT DESCRIBE THEM, BUT THEY SEPARATE THE POLHODES INTO FOUR GROUPS: TWO GROUPS LOOP AROUND THE MINOR AXIS (x AXIS) AND THE OTHER TWO AROUND THE MAJOR AXIS (z AXIS).

(c) IF THE BODY IS SET TO SPIN ABOUT ONE OF THE PRINCIPAL AXES, THE POINSOT ELLIPSOID WILL REMAIN IN CONTACT WITH THE INVARIABLE PLANE AT THE SAME POINT (ON THE x, y, OR z AXIS); THE ROTATION IS STEADY. IN ANY OTHER CASE, THE POINT OF CONTACT WILL BE LOCATED ON ONE OF THE POLHODES AND THE TIP OF ω WILL START DESCRIBING THAT POLHODE, WHILE THE POINSOT ELLIPSOID ROLLS ON THE INVARIABLE PLANE.

A ROTATION ABOUT THE MINOR OR THE MAJOR AXIS (x OR z AXIS) IS STABLE: IF THAT MOTION IS DISTURBED, THE TIP OF ω WILL MOVE TO A VERY SMALL POLHODE SURROUNDING THAT AXIS AND STAY CLOSE TO ITS ORIGINAL POSITION.

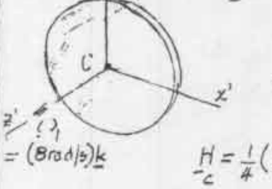
ON THE OTHER HAND, A ROTATION ABOUT THE INTERMEDIATE AXIS (z AXIS) IS UNSTABLE: IF THAT MOTION IS DISTURBED, THE TIP OF ω WILL MOVE TO ONE OF THE POLHODES LOCATED NEAR THAT AXIS AND START DESCRIBING IT, DEPARTING COMPLETELY FROM ITS ORIGINAL POSITION.AND CAUSING THE BODY TO TUMBLE.

18.147



GIVEN:
DISK OF MASS $m = 5$ kg
$\omega_1 = 8$ rad/s (constant)
$\omega_2 = 3$ rad/s (constant)

FIND:
$\underline{H}_C$



USING FRAME $Cx'y'z'$:

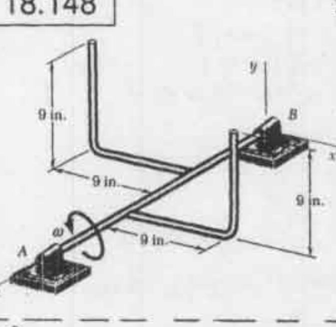
$$\bar{I}_{x'} = \bar{I}_{y'} = \frac{1}{4}mr^2 \qquad \bar{I}_{z'} = \frac{1}{2}mr^2$$

$$\underline{H}_C = \bar{I}_{y'}\omega_2\underline{j} + \bar{I}_{z'}\omega_1\underline{k} = \frac{1}{4}mr^2(\omega_2\underline{j} + 2\omega_1\underline{k})$$

$$\underline{H}_C = \frac{1}{4}(5\text{ kg})(0.25\text{ m})^2[(3\text{ rad/s})\underline{j} + 2(8\text{ rad/s})\underline{k}$$

$$\underline{H}_C = (0.234\text{ kg}\cdot\text{m}^2/\text{s})\underline{j} + (1.250\text{ kg}\cdot\text{m}^2/\text{s})\underline{k}$$ ◀

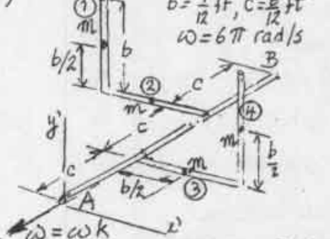
18.148



GIVEN:
TWO L-SHAPED ARMS, EACH WEIGHING 5 lb, ARE WELDED TO ONE-THIRD POINTS OF 24-in. SHAFT AB.
$\omega = 180$ rpm (CONSTANT)

FIND:
(a) $\underline{H}_A$
(b) ANGLE THAT $\underline{H}_A$ FORMS WITH AB.

(a)



$b = \frac{9}{12}$ ft, $c = \frac{8}{12}$ ft
$\omega = 6\pi$ rad/s

MOMENT OF INERTIA:

$$I_z = 2(I_z^{①} + I_z^{②}) = 2\left[\frac{1}{12}mb^2 + m\left(b^2 + \frac{b^2}{4}\right)\right] + 2\left[\frac{1}{12}mb^2 + m\frac{b^2}{4}\right]$$

$$I_z = \frac{10}{3}mb^2$$

PRODUCTS OF INERTIA
BECAUSE OF SYMMETRY OF EACH ELEMENT ABOUT ITS MASS CENTER:

$$I_{y'z} = m\bar{y}'_1\bar{z}_1 + m\bar{y}'_4\bar{z}_4 = m\left[\frac{b}{2}(-2c) + \frac{b}{2}(-c)\right] = -\frac{3}{2}mbc$$

$$I_{zx'} = m\bar{z}_1\bar{x}'_1 + m\bar{z}_2\bar{x}'_2 + m\bar{z}_3\bar{x}'_3 + m\bar{z}_4\bar{x}'_4 = m[-b(-2c) - \frac{b}{2}(-2c) + b(-c) + \frac{b}{2}(-c) = \frac{3}{2}mbc$$

EQS. (18.13):

$$H_{x'} = -I_{zx'}\omega_z = -\frac{3}{2}mbc\,\omega_z = -\frac{3}{2}\frac{2.5}{32.2}\left(\frac{9}{12}\right)\left(\frac{8}{12}\right)(6\pi\text{ rad/s}) = -1.0976\text{ ft}\cdot\text{lb}\cdot\text{s}$$

$$H_{y'} = -I_{y'z}\,\omega_z = +\frac{3}{2}mbc\,\omega_z = +1.0976\text{ ft}\cdot\text{lb}\cdot\text{s}$$

$$H_z = I_z\omega_z = \frac{10}{3}mb^2\omega_z = \frac{10}{3}\frac{2.5}{32.2}\left(\frac{9}{12}\right)^2(6\pi\text{ rad/s}) = 2.744\text{ ft}\cdot\text{lb}\cdot\text{s}$$

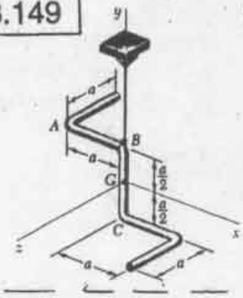
$$\underline{H}_A = -(1.098\text{ ft}\cdot\text{lb}\cdot\text{s})\underline{i} + (1.098\text{ ft}\cdot\text{lb}\cdot\text{s})\underline{j} + (2.74\text{ ft}\cdot\text{lb}\cdot\text{s})\underline{k}$$ ◀

(b) SINCE THE UNIT VECTOR OF AB IS $-\underline{k}$, AND RECALLING EQ. (3.32), WE HAVE

$$\cos\theta = \frac{\underline{H}_A\cdot(-\underline{k})}{|\underline{H}_A|} = \frac{-2.744}{\sqrt{2(1.0976)^2 + (2.744)^2}} = -\frac{2.744}{3.1526} = -0.87039$$

$$\theta = 150.5°$$ ◀

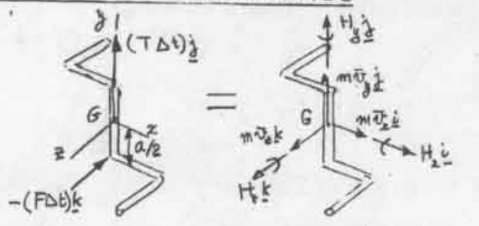
18.149



GIVEN:
ROD OF MASS m IS HIT AT C IN NEGATIVE z DIRECTION. IMPULSE $= -(F\Delta t)\,\mathbf{k}$.

FIND:
IMMEDIATELY AFTER IMPACT
(a) ANG. VELOCITY OF ROD,
(b) VELOCITY OF G.

IMPULSE-MOMENTUM PRINCIPLE



(WEIGHT IS OMITTED, SINCE NONIMPULSIVE)

(a) ANGULAR VELOCITY

EQUATE MOMENTS ABOUT G:

$$-\frac{a}{2}\mathbf{j} \times (-F\Delta t)\mathbf{k} = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$$

$$\frac{1}{2}aF\Delta t\,\mathbf{i} = H_x\mathbf{i} + H_y\mathbf{j} + H_z\mathbf{k}$$

THUS: $H_x = \frac{1}{2}aF\Delta t \qquad H_y = 0 \qquad H_z = 0$

MOMENTS AND PRODUCTS OF INERTIA:

$$\bar{I}_x = \frac{1}{12}\frac{m}{5}a^2 + 2\frac{m}{5}\left(\frac{a}{2}\right)^2 + 2\frac{m}{5}\left(\frac{1}{12}a^2 + 2\frac{a^2}{4}\right) = 0.35\,ma^2$$

$$\bar{I}_y = 2\frac{m}{5}\frac{a^2}{3} + 2\frac{m}{5}\left(\frac{1}{12}a^2 + a^2 + \frac{a^2}{4}\right) = \frac{2}{3}ma^2$$

$$\bar{I}_z = \frac{1}{12}\frac{m}{5}a^2 + 2\frac{m}{5}\left(\frac{1}{12}a^2 + 2\frac{a^2}{4}\right) + 2\frac{m}{5}\left(a^2 + \frac{a^2}{4}\right) = 0.75\,ma^2$$

$$\bar{I}_{xy} = \frac{m}{5}\left(-\frac{a}{2}\right)\left(\frac{a}{2}\right) + \frac{m}{5}(-a)\left(\frac{a}{2}\right) + \frac{m}{5}\left(\frac{a}{2}\right)\left(-\frac{a}{2}\right) + \frac{m}{5}(a)\left(-\frac{a}{2}\right) = -0.3\,ma^2$$

$$\bar{I}_{yz} = \frac{m}{5}\left(\frac{a}{2}\right)\left(-\frac{a}{2}\right) + \frac{m}{5}\left(-\frac{a}{2}\right)\left(\frac{a}{2}\right) = -0.1\,ma^2$$

$$\bar{I}_{zx} = \frac{m}{5}\left(-\frac{a}{2}\right)(-a) + \frac{m}{5}\left(\frac{a}{2}\right)(a) = 0.2\,ma^2$$

EQS. (18.17) AND DIVIDING BY ma^2:

$$H_x = \bar{I}_x\omega_x - \bar{I}_{xy}\omega_y - \bar{I}_{xz}\omega_z: \quad \frac{F\Delta t}{2ma} = 0.35\omega_x + 0.3\omega_y - 0.2\omega_z \qquad (1)$$

$$H_y = -\bar{I}_{xy}\omega_x + \bar{I}_y\omega_y - \bar{I}_{yz}\omega_z: \quad 0 = 0.3\omega_x + \frac{2}{3}\omega_y + 0.1\omega_z \qquad (2)$$

$$H_z = -\bar{I}_{zx}\omega_x - \bar{I}_{yz}\omega_y + \bar{I}_z\omega_z: \quad 0 = -0.2\omega_x + 0.1\omega_y + 0.75\omega_z \qquad (3)$$

SOLVING EQS. (1), (2), (3) SIMULTANEOUSLY:

$$\omega_x = \frac{30}{8}\frac{F\Delta t}{ma} \qquad \omega_y = -\frac{15}{8}\frac{F\Delta t}{ma} \qquad \omega_z = \frac{10}{8}\frac{F\Delta t}{ma}$$

THUS: $\boldsymbol{\omega} = \frac{F\Delta t}{8ma}(30\mathbf{i} - 15\mathbf{j} + 10\mathbf{k})$ ◀

(b) VELOCITY OF G

WE FIRST NOTE THAT THE GIVEN CONSTRAINTS REQUIRE THAT $\bar{v}_y = 0$. EQUATING THE COMPONENTS OF IMPULSE AND MOMENTUM:

x COMP.: $0 = m\bar{v}_x \qquad \bar{v}_x = 0$

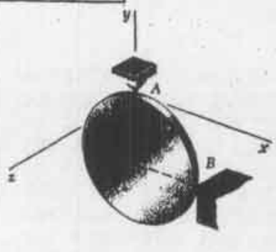
y COMP.: $T\Delta t = \bar{v}_y = 0 \qquad T\Delta t = 0$

z COMP.: $-F\Delta t = m\bar{v}_z \qquad \bar{v}_z = -\frac{F\Delta t}{m}$

THEREFORE:

$$\bar{\mathbf{v}} = -\frac{F\Delta t}{m}\mathbf{k}$$ ◀

18.150

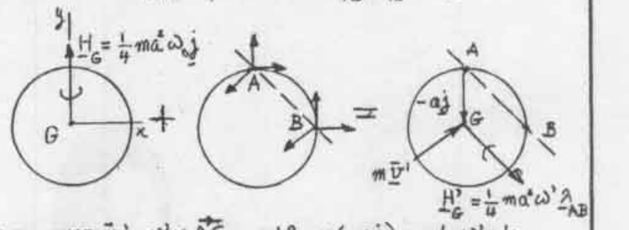


GIVEN:
DISK OF MASS m SUPPORTED BY BALL AND SOCKET AT A ROTATES WITH CONSTANT $\boldsymbol{\omega} = \omega_0\mathbf{j}$ WHEN OBSTRUCTION IS INTRODUCED AT B. IMPACT PERFECTLY PLASTIC ($e = 0$).

FIND:
IMMEDIATELY AFTER IMPACT
(a) ANGULAR VELOCITY OF DISK.
(b) VELOCITY OF G.

IMPULSE-MOMENTUM PRINCIPLE

WE NOTE THAT $\bar{I}_{DIAM} = \frac{1}{4}ma^2$ AND $\boldsymbol{\lambda}_{AB} = \frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j})$



WE NOTE THAT $\bar{\mathbf{v}}' = \boldsymbol{\omega}' \times \vec{AG} = \omega'\boldsymbol{\lambda}_{AB} \times (-a\mathbf{j}) = -\frac{1}{\sqrt{2}}\omega' a\mathbf{k}$

(a) EQUATE MOMENTS ABOUT AB OF ALL VECTORS AND COUPLES:

$$\boldsymbol{\lambda}_{AB}\cdot\mathbf{H}_G + 0 = \boldsymbol{\lambda}_{AB}\cdot(-a\mathbf{j} \times m\bar{\mathbf{v}}') + \boldsymbol{\lambda}_{AB}\cdot\mathbf{H}'_G$$

$$\frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j})\cdot\frac{1}{4}ma^2\omega_0\mathbf{j} = \frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j})\cdot\left[-a\mathbf{j} \times \left(-\frac{1}{\sqrt{2}}m\omega' a\mathbf{k}\right)\right] + \boldsymbol{\lambda}_{AB}\cdot\mathbf{H}'_G$$

$$-\frac{1}{4\sqrt{2}}ma^2\omega_0 = \frac{1}{2}ma^2\omega' + \frac{1}{4}ma^2\omega'$$

$$\omega' = -\frac{1}{3\sqrt{2}}\omega_0$$

$\boldsymbol{\omega}' = \omega'\boldsymbol{\lambda}_{AB} = -\frac{1}{3\sqrt{2}}\omega_0\frac{1}{\sqrt{2}}(\mathbf{i} - \mathbf{j}), \qquad \boldsymbol{\omega}' = \frac{1}{6}\omega_0(-\mathbf{i} + \mathbf{j})$ ◀

(b) RECALLING THAT $\bar{\mathbf{v}}' = \boldsymbol{\omega}' \times \vec{AG}$,

$\bar{\mathbf{v}}' = \frac{1}{6}\omega_0(-\mathbf{i} + \mathbf{j}) \times (-a\mathbf{j}) \qquad \bar{\mathbf{v}}' = \frac{1}{6}\omega_0 a\mathbf{k}$ ◀

18.151

GIVEN:
DISK OF PROB. 18.150

FIND:
KINETIC ENERGY LOST WHEN DISK HITS OBSTRUCTION.

BEFORE IMPACT:

$$T_0 = \frac{1}{2}\bar{I}_{DIAM}\omega_0^2 = \frac{1}{2}\frac{1}{4}ma^2\omega_0^2 = \frac{1}{8}ma^2\omega_0^2$$

AFTER IMPACT:

$$T' = \frac{1}{2}m\bar{v}'^2 + \frac{1}{2}\bar{I}_{DIAM}\omega'^2$$

BUT, FROM ANSWERS TO PROB. 18.150:

$$v'^2 = \left(\frac{1}{6}\omega_0 a\right)^2 = \frac{1}{36}\omega_0^2 a^2$$

$$\omega'^2 = \omega_x'^2 + \omega_y'^2 = \frac{\omega_0^2}{36}(1+1) = \frac{1}{18}\omega_0^2$$

THEREFORE:

$$T' = \frac{1}{2}m\left(\frac{1}{36}\omega_0^2 a^2\right) + \frac{1}{2}\frac{ma^2}{4}\left(\frac{1}{18}\omega_0^2\right) = \frac{1}{48}ma^2\omega_0^2$$

KINETIC ENERGY LOST

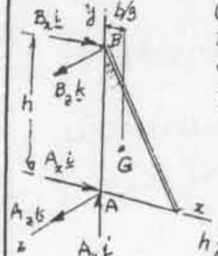
$$= T_0 - T' = \frac{1}{8}ma^2\omega_0^2 - \frac{1}{48}ma^2\omega_0^2$$

$$= \frac{5}{48}ma^2\omega_0^2$$ ◀

18.152

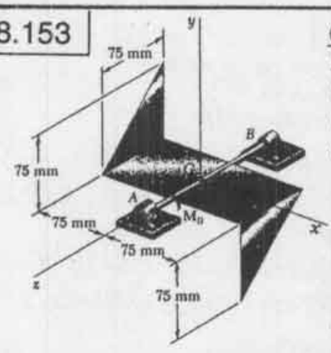
GIVEN:
TRIANGULAR PLATE OF MASS m WELDED TO SHAFT SUPPORTED BY BEARINGS AT A AND B.
PLATE ROTATES AT CONSTANT RATE ω.
FIND:
DYNAMIC REACTIONS AT A AND B.

COMPUTATION OF MOMENT AND PRODUCTS OF INERTIA
FROM BACK COVER:
$(I_y)_{AREA} = \frac{1}{12}b^3h,\quad A=\frac{1}{2}bh,\quad (I_y)_{MASS}=(I_y)_{AREA}\frac{m}{A}$
$(I_y)_{MASS} = \frac{1}{12}b^3h\left(\frac{m}{\frac{1}{2}bh}\right)=\frac{1}{6}mb^2$
FROM SAMPLE PROB. 9.6 (PAGE 485 OF STATICS):
$(I_{xy})_{AREA}=\frac{1}{24}b^2h^2,\quad (I_{xy})_{MASS}=(I_{xy})_{AREA}\frac{m}{A}=\frac{1}{24}b^2h^2\left(\frac{m}{\frac{1}{2}bh}\right)=\frac{1}{12}mbh$
WE ALSO NOTE THAT $I_{yz}=0$
ANGULAR MOMENTUM $\underline{H}_A$
SINCE $\omega_x=0,\ \omega_y=\omega,\ \omega_z=0$, EQS. (18.13) YIELD
$$H_x=-I_{xy}\omega_y=-\frac{1}{12}mbh\omega,\quad H_y=I_y\omega_y=\frac{1}{6}mb^2\omega,\quad H_z=0$$
$$\underline{H}_A = -\frac{1}{12}mbh\omega\underline{i} + \frac{1}{6}mb^2\omega\underline{j} \qquad (1)$$
EQUATIONS OF MOTION



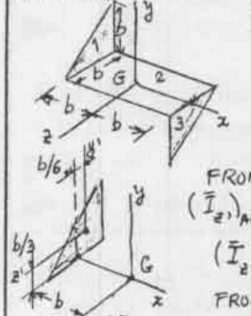
(WEIGHT OMITTED FOR DYNAMIC REACTIONS)
FRAME OF REFERENCE Axyz ROTATES WITH $\underline{\Omega}=\underline{\omega}=\omega\underline{j}$.
EQ. (18.28):
$$\Sigma\underline{M}_A = (\dot{\underline{H}}_A)_{Axyz} + \underline{\Omega}\times\underline{H}_A$$
$$= 0 + \omega\underline{j}\times\underline{H}_A$$
RECALLING (1) AND COMPUTING $\Sigma\underline{M}_A$:
$$h\underline{j}\times(B_x\underline{i}+B_z\underline{k})=\omega\underline{j}\times\left(-\frac{mbh}{12}\omega\underline{i}+\frac{mb^2}{6}\omega\underline{j}\right)$$
$$-hB_x\underline{k}+hB_z\underline{i}=\frac{1}{12}mbh\omega^2\underline{k}$$
EQUATING THE COEFF. OF THE UNIT VECTORS:
$$B_x=-\frac{1}{12}mb\omega^2 \qquad B_z=0$$
$$\underline{B}=-\frac{1}{12}mb\omega^2\underline{i} \quad ◄$$
ΣQ. (18.1):
$\Sigma\underline{F}=m\underline{\bar{a}}$ WHERE $\underline{\bar{a}}=-\bar{x}\omega^2\underline{i}=-\frac{1}{3}b\omega^2\underline{i}$
THUS:
$$\underline{A}+\underline{B}=-\frac{1}{3}b\omega^2\underline{i}$$
$$\underline{A}=-\frac{1}{3}b\omega^2\underline{i}-\left(-\frac{1}{12}mb\omega^2\underline{i}\right)$$
$$\underline{A}=-\frac{1}{4}mb\omega^2\underline{i} \quad ◄$$

18.153

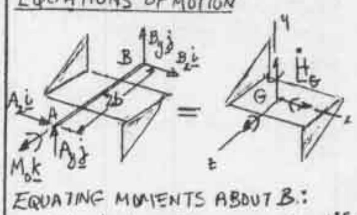


GIVEN:
SHEET-METAL COMPONENT OF MASS $m = 600$ g.
LENGTH AB = 150 mm.
COMPONENT AT REST WHEN $\underline{M}_0=(49.5\ \text{mN·m})\underline{k}$ IS APPLIED.
FIND:
DYNAMIC REACTIONS AT A AND B
(a) JUST AFTER COUPLE IS APPLIED
(b) 0.6 s LATER

$b=0.075$ m



MOMENT AND PRODUCTS OF INERTIA
RECTANGLE 2: MASS $=\frac{2}{3}m$
$$\bar{I}_z=\frac{1}{12}\left(\frac{2}{3}m\right)(2b)^2=\frac{2}{9}mb^2$$
$$I_{xz}=I_{yz}=0$$
TRIANGLE 1: MASS $=\frac{1}{6}m$
FROM BACK COVER:
$(\bar{I}_{z'})_{AREA}=\frac{1}{36}b^4,\ A=\frac{1}{2}b^2,\ (\bar{I}_{z'})_{MASS}=(\bar{I}_{z'})_{AREA}\frac{m/6}{A}$
$$(\bar{I}_{z'})_{MASS}=\frac{1}{36}b^4\left(\frac{\frac{1}{6}m}{\frac{1}{2}b^2}\right)=\frac{1}{108}mb^2$$
FROM SAMPLE PROB. 9.6 (PAGE 485 OF STATICS)
$$(\bar{I}_{y'z'})_{AREA}=-\frac{1}{72}b^4,\ (\bar{I}_{y'z'})_{MASS}=-\frac{1}{72}b^4\left(\frac{\frac{1}{6}m}{\frac{1}{2}b^2}\right)=-\frac{1}{216}mb^2$$
THEREFORE:
$$I_z=\bar{I}_{z'}+\frac{m}{6}d^2=\frac{1}{108}mb^2+\frac{m}{6}\left[b^2+\left(\frac{b}{3}\right)^2\right]=\left(\frac{1}{108}+\frac{10}{54}\right)mb^2=\frac{7}{36}mb^2$$
$$I_{yz}=\bar{I}_{y'z'}+\frac{m}{6}\bar{y}\bar{z}=-\frac{1}{216}mb^2+\frac{m}{6}\left(\frac{b}{3}\right)\left(-\frac{b}{6}\right)=-\frac{1}{72}mb^2$$
$$I_{xz}=\bar{I}_{x'z'}+\frac{m}{6}\bar{x}\bar{z}=0+\frac{m}{6}(-b)\left(-\frac{b}{6}\right)=\frac{1}{36}mb^2$$
TRIANGLE 3: BY SYMMETRY, SAME AS TRIANGLE 1.
FOR ENTIRE COMPONENT:
$$\bar{I}_z=\frac{2}{9}mb^2+2\left(\frac{7}{36}mb^2\right)=\frac{11}{18}mb^2$$
$$\bar{I}_{yz}=2\left(-\frac{1}{72}mb^2\right)=-\frac{1}{36}mb^2$$
$$\bar{I}_{xz}=2\left(\frac{1}{36}mb^2\right)=\frac{1}{18}mb^2$$
ANGULAR MOMENTUM $\underline{H}_G$
EQS. (18.7) WITH $\omega_x=\omega_y=0,\ \omega_z=\omega$:
$$H_x=-\bar{I}_{xz}\omega_z=-\frac{1}{18}mb^2\omega,\quad H_y=-\bar{I}_{yz}\omega_z=\frac{1}{36}mb^2\omega,\quad H_z=\bar{I}_z\omega_z=\frac{11}{18}mb^2\omega$$
THUS: $$\underline{H}_G=\frac{1}{36}mb^2\omega(-2\underline{i}+\underline{j}+22\underline{k}) \qquad (1)$$
EQUATIONS OF MOTION



EQ. (18.22), AND USING (1):
$$\dot{\underline{H}}_G=(\dot{\underline{H}}_G)_{Gxyz}+\underline{\Omega}\times\underline{H}_G$$
$$=\frac{1}{36}mb^2\alpha(-2\underline{i}+\underline{j}+22\underline{k})+\omega\underline{k}\times\frac{1}{36}mb^2\omega(-2\underline{i}+\underline{j}+22\underline{k})$$
EQUATING MOMENTS ABOUT B:
$$2b\underline{k}\times(A_x\underline{i}+A_y\underline{j})+M_0\underline{k}=\frac{1}{36}mb^2[(-2\underline{i}+\underline{j}+22\underline{k})\alpha-(2\underline{j}+\underline{i})\omega^2]$$
$$2bA_x\underline{j}-2bA_y\underline{i}+M_0\underline{k}=\frac{1}{36}mb^2[-(2\alpha+\omega^2)\underline{i}+(\alpha-2\omega^2)\underline{j}+22\alpha\underline{k}]$$
EQUATING THE COEFF. OF THE UNIT VECTORS:
(i) $-2bA_y=-\frac{1}{36}mb^2(2\alpha+\omega^2)$ $\qquad A_y=\frac{1}{72}mb(2\alpha+\omega^2)$ (2)
(j) $2bA_x=\frac{1}{36}mb^2(\alpha-2\omega^2)$ $\qquad A_x=\frac{1}{72}mb(\alpha-2\omega^2)$ (3)
(k) $M_0=\frac{11}{18}mb^2\alpha$ $\qquad \alpha=18M_0/11mb^2$ (4)
(CONTINUED)

18.153 continued

WE RECALL THE RESULTS OBTAINED:

$$A_y = \frac{1}{72} mb(2\alpha + \omega^2) \qquad (2)$$
$$A_z = \frac{1}{72} mb(\alpha - 2\omega^2) \qquad (3)$$
$$\alpha = 18 M_0 / 11 mb^2 \qquad (4)$$

WITH GIVEN DATA: $M_0 = 0.0495\ N\cdot m$, $m = 0.6\ kg$, $b = 0.075\ m$:

EQ (4): $\alpha = 18(0.0495)/11(0.6)(0.075)^2 = 24\ rad/s^2$

EQ. (3): $A_z = \frac{1}{72}(0.6)(0.075)(24 - 2\omega^2) = (15 - 1.25\omega^2)10^{-3} N \qquad (3')$

EQ. (2): $A_y = \frac{1}{72}(0.6)(0.075)(2 \times 24 + \omega^2) = (30 + 0.625\omega^2)10^{-3} N \qquad (2')$

(a) JUST AFTER COUPLE IS APPLIED:

LETTING $\omega = 0$ IN (3') AND (2'): $A_z = 15 \times 10^{-3} N$, $A_y = 30 \times 10^{-3} N$

THUS: $\underline{A} = (15.00\ mN)\underline{i} + (30.0\ mN)\underline{j}$ ◀

$\Sigma \underline{F} = m\underline{\bar{a}}$: $\underline{A} + \underline{B} = 0$, $\quad \underline{B} = -(15.00\ mN)\underline{i} - (30.0\ mN)\underline{j}$ ◀

(b) AFTER 0.6 s:

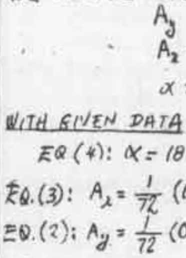
LETTING $\omega = \alpha t = (24\ rad/s^2)(0.6\ s) = 14.40\ rad/s$ IN (3') AND (2'):

$A_z = [15 - 1.25(14.40)^2]10^{-3} N = -244.2\ mN$, $A_y = [30 + 0.625(14.40)^2]10^{-3} N = 159.6\ mN$

THUS: $\underline{A} = -(244\ mN)\underline{i} + (159.6\ mN)\underline{j}$ ◀

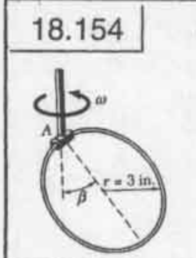
$\Sigma \underline{F} = m\underline{\bar{a}}$: $\underline{A} + \underline{B} = 0$, $\quad \underline{B} = (244\ mN)\underline{i} - (159.6\ mN)\underline{j}$ ◀

18.154



GIVEN:
RING ATTACHED BY COLLAR AT A TO VERTICAL SHAFT ROTATING AT CONSTANT RATE ω.

FIND:
(a) CONSTANT ANGLE β THAT PLANE OF RING FORMS WITH VERTICAL WHEN $\omega = 12\ rad/s$.
(b) MAX. VALUE OF ω FOR WHICH $\beta = 0$.

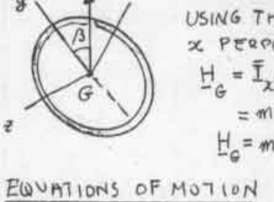


ANGULAR MOMENTUM $\underline{H}_G$

USING THE PRINCIPAL AXES Gxyz WITH x PERPENDICULAR TO PLANE OF RING:

$$\underline{H}_G = \bar{I}_x \omega_x \underline{i} + \bar{I}_y \omega_y \underline{j} + \bar{I}_z \omega_z \underline{k}$$
$$= mr^2\omega \sin\beta\, \underline{i} + \tfrac{1}{2} mr^2 \omega \cos\beta\, \underline{j}$$
$$\underline{H}_G = mr^2\omega(\sin\beta\, \underline{i} + \tfrac{1}{2}\cos\beta\, \underline{j}) \qquad (1)$$

EQUATIONS OF MOTION



$\bar{x} = r\sin\beta$
$\bar{y} = r\cos\beta$

EQ. (18.22) AND USING (1):

$$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega} \times \underline{H}_G$$
$$\dot{\underline{H}}_G = 0 + \underline{\omega} \times \underline{H}_G$$
$$= \omega(\sin\beta\, \underline{i} + \cos\beta\, \underline{j}) \times mr^2\omega(\sin\beta\, \underline{i} + \tfrac{1}{2}\cos\beta\, \underline{j})$$
$$= mr^2\omega^2(\tfrac{1}{2}\sin\beta\cos\beta - \sin\beta\cos\beta)\underline{k}$$
$$= -\tfrac{1}{2} mr^2\omega^2 \sin\beta\cos\beta\, \underline{k}$$

EQUATING MOMENTS ABOUT A:

$+\circlearrowleft\ W\bar{x} = (m\bar{x}\omega^2)\bar{y} + \dot{H}_G$

$$mg\, r\sin\beta = m(r\sin\beta)\omega^2(r\cos\beta) + \tfrac{1}{2} mr^2\omega^2 \sin\beta\cos\beta$$
$$g = \omega^2 r\cos\beta + \tfrac{1}{2}\omega^2 r\cos\beta$$
$$\cos\beta = \frac{2}{3}\frac{g}{r\omega^2} \qquad (2)$$

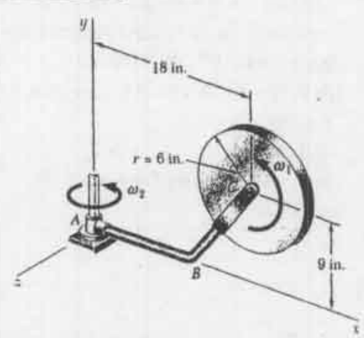
(a) LETTING $g = 32.2\ ft/s^2$, $r = 0.25\ ft$, $\omega = 12\ rad/s$:

$$\cos\beta = \frac{2}{3}\frac{32.2}{(0.25)(12)^2} = 0.59630 \qquad \beta = 53.4° \blacktriangleleft$$

(b) SOLVING EQ. (2) FOR ω^2 AND LETTING $g = 32.2\ ft/s^2$, $r = 0.25\ ft$, $\beta = 0$:

$$\omega^2 = \frac{2g}{3r} = \frac{2(32.2)}{3(0.25)} = 85.87 \qquad \omega = 9.27\ rad/s \blacktriangleleft$$

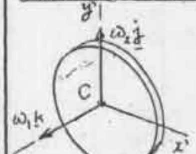
18.155



GIVEN:
10-lb DISK ROTATES AT CONSTANT RATE $\omega_1 = 15\ rad/s$.
ARM ABC ROTATES AT CONSTANT RATE $\omega_2 = 5\ rad/s$.

FIND:
FORCE-COUPLE SYSTEM REPRESENTING THE DYNAMIC REACTION AT SUPPORT A.

ANGULAR MOMENTUM OF DISK ABOUT C.



USING THE PRINCIPAL CENTROIDAL AXES $Cx'y'z'$:

$$\underline{\omega} = \omega_2 \underline{j} + \omega_1 \underline{k}$$

ANG. VELOCITY OF FRAME $Cx'y'z'$:

$$\underline{\Omega} = \omega_2 \underline{j}$$
$$\underline{H}_C = \bar{I}_{x'}\omega_{x'}\underline{i} + \bar{I}_{y'}\omega_{y'}\underline{j} + \bar{I}_{z'}\omega_{z'}\underline{k}$$
$$= 0 + \tfrac{1}{4}mr^2\omega_2\underline{j} + \tfrac{1}{2}mr^2\omega_1\underline{k}$$
$$\underline{H}_C = \tfrac{1}{4}mr^2(\omega_2\underline{j} + 2\omega_1\underline{k}) \qquad (1)$$

RATE OF CHANGE OF $\underline{H}_C$

EQ. (18.22) AND USING (1):

$$\dot{\underline{H}}_C = (\dot{\underline{H}}_C)_{Cx'y'z'} + \underline{\Omega} \times \underline{H}_C = 0 + \omega_2\underline{j} \times \tfrac{1}{4}mr^2(\omega_2\underline{j} + 2\omega_1\underline{k})$$
$$\dot{\underline{H}}_C = \tfrac{1}{2}mr^2\omega_1\omega_2\underline{i}$$

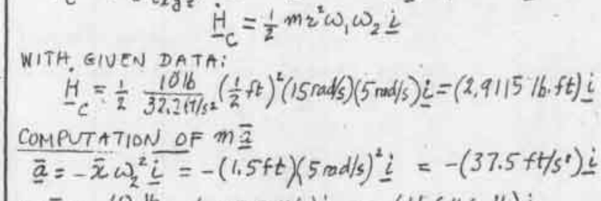
WITH GIVEN DATA:

$$\dot{\underline{H}}_C = \frac{1}{2}\frac{10\ lb}{32.2\ ft/s^2}\left(\frac{1}{2}ft\right)^2(15\ rad/s)(5\ rad/s)\underline{i} = (2.9115\ lb\cdot ft)\underline{i}$$

COMPUTATION OF $m\underline{\bar{a}}$

$$\underline{\bar{a}} = -\bar{x}\omega_2^2\underline{i} = -(1.5\ ft)(5\ rad/s)^2\underline{i} = -(37.5\ ft/s^2)\underline{i}$$
$$m\underline{\bar{a}} = \frac{10\ lb}{32.2\ ft/s^2}(-37.5\ ft/s^2)\underline{i} = -(11.646\ lb)\underline{i}$$

EQUATIONS OF MOTION



$\Sigma \underline{F} = \Sigma \underline{F}_{eff}$: $\underline{R}_A = m\underline{\bar{a}} = -(11.646\ lb)\underline{i}$

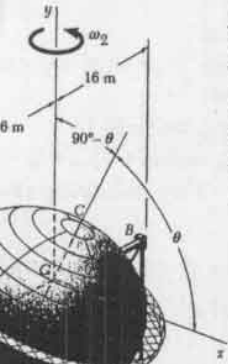
$\Sigma \underline{M}_A = \Sigma (\underline{M}_A)_{eff}$:

$$\underline{M}^R_A = \dot{\underline{H}}_C + (\bar{x}\underline{i} + \bar{y}\underline{j}) \times m\underline{\bar{a}}$$
$$= (2.9115\ lb\cdot ft)\underline{i} + [(1.5\ ft)\underline{i} + (0.75\ ft)\underline{j}] \times (-11.646\ lb)\underline{i}$$
$$= (2.9115\ lb\cdot ft)\underline{i} + (8.734\ lb\cdot ft)\underline{k}$$

FORCE-COUPLE SYSTEM AT A:

$$\underline{R}_A = -(11.65\ lb)\underline{i}\ ; \quad \underline{M}^R_A = (2.91\ lb\cdot ft)\underline{i} + (8.73\ lb\cdot ft)\underline{k} \blacktriangleleft$$

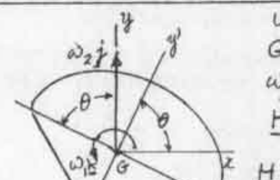
18.156



GIVEN:
SOLAR-ENERGY CONCENTRATOR: $m = 30$ Mg
RADII OF GYRATION
ABOUT CD: $\bar{k} = 12$ m
ABOUT AB: $\bar{k}' = 10$ m
$\omega_1 = 0.20$ rad/s (constant)
$\omega_2 = 0.25$ rad/s (constant)

FIND FOR $\theta = 60°$:
(a) FORCES EXERTED ON CONCENTRATOR AT A AND B,
(b) COUPLE $M_z \underline{k}$ APPLIED TO CONCENTRATOR AT THAT INSTANT.

ANGULAR MOMENTUM ABOUT G



USING THE PRINCIPAL AXES $Gx'y'z$.

$\omega_{x'} = -\omega_2 \cos\theta, \ \omega_{y'} = \omega_2 \sin\theta, \ \omega_z = \omega_1$

$\underline{H}_G = \bar{I}_{x'}\omega_{x'}\underline{i}' + \bar{I}_{y'}\omega_{y'}\underline{j}' + \bar{I}_z\omega_z\underline{k}$

$\underline{H}_G = -I'\omega_2\cos\theta\,\underline{i}' + I\omega_2\sin\theta\,\underline{j}' + I'\omega_1\underline{k}$

WHERE $I = m\bar{k}^2$ AND $I' = m\bar{k}'^2$

WE NOW RETURN TO THE REFERENCE FRAME $Gxyz$ ATTACHED TO THE STEEL FRAMEWORK ($\underline{\Omega} = \omega_2 \underline{j}$).

$\underline{H}_G = -I'\omega_2\cos\theta(\underline{i}\sin\theta - \underline{j}\cos\theta) + I\omega_2\sin\theta(\underline{i}\cos\theta + \underline{j}\sin\theta) + I'\omega_1\underline{k}$

$$\underline{H}_G = (I - I')\omega_2 \sin\theta\cos\theta\,\underline{i} + (I'\cos^2\theta + I\sin^2\theta)\omega_2\,\underline{j} + I'\omega_1\underline{k} \quad (1)$$

RATE OF CHANGE OF $\underline{H}_G$

WE NOTE THAT ω_1 AND ω_2 ARE CONSTANT, BUT THAT θ IS A FUNCTION OF t WITH DERIVATIVE $\dot{\theta} = \omega_1$

EQ. (18.22):

$\dot{\underline{H}}_G = (\dot{\underline{H}}_G)_{Gxyz} + \underline{\Omega}\times\underline{H}_G = (I - I')\omega_2(\cos^2\theta - \sin^2\theta)\dot{\theta}\,\underline{i} +$

$+ 2(I - I')\omega_2 \sin\theta\cos\theta\,\dot{\theta}\,\underline{j} + \omega_2\underline{j}\times[(I - I')\omega_2\sin\theta\cos\theta\,\underline{i} +$

$+ (I'\cos^2\theta + I\sin^2\theta)\omega_2\,\underline{j} + I'\omega_1\underline{k}]$

$= (I - I')\omega_1\omega_2(\cos 2\theta\,\underline{i} + \sin 2\theta\,\underline{j}) - \frac{1}{2}(I - I')\omega_2^2\sin 2\theta\,\underline{k} + I'\omega_1\omega_2\,\underline{i}$

$\dot{\underline{H}}_G = [I' + (I - I')\cos 2\theta]\omega_1\omega_2\,\underline{i} + (I - I')\omega_1\omega_2\sin 2\theta\,\underline{j} - \frac{1}{2}(I - I')\omega_2^2\sin 2\theta\,\underline{k}$

WITH GIVEN DATA: $I = m\bar{k}^2 = (30\times10^3\text{ kg})(12\text{ m})^2 = 4.32\times10^6\text{ kg}\cdot\text{m}^2$

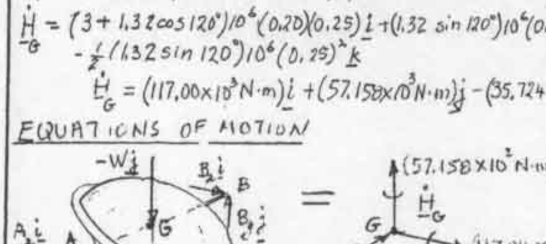
$I' = m\bar{k}'^2 = (30\times10^3\text{ kg})(10\text{ m})^2 = 3.00\times10^6\text{ kg}\cdot\text{m}^2$

$\omega_1 = 0.20$ rad/s, $\omega_2 = 0.25$ rad/s, $\theta = 60°$, $2\theta = 120°$

$\dot{\underline{H}}_G = (3 + 1.32\cos 120°)10^6(0.20)(0.25)\,\underline{i} + (1.32\sin 120°)10^6(0.20)(0.25)\,\underline{j} -$
$- \frac{1}{2}(1.32\sin 120°)10^6(0.25)^2\,\underline{k}$

$$\dot{\underline{H}}_G = (117.00\times10^3\text{ N}\cdot\text{m})\underline{i} + (57.158\times10^3\text{ N}\cdot\text{m})\underline{j} - (35.724\times10^3\text{ N}\cdot\text{m})\underline{k} \quad (2)$$

EQUATIONS OF MOTION



WHERE $W = (30\times10^3\text{ kg})(9.81\text{ m/s}^2) = 294.3\times10^3$ N

(CONTINUED)

18.156 continued

$\Sigma\underline{M}_B = \Sigma(\underline{M}_B)_{eff}$:

$(32\text{ m})\underline{k}\times(A_x\underline{i} + A_y\underline{j}) + (16\text{ m})\underline{k}\times(-294.3\times10^3\text{ N})\underline{j} + M_z\underline{k} = \dot{\underline{H}}_G$

$(32\text{ m})A_x\underline{j} - (32\text{ m})A_y\underline{i} + (16\text{ m})(294.3\times10^3\text{ N})\underline{i} + M_z\underline{k} =$
$= (117\times10^3\text{ N}\cdot\text{m})\underline{i} + (57.158\times10^3\text{ N}\cdot\text{m})\underline{j} - (35.724\times10^3\text{ N}\cdot\text{m})\underline{k}$

EQUATING THE COEFF. OF THE UNIT VECTORS:

($\underline{i}$) $-(32\text{ m})A_y + (16\text{ m})(294.3\times10^3\text{ N}) = 117\times10^3$ N·m $\quad A_y = 143.49\times10^3$ N

($\underline{j}$) $(32\text{ m})A_x = 57.158\times10^3$ N·m $\quad A_x = 1.7862\times10^3$ N

($\underline{k}$) $M_z = -35.724\times10^3$ N·m

(a) FORCES AT A AND B

$\underline{A} = A_x\underline{i} + A_y\underline{j} \qquad \underline{A} = (1.786\text{ kN})\underline{i} + (143.5\text{ kN})\underline{j}$ ◀

$\Sigma\underline{F} = m\bar{\underline{a}} = 0: \ \underline{A} + \underline{B} - W\underline{j} = 0$

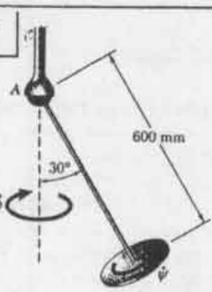
$\underline{B} = (294.3\text{ kN})\underline{j} - (1.786\text{ kN})\underline{i} - (143.5\text{ kN})\underline{j}$

$\underline{B} = -(1.786\text{ kN})\underline{i} + (150.8\text{ kN})\underline{j}$ ◀

(b) COUPLE $M_z\underline{k}$

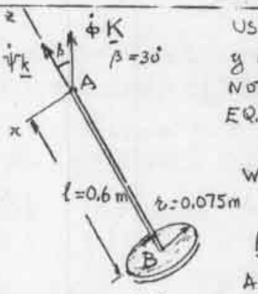
$M_z\underline{k} = -(35.724\times10^3\text{ N}\cdot\text{m})\underline{k} \qquad M_z\underline{k} = -(35.7\text{ kN}\cdot\text{m})\underline{k}$ ◀

18.157



GIVEN:
2-kg DISK OF 150-mm DIAMETER ATTACHED TO ROD SUPPORTED BY BALL AND SOCKET AT A.
$\dot{\phi} = 36$ rpm AS SHOWN

FIND:
RATE OF SPIN $\dot{\psi}$



USING THE FRAME $Axyz$ (WITH THE y AXIS POINTING TOWARD US), AND NOTING THAT THE PRECESSION IS STEADY, EQ. (18.43) YIELDS

$$\Sigma\underline{M}_A = \underline{\Omega}\times\underline{H}_A \quad (1)$$

WHERE

$\underline{H}_A = I_x\omega_x\underline{i} + I_z\omega_z\underline{k}$

$\underline{H}_A = I'(-\dot{\phi}\sin\beta)\underline{i} + I(\dot{\psi} + \dot{\phi}\cos\beta)\underline{k}$

AND $\underline{\Omega} = -\dot{\phi}\sin\beta\,\underline{i} + \dot{\phi}\cos\beta\,\underline{k}$

THUS: $\Sigma\underline{M}_A = (-\dot{\phi}\sin\beta\,\underline{i} + \dot{\phi}\cos\beta\,\underline{k})\times[I'(-\dot{\phi}\sin\beta)\underline{i} + I(\dot{\psi} + \dot{\phi}\cos\beta)\underline{k}]$

$$\Sigma\underline{M}_A = [I(\dot{\psi} + \dot{\phi}\cos\beta) - I'\dot{\phi}\cos\beta]\dot{\phi}\sin\beta\,\underline{j} \quad (2)$$

BUT $\Sigma\underline{M}_A = \vec{AB}\times -mg\underline{K} = -\ell\underline{k}\times -mg(-\sin\beta\,\underline{i} + \cos\beta\,\underline{k})$

$$= -mg\ell\sin\beta\,\underline{j} \quad (3)$$

EQUATING THE R.H. MEMBERS OF (2) AND (3):

$[I(\dot{\psi} + \dot{\phi}\cos\beta) - I'\dot{\phi}\cos\beta]\dot{\phi}\sin\beta = -mg\ell\sin\beta$

$[I\dot{\psi} + (I - I')\dot{\phi}\cos\beta]\dot{\phi} = -mg\ell$

$$\dot{\psi} = \frac{I' - I}{I}\dot{\phi}\cos\beta - \frac{mg\ell}{I\dot{\phi}} \quad (4)$$

FROM GIVEN DATA: $I = \frac{1}{2}mr^2 = \frac{1}{2}(2\text{ kg})(0.075\text{ m})^2 = 5.625\times10^{-3}$

$I' = m(\ell^2 + \frac{r^2}{4}) = 2[(0.6)^2 + \frac{1}{4}(0.075)^2] = 0.72281, \quad \frac{I' - I}{I} = 127.5$

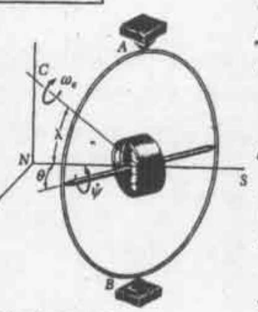
$\dot{\phi} = -36$ rpm $= -1.2\pi$ rad/s, $\beta = 30°$

EQ. (4): $\dot{\psi} = (127.5)(-1.2\pi)\cos 30° - \dfrac{2(9.81)(0.6)}{(5.625\times10^{-3})(-1.2\pi)}$

$= -416.27 + 555.13 = 138.86$ rad/s

$\dot{\psi} = 1326$ rpm ◀

18.158



GIVEN:
GYROCOMPASS CONSISTING OF ROTOR SPINNING AT RATE $\dot\psi$ ABOUT AXIS MOUNTED IN GIMBAL ROTATING FREELY ABOUT VERTICAL AB.
θ = ANGLE FORMED BY AXIS OF ROTOR AND MERIDIAN NS.
λ = LATITUDE = ANGLE FORMED BY NS AND LINE OC PARALLEL TO EARTH AXIS
$\underline{\omega}_e$ = ANG. VELOCITY OF EARTH ABOUT ITS AXIS.

SHOW THAT
(a) THE EQUATIONS OF MOTION OF THE GYROCOMPASS ARE

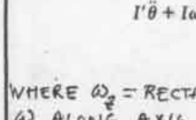
$$I'\ddot\theta + I\omega_z\omega_e\cos\lambda\sin\theta - I'\omega_e^2\cos^2\lambda\sin\theta\cos\theta = 0$$
$$I\dot\omega_z = 0$$

WHERE ω_z = RECTANGULAR COMPONENT OF TOTAL ANG. VELOCITY $\underline{\omega}$ ALONG AXIS OF ROTOR
(b) NEGLECTING TERMS IN ω_e^2 AND FOR SMALL VALUES OF θ,

$$\ddot\theta + \frac{I\omega_z\omega_e\cos\lambda}{I'}\theta = 0$$

AND THAT AXIS OF ROTOR OSCILLATES ABOUT THE LINE NS.

(a) ANGULAR MOMENTUM ABOUT O.



WE SELECT A FRAME OF REFERENCE Oxyz ATTACHED TO THE GIMBAL.
THE ANG. VELOCITY OF Oxyz WITH RESPECT TO A NEWTONIAN FRAME IS
$$\underline{\Omega} = \omega_e\underline{K} + \dot\theta\underline{j}$$
WHERE
$$\underline{K} = -\cos\lambda\sin\theta\,\underline{i} + \sin\lambda\,\underline{j} + \cos\lambda\cos\theta\,\underline{k}$$
THUS: $\underline{\Omega} = -\omega_e\cos\lambda\sin\theta\,\underline{i} + (\dot\theta + \omega_e\sin\lambda)\underline{j} + \omega_e\cos\lambda\cos\theta\,\underline{k}$ (1)

THE ANG. VELOCITY $\underline{\omega}$ OF THE ROTOR IS OBTAINED BY ADDING ITS SPIN $\dot\psi\underline{k}$ TO $\underline{\Omega}$. SETTING $\dot\psi + \omega_e\cos\lambda\cos\theta = \omega_z$, WE HAVE
$$\underline{\omega} = -\omega_e\cos\lambda\sin\theta\,\underline{i} + (\dot\theta + \omega_e\sin\lambda)\underline{j} + \omega_z\underline{k} \quad (2)$$
THE ANG. MOMENTUM $\underline{H}_O$ OF THE ROTOR IS
$$\underline{H}_O = I_x\omega_x\underline{i} + I_y\omega_y\underline{j} + I_z\omega_z\underline{k}$$
WHERE $I_x = I_y = I'$ AND $I_z = I$. RECALLING (2) WE WRITE
$$\underline{H}_O = -I'\omega_e\cos\lambda\sin\theta\,\underline{i} + I'(\dot\theta + \omega_e\sin\lambda)\underline{j} + I\omega_z\underline{k} \quad (3)$$

EQUATIONS OF MOTION
EQ. (18.28): $\Sigma\underline{M}_O = (\dot{\underline{H}}_O)_{Oxyz} + \underline{\Omega}\times\underline{H}_O$ OR, FROM (1) & (3):
$$\Sigma\underline{M}_O = -I'\omega_e\cos\lambda\cos\theta\,\dot\theta\,\underline{i} + I'\ddot\theta\underline{j} + I\dot\omega_z\underline{k} + \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -\omega_e\cos\lambda\sin\theta & \dot\theta + \omega_e\sin\lambda & \omega_e\cos\lambda\cos\theta \\ -I'\omega_e\cos\lambda\sin\theta & I'(\dot\theta + \omega_e\sin\lambda) & I\omega_z \end{vmatrix} \quad (4)$$

WE OBSERVE THAT THE ROTOR IS FREE TO SPIN ABOUT THE z AXIS AND FREE TO ROTATE ABOUT THE y AXIS. THEREFORE THE y AND z COMPONENTS OF $\Sigma\underline{M}_O$ MUST BE ZERO. IT FOLLOWS THAT THE COEFFICIENTS OF $\underline{j}$ AND $\underline{k}$ IN THE R.H. MEMBER OF EQ. (4) MUST ALSO BE ZERO.

(CONTINUED)

18.158 continued

SETTING THE COEFF. OF $\underline{j}$ IN THE R.H. MEMBER OF EQ. (4) EQUAL TO ZERO:
$$I'\ddot\theta + (-I'\omega_e\cos\lambda\sin\theta)(\omega_e\cos\lambda\cos\theta) - (-\omega_e\cos\lambda\sin\theta)I\omega_z = 0$$
$$I'\ddot\theta + I\omega_z\omega_e\cos\lambda\sin\theta - I'\omega_e^2\cos^2\lambda\sin\theta\cos\theta = 0 \quad (5)$$
(Q.E.D.)

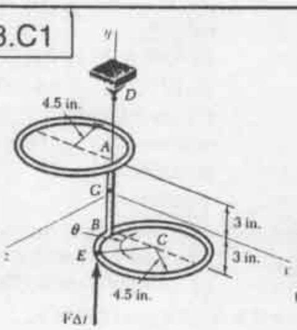
SETTING THE COEFF. OF $\underline{k}$ EQUAL TO ZERO:
$$I\dot\omega_z + (-\omega_e\cos\lambda\sin\theta)I'(\dot\theta + \omega_e\sin\lambda) - (-I'\omega_e\cos\lambda\sin\theta)(\dot\theta + \omega_e\sin\lambda) = 0$$
OBSERVING THAT THE LAST TWO TERMS CANCEL OUT, WE HAVE
$$I\dot\omega_z = 0 \qquad \text{(Q.E.D.)} \quad (6)$$

(b) IT FOLLOWS FROM EQ. (6) THAT
$$\omega_z = \text{CONSTANT} \quad (7)$$
REWRITE EQ. (5) AS FOLLOWS:
$$I'\ddot\theta + (I\omega_z - I'\omega_e\cos\lambda\cos\theta)\omega_e\cos\lambda\sin\theta = 0$$
IT IS EVIDENT THAT $\omega_z \ggg \omega_e$. WE CAN THEREFORE NEGLECT THE SECOND TERM IN THE PARENTHESIS AND WRITE
$$I'\ddot\theta + I\omega_z\omega_e\cos\lambda\sin\theta = 0$$
OR
$$\ddot\theta + \frac{I\omega_z\omega_e\cos\lambda}{I'}\sin\theta = 0 \quad (8)$$
WHERE THE COEFFICIENT OF $\sin\theta$ IS A CONSTANT. THE ROTOR, THEREFORE, OSCILLATES ABOUT THE LINE NS AS A SIMPLE PENDULUM.
FOR SMALL OSCILLATIONS, $\sin\theta \approx \theta$, AND EQ. (8) YIELDS
$$\ddot\theta + \frac{I\omega_z\omega_e\cos\lambda}{I'}\theta = 0 \qquad \text{(Q.E.D.)} \quad (9)$$

EQ. (9) IS THE EQUATION OF SIMPLE HARMONIC MOTION WITH PERIOD
$$\tau = 2\pi\sqrt{\frac{I'}{I\omega_z\omega_e\cos\lambda}} \quad (10)$$

SINCE ITS ROTOR OSCILLATES ABOUT THE LINE NS, THE GYROCOMPASS CAN BE USED TO DETERMINE THE DIRECTION OF THAT LINE. WE SHOULD NOTE, HOWEVER THAT FOR VALUES OF λ CLOSE TO 90° OR −90°, THE PERIOD OF OSCILLATION BECOMES VERY LARGE AND THE LINE ABOUT WHICH THE ROTOR OSCILLATES CANNOT BE DETERMINED. THE GYROCOMPASS, THEREFORE, CANNOT BE USED IN THE POLAR REGIONS.

18.C1



GIVEN:
FIGURE SHOWN MADE OF WIRE WEIGHING $\frac{5}{8}$ OZ/FT IS SUSPENDED FROM CORD AB. IMPULSE $\underline{F}\Delta t = (0.5\ \text{lb}\cdot\text{s})\underline{j}$ IS APPLIED AT E

FIND IMMEDIATELY AFTER IMPACT, FOR VALUES OF θ FROM 0 TO 180° IN 10° INCREMENTS
(a) VELOCITY OF G.
(b) ANGULAR VELOCITY

ANALYSIS

LET m' = MASS PER UNIT LENGTH
$2a$ = LENGTH OF ROD AB
r = RADIUS OF EACH RING

COMPUTATION OF MASSES:

AB: $m_{AB} = 2a\,m'$ (1)

EACH RING: $m_R = 2\pi r m'$ (2)

ENTIRE FIGURE: $m = m_{AB} + 2m_R$ (3)

MOMENTS OF INERTIA:

AB: $(I_x)_{AB} = (I_z)_{AB} = \frac{1}{3} m_{AB} a^2,\ (I_y)_{AB} = 0$ (4)

EACH RING: $(I_x)_R = \frac{1}{2} m_R r^2 + m_R a^2 = m_R\left(\frac{1}{2}r^2 + a^2\right)$ (5)

$(I_y)_R = m_R r^2 + m_R r^2 = 2m_R r^2$ (6)

$(I_z)_R = \frac{1}{2} m_R r^2 + m_R(r^2 + a^2) = m_R\left(\frac{3}{2}r^2 + a^2\right)$ (7)

ENTIRE FIGURE:

$I_x = (I_x)_{AB} + 2(I_x)_R,\ I_y = 2(I_y)_R,\ I_z = (I_z)_{AB} + 2(I_z)_R$ (8)

PRODUCTS OF INERTIA:

THE ONLY NON-ZERO PRODUCTS OF INERTIA ARE $(I_{xy})_R$

$I_{xy} = 2(I_{xy})_R = -2m_R\, r a$ (9)

IMPULSE-MOMENTUM PRINCIPLE:

EQUATING IMPULSE AND MOMENTUM AFTER IMPACT

$\underline{F}\Delta t = m\bar{\underline{v}}$: $\quad (F\Delta t)\underline{j} = m\bar{\underline{v}}$

$\bar{\underline{v}} = \frac{F\Delta t}{m}\underline{j}$ (FOR ALL VALUES OF θ) (10)

EQUATING MOMENT OF IMPULSE ABOUT G AND ANGULAR MOMENTUM $\underline{H}_G$ AFTER IMPACT (NOTE THAT THERE IS NO IMPULSIVE FORCE EXCEPT $\underline{F}$)

$\underline{r}_E \times F\Delta t\,\underline{j} = \underline{H}_G$

$\underline{H}_G = [r(1-\cos\theta)\underline{i} - a\underline{j} + r\sin\theta\,\underline{k}] \times F\Delta t\,\underline{j}$

$= -r F\Delta t \sin\theta\,\underline{i} + r F\Delta t(1-\cos\theta)\underline{k}$

THUS: $H_x = -r F\Delta t \sin\theta,\ H_y = 0,\ H_z = r F\Delta t(1-\cos\theta)$ (11)

USING EQS. (18.7) AND RECALLING THAT $I_{yz} = I_{zx} = 0$:

$I_x\omega_x - I_{xy}\omega_y = H_x$ (12)

$-I_{xy}\omega_x + I_y\omega_y = 0$ (13)

$I_z\omega_z = H_z$ (14)

(CONTINUED)

18.C1 continued

SOLVING EQS. (12) AND (13) SIMULTANEOUSLY FOR ω_x AND ω_y, AND EQ. (14) FOR ω_z, WE OBTAIN

$$\omega_x = \frac{I_y H_x}{I_x I_y - I_{xy}^2},\quad \omega_y = \frac{I_{xy} H_x}{I_x I_y - I_{xy}^2},\quad \omega_z = \frac{H_z}{I_z} \qquad (15)$$

OUTLINE OF PROGRAM

ENTER $m' = \frac{[(5/8)/16]\ \text{lb}}{32.2\ \text{ft/s}^2}$, $a = \frac{3}{12}$ ft, $r = \frac{4.5}{12}$ ft, $F\Delta t = 0.5$ lb·s

COMPUTE m_{AB}, m_R, AND m FROM EQS. (1), (2), AND (3)

COMPUTE $(I_x)_{AB}$ AND $(I_z)_{AB}$ FROM EQS. (4)

COMPUTE $(I_x)_R$, $(I_y)_R$, AND $(I_z)_R$ FROM EQS. (5), (6), AND (7)

COMPUTE I_x, I_y, AND I_z FROM EQS. (8) AND I_{xy} FROM EQ. (9)

COMPUTE $\bar{v} = F\Delta t/m$ AND PRINT

FOR $\theta = 0$ TO $\theta = 180°$ AND USING 10° INCREMENTS:

CALCULATE H_x AND H_z FROM EQS. (11)

CALCULATE ω_x, ω_y, AND ω_z FROM EQS. (15) AND TABULATE

PROGRAM OUTPUT

(a)

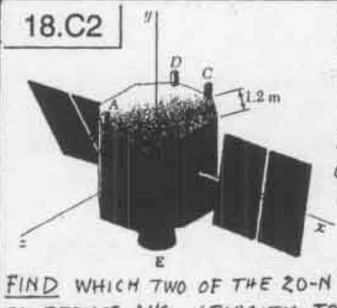
```
Velocity of mass center
vbar = 79.07 ft/s (directed upward)
```

(b)

Angular velocity

Theta degrees	(Omega)x rad/s	(Omega)y rad/s	(Omega)z rad/s
0.00	0.00	0.00	0.00
10.00	-54.88	18.29	1.81
20.00	-108.10	36.03	7.18
30.00	-158.03	52.68	15.94
40.00	-203.16	67.72	27.84
50.00	-242.12	80.71	42.50
60.00	-273.72	91.24	59.49
70.00	-297.00	99.00	78.29
80.00	-311.26	103.75	98.33
90.00	-316.06	105.35	118.99
100.00	-311.26	103.75	139.65
110.00	-297.00	99.00	159.68
120.00	-273.72	91.24	178.48
130.00	-242.12	80.71	195.47
140.00	-203.16	67.72	210.14
150.00	-158.03	52.68	222.03
160.00	-108.10	36.03	230.80
170.00	-54.88	18.29	236.17
180.00	0.00	-0.00	237.97

18.C2



GIVEN:
PROBE WITH $m = 2500$ kg, $k_x = 0.98$ m, $k_y = 1.06$ m, $k_z = 1.02$ m.
500-N MAIN THRUSTER E;
20-N THRUSTERS A, B, C, D CAN EXPEL FUEL IN y DIRECTION.
PROBE HAS ANG. VELOCITY $\underline{\omega} = \omega_x \underline{i} + \omega_z \underline{k}$

FIND WHICH TWO OF THE 20-N THRUSTERS SHOULD BE USED TO REDUCE ANG. VELOCITY TO ZERO AND FOR HOW LONG EACH OF THEM SHOULD BE ACTIVATED, ASSUMING

(a) $\underline{\omega} = (0.040 \text{ rad/s})\underline{i} + (0.060 \text{ rad/s})\underline{k}$, AS IN PROB. 18.33,
(b) $\underline{\omega} = (0.060 \text{ rad/s})\underline{i} - (0.040 \text{ rad/s})\underline{k}$, AS IN PROB. 18.34,
(c) $\underline{\omega} = (0.060 \text{ rad/s})\underline{i} + (0.020 \text{ rad/s})\underline{k}$,
(d) $\underline{\omega} = -(0.060 \text{ rad/s})\underline{i} - (0.020 \text{ rad/s})\underline{k}$.

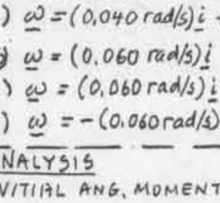
ANALYSIS

INITIAL ANG. MOMENTUM:

$$\underline{H}_G = I_x \omega_x \underline{i} + 0 + I_z \omega_z \underline{k} = m k_x^2 \omega_x \underline{i} + m k_z^2 \omega_z \underline{k}$$

THUS $H_x = m k_x^2 \omega_x \quad H_y = 0 \quad H_z = m k_z^2 \omega_z$ (1)

ANGULAR IMPULSE OF TWO 20-N THRUSTERS:



LET US ASSUME THAT A AND B ARE ACTIVATED.
ANG. IMPULSE ABOUT G

$$= \underline{r}_A \times (-F\Delta t_A)\underline{j} + \underline{r}_B \times (-F\Delta t_B)\underline{j}$$
$$= (-0.5a\underline{i} + 1.2071a\underline{k}) \times (-F\Delta t_A)\underline{j} + (0.5a\underline{i} + 1.2071a\underline{k}) \times (-F\Delta t_B)\underline{j}$$

ANG. IMP. $= 1.2071\, aF(\Delta t_A + \Delta t_B)\underline{i} + 0.5\, aF(\Delta t_A - \Delta t_B)\underline{k}$ (2)

IMPULSE-MOMENTUM PRINCIPLE
WE MUST HAVE $\underline{H}_G + \text{ANG. IMP.} = 0$
OR, USING COMPONENTS:

$$H_x + 1.2071aF(\Delta t_A + \Delta t_B) = 0 \qquad \Delta t_A + \Delta t_B = -\frac{H_x}{1.2071aF}$$
$$H_z + 0.5aF(\Delta t_A - \Delta t_B) = 0 \qquad \Delta t_A - \Delta t_B = -\frac{H_z}{0.5aF}$$

SOLVING THESE EQUATIONS SIMULTANEOUSLY:

$$\Delta t_A = -\frac{H_z + 0.41421 H_x}{aF}, \quad \Delta t_B = \frac{H_z - 0.41421 H_z}{aF} \qquad (3)$$

IF $\Delta t_A > 0$, ASSUMPTION IS CORRECT, A SHOULD BE USED;
IF $\Delta t_A < 0$, ASSUMPTION IS WRONG; C SHOULD BE USED AND ACTIVATED FOR $\Delta t_C = |\Delta t_A|$.
SIMILARLY, IF $\Delta t_B > 0$, B SHOULD BE USED, AND IF $\Delta t_B < 0$ D SHOULD BE USED WITH $\Delta t_D = |\Delta t_B|$.

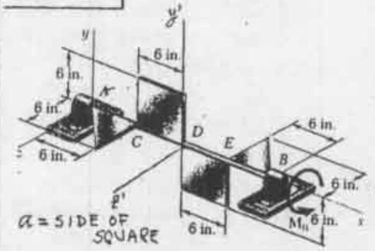
OUTLINE OF PROGRAM

ENTER PART: a, b, c, OR d
ENTER $m = 2500$ kg, $k_x = 0.98$ m, $k_z = 1.02$ m
ENTER $a = 1.2$ m, $F = 20$ N
ENTER VALUES OF ω_x AND ω_z
COMPUTE H_x AND H_z FROM EQS. (1)
COMPUTE Δt_A AND Δt_B FROM EQS. (3)
IF $\Delta t_A > 0$, PRINT Δt_A; IF NOT, PRINT $\Delta t_C = |\Delta t_A|$
IF $\Delta t_B > 0$, PRINT Δt_B; IF NOT, PRINT $\Delta t_D = |\Delta t_B|$

PROGRAM OUTPUT

(a) C AND B; $\Delta t_C = 8.160$ s; $\Delta t_B = 4.845$ s
(b) A AND D; $\Delta t_A = 1.849$ s; $\Delta t_D = 6.821$ s
(c) C AND D; $\Delta t_C = 4.654$ s; $\Delta t_D = 0.3188$ s
(d) A AND B; $\Delta t_A = 4.654$ s; $\Delta t_B = 0.3188$ s

18.C3



a = SIDE OF SQUARE

GIVEN:
A COUPLE $\underline{M}_0 = (0.03 \text{ lb·ft})\underline{i}$ IS APPLIED AT $t = 0$ TO 2.7-lb ASSEMBLY OF SHEET ALUMINUM OF UNIFORM THICKNESS

FIND:
(a) COMPONENTS ALONG THE ROTATING y AND z AXES OF THE DYNAMIC REACTIONS AT A AND B FROM $t = 0$ TO $t = 2$ s AT 0.1 s INTERVALS,
(b) THE TIME (WITH 3 SIGNIFICANT FIGURES) AT WHICH THE z COMPONENTS OF THESE REACTIONS ARE EQUAL TO ZERO.

ANALYSIS

WE COMPUTE THE MOMENT AND PRODUCTS OF INERTIA OF THE ASSEMBLY WITH RESPECT TO THE CENTROIDAL AXES $Dxy'z'$. WE FIRST COMPUTE THE MOMENT AND PRODUCTS OF AREAS

FOR EACH SQUARE: $(I_x)_{AREA} = \frac{1}{3}a^4$, $(I_{xy'})_{AREA} = -\frac{1}{4}a^4$, $(I_{xz'})_{AREA} = 0$

FOR EACH TRIANGLE: $(I_x)_{AREA} = \frac{1}{12}a^4$, $(I_{xy'})_{AREA} = 0$

$$(I_{xz'})_{AREA} = \tfrac{1}{2}a^2\bar{x}'\bar{z}' + \bar{I}_{x''z''} = -\tfrac{1}{2}a^2\left(\tfrac{4}{3}a\right)\left(\tfrac{1}{3}a\right) + \tfrac{1}{72}a^4 = -\tfrac{15}{72}a^4 \quad \text{[CF. SP 9.6]}$$

FOR ENTIRE ASSEMBLY:

$$(I_x)_{AREA} = 2\left(\tfrac{1}{3}a^4\right) + 2\left(\tfrac{1}{12}a^4\right) = \tfrac{5}{6}a^4$$
$$(I_{xy'})_{AREA} = 2\left(-\tfrac{1}{4}a^4\right) = -\tfrac{1}{2}a^4 \qquad (I_{xz'})_{AREA} = 2\left(-\tfrac{15}{72}a^4\right) = -\tfrac{15}{36}a^4$$

THE MASS MOMENT AND PRODUCTS OF INERTIA ARE OBTAINED BY MULTIPLYING THESE EXPRESSIONS BY THE MASS m OF THE ASSEMBLY AND DIVIDING BY ITS AREA, WHICH IS EQUAL TO $3a^2$:

$$I_x = \tfrac{5}{18}ma^2, \quad I_{xy'} = -\tfrac{1}{6}ma^2, \quad I_{xz'} = -\tfrac{5}{36}ma^2 \qquad (1)$$

WE DETERMINE $\underline{H}_D$ AND ITS DERIVATIVE $\dot{\underline{H}}_D$

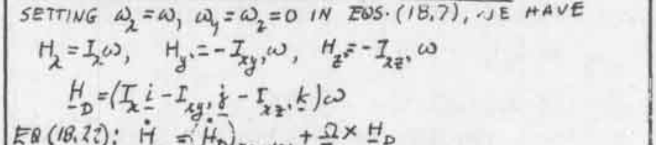
SETTING $\omega_x = \omega$, $\omega_y = \omega_z = 0$ IN EQS. (18.7), WE HAVE

$$H_x = I_x\omega, \quad H_y = -I_{xy'}\omega, \quad H_z = -I_{xz'}\omega$$
$$\underline{H}_D = (I_x\underline{i} - I_{xy'}\underline{j} - I_{xz'}\underline{k})\omega$$

EQ (18.22): $\dot{\underline{H}}_D = (\dot{\underline{H}}_D)_{Dxy'z'} + \underline{\Omega} \times \underline{H}_D$

$$\dot{\underline{H}}_D = (I_x\underline{i} - I_{xy'}\underline{j} - I_{xz'}\underline{k})\dot{\omega} + \omega\underline{i} \times (I_x\underline{i} - I_{xy'}\underline{j} - I_{xz'}\underline{k})\omega$$
$$= (I_x\underline{i} - I_{xy'}\underline{j} - I_{xz'}\underline{k})\alpha - I_{xy'}\omega^2\underline{k} + I_{xz'}\omega^2\underline{j}$$
$$\dot{\underline{H}}_D = I_x\alpha\underline{i} + (I_{xz'}\omega^2 - I_{xy'}\alpha)\underline{j} - (I_{xy'}\omega^2 + I_{xz'}\alpha)\underline{k}$$

EQUATIONS OF MOTION



$\Sigma\underline{M}_B = \Sigma(\underline{M}_B)_{eff}$:

$$M_0\underline{i} - 4a\underline{i} \times (A_y\underline{j} + A_z\underline{k}) = \dot{\underline{H}}_D$$
$$M_0\underline{i} - 4aA_y\underline{k} + 4aA_z\underline{j} = I_x\alpha\underline{i} + (I_{xz'}\omega^2 - I_{xy'}\alpha)\underline{j} - (I_{xy'}\omega^2 + I_{xz'}\alpha)\underline{k}$$

EQUATING THE COEFF. OF THE UNIT VECTORS:

(i) $M_0 = I_x\alpha \qquad \alpha = M_0/I_x$ (2)
FROM WHICH WE OBTAIN $\omega = \alpha t$ (3)
(j) $A_z = (I_{xz'}\omega^2 - I_{xy'}\alpha)/4a$ (4)
(k) $A_y = (I_{xy'}\omega^2 + I_{xz'}\alpha)/4a$ (5)

$\Sigma\underline{F} = \Sigma(\underline{F})_{eff} = 0: \quad \underline{A} + \underline{B} = 0$

THUS: $B_y = -A_y \qquad B_z = -A_z$ (6)

(CONTINUED)

18.C3 continued

OUTLINE OF PROGRAM

(a) ENTER $M_0 = 0.03$ lb·ft, $W = 2.7$ lb, $a = 0.5$ ft
COMPUTE $m = W/32.2$
COMPUTE I_x, I_{xy}, I_{xz} FROM EQS. (1)
COMPUTE α FROM EQ. (2)
FOR $t = 0$ TO $t = 2$s AT 0.1-s INTERVALS:
COMPUTE ω FROM EQ. (3)
COMPUTE A_y, A_z, B_y, B_z FROM EQS. (4), (5), AND (6) AND TABULATE VS t

(b) DETERMINE BY INSPECTION THE TIME INTERVAL IN WHICH A_z AND B_z CHANGE SIGN AND RUN THE PROGRAM OVER THAT INTERVAL, USING 0.01-s INCREMENTS. REPEAT THIS PROCEDURE, USING 0.001-s INCREMENTS. THE DESIRED VALUE OF t IS THAT FOR WHICH $|A_z|$ AND $|B_z|$ ARE SMALLEST.

PROGRAM OUTPUT

(a)

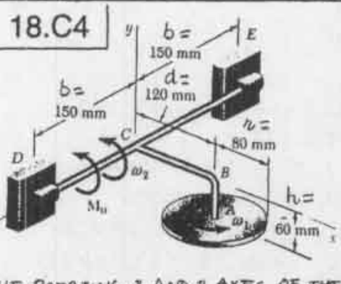
t s	Ay lb	Az lb	By lb	Bz lb
0.00000	-0.00750	0.00900	0.00750	-0.00900
0.10000	-0.00796	0.00861	0.00796	-0.00861
0.20000	-0.00935	0.00745	0.00935	-0.00745
0.30000	-0.01167	0.00552	0.01167	-0.00552
0.40000	-0.01492	0.00282	0.01492	-0.00282
0.50000	-0.01909	-0.00066	0.01909	0.00066
0.60000	-0.02419	-0.00491	0.02419	0.00491
0.70000	-0.03022	-0.00993	0.03022	0.00993
0.80000	-0.03718	-0.01573	0.03718	0.01573
0.90000	-0.04506	-0.02230	0.04506	0.02230
1.00000	-0.05387	-0.02964	0.05387	0.02964
1.10000	-0.06361	-0.03775	0.06361	0.03775
1.20000	-0.07427	-0.04664	0.07427	0.04664
1.30000	-0.08586	-0.05630	0.08586	0.05630
1.40000	-0.09838	-0.06673	0.09838	0.06673
1.50000	-0.11183	-0.07794	0.11183	0.07794
1.60000	-0.12620	-0.08992	0.12620	0.08992
1.70000	-0.14150	-0.10267	0.14150	0.10267
1.80000	-0.15773	-0.11619	0.15773	0.11619
1.90000	-0.17489	-0.13049	0.17489	0.13049
2.00000	-0.19297	-0.14556	0.19297	0.14556

(b)

t s	Ay lb	Az lb	By lb	Bz lb
0.40000	-0.01492	0.00282	0.01492	-0.00282
0.41000	-0.01529	0.00250	0.01529	-0.00250
0.42000	-0.01568	0.00218	0.01568	-0.00218
0.43000	-0.01607	0.00186	0.01607	-0.00186
0.44000	-0.01648	0.00152	0.01648	-0.00152
0.45000	-0.01689	0.00118	0.01689	-0.00118
0.46000	-0.01731	0.00082	0.01731	-0.00082
0.47000	-0.01774	0.00046	0.01774	-0.00046
0.48000	-0.01818	0.00010	0.01818	-0.00010
0.49000	-0.01863	-0.00028	0.01863	0.00028
0.50000	-0.01909	-0.00066	0.01909	0.00066

t s	Ay lb	Az lb	By lb	Bz lb
0.48000	-0.01818	0.00010	0.01818	-0.00010
0.48100	-0.01823	0.00006	0.01823	-0.00006
0.48200	-0.01827	0.00002	0.01827	-0.00002
0.48300	-0.01832	-0.00001	0.01832	0.00001
0.48400	-0.01836	-0.00005	0.01836	0.00005
0.48500	-0.01841	-0.00009	0.01841	0.00009
0.48600	-0.01845	-0.00013	0.01845	0.00013
0.48700	-0.01850	-0.00016	0.01850	0.00016
0.48800	-0.01854	-0.00020	0.01854	0.00020
0.48900	-0.01859	-0.00024	0.01859	0.00024
0.49000	-0.01863	-0.00028	0.01863	0.00028

18.C4



GIVEN:
DISK: $m = 2.5$ kg, $r = 80$ mm
$\omega_1 = 60$ rad/s AT $t = 0$ AND DECREASES AT RATE OF 15 rad/s². AT $t = 0$, $\omega_2 = 0$ AND COUPLE $\underline{M}_0 = (0.5\text{ N·m})\underline{k}$ IS APPLIED TO SHAFT DCE.
FIND:
(a) COMPONENTS ALONG THE ROTATING x AND y AXES OF THE DYNAMIC REACTIONS AT D AND E FROM $t = 0$ TO $t = 4$s AT 0.2-s INTERVALS,
(b) THE TIMES t_1 AND t_2 (WITH 3 SIGNIFICANT FIGURES) AT WHICH E_x AND E_y ARE RESPECTIVELY EQUAL TO ZERO.

ANALYSIS

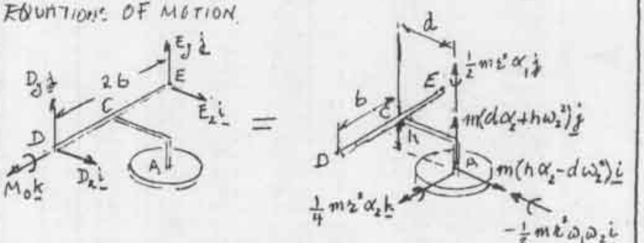
$$\underline{H}_A = \bar{I}_{y'}\omega_y\underline{j} + \bar{I}_{z'}\omega_z\underline{k} = \frac{1}{2}mr^2\omega_1\underline{j} + \frac{1}{4}mr^2\omega_2\underline{k}$$

EQ. (18.22): $\dot{\underline{H}}_A = (\dot{\underline{H}}_A)_{xyz} + \underline{\Omega}\times\underline{H}_A$

$$\dot{\underline{H}}_A = \frac{1}{2}mr^2\dot{\omega}_1\underline{j} + \frac{1}{4}mr^2\dot{\omega}_2\underline{k} + \omega_2\underline{k}\times\left(\frac{1}{2}mr^2\omega_1\underline{j} + \frac{1}{4}mr^2\omega_2\underline{k}\right)$$
$$= \frac{1}{2}mr^2\alpha_1\underline{j} + \frac{1}{4}mr^2\alpha_2\underline{k} - \frac{1}{2}mr^2\omega_1\omega_2\underline{i}$$
$$\dot{\underline{H}}_A = \frac{1}{2}mr^2(-\omega_1\omega_2\underline{i} + \alpha_1\underline{j} + 0.5\alpha_2\underline{k}) \quad (1)$$

$$m\bar{\underline{a}} = m(\underline{\alpha}_2\times\underline{r}_{A/C} - \omega_2^2\underline{r}_{A/C}) = m\alpha_2\underline{k}\times(d\underline{i} - h\underline{j}) - m\omega_2^2(d\underline{i} - h\underline{j})$$
$$= m(d\alpha_2\underline{j} + h\alpha_2\underline{i} - d\omega_2^2\underline{i} + h\omega_2^2\underline{j})$$
$$m\bar{\underline{a}} = m(h\alpha_2 - d\omega_2^2)\underline{i} + m(d\alpha_2 + h\omega_2^2)\underline{j} \quad (2)$$

EQUATIONS OF MOTION



$\Sigma\underline{M}_D = \Sigma(\underline{M}_D)_{eff}$:

$$-2b\underline{k}\times(E_x\underline{i} + E_y\underline{j}) + M_0\underline{k} = -\frac{1}{2}mr^2\omega_1\omega_2\underline{i} + \frac{1}{2}mr^2\alpha_1\underline{j} + \frac{1}{4}mr^2\alpha_2\underline{k} + (-b\underline{k} + d\underline{i} - h\underline{j})\times m[(h\alpha_2 - d\omega_2^2)\underline{i} + (d\alpha_2 + h\omega_2^2)\underline{j}]$$

$$-2bE_x\underline{j} + 2bE_y\underline{i} + M_0\underline{k} = -\frac{1}{2}mr^2\omega_1\omega_2\underline{i} + \frac{1}{2}mr^2\alpha_1\underline{j} + \frac{1}{4}mr^2\alpha_2\underline{k} - mb(h\alpha_2 - d\omega_2^2)\underline{j} + mb(d\alpha_2 + h\omega_2^2)\underline{i} + md(d\alpha_2 + h\omega_2^2)\underline{k} + mh(h\alpha_2 - d\omega_2^2)\underline{k}$$

EQUATE THE COEFF. OF THE UNIT VECTORS:

$$(\underline{k})\quad M_0 = m\left(\frac{1}{4}r^2 + d^2 + h^2\right)\alpha_2 \qquad \alpha_2 = \frac{M_0}{m(\frac{1}{4}r^2 + d^2 + h^2)} \quad (3)$$

$$(\underline{j})\quad E_x = \frac{m}{2b}\left(-\frac{1}{2}r^2\alpha_1 + bh\alpha_2 - bd\omega_2^2\right) \quad (4)$$

$$(\underline{i})\quad E_y = \frac{m}{2b}\left(-\frac{1}{2}r^2\omega_1\omega_2 + bd\alpha_2 + bh\omega_2^2\right) \quad (5)$$

$\Sigma\underline{F} = \Sigma(\underline{F})_{eff}$: $\underline{D} + \underline{E} = m\bar{\underline{a}}$

$$D_x\underline{i} + D_y\underline{j} + E_x\underline{i} + E_y\underline{j} = m(h\alpha_2 - d\omega_2^2)\underline{i} + m(d\alpha_2 + h\omega_2^2)\underline{j}$$

EQUATE THE COEFF. OF THE UNIT VECTORS:

$$(\underline{i})\quad D_x = m(h\alpha_2 - d\omega_2^2) - E_x \quad (6)$$

$$(\underline{j})\quad D_y = m(d\alpha_2 + h\omega_2^2) - E_y \quad (7)$$

WE RECALL FROM THE GIVEN DATA THAT

$$m = 2.5\text{ kg},\ r = 0.08\text{ m},\ b = 0.15\text{ m},\ d = 0.12\text{ m},\ h = 0.06\text{ m} \quad (8)$$
$$M_0 = 0.5\text{ N·m} \qquad \omega_0 = 60\text{ rad/s} \qquad \alpha_1 = -15\text{ rad/s}^2$$

AND NOTE THAT AT TIME t

$$\omega_1 = \omega_0 + \alpha_1 t \qquad \omega_2 = \alpha_2 t \quad (9)$$

(CONTINUED)

18.C4 continued

OUTLINE OF PROGRAM

(a) ENTER DATA SHOWN IN (B) ON PREVIOUS PAGE
COMPUTE α_2 FROM EQ. (3)
FOR $t=0$ TO $t=4$ s AT 0.2-s INTERVALS
COMPUTE ω_1 AND ω_2 FROM EQS. (9)
COMPUTE E_x AND E_y FROM EQS. (4) AND (5)
COMPUTE D_x AND D_y FROM EQS. (6) AND (7)
AND TABULATE VS t.

(b) TO FIND THE TIME t_1 AT WHICH $E_x=0$, DETERMINE BY INSPECTION THE TIME INTERVAL IN WHICH E_x CHANGES SIGN AND RUN THE PROGRAM OVER THAT INTERVAL, USING 0.01-s INCREMENTS. REPEAT THIS PROCEDURE USING 0.001-s INCREMENTS. SELECT FOR t_1 THE TIME AT WHICH $|E_x|$ IS SMALLEST.
A SIMILAR PROCEDURE IS USED TO DETERMINE THE TIME t_2 AT WHICH $E_y=0$.

PROGRAM OUTPUT

(a)

t (s)	Dx (N)	Dy (N)	Ex (N)	Ey (N)
0.0000	0.3653	1.5306	1.1653	1.5306
0.2000	-0.2594	4.9450	0.5406	-1.2591
0.4000	-2.1337	8.6576	-1.3337	-3.0975
0.6000	-5.2574	12.6685	-4.4574	-3.9846
0.8000	-9.6305	16.9775	-8.8305	-3.9204
1.0000	-15.2532	21.5848	-14.4532	-2.9050
► 1.2000	-22.1253	26.4902	-21.3253	-0.9384
1.4000	-30.2469	31.6939	-29.4469	1.9796
1.6000	-39.6181	37.1958	-38.8180	5.8488
1.8000	-50.2386	42.9958	-49.4386	10.6693
2.0000	-62.1087	49.0941	-61.3087	16.4411
2.2000	-75.2282	55.4906	-74.4282	23.1641
2.4000	-89.5972	62.1854	-88.7973	30.8384
2.6000	-105.2158	69.1783	-104.4158	39.4640
2.8000	-122.0838	76.4694	-121.2837	49.0409
3.0000	-140.2012	84.0588	-139.4012	59.5690
3.2000	-159.5681	91.9463	-158.7681	71.0483
3.4000	-180.1846	100.1321	-179.3845	83.4790
3.6000	-202.0505	108.6160	-201.2504	96.8609
3.8000	-225.1659	117.3982	-224.3658	111.1942
4.0000	-249.5307	126.4786	-248.7306	126.4786

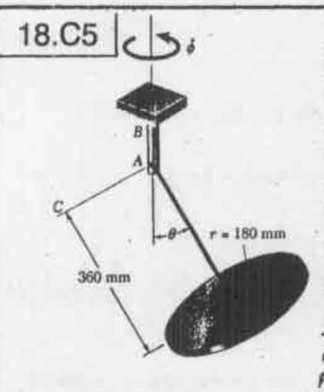
(b) LAST STEP IN DETERMINATION OF t_1

t (s)	Dx (N)	Dy (N)	Ex (N)	Ey (N)
0.2700	-0.7733	6.2105	0.0267	-2.0107
0.2710	-0.7817	6.2289	0.0183	-2.0206
0.2720	-0.7902	6.2472	0.0098	-2.0305
► 0.2730	-0.7987	6.2656	0.0013	-2.0403
0.2740	-0.8073	6.2839	-0.0073	-2.0501
0.2750	-0.8158	6.3023	-0.0158	-2.0599
0.2760	-0.8244	6.3207	-0.0244	-2.0697
0.2770	-0.8331	6.3391	-0.0331	-2.0795
0.2780	-0.8418	6.3575	-0.0418	-2.0892
0.2790	-0.8505	6.3759	-0.0505	-2.0989
0.2800	-0.8592	6.3943	-0.0592	-2.1086

LAST STEP IN DETERMINATION OF t_2

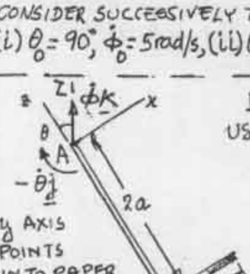
t (s)	Dx (N)	Dy (N)	Ex (N)	Ey (N)
1.2700	-24.8258	28.2776	-24.0258	-0.0253
1.2710	-24.8654	28.3034	-24.0655	-0.0114
► 1.2720	-24.9052	28.3292	-24.1052	0.0025
1.2730	-24.9449	28.3550	-24.1449	0.0165
1.2740	-24.9847	28.3808	-24.1847	0.0304
1.2750	-25.0245	28.4066	-24.2245	0.0444
1.2760	-25.0644	28.4325	-24.2644	0.0584
1.2770	-25.1042	28.4583	-24.3042	0.0724
1.2780	-25.1442	28.4842	-24.3441	0.0865
1.2790	-25.1841	28.5100	-24.3841	0.1006

18.C5



GIVEN:
DISK WELDED TO ROD AC OF NEGLIGIBLE MASS CONNECTED BY CLEVIS TO SHAFT AB. ROD AND DISK FREE TO ROTATE ABOUT AC; SHAFT AB FREE TO ROTATE ABOUT VERTICAL AXIS.
INITIALLY, $\theta=\theta_0$, $\dot\theta=0$, $\dot\phi=\dot\phi_0$

FIND:
(a) MINIMUM VALUE θ_m OF θ DURING ENSUING MOTION AND TIME REQUIRED FOR θ TO RETURN TO θ_0 (PERIOD).
(b) ANG. VEL. $\dot\phi$ FOR VALUES OF θ FROM θ_0 TO θ_m USING 2° INCREMENTS
CONSIDER SUCCESSIVELY THE INITIAL CONDITIONS
(i) $\theta_0=90°$, $\dot\phi_0=5$ rad/s, (ii) $\theta_0=90°$, $\dot\phi_0=10$ rad/s, (iii) $\theta_0=60°$, $\dot\phi_0=5$ rad/s.



y AXIS POINTS INTO PAPER

ANALYSIS

USING THE ROTATING FRAME Axyz:

$\underline{\omega}=\dot\phi\sin\theta\,\underline{i}-\dot\theta\underline{j}+\dot\phi\cos\theta\,\underline{k}$

$H_x=I_x\omega_x=I'\dot\phi\sin\theta$

$H_z=I_z\omega_z=I\dot\phi\cos\theta$

WHERE $I=\frac{1}{2}ma^2$ (1)

$I'=\frac{1}{4}ma^2+m(2a)^2=\frac{17}{4}ma^2$ (2)

CONSERVATION OF ANG. MOM. ABOUT Z

SINCE THE FORCES CONSIST OF REACTION AT A AND WEIGHT $\underline{W}=-mg\underline{K}$ AT G, WE HAVE $\Sigma M_Z=0$ AND H_Z = CONSTANT

SINCE $H_Z=H_x\sin\theta+H_z\cos\theta=I'\dot\phi\sin^2\theta+I\dot\phi\cos^2\theta$, WE HAVE

$$(I'\sin^2\theta+I\cos^2\theta)\dot\phi=(I'\sin^2\theta_0+I\cos^2\theta_0)\dot\phi_0$$

SETTING $Q=I'\sin^2\theta+I\cos^2\theta$ (3)

AND $Q_0=I'\sin^2\theta_0+I\cos^2\theta_0$ (4)

AND SOLVING FOR $\dot\phi$: $\dot\phi=(Q_0/Q)\dot\phi_0$ (5)

CONSERVATION OF ENERGY

$T+V=E=$ CONSTANT: $\frac{1}{2}(I_x\omega_x^2+I_y\omega_y^2+I_z\omega_z^2)+W(-2a\cos\theta)=E$

$$\tfrac{1}{2}(I'\dot\phi^2\sin^2\theta+I'\dot\theta^2+I\dot\phi^2\cos^2\theta)-2mga\cos\theta=E$$

$$(I'\sin^2\theta+I\cos^2\theta)\dot\phi^2+I'\dot\theta^2-4mga\cos\theta=2E$$

RECALLING (3) AND SUBSTITUTING FOR $\dot\phi$ FROM (5):

$$(Q_0^2\dot\phi_0^2/Q)+I'\dot\theta^2-4mga\cos\theta=2E \qquad (6)$$

SOLVING FOR $\dot\theta^2$:

$$\dot\theta^2=\frac{1}{I'}\left(2E+4mga\cos\theta-\frac{Q_0^2\dot\phi_0^2}{Q}\right)$$

WHICH IS OF THE FORM $\dot\theta^2=f(\theta)$ (7)

WHERE $f(\theta)=\frac{1}{I'}\left(2E+4mga\cos\theta-\frac{Q_0^2\dot\phi_0^2}{Q}\right)$ (8)

AND Q IS THE FUNCTION OF θ DEFINED IN (3). THE CONSTANT 2E IS OBTAINED BY MAKING $\theta=\theta_0$, $\dot\theta=0$ AND $Q=Q_0$ IN EQ. (6): $E=\frac{1}{2}Q_0\dot\phi_0^2-2mga\cos\theta_0$ (9)

FROM (7) WE WRITE

$\frac{d\theta}{dt}=\dot\theta=\sqrt{f(\theta)}$ $\qquad t=\int_{\theta_0}^{\theta}\frac{d\theta}{\sqrt{f(\theta)}}$ (10)

(a) THE TIME $\frac{1}{2}\tau$ NEEDED FOR θ TO DECREASE TO θ_m IS OBTAINED THROUGH NUMERICAL INTEGRATION, θ_m BEING DEFINED BY THE FACT THAT $f(\theta_m)=0$ (f CHANGES SIGN)

(b) FOR EACH DESIRED VALUE OF θ, COMPUTE Q FROM EQ. (3) AND $\dot\phi$ FROM EQ. (5).

(CONTINUED)

18.C5 continued

OUTLINE OF PROGRAM

ENTER $a = 0.18$ m, $g = 9.81$ m/s^2. ASSUME $m = 1$.
ENTER INITIAL CONDITIONS: θ_0 AND $\dot\phi_0$
ENTER DECREMENT $d\theta$ YOU WISH TO USE
COMPUTE I AND I' FROM (1) AND (2)
COMPUTE Q_0 FROM (4) AND E FROM (9)
FOR $\theta = \theta_0$ TO $\theta = \theta_m$ (WHEN $f(\theta)$ CHANGES SIGN), AND USING DECREMENTS $d\theta$:
- COMPUTE Q FROM (3)
- COMPUTE $f(\theta)$ FROM (8)
- CARRY OUT NUMERICALLY THE INTEGRATION INDICATED IN (10)
- AT 2° INTERVALS, COMPUTE $\dot\phi$ FROM (5) AND PRINT THE VALUES OF θ AND $\dot\phi$

THE PERIOD OF THE OSCILLATION IN θ IS OBTAINED BY DOUBLING THE VALUE OF t WHEN θ REACHES ITS MINIMUM VALUE θ_m.

PROGRAM OUTPUT

(i)

THO= 90
PHIDO= 5
DTH= .1

Theta (degrees)	Precession Rate (rad/s)
90.000	5.000
88.000	5.005
86.000	5.022
84.000	5.049
82.000	5.087
80.000	5.137
78.000	5.198
76.000	5.272
74.000	5.359
72.000	5.460
70.000	5.575
68.000	5.707
66.000	5.855
64.000	6.021
62.000	6.207
60.000	6.415
58.000	6.647
56.000	6.905
54.000	7.193
52.000	7.513
50.000	7.869
48.000	8.265
46.000	8.708
44.000	9.201
42.000	9.752
40.000	10.369
38.000	11.060
36.000	11.835
34.000	12.705
32.000	13.683
31.959	13.704

Theta min = 32.0 degrees
Period = 0.736 s

(ii)

THO= 90
PHIDO= 10
DTH= .1

Theta (degrees)	Precession Rate (rad/s)
90.000	10.000
88.000	10.011
86.000	10.043
84.000	10.097
82.000	10.174
80.000	10.273
78.000	10.397
76.000	10.545
74.000	10.719
72.000	10.920
70.000	11.151
68.000	11.413
66.000	11.709
64.000	12.042
62.052	12.404

Theta min = 62.1 degrees
Period = 0.577 s

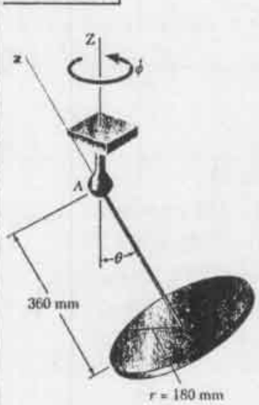
(iii)

THO= 60
PHIDO= 5
DTH= .1

Theta (degrees)	Precession Rate (rad/s)
60.000	5.000
58.000	5.181
56.000	5.382
54.000	5.606
52.000	5.855
50.000	6.133
48.000	6.442
46.000	6.787
44.000	7.171
42.000	7.601
40.000	8.082
38.000	8.620
36.894	8.945

Theta min = 36.9 degrees
Period = 0.725 s

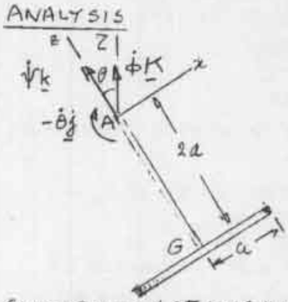
18.C6



GIVEN:
DISK WELDED TO ROD AG OF NEGLIGIBLE MASS SUPPORTED BY BALL AND SOCKET AT A.
INITIALLY, $\theta = \theta_0$, $\dot\theta = 0$, $\dot\psi = \dot\psi_0$, AND $\dot\phi = \dot\phi_0$.

FIND:
(a) MINIMUM VALUE θ_m OF θ IN ENSUING MOTION AND PERIOD (TIME REQUIRED FOR θ TO RETURN TO θ_0).
(b) $\dot\psi$ AND $\dot\phi$ FOR VALUES OF θ FROM θ_0 TO θ_m USING 2° INCREMENTS.
CONSIDER SUCCESSIVELY THE INITIAL CONDITIONS
(i) $\theta_0 = 90°$, $\dot\psi_0 = 50$ rad/s, $\dot\phi_0 = 0$
(ii) $\theta_0 = 90°$, $\dot\psi_0 = 0$, $\dot\phi_0 = 5$ rad/s
(iii) $\theta_0 = 90°$, $\dot\psi_0 = 50$ rad/s, $\dot\phi_0 = 5$ rad/s
(iv) $\theta_0 = 90°$, $\dot\psi_0 = 10$ rad/s, $\dot\phi_0 = 5$ rad/s
(v) $\theta_0 = 60°$, $\dot\psi_0 = 0$, $\dot\phi_0 = 5$ rad/s
(vi) $\theta_0 = 60°$, $\dot\psi_0 = 50$ rad/s, $\dot\phi_0 = 5$ rad/s

ANALYSIS



USING THE ROTATING FRAME Axyz WITH y AXIS POINTING INTO THE PAPER:

$\underline{\omega} = \dot\phi\sin\theta\,\underline{i} - \dot\theta\,\underline{j} + (\dot\psi + \dot\phi\cos\theta)\underline{k}$

$\underline{H}_A = I_x\omega_x\underline{i} + I_y\omega_y\underline{j} + I_z\omega_z\underline{k}$
$= I'\dot\phi\sin\theta\,\underline{i} - I'\dot\theta\,\underline{j} + I(\dot\psi + \dot\phi\cos\theta)\underline{k}$

WHERE $I = \frac{1}{2}ma^2$ (1)
$I' = \frac{1}{4}ma^2 + m(2a)^2 = \frac{17}{4}ma^2$ (2)

CONSERVATION OF ANGULAR MOMENTUM

SINCE THE ONLY EXTERNAL FORCES ARE THE REACTION AT A AND THE WEIGHT $\underline{W} = -mg\underline{K}$ AT G, WE HAVE $\Sigma M_Z = 0$ AND $\Sigma M_z = 0$. SINCE Z IS PART OF A NEWTONIAN FRAME OF REFERENCE, IT FOLLOWS THAT H_Z = CONSTANT. BECAUSE OF THE AXISYMMETRY OF THE DISK, IT ALSO FOLLOWS THAT H_z = CONSTANT (SEE PROB. 18.139). WE WRITE

$H_z = \text{CONST}: \quad I(\dot\psi + \dot\phi\cos\theta) = \beta$ (3)

WHERE FROM INIT. COND.: $\beta = I(\dot\psi_0 + \dot\phi_0\cos\theta_0)$ (4)

$H_Z = \text{CONST.}: \quad H_x\sin\theta + H_z\cos\theta = \alpha$

$I'\dot\phi\sin^2\theta + I(\dot\psi + \dot\phi\cos\theta)\cos\theta = \alpha$

RECALLING (3) WE HAVE $I'\dot\phi\sin^2\theta + \beta\cos\theta = \alpha$ (5)

FROM INITIAL CONDITIONS: $\alpha = I'\dot\phi_0\sin^2\theta_0 + \beta\cos\theta_0$ (6)

SOLVING (5) FOR $\dot\phi$: $\dot\phi = \dfrac{\alpha - \beta\cos\theta}{I'\sin^2\theta}$ (7)

CONSERVATION OF ENERGY

$T = \frac{1}{2}(I_x\omega_x^2 + I_y\omega_y^2 + I_z\omega_z^2)$
$= \frac{1}{2}[I'\dot\phi^2\sin^2\theta + I'\dot\theta^2 + I(\dot\psi + \dot\phi\cos\theta)^2]$

SUBSTITUTE FOR ( ) FROM (3):

$T = \frac{1}{2}\left(I'\dot\phi^2\sin^2\theta + I'\dot\theta^2 + \frac{\beta^2}{I}\right) \qquad V = -mg(2a)\cos\theta$

$T + V = E: \quad \frac{1}{2}\left(I'\dot\phi^2\sin^2\theta + I'\dot\theta^2 + \frac{\beta^2}{I}\right) - 2mga\cos\theta = E$ (8)

FROM INIT COND.: $E = \frac{1}{2}\left(I'\dot\phi_0^2\sin^2\theta_0 + \frac{\beta^2}{I}\right) - 2mga\cos\theta_0$ (9)

SOLVING (8) FOR $\dot\theta^2$: $\dot\theta^2 = f(\theta)$ (10)

WHERE

$f(\theta) = \frac{1}{I'}\left(2E - \frac{\beta^2}{I} + 4mga\cos\theta\right) - \dot\phi^2\sin^2\theta$ (11)

(CONTINUED)

SUBSTITUTING FOR $\dot{\phi}$ FROM (7) INTO (11), WE HAVE

$$f(\theta)=\frac{1}{I'}\left(2E-\frac{\beta^2}{I}+4mga\cos\theta\right)-\left(\frac{\alpha-\beta\cos\theta}{I'\sin\theta}\right)^2 \quad (12)$$

FROM EQ. (10) WE WRITE

$$\frac{d\theta}{dt}=\dot{\theta}=\sqrt{f(\theta)} \qquad t=\int_{\theta_0}^{\theta}\frac{d\theta}{\sqrt{f(\theta)}} \quad (13)$$

(a) THE TIME $\frac{1}{2}\tau$ NEEDED FOR θ TO DECREASE TO θ_m IS OBTAINED THROUGH NUMERICAL INTEGRATION, θ_m BEING DEFINED BY THE FACT THAT $f(\theta_m)=0$, THAT IS, THAT $f(\theta)$ CHANGES SIGN FOR $\theta=\theta_m$

(b) FOR EACH DESIRED VALUE OF θ, COMPUTE $\dot{\phi}$ FROM EQ. (7).

OUTLINE OF PROGRAM

ENTER $a=0.18$ m, $g=9.81$ m/s^2. ASSUME $m=1$.
ENTER INITIAL CONDITIONS: θ_0, $\dot{\psi}_0$, AND $\dot{\phi}_0$.
ENTER DECREMENT $d\theta$ YOU WISH TO USE
COMPUTE I AND I' FROM (1) AND (2)
COMPUTE β FROM (4), α FROM (6), AND E FROM (9)
FOR $\theta=\theta_0$ TO $\theta=\theta_m$ (WHEN $f(\theta)$ CHANGES SIGN), AND USING DECREMENTS $d\theta$:
COMPUTE $\dot{\phi}$ FROM (7)
COMPUTE $f(\theta)$ FROM (11)
CARRY OUT NUMERICALLY THE INTEGRATION DEFINED IN EQ. (13)
AT 2° INTERVALS, PRINT THE VALUES OF θ, $\dot{\phi}$, AND, FROM (3), OF $\dot{\psi}=\frac{\beta}{I}-\dot{\phi}\cos\theta$

THE PERIOD OF THE OSCILLATION IN θ IS OBTAINED BY DOUBLING THE VALUE OF t CORRESPONDING TO $\theta=\theta_m$.

PROGRAM OUTPUT

(i)

TH0=90 PSID0=50 PHID0= 0
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
90.00	0.00	50.00
88.00	-0.21	50.01
86.00	-0.41	50.03
84.00	-0.62	50.06
82.00	-0.83	50.12
80.00	-1.05	50.18
78.00	-1.28	50.27
76.00	-1.51	50.37
74.00	-1.75	50.48
72.00	-2.01	50.62
70.00	-2.28	50.78
68.00	-2.56	50.96
66.00	-2.87	51.17
64.00	-3.19	51.40
62.00	-3.54	51.66
60.00	-3.92	51.96
58.00	-4.33	52.30
56.00	-4.79	52.68
54.00	-5.28	53.11
52.00	-5.83	53.59
50.00	-6.44	54.14
48.00	-7.13	54.77
46.00	-7.90	55.49
44.11	-8.72	56.26

Theta min = 44.1 degrees
Period = 0.668 s

(ii)

TH0=90 PSID0= 0 PHID0= 5
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
90.00	5.00	0.00
88.00	5.01	-0.17
86.00	5.02	-0.35
84.00	5.06	-0.53
82.00	5.10	-0.71
80.00	5.16	-0.90
78.00	5.23	-1.09
76.00	5.31	-1.28
74.00	5.41	-1.49
72.00	5.53	-1.71
70.00	5.66	-1.94
68.00	5.82	-2.18
66.00	5.99	-2.44
64.00	6.19	-2.71
62.00	6.41	-3.01
60.00	6.67	-3.33
58.00	6.95	-3.68
56.00	7.27	-4.07
54.00	7.64	-4.49
52.00	8.05	-4.96
50.00	8.52	-5.48
48.00	9.05	-6.06
46.00	9.66	-6.71
44.00	10.36	-7.45
42.00	11.17	-8.30
40.00	12.10	-9.27
38.23	13.06	-10.26

Theta min = 38.2 degrees
Period = 0.687 s

(iii)

TH0=90 PSID0=50 PHID0= 5
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
90.00	5.00	50.00
88.00	4.80	49.83
86.00	4.61	49.68
84.00	4.43	49.54
82.00	4.26	49.41
80.00	4.10	49.29
78.00	3.95	49.18
76.00	3.80	49.08
74.00	3.66	48.99
72.00	3.52	48.91
70.00	3.38	48.84
68.00	3.25	48.78
66.00	3.12	48.73
64.00	3.00	48.69
62.00	2.87	48.65
60.00	2.75	48.63
58.00	2.62	48.61
56.00	2.49	48.61
54.00	2.36	48.61
52.00	2.22	48.63
50.00	2.08	48.66
48.00	1.93	48.71
46.00	1.77	48.77
44.00	1.59	48.85
42.00	1.40	48.96
40.00	1.20	49.08
38.00	0.96	49.24
36.00	0.70	49.44
34.00	0.39	49.67
32.00	0.04	49.97
30.00	-0.38	50.33
28.00	-0.88	50.78
26.00	-1.49	51.34
24.00	-2.26	52.06
22.00	-3.24	53.00
20.00	-4.51	54.24
18.00	-6.23	55.92
16.00	-8.62	58.28
14.00	-12.09	61.73
12.00	-17.44	67.06
10.00	-26.30	75.90
8.00	-42.61	92.19
6.00	-77.82	127.40
5.62	-89.03	138.60

Theta min = 5.62 degrees
Period = 0.542 s

(iv)

TH0=90 PSID0=10 PHID0= 5
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
90.00	5.00	10.00
88.00	4.96	9.83
86.00	4.94	9.66
84.00	4.93	9.48
82.00	4.93	9.31
80.00	4.94	9.14
78.00	4.97	8.97
76.00	5.01	8.79
74.00	5.06	8.61
72.00	5.13	8.42
70.00	5.21	8.22
68.00	5.30	8.01
66.00	5.42	7.80
64.00	5.55	7.57
62.00	5.71	7.32
60.00	5.88	7.06
58.00	6.09	6.78
56.00	6.32	6.47
54.00	6.58	6.13
52.00	6.89	5.76
50.00	7.23	5.35
48.00	7.63	4.90
46.00	8.08	4.38
44.00	8.61	3.81
42.00	9.21	3.15
40.00	9.92	2.40
38.00	10.75	1.53
36.00	11.72	0.52
34.00	12.87	-0.67
32.00	14.25	-2.09
30.00	15.93	-3.79
28.23	17.71	-5.60

Theta min = 28.2 degrees
Period = 0.655 s

(v)

TH0=60 PSID0= 0 PHID0= 5
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
60.00	5.00	0.00
58.00	5.20	-0.26
56.00	5.43	-0.54
54.00	5.69	-0.84
52.00	5.98	-1.18
50.00	6.32	-1.56
48.00	6.70	-1.98
46.00	7.14	-2.46
44.00	7.64	-2.99
42.00	8.22	-3.61
40.33	8.77	-4.18

Theta min = 40.3 degrees
Period = 0.661 s

(vi)

TH0=60 PSID0=50 PHID0= 5
DTH=0.10

Theta degrees	Spin rad/s	Precess. rad/s
60.00	5.00	50.00
58.00	4.96	49.87
56.00	4.92	49.75
54.00	4.90	49.62
52.00	4.89	49.49
50.00	4.89	49.36
48.00	4.90	49.22
46.00	4.92	49.08
44.00	4.96	48.93
42.00	5.02	48.77
40.00	5.10	48.59
38.00	5.20	48.40
36.00	5.33	48.19
34.00	5.49	47.95
32.00	5.70	47.67
30.00	5.96	47.34
28.00	6.28	46.95
26.00	6.70	46.48
24.00	7.23	45.90
22.00	7.92	45.16
20.00	8.84	44.19
18.00	10.10	42.90
16.00	11.86	41.10
14.00	14.44	38.49
12.00	18.43	34.47
10.00	25.06	27.82
8.00	37.27	15.59
6.01	63.40	-10.55

Theta min = 6.01 degrees
Period = 0.520 s

19.1

GIVEN:

PARTICLE IN SIMPLE HARMONIC MOTION
AMPLITUDE = 40 in., PERIOD = 1.4 s.

FIND:

MAXIMUM VELOCITY, v_m
MAIMUM ACCELERATION, a_m

SIMPLE HARMONIC MOTION

$x = x_m \sin(\omega_n t + \phi)$

$\omega_n = 2\pi/\tau_n = 2\pi/(1.4\,s) = 4.480$ rad/s

x_m = AMPLITUDE = 40 in. = 3.333 ft

$x = (3.333)\sin(4.480t + \phi)$

$\dot{x} = x_m \omega_n \cos(\omega_n t + \phi)$ $\dot{x}_m = v_m = x_m \omega_n$

$v_m = (3.333\,ft)(4.480\,r/s)$

$v_m = 14.96$ ft/s

$\ddot{x} = -x_m \omega_n^2 \sin(\omega_n t + \phi)$ $\ddot{x}_m = a_m = -x_m \omega_n^2$

$a_m = (3.333\,ft)(4.480\,r/s)^2$

$a_m = 67.1$ ft/s²

19.2

GIVEN:

PARTICLE IN SIMPLE HARMONIC MOTION
MAXIMUM ACCELERATION 7.2 m/s²
FREQUENCY $f_n = 8$ Hz.

FIND:

AMPLITUDE, x_m
MAXIMUM VELOCITY, v_m

SIMPLE HARMONIC MOTION

$x = x_m \sin(\omega_n t + \phi)$

$\omega_n = 2\pi f_n = (2\pi)(8\,Hz.) = 16\pi$ rad/s

$\dot{x} = x_m \omega_n \cos(\omega_n t + \phi)$ $v_m = x_m \omega_n$

$\ddot{x} = -x_m (\omega_n)^2 \sin(\omega_n t + \phi)$ $a_m = x_m \omega_n^2$

$a_m = 7.2\,m/s^2 = x_m (16\pi\,rad/s)^2$

$x_m = (7.2\,m/s^2)/(16\pi\,rad/s)^2$

$x_m = 2.849 \times 10^{-3}$ m $x_m = 2.85$ mm

$v_m = x_m \omega_n = (2.849\,mm)(16\pi\,rad/s)$

$v_m = 143.2$ mm/s

19.3

GIVEN:

PARTICLE IN SIMPLE HARMONIC MOTION
AMPLITUDE = 300 mm
MAXIMUM ACCELERATION = 5 m/s²

FIND:

MAXIMUM VELOCITY, v_m
FREQUENCY, f

SIMPLE HARMONIC MOTION

$x = x_m \sin(\omega_n t + \phi)$ $x_m = 0.300$ m

$x = (0.300)\sin(\omega_n t + \phi)$ (m)

$\dot{x} = (0.3)(\omega_n)\cos(\omega_n t + \phi)$ (m/s)

$\ddot{x} = -(0.3)(\omega_n)^2 \sin(\omega_n t + \phi)$ (m/s)

$|a_m| = (0.3\,m/s)(\omega_n)^2$ $a_m = 5\,m/s^2$

$\omega_n^2 = |a_m|/(0.3\,m) = (5\,m/s^2)/(0.3\,m) = 16.667\,rad/s^2$

$\omega_n = 4.082$ rad/s $f_n = \omega_n/2\pi$

$f_n = (4.082\,rad/s)/(2\pi\,rad/cycle) = 0.6497$ Hz

$f_n = 0.650$ Hz

$v_m = x_m \omega_n = (0.3\,m)(4.082\,rad/s)$

$v_m = 1.225$ m/s

19.4

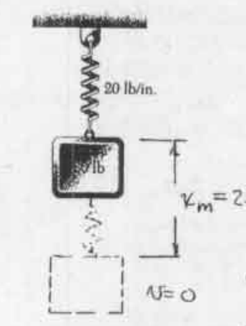
GIVEN:

BLOCK W = 30 lb
SPRING $k = 20$ lb/in.
INITIAL DEFLECTION = 2.1 in.
RELEASED FROM REST

FIND:

(a) PERIOD τ_n, AND FREQUENCY, f_n
(b) MAXIMUM VELOCITY, v_m
AND ACCELERATION, a_m

(a)



$x = x_m \sin(\omega_n t + \phi)$

$\omega_n = \sqrt{k/m}$ $k = 20\,lb/in = 240\,lb/ft$

$\omega_n = \sqrt{(240\,lb/ft)/(30\,lb)/(32.2\,ft/s^2)}$

$\omega_n = 16.050$ rad/s $\tau_n = \frac{2\pi}{\omega_n}$

$\tau_n = 2\pi/16.050 = 0.3915$ s

$\tau_n = 0.391$ s

$f_n = 1/\tau_n = 1/0.391 = 2.55$ Hz

(b) $x_m = 2.1$ in. $= 0.175$ ft

$x = 0.175\sin(16.050t + \phi)$

MAXIMUM VELOCITY

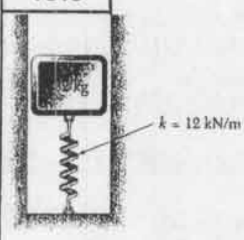
$v_m = x_m \omega_n = (0.175\,ft)(16.050\,rad/s)$

$v_m = 2.81$ ft/s

$a_m = x_m \omega_n^2 = (0.175\,ft)(16.050\,rad/s)^2$

$a_m = 45.1$ ft/s²

19.5



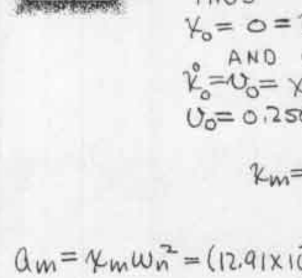
GIVEN:

BLOCK $m = 32$ kg
SPRING $k = 12$ kN/m
INITIAL VELOCITY
$v_0 = 250$ mm/s
INITIAL DISPLACEMENT = 0

FIND:

(a) PERIOD τ_n AND FREQ, f_n
(b) AMPLITUDE x_m
MAXIMUM ACCELERATION, a_m

(a)



$x = x_m \sin(\omega_n t + \phi)$

$\omega_n = \sqrt{k/m} = \sqrt{\frac{12 \times 10^3\,N/m}{32\,kg}}$

$\omega_n = 19.365$ rad/s

$\tau_n = 2\pi/\omega_n$

$\tau_n = 2\pi/19.365$

$\tau_n = 0.3245$

$f_n = 1/\tau_n = 1/0.324 = 3.08$ Hz

(b) @ $t = 0$, $x_0 = 0$, $\dot{x}_0 = v_0 = 250$ mm/s

THUS

$x_0 = 0 = x_m \sin(\omega_n(0) + \phi)$

AND $\phi = 0$

$\dot{x}_0 = v_0 = x_m \omega_n \cos(\omega_n(0) + 0) = x_m \omega_n$

$v_0 = 0.250\,m/s = x_m (19.365\,rad/s)$

$x_m = (0.250\,m/s)/(19.365\,rad/s)$

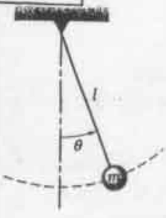
$x_m = 12.91 \times 10^{-3}$ m

$x_m = 12.91$ mm

$a_m = x_m \omega_n^2 = (12.91 \times 10^{-3}\,m)(19.365\,rad/s)^2$

$a_m = 4.84$ m/s²

19.6



GIVEN:

PENDULUM IN SIMPLE HARMONIC MOTION

PERIOD $\tau_n = 1.3\,s$

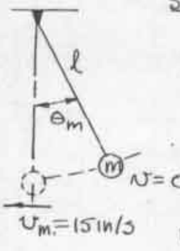
MAXIMUM VELOCITY, $v_m = 15$ in./s

FIND:

(a) AMPLITUDE OF THE MOTION, θ_m IN DEGREES

(b) THE MAXIMUM TANGENTIAL ACCELERATION $(a_t)_m$

(a)



SIMPLE HARMONIC MOTION

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

$$\omega_n = 2\pi/\tau_n = (2\pi)/(1.3\,s)$$

$$\omega_n = 4.833 \text{ rad/s}$$

$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\dot{\theta}_m = \theta_m \omega_n$$

$$v_m = l\dot{\theta}_m = l\theta_m \omega_n$$

$$\theta_m = v_m / l\omega_n \qquad (1)$$

FOR A SIMPLE PENDULUM

$$\omega_n = \sqrt{g/l}$$

THUS

$$l = g/\omega_n^2 = \frac{32.2 \text{ ft/s}^2}{(4.833 \text{ rad/s})^2}$$

$$l = 1.378 \text{ ft}$$

FROM (1) $\theta_m = v_m / l\omega_n = (15/12 \text{ ft/s})/(1.378 \text{ ft})(4.833 \tfrac{\text{rad}}{\text{s}})$

$$\theta_m = 0.18769 \text{ rad} = 10.75°$$

(b) $a_t = l\ddot{\theta}$

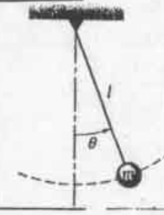
MAX TANGENTIAL ACCELERATION OCCURS WHEN $\ddot{\theta}$ IS MAXIMUM, $\ddot{\theta} = -\theta_m \omega_n^2 \sin(\omega_n t + \phi)$

$$\ddot{\theta}_{MAX} = \theta_m \omega_n^2, \quad (a_t)_{MAX} = l\,\theta_m \omega_n^2$$

$$(a_t)_{MAX} = (1.378 \text{ ft})(0.1879 \text{ rad})(4.833 \tfrac{\text{rad}}{\text{s}})^2$$

$$(a_t)_m = 6.04 \text{ ft/s}^2$$

19.7



GIVEN:

SIMPLE PENDULUM

$l = 800$ mm, $\theta_{MAX} = 6°$

FIND:

(a) FREQUENCY OF OSCILLATION, f_n

(b) MAXIMUM VELOCITY v_m OF THE BOB

(a) $\omega_n = \sqrt{g/l} = \sqrt{(9.81 \text{ m/s}^2)/(0.8 \text{ m})}$

$$\omega_n = 3.502 \text{ rad/s}$$

$$f_n = \omega_n / 2\pi = (3.502 \text{ rad/s})/2\pi$$

$$f_n = 0.557 \text{ Hz}$$

(b) $\theta = \theta_m \sin(\omega_n t + \phi)$

$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

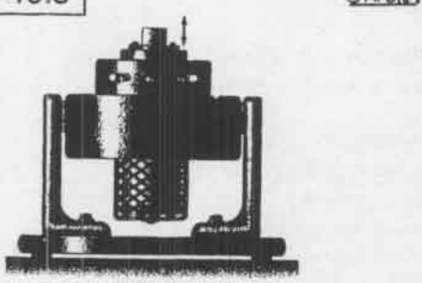
$$\dot{\theta}_m = \theta_m \omega_n$$

$$v_m = l\dot{\theta}_m = l\,\theta_m \omega_n = (0.8 \text{ m})(6°)\left(\frac{\pi \text{ rad}}{180°}\right)(3.502 \tfrac{\text{rad}}{\text{s}})$$

$$v_m = 293.4 \times 10^{-3} \text{ m/s}$$

$$v_m = 293 \text{ mm/s}$$

19.8



GIVEN:

PACKAGE A IN SIMPLE HARMONIC MOTION AT A FREQUENCY WHICH IS THE SAME AS THE MOTOR WHICH DRIVES IT.

PEAK ACCERATION $= 150$ ft/s²

AMPLITUDE $= 2.3$ in.

FIND:

REQUIRED SPEED OF THE MOTOR IN RPM

MAXIMUM VELOCITY OF THE TABLE (PACKAGE)

IN SIMPLE HARMONIC MOTION

$$a_{MAX} = x_{MAX}\,\omega_n^2$$

$$150 \text{ ft/s}^2 = \left(\frac{2.3}{12} \text{ ft}\right)\omega_n^2$$

$$\omega_n^2 = (782.6 \text{ rad/s})^2$$

$$\omega_n = 27.98 \text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{27.98}{2\pi} = 4.452 \text{ Hz (CYCLE/s)}$$

$$1 \text{ RPM} = 1 \text{ CYCLE}/((1 \text{ min.})(60 \text{ s/min})) = 1/60 \text{ (Hz.)}$$

$$(f \text{ Hz})/(1/60 \text{ Hz}) = \frac{4.452}{(1/60)} = 267 \text{ RPM}$$

MAXIMUM VELOCITY $v_{MAX} = x_{MAX}\,\omega_n = \left(\frac{2.3}{12} \text{ ft}\right)(27.98 \text{ rad/s})$

$$v_{MAX} = 5.36 \text{ ft/s}$$

19.9

GIVEN:

PARTICLE MOTION

$$x = 5\sin 2t + 4\cos 2t \quad (\text{m, s})$$

FIND:

(a) PERIOD, τ_n

(b) AMPLITUDE, x_m

(c) PHASE ANGLE, ϕ

FOR SIMPLE HARMONIC MOTION

$$x = x_m \sin(\omega_n t + \phi)$$

DOUBLE ANGLE FORMULA (TRIGONOMETRY)

$$\sin(A+B) = (\sin A)(\cos B) + (\sin B)(\cos A)$$

LET $A = \omega_n t$, $B = \phi$

THEN $x = x_m \sin(\omega_n t + \phi)$

$$x = x_m(\sin\omega_n t)(\cos\phi) + x_m(\sin\phi)(\cos\omega_n t)$$

$$x = (x_m\cos\phi)(\sin\omega_n t) + (x_m\sin\phi)(\cos\omega_n t)$$

GIVEN $x = 5\sin 2t + 4\cos 2t$

COMPARING, $\omega_n = 2 \qquad x_m\cos\phi = 5 \quad (1)$

$$x_m\sin\phi = 4 \quad (2)$$

(a) $\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{(2 \text{ rad/s})} = \pi \text{ s}$

$$\tau = 3.14 \text{ s}$$

(b) SQUARING (1) AND (2) AND ADDING,

$$x_m^2\cos^2\phi + x_m^2\sin^2\phi = 4^2 + 5^2$$

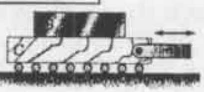
$$x_m^2(\cos^2\phi + \sin^2\phi) = x_m^2 = 41 \text{ m}^2$$

$$x_m = 6.40 \text{ m}$$

(c) DIVIDE (2) BY (1)

$$\tan\phi = \frac{4}{5} \qquad \phi = 38.7°$$

19.10



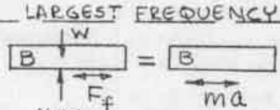
GIVEN:

TABLE C MOVES IN SIMPLE HARMONIC MOTION WITH AMPLITUDE 3 in

$\mu_s = 0.65$ BETWEEN BLOCK B AND C

FIND:

LARGEST FREQUENCY ALLOWED FOR NO SLIDING



NEWTON'S LAW

$F_f = ma$

BLOCK B MOVES IN SIMPLE HARMONIC MOTION WITH THE SAME FREQUENCY AS C WHEN THERE IS NO SLIDING

$x = x_m \sin(\omega_n t + \phi)$

$\ddot{x} = -x_m \omega_n^2 \sin(\omega_n t + \phi)$

MAXIMUM ACCELERATION

$a = x_m \omega_n^2$

$F_f = m x_m \omega_n^2$

FOR NO SLIDING

$F_f > \mu_s W$ OR $\mu_s W > \frac{W}{g} x_m \omega_n^2$

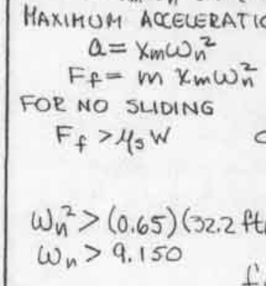
$\omega_n^2 > \mu_s g / x_m$

$\omega_n^2 > (0.65)(32.2\ ft/s^2)/(3/12\ ft) = 83.72\ (rad/s)^2$

$\omega_n > 9.150$

$f_n = \omega_n / 2\pi = (9.150)/2\pi = 1.456\ Hz$ ◀

19.11



GIVEN:

INITIAL DISPLACEMENT OF THE BLOCK = 300 mm DOWNWARD

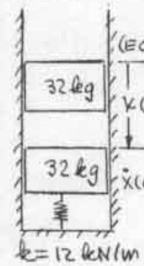
FIND:

1.5 s AFTER THE BLOCK IS RELEASED,

(a) TOTAL DISTANCE TRAVELED BY THE BLOCK

(b) ACCELERATION OF THE BLOCK

(a)



$x = x_m \sin(\omega_n t + \phi)$

$\omega_n = \sqrt{k/m} = \sqrt{\frac{(12 \times 10^3\ N/m)}{(32\ kg)}}$

$\omega_n = 19.365\ rad/s$

$\tau_n = 2\pi/\omega_n = (2\pi)/(19.365)$

$\tau_n = 0.3245\ s$

INITIAL CONDITIONS

$x(0) = 0.3\ m$, $\dot{x}(0) = 0$

$0.3 = x_m \sin(0 + \phi)$

$\dot{x}(0) = 0 = x_m \omega_n \cos(0 + \phi)$

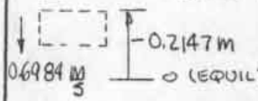
$\phi = \pi/2$

$x_m = 0.3$

$x(t) = (0.3)\sin(19.365 t + \pi/2)$, $\tau_n = 0.3245\ s$

$x(1.5\ s) = (0.3)\sin[(19.365)(1.5) + \pi/2] = -0.2147\ m$

$\dot{x}(1.5\ s) = (0.3)(19.365)\cos[(19.365)(1.5) + \frac{\pi}{2}] = 4.057\ m/s$ ↑



IN ONE CYCLE, BLOCK TRAVELS

$(4)(0.3\ m) = 1.2\ m$

TO TRAVEL 4 CYCLES IT TAKES

$(4\ cyc)(0.3245\ s/cyc) = 1.2980\ s$

At $t = 1.5\ s$ THUS, TOTAL DISTANCE TRAVELED IS $4(1.2) + 0.6 + (0.3 - 0.2147) = 5.49\ m$ ◀

(b)

$\ddot{x}(1.5) = -(0.3)(19.365)^2 \sin[(19.365)(1.5) + \frac{\pi}{2}] = 80.5\ m/s^2$ ↓ ◀

19.12

GIVEN:

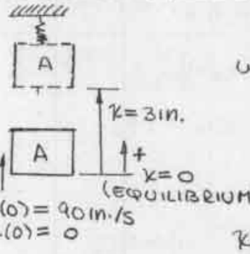
$W_A = 3\ lb$, $k = 2\ lb/in$

INITIAL VELOCITY OF A = 90 in./s ↑

FIND:

(a) TIME REQUIRED FOR THE BLOCK TO MOVE 3 in UPWARD

(b) CORRESPONDING VELOCITY AND ACCELERATION



$\dot{x}(0) = 90\ in./s$

$x(0) = 0$

$x = x_m \sin(\omega_n t + \phi)$

$\omega_n = \sqrt{\frac{k}{m}}$, $k = 2\ lb/in = 24\ \frac{lb}{ft}$

$\omega_n = \sqrt{\frac{24\ lb/ft}{(3\ lb/32.2\ ft/s^2)}}$

$\omega_n = 16.05\ rad/s$

$x(0) = 0 = x_m \sin(0 + \phi)$

$\phi = 0$

$\dot{x}(0) = x_m \omega_n \cos(0 + 0)$ $\quad \dot{x}(0) = \frac{90}{12} = 7.5\ ft/s$

$7.5 = x_m (16.05)$ $\quad x_m = 0.4673\ ft$

$x = (0.4673)\sin(16.05 t)\ (ft, s)$ (1)

(a) AT $x = 3/12 = 0.25\ ft$

$0.25 = 0.4673 \sin(16.05 t)$

$t = \frac{\sin^{-1}(0.25)/(0.4673)}{16.05} = 0.0352\ s$ ◀

(b) $\dot{x} = x_m \omega_n \cos(\omega_n t)$ $\quad \ddot{x} = -x_m \omega_n^2 \sin \omega_n t$

$t = 0.0352$, $\dot{x} = (0.4673)(16.05)\cos[(16.05)(.0352)]$

$\dot{x} = 6.34\ ft/s$ ↑ ◀

$\ddot{x} = -(.4673)(16.05)^2 \sin[(16.05)(.0352)] = -64.4\ ft/s^2$ ↓ ◀

19.13

REFER TO FIGURE IN PROBLEM 19.12 ABOVE

GIVEN:

$W_A = 3\ lb$, $k = 2\ lb/in$, $v_0 = 90\ in./s$ ↑ (SAME AS 19.12)

FIND:

AFTER 0.90 s, POSITION, VELOCITY AND ACCELERATION OF THE BLOCK

$x = x_m \sin(\omega_n t + \phi)$

$\dot{x} = x_m \omega_n \cos(\omega_n t + \phi)$

$\ddot{x} = -x_m \omega_n^2 \sin(\omega_n t + \phi)$

SINCE THE GIVEN DATA IS THE SAME AS IN PROBLEM 19.12 ABOVE, THE EQUATION OF MOTION IS THE SAME AS EQUATION (1) IN 19.12

$\phi = 0$, $x_m = 0.4673\ ft$, $\omega_n = 16.05\ rad/s$ AND $x, \dot{x}, \ddot{x}$ ARE + ↑

$x = (0.4673)\sin(16.05 t)\ (ft, s)$ (1)

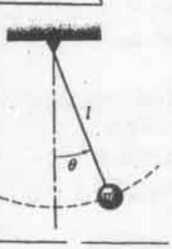
AT 0.90 s

$x = (0.4673)\sin[(16.05)(0.90)] = 0.445\ ft$ ↑ ◀

$\dot{x} = (0.4673)(16.05)\cos[(16.05)(0.90)] = -2.27\ ft/s$ ↓ ◀

$\ddot{x} = -(0.4673)(16.05)^2 \sin[(16.05)(0.090)] = 114.7\ ft/s^2$ ↓ ◀

19.14



GIVEN:

$\ell = 800$ mm

AT $t=0$, $\theta = +5°$, $\dot\theta = 0$

ASSUME SIMPLE HARMONIC MOTION

FIND:

1.6 S AFTER RELEASE

(a) θ

(a) v AND a OF THE BOB.

$\theta = \theta_m \sin(\omega_n t + \phi)$ $\quad \omega_n = \sqrt{\frac{g}{\ell}} = \sqrt{\frac{9.81\,m/s}{0.8\,m}}$

INITIAL CONDITIONS

$\theta(0) = 5° = (5)(\pi)/180$ RAD $\quad \omega_n = 3.502$ RAD/S

$\dot\theta(0) = 0$

$\theta(0) = \frac{5\pi}{180} = \theta_m \sin(0+\phi)$

$\dot\theta(0) = 0 = \theta_m \omega_n \cos(0+\phi)$ $\quad \phi = \pi/2$

$\theta_m = \frac{5\pi}{180}$ RAD

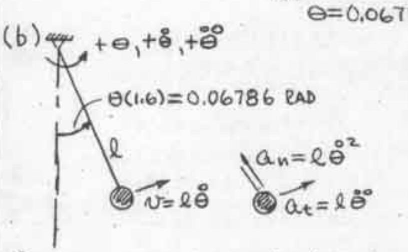
$\theta = \frac{5\pi}{180}\sin(3.502t + \frac{\pi}{2})$

(a)

AT $t = 1.6$ s $\quad \theta = \frac{5\pi}{180}\sin[(3.502)(1.6) + \frac{\pi}{2}]$

$\theta = 0.06786$ RAD $= 3.89°$

(b)



$\dot\theta = \theta_m \omega_n \cos(\omega_n t + \phi) = \left(\frac{5\pi}{180}\right)(3.502)\cos[(3.502)(1.6) + \frac{\pi}{2}]$

$\dot\theta(1.6\,s) = 0.19223$ RAD/S

$v = \ell\dot\theta = (0.800\text{ m})(0.19223\text{ RAD/S}) = 0.1538$ m/s

$\ddot\theta = -\theta_m \omega_n^2 \sin(\omega_n t + \phi) = -\left(\frac{5\pi}{180}\right)(3.502)^2 \sin[(3.502)(1.6) + \frac{\pi}{2}]$

$\ddot\theta = -0.8319$ RAD/S^2

$a = \sqrt{(a_t)^2 + (a_n)^2}$

$a_t = \ell\ddot\theta = (0.8\,m)(-0.8319\text{ RAD/S}^2) = 0.6655\ \frac{m}{s^2}$

$a_n = \ell\dot\theta^2 = (0.8\,m)(0.19223\text{ RAD/S})^2 = 0.02956\ \frac{m}{s^2}$

$a = \sqrt{(0.6655)^2 + (0.02956)^2} = 0.6662$ m/s^2

$a = 0.666$ m/s^2

19.15

GIVEN:

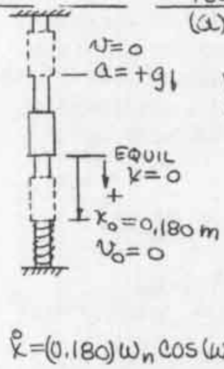
$m = 5$ kg, UNATTACHED TO THE SPRING WHEN COLLAR IS PUSHED DOWN 180 MM OR MORE AND RELEASED IT LOSES CONTACT WITH THE SPRING

FIND:

(a) THE SPRING CONSTANT k

(b) POSITION, VELOCITY AND ACCELERATION 0.16 S AFTER IT IS PUSHED DOWN 180 mm AND RELEASED.

(a)



$x = x_m \sin(\omega_n t + \phi)$

$x_0 = x_m \sin(0+\phi) = 0.180$ m

$\dot x_0 = 0 = x_m \cos(0+\phi)$

$\phi = \pi/2$

$x_m = 0.180$ m

$x = 0.180\sin(\omega_n t + \frac{\pi}{2})$

WHEN THE COLLAR JUST LEAVES THE SPRING, its ACCELERATION IS $g\downarrow$ AND $v=0$

$\dot x = (0.180)\omega_n \cos(\omega_n t + \frac{\pi}{2})$

$v=0 \quad 0 = (0.180)\omega_n \cos(\omega_n t + \pi/2)$

$(\omega_n t + \pi/2) = \pi/2$

$a = -g = (-0.180)(\omega_n)^2 \sin(\omega_n t + \pi/2)$

$-g = (-0.180)(\omega_n^2) \quad \omega_n = \sqrt{\frac{9.81\,m/s^2}{0.180\,m}}$

$\omega_n = 7.382$ RAD/S

$\omega_n = \sqrt{k/m}$

$k = m\omega_n^2 = (5\,kg)(7.382\text{ RAD/S})^2 = 272.5$ N/m

$k = 273$ N/m

(b) $\omega_n = 7.382$ RAD/S

$x = 0.180\sin[(7.382)t + \pi/2]$

At $t = 0.16$ s

POSITION

$x = 0.180\sin[(7.382)(0.16) + \pi/2] = 0.06838$ M

$x = 68.4$ mm

BELOW EQUILIBRIUM POSITION

VELOCITY

$\dot x = (0.180)(7.382)\cos[(7.382)(0.16) + \pi/2] = -1.229$ m/s

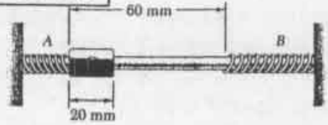
$\dot x = 1.229$ m/s $\uparrow$

ACCELERATION

$\ddot x = -(0.180)(7.382)^2 \sin[(7.382)(0.16) + \frac{\pi}{2}] = -3.726\ \frac{m}{s^2}$

$\ddot x = 3.73$ m/s^2 $\uparrow$

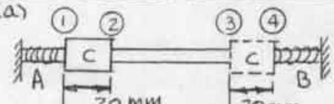
19.16



GIVEN:
$m_c = 8\,kg$
$k = 600\ N/m$
FOR EACH SPRING
INITIAL DEFLECTION OF SPRING A $= 20\,mm$.
NO FRICTION

FIND:
(a) PERIOD
(b) POSITION OF C AFTER 1.5 S

(a)



FOR EITHER SPRING
$\tau_n = \dfrac{2\pi}{\sqrt{k/m_c}}$

$\tau_n = \dfrac{2\pi}{\sqrt{600\ N/m / 8\,kg}}$

$\tau_n = 0.7255\,s$

COMPLETE CYCLE IS 1234 4321
TIME FROM 1 TO 2 IS $\tau_n/4$ WHICH IS THE SAME AS TIME FROM 3 TO 4, 4 TO 3 AND 2 TO 1
THUS THE TIME DURING WHICH THE SPRINGS ARE COMPRESSED IS $4(\tau_n/4) = \tau_n = 0.7255\,s$

VELOCITY AT 2 (OR 3).
$v_1 = 0 \quad T_1 = 0 \quad V_1 = \frac{1}{2} k x^2 = \frac{1}{2}(600\,N/m)(0.020\,m)^2$

$V_1 = 0.120\,J$

$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}(8\,kg)(v_2)^2 \quad T_2 = 4v_2^2$

$V_2 = 0$

$T_1 + V_1 = T_2 + V_2 \quad 0 + 0.120 = 4v_2^2 \quad v_2 = 0.1732\,m/s$

TIME FROM 2 TO 3 IS $t_{2\text{-}3} = \dfrac{(0.020\,m)}{(0.1732\,m/s)} = 0.11545\,s$

AND IS THE SAME AS THE TIME FROM 3 TO 2.
THUS

TOTAL TIME FOR A COMPLETE CYCLE IS
$\tau_c = \tau_n + 2t_{2\text{-}3} = 0.7255 + 2(0.11545) = 0.9564$

$\tau_c = 0.956\,s$

(b) FROM (a), IN 0.9564 THE SPRING A IS AGAIN FULLY COMPRESSED. SPRING B IS COMPRESSED THE SECOND TIME IN 1.5 CYCLES OR $(1.5)(0.9564) = 1.4346\,s$. AT 1.5 S THE COLLAR IS STILL IN CONTACT WITH SPRING B MOVING TO THE LEFT AND IS AT A DISTANCE Δx FROM THE MAXIMUM DEFLECTION OF B EQUAL TO

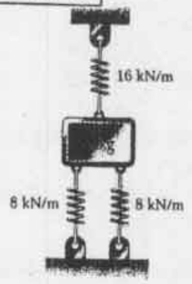
$\Delta x = 20 - 20\cos\left[\frac{2\pi}{0.7255}(1.5 - 1.4346)\right]$

$\Delta x = 20 - 16.877 = 3.123\,mm$

THUS COLLAR C IS $60 - 3.123 = 56.877\,mm$ FROM ITS INITIAL POSITION

56.9 mm
FROM INITIAL POSITION

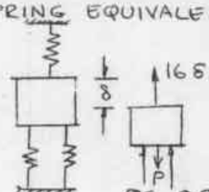
19.17



GIVEN:
MASS AND SPRINGS AS SHOWN AFTER THE MASS IS PULLED DOWN AND RELEASED FROM REST THE AMPLITUDE OF THE RESULTING MOTION IS 45 mm

FIND:
(a) THE PERIOD AND FREQUENCY OF THE MOTION
(b) THE MAXIMUM VELOCITY AND ACCELERATION OF THE BLOCK

(a) DETERMINE THE CONSTANT k OF A SINGLE SPRING EQUIVALENT TO THE THREE SPRINGS



$P = k\delta = 16\delta + 8\delta + 8\delta$
$k = 32\ kN/m$

$\omega_n = \sqrt{k/m} = \sqrt{\dfrac{32\times10^3\,N/m}{35\,kg}}$

$(1N = 1\,kg \cdot 1\,m/s^2)$

$\omega_n = 30.237\ rad/s$

$\tau_n = 2\pi/\omega_n = 2\pi/30.23 = 0.208\,s$

$f_n = 1/\tau_n = 4.81\,Hz$

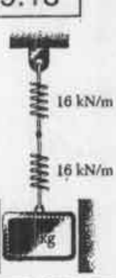
(b) $x = x_m \sin(\omega_n t + \phi) \quad x_0 = 0.045\,m = x_m$
$\omega_n = 30.24\ rad/s$

$x = 0.045\sin(30.24t + \phi)$

$\dot{x} = (0.045)(30.24)\cos(30.24t + \phi) \quad v_{max} = 1.361\,m/s$

$\ddot{x} = -(0.045)(30.237)^2\sin(30.24t + \phi) \quad a_{max} = 41.1\,\frac{m}{s^2}$

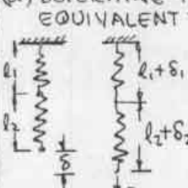
19.18



GIVEN:
MASS AND SPRINGS AS SHOWN AMPLITUDE OF MOTION IS 45 mm AFTER MASS IS PULLED DOWN AND RELEASED FROM REST

FIND:
(a) PERIOD AND FREQUENCY OF MOTION
(b) MAXIMUM VELOCITY AND ACCELERATION

(a) DETERMINE THE CONSTANT k OF A SINGLE SPRING EQUIVALENT TO THE TWO SPRINGS SHOWN



$\delta = \delta_1 + \delta_2 = \dfrac{P}{16\,kN/m} + \dfrac{P}{16\,kN/m} = \dfrac{P}{k}$

$\frac{1}{k} = \frac{1}{16} + \frac{1}{16} \quad k = 8\,kN/m$

$\tau_n = \dfrac{2\pi}{\sqrt{k/m}} = \dfrac{2\pi}{\sqrt{\frac{8\times10^3}{35}}} = 0.416\,s$

$f_n = \frac{1}{\tau_n} = \frac{1}{0.416} = 2.41\,Hz$

(b) $\omega_n = 2\pi f_n = 2\pi(2.41) = 15.12\ rad/s$

$x = 0.045\sin(15.12t + \phi)$

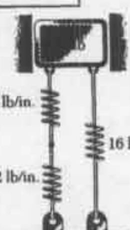
$\dot{x} = (0.045)(15.12)\cos(15.12t + \phi)$

$v_{max} = 0.680\,m/s$

$\ddot{x} = -(0.045)(15.12)^2\sin(15.12t + \phi)$

$a_{max} = 10.29\,m/s^2$

19.19



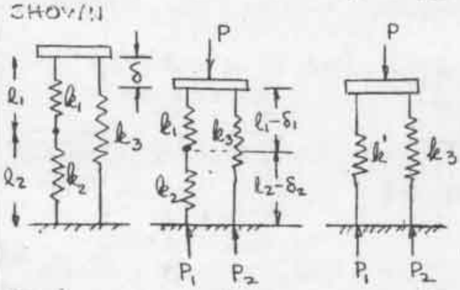
GIVEN:

30 lb BLOCK
AT $t=0$, $x=1.75$ in. DOWNWARD, $v=0$

FIND:

(a) PERIOD AND FREQUENCY OF MOTION
(b) MAXIMUM VELOCITY AND ACCELERATION

DETERMINE THE CONSTANT k OF A SINGLE SPRING EQUIVALENT TO THE THREE SPRINGS SHOWN



$k_1 = 20$ lb/in.
$k_2 = 12$ lb/in
$k_3 = 16$ lb/in

SPRINGS 1 AND 2 (FORCE IN EACH SPRING IS THE SAME)

$\delta = \delta_1 + \delta_2 \quad \frac{P_1}{k'} = \frac{P_1}{k_1} + \frac{P_1}{k_2}$

$k' = \frac{k_1 k_2}{k_1 + k_2}$

k' IS THE SPRING CONSTANT OF A SINGLE SPRING EQUIVALENT TO SPRINGS 1 AND 2

SPRINGS k' AND 3 (DEFLECTION IN EACH SPRING IS THE SAME)

$P = P_1 + P_2 \quad P = k\delta \quad P_1 = k'\delta \quad P_2 = k_3\delta$

$k\delta = k'\delta + k_3\delta$

$k = k' + k_3 = \frac{k_1 k_2}{k_1 + k_2} + k_3$

$k = \frac{(20)(12)}{(20+12)} + 16 = 23.5$ lb/in. $= 282$ lb/ft

(a) $\tau_n = \frac{2\pi}{\sqrt{k/m}} = \frac{2\pi}{\sqrt{\frac{(282 \text{ lb/ft})}{(30 \text{ lb}/32.2 \text{ ft/s}^2}}} = 0.361$ s

$f_n = 1/\tau_n = 1/0.361 = 2.77$ Hz

(b) $x = x_m \sin(\omega_n t + \phi)$

$x_m = 1.75$ in $= 0.1458$ ft

$\omega_n = 2\pi f_n = (2\pi)(2.77) = 17.40$ rad/s

$x = (0.1458)(\sin 17.41t + \phi)$

$\dot{x} = (0.1458)(17.40)\sin(17.41t + \phi)$

$v_{MAX} = (0.1458)(17.41) = 2.54$ ft/s

$\ddot{x} = -(0.1458)(17.40)^2\cos(0.1741t + \phi)$

$a_{MAX} = 44.14$ ft/s

$a_{MAX} = 44.1$ ft/s^2

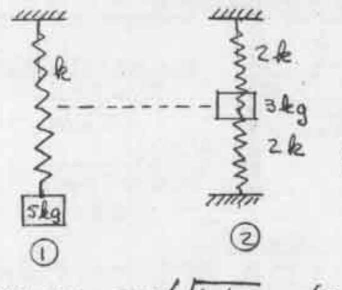
19.20

GIVEN:

5-kg BLOCK ATTACHED TO A SPRING FIXED AT THE OTHER END VIBRATES WITH A PERIOD $\tau_n = 6.8$ s. SPRING CONSTANT k IS INVERSELY PROPORTIONAL TO THE SPRINGS' LENGTH.

FIND:

THE PERIOD FOR A 3 kg BLOCK ATTACHED TO THE CENTER OF THE SAME SPRING FIXED AT BOTH ENDS



EQUIVALENT SPRING CONSTANT
$k' = 2k + 2k = 4k$
(DEFLECTION OF EACH SPRING IS THE SAME)

$(\tau_n)_1 = 6.8 = 2\pi/\sqrt{k/(5 \text{ kg})} \quad (\tau_n)_2 = 2\pi/\sqrt{4k/(3 \text{ kg})}$

$k = \frac{(2\pi)^2}{(6.8 \text{ s})^2}(5 \text{ kg}) \quad (\tau_n)_2 = 2\pi/\sqrt{4(4.269 \text{ N/m})/(3 \text{ kg})}$

$k = 4.269$ N/m $\quad (\tau_n)_2 = 2.63$ s

19.21

GIVEN:

SYSTEM AS SHOWN IS MOVED 0.8 in. DOWNWARD AND RELEASED FROM REST.
PERIOD FOR RESULTING MOTION IS $\tau_n = 1.5$ s

FIND:

(a) CONSTANT k
(b) MAXIMUM VELOCITY AND ACCELERATION OF THE BLOCK

SINCE THE FORCE IN EACH SPRING IS THE SAME, THE CONSTANT k' OF A SINGLE EQUIVALENT SPRING IS

$\frac{1}{k'} = \frac{1}{2k} + \frac{1}{k} + \frac{1}{k} \quad k' = k/2.5$ (SEE PROB 19.19)

(a) $\tau_n' = 1.5$ s. $= 2\pi/\sqrt{k'/m}$; $k = \left(\frac{2\pi}{1.5}\right)^2\left(\frac{W}{g}\right)(2.5)$

$k = \frac{(2\pi)^2}{(1.5 \text{ s})^2}\left(\frac{30 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(2.5) = 40.868$ lb/ft

$k = 40.9$ lb/ft

(b) $x = x_m \sin(\omega_n t + \phi)$

$\dot{x} = x_m \omega_n \cos(\omega_n t + \phi) \quad v_{MAX} = x_m \omega_n$

$\omega_n = \frac{2\pi}{\tau_n} = \frac{2\pi}{1.5 \text{ s}} = 4.189$ rad/s

$x_m = 0.8$ in. $= 0.06667$ ft

$v_{MAX} = (.06667 \text{ ft})(4.189 \text{ rad/s})$

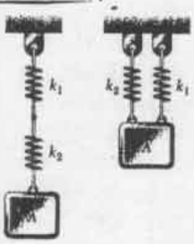
$v_{MAX} = 0.279$ ft/s

$\ddot{x} = -x_m \omega_n^2 \cos(\omega_n t + \phi)$

$|a_{MAX}| = x_m \omega_n^2 = (0.06667 \text{ ft})(4.189)^2$

$|a_{MAX}| = 1.170$ ft/s^2

19.22



GIVEN:

PERIOD FOR SPRINGS IN SERIES, $\tau_S = 5\,s$

PERIOD FOR SPRINGS IN PARALLEL, $\tau_P = 2\,s$

FIND:

RATIO OF SPRING CONSTANTS k_1/k_2

EQUIVALENT SPRINGS

SERIES, $k_S = \dfrac{k_1 k_2}{(k_1 + k_2)}$

PARALLEL, $k_P = k_1 + k_2$

$$\tau_S = \frac{2\pi}{\omega_S} = \frac{2\pi}{\sqrt{k_S/m}} \qquad \tau_P = \frac{2\pi}{\omega_P} = \frac{2\pi}{\sqrt{k_P/m}}$$

$$\left(\frac{\tau_S}{\tau_P}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{k_P}{k_S} = \frac{(k_1+k_2)}{(k_1 k_2)/(k_1+k_2)} = \frac{(k_1+k_2)^2}{k_1 k_2}$$

$$(6.25)(k_1 k_2) = k_1^2 + 2k_1 k_2 + k_2^2$$

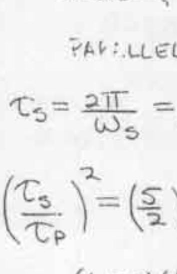
$$k_1^2 - (4.25)k_2 k_1 + k_2^2 = 0$$

$$k_1 = \frac{(4.25)k_2 \mp \sqrt{(4.25)^2 k_2^2 - 4k_2^2}}{2}$$

$$\frac{k_1}{k_2} = 2.125 \mp \sqrt{3.516}$$

$$k_1/k_2 = 4$$

19.23



GIVEN:

PERIOD $= 0.7\,s = \tau$

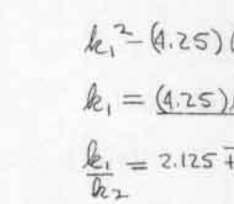
AFTER k_2 IS REMOVED PERIOD $= 0.9\,s = \tau'$

FIND:

(a) k_1

(b) MASS OF A

EQUIVALENT SPRINGS



$$F_1 = k_1\delta \quad F_2 = k_2\delta$$

$$F_1 + F_2 = F = k_e\delta$$

$$k_1\delta + k_2\delta = k_e\delta$$

$$k_e = k_1 + k_2$$

(a) BOTH SPRINGS $\tau = \dfrac{2\pi}{\sqrt{k_e/m}} = 0.7\,s$

k_1 ALONE, $\tau' = \dfrac{2\pi}{\sqrt{k_1/m}} = 0.9\,s$

$$\frac{\tau}{\tau'} = \frac{0.7}{0.9} = \sqrt{\frac{k_1}{k_e}} = \sqrt{\frac{k_1}{k_1+k_2}}$$

$$\left(\frac{.7}{.9}\right)^2 = 0.6049 = \frac{k_1}{k_1+1.2}$$

19.23 CONTINUED

$$(0.6049)(k_1 + 1.2) = k_1$$

$$k_1 = 1.838\ kN/m$$

(b) $\tau' = \dfrac{2\pi}{\sqrt{k_1/m}}$ $\quad (0.9\,s)^2 = \dfrac{(2\pi)^2 m}{(1.838 \times 10^3\ N/m)}$

$$m = \frac{(0.9\,s)^2(1.838 \times 10^3\ N/m)}{(2\pi)^2}$$

$$m = 37.7\ kg$$

19.24

GIVEN:

PERIOD FOR SYSTEM SHOWN IS $\tau = 1.6\,s$

PERIOD AFTER A 7-kg COLLAR IS PLACED ON A, IS $\tau' = 2.1\,s$

FIND:

(a) MASS OF A

(b) k

INITIALLY $\tau = \dfrac{2\pi}{\sqrt{k/m_A}} = 1.6\,s$

AFTER 7 kg MASS IS ADDED TO A,

$$\tau' = \frac{2\pi}{\sqrt{k/(m_A+7)}} = 2.1\,s$$

(a)

$$\frac{\tau'}{\tau} = \sqrt{\frac{(m_A+7)}{m_A}}$$

$$\left(\frac{2.1}{1.6}\right)^2 = \frac{m_A + 7}{m_A}$$

$$(1.7227)(m_A) = m_A + 7$$

$$m_A = 9.69\ kg$$

(b)

$$\tau = \frac{2\pi}{\sqrt{k/m_A}}$$

$$k = (2\pi)^2(m_A)/(\tau)^2$$

$$k = (2\pi)^2(9.69\ kg)/(1.6\,s)^2$$

$$k = 149.4\ kg/s^2$$

$$k = 149.4\ N/m$$

19.25

GIVEN:

FOR SYSTEM SHOWN
PERIOD $\tau = 0.2$ S
AFTER k_2 IS REMOVED AND BLOCK A IS CONNECTED TO k_1, PERIOD $\tau' = 0.12$ S

FIND:
(a) k_1
(b) WEIGHT OF BLOCK A.

EQUIVALENT SPRING CONSTANT FOR SPRINGS IN SERIES.

$$k_e = \frac{k_1 k_2}{(k_1 + k_2)}$$

FOR k_1 AND k_2

$$\tau = \frac{2\pi}{\sqrt{k_e/m_A}} = \frac{2\pi}{\sqrt{(k_1 k_2)/(m_A)(k_1+k_2)}}$$

FOR k_1 ALONE

$$\tau' = \frac{2\pi}{\sqrt{k_1/m_A}}$$

(a)

$$\frac{\tau}{\tau'} = \sqrt{\frac{(k_1+k_2)(k_1)}{(k_1 k_2)}} = \sqrt{\frac{k_1+k_2}{k_2}}$$

$$k_2\left(\frac{\tau}{\tau'}\right)^2 = k_1 + k_2$$

$$\tau/\tau' = 0.2/0.12 = 1.6667, \quad k_2 = 20 \text{ lb/in}$$

$$(20 \text{ lb/in.})(1.6667 \text{ s})^2 = k_1 + 20 \text{ lb/in}$$

$$k_1 = 35.6 \text{ lb/in.}$$

(b)

$$\tau' = \frac{2\pi}{\sqrt{k_1/m_A}} \qquad m_A = W_A/g$$

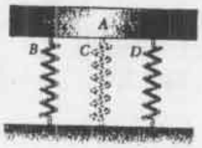
$$m_A = \frac{(\tau')^2 k_1}{(2\pi)^2} \qquad k_1 = 35.6 \text{ lb/in.} = 426.7 \frac{\text{lb}}{\text{ft}}$$

$$W_A = \frac{(32.2 \text{ ft/s}^2)(0.12 \text{ s})^2(426.7 \text{ lb/ft})}{(2\pi)^2}$$

$$W_A = 5.01 \text{ lb}$$

19.26

GIVEN:



$W_A = 100$ lb
$k_B = k_D = k = 120$ lb/ft
FREQUENCY REMAINS THE SAME WHEN AN 80 lb BLOCK IS ADDED TO A AND A SPRING OF CONSTANT k_C IS ADDED TO THE SYSTEM

FIND: k_C

FREQUENCY OF THE ORIGINAL SYSTEM

SPRINGS B AND D ARE IN PARALLEL

$$k_e = k_B + k_D = 2(120 \text{ lb/ft}) = 240 \text{ lb/ft}$$

$$\omega_n^2 = \frac{k_e}{m_A} = \frac{240 \text{ lb/ft}}{(100 \text{ lb}/32.2 \text{ ft/s}^2)}$$

$$\omega_n^2 = 77.28 (\text{rad/s})^2$$

FREQUENCY OF NEW SYSTEM

SPRINGS A, B AND C ARE IN PARALLEL

$$k_e' = k_B + k_D + k_C = (2)(120) + k_C$$

$$(\omega_n')^2 = \frac{k_e'}{m_A + m_B} = \frac{(240 + k_C)(32.2 \text{ ft/s}^2)}{(100 \text{ lb} + 80 \text{ lb})}$$

$$(\omega_n')^2 = (0.1789)(240 + k_C)$$

$$\omega_n^2 = (\omega_n')^2$$

$$77.28 = (0.1789)(240 + k_C)$$

$$k_C = 191.97 \text{ lb/ft}$$

$$k_C = 192.0 \text{ lb/ft}$$

19.27

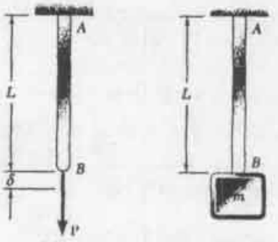
GIVEN:

$\delta = PL/AE$

$L = 450$ mm

$E = 200$ GPa

ROD DIAMETER $= 8$ mm. $m = 8$ kg



FIND:

(a) EQUIVALENT SPRING CONSTANT OF THE ROD, (k_e)

(b) FREQUENCY OF VERTICAL VIBRATIONS OF THE 8-kg MASS

(a) $P = k_e\,\delta \qquad \delta = PL/AE,\ P = \left(\frac{AE}{L}\right)\delta$

$k_e = \frac{AE}{L} \qquad A = \pi d^2/4 = \pi(8\times10^{-3}\,m)^2/4$

$A = 5.027\times10^{-5}\,m^2$

$L = 0.450\,m$

$E = 200\times10^9\,N/m^2$

$k_e = \frac{(5.027\times10^{-5}\,m^2)(200\times10^9\,N/m^2)}{(0.450\,m)}$

$k_e = 22.34\times10^6\,N/m$

$k_e = 22.3\,MN/m$

(b) $f_n = \frac{\sqrt{k_e/m}}{2\pi} = \frac{\sqrt{22.3\times10^6/8}}{2\pi} = 265.96\,Hz$

$f_n = 266\,Hz$

19.28

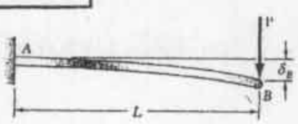
GIVEN:

$\delta_B = PL^3/3EI$

$L = 10$ ft

$E = 29\times10^6\,lb/in^2$

$I = 12.4\,in^4$



FIND:

(a) EQUIVALENT SPRING CONSTANT (k_e)

(b) FREQUENCY OF A 520-lb BLOCK AT B

(a) $P = k_e\,\delta_B \quad \delta_B = PL^3/3EI,\ P = \left(\frac{3EI}{L^3}\right)\delta_B$

$k_e = \frac{3EI}{L^3} = \frac{(3)(29\times10^6\,lb/in^2)(12.4\,in^4)}{(10\times12\,in)^3}$

$k_e = 624.3\,lb/in$

$k_e = 624.3\,lb/in.$

(b) $f_n = \frac{\sqrt{k_e/m}}{2\pi} \qquad k_e = 624.3\,lb/in = 7.492\times10^3\,lb/ft$

$f_n = \frac{\sqrt{(7.492\times10^3\,lb/ft)/[(520\,lb)/(32.2\,ft/s^2)]}}{2\pi}$

$f_n = 3.428\,Hz$

$f_n = 3.43\,Hz$

19.29

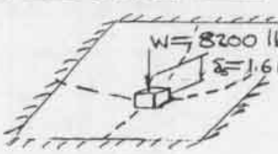
GIVEN:

STATIC DEFLECTION OF THE FLOOR OF A BUILDING UNDER AN 8200-lb PIECE OF MACHINERY EQUALS $\delta_s = 1.6$ in.

FIND:

(a) EQUIVALENT SPRING CONSTANT k_e

(b) THE SPEED IN RPM OF THE MACHINERY THAT SHOULD BE AVOIDED SO AS NOT TO COINCIDE WITH THE NATURAL FREQUENCY OF THE SYSTEM.



(a) $W = k_e\,\delta_s$

$k_e = \frac{W}{\delta} = \frac{8200\,lb}{1.6\,in.}$

$k_e = 5130\,lb/in.$

(b) $f_n = \frac{\sqrt{k_e/m}}{2\pi} = \frac{\sqrt{\frac{(5130\times12\,lb/ft)}{(8200\,lb/32.2\,ft/s^2)}}}{2\pi}$

$f_n = 2.473\,Hz$

$1\,Hz = 1\,CYCLE/S = 60\,RPM$

$SPEED = (2.473\,Hz)\left(\frac{60\,RPM}{Hz}\right) = 148.4\,RPM$

19.30

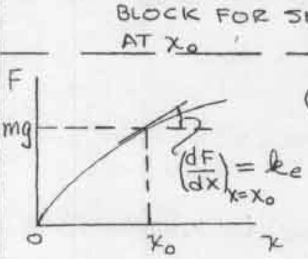
GIVEN:

FORCE-DEFLECTION EQUATION FOR A NON-LINEAR SPRING, $F = 5x^{1/2}$ (N, m)

FIND:

(a) STATIC DEFLECTION x_o UNDER A 120-g BLOCK

(b) FREQUENCY OF VIBRATION OF THE BLOCK FOR SMALL OSCILLATIONS AT x_o



(a) $mg = (0.120\,kg)(9.81\,m/s^2)$

$mg = 1.177\,N$

$F = mg = 5x_o^{1/2}$

$x_o = \left(\frac{1.177}{5}\right)^2 = 0.0554\,m$

$x_o = 55.4\,mm$

(b) AT x_o, $\left(\frac{dF}{dx}\right)_{x_o} = \frac{5}{2}(x_o)^{-\frac{1}{2}} = \frac{5}{2}(.0554)^{-1/2}$

$\left(\frac{dF}{dx}\right)_{x_o} = 10.618\,N/m$

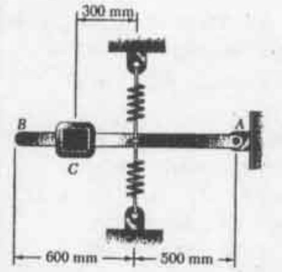
$k_e = 10.618\,N/m$

$f_n = \frac{\sqrt{k_e/m}}{2\pi} = \frac{\sqrt{(10.618\,N/m)/(.120\,kg)}}{2\pi}$

$f_n = 1.4971\,Hz$

$f_n = 1.497\,Hz.$

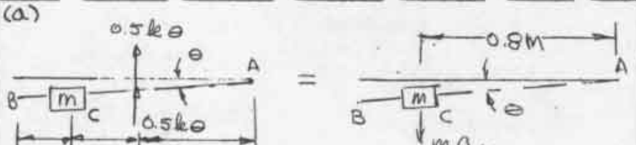
19.31



GIVEN:
$\tau_n = 3\ s$
$k = 900\ N/m$, EACH SPRING, TENSION OR COMPRESSION
NEGLECT THE MASS OF ROD AB

FIND:
(a) MASS m AT C
(b) $(v_C)_{MAX}$ IF B IS DEPRESSED 40 MM AND RELEASED

(a)



$a_t = (0.8)\ddot{\theta}$

NEWTONS LAW $\Sigma M_A = -(0.5)^2 k\theta - (0.5)^2 k\theta = (0.8)^2 m\ddot{\theta}$

$(0.5\ m^2)(900\ N/m)\,\theta = (0.64\ m^2)\ m\ddot{\theta}$

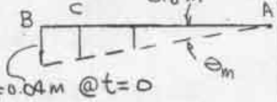
$\ddot{\theta} + \dfrac{(450\ N\cdot m)}{(.64\ m^2)\,m}\ \theta = 0$

$\omega_n^2 = \dfrac{(703.1\ N/m)}{m}$ $\quad \tau_n = \dfrac{2\pi}{\omega_n} = 2\pi\sqrt{\dfrac{m}{703.1\ N/m}}$

$m = \dfrac{\tau_n^2}{(2\pi)^2}(703.1\ N/m) = \dfrac{(3\ s)^2(703.1\ N/m)}{(2\pi)^2}$

$m = 160.3\ kg$

(b) $\theta = \theta_m \sin(\omega_n t + \phi)$



AT $t = 0$
$\theta = \dfrac{(0.04\ m)}{1.1} = 0.03636\ RAD$ $\quad x_B = 0.04\ m$ @ $t = 0$
$\dot{\theta} = 0$ $\quad \dot{x}_B = 0$
$0.03636 = \theta_m \sin\phi$
$0 = \omega_n \theta_m \cos\phi$ $\quad \phi = \pi/2,\ \theta_m = 0.03636\ RAD$
$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$
$\omega_n = \sqrt{\dfrac{(450)}{(.64)(160.3)}} = 2.094\ RAD/S$

$\dot{\theta} = (0.03636)(2.094\ RAD/S)\cos(2.094t + \pi/2)$
$\dot{\theta}_{MAX} = (0.03636)(2.094) = 0.07615\ RAD/S$

$(v_C)_{MAX} = (0.800)(\dot{\theta}_{MAX}) = (0.800)(0.7615)\ RAD/S)$

$(v_C)_{MAX} = 0.0609\ m/s$

19.32

GIVEN:
δ_{ST}, STATIC DEFLECTION OF A BEAM UNDER LOAD W

SHOW:
THAT, $f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{\delta_{ST}}}$

$k = W/\delta_{ST}$
$m = W/g$

$\omega_n^2 = k/m = \dfrac{W/\delta_{ST}}{W/g} = g/\delta_{ST}$

$f_n = \dfrac{\omega_n}{2\pi} = \dfrac{1}{2\pi}\sqrt{g/\delta_{ST}}$

*19.33

GIVEN:
$$\tau_n = 4\sqrt{\frac{l}{g}}\int_0^{\pi/2}\frac{d\phi}{\sqrt{\sin^2(\theta_m/2) - \sin^2\phi}}$$

SHOW:
BY EXPANDING THE INTEGRAND OF THE ABOVE EQUATION, $\tau_n = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\theta_m}{2}\right)$

USING THE BINOMIAL THEOREM, WE WRITE

$$\frac{1}{\sqrt{1 - \sin^2(\frac{\theta_m}{2})\sin^2\phi}} = \left[1 - \sin^2\left(\frac{\theta_m}{2}\right)\sin\phi\right]^{-\frac{1}{2}}$$
$$= 1 + \frac{1}{2}\sin^2\frac{\theta_m}{2}\sin^2\phi + \cdots$$

WHERE WE NEGLECT TERMS OF ORDER HIGHER THAN 2
SETTING $\sin^2\phi = \frac{1}{2}(1 - \cos 2\phi)$, WE HAVE

$$\tau_n = 4\sqrt{\frac{l}{g}}\int_0^{2\pi}\left\{1 + \frac{1}{2}\sin^2\frac{\theta_m}{2}\left[\frac{1}{2}(1 - \cos 2\phi)\right]\right\}d\phi$$
$$= 4\sqrt{\frac{l}{g}}\int_0^{2\pi}\left\{1 + \frac{1}{4}\sin^2\frac{\theta_m}{2} - \frac{1}{4}\sin^2\frac{\theta_m}{2}\cos 2\phi\right\}d\phi$$
$$= 4\sqrt{\frac{l}{g}}\left[\phi + \frac{1}{4}\left(\sin^2\frac{\theta_m}{2}\right)\phi - \frac{1}{8}\sin^2\frac{\theta_m}{2}\sin 2\phi\right]_0^{\pi/2}$$
$$= 4\sqrt{\frac{l}{g}}\left[\frac{\pi}{2} + \frac{1}{4}\left(\sin^2\frac{\theta_m}{2}\right)\frac{\pi}{2} + 0\right]$$

$$\tau_n = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\theta_m}{2}\right)$$

*19.34

GIVEN:
$\tau_n = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\theta_m}{2}\right)$ (PROB. 19.33)

FIND:
AMPLITUDE θ_m OF A PENDULUM FOR WHICH THE PERIOD OF A SIMPLE PENDULUM IS $\frac{1}{2}$ PERCENT LONGER THAN THE PERIOD OF THE SAME PENDULUM FOR SMALL OSCILLATIONS

FOR SMALL OSCILLATIONS $(\tau_n)_0 = 2\pi\sqrt{\frac{l}{g}}$

WE WANT $\tau_n = 1.005(\tau_n)_0 = 1.005\ 2\pi\sqrt{\frac{l}{g}}$

USING THE FORMULA OF PROB 19.33, WE WRITE

$\tau_n = (\tau_n)_0\left(1 + \frac{1}{4}\sin^2\frac{\theta_m}{2}\right) = 1.005(\tau_n)_0$

$\sin^2\frac{\theta_m}{2} = 4[1.005 - 1] = 0.02$

$\sin\frac{\theta_m}{2} = \sqrt{.02}$

$\frac{\theta_m}{2} = 8.130°$

$\theta_m = 16.3°$

• 19.35 GIVEN:

DATA OF TABLE 19.1

PENDULUM LENGTH, $\ell = 750$ mm

FIND:

(a) PERIOD FOR SMALL OSCILLATIONS

(b) PERIOD FOR AMPLITUDE $\theta_m = 60°$

(c) PERIOD FOR AMPLITUDE $\theta_m = 90°$

(a) $\tau_n = 2\pi\sqrt{\frac{\ell}{g}}$ (EQ. 19.18 FOR SMALL OSCILLATIONS)

$$\tau_n = 2\pi\sqrt{\frac{0.750\text{ m}}{9.81\text{ m/s}^2}} = 1.737\text{ s}$$

$$\tau_n = 1.737\text{ s}$$

(b) FOR LARGE OSCILLATIONS (EQ. 19.20)

$$\tau_n = \left(\frac{2k}{\pi}\right)\left(2\pi\sqrt{\frac{\ell}{g}}\right) = \frac{2k}{\pi}(1.737\text{ s})$$

FOR $\theta_m = 60°$, $k = 1.686$ (TABLE 19.1)

$$\tau_n(60°) = \frac{2(1.686)(1.737\text{ s})}{\pi} = 1.864\text{ s}$$

$$\tau_n(60) = 1.864\text{ s}$$

(c) FOR $\theta_m = 90°$, $k = 1.854$

$$\tau_n = \frac{2(1.854)(1.737\text{ s})}{\pi} = 2.05\text{ s}$$

• 19.36 GIVEN:

DATA OF TABLE 19.1

PERIOD = 2 s, AMPLITUDE = 90°

FIND:

LENGTH ℓ OF A SIMPLE PENDULUM (in.)

FOR LARGE OSCILLATIONS (EQ 19.20)

$\tau_n = \left(\frac{2k}{\pi}\right)\left(2\pi\sqrt{\frac{\ell}{g}}\right)$ FOR $\theta_m = 90°$

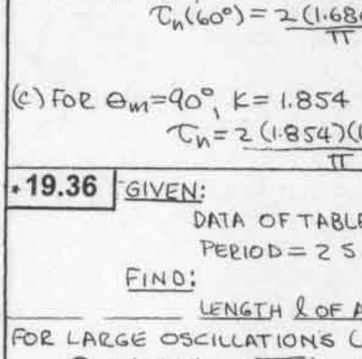
$k = 1.854$ (TABLE 19.1)

$$(2\text{ s}) = (2)(1.854)(2)\sqrt{\frac{\ell}{32.2\text{ ft/s}^2}}$$

$$\ell = \frac{(2\text{ s})^2(32.2\text{ ft/s}^2)}{[(4)(1.854)]^2} = 2.342\text{ ft}$$

$$\ell = 28.1\text{ in.}$$

19.37



GIVEN:

5-kg ROD AC

SPRING B, $k = 500$ N/m

SPRING C, $k = 620$ N/m

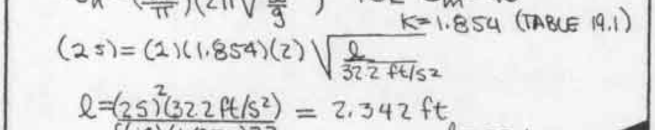
(TENSION OR COMP.)

FIND:

WHEN END C IS DEPRESSED SLIGHTLY

(a) FREQUENCY OF VIBRATION

(b) AMPLITUDE OF POINT C KNOWING THAT ITS MAXIMUM VELOCITY IS 0.9 m/s



$$F_B = k_B(x_B + (\delta_{ST})_B) = k_B(0.7\theta + (\delta_{ST})_B)$$

$$F_C = k_C(x_C + (\delta_{ST})_C) = k_C(1.4\theta + (\delta_{ST})_C)$$

19.37 CONTINUED

IN MOTION $\circlearrowleft \Sigma M_A = (\Sigma M_A)_{eff}$

$$(0.7)\left[k_B[0.7\theta + (\delta_{ST})_B] - mg\right] + 1.4[k_C(1.4\theta + (\delta_{ST})_C] = -\bar{I}\alpha - (0.7)(ma_t) \quad (1)$$

BUT IN EQUILIBRIUM ($\theta = 0$)

$$\circlearrowleft \Sigma M_A = 0 = 0.7[k_B(\delta_{ST})_B - mg] + 1.4\,k_C(\delta_{ST})_C \quad (2)$$

SUBSTITUTING (2) INTO (1)

$$\bar{I}\alpha + 0.7ma_t + (0.7)^2k_B\theta + (1.4)^2k_C\theta = 0$$

KINEMATICS ($\alpha = \ddot{\theta}$)

$$a_t = 0.7\alpha = 0.7\ddot{\theta}$$

$$[\bar{I} + m(0.7)^2]\ddot{\theta} + [(0.7)^2k_B + (1.4)^2k_C]\theta = 0$$

$$\bar{I} = \frac{1}{12}m\ell^2 = \frac{1}{12}(5\text{ kg})(1.4\text{ m})^2 = 0.8167\text{ kg-m}^2$$

$$(.7)^2m = (0.49\text{ m}^2)(5\text{ kg}) = 2.45\text{ kg}\cdot\text{m}^2$$

$$(0.7)^2k_B + (1.4)^2k_C = (.49\text{ m}^2)(500\text{ N/m}) + (1.96\text{ m}^2)(620\text{ N/m})$$

$$= 245 + 1215.2 = 1460.2\text{ N}\cdot\text{m}$$

$$[0.8167 + 2.45]\ddot{\theta} + 1460.2\theta = 0$$

$$\ddot{\theta} + \frac{(1460.2\text{ N}\cdot\text{m})}{(3.267\text{ kg}\cdot\text{m}^2)}\theta = 0$$

$$\ddot{\theta} + 447\theta = 0 \qquad \left(\frac{\text{N}}{\text{kg}\cdot\text{m}} = \text{s}^{-2}\right)$$

$$\omega_n = \sqrt{447\text{ s}^{-2}} = 21.14\text{ RAD/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{21.14}{2\pi} = 3.36\text{ Hz}$$

(b) $\theta = \theta_m\sin(\omega_n t + \phi)$

$$\dot{\theta} = (\theta_m)(\omega_n)\cos(\omega_n t + \phi)$$

MAXIMUM ANGULAR VELOCITY, $\dot{\theta}_m = \theta_m\omega_n$

MAXIMUM VELOCITY AT C

$$(\dot{x}_C)_m = 1.4\,\dot{\theta}_m = (1.4\text{ m})(\theta_m)(\omega_n)$$

$$\theta_m = \frac{(0.9\text{ m/s})}{(1.4\text{ m})(21.14\text{ RAD/s})} = 0.03041\text{ RAD} \qquad \omega_n = 21.14\text{ RAD/s}$$

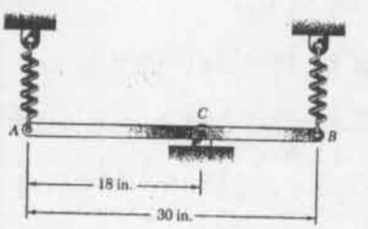
MAXIMUM AMPLITUDE AT C

$$(x_C)_m = (1.4\text{ m})(\theta_m) = (1.4\text{ m})(0.03041)$$

$$(x_C)_m = 0.0426\text{ m}$$

19.38

GIVEN:



18-lb ROD

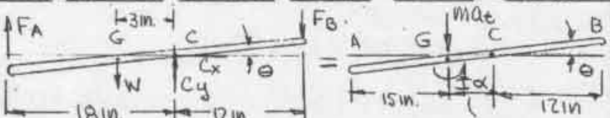
$k = 6$ lb/in FOR EACH SPRING

END A IS DEPRESSED SLIGHTLY AND RELEASED

FIND:

(a) FREQUENCY

(b) AMPLITUDE OF ANGULAR MOTION KNOWING THAT THE MAXIMUM VELOCITY OF A IS 22 in./s



$$F_A = k\left[\left(\tfrac{18}{12}\text{ ft}\right)\theta + (\delta_{ST})_A\right] = k\left[(1.5\text{ ft})\theta + (\delta_{ST})_A\right]$$

$$F_B = k\left[\left(\tfrac{12}{12}\text{ ft}\right)\theta + (\delta_{ST})_B\right] = k\left[(1\text{ ft})(\theta) + (\delta_{ST})_B\right]$$

(a) $\Sigma M_C = (\Sigma M)_{eff}$

$$-\tfrac{18}{12}(k)\left[(1.5\text{ ft})\theta + (\delta_{ST})_A\right] + \tfrac{3}{12}W - \tfrac{12}{12}(k)\left[(1\text{ ft})\theta + (\delta_{ST})_B\right] = \bar{I}\alpha + \tfrac{3}{12}m a_t \quad (1)$$

BUT IN EQUILIBRIUM ($\theta = 0$)

$$\Sigma M_C = 0 = -\tfrac{18}{12}k(\delta_{ST})_A + \tfrac{3}{12}W - \tfrac{12}{12}k(\delta_{ST})_C \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$\bar{I}\alpha + (0.25)m a_t + (3.25)k\theta = 0$$

KINEMATICS ($\alpha = \ddot{\theta}$)

$$a_t = \tfrac{3}{12}\ddot{\theta} = 0.25\ddot{\theta} \qquad \bar{I} = \tfrac{1}{12}\tfrac{W}{g}l^2$$

$$\bar{I} + (0.25)^2 m = \frac{1}{12}\frac{18\text{ lb}}{(32.2\text{ ft/s}^2)}\left(\frac{30}{12}\text{ ft}\right)^2 + (0.25\text{ ft})^2\frac{(18\text{ lb})}{(32.2\text{ ft/s}^2)}$$

$$= 0.2912 + 0.03494 = 0.3261\text{ lb}\cdot\text{s}^2\cdot\text{ft}$$

$$(0.3261\text{ lb}\cdot\text{s}^2\cdot\text{ft})\ddot{\theta} + (3.25\text{ ft}^2)\left(6\cdot 12\tfrac{\text{lb}}{\text{ft}}\right)\theta = 0$$

$$\ddot{\theta} + (717.6\text{ s}^{-2})\theta = 0$$

$$\omega_n = \sqrt{717.6} = 26.78\text{ RAD/s} \qquad f_n = \omega_n/2\pi$$

$$f_n = \frac{26.78}{2\pi} = 4.26\text{ HZ}$$

(b) $\theta = \theta_m \sin(\omega_n t + \phi)$

$$\dot{\theta} = \theta_m\omega_n\cos(\omega_n t + \phi)$$

$$\dot{\theta}_{MAX} = \theta_m\omega_n \qquad (\dot{x}_A)_{MAX} = \left(\tfrac{18}{12}\text{ ft}\right)(\dot{\theta})_{MAX}$$

$$(\dot{x}_A)_{MAX} = (22\text{ in/s})/(12\text{ in/ft}) = 1.833\text{ ft/s}$$

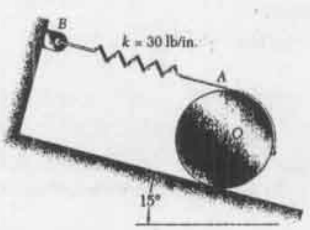
$$1.833\text{ ft/s} = (1.5\text{ ft})(\theta_m)(26.78\text{ RAD/s})$$

$$\theta_m = 0.04564\text{ RAD}$$

$$\theta_m = 2.61°$$

19.39

GIVEN:



30-lb CYLINDER ROLLS WITHOUT SLIDING.

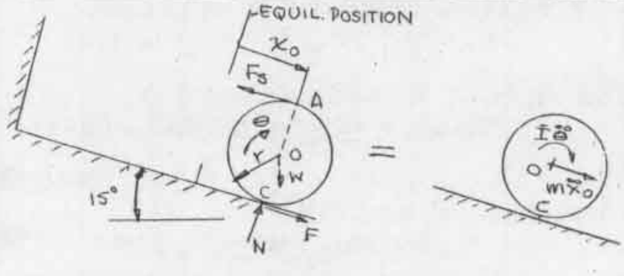
DATA AS SHOWN.

INITIAL DISPLACEMENT = 2 in. DOWN

(a) FIND:

(a) PERIOD

(b) MAXIMUM ACCELERATION OF O



SPRING DEFLECTION, $x_A = x_o + x_{A/O}$

$$x_{A/O} = r\theta \qquad \theta = x_o/r$$

$$x_A = 2x_o$$

$$F_S = k(x_A + \delta_S) = k(2x_o + \delta_{ST})$$

$\Sigma M_C = (\Sigma M)_{eff}$

$$-2rk(2x_o + \delta_{ST}) + rW\sin 15° = rm\ddot{x}_o + \bar{I}\ddot{\theta} \quad (1)$$

BUT IN EQUILIBRIUM, $x_o = 0$

$$\Sigma M_C = 0 = -2rk\delta_{ST} + rW\sin 15° \quad (2)$$

SUBSTITUTE (2) INTO (1) AND NOTING THAT $\theta = x_o/r$, $\ddot{\theta} = \ddot{x}_o/r$

$$rm\ddot{x}_o + \bar{I}\frac{\ddot{x}_o}{r} + 4rkx_o = 0 \qquad \bar{I} = \tfrac{1}{2}mr^2$$

$$\tfrac{3}{2}mr\ddot{x}_o + 4rkx_o = 0$$

$$\ddot{x}_o + \left(\frac{8}{3}\frac{k}{m}\right)x_o = 0$$

$$\omega_n = \sqrt{\frac{8}{3}\frac{k}{m}} = \sqrt{\frac{(8)(30\times 12\text{ lb/ft})}{(3)(30\text{ lb})/(32.2\text{ ft/s}^2)}} = 32.1\text{ s}^{-1}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{32.1} = 0.1957\text{ s}$$

$$\tau_n = 0.1957\text{ s}$$

(b) $x_o = (x_o)_m \sin(\omega_n t + \phi)$

@ $t = 0$ $\quad x_o = \frac{2}{12}$ ft $\quad \dot{x}_o = 0$

$$\dot{x}_o = (x_o)_m\omega_n\cos(\omega_n t + \phi), \quad t = 0,\ 0 = (x_o)_m\omega_n\cos\phi$$

THUS $\phi = \pi/2$

$$t = 0 \quad x_o(0) = \tfrac{1}{6}\text{ ft} = (x_o)_m\sin\phi = (x_o)_m(1)$$

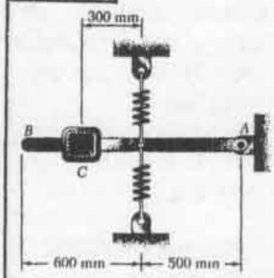
$$(x_o)_m = \tfrac{1}{6}\text{ ft}$$

$$\ddot{x}_o = -(x_o)_m\omega_n^2\sin(\omega_n t + \phi)$$

$$(a_o)_{MAX} = (\ddot{x}_o)_{MAX} = -(x_o)_m\omega_n^2 = -\left(\tfrac{1}{6}\text{ ft}\right)(32.1\text{ s}^{-1})^2 = 171.7\text{ ft/s}^2$$

$$(a_o)_{MAX} = 171.7\text{ ft/s}^2$$

19.40



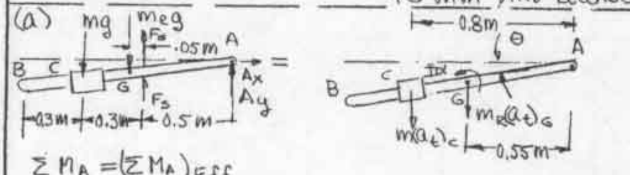
GIVEN:

750-g ROD AB
$k = 300$ N/m FOR EACH SPRING.
PERIOD $\tau = 0.4$ S

FIND:

(a) MASS m OF BLOCK C
(b) MAXIMUM VELOCITY OF BLOCK C IF END B IS DEPRESSED 40 mm AND RELEASED

(a)



$$\Sigma M_A = (\Sigma M_A)_{eff}$$

$$0.8\,mg + 0.55 m_R g - (0.5)(2F_s) = \bar{I}\alpha + (0.55) m_R (a_t)_G + (0.8) m (a_t)_C \quad (1)$$

$$F_s = k(0.5\theta + \delta_{ST})$$

BUT AT EQUILIBRIUM ($\theta = 0$)
$F_s = k(\delta_{ST})$ AND $\Sigma M_A = 0$

$$\Sigma M_A = 0.8 mg + 0.55 m_R g - (0.5)(2) k\delta_{ST} = 0 \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$\bar{I}\alpha + (0.55) m_R (a_t)_G + (0.8) m (a_t)_C + (0.5)^2 2 k\theta = 0$$

$$\alpha = \ddot{\theta} \quad (a_t)_G = (0.55)(\ddot{\theta}) \quad (a_t)_C = (0.8)(\ddot{\theta})$$

$$\bar{I} = \frac{1}{12} m_R l_R^2 = \frac{1}{12}(0.750\ kg)(1.1\ m)^2$$

$$\bar{I} = 0.07563\ kg\cdot m^2$$

$$(0.07563 + (0.55)^2(0.750) + (0.8)^2 m)\ddot{\theta} + (0.5)^2(2)(300)\theta = 0$$

$$\ddot{\theta} + \frac{(150\ N\cdot m)}{(0.3025\ kg\ m^2 + (0.64\ m^2)m)}\theta = 0$$

$$\omega_n^2 = (2\pi f_n)^2 = \frac{(2\pi)^2}{\tau_n^2} = \frac{(2\pi)^2}{(0.4 s)^2} = 246.7\ (s^2)^{-1}$$

$$\omega_n^2 = \frac{150\ N\cdot m}{(0.3025\ kg\ m^2 + (0.64\ m^2)m)} = 246.7\ (s^2)^{-1}$$

$$150\ N = 246.7\ (s^2)^{-1}[0.3025\ kg\cdot m + (.64\ m)m]$$

$$m = \frac{(150 - 74.64)(N)}{(246.7)(.64)(m/s^2)} = 0.477\ kg$$

(b) $(v_C)_{MAX} = (0.8)(\dot{\theta})_{MAX}$ $\qquad \theta_m = \frac{y_B}{1.1} = \frac{0.04}{1.1}$

$\theta = \theta_m \sin(\omega_n t + \phi)$ $\qquad \theta_m = 0.03636$ RAD

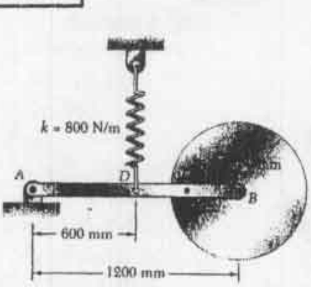
$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\dot{\theta}_{MAX} = \theta_m \omega_n = (0.03636)(246.7)^{1/2} = 0.5712\ RAD/s$$

$$(v_C)_{MAX} = (0.8 m)(.5712\ s^{-1}) = 0.4569\ m/s$$

$$(v_C)_{MAX} = 457\ \frac{mm}{s}$$

19.41



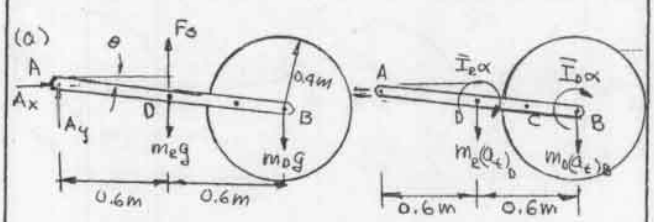
GIVEN:

8-kg ROD AB
12-kg DISK OF RADIUS 0.4 m
DISK IS RIGIDLY ATTACHED TO THE ROD AT B AND C.
POINT B MOVED DOWN 0.025 m AND RELEASED

FIND:

(a) THE PERIOD
(b) MAXIMUM VELOCITY OF B

(a)



$$\Sigma M_A = (\Sigma M_A)_{eff} \qquad F_s = k(0.6\theta + \delta_{ST})$$

$$0.6(m_R g - F_s) + 1.2 m_D g = (\bar{I}_R + \bar{I}_D)\alpha + 0.6(m_R)(a_t)_D + 1.2(m_D)(a_t)_B \quad (1)$$

AT EQUILIBRIUM ($\theta = 0$) $F_s = k\delta_{ST}$

$$\Sigma M_A = 0 = 0.6(m_R g - k(\delta_{ST})) + 1.2 m_D g \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$(\bar{I}_R + \bar{I}_D)\alpha + 0.6 m_R (a_t)_D + 1.2 m_D (a_t)_B + (0.6)^2 k\theta = 0$$

$$\alpha = \ddot{\theta} \quad (a_t)_B = 0.6\ddot{\theta} \quad (a_t)_D = 1.2\ddot{\theta}$$

$$\bar{I}_R = \frac{1}{12} m_R l^2 = \frac{1}{12}(8)(1.2)^2 = 0.960\ kg\cdot m$$

$$\bar{I}_D = \frac{1}{2} m_D R^2 = \frac{1}{2}(12)(0.4)^2 = 0.960\ kg\cdot m$$

$$[0.960 + 0.960 + (0.6)^2(8) + (1.2)^2(12)]\ddot{\theta} + (0.6)^2(800)\theta = 0$$

$$\ddot{\theta} + \frac{288\ N\cdot m}{(22.08\ kg\cdot m^2)}\theta = 0 \qquad \omega_n = \sqrt{\frac{288}{22.08}} = 3.612\ r/s$$

$$\tau_n = \frac{2\pi}{\omega_n} = 1.740\ s$$

(b) $(v_B)_{MAX} = (1.2)(\dot{\theta}_{MAX})$ $\qquad \theta_m = y_B/1.2$

$$\theta_m = \frac{0.025}{1.2} = 0.02083\ RAD$$

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

$$\dot{\theta} = \theta_m \omega_n \cos(\omega_n t + \phi)$$

$$\dot{\theta}_{MAX} = \theta_m \omega_n = (0.02083)(3.612) = 0.07524\ RAD/s$$

$$(v_B)_{MAX} = (1.2)(\dot{\theta}_{MAX}) = (1.2\ m)(0.07524\ RAD/s)$$

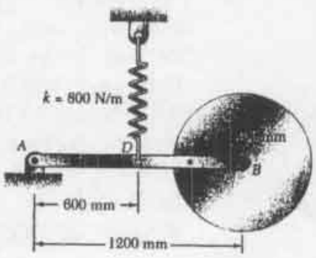
$$(v_B)_{MAX} = 90.29\ m/s$$

$$(v_B)_{MAX} = 90.3\ m/s$$

19.42

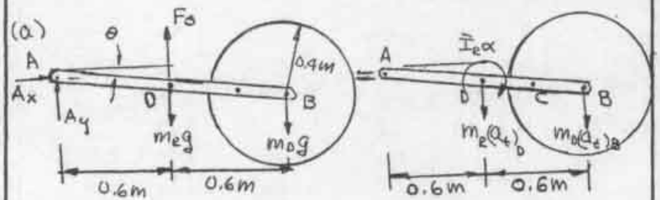
GIVEN:

8kg ROD AB
12kg DISK OF RADIUS 0.4 M
PIN C REMOVED AND DISK CAN ROTATE FREELY ABOUT PIN B.
POINT B MOVED DOWN 0.025 M AND RELEASED



FIND:
(a) PERIOD
(b) MAX VELOCITY OF B

(a)



NOTE: THIS PROBLEM IS THE SAME AS PROB 19.41 EXCEPT THAT THE DISK DOES NOT ROTATE, SO THAT THE EFFECTIVE MOMENT $I_D\alpha = 0$.

$\Sigma M_A = (\Sigma M_A)_{eff} \quad F_S = k(0.60 + \delta_{ST})$

$(0.6)(m_R g - F_S) + 1.2 m_D g = \bar{I}_R\alpha + (0.6)(m_R)(a_t)_D + 1.2(m_D)(a_t)_B \quad (1)$

AT EQUILIBRIUM $(\theta = 0)$ $F_S = k\delta_{ST}$

$\Sigma M_A = 0 = 0.6(m_R g - \delta_{ST}) + 1.2 m_D g \quad (2)$

SUBSTITUTE (2) INTO (1)

$\bar{I}_R\alpha + 0.6\, m_R(a_t)_D + 1.2 m_D(a_t)_B + (0.6)^2 k\theta = 0$

$\alpha = \ddot{\theta} \quad (a_t)_B = 0.6\ddot{\theta} \quad (a_t)_D = 1.2\ddot{\theta}$

$\bar{I}_R = \frac{1}{12} m_R l^2 = \frac{1}{12}(8)(1.2)^2 = 0.960 \text{ kg}\cdot\text{m}$

$[0.960 + (0.6)^2(8) + (1.2)^2(12)]\ddot{\theta} + (0.6)^2(800)\theta = 0$

$\ddot{\theta} + \frac{(288 \text{ N}\cdot\text{m})}{21.12 \text{ kg}\cdot\text{m}^2}\theta = 0 \quad \omega_n = \sqrt{\frac{288}{21.12}} = 3.693 \frac{1}{s}$

$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.693} = 1.701 \text{ s}$

(b) $(v_B)_{MAX} = (1.2)(\dot{\theta})_{MAX} \quad \theta_m = \frac{y_B}{1.2} = \frac{.025}{1.20} = 0.02083 \text{ RAD}$

$\theta = \theta_m \sin(\omega_n t + \phi)$

$\dot{\theta} = \theta_m\omega_n \cos(\omega_n t + \phi)$

$\dot{\theta}_{MAX} = \theta_m\omega_n = (0.02083)(3.693) = 0.07694 \text{ RAD/S}$

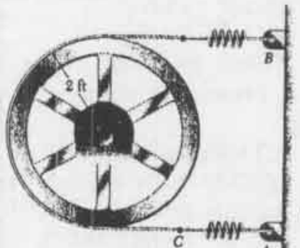
$(v_B)_{MAX} = (1.2)(\dot{\theta}_{MAX}) = (1.2)(.07694) = 0.9233 \frac{m}{s}$

$(v_B)_{MAX} = 92.3 \text{ mm/s}$

19.43

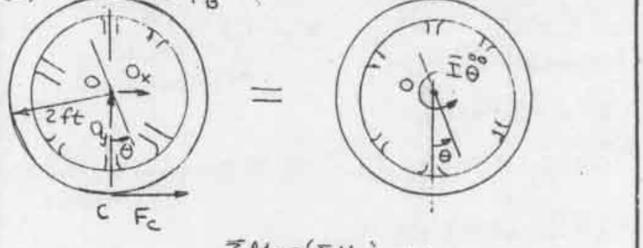
GIVEN:

600-lb FLYWHEEL OF RADIUS OF GYRATION = 20 in
$k = 75$ lb/in. FOR EACH SPRING.
POINT C IS PULLED TO THE RIGHT 1 in. AND RELEASED



FIND:
(a) PERIOD OF VIBRATION
(b) MAXIMUM ANGULAR VELOCITY OF THE FLYWHEEL

(a)



$\Sigma M_O = (\Sigma M_O)_{eff}$

$F_C = k((\delta_{ST})_C - 2\theta) \quad F_B = k(2\theta + (\delta_{ST})_B)$

$2F_C - 2F_B = \bar{I}\ddot{\theta}$

$2k[(\delta_{ST})_C - 2\theta] - 2k[(\delta_{ST})_B + 2\theta] = \bar{I}\ddot{\theta} \quad (1)$

AT EQUILIBRIUM $(\theta = 0)$ $F_B = k(\delta_{ST})_B$, $F_C = k(\delta_{ST})_C$

$\Sigma M_A = 0 = 2k(\delta_{ST})_C - 2k(\delta_{ST})_B \quad (2)$

SUBSTITUTE (2) INTO (1)

$\bar{I}\ddot{\theta} + 8k\theta = 0$

$\bar{I} = m\bar{k}^2 = \frac{(600 \text{ lb})}{(32.2 \text{ ft/s}^2)}(20/12 \text{ ft})^2 = 51.76 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$

$k = (8 \text{ ft}^2)(75 \times 12 \text{ lb/ft}) = 7200 \text{ lb}\cdot\text{ft}$

$\omega_n^2 = \frac{8k}{\bar{I}} = \frac{7200 \text{ lb ft}}{51.76 \text{ lb ft s}^2} = 139.1 \text{ s}^{-2}$

$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{139.1}} = 0.533 \text{ s}$

(b) $\theta = \theta_m \sin(\omega_n t + \phi)$

$\dot{\theta} = \theta_m\omega_n \cos(\omega_n t + \phi)$

$\dot{\theta}_{MAX} = \theta_m\omega_n \quad \theta_m = x_C/r = \frac{1/12}{2}$

$\omega_n = \sqrt{139.1} = 11.79 \text{ RAD/S} \quad \theta_M = 0.04167 \text{ RAD}$

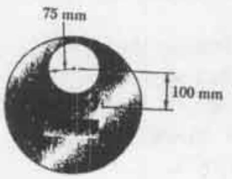
$\dot{\theta}_{MAX} = (0.04167)(11.79) = 0.491 \text{ rad/s}$

$\omega = 0.491 \text{ rad/s}$

19.44

GIVEN:

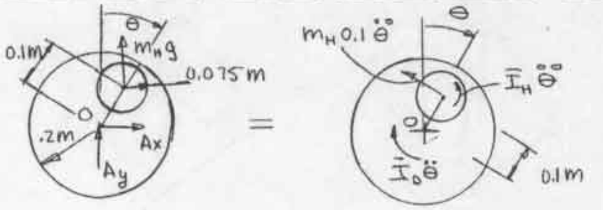
DISK ATTACHED TO A FRICTIONLESS PIN AT ITS GEOMETRIC CENTER AS SHOWN



FIND:

(a) PERIOD OF SMALL OSCILLATIONS

(b) LENGTH OF A SIMPLE PENDULUM OF THE SAME PERIOD



$\Sigma M_O = (\Sigma M_O)_{eff}$

$\circlearrowleft + \quad -m_H g(0.1)\sin\theta = \bar{I}_D\ddot{\theta} - \bar{I}_H\ddot{\theta} - (0.1)^2 m_H\ddot{\theta}$

$m_D = \rho t\pi R^2 = (\rho t\pi)(.2)^2 = (0.04)\pi\rho t$

$m_H = \rho t\pi r^2 = (\rho t\pi)(0.075)^2 = (0.005625)\pi\rho t$

$I_D = \frac{1}{2}m_D R^2 = \frac{1}{2}(.04\pi\rho t)(.2)^2 = 800\times10^{-6}\pi\rho t$

$I_H = \frac{1}{2}m_H r^2 = \frac{1}{2}(0.005625\pi\rho t)(0.075)^2 = 15.82\times10^{-6}\pi\rho t$

SMALL ANGLES $\sin\theta \approx \theta$

$[800\times10^{-6}\pi - 15.82\times10^{-6}\pi - (0.1)^2(0.005625\pi)]\rho t\ddot{\theta} + (0.005625)\pi\rho t(9.81)(0.1)\theta = 0$

$727.9\times10^{-6}\ddot{\theta} + 5.518\times10^{-3}\theta = 0$

$\omega_n^2 = \dfrac{5.518\times10^{-3}}{727.9\times10^{-6}} = 7.581$

$\omega_n = 2.753$ RAD/s

$\tau_n = \dfrac{2\pi}{\omega_n} \qquad \tau_n = \dfrac{2\pi}{2.753} = 2.28\ s$ ◀

(b) PERIOD OF A SIMPLE PENDULUM

$\tau_n = 2\pi\sqrt{l/g}$

$l = (\tau_n/2\pi)^2 g$

$l = [(2.753)/2\pi]^2(9.81\ m/s^2)$

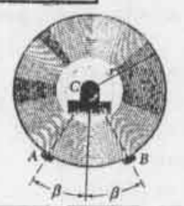
$l = 1.294\ m$ ◀

19.45

GIVEN:

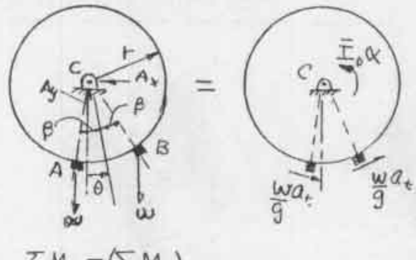
WEIGHTS w AT A AND B, AND DISK W

FOR $\beta = 0$, PERIOD $= \tau_0$



FIND:

ANGLE β FOR A PERIOD OF $2\tau_0$



$\alpha = \ddot{\theta}$

$\bar{I}_0 = \frac{1}{2}\frac{W}{g}r^2$

$\Sigma M_C = (\Sigma M_C)_{eff}$

$\circlearrowleft + \quad wr\sin(\beta-\theta) - wr\sin(\beta+\theta) = \frac{2w}{g}ra_t + \bar{I}\alpha$

$r[\sin(\beta-\theta) - \sin(\beta+\theta)] = -2wr\sin\theta\cos\beta$

$\sin\theta \approx \theta \qquad a_t = r\ddot{\theta}$

$\left(\frac{2w}{g}r^2 + \frac{W}{2g}r^2\right)\ddot{\theta} + (2wr\cos\beta)\theta = 0$

$$\omega_n = \sqrt{\frac{2wg\cos\beta}{(2w+W/2)r}} = \sqrt{\frac{4g\cos\beta}{(4+W/w)r}} \quad (1)$$

$\beta = 0 \quad \tau_0 = \dfrac{2\pi}{\omega_0} = 2\pi\Big/\sqrt{\dfrac{4g}{(4+W/w)r}}$

$\tau_n = 2\pi\Big/\sqrt{\dfrac{\cos\beta}{(4+W/w)r}} = 2\tau_0 = 4\pi\Big/\sqrt{\dfrac{4g}{(4+W/w)r}}$

$\cos\beta = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\beta = 75.5°$ ◀

19.46

REFER TO FIGURE IN PROB 19.45 ABOVE

GIVEN:

$w = 0.1\ lb,\ W = 3\ lb,\ r = 4\ in.,\ \beta = 60°$

FIND:

FREQUENCY OF SMALL OSCILLATIONS

FROM DERIVATION IN PROB 19.45 (EQ. 1)

$$\omega_n = \sqrt{\frac{24g\cos\beta}{(24+W/w)r}}$$

$$\omega_n = \sqrt{\frac{(4)(32.2\ ft/s^2)\cos60°}{(4+3/0.1)(4/12)}} = 2.384\ r/s$$

$f_n = \omega_n/2\pi = 2.384/2\pi$

$f_n = 0.379\ Hz.$ ◀

19.47

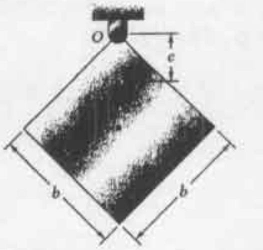
GIVEN:

SQUARE PLATE, $b = 0.3\,m$

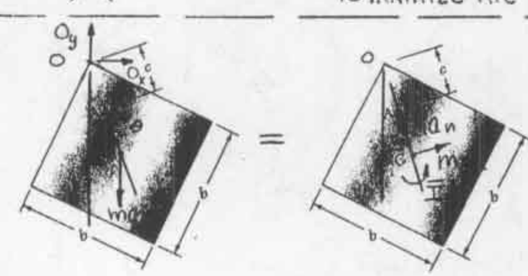
FIND:

(a) PERIOD OF SMALL OSCILLATIONS ABOUT O

(b) DISTANCE C FROM O TO A POINT A FROM WHICH THE PLATE SHOULD BE SUSPENDED TO MINIMIZE THE PERIOD



(a)



$\Sigma M_O = (\Sigma M_O)_{eff} \qquad \alpha = \ddot{\theta} \qquad a_t = (OG)(\alpha)$

$\bar{I} = \frac{1}{6} m b^2 \qquad OG = b\sqrt{2}/2 \qquad a_t = (b\sqrt{2}/2)\ddot{\theta}$

$+\!\circlearrowleft\ (OG)(\sin\theta)(mg) = -(OG)\, m a_t - \bar{I}\alpha \qquad \sin\theta \approx \theta$

$b\sqrt{2}/2\, m\,(b\sqrt{2}/2)\ddot{\theta} + \frac{1}{6} m b^2 \ddot{\theta} + (b\sqrt{2}/2) m g\,\theta = 0$

$(b)(\frac{1}{2} + \frac{1}{6}) m \ddot{\theta} + \sqrt{2}/2\, m g\, \theta = 0$

$\ddot{\theta} + \frac{(\sqrt{2}/2)\,g}{(2/3)\,b}\,\theta = 0 \qquad b = 0.3\,m$

$(\tau_n)_O = \frac{2\pi}{(\omega_n)_O} = 2\pi\sqrt{\frac{(2/3)b}{(\sqrt{2}/2)g}} = 2\pi\sqrt{\frac{4(.3\,m)}{(3\sqrt{2})(9.81\,\frac{m}{s^2})}}$

$(\tau_n)_O = 1.067\ s$

(b) SUSPENDED ABOUT A

$\Sigma M_A = (\Sigma M_A)_{eff} \qquad a_t = (OG - c)\alpha$

$+\!\circlearrowleft\ (OG - c)(\sin\theta)(mg) = -(OG - c)\, m a_t - \bar{I}\alpha$

$((b\sqrt{2}/2 - c)^2 + \frac{1}{6} b^2)\, m\ddot{\theta} + (c\frac{\sqrt{2}}{2} - c)\, m g\, \theta = 0$

$(\tau_n)_A^2 = \frac{(2\pi)^2}{\omega_n^2} = \frac{4\pi^2\left[(b\sqrt{2}/2 - c)^2 + b^2/6\right]}{(b\frac{\sqrt{2}}{2} - c)}$

FOR MINIMUM PERIOD $\frac{d(\tau_n^2)_A}{dc} = 0$

$0 = \frac{2(b\sqrt{2}/2 - c)(-1)(b\frac{\sqrt{2}}{2} - c) - (-1)\left[(b\sqrt{2}/2 - c)^2 + b^2/6\right]}{(b\sqrt{2}/2 - c)^2}$

$(b\sqrt{2}/2 - c)^2 + b^2/6 = 0 \qquad b = 0.3\,m$

$b\sqrt{2}/2 - c = \frac{b}{\sqrt{6}} \qquad c = 0.3\left[\frac{\sqrt{2}}{2} - \frac{1}{\sqrt{6}}\right] = 0.08966\,m$

$c = 89.7\ mm$

19.48

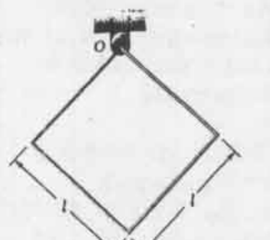
GIVEN:

THIN WIRE, $l = 1.2\,ft$

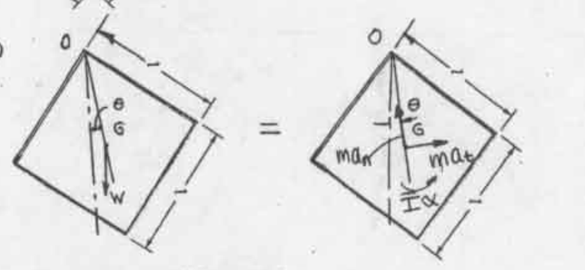
FIND:

(a) PERIOD ABOUT O

(b) PERIOD ABOUT A POINT AT THE MIDPOINT OF ONE OF THE SIDES



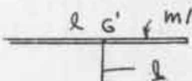
(a)



m = MASS OF THE FRAME

$\Sigma M_O = (\Sigma M_O)_{eff} \qquad \alpha = \ddot{\theta} \qquad a_t = (OG)(\alpha)$

$OG = l\sqrt{2}/2 \qquad a_t = (l\sqrt{2}/2)\ddot{\theta}$



FOR ONE LEG $\quad (I_G)_1 = I_{G'} + (m/4)(l/2)^2$

$I_{G'} = \frac{1}{12}\frac{m}{4} l^2 \qquad (I_G)_1 = \frac{m l^2}{4}\left[\frac{1}{12} + (\frac{1}{2})^2\right] = \frac{m}{4} l^2 (\frac{1}{3})$

FOR COMPLETE WIRE FRAME

$\bar{I} = 4(I_G)_1 = (4)\frac{m}{4} l^2 (\frac{1}{3}) = \frac{1}{3} m l^2$

$+\!\circlearrowleft$

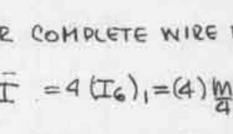
$-mg\, l\frac{\sqrt{2}}{2}\sin\theta = \bar{I}\alpha + (m a_t)\, l\frac{\sqrt{2}}{2} \qquad \sin\theta \approx \theta$

$(\frac{1}{3} + \frac{1}{2}) m l^2 \ddot{\theta} + m g\, l\sqrt{2}/2\, \theta = 0$

$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{(\sqrt{2}/2)\,g}{(5/6)\,l}}} = 2\pi\sqrt{\frac{5\,(1.2\,ft)}{(3\sqrt{2})(32.2\,ft/s^2)}}$

$\tau_n = 1.317\ s$

(b) FOR FRAME SUSPENDED FROM MIDPOINT



$OG = \frac{l}{2}$

$\bar{I} = (4)\frac{m l^2}{4}\left[\frac{1}{12} + (\frac{1}{2})^2\right] = \frac{1}{3} m l^2$

$-mg\,\frac{l}{2}\sin\theta = \bar{I}\ddot{\theta} + (m\frac{l}{2}\ddot{\theta})\frac{l}{2} = (\frac{1}{3} + \frac{1}{4}) m l^2 \ddot{\theta}$

$\frac{7}{12} m l^2 \ddot{\theta} + m g\,\frac{l}{2}\,\theta = 0$

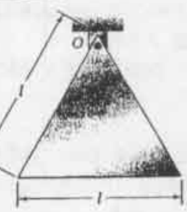
$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{6}{7}\frac{g}{l}}} = 2\pi\sqrt{(\frac{7}{6})(\frac{1.2\,ft}{32.2\,ft/s^2})} = 1.310\ s$

$\tau_n = 1.310\ s$

19.49

GIVEN:

UNIFORM EQUILATERAL TRIANGLE OF SIDE $\ell = 0.3\,m$

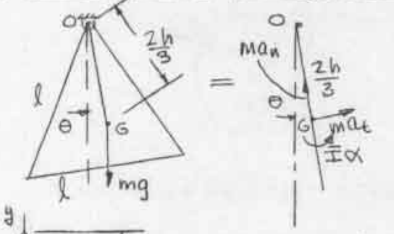


FIND:

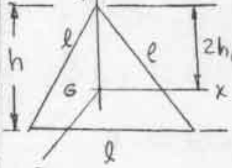
(a) PERIOD IF PLATE IS SUSPENDED FROM ONE OF ITS VERTICES

(b) PERIOD IF PLATE IS SUSPENDED FROM THE MIDPOINT OF ONE OF ITS SIDES

(a)



$h = \frac{\sqrt{3}}{2}\ell$



$I_{x,MASS} = \rho t I_{x,AREA} = \frac{\rho t \ell h^3}{36}$

$m = \rho t A = \rho t \frac{\ell h}{2}$

$I_{x,MASS} = \frac{mh^2}{18}$

$I_{y,MASS} = \rho t I_{y,AREA} \qquad I_{y,AREA} = \frac{h\ell^3}{48}$

$I_{y,MASS} = \frac{m\ell^2}{24}$

$I_z = \bar{I} = I_x + I_y = \frac{mh^2}{18} + \frac{m\ell^2}{24}$

$h = \ell\sqrt{3}/2 \qquad \bar{I} = m\ell^2\left[\frac{3/4}{18} + \frac{1}{24}\right] = m\ell^2/12$

(a)

$\alpha = \ddot{\theta} \qquad a_t = \frac{\sqrt{3}}{3}\ell\ddot{\theta} \qquad \sin\theta \approx \theta$

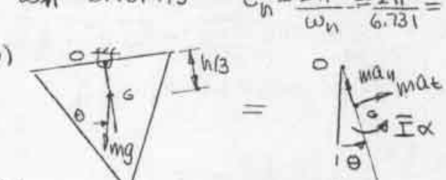
$\Sigma M_O = (\Sigma M_O)_{eff} \qquad -mg\frac{\sqrt{3}}{3}\ell\sin\theta = \bar{I}\alpha + \frac{\sqrt{3}}{3}\ell m a_t$

$\left(\frac{1}{12} + \frac{1}{3}\right)m\ell^2\ddot{\theta} + mg\sqrt{3}/3\,\ell\,\theta = 0$

$\omega_n^2 = \frac{\sqrt{3}/3}{5/12}\frac{g}{\ell} = \frac{(\sqrt{3})(4)}{5}\frac{(9.81\,m/s^2)}{(0.3\,m)} = 45.31\,s^{-2}$

$\omega_n = 6.731\,r/s \qquad \tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{6.731} = 0.933\,s$

(b)



$\Sigma M_O = (\Sigma M_O)_{eff} \qquad -mg\,h/3\sin\theta = \bar{I}\ddot{\theta} + m\left(\frac{h}{3}\right)^2\ddot{\theta}$

$h = \ell\sqrt{3}/2 \quad \bar{I} = m\ell^2/12 \quad \left(\frac{1}{12} + \left(\frac{\sqrt{3}}{6}\right)^2\right)m\ell^2\ddot{\theta} + mg\ell\frac{\sqrt{3}}{6}\theta = 0$

$\frac{1}{6}\ddot{\theta} + \frac{\sqrt{3}}{6}\frac{g}{\ell} = 0$

$\omega_n^2 = \sqrt{3}\frac{g}{\ell} = \frac{(\sqrt{3})9.81\,m/s^2}{0.3\,m} = 56.63\,s^{-1} \qquad \omega_n = 7.5258\,r/s$

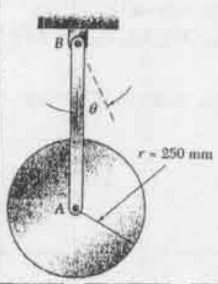
$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{2.756} = 0.835\,s$

19.50

GIVEN:

ROD AB OF NEGLIGIBLE MASS ATTACHED TO A DISK OF MASS m. $AB = \ell = 0.650\,m$

$r = 0.250\,m$

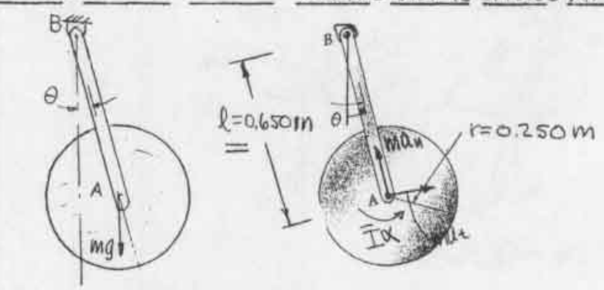


FIND:

THE PERIOD OF SMALL OSCILLATIONS IF

(a) THE DISK IS FREE TO ROTATE IN A BEARING AT A

(b) THE DISK IS RIVETED AT A



$\bar{I} = \frac{1}{2}mr^2 = \frac{1}{2}(0.250)^2 m = \frac{m}{32}$

$a_t = \ell\alpha = 0.650\,\alpha \qquad \alpha = \ddot{\theta}$

(a) THE DISK IS FREE TO ROTATE AND IS IN CURVILINEAR TRANSLATION

THUS $\bar{I}\alpha = 0$

$\Sigma M_B = (\Sigma M_B)_{eff}$

$-mg\ell\sin\theta = \ell m a_t \qquad \sin\theta \approx \theta$

$m\ell^2\ddot{\theta} + mg\ell\theta = 0$

$\omega_n^2 = \frac{g}{\ell} = \frac{9.81\,m/s^2}{0.650\,m} = 15.092$

$\omega_n = 3.885 \qquad \tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.885} = 1.617\,s$

(b) WHEN THE DISK IS RIVETED AT A, IT ROTATES AT AN ANGULAR ACCELERATION α

$\Sigma M_B = \Sigma(M_B)_{eff}$

$-mg\ell\sin\theta = \bar{I}\alpha + \ell m a_t \qquad \bar{I} = \frac{1}{2}mr^2$

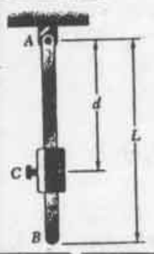
$\left(\frac{1}{2}mr^2 + m\ell^2\right)\ddot{\theta} + mg\ell\theta = 0$

$\omega_n^2 = \frac{g\ell}{(r^2/2 + \ell^2)} = \frac{(9.81\,m/s^2)(0.650\,m)}{[(0.250^2)/2 + (0.650)^2]} = 14.053\,s^{-2}$

$\omega_n = 3.749\,r/s$

$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{3.749} = 1.676\,s$

19.51



GIVEN:

COLLAR C WEIGHT, $W_C = 2\,lb$

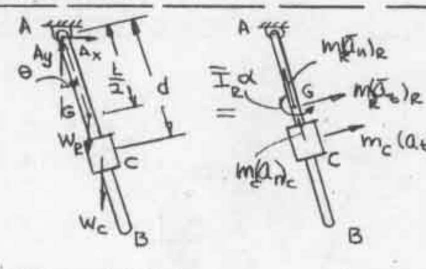
ROD AB WEIGHT, $W_R = 6\,lb$, $L = 3\,ft$

FIND:

PERIOD OF SMALL OSCILLATIONS WHEN,

(a) $d = 3\,ft$

(b) $d = 2\,ft$



$\Sigma M_A = (\Sigma M_A)_{eff}$

$-W_R \frac{L}{2}\sin\theta - W_c d\sin\theta = \bar{I}_R\alpha + m_R\frac{L}{2}(\bar{a}_t)_R + m_c d(a_t)_c$

$\sin\theta \approx \theta \quad \alpha = \ddot{\theta},\ (\bar{a}_t)_R = \frac{L}{2}\alpha = \frac{L}{2}\ddot{\theta},\ (a_t)_c = d\alpha = d\ddot{\theta}$

$(\bar{I}_R + m_R(\frac{L}{2})^2 + m_c d^2)\ddot{\theta} + (m_R g\frac{L}{2} + m_c g d)\theta = 0$

$\bar{I}_R = \frac{1}{12} m_R L^2 \quad \bar{I}_R + m_R(\frac{L}{2})^2 = \frac{m_R L^2}{3}$

$(m_R L^2/3 + m_c d^2)\ddot{\theta} + (m_R g\frac{L}{2} + m_c g d)\theta = 0$

$\ddot{\theta} + \dfrac{(L/2 + \frac{m_c}{m_R}d)g}{(L^2/3 + \frac{m_c}{m_R}d^2)}\theta = 0$

$\frac{m_c}{m_R} = \frac{W_c}{W_R} = \frac{2}{6} \qquad L = 3\,ft$

$\ddot{\theta} + \dfrac{(\frac{3}{2} + \frac{1}{3}d)g}{(3 + \frac{1}{3}d^2)}\theta = 0$

$\tau_n = 2\pi/\omega_n = 2\pi\sqrt{\dfrac{(3 + d^2/3)}{(\frac{3}{2} + d/3)(g)}}$

(a) $d = 3\,ft$

$\tau_n = 2\pi\sqrt{\dfrac{(3+3)}{(\frac{3}{2}+1)(32.2)}} = 1.715\,s$

(b) $d = 2\,ft$

$\tau_n = 2\pi\sqrt{\dfrac{(3+4/3)}{(3/2+2/3)(32.2)}} = 1.566\,s$

19.52

GIVEN:

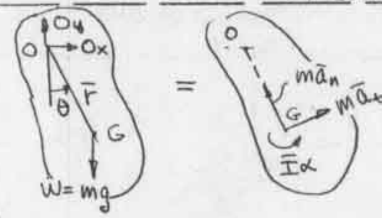
COMPOUND PENDULUM WHICH OSCILLATES ABOUT O

$\bar{k}$ = CENTRODAL RADIUS OF GYRATION

$GA = \bar{k}^2/\bar{r}$

SHOW THAT:

PERIOD EQUALS THE PERIOD OF A SIMPLE PENDULUM OF LENGTH OA.

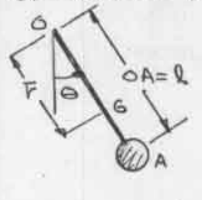


$\Sigma M_O = \Sigma(M_O)_{eff}: \ -W\bar{r}\sin\theta = \bar{I}\alpha + m\bar{a}_t\bar{r}$

$-mg\bar{r}\sin\theta = m\bar{k}^2\ddot{\theta} + m\bar{r}^2\ddot{\theta}$

$\ddot{\theta} + \dfrac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}\sin\theta = 0 \qquad (1)$

FOR A SIMPLE PENDULUM OF LENGTH $OA = l$



$\ddot{\theta} + \frac{g}{l}\theta = 0 \qquad (2)$

COMPARING EQUATIONS (1) AND (2)

$l = \dfrac{\bar{r}^2 + \bar{k}^2}{\bar{r}}$

$GA = l - \bar{r} = \bar{k}^2/\bar{r}$ (QED)

19.53

GIVEN;

COMPOUND PENDULUM AS IN PROB. 19.52 SHOWN ABOVE

SHOW THAT:

SMALLEST PERIOD OF OSCILLATION OCCURS WHEN $\bar{r} = \bar{k}$

SEE SOLUTION TO PROB 19.52 FOR DERIVATION OF

$\ddot{\theta} + \dfrac{g\bar{r}}{\bar{r}^2 + \bar{k}^2}\sin\theta = 0$

FOR SMALL OSCILLATIONS $\sin\theta \approx \theta$ AND

$\tau_n = \dfrac{2\pi}{\omega_n} = 2\pi\sqrt{\dfrac{\bar{r}^2 + \bar{k}^2}{g\bar{r}}} = \dfrac{2\pi}{\sqrt{g}}\sqrt{\bar{r} + \dfrac{\bar{k}^2}{\bar{r}}}$

FOR SMALLEST τ_n WE MUST HAVE $\bar{r} + \frac{\bar{k}^2}{\bar{r}}$ A MINIMUM

$\dfrac{d(\bar{r} + \frac{\bar{k}^2}{\bar{r}})}{d\bar{r}} = 1 - \dfrac{\bar{k}^2}{\bar{r}^2} = 0$

$\bar{r}^2 = \bar{k}^2$

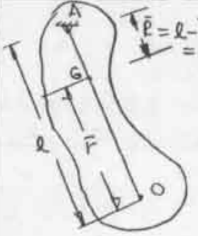
$\bar{r} = \bar{k}$ (QED)

19.54 GIVEN:

COMPOUND PENDULUM OF PROB. 19.52 SUSPENDED FROM A

SHOW THAT:

PERIOD IS THE SAME AS BEFORE AND THE NEW CENTER OF OSCILLATION IS AT O.



SAME DERIVATION AS IN PROB. 19.52 WITH $\bar{r}$ REPLACED BY $\bar{R}$. THUS,

$$\ddot{\theta} + \frac{g\bar{R}}{\bar{R}^2+\bar{k}^2}\,\theta = 0$$

LENGTH OF THE EQUIVALENT SIMPLE PENDULUM IS

$$L = \frac{\bar{R}^2+\bar{k}^2}{\bar{R}} = \bar{R} + \frac{\bar{k}^2}{\bar{R}}$$

$$L = (\ell-\bar{r}) + \frac{\bar{k}^2}{\bar{k}^2/\bar{r}} = \ell$$

THUS THE LENGTH OF THE EQUIVALENT SIMPLE PENDULUM IS THE SAME AS IN PROB. 19.52. IT FOLLOWS THAT THE PERIOD IS THE AND THAT THE NEW CENTER OF OSCILLATION IS AT O (QED)

19.55 GIVEN:

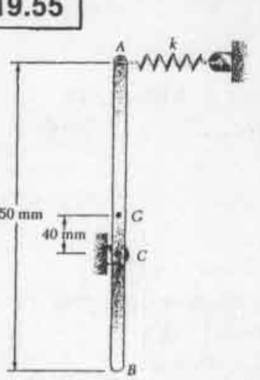
8-kg BAR AB

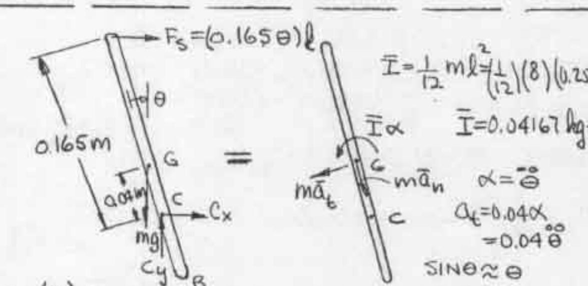
$k = 500$ N/m

FIND:

(a) FREQUENCY OF SMALL OSCILLATIONS

(b) SMALLEST k FOR WHICH OSCILLATIONS WILL OCCUR





$\bar{I} = \frac{1}{12}m\ell^2 = \frac{1}{12}(8)(0.250)^2$ $\quad \bar{I} = 0.04167$ kg·m²

$\alpha = \ddot{\theta}$

$a_t = 0.04\alpha = 0.04\ddot{\theta}$

$\sin\theta \approx \theta$

$\Sigma M_C = \Sigma(M_C)_{eff}$

$$-(0.165)^2 k\theta + 0.04\,mg\,\theta = \bar{I}\ddot{\theta} + (0.04)^2 m\ddot{\theta}$$

$$(0.04167+0.01280)\ddot{\theta} + (0.02722k - 0.32g)\theta = 0 \quad (1)$$

(a) $k = 500$ N/m

$$0.05447\,\ddot{\theta} + (10.47)\theta = 0$$

$$f_n = \frac{\omega_n}{2\pi} = \left(\sqrt{10.47/0.05447}\right)/2\pi = 2.21 \text{ Hz}$$

(b) FOR $\tau_n \to \infty$ $\omega_n \to 0$ OSCILLATIONS WILL NOT OCCUR

FROM EQUATION (1), $\omega_n^2 = \dfrac{0.02722k - 0.32g}{(0.05447)} = 0$

$$k - \frac{0.32g}{0.02722} = \frac{(0.32)(9.81)}{(0.02722)} = 115.3 \text{ N/m}$$

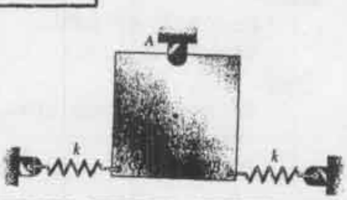
19.56 GIVEN:

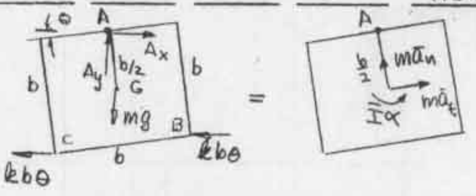
45-lb SQUARE PLATE WITH 1.2 ft SIDES

$k = 8$ lb/in. EACH

FIND:

FREQUENCY OF VIBRATION





$\alpha = \ddot{\theta}$

$a_t = \frac{b}{2}\alpha = \frac{b}{2}\ddot{\theta}$

$\sin\theta \approx \theta$

$\Sigma M_O = \Sigma(M_O)_{eff}$ $\quad -mg\frac{b}{2}\theta - 2kb^2\theta = \bar{I}\alpha + \left(\frac{b}{2}\right)^2 m\alpha$

$$\bar{I} + m\left(\frac{b}{2}\right)^2 = \frac{1}{6}mb^2 + m\frac{b^2}{4} = \frac{5}{12}mb^2$$

$$\frac{5}{12}mb^2\ddot{\theta} + \left(mg\frac{b}{2} + 2kb\right)\theta = 0$$

$$f_n = \omega_n/2\pi = \sqrt{\frac{(mg/2 + 2kb)}{\frac{5}{12}mb}}\Big/2\pi$$

$mg = 45$ lb

$k = 8\frac{\text{lb}}{\text{in}} = 96\frac{\text{lb}}{\text{ft}}$

$b = 1.2$ ft

$$f_n = \sqrt{\frac{[45/2 + (2)(96)(1.2)](16)}{\left[\frac{5}{12}\frac{45 \text{ lb}}{32.2 \text{ ft/s}^2}(1.2 \text{ ft})\right]}}\Big/2\pi$$

$$f_n = 3.03 \text{ Hz}$$

19.57 GIVEN:

$m = 12$ kg FOR AC

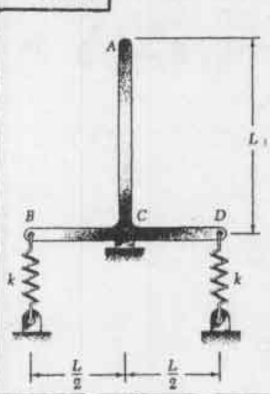
$m = 12$ kg FOR BD

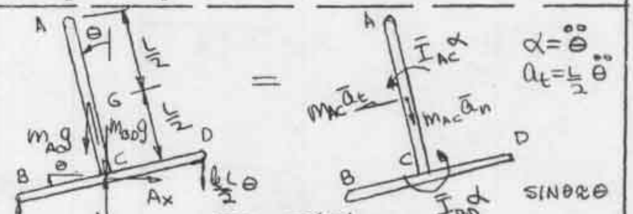
$L = 0.8$ m

$k = 500$ N/m

FIND:

FREQUENCY OF SMALL OSCILLATIONS





$\alpha = \ddot{\theta}$

$a_t = \frac{L}{2}\ddot{\theta}$

$\sin\theta \approx \theta$

$\Sigma M_O = \Sigma(M_O)_{eff}$

$$\left[m_{AC}g\frac{L}{2} - 2k\left(\frac{L}{2}\right)^2\right]\theta = (\bar{I}_{AC} + \bar{I}_{BD})\alpha + m_{AC}\left(\frac{L}{2}\right)^2\alpha$$

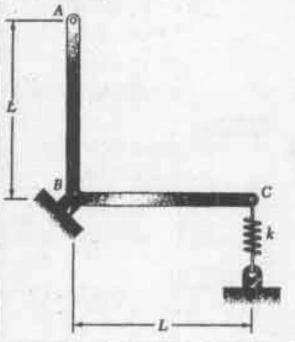
$m_{BD} = m_{AC} = m$ $\quad \bar{I}_{BD} = \bar{I}_{AC} = \bar{I} = \frac{1}{12}mL^2$

$$\left(\frac{1}{6}+\frac{1}{4}\right)mL^2\ddot{\theta} + \left[2k\left(\frac{L}{2}\right)^2 - mg\frac{L}{2}\right]\theta = 0$$

$$f_n = \omega_n/2\pi = \sqrt{\frac{2(500)(0.4)^2 - (12)(9.81)(.4)\,(\text{N}\cdot\text{m})}{\left(\frac{5}{12}\right)(12)(0.8)^2\,(\text{kg}\cdot\text{m}^2)}}$$

$$f_n = 0.945 \text{ Hz.}$$

19.58

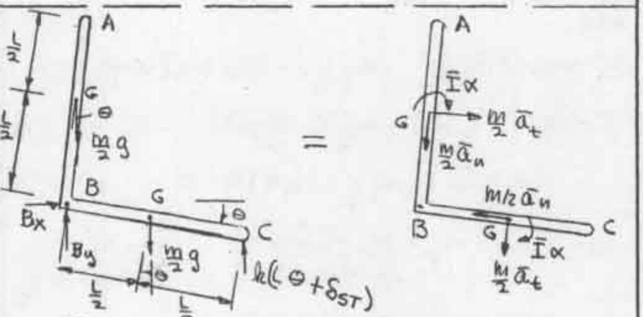


GIVEN:

ROD ABC OF TOTAL MASS m

FIND:

FREQUENCY OF SMALL OSCILLATIONS IN TERMS OF m, L AND k.



$\alpha = \ddot{\theta}$ $\bar{a}_t = \frac{L}{2}\ddot{\theta}$ $\bar{I} = \frac{1}{12}\frac{m}{2}L^2 = \frac{mL^2}{24}$

$\Sigma M_B = \Sigma (M_B)_{eff}$ $\sin\theta \approx \theta$ $\cos\theta \approx 1$

$$\frac{mg}{2}\frac{L}{2}\sin\theta + \frac{mg}{2}\frac{L}{2}\cos\theta - kL(L\theta + \delta_{ST})\cos\theta = 2\bar{I}\ddot{\theta} + 2\frac{m}{2}\bar{a}_t\frac{L}{2}$$

$$\frac{mgL}{4}\theta + \frac{mgL}{4} - kL^2\theta - kL^2\delta_{ST} = \frac{mL^2}{12}\ddot{\theta} + \frac{mL^2}{4}\ddot{\theta} \quad (1)$$

BUT FOR EQUILIBRIUM ($\theta = 0$)

$$\Sigma M_B = 0 = \frac{m}{2}g\frac{L}{2} - kL^2\delta_{ST} \quad (2)$$

SUBSTITUTE (2) INTO (1)

$$\left(\frac{mgL}{4} - kL^2\right)\theta = \frac{mL^2}{3}\ddot{\theta}$$

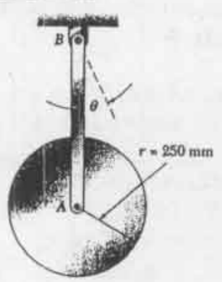
$$\ddot{\theta} + \frac{(kL^2 - mgL/4)}{mL^2/3}\theta = 0$$

$$\omega_n^2 = 3\frac{k}{m} - \frac{3}{4}\frac{g}{L} \quad \omega_n = \sqrt{3}\sqrt{\frac{k}{m} - \frac{g}{4L}}$$

$$f_n = \omega_n / 2\pi$$

$$f_n = \frac{\sqrt{3}}{2\pi}\sqrt{\frac{k}{m} - \frac{g}{4L}}$$

19.59



GIVEN:

ROB AB LENGTH $l = 0.650$ m

MASS OF AB IS NEGLIGIBLE

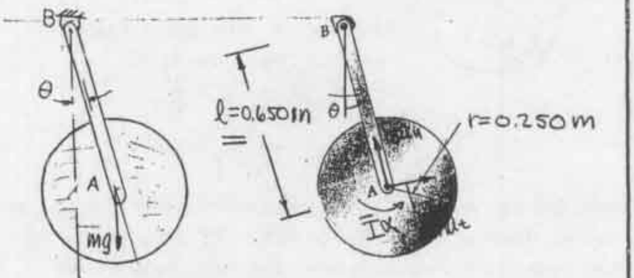
AB IS DISPLACED 2° FROM THE POSITION SHOWN AND RELEASED

FIND:

MAXIMUM VELOCITY OF A IF THE DISK IS,

(a) FREE TO ROTATE ABOUT A

(b) RIVETED TO AB AT A



$$\bar{I} = \frac{1}{2}mr^2 = \frac{1}{2}(0.250)^2 m = \frac{m}{32} \quad \alpha = \ddot{\theta}$$

$$a_t = l\alpha = 0.650\,\ddot{\theta}$$

(a) THE DISK IS FREE TO ROTATE AND IS IN CURVILINEAR TRANSLATION. THUS $\bar{I}\alpha = 0$

$\Sigma M_B = \Sigma (M_B)_{eff}$

$$-mgl\sin\theta = lma_t$$

$$ml^2\ddot{\theta} + mgl\theta = 0 \quad \omega_n^2 = \frac{g}{l}$$

FROM 19.17, THE SOLUTION TO THIS EQUATION IS

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

At $t = 0$, $\theta = 2\cdot\frac{\pi}{180} = \frac{\pi}{90}$ RAD, $\dot{\theta} = 0$

$$\dot{\theta} = \theta_m\omega_n\cos(\omega_n t + \phi)$$

$t = 0$ $0 = \theta_m\omega_n\cos\phi \quad \phi = \frac{\pi}{2}$

$$\frac{\pi}{90} = \theta_m\sin\left(0 + \frac{\pi}{2}\right) \quad \theta_m = \frac{\pi}{90}\text{ RAD}$$

THUS $\theta = \frac{\pi}{90}\sin\left(\omega_n t + \frac{\pi}{2}\right)$

$(v_A)_{MAX} = l\dot{\theta}_{MAX} = l\theta_m\omega_n$ $l = 0.650$ m

$\theta_m = \frac{\pi}{90}$ $\omega_n = \sqrt{\frac{g}{l}}$

$$(v_A)_{MAX} = (0.650\text{ m})\left(\frac{\pi}{90}\right)\left(\sqrt{\frac{9.81\text{ m/s}^2}{0.650\text{ m}}}\right)$$

$(v_A)_{MAX} = 0.08815$ m/s $(v_A)_{MAX} = 88.1$ mm/s

(b) FOR DISK RIVETED AT A ($\bar{I}\alpha$ INCLUDED)

$\Sigma M_B = \Sigma (M_B)_{eff}$ $-mgl\sin\theta = \bar{I}\alpha + lma_t$

$$\left(\frac{1}{2}mr^2 + ml^2\right)\ddot{\theta} + mgl\theta = 0$$

$$\omega_n^2 = \frac{gl}{r^2/2 + l^2}$$

$\theta = \frac{\pi}{90}\sin\left(\omega_n t + \frac{\pi}{2}\right)$ (SEE (a))

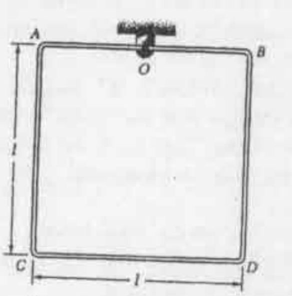
$$(v_A)_{MAX} = l\dot{\theta}_{MAX} = l\theta_m\omega_n$$

$$(v_A)_{MAX} = (0.650\text{ m})\sqrt{\frac{(9.81\text{ m/s}^2)(0.650\text{ m})}{(0.250^2/2 + 0.650^2)\text{ m}^2}} = 0.0851\text{ m/s}$$

$(v_A)_{MAX} = 85.1$ mm/s

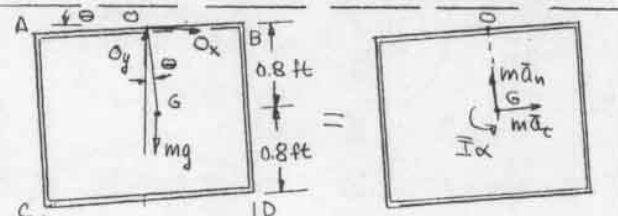
19.60

GIVEN:



THIN WIRE OF SIDE $\ell = 1.2\text{ ft}$.
CORNER B PUSHED DOWN 0.6 in. AND RELEASED

FIND:
(a) MAXIMUM VELOCITY OF POINT B
(b) CORRESPONDING MAGNITUDE OF THE ACCELERATION



M = TOTAL MASS

$$\bar{I} = \frac{1}{12}m\ell^2 + m\left(\frac{\ell}{2}\right)^2 = \frac{m\ell^2}{3}$$

$$\alpha = \ddot{\theta} \qquad \bar{a}_t = \frac{\ell}{2}\alpha = \frac{\ell}{2}\ddot{\theta} \qquad \theta \approx \sin\theta$$

$$\circlearrowleft\ \Sigma M_O = \Sigma(M_O)_{eff} \qquad -mg\sin\theta\,\frac{\ell}{2} = \bar{I}\alpha + m\left(\frac{\ell}{2}\right)^2\alpha$$

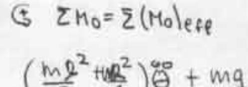
$$\left(\frac{m\ell^2}{3} + \frac{m\ell^2}{4}\right)\ddot{\theta} + mg\frac{\ell}{2}\theta = 0$$

$$\omega_n^2 = \frac{g\ell/2}{\frac{7}{12}\ell^2} = \frac{6}{7}\frac{g}{L} = \frac{6}{7}\left(\frac{32.2\text{ ft/s}^2}{1.6\text{ ft}}\right) = 23.0\text{ s}^{-2}$$

$$\omega_n = 4.796\text{ RAD/S}$$

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

(a)



$$x_B = \ell/2\,\theta$$

$$x_B = \ell/2\,\theta_m \sin(\omega_n t + \phi)$$

$$x_B = (x_B)_m \sin(\omega_n t + \phi)$$

AT $t = 0$ $\quad x_B = 0.6\text{ in.} = 0.05\text{ ft}$

$\dot{x}_B = 0$

$\underline{t=0}$ $\quad \dot{x}_B = 0 = (x_B)_m\,\omega_n\cos(0+\phi) \qquad \phi = \pi/2$

$$x_B = 0.05\text{ ft} = (x_B)_m\sin\left(0 + \frac{\pi}{2}\right), \ (x_B)_m = 0.05\text{ ft})$$

$$(\dot{x}_B)_m = (x_B)_m\,\omega_n = (0.05\text{ ft})(4.796\text{ r/s}) = 0.2398\text{ ft/s}$$

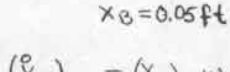
$$(\dot{x}_B)_m = 0.2398\text{ ft/s} = 2.88\ \frac{\text{in.}}{\text{s}}$$

(b) $x_B = (0.05\text{ ft})\sin(4.796t + \pi/2)$

$$\dot{x}_B = (0.2398\text{ ft/s})\cos\left(4.796t + \frac{\pi}{2}\right)$$

$$\ddot{x}_B = -(0.2398\text{ ft/s})(4.796\text{ s}^{-1})\sin\left(4.796t + \frac{\pi}{2}\right)$$

MAX VELOCITY WHEN $\left(4.796t + \frac{\pi}{2}\right) = 0$ OR $x_B = 0$



AND $\dot{x}_B = 0.2398\ \frac{\text{ft}}{\text{s}}$
AND $\ddot{x}_B = 0$

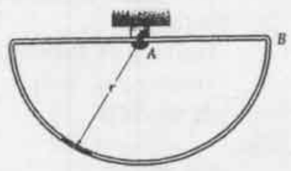
$$a_t = \ddot{x}_B = 0$$

$$a_n = (\dot{x}_B)^2/(\ell/2) = (0.2398\text{ ft/s})^2/(0.8\text{ ft}) = (0.07188\text{ ft/s}^2)$$

$$a_B = \sqrt{a_t^2 + a_n^2} = 0.07188\text{ ft/s}^2 = 0.863\ \frac{\text{in.}}{\text{s}^2}$$

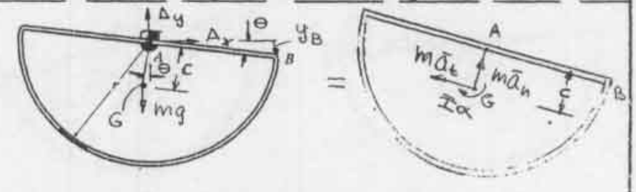
19.61

GIVEN:



THIN WIRE OF RADIUS $r = 0.220\text{ m}$.
POINT B IS PUSHED DOWN 0.020 m AND RELEASED

FIND:
v_B AT 8 S.

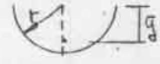


DETERMINE LOCATION OF THE CENTROID G.

LET ρ = MASS PER UNIT LENGTH

THEN TOTAL MASS $m = \rho(2r + \pi r) = \rho r(2+\pi)$

ABOUT C $\quad mgc = 0 + \pi r\rho\left(\frac{2r}{\pi}\right)g = 2r^2\rho g$



$$\bar{y} = \frac{2r}{\pi}$$

$$\rho r(2+\pi)c = 2r^2\rho$$

$$c = \frac{2r}{(2+\pi)}$$

$$\circlearrowleft\ \Sigma M_O = \Sigma(M_O)_{eff} \qquad \alpha = \ddot{\theta} \quad a_t = c\alpha = c\ddot{\theta}$$

$$-mgc\sin\theta = \bar{I}\alpha + mca_n \qquad \sin\theta \approx \theta$$

$$(\bar{I} + mc^2)\ddot{\theta} + mgc\theta = 0 \qquad I_O\ddot{\theta} + mgc\theta = 0$$

BUT $\bar{I} + mc^2 = I_O$

$$I_O = (I_O)_{\text{SEMI CIRC}} + (I_O)_{\text{LINE}} = m_{\text{S-CIRC}}r^2 + m_{\text{LINE}}\frac{(2r)^2}{12}$$

$$m_{\text{S-CIRC}} = \rho\pi r \qquad m_{\text{LINE}} = \rho 2r \qquad \rho = \frac{m}{r(2+\pi)}$$

$$I_O = \rho\left[\pi r \cdot r^2 + \frac{2r \cdot r^2}{3}\right] = \frac{mr^2}{(2+\pi)}\left[\pi + \frac{2}{3}\right]$$

$$\frac{mr^2}{(2+\pi)}\left(\pi + \frac{2}{3}\right)\ddot{\theta} + mg\frac{2r}{(2+\pi)}\theta = 0$$

$$\omega_n^2 = \frac{2g}{\left(\pi + \frac{2}{3}\right)r} = \frac{2(9.81\text{ m/s}^2)}{\left(\pi + \frac{2}{3}\right)(0.220\text{ m})}$$

$$\omega_n^2 = 23.42\text{ s}^{-2} \qquad \omega_n = 4.839\text{ RAD/S}$$

$$\theta = \theta_m\sin(\omega_n t + \phi) \qquad y_B = r\theta$$

$$y_B = r\theta_m\sin(\omega_n t + \phi) = (y_B)_m\sin(\omega_n t + \phi)$$

At $t = 0$ $\quad y_B = 0.02\text{ m} \quad \dot{y}_B = 0$

$(t=0)$ $\quad \dot{y}_B = 0 = (y_B)_m\cos(0+\phi) \qquad \phi = \frac{\pi}{2}$

$$y_B = 0.02 = (y_B)_m\sin\left(0 + \frac{\pi}{2}\right) \qquad (y_B)_m = 0.02\text{ m}$$

$$y_B = 0.02\sin\left(\omega_n t + \frac{\pi}{2}\right) \qquad \omega_n = 4.839\text{ RAD}$$

$$\dot{y}_B = (0.02)(\omega_n)\cos\left(\omega_n t + \frac{\pi}{2}\right) = -(0.02)\omega_n\sin\omega_n t$$

At $t = 8\text{ s}$ $\quad v_B = \dot{y}_B = -(0.02)(4.839)\sin[(4.839)(8)]$

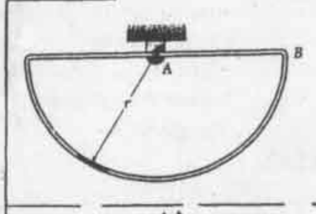
$$v_B = -0.0821\text{ m/s} \qquad v_B = 82.1\text{ mm/s}$$

19.62

GIVEN:

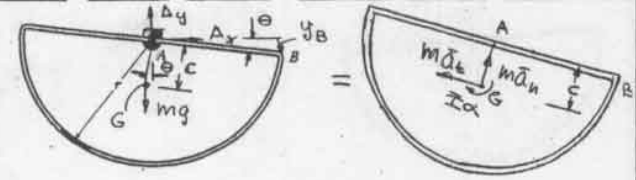
THIN WIRE OF RADIUS $r = 16$ in.

POINT B IS PUSHED DOWN 1.5 in. AND RELEASED



FIND:

a_B AT 10 s



DETERMINE LOCATION OF THE CENTROID G.

LET ρ = MASS PER UNIT LENGTH

THEN TOTAL MASS $m = \rho(2r + \pi r) = \rho r(2+\pi)$

ABOUT C $\quad mgc = 0 + \pi r \rho \left(\frac{2r}{\pi}\right) g = 2r^2 \rho g$

$\bar{y} = \frac{2r}{\pi}$

$$\rho r(2+\pi) c = 2r^2 \rho$$

$$c = \frac{2r}{(2+\pi)}$$

$\Sigma M_0 = \Sigma (M_0)_{eff} \quad \alpha = \ddot{\theta} \quad a_t = c\alpha = c\ddot{\theta}$

$-mgc \sin\theta = \bar{I}\alpha + mca_n \quad \sin\theta \approx \theta$

$(\bar{I} + mc^2)\ddot{\theta} + mgc\theta = 0 \quad I_0\ddot{\theta} + mgc\theta = 0$

BUT $\bar{I} + mc^2 = I_0$

$I_0 = (I_0)_{semi\ circ} + (I_0)_{LINE} = m_{s\text{-}circ} r^2 + m_{LINE}\frac{(2r)^2}{12}$

$m_{s\text{-}circ} = \rho\pi r \quad m_{LINE} = \rho 2r \quad \rho = m/(2+\pi)r$

$I_0 = \rho[\pi r^2 \cdot r + 2r \cdot r^2/3] = \frac{mr^2}{(2+\pi)}[\pi + \frac{2}{3}]$

$\frac{mr^2}{(2+\pi)}[\pi + \frac{2}{3}]\ddot{\theta} + mg\frac{2r}{(2+\pi)}\theta = 0$

$$\omega_n^2 = \frac{2g}{(\pi + \frac{2}{3})r} = \frac{2(32.2\ \text{ft/s}^2)}{(\pi + \frac{2}{3})(16/12\ \text{ft})}$$

$$\omega_n^2 = 12.683 \qquad \omega_n = 3.561\ \text{r/s}$$

$\theta = \theta_m \sin(\omega_n t + \phi) \quad y_B = r\theta$

AT $t = 0 \quad y_B = 1.5/12 = 0.125$ ft $\quad \dot{y}_B = 0$

$\dot{y}_B = 0 = (y_B)_m \cos(0 + \phi), \ \phi = \frac{\pi}{2}$

$y_B = 0.125\ \text{ft} = (y_B)_m \sin(0 + \frac{\pi}{2}), (y_B) = 0.125$ ft

$y_B = 0.125 \sin(\omega_n t + \frac{\pi}{2}) \quad \omega_n = 3.561$ r/s

$\dot{y}_B = 0.125\,\omega \cos(\omega_n t + \pi/2) = -.125\omega_n \sin\omega_n t$

$\ddot{y}_B = (-0.125)(\omega_n^2)\sin(\omega_n t + \frac{\pi}{2}) = 0.125\omega_n^2 \cos\omega_n t$

AT $t = 10$ s $\quad (a_B)_t = \ddot{y}_B = (0.125)(3.561)^2\cos[(3.561)(10)] = -0.7811$

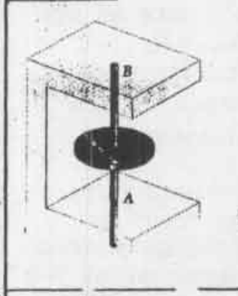
$(v_B) = \dot{y}_B = (0.125)(3.561)\sin[(3.561)(10)] = 0.3874$ ft/s

$a_B = \left[(a_B)_t^2 + \frac{v_B^2}{r}\right]^{1/2} = \left[(0.7811)^2 + \left\{\frac{(0.3874)^2}{(16/12)}\right\}^2\right]^{1/2} = 0.789\ \text{ft/s}^2$

19.63

GIVEN:

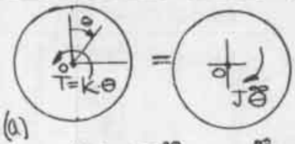
DISK OF RADIUS $r = 120$ mm IS WELDED TO ROD AB WHICH IS FIXED AT A AND B. DISK ROTATES 8° WHEN A 500-mN·m IS APPLIED PERIOD $\tau_n = 1.3$ s WHEN THE COUPLE IS REMOVED



FIND:

(a) THE MASS OF THE DISK

(b) PERIOD IF ONE ROD IS REMOVED



$K = \frac{T}{\theta} = \frac{0.5\ \text{N.m}}{(8)(\pi/180)}$

$K = 3.581$ N·m/rad

(a) $\quad \Sigma M_0 = \Sigma(M_0)_{eff}$

$-K\theta = J\ddot{\theta} \qquad \ddot{\theta} + \frac{K}{J}\theta = 0$

$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{J/K} \qquad J = \frac{\tau_n^2 K}{(2\pi)^2} = \frac{(1.3s)^2(3.581\ \text{N.m/r})}{(2\pi)^2}$

$J = 0.1533\ \text{N.m.s}^2 = \frac{1}{2}mr^2 = \frac{1}{2}m(0.120\text{m})^2$

$m = \frac{(0.1533\ \text{N.m.s}^2)(2)}{(0.120\ \text{m})^2} = 21.3$ kg

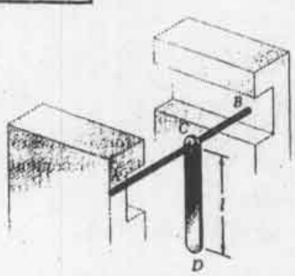
(b) WITH ONE ROD REMOVED $\quad K' = K/2 = \frac{3.581}{2} = 1.791\ \frac{\text{N.m}}{\text{rad}}$

$\tau = 2\pi\sqrt{J/K'} = 2\pi\sqrt{\frac{(0.1533\ \text{N.m.s}^2)}{1.791\ \text{N.m/rad}}} = 1.838$ s

19.64

GIVEN:

10-lb ROD CD OF LENGTH $l = 2.2$ ft WELDED ROD FIXED AT A AND B WITH $K = 18$ lb·ft/rad



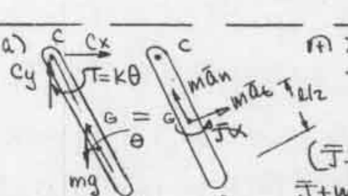
FIND:

PERIOD OF SMALL OSCILLATIONS IF THE EQUILIBRIUM POSITION IS,

(a) VERTICAL AS SHOWN

(b) HORIZONTAL

(a)



$\Sigma M_C = \Sigma(M_C)_{eff}$

$-k\theta - mg\sin\theta = \bar{J}\alpha + ml\bar{a}_t$

$\alpha = \ddot{\theta} \quad a_t = \frac{l}{2}\alpha = \frac{l}{2}\ddot{\theta}$

$(\bar{J} + m(\frac{l}{2})^2)\ddot{\theta} + (k + mgl/2)\theta = 0$

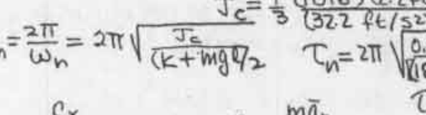
$\bar{J} + m(\frac{l}{2})^2 = J_C = \frac{1}{3}ml^2$

$J_C = \frac{1}{3}\frac{(10\ \text{lb})(2.2\ \text{ft})^2}{(32.2\ \text{ft/s}^2)} = 0.501\ \text{lb.ft.s}^2$

$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{J_c}{(K + mgl/2)}} \qquad \tau_n = 2\pi\sqrt{\frac{0.501\ \text{lb.ft.s}^2}{[(18) + (10)(1.1)]\ \text{lb.ft}}}$

$\tau_n = 0.826$ s

(b)



$\alpha = \ddot{\theta} \quad a_t = l/2\,\ddot{\theta}$

$\Sigma M_C = \Sigma(M_C)_{eff}$

$-k(\theta + \theta_{ST}) + mgl/2 = \bar{J}\ddot{\theta} + m(l/2)^2\ddot{\theta} = J_C\ddot{\theta}$

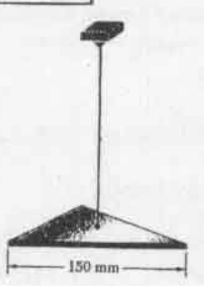
BUT IN EQUILIBRIUM $(\theta = 0)$ $\Sigma M_C = 0 = -k\theta_{ST} + mgl/2$

THUS $J_C\ddot{\theta} + K\theta = 0$

$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\bar{I}/K} = 2\pi\sqrt{\frac{0.501\ \text{lb.ft.s}^2}{18\ \text{lb.ft}}} = 1.048$ s

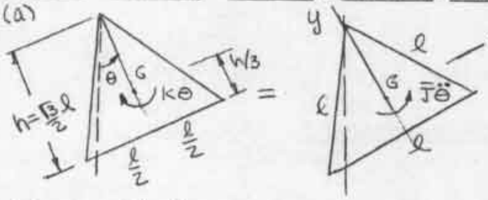
19.65

GIVEN:
1.8-kg PLATE IN THE SHAPE OF AN EQUILATERAL TRIANGLE SUSPENDED FROM A WIRE AT ITS CENTER OF GRAVITY
FOR THE WIRE $K = 35\ mN\cdot m/rad$
PLATE IS ROTATED 360° AND RELEASED.

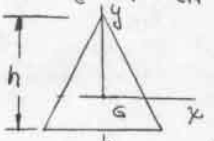


FIND:
(a) PERIOD OF OSCILLATION
(b) MAXIMUM VELOCITY OF ONE OF THE VERTICES

(a)



$\circlearrowleft \Sigma M_G = \Sigma (M_G)_{eff} \qquad -K\theta = J_{mass}\alpha \quad \alpha = \ddot{\theta}$



$\bar{J}_M = (I_x)_m + (I_y)_m \quad A = \frac{1}{2}bh \quad m = \frac{\rho t\, bh}{2}$

$(I_x)_m = \frac{1}{36}bh^3\rho t \qquad \rho t = \frac{2m}{bh}$

$(I_y)_m = 2\left(\frac{1}{12}h\left(\frac{b}{2}\right)^3\right)\rho t$

$(I_y)_m = \frac{hb^3}{48}\rho t$

$\bar{J}_m = \frac{1}{36}bh^3\rho t + \frac{1}{48}hb^3\rho t = m\left[\frac{h^2}{18} + \frac{b^2}{24}\right]$

$\bar{J}_m = (1.8\ kg)\left[\frac{(\sqrt{3}/2)^2}{18} + \frac{1}{24}\right](0.150)^2 = 3.375\times10^{-3}\ kg\text{-}m^2$

$\bar{J}_m\ddot{\theta} + K\theta_m = 0 \qquad 3.375\times10^{-3}\ddot{\theta} + 35\times10^{-3}\theta = 0$

$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{3.375\times10^{-3}\ kg\text{-}m^2}{35\times10^{-3}\ N\cdot m}}$

$\tau_n = 1.951\ s$

(b)

$\theta = \theta_m \sin(\omega_n t + \phi)$

$\omega_n = \sqrt{\frac{35\times10^{-3}\ N\cdot m}{3.375\ kg\cdot m^2}} = 3.22\ rad/s^2$

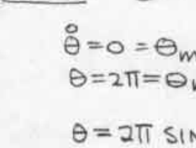
AT $t=0 \quad \theta = 2\pi\ rad \quad \dot{\theta} = 0$

$\dot{\theta} = 0 = \theta_m\omega_n\cos(0+\phi) \qquad \phi = \pi/2$

$\theta = 2\pi = \theta_m \sin(0 + \frac{\pi}{2}) \qquad \theta_m = 2\pi\ rad$

$\theta = 2\pi\sin(3.22t + \frac{\pi}{2}) = 2\pi\cos(3.22t)$

$\dot{\theta}_{MAX} = \theta_m\omega_n = (2\pi)(3.22) = 20.23\ rad/s$



$r = \frac{h/3}{\sin 30°} = 2h/3 = \sqrt{3}/3\,\ell$

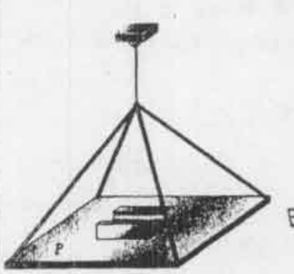
$(v_A)_{MAX} = r\dot{\theta}_{MAX}$

$(v_A)_{MAX} = (\sqrt{3}/3)(0.150\ m)(20.23\ rad/s)$

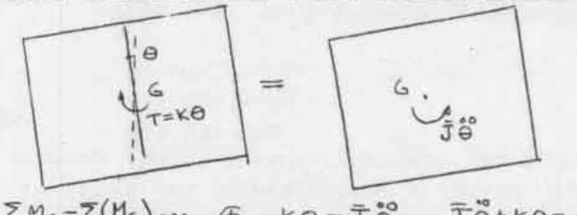
$(v_A)_{MAX} = 1.752\ m/s$

19.66

GIVEN:
ROTATION ABOUT A VERTICAL AXIS
$\tau_n = 2.2\ s$, EMPTY PLATFORM
$\tau_n' = 3.8\ s$, WHEN OBJECT A IS ADDED
$K = 20\ lb\cdot ft/rad$ FOR WIRE



FIND:
CENTROIDAL MOMENT OF INERTIA FOR OBJECT A



$\Sigma M_G = \Sigma(M_G)_{eff} \quad \circlearrowleft\ -K\theta = \bar{J}\ddot{\theta} \qquad \bar{J}\ddot{\theta} + K\theta = 0$

EMPTY PLATFORM $\quad \bar{J}_P$ = CENTROIDAL J OF PLATFORM

$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{K/\bar{J}_P}} \qquad \bar{J}_P = \frac{K\tau_n^2}{4\pi^2} = \frac{(20\ lb\,ft/rad)(2.2\ s)^2}{4\pi^2}$

$\bar{J}_P = 2.452\ lb\cdot ft\cdot s^2$

PLATFORM WITH OBJECT A $\quad \bar{J}_A$ = CENTROIDAL J OF A

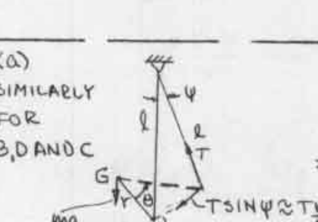
$\tau_n' = \frac{2\pi}{\omega_n'} = \frac{2\pi}{\sqrt{K/(\bar{J}_P + \bar{J}_A)}} \qquad \bar{J}_P + \bar{J}_A = \frac{K(\tau_n')^2}{4\pi^2}$

$\bar{J}_A = \frac{(20\ lb\,ft/rad)(3.8\ s)^2}{4\pi^2} - 2.452\ lb\cdot ft\cdot s^2$

$\bar{J}_A = 7.315 - 2.452 = 4.863\ lb\cdot ft\cdot s^2 \qquad \bar{J}_A = 4.86\ lb\,ft\,s^2$

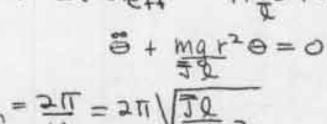
19.67

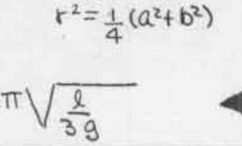
GIVEN:
THIN RECTANGULAR PLATE SUSPENDED AS SHOWN



FIND:
PERIOD FOR A
(a) ROTATION ABOUT A VERTICAL AXIS THROUGH G
(b) HORIZONTAL DISPLACEMENT PERPENDICULAR TO AB.
(C) HORIZONTAL DISPLACEMENT PERPENDICULAR TO BC

(a)
SIMILARLY FOR B, D AND C





$\Sigma M_G = \Sigma(M_G)_{eff} \qquad -4T\frac{r\theta}{\ell}\cdot r = \bar{J}\ddot{\theta} \qquad T = mg/4$

$\ddot{\theta} + \frac{mg\,r^2}{\bar{J}\ell}\theta = 0 \qquad \bar{J} = \frac{1}{12}m(a^2+b^2)$

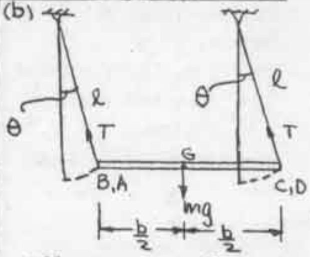
$r^2 = \frac{1}{4}(a^2+b^2)$

$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{\bar{J}\ell}{mgr^2}}$

$\tau_n = 2\pi\sqrt{\frac{\frac{1}{12}m(a^2+b^2)\ell}{mg\frac{1}{4}(a^2+b^2)}} = 2\pi\sqrt{\frac{\ell}{3g}}$

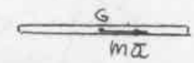
19.67 CONTINUED

(b)



THE PLATE IS IN CURVILINEAR TRANSLATION

$=$



$\sin\theta \approx \theta$
$\cos\theta \approx 1$

$\ell\ddot{\theta}\cos\theta \approx \ell\ddot{\theta} = \bar{a}$

$+\uparrow \Sigma F_V = 0 = 4(T\cos\theta) - mg = 0 \quad T = mg/4$

$+\rightarrow \Sigma F_H = \Sigma (F_H)_{eff} \quad -4T\sin\theta = m\bar{a}$

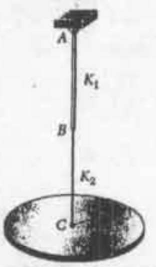
$\ell\ddot{\theta} + g\theta = 0$

$\tau_n = 2\pi\sqrt{\frac{\ell}{g}}$

(c) SINCE THE OSCILLATION ABOUT AXES PARALLEL TO AB (AND CD) IS INDEPENDENT OF THE LENGTH OF THE SIDES OF THE PLATE, THE PERIOD OF VIBRATION ABOUT AXES PARALLEL TO BC (AND AD) IS THE SAME

$\tau_n = 2\pi\sqrt{\ell/g}$

19.68



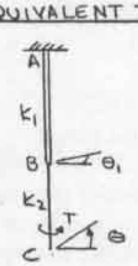
GIVEN:

2.2-kg CIRCULAR DISK
$r = 0.8\,m$
WIRE AB, $k_1 = 10\,N\cdot m/rad$
WIRE BC, $k_2 = 5\,N\cdot m/rad$

FIND:

PERIOD OF OSCILLATION ABOUT AXIS AC

EQUIVALENT TORSIONAL SPRING CONSTANT



$T = k_e\theta, \quad T = k_2(\theta - \theta_1), \quad T = k_1\theta_1$

$k_2\theta = (k_1 + k_2)\theta_1$

$\theta_1 = \frac{k_2}{k_1 + k_2}\theta$

$T = k_e\theta = k_1\theta$

$k_e\theta = k_1\frac{k_2}{k_1 + k_2}\theta$

$k_e = \frac{k_1 k_2}{k_1 + k_2}$

NEWTONS LAW

$\Sigma M_C = \Sigma (M_C)_{eff}$
$+$
$-k_e\theta = \bar{J}\ddot{\theta}$

$\bar{J} = \frac{1}{2}mr^2$

$\frac{1}{2}mr^2\ddot{\theta} + k_e\theta = 0$

$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{2k_e}{mr^2}}} = 2\pi\sqrt{\frac{(2.2\,kg)(0.8\,m)^2}{2[(10)(5)/(10+5)]\,N\cdot m}}$

$\tau_n = 2.89\,s$

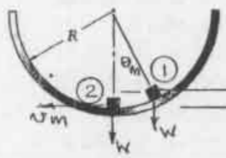
19.69

GIVEN:

PARTICLE WHICH MOVES WITHOUT FRICTION INSIDE A CURVED SURFACE

FIND:

PERIOD OF SMALL OSCILLATIONS



DATUM AT ②

POSITION ①

$T_1 = 0$
$V_1 = WR(1 - \cos\theta_m)$
SMALL OSCILLATIONS
$(1 - \cos\theta_m) = 2\sin^2\frac{\theta_m}{2} \approx \theta_m^2/2$
$V_1 = \frac{WR\theta_m^2}{2}$

POSITION ②

$v_m = R\dot{\theta}_m \quad T_2 = \frac{1}{2}mv_m^2 = \frac{1}{2}mR^2\dot{\theta}_m^2$
$V_2 = 0$

CONSERVATION OF ENERGY $\quad T_1 + V_1 = T_2 + V_2$

$0 + WR\frac{\theta_m^2}{2} = \frac{1}{2}mR^2\dot{\theta}_m^2 + 0 \quad \dot{\theta}_m = \omega_n\theta_m$
$W = mg$

$mgR\frac{\theta_m^2}{2} = \frac{1}{2}mR^2\omega_n^2\theta_m^2$

$\omega_n = \sqrt{g/R}$

$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{R}{g}}$

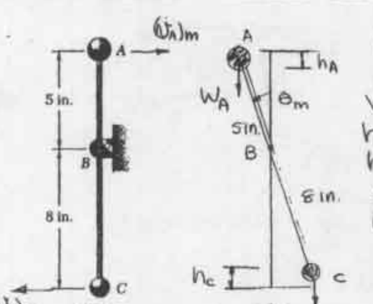
19.70

GIVEN:

14 OZ. SPHERE A
10 OZ. SPHERE C
ROD AC OF NEGLIGIBLE WEIGHT

FIND:

PERIOD OF SMALL OSCILLATIONS OF THE ROD



DATUM AT ①

POSITION ①

$T_1 = 0$
$V_1 = W_C h_C - W_A h_A$
$h_C = BC(1 - \cos\theta_m)$
$h_A = BA(1 - \cos\theta_m)$
SMALL ANGLES
$1 - \cos\theta_m \approx \theta_m^2/2$

$V_1 = [(W_C)(BC) - (W_A)(BA)]\frac{\theta_m^2}{2}$

$V_1 = \left[\left(\frac{10}{16}\,lb\right)\left(\frac{8}{12}\,ft\right) - \left(\frac{14}{16}\,lb\right)\left(\frac{5}{12}\,ft\right)\right]\frac{\theta_m^2}{2}$

$V_1 = (0.4167 - 0.3646)\frac{\theta_m^2}{2} = 0.05208\frac{\theta_m^2}{2}$

$V_2 = 0 \quad T_2 = \frac{1}{2}m_C(v_C)_m^2 + \frac{1}{2}m_A(v_A)_m^2, \quad (v_C)_m = \frac{8}{12}\dot{\theta}_m$

$T_2 = \frac{1}{2}m_C\left(\frac{8}{12}\right)^2\dot{\theta}_m^2 + \frac{1}{2}m_A\left(\frac{5}{12}\right)^2\dot{\theta}_m^2 \quad (v_A)_m = \frac{5}{12}\dot{\theta}_m$

$T_2 = \frac{1}{2g}\left[(10/16)(8/12)^2 + (14/16)(5/12)^2\right]\omega_n^2\theta_m^2 \qquad \dot{\theta}_m = \omega_n\theta_m$

$T_2 = \frac{1}{2g}[0.2778 + 0.1519]\omega_n^2\theta_m^2 = \frac{1}{2g}(0.4297)\omega_n^2\theta_m^2$

$T_1 + V_1 = T_2 + V_2 \quad 0 + 0.05208\frac{\theta_m^2}{2} = \frac{0.4297}{2g}\omega_n^2\theta_m^2$

$\omega_n^2 = \frac{(32.2)(.05208)}{(0.4297)} = 3.902, \quad \tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{3.902}} = 3.18\,s$

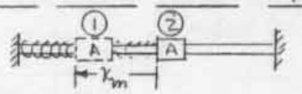
19.71

GIVEN:

1.8 kg COLLAR A ATTACHED TO A SPRING $k = 800$ N/m, NO FRICTION COLLAR MOVED 70 mm TO THE LEFT AND RELEASED

FIND:

THE MAXIMUM VELOCITY
THE MAXIMUM ACCELERATION



DATUM AT ①

POSITION ①

$T_1 = 0 \quad V_1 = \frac{1}{2} k x_m^2$

POSITION ②

$T_2 = \frac{1}{2} m v_2^2 \quad V_2 = 0 \quad v_2 = \dot{x}_m$

$\dot{x}_m = \omega_n x_m$

$T_1 + V_1 = T_2 + V_2 \quad 0 + \frac{1}{2} k x_m^2 = \frac{1}{2} m \dot{x}_m^2 + 0$

$\frac{1}{2} k x_m^2 = \frac{1}{2} m \omega_n^2 x_m^2 \quad \omega_n^2 = \frac{k}{m} = \frac{800 \text{ N/m}}{1.8 \text{ kg}}$

$\omega_n^2 = 444.4 \text{ s}^{-2} \quad \omega_n = 21.08 \text{ rad/s}$

$\dot{x}_m = \omega_n x_m = (21.08 \text{ s}^{-1})(0.070 \text{ m}) = 1.476 \text{ m/s}$ ◀

$\ddot{x}_m = \omega_n^2 x_m = (21.08 \text{ s}^{-2})(0.070 \text{ m}) = 31.1 \text{ m/s}^2$ ◀

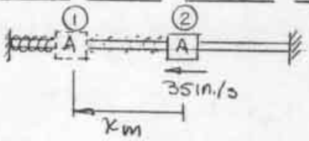
19.72

GIVEN:

3-lb COLLAR A, ATTACHED TO A SPRING OF CONSTANT $k = 5$ lb/in.
COLLAR INITIAL VELOCITY $= 35$ in./s. NO FRICTION

FIND:

THE AMPLITUDE
THE MAXIMUM ACCELERATION



DATUM AT ①

POSITION ①

$35 \text{ in./s} = 2.917 \text{ ft/s}$

$k = 5 \text{ lb/in.} = 60 \text{ lb/ft}$

$T_1 = 0 \quad V_1 = \frac{1}{2} k x_m^2 = \frac{1}{2}(60 \text{ lb/ft}) x_m^2$

POSITION ②

$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}\left(\frac{3 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(2.917 \text{ ft/s})^2 = 0.79257 \text{ lb-ft}$

$V_2 = 0$

$T_1 + V_1 = T_2 + V_2 \quad 0 + \frac{60}{2} x_m^2 = \frac{0.79257}{2}$

$x_m = 0.01149 \text{ ft} = 1.379 \text{ in.}$ ◀

AT POSITION 2 $\quad T_2 = \frac{1}{2} m \dot{x}_2^2 = \frac{1}{2}\left(\frac{3 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(\omega_n^2)(x_m^2)$

$T_2 = \frac{0.09317}{2} \omega_n^2 x_m^2$

$T_1 + V_1 = T_2 + V_2$

$0 + \frac{60}{2} x_m^2 = \frac{0.09317}{2} \omega_n^2 x_m^2$

$\omega_n^2 = 644 \text{ s} \quad \omega_n = 25.38 \text{ rad/s}$

$\ddot{x}_m = \omega_n^2 x_m = (644 \text{ s}^{-2})(0.1149 \text{ ft})$

$\ddot{x}_m = 74.0 \text{ ft/s}^2 = 888 \text{ in./s}^2$ ◀

19.73

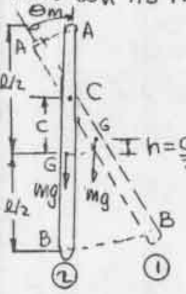
GIVEN:

ROTATION OF A UNIFORM ROD ABOUT A HORIZONTAL AXIS AT C. SMALL OSCILLATIONS

FIND:

VALUE OF THE DISTANCE c FOR WHICH THE FREQUENCY OF SMALL OSCILLATIONS WILL BE MAXIMUM

FIND ω_n AS A FUNCTION OF c.



$h = \frac{c}{2}(1 - \cos\theta_m)$

DATUM AT ②

POSITION ①

$T_1 = 0 \quad V_1 = mgh$

$V_1 = mgc(1 - \cos\theta_m)$

$1 - \cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$

$V_1 = mgc\frac{\theta_m^2}{2}$

POSITION ②

$T_2 = \frac{1}{2} I_C \dot{\theta}_m^2$

$I_C = \bar{I} + mc^2 = \frac{1}{12} ml^2 + mc^2$

$T_2 = \frac{1}{2} m(l^2/12 + c^2)\dot{\theta}_m^2 \quad V_2 = 0$

$T_1 + V_1 = T_2 + V_2 \quad 0 + mgc\frac{\theta_m^2}{2} = m(l^2/12 + c^2)\frac{\dot{\theta}_m^2}{2} + 0$

$\dot{\theta}_m = \omega_n \theta_m$

$gc = m(l^2/12 + c^2)\omega_n^2$

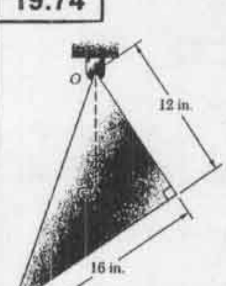
$\omega_n^2 = \frac{gc}{(l^2/12 + c^2)}$

MAXIMUM C, WHEN

$\frac{d\omega_n^2}{dc} = 0 = \frac{g(l^2/12 + c^2) - 2c^2 g}{(l^2/12 + c^2)} = 0$

$l^2/12 - c^2 = 0 \quad c = l/\sqrt{12}$ ◀

19.74

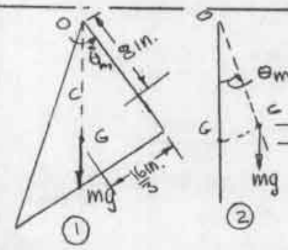


GIVEN:

THIN PLATE CUT INTO THE SHAPE OF A RIGHT TRIANGLE AND SUSPENDED FROM O IN A VERTICAL PLANE

FIND:

PERIOD FOR SMALL OSCILLATIONS



$h = c(1 - \cos\theta_m)$

DATUM AT ①

POSITION ②

$T_2 = 0$

$V_2 = mgc(1 - \cos\theta_m)$

$1 - \cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$

$V_2 = mgc\,\theta_m^2/2$

19.74 CONTINUED

POSITION ①

$T_1 = \frac{1}{2} J_O \dot{\theta}_m^2 \quad V_1 = 0$

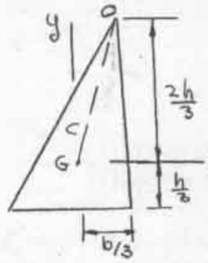
CONSERVATION OF ENERGY $\quad T_1 + V_1 = T_2 + V_2$

$\frac{1}{2} J_O \dot{\theta}^2 + 0 = 0 + mgc \frac{\theta_m^2}{2} \qquad \dot{\theta}_m = \omega_n \theta_m$

$J_O \omega_n^2 \theta_m^2 = mgc\theta_m^2$

$\omega_n^2 = \frac{mgc}{J_O}$

DETERMINE C AND J_O



$c = \left[\left(\frac{2h}{3}\right)^2 + \left(\frac{b}{3}\right)^2\right]^{\frac{1}{2}}$

$c = \frac{1}{3}\left[4h^2 + b^2\right]^{1/2}$

$h = 12 \text{ in.} = 1 \text{ ft}$

$b = 16 \text{ in} = 4/3 \text{ ft}$

$c = \left[4(1)^2 + (4/3)^2\right]^{1/2} = \frac{\sqrt{52}}{9}$ (with $\frac{}{3}$)

$J_O = \bar{J} + mc^2$

$\bar{J} = \rho t\left[\bar{I}_x + \bar{I}_y\right] = \rho t\left[\frac{1}{36} bh^3 + \frac{1}{36} hb^3\right]$

$m = \rho t \frac{1}{2} bh$

$\rho t = \frac{2m}{bh} \qquad \bar{J} = \frac{m}{18}\left[h^2 + b^2\right] = \frac{m}{18}\left[1 + (4/3)^2\right] = \frac{25}{162} m$

$J_O = \frac{25}{162} m + \frac{52}{81} m = \frac{129}{162} m \quad \text{lb}\cdot\text{ft}\cdot\text{s}^2$

$\omega_n^2 = \frac{mgc}{J_O} = \frac{m(32.2)\left(\frac{\sqrt{52}}{9}\right)}{m(129/162)} = 32.4 \text{ s}^{-2} \quad \omega_n = 5.592 \frac{r}{s}$

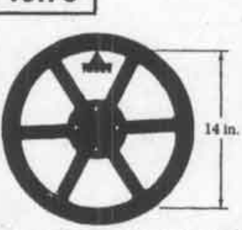
$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{5.592} = 1.104 \text{ s}$ ◀

19.75

GIVEN:

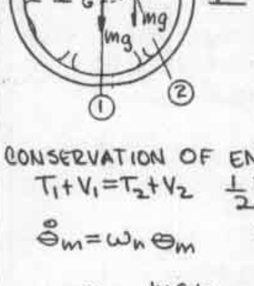
85-lb FLYWHEEL

PERIOD = 1.26 S FOR SMALL OSCILLATIONS



FIND:

CENTROIDAL MOMENT OF INERTIA, $\bar{J}$



DATUM AT ①

POSITION ①

$T_1 = \frac{1}{2} J_O \dot{\theta}_m^2 \quad V_1 = 0$

POSITION ②

$T_2 = 0 \quad V_2 = mgh$

$h = r(1 - \cos\theta_m) = r\,2\sin^2\theta_m/2$

$\approx r\theta_m^2/2$

$V_2 = mgr\theta_m^2/2$

CONSERVATION OF ENERGY

$T_1 + V_1 = T_2 + V_2 \quad \frac{1}{2} J_O \dot{\theta}_m^2 + 0 = 0 + mgr\theta_m^2/2$

$\dot{\theta}_m = \omega_n \theta_m \qquad J_O \omega_n^2 \theta_m^2 = mgr\theta_m^2$

$\omega_n^2 = \frac{mgr}{J_O} \qquad \tau_n^2 = \frac{4\pi^2}{\omega_n^2} = \frac{(4\pi^2) J_O}{mgr}$

$J_O = \bar{J} + mr^2 \qquad \bar{J} + mr^2 = \frac{(\tau_n^2)(mgr)}{4\pi^2}$

$\bar{J} = \frac{(\tau_n^2)(mgr)}{4\pi^2} - mr^2 = \frac{(1.26 s)^2 (85 lb)\left(\frac{7}{12} ft\right)}{4\pi^2} - \frac{(85 lb)}{(32.2 ft/s^2)}\left(\frac{7}{12} ft\right)^2$

$\bar{J} = 1.994 - 0.8983 = 1.096 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

19.76

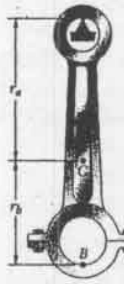
GIVEN:

FOR SMALL OSCILATIONS

PERIOD ABOUT A $= \tau_A = 0.895$ s

PERIOD ABOUT B $= \tau_B = 0.805$ s

$r_a + r_b = 0.270$ m

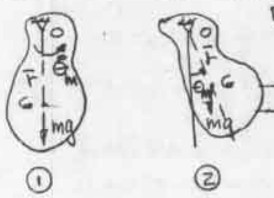


FIND:

(a) LOCATION OF THE MASS CENTER G

(b) CENTROIDAL RADIUS OF GYRATION $\bar{k}$.

CONSIDER GENERAL PENDULUM OF CENTROIDAL RADIUS OF GYRATION $\bar{k}$.



DATUM AT ①

POSITION ①

$T_1 = \frac{1}{2} J_O \dot{\theta}_m^2$

$V_1 = 0$

POSITION ②

$T_2 = 0 \quad V_2 = mgh$

$h = \bar{r}(1 - \cos\theta_m) = \bar{r}\,2\sin^2\frac{\theta_m}{2}$

$\approx \bar{r}\frac{\theta_m^2}{2}$

$V_2 = mg\frac{\theta_m^2}{2}$

$T_1 + V_1 = T_2 + V_2$

$\frac{1}{2} J_O \dot{\theta}_m^2 + 0 = 0 + \frac{1}{2} mg\bar{r}\theta_m^2$

$\dot{\theta}_m = \omega_n \theta_m$

$J_O \omega_n^2 \theta_m^2 = mg\bar{r}\theta_m^2$

$\omega_n^2 = \frac{mg\bar{r}}{J_O} \qquad \tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{J_O}{mg\bar{r}}}$

$J_O = \bar{J} + m\bar{r}^2 = m\bar{k}^2 + m\bar{r}^2$

(a) $\qquad \tau_n = 2\pi\sqrt{\frac{\bar{k}^2 + \bar{r}^2}{g\bar{r}}}$

FOR THE ROD SUSPENDED AT A

$\tau_n = 0.895 \text{ s} = 2\pi\sqrt{\frac{\bar{k}^2 + r_a^2}{g r_a}} \quad \bar{r} = r_a \qquad (1)$

FOR THE ROD SUSPENDED AT B

$\tau_n = 0.805 \text{ s} = \sqrt{\frac{\bar{k}^2 + r_b^2}{g r_b}} \quad \bar{r} = r_b \qquad (2)$

BUT $r_a + r_b = 0.270$ m $\qquad (3)$

FROM (1) $\bar{k}^2 + r_a^2 = g r_a \left(\frac{0.895}{2\pi}\right)^2 \qquad (1')$

FROM (2) $\bar{k}^2 + r_b^2 = g r_b \left(\frac{0.805}{2\pi}\right)^2 \qquad (2')$

SUBTRACING (2') FROM (1')

$r_a^2 - r_b^2 = (g/4\pi^2)(0.801 r_a - 0.648 r_b) \quad (4)$

DIVIDING (4) BY (3) MEMBER BY MEMBER

$r_a - r_b = \frac{1}{0.270}(g/4\pi^2)(0.801 r_a - 0.648 r_b)$

$r_a - r_b = \frac{9.81/4\pi^2}{0.270}(0.801 r_a - 0.648 r_b) = 0.7372 r_a - 0.5963 r_b$

$r_b = 0.6510 r_a \qquad (5)$

SUBSTITUTE FOR r_b FROM (5) INTO (3)

$r_a + 0.6510 r_a = 0.270 \qquad r_a = 0.1635$ m

$r_a = 163.5$ mm ◀

(b) FROM (1')

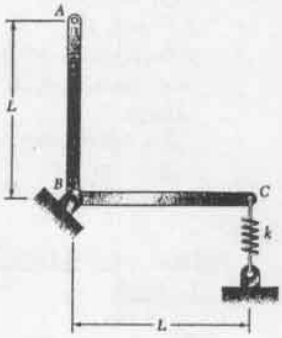
$\bar{k}^2 = (9.81)(0.1635)(0.895/2\pi)^2 - (0.1635)^2$

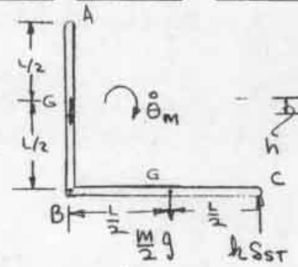
$\bar{k}^2 = 0.03254 - 0.02673 = 0.05812 \text{ m}^2 \quad \bar{k} = 76.2$ mm ◀

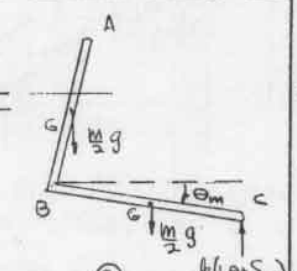
19.77

GIVEN: ROD ABC OF MASS m IN A VERTICAL PLANE PINNED AT B AND SUPPORTED BY A SPRING AT C OF CONSTANT k.

FIND: FREQUENCY OF SMALL OSCILLATIONS IN TERMS OF m, L AND k.







POSITION ①

$$T_1 = \frac{1}{2} I_B \dot{\theta}_m^2 \qquad V_1 = \frac{1}{2} k(\delta_{ST})^2$$

POSITION ②

$$T_2 = 0 \qquad V_2 = -\frac{m}{2}gh - \frac{m}{2}g\frac{L}{2}\sin\theta_m + \frac{1}{2}k(L\theta_m + \delta_{ST})^2$$

$$h = \frac{L}{2}(1-\cos\theta_m) = 2\frac{L}{2}\sin^2\frac{\theta_m}{2}$$

$$h \approx \frac{\theta_m^2 L}{4} \qquad \sin\theta_m \approx \theta_m \qquad V_2 = -\frac{m}{2}g\frac{L}{4}\theta_m^2 - \frac{m}{2}g\frac{L}{2}\theta_m + \frac{1}{2}k(L\theta_m + \delta_{ST})^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}I_B\dot{\theta}_m^2 + \frac{1}{2}k\cancel{\delta_{ST}^2} = 0 - \frac{m}{2}g\frac{L}{4}\theta_m^2 - \frac{mg}{2}\frac{L}{2}\theta_m + \frac{1}{2}k\left[L^2\theta_m^2 + \cancel{\delta_{ST}^2} + 2L\delta_{ST}\theta_m\right] \qquad (1)$$

WHEN THE ROD IS IN EQUILIBRIUM,

$$\curvearrowleft \Sigma M_B = 0 = \frac{m}{2}g\frac{L}{2} - kL\delta_{ST} \qquad (2)$$

SUBSTITUTE (2) INTO (1)

$$I_B\dot{\theta}_m^2 = \left(kL^2 - \frac{mgL}{4}\right)\theta_m^2 \qquad \dot{\theta}_m = \omega_n\theta_m$$

$$I_B\omega_n^2\theta_m^2 = \left(kL^2 - \frac{mgL}{4}\right)\theta_m^2$$

$$\omega_n^2 = \frac{kL^2 - mgL/4}{I_B}$$

$$I_B = 2\left(\frac{1}{3}\frac{m}{2}L^2\right) = \frac{mL^2}{3}$$

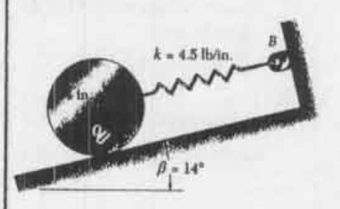
$$\omega_n^2 = \frac{kL^2 - mgL/4}{mL^2/3} = 3\frac{k}{m} - \frac{3g}{4L}$$

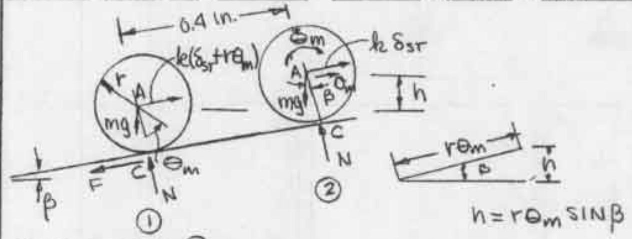
$$f_n = \frac{\omega_n}{2\pi} = \frac{\sqrt{3}}{2\pi}\sqrt{\frac{k}{m} - \frac{g}{4L}}$$ ◀

19.78

GIVEN: 15-lb DISK WHICH ROLLS WITHOUT SLIDING. POINT A IS MOVED DOWN 0.4 in. AND RELEASED

FIND: (a) PERIOD (b) MAXIMUM VELOCITY OF POINT A.





$$h = r\theta_m\sin\beta$$

(a) POSITION ①

$$T_1 = 0 \qquad V_1 = \frac{1}{2}k(\delta_{ST} + r\theta_m)^2$$

POSITION ②

$$T_2 = \frac{1}{2}\bar{J}\dot{\theta}_m^2 + \frac{1}{2}m\bar{v}_m^2 \qquad V_2 = mgh + \frac{1}{2}k(\delta_{ST})^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}k(\delta_{ST} + r\theta_m)^2 = \frac{1}{2}\bar{J}\dot{\theta}_m^2 + \frac{1}{2}m\bar{v}_m^2 + mgh + \frac{1}{2}k(\delta_{ST})^2$$

$$\cancel{k\delta_{ST}^2} + 2k\delta_{ST}r\theta_m + kr^2\theta_m^2 = \bar{J}\dot{\theta}_m^2 + m\bar{v}_m^2 + 2mgh + \cancel{k\delta_{ST}^2} \qquad (1)$$

WHEN THE DISK IS IN EQUILIBRIUM

$$\circlearrowleft \Sigma M_C = 0 = mg\sin\beta\, r - k\delta_{ST}r$$

ALSO $h = r\sin\beta\,\theta_m$

THUS

$$mgh - k\delta_{ST}r = 0 \qquad (2)$$

SUBSTITUTE (2) INTO (1)

$$kr^2\theta_m^2 = \bar{J}\dot{\theta}_m^2 + m\bar{v}_m^2$$

$$\dot{\theta}_m = \omega_n\theta_m \qquad \bar{v}_m = r\dot{\theta}_m = r\omega_n\theta_m$$

$$kr^2\theta_m^2 = (\bar{J} + mr^2)\theta_m^2\omega_n^2$$

$$\omega_n^2 = \frac{kr^2}{\bar{J} + mr^2}$$

$$\omega_n^2 = \frac{kr^2}{\frac{1}{2}mr^2 + mr^2} = \frac{2}{3}\frac{k}{m} \qquad \bar{J} = \frac{1}{2}mr^2$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{\frac{2}{3}\frac{(4.5\times 12\ \text{lb/ft})}{(15\ \text{lb}/32.2\ \text{ft/s}^2)}}} = 0.715\ \text{s}$$ ◀

(b)

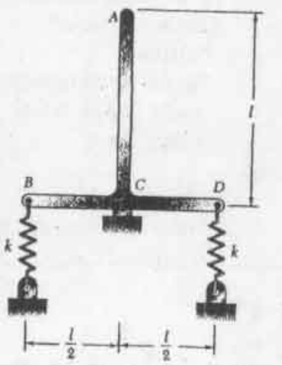
$$v_m = r\dot{\theta}_m$$

$$\dot{\theta}_m = \theta_m\omega_n$$

$$v_m = r\theta_m\omega_n \qquad r\theta_m = \frac{0.4}{12}\ \text{ft}$$

$$v_m = \left(\frac{0.4}{12}\ \text{ft}\right)\left(\frac{2\pi}{0.715\ \text{s}}\right) = 0.293\ \text{ft/s}$$ ◀

19.79



GIVEN:

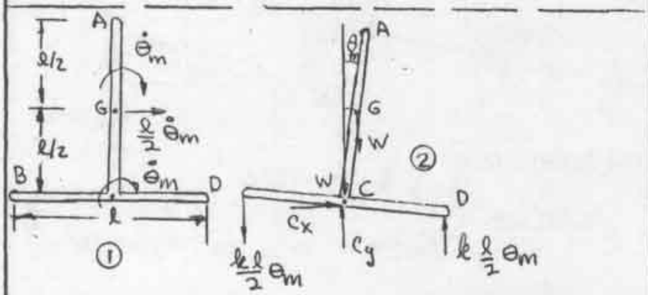
$W_{AC} = W_{BD} = W = 1.2$ lb

$l = 8$ in.

$k = 0.6$ lb/in

FIND:

FREQUENCY OF SMALL OSCILLATIONS



$\sin\theta \approx \theta$

$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$

POSITION ①

$$T_1 = 2\left(\tfrac{1}{2}\bar{J}\,\dot{\theta}_m^2\right) + \tfrac{1}{2}m\left(\tfrac{l}{2}\dot{\theta}_m\right)^2$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0 \quad V_2 = -W\frac{l}{2}(1-\cos\theta_m) + \frac{2}{2}k\left(\frac{l}{2}\theta_m\right)^2$$

$$V_2 = -\frac{Wl}{2}\frac{\theta_m^2}{2} + \frac{kl^2}{4}\theta_m^2$$

CONSERVATION OF ENERGY

$$T_1 + V_1 = T_2 + V_2$$

$$\tfrac{1}{2}(2\bar{J})\dot{\theta}_m^2 + \tfrac{1}{2}m\frac{l^2}{4}\dot{\theta}_m^2 + 0 = 0 - \frac{Wl}{2}\frac{\theta_m^2}{2} + \frac{kl^2}{4}\theta_m^2$$

$$\dot{\theta}_m = \omega_n\theta_m \quad \bar{J} = \frac{1}{12}\frac{W}{g}l^2$$

$$\left(\frac{W}{6g} + \frac{W}{4g}\right)l^2\omega_n^2\theta_m^2 = \left(-\frac{Wl}{2} + \frac{kl^2}{2}\right)\theta_m^2$$

$$\omega_n^2 = \frac{-\frac{W}{2} + \frac{kl}{2}}{\frac{5}{12}(W/g)l} = \frac{6}{5}\left(-\frac{g}{l} + \frac{k}{W/g}\right)$$

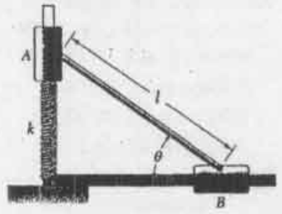
$$\omega_n^2 = \frac{6}{5}\left(\frac{-32.2\text{ ft/s}^2}{(8/12\text{ ft})} + \frac{(0.6\times 12\text{ lb/ft})}{(1.2\text{ lb}/32.2\text{ ft/s}^2)}\right)$$

$$\omega_n^2 = \frac{6}{5}(-48.3 + 193.2) = 173.9\text{ s}^{-2}$$

$$\omega_n = 13.19\text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{13.19}{2\pi} = 2.10\text{ Hz.}$$

19.80



GIVEN:

8 kg ROD AB

$l = 0.6$ m

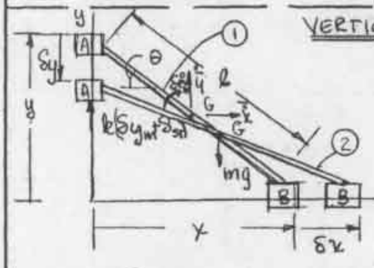
COLLARS A AND B OF NEGLIGIBLE MASS

$k = 1.2$ kN/m

$\theta = 40°$ AT EQUILIBRIUM

FIND:

PERIOD OF VIBRATION



VERTICAL ROD

$y = l\sin\theta$

$\delta y = l\cos\theta\,\delta\theta$

$\delta\dot{y} = l\cos\theta\,\delta\dot{\theta}$

$x = l\cos\theta$

$\delta x = -l\sin\theta\,\delta\theta$

$\delta\dot{x} = -l\sin\theta\,\delta\dot{\theta}$

$\bar{y} = y/2 \quad \bar{x} = x/2$

POSITION ① (MAXIMUM VELOCITY $\delta\dot{\theta}_m$)

$$T_1 = \tfrac{1}{2}\bar{J}(\delta\dot{\theta})_m^2 + \tfrac{1}{2}m[(\delta\dot{\bar{x}})^2 + (\delta\dot{\bar{y}})_m^2]$$

$$T_1 = \tfrac{1}{2}\left(\tfrac{1}{12}ml^2\right)(\delta\dot{\theta})_m^2 + \tfrac{1}{2}m\left[\left(\tfrac{l}{2}\sin\theta\right)^2 + \left(\tfrac{l}{2}\cos\theta\right)^2\right](\delta\dot{\theta})_m^2$$

$$T_1 = \tfrac{1}{2}ml^2\left[\tfrac{1}{12} + \tfrac{1}{4}\right](\delta\dot{\theta}_m)^2 = \tfrac{1}{2}ml^2\tfrac{1}{3}(\delta\dot{\theta}_m)^2$$

$$V_1 = \tfrac{1}{2}k(\delta_{ST})^2 + mg\bar{y}$$

POSITION ② (ZERO VELOCITY, MAXIMUM $\delta\theta_m$)

$$T_2 = 0$$

$$V_2 = \tfrac{1}{2}k(\delta y + \delta_{ST})^2 + mg(\bar{y} - \delta\bar{y}_m)$$

$$T_1 + V_1 = T_2 + V_2$$

$$\tfrac{1}{2}ml^2\tfrac{1}{3}(\delta\dot{\theta}_m)^2 + \tfrac{1}{2}k\delta_{ST}^2 + mg\bar{y} = 0 + \tfrac{1}{2}k(\delta y + \delta_{ST})^2 + mg(\bar{y} - \delta\bar{y}_m)$$

$$ml^2\tfrac{1}{3}(\delta\dot{\theta}_m)^2 + k\delta_{ST}^2 + mg\bar{y} = k(\delta y^2 + 2\delta y\,\delta_{ST} + \delta_{ST}^2) + mg(\bar{y} - \delta y_m) \quad (1)$$

BUT WHEN THE ROD IS IN EQUILIBRIUM,

$$\circlearrowleft \Sigma M_B = mg\frac{x}{2} - k\delta_{ST}x = 0 \qquad mg = 2k\delta_{ST} \qquad (2)$$

SUBSTITUTE (2) INTO (1)

$$ml^2\tfrac{1}{3}(\delta\dot{\theta}_m)^2 = k\delta y_m^2 \qquad \delta y_m = l\cos\theta\,\delta\theta_m$$

$$ml^2\tfrac{1}{3}(\delta\dot{\theta}_m)^2 = kl^2\cos^2\theta(\delta\theta_m)^2$$

FOR SIMPLE HARMONIC MOTION

$$\delta\theta = \delta\theta_m\sin(\omega_n t + \phi)$$

$$\delta\dot{\theta}_m = \delta\theta_m\omega_n$$

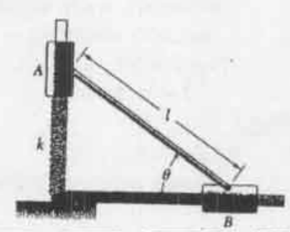
$$\tfrac{1}{3}m(\delta\theta_m)^2\omega_n^2 = k\cos^2\theta(\delta\theta_m)^2$$

$$\omega_n^2 = 3\frac{k}{m}\cos^2\theta = 3\frac{(1200\text{ N/m})}{8\text{ kg}}\cos^2 40°$$

$$\omega_n^2 = 264.07$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{264.07}} = 0.387\text{ s}$$

19.81



GIVEN:

$l = 0.6\ m$

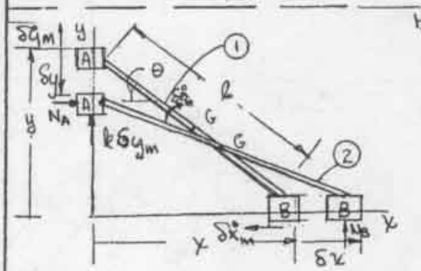
$m_A = m_B = m = 8\ kg$

$k = 1.2\ kN/m$

$\theta = 40^\circ$

ROD AB OF NEGLIGIBLE MASS

FIND: PERIOD OF VIBRATION



HORIZONTAL ROD

$y = l \sin\theta$

$\delta y = l\cos\theta\ \delta\theta$

$\delta \dot{y} = l\cos\theta\ \delta\dot{\theta}$

$x = l\cos\theta$

$\delta x = -l\sin\theta\ \delta\theta$

$\delta\dot{x} = -l\sin\theta\ \delta\dot{\theta}$

POSITION ① (MAXIMUM VELOCITY, $\delta\dot{\theta}_m$)

$$T_1 = \frac{1}{2}m(\delta\dot{y}_m)^2 + \frac{1}{2}m(\delta\dot{x}_m)^2$$

$$T_1 = \frac{1}{2}m\left[(l\cos\theta)^2 + (l\sin\theta)^2\right](\delta\dot{\theta}_m)^2$$

$$T_1 = \frac{1}{2}ml^2(\delta\dot{\theta}_m)^2$$

$$V_1 = 0$$

POSITION ② (ZERO VELOCITY, MAXIXUM $\delta\theta$)

$$T_2 = 0$$

$$V_2 = \frac{1}{2}k\,\delta y_m^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}ml^2(\delta\dot{\theta}_m)^2 + 0 = \frac{1}{2}k\,\delta y_m^2$$

$$\delta y_m = l\cos\theta\ \delta\theta_m$$

$$ml^2(\delta\dot{\theta}_m)^2 = kl^2\cos^2\theta(\delta\theta_m)^2$$

SIMPLE HARMONIC MOTION

$$\delta\theta = \delta\theta_m \sin(\omega_n t + \phi)$$

$$\delta\dot{\theta}_m = \delta\theta_m\omega_n$$

$$ml^2(\delta\theta_m)^2\omega_n^2 = kl^2\cos^2\theta(\delta\theta_m)^2$$

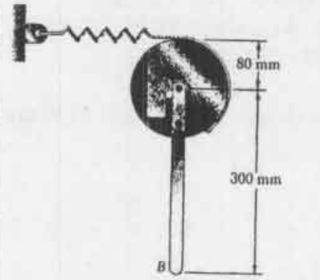
$$\omega_n^2 = \frac{k}{m}\cos^2\theta$$

$$\omega_n^2 = \frac{1200\ N/m}{8\ kg}\cos^2 40 = 88.02\ s^{-2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{88.02}} = 0.6697\ s$$

$$\tau_n = 0.670\ s.$$

19.82



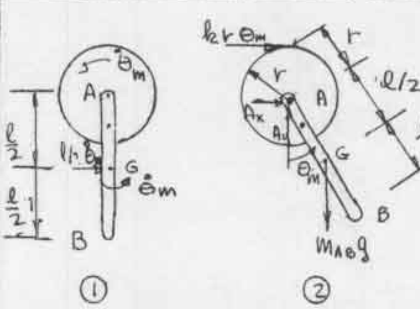
GIVEN:

$m_{AB} = 3\ kg$

$m_{DISK} = 5\ kg$

SPRING IS UNSTRETCHED IN THE POSITION SHOWN, $k = 280\ N/m$

FIND: PERIOD OF SMALL OSCILLATIONS



$r = 0.08\ m$

$l = 0.3\ m$

POSITION ①

$$T_1 = \frac{1}{2}\bar{J}_{DISK}\dot{\theta}_m^2 + \frac{1}{2}(J_A)_{ROD}\dot{\theta}_m^2$$

$$V_1 = 0 \qquad J_{DISK} = \frac{1}{2}m_D r^2$$

$$(J_A)_{ROD} = \frac{1}{3}m_{AB}l^2$$

POSITION ②

$$T_2 = 0$$

$$V_2 = \frac{1}{2}k(r\theta_m)^2 + m_{AB}g\frac{l}{2}(1-\cos\theta_m)$$

$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_2 = \frac{1}{2}kr^2\theta_m^2 + \frac{m_{AB}g(l/2)}{2}\theta_m^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2}\left(\frac{1}{2}m_D r^2 + \frac{1}{3}m_{AB}l^2\right)\dot{\theta}_m^2 + 0 = 0 + \frac{1}{2}kr^2\theta_m^2 + \frac{1}{2}m_{AB}g\frac{l}{2}\theta_m^2$$

$$\dot{\theta}_m = \omega_n\theta_m$$

$$\left(\frac{1}{2}m_D r^2 + \frac{1}{3}m_{AB}l^2\right)\omega_n^2\theta_m^2 = \left(kr^2 + m_{AB}g\frac{l}{2}\right)\theta_m^2$$

$$\omega_n^2 = \frac{kr^2 + m_{AB}gl}{\frac{1}{2}m_D r^2 + \frac{1}{3}m_{AB}l^2}$$

$$\omega_n^2 = \frac{(280\ N/m)(0.08\ m)^2 + (3\ kg)(9.81\ m/s^2)(0.3/2\ m)}{\frac{1}{2}(5\ kg)(0.08\ m)^2 + \frac{1}{3}(3\ kg)(0.300\ m)^2}$$

$$\omega_n^2 = \frac{6.207}{0.106} = 58.55$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{58.55}} = 0.821\ s$$

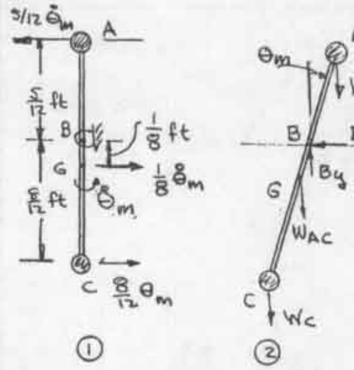
19.83

GIVEN:

$W_A = 14$ oz. $W_C = 10$ oz.
ROD AC WEIGHT = 20 OZ
VERTICAL PLANE

FIND:

PERIOD OF SMALL OSCILLATIONS



POSITION ①

$$T_1 = \frac{1}{2}\frac{W_A}{g}\left(\frac{5}{12}\dot\theta_m\right)^2 + \frac{1}{2}\frac{W_C}{g}\left(\frac{8}{12}\dot\theta_m\right)^2 + \frac{1}{2}\frac{W_{AC}}{g}\left(\frac{1}{8}\dot\theta_m\right)^2 + \frac{1}{2}\bar I_{AC}\dot\theta_m^2$$

$$\bar I_{AC} = \frac{1}{12}\frac{W_{AC}}{g}\left(\frac{13}{12}\right)^2$$

$$T_1 = \frac{1}{2g}\left[\frac{14}{16}\left(\frac{5}{12}\right)^2 + \frac{10}{16}\left(\frac{8}{12}\right)^2 + \frac{20}{16}\left(\frac{1}{8}\right)^2 + \frac{1}{12}\left(\frac{20}{16}\right)\left(\frac{13}{12}\right)^2\right]\dot\theta_m^2$$

$$T_1 = \frac{1}{2(32.2\ \text{ft/s}^2)}\left[0.1519 + 0.2778 + 0.01953 + 0.1223\right]\dot\theta_m^2$$

$$T_1 = \frac{1}{2}\left(\frac{0.5715\ \text{lb}\cdot\text{ft}^2}{32.2\ \text{ft/s}^2}\right)\dot\theta_m^2 = \frac{1}{2}(0.01775)\dot\theta_m^2\ (\text{lb}\cdot\text{ft})$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0$$

$$V_2 = -W_A\frac{5}{12}(1-\cos\theta_m) + W_C\frac{8}{12}(1-\cos\theta_m) + W_{AC}\frac{1}{8}(1-\cos\theta_m)$$

$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_2 = \left[-\left(\frac{14}{16}\right)\left(\frac{5}{12}\right) + \left(\frac{10}{16}\right)\left(\frac{8}{12}\right) + \left(\frac{20}{16}\right)\left(\frac{1}{8}\right)\right]\frac{\theta_m^2}{2}\ (\text{lb}\cdot\text{ft})$$

$$V_2 = [-0.3646 + 0.4167 + 0.1563]\frac{\theta_m^2}{2}$$

$$V_2 = 0.2084\,\theta_m^2/2$$

$T_1 + V_1 = T_2 + V_2$

$$\frac{1}{2}(0.01775)\dot\theta_m^2 + 0 = 0 + \frac{0.2084}{2}\theta_m^2$$

$\dot\theta_m = \omega_n\theta_m$

$$\omega_n^2 = \frac{0.2084}{0.01775} = 11.738$$

$$\tau_n = \frac{2\pi}{\sqrt{\omega_n}} = \frac{2\pi}{\sqrt{11.738}} = 1.834\ \text{s}$$

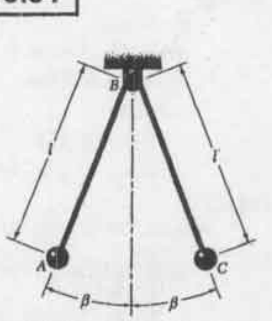
19.84

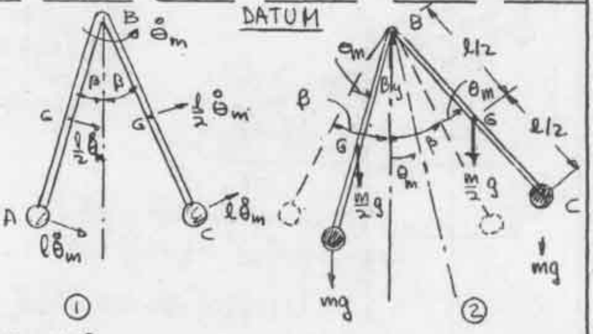
GIVEN:

SPHERES AND ROD ABC
ALL OF MASS m
$\beta = 40°$
$l = 0.5$ m

FIND:

FREQUENCY OF SMALL OSCILLATIONS





POSITION ①

$$T_1 = \frac{1}{2}m(v_A)_m^2 + \frac{1}{2}m(v_C)_m^2 + \frac{1}{2}(2I_G)(\dot\theta_m^2) + \frac{1}{2}\left(2\frac{m}{2}\frac{l^2}{4}\dot\theta_m^2\right)$$

$$I_G = \frac{1}{12}\frac{m}{2}l^2 \quad (v_A)_m = (v_C)_m = l\dot\theta_m$$

$$T_1 = ml^2\dot\theta_m^2 + \left(\frac{ml^2}{24} + \frac{ml^2}{8}\right)\dot\theta_m = \frac{7}{6}ml^2\dot\theta_m$$

$$V_1 = -2mgl\cos\beta - mg\frac{l}{2}\cos\beta = -\frac{5}{2}mgl\cos\beta$$

POSITION ②

$$T_2 = 0$$

$$V_2 = -mgl\cos(\beta-\theta_m) - \frac{m}{2}g\frac{l}{2}\cos(\beta-\theta_m) - mgl\cos(\beta+\theta_m) - \frac{m}{2}g\frac{l}{2}\cos(\beta+\theta_m)$$

$$V_2 = -\frac{5}{4}mgl[\cos\beta\cos\theta_m + \sin\beta\sin\theta_m + \cos\beta\cos\theta_m - \sin\beta\sin\theta_m]$$

$$V_2 = -\frac{5}{2}mgl\cos\beta\cos\theta_m$$

$$\cos\theta_m \approx 1 - \theta_m^2/2 \quad \text{(SMALL ANGLES)}$$

$$V_2 = -\frac{5}{2}mgl\cos\beta\,[1-\theta_m^2/2]$$

$T_1 + V_1 = T_2 + V_2$

$$\frac{7}{6}ml^2\dot\theta_m^2 - \frac{5}{2}mgl\cos\theta = 0 - \frac{5}{2}mgl\cos\beta\left(1-\frac{\theta_m^2}{2}\right)$$

$\dot\theta_m = \omega_n\theta_m$

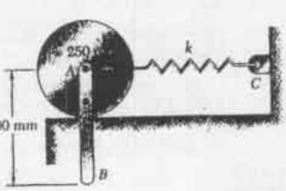
$$\frac{7}{6}l\,\omega_n^2\theta_m^2 = \frac{5}{4}\cdot g\cos\beta\,\theta_m^2$$

$$\omega_n^2 = \frac{15}{14}\frac{g}{l}\cos\beta$$

$$\omega_n^2 = \frac{15}{14}\left(\frac{9.81\ \text{m/s}^2}{0.5\ \text{m}}\right)\cos 40° = 16.10\ \text{s}^{-2}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{\sqrt{16.10}}{2\pi} = 0.639\ \text{Hz}$$

19.85



GIVEN:

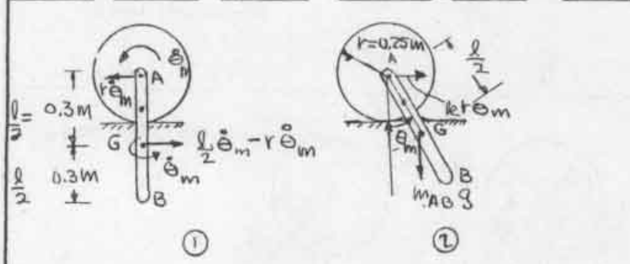
0.8 kg ROD BOLTED TO A 1.2 kg DISK

$k = 12$ N/m

DISK ROLLS WITHOUT SLIDING.

FIND:

PERIOD OF SMALL OSCILLATIONS



POSITION ①

$$T_1 = \frac{1}{2}(\bar{I}_G)_{AB}\dot{\theta}_m^2 + \frac{1}{2}m_{AB}\left(\frac{l}{2}-r\right)^2\dot{\theta}_m^2 + \frac{1}{2}(\bar{I}_G)_{DISK}\dot{\theta}_m^2 + \frac{1}{2}m_{DISK}r^2\dot{\theta}_m^2$$

$$(I_G)_{AB} = \frac{1}{12}ml^2 = \frac{1}{12}(0.8)(0.6)^2 = 0.024 \text{ kg-m}^2$$

$$m_{AB}\left(\frac{l}{2}-r\right)^2 = (0.8)(0.3-0.25)^2 = 0.002 \text{ kg-m}^2$$

$$(I_G)_{DISK} = \frac{1}{2}m_{DISK}r^2 = \frac{1}{2}(1.2)(0.25)^2 = 0.0375 \text{ kg m}^2$$

$$m_{DISK}r^2 = 1.2(0.25)^2 = 0.0750 \text{ kg-m}^2$$

$$T_1 = \frac{1}{2}[0.024 + 0.002 + 0.0375 + 0.0750]\dot{\theta}_m^2$$

$$T_1 = \frac{1}{2}[0.1385]\dot{\theta}_m^2$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0$$

$$V_2 = \frac{1}{2}k(r\theta_m)^2 + m_{AB}g\frac{l}{2}(1-\cos\theta_m)$$

$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2} \quad \text{(SMALL ANGLES)}$$

$$V_2 = \frac{1}{2}(12 \text{ N/m})(0.25 \text{ m})^2\theta_m^2 + (.8 \text{ kg})(9.81 \tfrac{\text{m}}{\text{s}^2})\left(\frac{0.6\text{m}}{2}\right)\frac{\theta_m^2}{2}$$

$$V_2 = \frac{1}{2}[0.750 + 2.354]\theta_m^2 = \frac{1}{2}(3.104)\theta_m^2 \text{ N·m}$$

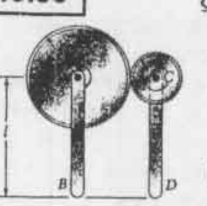
$$T_1 + V_1 = T_2 + V_2 \qquad \dot{\theta}_m^2 = \omega_n^2\theta_m^2$$

$$\frac{1}{2}(0.1385)\theta_m^2\omega_n^2 + 0 = 0 + \frac{1}{2}(3.104)\theta_m^2$$

$$\omega_n^2 = \frac{(3.104 \text{ N·m})}{(0.1385 \text{ kg·m}^2)} = 22.41 \text{ s}^{-2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{22.41}} = 1.327 \text{ s}$$

19.86



GIVEN:

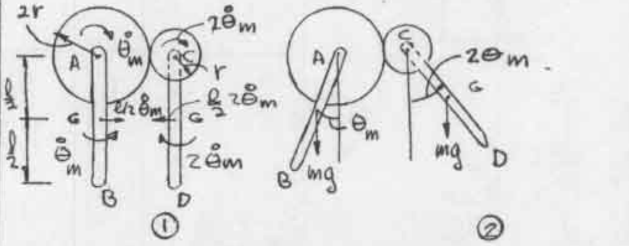
RODS AB AND CD EACH OF MASS m AND LENGTH l ATTACHED TO GEARS A AND C

MASS OF GEAR A = 4 m

MASS OF GEAR C = m

FIND:

PERIOD OF SMALL OSCILLATIONS



KINEMATICS $\quad 2r\theta_A = r\theta_C \qquad 2\theta_A = \theta_C$

$$2\dot{\theta}_A = \dot{\theta}_C$$

LET $\theta_A = \theta_m \qquad 2\theta_m = (\theta_C)_m$

$$2\dot{\theta}_m = (\dot{\theta}_C)_m$$

POSITION ①

$$T_1 = \frac{1}{2}\bar{I}_A\dot{\theta}_m^2 + \frac{1}{2}\bar{I}_C(2\dot{\theta}_m)^2 + \frac{1}{2}\bar{I}_{AB}\dot{\theta}_m^2 + \frac{1}{2}\bar{I}_{CD}(2\dot{\theta}_m)^2 + \frac{1}{2}m_{AB}\left(\frac{l}{2}\dot{\theta}_m\right)^2 + \frac{1}{2}m_{CD}\left(\frac{l}{2}2\dot{\theta}_m\right)^2$$

$$\bar{I}_A = \frac{1}{2}(4m)(2r)^2 = 8mr^2$$

$$I_C = \frac{1}{2}(m)(r)^2 = \frac{1}{2}mr^2$$

$$\bar{I}_{AB} = \frac{1}{12}ml^2 \quad \bar{I}_{CD} = \frac{1}{12}ml^2$$

$$T_1 = \frac{1}{2}m[8r^2 + (r^2/2)4 + l^2/12 + l^2/3 + l^2/4 + l^2]$$

$$T_1 = \frac{1}{2}m[10r^2 + \tfrac{5}{3}l^2]\dot{\theta}_m^2$$

$$V_1 = 0$$

POSITION ②

$$T_1 = 0$$

$$V_1 = mg\frac{l}{2}(1-\cos\theta_m) + \frac{mgl}{2}(1-\cos 4\theta_m)$$

SMALL ANGLES $\quad 1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$

$$1-\cos 4\theta_m = 2\sin^2 2\theta_m \approx 2\theta_m^2$$

$$V_1 = \frac{1}{2}mgl\left(\frac{\theta_m^2}{2} + 2\theta_m^2\right) = \frac{1}{2}mgl\,\frac{5\theta_m^2}{2}$$

$$T_1 + V_1 = T_2 + V_2 \qquad \dot{\theta}_m^2 = \omega_n^2\theta_m^2$$

$$\frac{1}{2}m[10r^2 + \tfrac{5}{3}l^2]\omega_n^2\theta_m^2 + 0 = 0 + \frac{1}{2}mgl\frac{5\theta_m^2}{2}$$

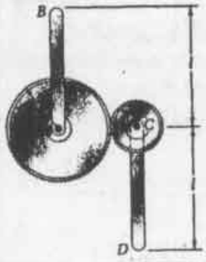
$$\omega_n^2 = \frac{\frac{5}{2}gl}{10r^2 + \frac{5}{3}l^2} = \frac{3gl}{12r^2 + 2l^2}$$

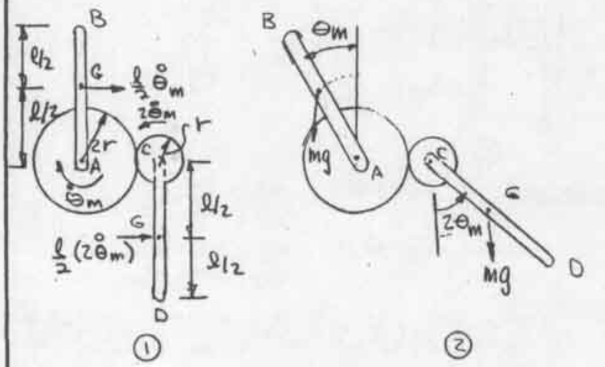
$$\tau_n = \frac{2\pi}{\sqrt{\omega_n}} = 2\pi\sqrt{\frac{12r^2 + 2l^2}{3gl}}$$

19.87

GIVEN:
RODS AB AND BC EACH OF MASS m
GEAR A OF MASS $4m$
GEAR C OF MASS m

FIND:
PERIOD OF SMALL OSCILLATIONS





KINEMATICS $2r\theta_A = r\theta_C$ $2\theta_A=\theta_C$
$2\dot\theta_A=\dot\theta_C$
LET $\theta_A=\theta_m$ $2\theta_m=(\theta_C)_m$
$2\dot\theta_m=(\dot\theta_C)_m$

POSITION ①

$$T_1=\frac12\bar I_A\dot\theta_m^2+\frac12\bar I_C(2\dot\theta_m)^2+\frac12\bar I_{AB}\dot\theta_m^2+\frac12\bar I_{CD}(2\dot\theta_m)^2+\frac12 m_{AB}\left(\frac{l}{2}\dot\theta_m\right)^2+\frac12 m_{CD}\left(\frac{l}{2}2\dot\theta_m\right)^2$$

$\bar I_A=\frac12(4m)(2r)^2=8mr^2$
$\bar I_C=\frac12(m)(r^2)=\frac12 mr^2$
$\bar I_{AB}=\frac1{12}ml^2$ $\bar I_{CD}=\frac1{12}ml^2$

$T_1=\frac12 m[8r^2+(r^2/2)4+l^2/12+l^2/3+l^2/4+l^2]\dot\theta_m^2$

$T_1=\frac12 m[10r^2+\frac53 l^2]\dot\theta_m^2$ $V_1=0$

POSITION ②
$T_2=0$ $V_2=-mg\frac{l}{2}(1-\cos\theta_m)+\frac{mgl}{2}(1-\cos 2\theta_m)$

SMALL ANGLES $1-\cos\theta_m=2\sin^2\frac{\theta_m}{2}\approx\frac{\theta_m^2}{2}$

$1-\cos 2\theta_m=2\sin^2\theta_m\approx 2\theta_m^2$
$V_2=-mg\frac{l}{2}\frac{\theta_m^2}{2}+\frac{mgl}{2}2\theta_m^2=\frac12 mgl\frac32\theta_m^2$

$T_1+V_1=T_2+V_2$ $\dot\theta_m=\omega_n\theta_m$

$\frac12 m[10r^2+\frac53 l^2]\theta_m^2\omega_n^2+0=0+\frac12 mgl\frac32\theta_m^2$

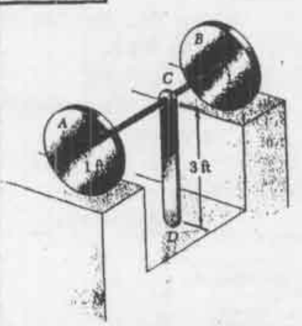
$\omega_n^2=\frac{\frac32 gl}{10r^2+\frac53 l^2}=\frac{9gl}{60r^2+10l^2}$

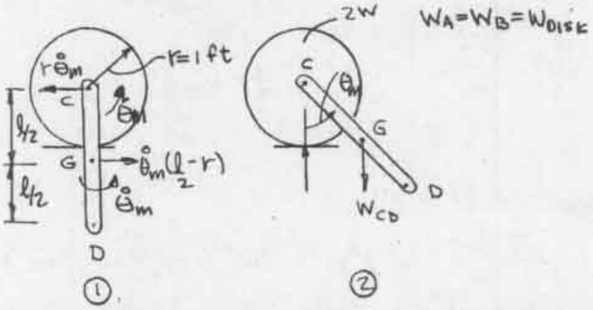
$\tau_n=\frac{2\pi}{\sqrt{\omega_n}}=2\pi\sqrt{\frac{60r^2+10l^2}{9gl}}$

19.88

GIVEN:
10-lb ROD CD
DISKS A AND B EACH WEIGH 20 lb
AC OF NEGLIGIBLE WEIGHT
NO SLIDING

FIND:
PERIOD OF SMALL OSCILLATIONS





POSITION ①
$$T_1=\frac12 2(\bar I_A)_{DISK}\dot\theta_m^2+\frac12\left(\frac{2W_{DISK}}{g}\right)(r\dot\theta_m)^2+\frac12\bar I_{CD}\dot\theta_m^2+\frac12\frac{W_{CD}}{g}\left(\frac{l}{2}-r\right)^2\dot\theta_m^2$$

$(\bar I_A)_{DISK}=\frac12\frac{W_{DISK}}{g}r^2=\frac12\left(\frac{20}{g}\right)(1)^2=\frac{10}{g}$

$\bar I_{CD}=\frac1{12}\frac{W_{CD}}{g}l^2=\frac1{12}\left(\frac{10}{g}\right)(3)^2=\frac{15}{2g}$

$T_1=\frac1{2g}[20+40+\frac{15}{2}+\frac52]\dot\theta_m^2$

$T_1=\frac1{2g}(70)\dot\theta_m^2$

$V_1=0$

POSITION ②
$T_2=0$

$V_2=W_{CD}\frac{l}{2}(1-\cos\theta_m)$

SMALL ANGLES $1-\cos\theta_m=2\sin^2\frac{\theta_m}{2}\approx\frac{\theta_m^2}{2}$

$V_2=\frac12 W_{CD}l\frac{\theta_m^2}{2}=\frac12(10)(1.5)\theta_m^2=\frac12 15\theta_m^2$

$T_1+V_1=T_2+V_2$ $\dot\theta_m=\omega_n\theta_m$

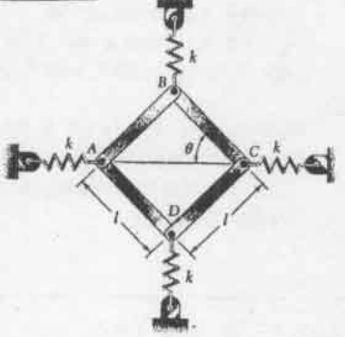
$\frac1{2g}(10)\omega_n^2\theta_m^2+0=0+\frac12 15\theta_m^2$

$\omega_n^2=\frac{15g}{70}$

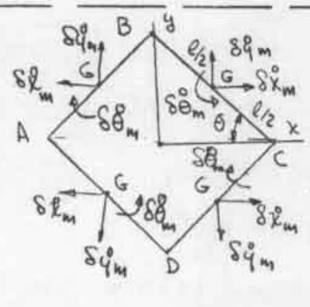
$\tau_n=\frac{2\pi}{\sqrt{\omega_n}}=2\pi\sqrt{\frac{70}{(15)(32.2)}}=2.39\text{ s.}$

19.89

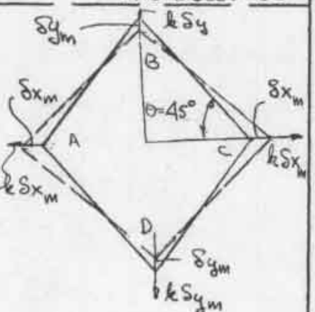
GIVEN:
FOUR BARS OF EQUAL MASS m AND EQUAL LENGTH l
$\theta = 45°$
HORIZONTAL PLANE



FIND:
PERIOD OF VIBRATION IF A AND C ARE GIVEN SMALL, EQUAL DISPLACEMENTS AND RELEASED



①



②

KINEMATICS

BC $\quad x_G = l/2\cos\theta \quad \delta x_G = -l/2\sin\theta\,\delta\theta$

$\delta\dot{x}_G = -l/2\sin\theta\,\delta\dot{\theta}$

$y_G = l/2\sin\theta \quad \delta y_G = l/2\cos\theta\,\delta\theta$

$\delta\dot{y}_G = l/2\cos\theta\,\delta\dot{\theta}$

$x_C = l\cos\theta \quad \delta x_C = -l\sin\theta\,\delta\theta$

$\delta\dot{x}_C = -l\sin\theta\,\delta\dot{\theta}$

$x_B = l\sin\theta \quad \delta x_B = l\cos\theta\,\delta\theta$

$\delta\dot{x}_B = l\cos\theta\,\delta\dot{\theta}$

$y_C = 0$

$y_B = l\sin\theta \quad \delta y_B = l\cos\theta\,\delta\theta$

$\delta\dot{y}_B = l\cos\theta\,\delta\dot{\theta}_m$

THE KINETIC ENERGY IS THE SAME FOR ALL FOUR BARS.

POSITION ①

$$T_1 = 4\left[\tfrac{1}{2}\bar{I}(\delta\dot{\theta}_m)^2 + \tfrac{1}{2}m\left[(\delta\dot{x}_G)_m^2 + (\delta\dot{y}_G)_m^2\right]\right]$$

$\bar{I} = \frac{1}{12}ml^2$

$$T_1 = 2ml^2\left[\tfrac{1}{12} + \tfrac{1}{4}(\sin^2\theta_m + \cos^2\theta_m)\right]\delta\dot{\theta}_m^2$$

$T_1 = \frac{2}{3}ml^2$

$V_1 = 0$

POSITION ② $\quad T_2 = 0$

$$V_2 = (2)\tfrac{1}{2}k(\delta x_m)^2 + (2)\tfrac{1}{2}k(\delta y_m)^2$$

$$V_2 = k(l^2\sin^2\theta + l^2\cos^2\theta)\delta\theta_m^2 = kl^2\delta\theta_m^2$$

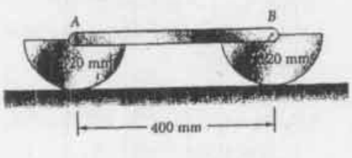
$T_1 + V_1 = T_2 + V_2 \quad \frac{2}{3}ml^2(\delta\dot{\theta}_m)^2 + 0 = 0 + kl^2(\delta\theta_m)^2$

$\delta\dot{\theta}_m = \omega_n\delta\theta_m \quad \omega_n^2 = \frac{3}{2}k/m$

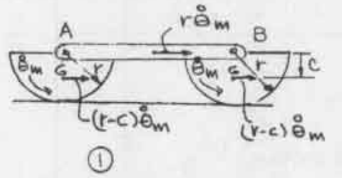
$$\tau_n = 2\pi\sqrt{\frac{2m}{3k}}$$

19.90

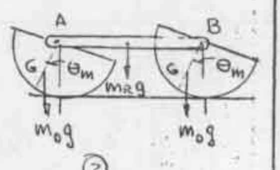
GIVEN:
DISKS OF MASS 3 kg EACH
MASS OF ROD AB $= 2$ kg
NO SLIDING



FIND:
PERIOD FOR SMALL OSCILLATIONS



①



②

POSITION ①

$$T_1 = 2(\tfrac{1}{2})\bar{I}_D\dot{\theta}_m^2 + 2(\tfrac{1}{2})m_D(r-c)^2\dot{\theta}_m^2 + \tfrac{1}{2}m_r r^2\dot{\theta}_m^2$$

FOR ONE DISK

$$\bar{I}_D = (I_D)_A - m_D c^2 = \tfrac{1}{2}m_D r^2 - m_D\left(\tfrac{4r}{3\pi}\right)^2 = m_D\left[\tfrac{r^2}{2} - \tfrac{16r^2}{9\pi^2}\right]$$

$\bar{I}_D = 0.3199\,m_D r^2$

$m_D(r-c)^2 = m_D r^2\left(1 - \frac{4}{3\pi}\right)^2 = 0.3313\,m_D r^2$

$T_1 = [(0.3199 + 0.3313)m_D r^2 + 0.5m_r r^2]\dot{\theta}_m^2$

$T_1 = [0.6512\,m_D + 0.5\,m_r]r^2$

POSITION ②

$T_2 = 0$

$V_2 = 2m_D g\,c(1 - \cos\theta_m)$

$c = \frac{4r}{3\pi} \qquad 1 - \cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$ (SMALL ANGLES)

$V_2 = 2m_D g\,\frac{4r}{3\pi}\,\frac{\theta_m^2}{2}$

$V_2 = m_D r\,\frac{g(4)}{3\pi} = m_D r\,\frac{(9.81)(4)}{(3\pi)} = 4.164\,m_D r$

$T_1 + V_1 = T_2 + V_2$

$0 + (0.6512m_D + 0.5m_r)r^2\dot{\theta}_m^2 = 0 + 4.164\,m_D r$

$\dot{\theta}_m = \omega_n\theta_m$

$$\omega_n^2 = \frac{4.164\,m_D r}{(0.6512m_D + .5m_r)r^2} = \frac{(4.164)(3)}{[(0.6512)(3) + 0.5(2)]0.120}$$

$$\omega_n^2 = \frac{12.490}{0.3544} = 35.24$$

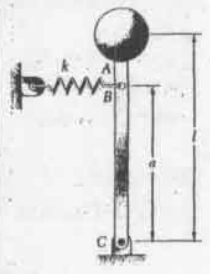
$\omega_n = 5.936$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{5.936} = 1.058\text{ s}$$

19.91

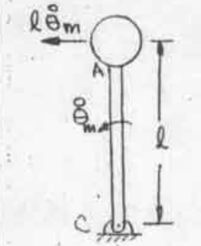
GIVEN:

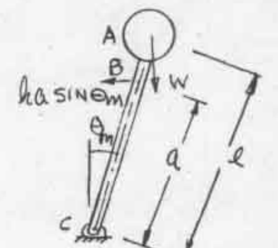
SPHERE OF WEIGHT W
BAR ABC OF NEGLIGIBLE WEIGHT



FIND:

(a) FREQUENCY OF SMALL OSCILLATIONS
(b) SHALLEST VALUE OF a FOR WHICH OSCILLATIONS WILL OCCUR.





(a) ① ②

POSITION ①

$$T_1 = \frac{1}{2} m (l\dot{\theta}_m)^2 = \frac{1}{2} m l^2 \dot{\theta}_m^2$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0 \qquad V_2 = \frac{1}{2} k (a \sin\theta_m)^2 - W l (1-\cos\theta_m)$$

SMALL ANGLES $\sin\theta_m \approx \theta_m$

$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \frac{\theta_m^2}{2}$$

$$V_2 = \frac{1}{2} k a^2 \frac{\theta_m^2}{2} - W l \frac{\theta_m^2}{2} = \frac{1}{2}[ka^2 - Wl]\theta_m^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\frac{1}{2} m l^2 \dot{\theta}_m^2 + 0 = 0 + \frac{1}{2}[ka^2 - Wl]\theta_m^2$$

$$\dot{\theta}_m = \omega_n \theta_m \qquad m = W/g$$

$$\frac{W}{g} l^2 \omega_n^2 \theta_m^2 = ka^2 - Wl$$

$$\omega_n^2 = \frac{ka^2 - Wl}{\left(\frac{W}{g}\right) l^2} = g/l\left[\frac{ka^2}{Wl} - 1\right]$$

$$f_n = \frac{1}{2\pi}\omega_n = \frac{1}{2\pi}\sqrt{g/l\left(\frac{ka^2}{Wl} - 1\right)}$$ ◀

(b) $f_n = 0$

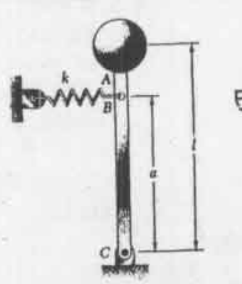
$$\frac{ka^2}{Wl} - 1 > 0$$

$$a > \sqrt{Wl/k}$$ ◀

19.92

GIVEN:

SPHERE OF WEIGHT W
$f_n = 1.5$ Hz WHEN $W = 2$ lb
$f_n = 0.8$ Hz WHEN $W = 4$ lb



FIND:

FOR GIVEN k, a AND l, THE LARGEST VALUE OF W FOR WHICH OSCILLATIONS WILL OCCUR

SEE SOLUTION TO PROB 19.91 FOR THE FRQUENCY IN TERMS OF W, k, a AND l

$$f_n = \frac{1}{2\pi}\sqrt{g/l\left(\frac{ka^2}{Wl} - 1\right)}$$

$f_n = 1.5$ Hz $W = 2$ lb $\qquad 1.5 = \frac{1}{2\pi}\sqrt{g/l\left(\frac{ka^2}{2l} - 1\right)}$ (1)

$f_n = 0.8$ Hz $W = 4$ lb $\qquad 0.8 = \frac{1}{2\pi}\sqrt{g/l\left(\frac{ka^2}{4l} - 1\right)}$ (2)

DIVIDE (1) BY (2)

$$\left(\frac{1.5}{0.8}\right)^2 = \left(\frac{ka^2}{2l} - 1\right) \Big/ \left(\frac{ka^2}{4l} - 1\right)$$

$$3.516\frac{ka^2}{4l} - 3.516 = \frac{ka^2}{2l} - 1$$

$$\frac{ka^2}{l}\left[\frac{3.516}{4} - \frac{1}{2}\right] = 2.516$$

$$\frac{ka^2}{l} = 6.640$$

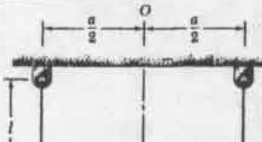
$$f_n = \frac{1}{2\pi}\sqrt{g/l\left(\frac{6.640}{W} - 1\right)}, \quad f_n = 0, \quad \frac{6.640}{W} - 1 = 0$$

$$W \le 6.64 \text{ lb}$$ ◀

19.93

GIVEN:

PIPE SUSPENDED FROM TWO CABLES AT A AND B



FIND:

FREQUENCY VIBRATION FOR A SMALL ROTATION ABOUT OO'

TOP VIEW SIDE VIEW

$$AA' = BB' = \frac{a}{2}\theta_m = l\alpha_m \qquad \alpha_m = \frac{a}{2l}\theta_m$$

POSITION ①

$$T_1 = 0 \quad V_1 = mg\,y_c = mgl(1-\cos\alpha)$$

SMALL ANGLES $1-\cos\alpha_m = 2\sin^2\frac{\alpha_m}{2} \approx \frac{\alpha_m^2}{2} = \frac{a^2}{8l^2}\theta_m^2$

$$V_1 = mgl(a^2/8l^2)\theta_m^2$$

POSITION ② $\quad T_2 = \frac{1}{2}\bar{I}\dot{\theta}_m^2 = \frac{1}{2}\left(\frac{1}{12}ma^2\right)\dot{\theta}_m^2 \quad V_2 = 0$

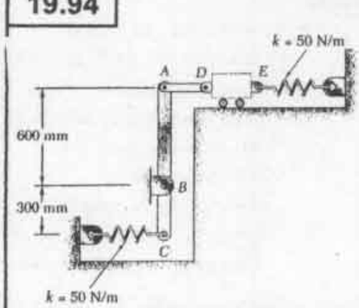
$$\dot{\theta}_m = \omega_n\theta_m$$

$$T_1 + V_1 = T_2 + V_2$$

$$mgl(a^2/8l^2) + 0 + \frac{1}{24}ma^2\omega_n^2\theta_m^2$$

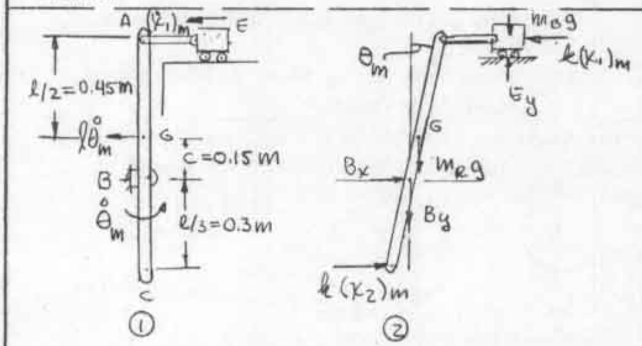
$$\omega_n^2 = 3g/l \qquad f_n = \frac{1}{2\pi}\sqrt{3g/l}$$ ◀

19.94



GIVEN:
2kg ROD ABC
2kg BLOCK DE
SPRINGS ACT IN TENSION OR COMPRESSION

FIND:
FREQUENCY OF SMALL VIBRATIONS



POSITION ①

$$T_1 = \frac{1}{2}(\bar{I}_R)\dot{\theta}_m^2 + \frac{1}{2}(m_R c^2)\dot{\theta}_m + \frac{1}{2}m_E(\dot{x}_1)_m^2$$

$$\bar{I}_R = \frac{1}{12}m_R l^2 = \frac{1}{12}(2\,kg)(0.9\,m)^2 = 0.135\,kg\,m^2$$

$$m_R c^2 = 2\,kg\,(0.15\,m)^2 = 0.0225\,kg\cdot m^2$$

$x_1 = 0.6\theta$ $\quad m_E(\dot{x}_1)_m^2 = (2\,kg)(0.6\dot{\theta}_m\,m)^2$

$\dot{x}_1 = 0.6\dot{\theta}$

$$m_E(\dot{x}_1)_m^2 = 0.72\,\dot{\theta}_m^2\,m^2$$

$$T_1 = \frac{1}{2}[0.135 + 0.0225 + 0.72]\dot{\theta}_m^2$$

$$T_1 = \frac{1}{2}(0.9)\dot{\theta}_m$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0$$

$$V_2 = \frac{1}{2}k(x_1)_m^2 + \frac{1}{2}k(x_2)_m^2 - m_R g c(1-\cos\theta_m)$$

$(x_1)_m = 0.6\theta_m \quad x_2 = 0.3\theta_m$

$$1-\cos\theta_m = 2\sin^2\frac{\theta_m}{2} \approx \theta_m^2$$

$$V_2 = \frac{1}{2}[(50\,N/m)(0.6\,m)^2\theta_m^2 + (50\,N/m)(0.3\,m)^2\theta_m^2 - (2\,kg)(9.81\,m/s^2)(0.15)\theta_m^2$$

$$V_2 = \frac{1}{2}[18 + 4.5 - 2.943]\theta_m^2$$

$$V_2 = \frac{1}{2}(19.55)\theta_m^2$$

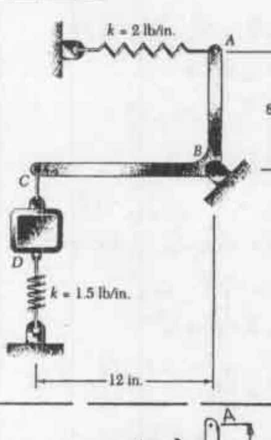
$T_1 + V_1 = T_2 + V_2 \quad \frac{1}{2}(0.9)\dot{\theta}_m^2 + 0 = 0 + \frac{1}{2}(19.55)\theta_m^2$

$\dot{\theta}_m = \omega_n^2\theta_m^2 \quad (0.9)(\omega_n^2)\theta_m^2 = (19.55)\theta_m^2$

$$\omega_n^2 = \frac{19.55}{0.9} = 21.73\,s^{-2}$$

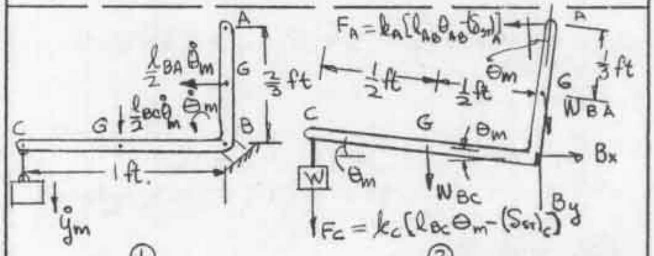
$$f_n = \frac{\omega_n}{2\pi} = \frac{\sqrt{21.73}}{2\pi} = 0.742\,Hz.$$

19.95



GIVEN:
$W_{ABC} = 1.4\,lb$
$W = 3\,lb$
SPRINGS ACT IN TENSION OR COMPRESSION

FIND:
FREQUENCY OF SMALL OSCILLATIONS



$k_c = (1.5)12 = 18\,lb/ft$

$k_A = 2(12) = 24\,lb/ft$

POSITION ①

$$T_1 = \frac{1}{2}\bar{I}_{BC}(\dot{\theta}_m)^2 + \frac{1}{2}m_{BC}\left(\frac{l_{BC}}{2}\right)^2\dot{\theta}_m^2 + \frac{1}{2}\bar{I}_{BA}\dot{\theta}_m^2 + \frac{1}{2}m_{BA}\left(\frac{l_{AB}}{2}\right)^2\dot{\theta}_m^2 + \frac{1}{2}m\cdot\dot{y}_m^2$$

$$\bar{I}_{BC} + m_B\left(\frac{l_{BC}}{2}\right)^2 = \frac{1}{12}m_{BC}l_{BC}^2 + \frac{1}{4}m_{BC}l_{BC}^2 = \frac{1}{3}m_{BC}l_{BC}^2$$

$$\bar{I}_{BA} + m_{BA}\left(\frac{l_{AB}}{2}\right)^2 = \frac{1}{12}m_{AB}l_{AB}^2 + \frac{1}{4}m_{AB}l_{AB}^2 = \frac{1}{3}m_{AB}l_{AB}^2$$

$$W_{BC} = \frac{12}{20}W_{ABC} = \frac{3}{5}(1.4\,lb) = 0.840\,lb$$

$$W_{BA} = \frac{8}{20}W_{ABC} = \frac{2}{5}(1.4\,lb) = 0.560\,lb$$

$$\frac{1}{3}m_{BC}l_{BC}^2 = \frac{1}{3}\frac{(0.840\,lb)}{(32.2\,ft/s^2)}(1\,ft)^2 = 0.008696\,lb\cdot s^2\cdot ft$$

$$\frac{1}{3}m_{BA}l_{BA}^2 = \frac{1}{3}\frac{(0.56\,lb)}{(32.2\,ft/s^2)}\left(\frac{2}{3}ft\right)^2 = 0.002577\,lb\cdot s^2\cdot ft$$

$$\dot{y}_m = l_{BC}\dot{\theta}_m$$

$$m\dot{y}_m^2 = m l_{BC}^2\dot{\theta}_m^2 = \frac{3\,lb}{(32.2\,ft/s^2)}(1\,ft)^2\dot{\theta}_m^2 = 0.09317\,\dot{\theta}_m^2$$

$$T_1 = \frac{1}{2}[0.008696 + 0.002577 + 0.09317]\dot{\theta}_m^2 = 0.1044\frac{\dot{\theta}_m^2}{2}$$

$$V_1 = \frac{1}{2}k_c(\delta_{st})_c^2 + \frac{1}{2}k_A(\delta_{st})_A^2$$

POSITION ② $\quad T_2 = 0$

$$V_2 = W l_{BC}\theta_m + W_{BC}l_{BC}/2\,\theta_m - W_{BA}(l_{AB}/2)(1-\cos\theta_m) + \frac{1}{2}k_c(l_{BC}\theta_m + (\delta_{st})_c)^2 + \frac{1}{2}k_A(l_{AB}\theta_m + (\delta_{st})_A)^2$$

WHEN THE SYSTEM IS IN EQUILIBRIUM ($\theta = 0$)

$$\Sigma M_B = 0 = W l_{BC} + W_{BC}\frac{l_{BC}}{2} - k_c(\delta_{st})_c l_{BC} - k_A(\delta_{st})_A l_{AB} \quad (1)$$

19.95 CONTINUED

$1-\cos\Theta_m = 2\sin^2\frac{\Theta_m}{2} \approx \frac{\Theta_m^2}{2}$

$V_2 = [W\ell_{BC} + W_{BC}(\ell_{BC}/2)]\Theta_m - [W_{BA}(\ell_{AB}/2)(\Theta_m^2/2)] +$
$+ \frac{1}{2}k_C\ell_{BC}^2\Theta_m^2 - \cancel{k_C(\delta_{ST})_C\ell_{BC}\Theta_m} + \frac{1}{2}k_C(\delta_{ST})_C^2 +$
$+ \frac{1}{2}k_A\ell_{AB}^2\Theta_m^2 - \cancel{k_A(\delta_{ST})_A\ell_{AB}\Theta_m} + \frac{1}{2}k_A(\delta_{ST})_A^2$

TAKING EQUATION (1) INTO ACCOUNT

$$V_2 = -[W_{BA}(\ell_{AB}/2)\,\Theta_m^2/2 + \frac{1}{2}k_C\ell_{BC}^2\Theta_m^2 + \frac{1}{2}k_C(\delta_{ST})_C^2 + \frac{1}{2}k_A\ell_{AB}^2\Theta_m^2 + \frac{1}{2}k_A(\delta_{ST})_A^2$$

$$V_2 = \frac{1}{2}\left[-0.560\left(\tfrac{1}{3}\right) + 18(1)^2 + 24\left(\tfrac{2}{3}\right)^2\right]\Theta_m^2 + \frac{1}{2}k_C(\delta_{ST})_C^2 + \frac{1}{2}k_A(\delta_{ST})_A^2$$

$$V_2 = \frac{1}{2}[-0.1867 + 18 + 10.67]\Theta_m^2 + \frac{1}{2}k_C(\delta_{ST})_C^2 + \frac{1}{2}k_A(\delta_{ST})_A^2$$

$$V_2 = \frac{1}{2}[28.48]\Theta_m^2 + \frac{1}{2}k_C(\delta_{ST})_C^2 + \frac{1}{2}k_A(\delta_{ST})_A^2$$

$T_1 + V_1 = T_2 + V_2$

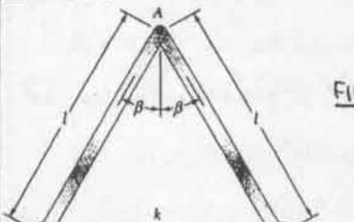
$$\frac{1}{2}(0.1044)\dot{\Theta}_m^2 + \cancel{\frac{1}{2}k_C(\delta_{ST})_C^2} + \cancel{\frac{1}{2}k_A(\delta_{ST})_A^2} = 0 + \frac{1}{2}(28.48)\Theta_m^2 + \cancel{\frac{1}{2}k_C(\delta_{ST})_C^2} + \cancel{\frac{1}{2}k_A(\delta_{ST})_A^2}$$

$\dot{\Theta}_m = \omega_n\Theta_m$

$0.1044\,\omega_n^2\Theta_m^2 = 28.48\,\Theta_m^2$

$\omega_n^2 = \frac{28.48}{0.1044} = 272.8\ s^{-2} \quad f_n = \frac{\sqrt{272.8}}{2\pi} = 2.63\ \text{Hz.}$

*19.96

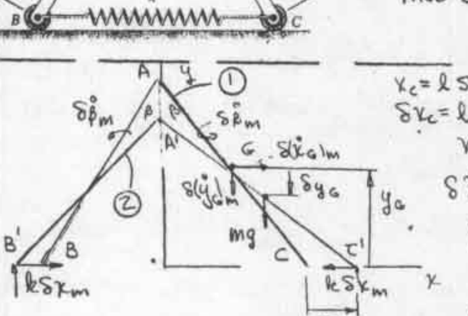


GIVEN:

RODS AB AND AC EACH OF MASS m AND LENGTH ℓ

FIND:

PERIOD WHEN A IS GIVEN A SMALL DOWN DEFLECTION AND RELEASED



$x_C = \ell\sin\beta$
$\delta x_C = \ell\cos\beta\,\delta\beta$
$x_G = \frac{\ell}{2}\cos\beta$
$\delta\dot{x}_G = -\frac{\ell}{2}\sin\beta\,\delta\dot{\beta}$
$y_G = \frac{\ell}{2}\sin\beta$
$\delta y_G = \frac{\ell}{2}\cos\beta\,\delta\beta$
$\delta\dot{y}_G = \frac{\ell}{2}\cos\beta\,\delta\dot{\beta}$

POSITION (1)

$T_1 = 2\left[\frac{1}{2}\bar{I}(\delta\dot{\beta}_m)^2 + \frac{1}{2}m\left((\delta\dot{x}_m)_G^2 + (\delta\dot{y}_m)_G^2\right)\right]$
$= \left(\frac{1}{12}m\ell^2 + m\frac{\ell^2}{4}(\sin^2\beta + \cos^2\beta)\right)\delta\dot{\beta}_m^2 = \frac{1}{3}m\ell^2\delta\dot{\beta}_m$

$V_1 = 0$

POSITION (2) $\quad T_1 = 0$

$V_2 = \frac{1}{2}k(2\delta x_m)^2 = \frac{1}{2}(4\ell^2\cos^2\beta) = 2\ell^2\cos^2\beta$

$\delta\dot{\beta}_m = \omega_n\delta\beta_m$

$T_1 + V_1 = T_2 + V_2 \qquad \frac{1}{3}m\ell^2\omega_n^2\delta\beta_n^2 + 0 = 0 + 2k\ell^2\cos^2\beta\,\delta\beta_n^2$

$\omega_n^2 = \frac{6k\cos^2\beta}{m}$

$\tau_n = \frac{2\pi}{\sqrt{6k/m\cos^2\beta}} = \frac{2\pi}{\cos\beta}\sqrt{\frac{m}{6k}}$

*19.97

GIVEN:

SUBMERGED SPHERE
V = VOLUME OF THE SPHERE
KINETIC ENERGY $= \frac{1}{4}\rho V v^2$
WHERE ρ = MASS DENSITY
AND v = VELOCITY OF THE SPHERE
SPHERE MASS = 500 g
k = 500 N/m, HOLLOW OF SPHERE RADIUS = 80 mm

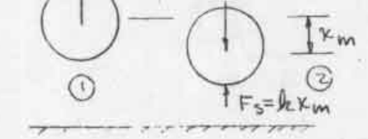
FIND:

(a) PERIOD WHEN DISPLACED VERTICALLY AND RELEASED

(b) PERIOD WHEN THE TANK IS ACCELERATED UPWARD AT 8 m/s²

THIS IS NOT A DAMPED VIBRATION. HOWEVER THE KINETIC ENERGY OF THE FLUID MUST BE INCLUDED

(a)



POSITION (2)
$T_2 = 0$
$V_2 = \frac{1}{2}kx_m^2$

POSITION (1) $\quad T_1 = T_{SPHERE} + T_{FLUID} = \frac{1}{2}m_s v_m^2 + \frac{1}{4}\rho V v_m^2$
$V_1 = 0$

$T_1 + V_1 = T_2 + V_2 \qquad \frac{1}{2}m_s v_m^2 + \frac{1}{4}\rho V v_m^2 + 0 = 0 + \frac{1}{2}kx_m^2$

$v_m = \dot{x}_m = x_m\omega_n$

$\frac{1}{2}(m_s + \frac{1}{2}\rho V)x_m^2\omega_n^2 = \frac{1}{2}kx_m^2, \quad \omega_n^2 = \frac{k}{m_s + \frac{1}{2}\rho V}$

$\omega_n^2 = \frac{500\ \text{N/m}}{(0.5\ \text{kg} + (\frac{1}{2}\rho V))} \qquad \frac{1}{2}\rho V = \frac{1}{2}\left(1000\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{4}{3}\pi(0.08\ \text{m})^3\right)$

$\omega_n^2 = 318\ s^{-2} \qquad \frac{1}{2}\rho V = 1.0723\ \text{kg}$

$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{318}} = 0.352\ \text{s}$

(b) ACCELERATION DOES NOT CHANGE MASS, $\tau_n = 0.352$ s

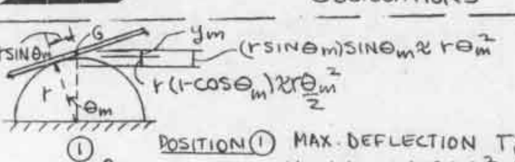
*19.98

GIVEN:

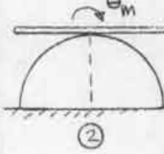
PLATE ON A SEMI-CIRCULAR CYLINDER AS SHOWN

FIND:

PERIOD FOR SMALL OSCILLATIONS



POSITION (1) MAX. DEFLECTION $T_1 = 0$
$V_1 = Wy_m = mgr\Theta_m^2/2$



POSITION (2) $(\Theta = 0)$
$T_2 = \frac{1}{2}\bar{I}\dot{\Theta}_m^2 = \frac{1}{2}\left(\frac{1}{12}\right)m\ell^2\dot{\Theta}_m^2$
$\dot{\Theta}_m = \omega_n\Theta_m$
$T_2 = \frac{1}{2}\left(\frac{1}{12}\right)m\ell^2\omega_n^2\Theta_m^2$

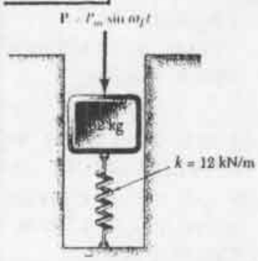
$T_1 + V_1 = T_2 + V_2$

$0 + \frac{1}{2}mgr\Theta_m^2 = \frac{1}{2}\left(\frac{1}{12}\right)m\ell^2\omega_n^2\Theta_m^2$

$\omega_n^2 = \frac{12gr}{\ell^2} \qquad \tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\ell^2/12gr}$

$\tau_n = \frac{\pi\ell}{\sqrt{3gr}}$

19.99



GIVEN:
SYSTEM AS SHOWN WITH $\omega_f = 10$ rad/s
NO FRICTION
AMPLITUDE = 15 mm.

FIND: P_m

EQ. (19.33) $\quad x_m = \dfrac{P_m/k}{1-(\omega_f/\omega_n)^2} \qquad P_m = x_m(1-(\omega_f/\omega_n)^2)k$

$\omega_n = \sqrt{k/m} = \sqrt{\dfrac{12\times10^3\ \text{N/m}}{32\ \text{kg}}} = 19.365$ rad/s

$P_m = (0.015\ \text{m})(12000\ \text{N/m})(1-(10/19.365)^2)$

$P_m = 132.0\ \text{N}$

19.100

GIVEN:
9-lb COLLAR ATTACHED TO A SPRING, $k = 2.5$ lb/in.
$P_m = 3$ lb., NO FRICTION

FIND:
AMPLITUDE IF
(a) $\omega_f = 5$ rad/s
(b) $\omega_f = 10$ rad/s

EQ. (19.33)
$x_m = \dfrac{P_m/k}{1-(\omega_f/\omega_n)^2}$

$\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{(2.5\times12\ \text{lb/ft})}{(9\ \text{lb}/32.2\ \text{ft/s}^2)}} = 10.36\ \text{s}^{-1}$

$P_m/k = 3\ \text{lb}/(2.5\times12\ \text{lb/ft}) = 0.100$ ft

$x_m = \dfrac{0.100}{1-(\omega_f/10.36)^2} \qquad$ (a) $\omega_f = 5\ \text{s}^{-1}$

$x = \dfrac{0.100\ \text{ft}}{1-(5/10.36)^2} = 0.1304$ ft (IN PHASE)

(b) $\omega_f = 10$ rad/s $\quad x = \dfrac{0.100\ \text{ft}}{1-(10/10.36)^2} = 1.464$ ft (IN PHASE)

19.101

REFER TO FIGURE FOR PROB 19.100 SHOWN ABOVE

GIVEN:
9-lb COLLAR ATTACHED TO A SPRING OF CONSTANT k. $P_m = 2$ lb, $\omega_f = 5$ rad/s
AMPLITUDE OF MOTION, $x_m = 6$ in.

FIND:
k, IF (a) x_m IS IN PHASE WITH P
(b) x_m IS OUT OF PHASE WITH P

EQ. (19.33) $\quad x_m = \dfrac{P_m/k}{1-\omega_f^2/\omega_n^2} \qquad \omega_n^2 = k/m$

$x_m = P_m/(k - m\omega_f^2)$

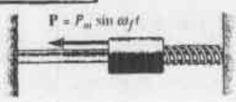
$k = \dfrac{P_m}{x_m} + m\omega_f^2$

(a) IN PHASE $\quad k = \dfrac{2\ \text{lb}}{1/2\ \text{ft}} + \dfrac{(9\ \text{lb})(5\ \text{s}^{-1})^2}{32.2\ \text{ft/s}^2} = 10.99$ lb/ft

(b) OUT OF PHASE $\quad x_m = -1/2$ ft

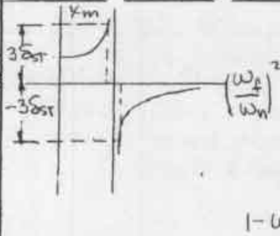
$k = \dfrac{2\ \text{lb}}{-(1/2)\ \text{ft}} + \dfrac{9\ (\text{lb})(5\ \text{s}^{-1})^2}{32.2\ \text{ft/s}^2} = 2.99$ lb/ft

19.102



GIVEN:
COLLAR OF MASS m ATTACHED TO A SPRING OF CONSTANT k

FIND:
RANGE OF ω_f FOR WHICH AMPLITUDE EXCEEDS THREE TIMES THE STATIC DEFLECTION CAUSED BY P_m



EQ (19.33)
$x_m = \dfrac{P_m/k}{1-\omega_f^2/\omega_n^2}$

$P_m/k = \delta_{ST} \qquad \omega_n^2 = \dfrac{k}{m}$

$\dfrac{\delta_{ST}}{1-\omega_f^2/\omega_n^2} \geq 3\delta_{ST}$

$1-\omega_f^2/\omega_n^2 \leq 1/3$

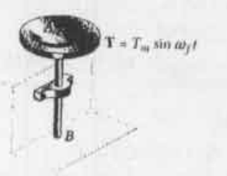
$\dfrac{\omega_f^2}{\omega_n^2} > \dfrac{2}{3}$

ALSO $\quad \dfrac{\delta_{ST}}{1-\omega_f^2/\omega_n^2} < -3\delta_{ST}$

$1-\omega_f^2/\omega_n^2 < -\dfrac{1}{3} \qquad \dfrac{\omega_f^2}{\omega_n^2} < \dfrac{4}{3}$

$\dfrac{4}{3} > \dfrac{\omega_f^2}{\omega_n^2} > \dfrac{2}{3} \qquad \sqrt{\dfrac{4k}{3m}} > \omega_f > \sqrt{\dfrac{2k}{3m}}$

19.103



GIVEN:
8-kg DISK
RADIUS $r = 200$ mm
WELDED TO A SHAFT FIXED AT B.
DISK ROTATES 3° WHEN A STATIC COUPLE 50 N·m IS APPLIED. $T_m = 60$ N·m

FIND:
RANGE OF VALUES OF ω_f FOR WHICH THE AMPLITUDE IS LESS THAN THE STATIC DEFLECTION CAUSED BY T_m

ANALOGOUS TO EQ (19.33)

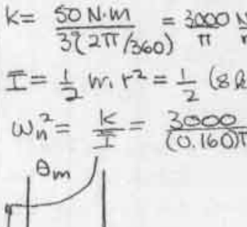
$\Theta_m = \dfrac{T_m/K}{1-\omega_f^2/\omega_n^2} = \dfrac{\Theta_{ST}}{1-\omega_f^2/\omega_n^2}$

WHERE $\omega_n^2 = \dfrac{K}{\bar{I}}$ $\quad K$ IS THE TORSIONAL SPRING CONSTANT AND $\bar{I}$ IS THE CENTROIDAL MOMENT OF INERTIA OF THE DISK (SEE SAMPLE PROB 9.3)

$K = \dfrac{50\ \text{N·m}}{3(2\pi/360)} = \dfrac{3000}{\pi}\ \dfrac{\text{N·m}}{\text{rad}}$

$\bar{I} = \frac{1}{2} m r^2 = \frac{1}{2}(8\ \text{kg})(0.2\ \text{m})^2 = 0.160\ \text{kg·m}^2$

$\omega_n^2 = \dfrac{K}{\bar{I}} = \dfrac{3000}{(0.160)\pi}\ \dfrac{\text{N·m}}{\text{kg·m}^2} = 5968\ \text{s}^{-2}$



$\Theta_m = \dfrac{\Theta_{ST}}{1-\omega_f^2/\omega_n^2} < -\Theta_{ST}$

$1 < -1 + \omega_f^2/\omega_n^2$

$\omega_f^2 > 2\omega_n^2$

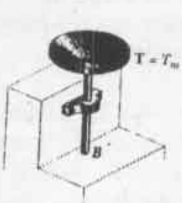
$\omega_f > \sqrt{2}\sqrt{5968\ \text{s}^{-2}} = 109.3$ rad/s

NOTE: ω_f IS INDEPENDENT OF T_m

19.104

GIVEN:

8 kg DISK, r = 200 mm, WELDED TO A SHAFT FIXED AT B. DISK ROTATES 3° WHEN A STATIC COUPLE OF 50 N·m IS APPLIED. $T_m = 60$ N·m



FIND:

RANGE OF VALUES FOR WHICH THE AMPLITUDE OF VIBRATION IS LESS THAN 3.5°

ANALOGOUS TO EQ (19.33) $\theta_m = \dfrac{T_m/K}{1-\omega_f^2/\omega_n^2} = \dfrac{\theta_{ST}}{1-\omega_f^2/\omega_n^2}$

$\omega_n^2 = K/\bar{I}$ WHERE K IS THE TORSIONAL SPRING CONSTANT AND $\bar{I}$ IS THE CENTROIDAL MOMENT OF INERTIA OF THE DISK. (SEE SAMPLE PROB 19.3)

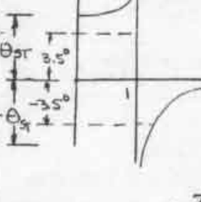
$K = \dfrac{50\text{ N·m}}{(3°)(\pi/360)} = \dfrac{3000}{\pi}\ \dfrac{\text{N·m}}{\text{rad}}$

$\bar{I} = \frac{1}{2} m r^2 = \frac{1}{2}(8\text{ kg})(0.2\text{ m})^2 = 0.160\text{ kg·m}^2$

$\omega_n^2 = K/\bar{I} = (3000/\pi)/(0.160) = 5968\text{ s}^{-2}$

$\theta_{ST} = T_m/K = 60\text{ N·m}/(3000/\pi) = 0.06283$ rad

$\theta_m = 3.5° = 0.06109$ rad



$\dfrac{\theta_{ST}}{1-\omega_f^2/\omega_n^2} < \theta_m$

$\dfrac{0.06283}{1-(\omega_f^2/5968)} > -0.06109$

$.06283 < [(\omega_f^2/5968)-1][0.06109]$

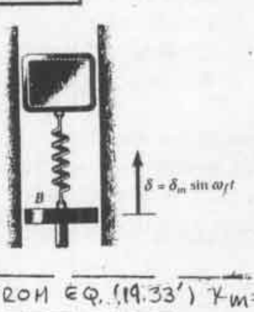
$1.02857 < (\omega_f^2/5968) - 1$

$\omega_f^2 > 12107\text{ s}^{-2}$ $\quad \omega_f > 110.0$ RAD/S

19.105

GIVEN:

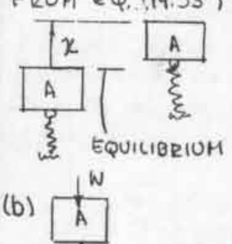
4-lb BLOCK A
SPRING $k = 8$ lb/ft
$\delta_m = 4$ in., $\omega_f = 5$ rad/s



FIND:

(A) AMPLITUDE OF MOTION OF THE BLOCK
(B) AMPLITUDE OF FLUCTUATING FORCE OF THE SPRING ON BLOCK

FROM EQ. (19.33') $x_m = \delta_m/(1-(\omega_f/\omega_n)^2)$



(a) $\omega_n^2 = \dfrac{k}{m} = \dfrac{(8\text{ lb/ft})}{(4\text{ lb}/32.2\text{ ft/s}^2)}$

$\omega_n^2 = 64.4\text{ s}^{-2}$

$x_m = \dfrac{(4/12\text{ ft})}{1-(25/64.4)} = 0.5448$ ft

$x_m = 0.545$ ft

SINCE $\omega_f^2 < \omega_n^2$, x AND δ ARE IN PHASE AND NET SPRING DEFLECTION IS $x - \delta$ AND $F = k(x-\delta)$

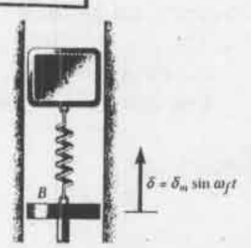
$F_m = (8\text{ lb/ft})(0.5448\text{ ft} - 0.333\text{ ft})$

$F_m = 1.692$ lb

19.106

GIVEN:

8 kg BLOCK A
SPRING $k = 1.6$ kN/m
$\delta_m = 150$ mm



FIND:

VALUES OF ω_f FOR WHICH THE FLUCTUATING FORCE OF THE SPRING ON THE BLOCK IS LESS THAN 120 N.

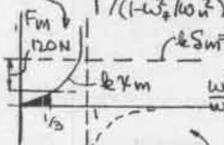
FROM EQ. (19.33') $x_m = \delta_m/(1-\omega_f^2/\omega_n^2)$

$\omega_n^2 = k/m = \dfrac{1.6\times 10^3\text{ N/m}}{8\text{ kg}} = 200\text{ s}^{-2}$

IN PHASE

$F_m = k(x_m - \delta_m) = k\delta_m/(1-\omega_f^2/\omega_n^2) - k\delta_m < 120$ N

$1/(1-\omega_f^2/\omega_n^2) - 1 < 120/[(1600)(150)] = 1/2$



$2/3 < 1 - \omega_f^2/\omega_n^2$

$\omega_f^2 < \frac{1}{3}\omega_n^2$

$\omega_f^2 < 200/3 \quad \omega_f < 8.16$ rad/s

OUT OF PHASE

$F_m = k(x_m + \delta_m) = 1600(x_m + 0.150)$

$= 1600 x_m + 240\text{ N} > 120$ N

THERE IS NO VALUE FOR x_m WHICH WILL MAKE $F_m < |120|$ WHEN x AND δ ARE OUT OF PHASE

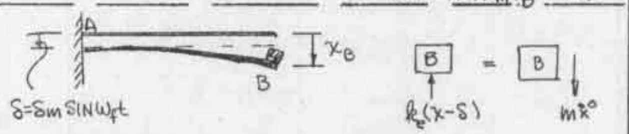
19.107

GIVEN:

$(\delta_{ST})_B = 2$ in.
$\delta_m = 0.5$ in.

FIND:

RANGE OF ω_f FOR WHICH $|x_m|_B < 1$ in.



$m\ddot{x}_B = -k_e(x_B - \delta)$

$m\ddot{x}_B + k_e x_B = k_e \delta_m \sin \omega_f t$

THUS, FROM EQ (19.31) AND (19.33')

$(x_m)_B = \dfrac{\delta_m}{1-\omega_f^2/\omega_n^2}$

$\omega_n^2 = \dfrac{k_e}{m} = \dfrac{g}{\delta_{ST}} = \dfrac{32.2\text{ ft/s}^2}{2/12\text{ ft}} = 193.2\text{ s}^{-2}$

$(x_m)_B = \dfrac{0.5}{1-\omega_f^2/193.2}$

$|1| < \dfrac{1/2}{1-\omega_f^2/193.2}$

IN PHASE $\quad +1(1-\omega_f^2/193.2) < \frac{1}{2}$

$\omega_f^2 < \dfrac{193.2}{2}$

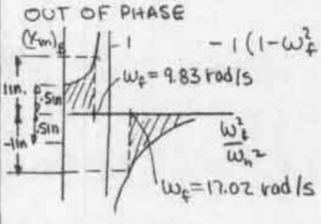
$\omega_f < 9.83$ rad/s

OUT OF PHASE

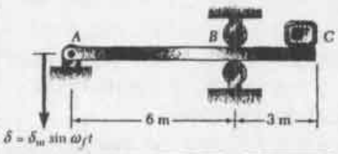
$-1(1-\omega_f^2/193.2) > \frac{1}{2}$

$\omega_f^2 > (3/2)(193.2)$

$\omega_f > 17.02$ rad/s



19.108



GIVEN:

$(\delta_{ST})_C = 15\text{ mm}$

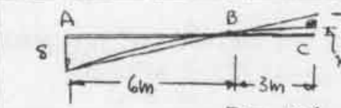
$m_C = 120\text{ kg}$

$\omega_f = 18\text{ rad/s}$

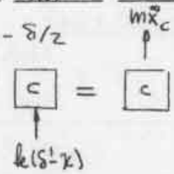
$\delta_m = 10\text{ mm}$

FIND:

$(a_c)_m$



$\delta' = -\delta/2$



$\Sigma F = ma$

$m\ddot{x}_c = k(\delta' - x_c) \quad \delta' = -\delta/2$

$\ddot{x}_c + \frac{k}{m} x_c = -k\delta/2 = -k\frac{\delta_m}{2}\sin\omega_f t$

FROM EQ (19.31 AND 19.33') $(x_c)_m = \dfrac{-\delta_m/2}{1 - \omega_f^2/\omega_n^2}$

THUS $(x_c)_m = \dfrac{-0.010/2\text{ m}}{1 - (18)^2/654}$ $\qquad \omega_n^2 = \dfrac{g}{(\delta_{ST})_c} = \dfrac{9.81\text{ m/s}^2}{0.015\text{ m}}$

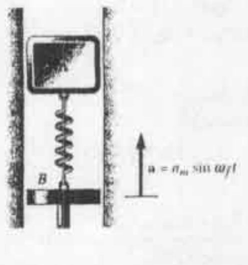
$(x_c)_m = -0.009909\text{ m}$ $\qquad \omega_n^2 = 654\text{ s}^{-2}$

$x_c = (x_c)_m \sin\omega_f t$

$\ddot{x}_c = (a_c)_m = -(x_c)_m\omega_f^2 \qquad (a_c)_m = (0.00909\text{ m})(18\text{ s}^{-1})^2$

$(a_c)_m = 3.21\text{ m/s}^2$

19.109



GIVEN:

8-kg BLOCK A

SPRING $k = 120\text{ N/m}$

$a_m = 1.5\text{ m/s}^2$

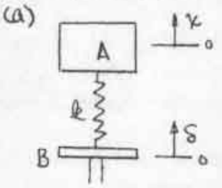
$\omega_f = 5\text{ rad/s}$

FIND:

(a) MAXIMUM DISPLACEMENT OF A

(b) AMPLITUDE OF THE FLUCTUATING FORCE EXERTED BY THE SPRING ON THE BLOCK

(a)



SUPPORT MOTION

$a = \ddot{\delta} = a_m \sin\omega_f t$

$\delta = -(a_m/\omega_f^2)\sin\omega_f t$

$\delta_m = -a_m/\omega_f^2 = -\dfrac{1.5\text{ m/s}^2}{(5\text{ s}^{-1})^2}$

$\delta_m = -0.060\text{ m}$

FROM EQ. (19.31 AND 19.33')

$x_m = \dfrac{\delta_m}{1 - \omega_f^2/\omega_n^2}$ $\qquad \omega_n^2 = k/m = \dfrac{120\text{ N/m}}{8\text{ kg}}$

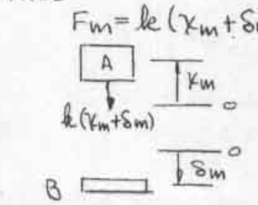
$x_m = \dfrac{-0.060}{1 - (25/15)} = 0.090\text{ m}$ $\qquad \omega_n^2 = 15\text{ s}^{-2}$

$x_m = 0.090\text{ m}$

(b) x IS OUT OF PHASE WITH δ FOR $\omega_f = 5\text{ rad/s}$

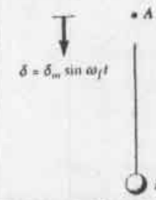
THUS

$F_m = k(x_m + \delta_m) = 120\text{ N/m}\,(0.09\text{ m} + 0.060\text{ m})$



$F_m = 18\text{ N}$

19.110



GIVEN:

0.8-lb BALL

ELASTIC CORD AB

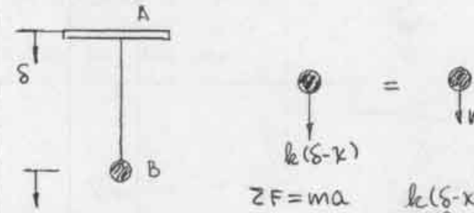
$k = 5\text{ lb/ft}$

$\delta_m = 8\text{ in}$

$f_f = 0.5\text{ Hz}$

FIND:

AMPLITUDE OF THE MOTION OF B



$\Sigma F = ma \qquad k(\delta - x) = m\ddot{x}$

$\ddot{x} + (k/m)x = \delta$

FROM EQ. (19.31 AND 19.33')

$x_m = \delta_m/(1 - \omega_f^2/\omega_n^2)$

$\omega_f = 2\pi f_f = 2\pi(\tfrac{1}{2}) = \pi\text{ s}^{-1} \quad \omega_f^2 = \pi^2\text{ s}^{-2}$

$\omega_n^2 = k/m = (5\text{ lb/ft})/(0.8\text{ lb}/32.2\text{ ft/s}^2) = 201.25\text{ s}^{-2}$

$\delta_m = (8)(1/12) = 2/3\text{ ft}$

$x_m = \dfrac{2/3\text{ ft}}{(1 - \pi^2/201.25)} = 0.7011\text{ ft}$ (IN PHASE)

CHECK TO SEE WHETHER CORD GOES SLACK

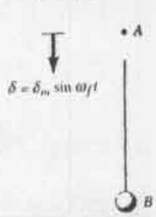
STATIC DEFLECTION $\delta_{ST} = W/k = 0.8\text{ lb}/5\text{ lb/ft} = 0.16\text{ ft}$

SINCE X AND δ ARE IN PHASE THE MAXIMUM DEFLECTION OF THE CORD IS $x_m - \delta_m = 0.7011 - 0.6667 = 0.0344\text{ ft}$

WHICH IS LESS THAN THE STATIC DEFLECTION OF 0.16 ft

$x_m = 0.701\text{ ft}$

19.111



GIVEN:

0.8-lb BALL

ELASTIC CORD AB

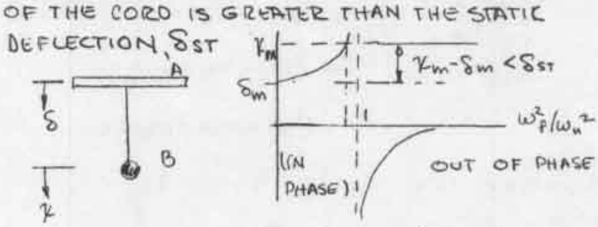
$k = 5\text{ lb/ft}$

$\delta_m = 8\text{ in.}$

FIND:

MAXIMUM ω_f IF CORD IS NOT TO GO SLACK

CORD BECOMES SLACK WHEN THE NET DEFLECTION OF THE CORD IS GREATER THAN THE STATIC DEFLECTION, δ_{ST}



$\delta_{ST} = \dfrac{W}{k} = \dfrac{0.8\text{ lb}}{5\text{ lb/ft}} = 0.16\text{ ft} \qquad \omega_n^2 = \dfrac{k}{m} = \dfrac{(5\text{ lb/ft})}{(0.8\text{ lb}/32.2\text{ ft/s}^2)}$

FROM EQ (19.31 AND 19.33') $x_m = \delta_m/(1 - \omega_f^2/\omega_n^2)$

$x_m - \delta_m < \delta_{ST} \qquad \delta_m\left[\dfrac{1}{(1 - \omega_f^2/\omega_n^2)} - 1\right] < \delta_{ST}$

$\dfrac{1}{1 - (\omega_f^2/201.25)} < \dfrac{0.16}{2/3} + 1$

$\dfrac{1}{1.24} < 1 - \dfrac{\omega_f^2}{201.25} \qquad -\omega_f^2 > 201.25(0.806 - 1)$

$\omega_f^2 < 38.95\text{ s}^{-2}$

$\omega_f < 6.24\text{ rad/s}$

19.112

GIVEN:

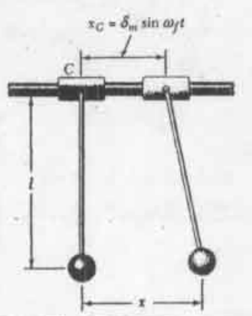
1.2 kg BOB

$l = 0.600\ m$

1.4 kg COLLAR C

$\delta_m = 0.010\ m$

$f_f = 0.5\ Hz$

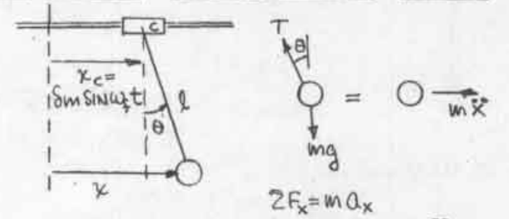


FIND:

(a) AMPLITUDE OF MOTION OF THE BOB

(b) FORCE APPLIED TO THE COLLAR TO MAINTAIN THE MOTION

(a)



$\Sigma F_x = m a_x$

$-T\sin\theta = m\ddot{x}$

$\Sigma F_y = T\cos\theta - mg = 0$ SMALL ANGLES $\cos\theta \approx 1$, ACCELERATION IN THE y DIRECTION IS SECOND ORDER AND IS NEGLECTED

$T = mg$

$m\ddot{x} = -mg\sin\theta$

$\sin\theta = \dfrac{x - x_c}{l}$

$m\ddot{x} + \dfrac{mg}{l}x = \dfrac{g}{l}x_c = \dfrac{mg}{l}\delta_m \sin\omega_f t$ $\quad \omega_n^2 = g/l$

$\ddot{x} + \omega_n^2 x = \omega_n^2 \delta_m \sin\omega_f t$

FROM EQ (19.33')

$$x_m = \frac{\delta_m}{1 - \omega_f^2/\omega_n^2}$$

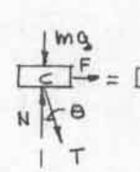
$\omega_f^2 = (2\pi f_n)^2 = 4\pi^2 \frac{1}{4} = \pi^2\ s^{-2}$

$\omega_n^2 = g/l = \dfrac{9.81\ m/s^2}{0.6\ m} = 16.35\ s^{-2}$

$$x_m = \frac{0.010\ m}{1 - \pi^2/16.35} = 0.025229\ m$$

$x_m = 25.2\ mm$

(b)



$a_c = \ddot{x}_c = -\delta_m \omega_f^2 \sin\omega_f t$

$\pm\!\!\rightarrow \Sigma F_x = m_c a_c$

$F - T\sin\theta = m_c a_c$

FROM PART (a) $T = mg$, $\sin\theta = \dfrac{x - x_c}{l}$

THUS

$F = -mg\left[\dfrac{x - x_c}{l}\right] + m_c \ddot{x}_c$

$F = -m\omega_n^2 x + m\omega_n^2 x_c + m_c \ddot{x}_c$

$F = -m\omega_n^2 x_m \sin\omega_f t + m\omega_n^2 \delta_m \sin\omega_f t - m_c \omega_f^2 \delta_m \sin\omega_f t$

$F = [-(1.2\ kg)(16.35\ s^{-2})(0.025229\ m) + (1.2\ kg)(16.35\ s^{-2})(0.01\ m) - (1.4\ kg)(\pi^2 s^{-2})(0.01\ m)]\sin\pi t$

$F = -0.437 \sin\pi t\ (N)$

19.113

GIVEN:

MOTOR OF MASS M SUPPORTED BY SPRINGS WITH EQUIVALENT CONSTANT k

EQUIVALENT ROTOR MASS UNBALANCE m AT A DISTANCE r FROM THE AXIS OF ROTATION.

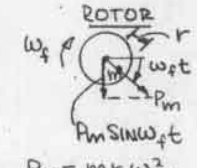
ANGULAR VELOCITY OF MOTOR, ω_f

SHOW THAT:

AMPLITUDE OF THE MOTION OF THE MOTOR

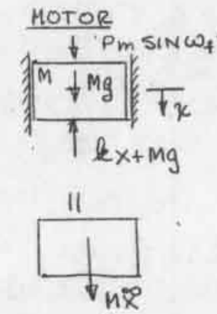
$\omega_n = \sqrt{k/M}$ $\quad x_m = \dfrac{r(m/M)(\omega_f/\omega_n)^2}{1 - (\omega_f/\omega_n)^2}$

ROTOR



$P_m = m r \omega_f^2$

MOTOR



$+\!\downarrow \Sigma F = ma \quad P_m \sin\omega_f t - kx = M\ddot{x}$

$M\ddot{x} + kx = P_m \sin\omega_f t$

$\ddot{x} + k/M\, x = P_m/M \sin\omega_f t$

$\omega_n^2 = k/M$

FROM EQ. (19.33)

$$x_m = \frac{P_m/k}{1 - (\omega_f/\omega_n)^2}$$

BUT $P_m/k = m r \omega_f^2/k$ $\quad k = M\omega_n^2$

$P_m/k = r(m/M)(\omega_f/\omega_n)^2$

THUS

$$x_m = \frac{r(m/M)(\omega_f/\omega_n)^2}{1 - (\omega_f/\omega_n)^2}$$ QED

19.114

GIVEN:

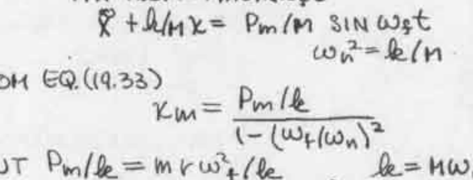
100-kg MOTOR; UNBALANCED 15-kg ROTOR. SPEED INCREASED AND AT VERY HIGH SPEEDS THE AMPLITUDE NEARS 3.3 mm

FIND:

THE DISTANCE BETWEEN THE MASS CENTER OF THE ROTOR AND ITS AXIS OF ROTATION

USE THE EQUATION DERIVED IN PROB 19.113 (ABOVE)

$$x_m = \frac{r(m/M)(\omega_f/\omega_n)^2}{1 - (\omega_f/\omega_n)^2} = \frac{r(m/M)}{\frac{1}{(\omega_f/\omega_n)^2} - 1}$$



FOR VERY HIGH SPEEDS $\dfrac{1}{(\omega_f/\omega_n)^2} \rightarrow 0$ AND

$x_m \rightarrow r\, m/M$, THUS $3.3\ mm = r(15/100)$

$r = 22\ mm$

19.115 GIVEN:

SPRING SUPPORTED MOTOR WHOSE SPEED IS INCREASED FROM 200 TO 300 RPM. AMPLITUDE DUE TO UNBALANCE INCREASES CONTINUOUSLY FROM 2.5 TO 8 MM.

FIND:

SPEED AT RESONANCE

FROM PROB 19.113

$$x_m = \frac{r(m/M)(\omega_f/\omega_n)^2}{1-(\omega_f/\omega_n)^2}$$

$$2.5 = \frac{r(m/M)(200/\omega_n)^2}{1-(200/\omega_n)^2}$$

$$8 = \frac{r(m/M)(300/\omega_n)^2}{1-(300/\omega_n)^2}$$

$$\frac{2.5}{8.0} = \frac{1-(300/\omega_n)^2}{1-(200/\omega_n)^2}\left(\frac{200}{300}\right)^2$$

$$0.703 - 0.703(200/\omega_n)^2 = 1-(300/\omega_n)^2$$

$$\frac{1}{\omega_n^2}\left[90\times10^3 - 28.125\times10^3\right] = 0.2969$$

$$\omega_n^2 = 208.4 \quad \omega_n = 457 \text{ rpm}$$

RESONANCE WHEN $\omega_f = \omega_n$ $\qquad \omega_f = 457$ rpm

19.116 GIVEN:

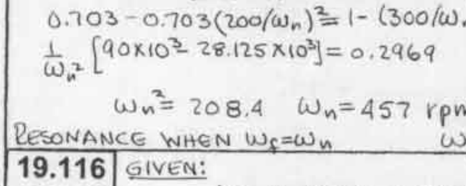
400-lb MOTOR SUPPORTED BY SPRINGS WITH TOTAL $k = 1200$ lb/in. ROTOR UNBALANCE IS 1 OZ, 8 IN. FROM THE AXIS OF ROTATION

FIND:

RANGE OF ALLOWABLE VALUES OF MOTOR SPEED IF THE AMPLITUDE OF VIBRATION IS NOT TO EXCEED 0.06 IN

FROM PROB 19.113

$$x_m = \frac{r(m/M)(\omega_f/\omega_n)^2}{1-(\omega_f/\omega_n)^2}$$



$k = 1200$ lb/in $= 14400 \frac{\text{lb}}{\text{ft}}$

$$\omega_n^2 = k/M = \frac{(14400 \text{ lb/ft})}{(400 \text{ lb}/32.2 \text{ ft/s}^2)} = 1159.2 \text{ s}^{-2}$$

$$r\,m/M = (8/12 \text{ ft})\left(\frac{1/16 \text{ lb}}{400 \text{ lb}}\right) = 104.17\times10^{-6} \text{ ft}$$

$x_m = 0.06$ in $\qquad (0.06/12 \text{ ft}) \le \dfrac{104.17\times10^{-6} \text{ ft}\,(\omega_f/\omega_n)^2}{1-(\omega_f/\omega_n)^2}$

$$47.998 - 47.998(\omega_f/\omega_n)^2 \le (\omega_f/\omega_n)^2$$

$$(\omega_f/\omega_n)^2 \le \frac{47.998}{48.998}$$

$\omega_f/\omega_n < 0.9897 \qquad \omega_f \le 0.9897(1159.2)^{1/2}$

$$\omega_f \le 33.69 \text{ rad/s}$$

$$\omega_f \le \left(33.69 \frac{\text{rad}}{\text{s}}\right)\left(60\frac{\text{s}}{\text{m}}\right)\left(\frac{1}{2\pi \text{ RAD/REV}}\right) = 322 \text{ RPM}$$

$x_m = -0.06$ in $\quad (-0.06/12 \text{ ft}) \ge \dfrac{104.17\times10^{-6} \text{ ft}\,(\omega_f^2/\omega_n^2)}{1-\omega_f^2/\omega_n^2}$

$$(\omega_f/\omega_n)^2 \ge \frac{47.998}{46.998} =$$

$\omega_f \ge (1.0106)(1159.2)^{1/2} \qquad \omega_f \ge 34.40 \text{ rad/s} = 329$ rpm

19.117 GIVEN:

220-lb MOTOR UNBALANCE OF THE ROTOR = 2 OZ, 4 IN FROM THE AXIS OF ROTATION RESONANCE AT 400 RPM

FIND:

AMPLITUDE AT (a) 800 rpm, (b) 200 rpm, (c) 425 rpm

FROM PROB 19.113

$$x_m = \frac{r(m/M)(\omega_f/\omega_n)^2}{1-(\omega_n/\omega_f)^2}$$

RESONANCE AT 400 rpm MEANS THAT $\omega_n = 400$ rpm

$r(m/M) = (4 \text{ in.})(2/16)/(220) = 2.2727\times10^{-3}$ in.

(a) $(\omega_f/\omega_n)^2 = (800/400)^2 = 4$

$$x_m = \frac{2.2727\times10^{-3} \text{ in.}(4)}{1-4} = 0.00303 \text{ in}$$

(b) $(\omega_f/\omega_n)^2 = (200/400)^2 = 1/4$

$$x_m = \frac{2.2727\times10^{-3}(\frac{1}{4})}{1-1/4} = 0.000758 \text{ in.}$$

(c) $(\omega_f/\omega_n)^2 = (425/400)^2 = 1.1289$

$$x_m = \frac{2.272\times10^{-3}(1.1289)}{1-1.1289} = -0.01990 \text{ in.}$$

19.118 GIVEN:

180-kg MOTOR UNBALANCE OF THE ROTOR = 28 g 150 mm FROM AXIS OF ROTATION STATIC DEFLECTION $\delta_{ST} = 12$ mm

FIND:

MASS OF A PLATE ADDED TO THE BASE OF THE MOTOR SO THAT AMPLITUDE OF VIBRATION IS LESS THAN 60×10^{-6} m FOR MOTOR SPEEDS ABOVE 300 rpm.

FROM PROB 19.113

$$x_m = \frac{r(m/M)(\omega_f/\omega_n)^2}{1-\omega_f^2/\omega_n^2}$$

SINCE $M\omega_n^2 = k$

$$x_m = (mr/k)\omega_f^2 / (1-\omega_f^2/\omega_n^2)$$

BEFORE THE PLATE IS ADDED, $\omega_n^2 = \dfrac{g}{\delta_{ST}} = \dfrac{9.81 \text{ m/s}^2}{0.012 \text{ m}}$

$k = M\omega_n^2 = (180 \text{ kg})(817.5 \text{ s}^{-2}) \quad \omega_n^2 = 817.5 \text{ s}^{-2}$

$k = 147.15\times10^3$ N/m

$mr/k = (28\times10^{-3} \text{ kg})(0.150 \text{ m})/(147.15\times10^3 \text{ N/m})$

$= 28.542\times10^{-9} \text{ m}\cdot\text{s}^2$

AFTER THE PLATE IS ADDED THE NATURAL FREQUENCY OF THE SYSTEM CHANGES SINCE THE MASS CHANGES $\omega_n'^2 = k/M'$

SINCE THE VIBRATION IS TO BE LESS THAN 60×10^{-6} m FOR MOTOR SPEEDS ABOVE 300 rpm, WE HAVE

$$x_m = -60\times10^{-6} \text{ m} = \frac{(28.542\times10^{-9} \text{ m}\cdot\text{s}^2)(300\cdot\frac{2\pi}{60}\text{s}^{-1})^2}{1-\left(\frac{300\cdot2\pi/60}{\omega_n'}\right)^2}$$

$$-2.1299 + 2.1299\left(\frac{986.96}{\omega_n'^2}\right) = 1$$

$$\omega_n'^2 = \frac{2.1299(986.96)}{3.1299} = 671.6 \text{ s}^{-2} = \frac{k}{M'}$$

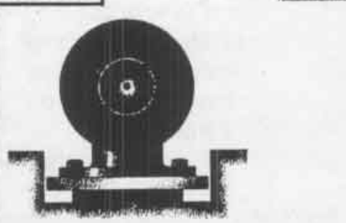
$M' = (147.15\times10^3 \text{ N/m})/(671.6 \text{ s}^{-2}) = 219.1$ kg

$\Delta M = M' - M = 219 - 180 = 39.1$ kg

19.119

GIVEN:

400-lb MOTOR
UNBALANCE OF 3 OZ.
6 IN FROM AXIS OF ROTATION
FORCE TRANSMITTED TO FOUNDATION LIMITED TO 0.2 lb WHEN MOTOR IS RUN AT 100 rpm AND ABOVE



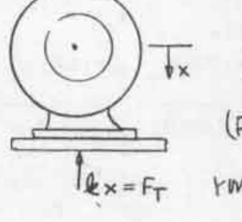
FIND:

(a) MAXIMUM ALLOWABLE SPRING CONSTANT k OF A PAD PLACED BETWEEN THE MOTOR AND THE FOUNDATION

(b) CORRESPONDING AMPLITUDE OF THE FLUCTUATING FORCE WHEN THE MOTOR IS RUN AT 200 rpm

(a) FROM PROB. (19.113)

$$x_m = \frac{r(m/M)(\omega_f/\omega_n)^2}{1-(\omega_f/\omega_n)^2}$$



$$(F_T)_m = k x_m \qquad \frac{k}{M} = \omega_n^2$$

$$(F_T)_m = \frac{rm\,\omega_f^2}{1-\omega_f^2 M/k} \qquad (1)$$

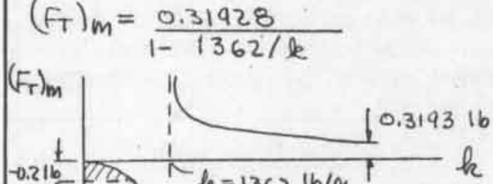
$$rm = (6/12\text{ ft})(3/16\text{ lb})/(32.2\text{ ft/s}^2)$$

$$rm = 0.002912\text{ lb}\cdot\text{s}^2$$

AT $\omega_f = 100\text{ rpm} = 100(2\pi/60) = 10.472\text{ rad/s}$

$$(F_T)_m = \frac{(0.002912\text{ lb s}^2)(10.472\text{ s}^{-1})^2}{1-\dfrac{(10.472\text{ s}^{-1})^2(400\text{ lb}/32.2\text{ ft/s}^2)}{k}}$$

$$(F_T)_m = \frac{0.31928}{1-1362/k}$$



$$-.2 = 0.319\ /(1-1362/k)$$

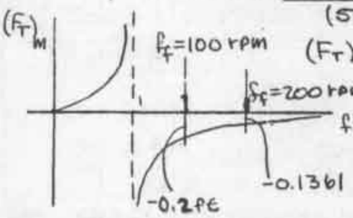
$$-0.2 + 0.2(1362)/k_m = 0.31928$$

$$k_m = \frac{(0.2)(1362)}{0.51928} = 525\text{ lb/ft}$$

(b) AT 200 rpm, $\omega_f = (200)(2\pi)/60 = 20.94$ rad/s
FROM (1), AND USING k FOUND IN PART (a)

$$(F_T)_m = \frac{(0.002912\text{ lb}\cdot\text{s}^2)(20.94\text{ s}^{-1})^2}{1-\dfrac{(20.94\text{ s}^{-1})^2(400\text{ lb}/32.2\text{ ft/s}^2)}{(525\text{ lb/ft})}}$$

$$(F_T)_m = -0.1361\text{ lb}$$



19.120

GIVEN:

180-kg MOTOR, SUPPORTED BY SPRINGS OF TOTAL CONSTANT $k = 150$ kN/m
UNBALANCE OF THE ROTOR IS 28-g AT 150 mm

FIND:

RANGE OF SPEEDS FOR WHICH THE FLUCTUATING FORCE $(F_T)_m$ IS LESS THAN 20 N

FROM PROB. (19.113)

$$x_m = \frac{r(m/M)(\omega_f^2)}{1-(\omega_f/\omega_n)^2}$$

$$(F_T)_m = k x_m \qquad k/M = \omega_n^2$$

$$F_T = rm\,\omega_f^2/(1-(\omega_f/\omega_n)^2)$$

$$rm = (0.150\text{ m})(0.028\text{ kg}) = 0.0042\text{ m}\cdot\text{kg}$$

$$\omega_n^2 = k/M = (150\times10^3\text{ N/m})/(180\text{ kg}) = 833.3\text{ s}^{-2}$$

$$(F_T)_m = [(0.0042)(\omega_f^2)]/(1-\omega_f^2/833.3)$$

$$(F_T)_m(1-\omega_f^2/833.3) = 0.0042\,\omega_f^2$$

$$(F_T)_m = [(F_T)_m/833.3]\omega_f^2 + 0.0042\,\omega_f^2$$

$$\omega_f^2 = (F_T)_m/[(F_T)_m/(833.3) + 0.0042]$$

FOR $(F_T)_m = 20$ N

$$\omega_f^2 < \frac{20}{0.024+0.0042} = 709.2\text{ s}^{-2}$$

$\omega_f \le 26.63$ rad/s

$$\omega < 26.63\left(\frac{60}{2\pi}\right) = 254\text{ rpm}$$

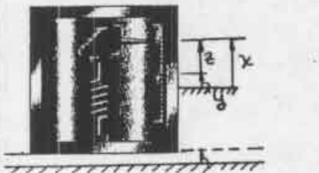
FOR $(F_T)_m = -20$ N

$$\omega_f^2 > \frac{-20}{-0.024+.0042} = 1010\text{ s}^{-1}$$

$\omega_f > 31.78$ rad/s

$$\omega > 31.78\left(\frac{60}{2\pi}\right) = 303\text{ rpm}$$

19.121

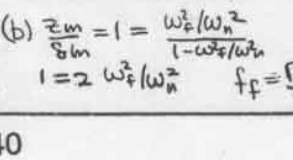


GIVEN:

$f_n = 120$ Hz
z_m = AMPLITUDE RELATIVE TO THE BOX IS USED AS A MEASURE OF δ_m

FIND:

(a) % ERROR FOR $f_f = 600$ Hz
(b) f_f FOR ZERO ERROR



$$x = \left(\frac{\delta_m}{1-\omega_f^2/\omega_n^2}\right)\sin\omega_f t$$

$y = \delta_m \sin\omega_f t$
z = RELATIVE MOTION

$$z = x - y = \left[\frac{\delta_m}{1-\omega_f^2/\omega_n^2} - \delta_m\right]\sin\omega_f t$$

$$z_m = \delta_m\left[\frac{1}{1-\omega_f^2/\omega_n^2} - 1\right] = \frac{\delta_m\,\omega_f^2/\omega_n^2}{1-\omega_f^2/\omega_n^2}$$

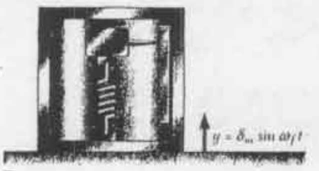
(a) $$\frac{z_m}{\delta_m} = \frac{\omega_f^2/\omega_n^2}{1-\omega_f^2/\omega_n^2} = \frac{(600/120)^2}{1-(600/120)^2} = \frac{25}{24} = 1.0417$$

ERROR = 4.17%

(b) $$\frac{z_m}{\delta_m} = 1 = \frac{\omega_f^2/\omega_n^2}{1-\omega_f^2/\omega_n^2}$$

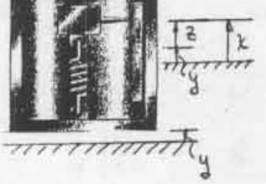
$$1 = 2\,\omega_f^2/\omega_n^2 \qquad f_f = \frac{\sqrt2}{2}f_n = \frac{\sqrt2}{2}(120) = 84.9\text{ Hz}$$

19.122



GIVEN:
$f_n = 2200$ Hz
z_m = AMPLITUDE RELATIVE TO THE BOX
$z_m \omega_n^2$ USED AS A MEASURE OF THE ACCELERATION OF THE VIBRATING SURFACE, $a_m = \delta_m \omega_f^2$

FIND:
% ERROR IN a_m WHEN $f_f = 600$ Hz



ABSOLUTE MOTION OF THE MASS
$$x = \frac{\delta_m}{1-\omega_f^2/\omega_n^2} \sin \omega_f t$$
$y = \delta_m \sin \omega_f t$
RELATIVE MOTION OF THE MASS
$$z = x - y = \left[\frac{\delta_m}{1-\omega_f^2/\omega_n^2} - \delta_m\right] \sin \omega_f t$$

$$z_m = \frac{\delta_m \frac{\omega_f^2}{\omega_n^2}}{1-\omega_f^2/\omega_n^2} \qquad z_m \omega_n^2 = \frac{\delta_m \omega_f^2}{1-\omega_f^2/\omega_n^2}$$

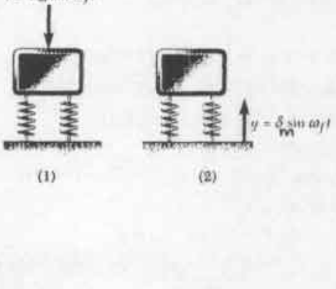
$a_m = \delta_m \omega_f^2 \qquad z_m \omega_n^2 = a_m/(1-\omega_f^2/\omega_n^2)$

$$(\omega_f/\omega_n)^2 = \left(\frac{600}{2200}\right)^2 = 0.07438$$

$$\frac{z_m \omega_n^2}{a_m} = \frac{1}{1-0.07438} = 1.0804$$

ERROR = 8.04 %

19.123



GIVEN:
TWO SYSTEMS AS SHOWN

FIND: TRANSMISSIBILITY, i.e.
(1) THE RATIO OF THE TRANSMITTED FORCE TO THE IMPRESSED FORCE
(2) RATIO OF THE TRANSMITTED DISPLACEMENT TO THE IMPRESSED DISPLACEMENT

SHOW THAT:
TO REDUCE TRANSMISSIBILITY, $\omega_f/\omega_n > \sqrt{2}$

(1) FROM EQ. (19.33) $x_m = \dfrac{P_m/k}{1-(\omega_f/\omega_n)^2}$

FORCE TRANSMITTED $(P_T)_m = k x_m = k\left[\dfrac{P_m/k}{1-(\omega_f/\omega_n)^2}\right]$

THUS
$$\text{TRANSMISSIBILITY} = \frac{(P_T)_m}{P_m} = \frac{1}{1-(\omega_f/\omega_n)^2}$$

(2) FROM EQ. (19.33')

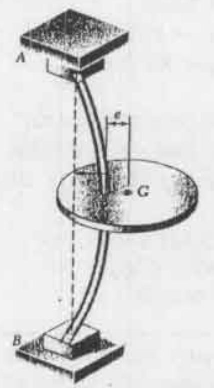
DISPLACEMENT TRANSMITTED $x_m = \dfrac{\delta_m}{1-(\omega_f/\omega_n)^2}$

$$\text{TRANSMISSIBILITY} = \frac{x_m}{\delta_m} = \frac{1}{1-(\omega_f/\omega_n)^2}$$

FOR $\dfrac{(P_T)_m}{P_m}$ OR $\dfrac{x_m}{\delta_m}$ TO BE LESS THAN 1

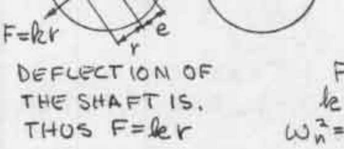
$$\left|\frac{1}{1-(\omega_f/\omega_n)^2}\right| < 1$$
$$1 < |1-(\omega_f/\omega_n)^2|$$
$$(\omega_f/\omega_n)^2 > 2 \qquad \omega_f/\omega_n > \sqrt{2} \quad \text{QED}$$

19.124



GIVEN:
60-lb DISK
$e = 0.006$ in.
$k = 40{,}000$ lb/ft FOR THE SHAFT WHICH ROTATES AT A CONSTANT ANGULAR VELOCITY ω_f ABOUT AB

FIND:
(a) ω_f FOR RESONANCE
(b) DEFLECTION r WHEN $\omega_f = 1200$ rpm



G DESCRIBES A CIRCLE ABOUT THE AXIS AB OF RADIUS $r+e$.
THUS $a_n = (r+e)\omega_f^2$

DEFLECTION OF THE SHAFT IS, THUS $F = kr$

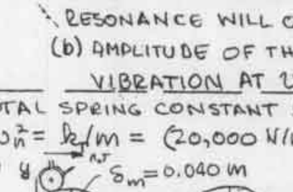
$F = ma_n$
$kr = m(r+e)\omega_f^2$
$\omega_n^2 = \dfrac{k}{m} \qquad m = k/\omega_n^2$
$kr = \dfrac{k}{\omega_n^2}(r+e)\omega_f^2$

$$r = \frac{e\frac{\omega_f^2}{\omega_n^2}}{1-\omega_f^2/\omega_n^2}$$

(a) RESONANCE OCCURS WHEN $\omega_f = \omega_n$, i.e. $r \to \infty$
$$\omega_n = \sqrt{k/m} = \sqrt{\frac{(40{,}000 \text{ lb/ft})}{(60 \text{ lb}/32.2 \text{ ft/s}^2)}} = 146.5 \text{ rad/s}$$
$$\omega_n = \omega_f = (146.5)\frac{(60)}{(2\pi)} = 1399.1$$

(b)
$$r = \frac{e\frac{\omega_f^2}{\omega_n^2}}{1-\omega_f^2/\omega_n^2} = \frac{(0.006 \text{ in.})\left(\frac{1200}{1399.1}\right)^2}{1-(1200/1399.1)^2} = 0.01669 \text{ in.}$$

19.125

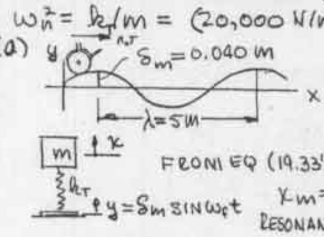


GIVEN:
MASS OF TRAILER AND LOAD = 250 kg
TRAILER SUPPORTED BY TWO SPRINGS EACH OF $k = 10$ kN/m
ROAD SURFACE IS A SINE CURVE WITH AMPLITUDE OF 40 mm AND WAVELENGTH 5 m

FIND:
(a) SPEED v AT WHICH RESONANCE WILL OCCUR
(b) AMPLITUDE OF THE TRAILER VIBRATION AT $v = 50$ km/h

TOTAL SPRING CONSTANT $k_T = 2k = 20$ kN/m
$$\omega_n^2 = k_T/m = (20{,}000 \text{ N/m})/(250 \text{ kg}) = 80 \text{ s}^{-2}$$

(a)



$y = \delta_m \sin \frac{x}{\lambda} \qquad x = vt$
$y = \delta_m \sin (v/\lambda) t \qquad \tau = \lambda/v$
$\omega_f = 2\pi/\tau$, THUS
$y = 0.04 \sin \omega_f t$ WHERE $\omega_f = \frac{2\pi v}{\lambda} = \frac{2\pi v}{5}$

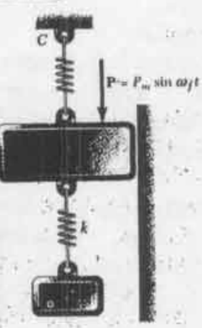
FROM EQ (19.33')
$x_m = \delta_m/(1-\omega_f^2/\omega_n^2)$
RESONANCE, $\omega_f = \dfrac{2\pi v}{5 \text{ m}} = \omega_n = \sqrt{80 \text{ s}^{-2}}$
$v = 7.11$ m/s $= 25.6$ km/h

(b) $\omega_f = \dfrac{(2\pi)(50/3.6 \frac{\text{m}}{\text{s}})}{5} = 17.45 \text{ s}^{-1}$, $x_m = \dfrac{0.04}{1-17.45^2/80} = -0.01425$ m

19.126

GIVEN:

$P_m = 20\,N$
$\omega_f = 2\ rad/s$
$m_B = 22\,kg$

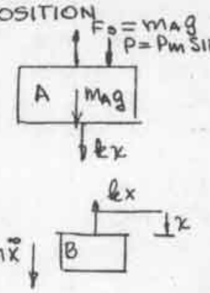


FIND:

(a) VALUE OF k WHICH WILL PREVENT STEADY STATE VIBRATION OF A

(b) CORRESPONDING AMPLITUDE OF BLOCK B

AT STEADY STATE BLOCK A DOES NOT MOVE AND IS THEREFORE REMAINS IN ITS ORIGINAL EQUILIBRIUM POSITION



(a) BLOCK A

$+\downarrow\ \Sigma F = 0$

$kx = -P_m \sin \omega_f t \quad (1)$

BLOCK B

$+\downarrow\ \Sigma F = m_B \ddot{x}$

$m_B \ddot{x} + kx = 0$

$x = x_m \sin \omega_n t,\ \omega_n^2 = k/m_B$

FROM (1)

$k x_m \sin \omega_n t = -P_m \sin \omega_f t$

$\omega_n = \omega_f = 2\ rad/s \quad ,\ k x_m = -P_m$

$\sqrt{\frac{k}{m_B}} = 2\,s^{-1}$ $\quad k = (4\,s^{-2})(22\,kg)$

$k = 88\ N/m$

(b) $k x_m = -P_m \quad x_m = -\frac{20\,N}{88\,N/m} = -0.227\,m$

19.127

GIVEN:

HEAVY DAMPING, $C > C_c$

SHOW THAT:

A BODY NEVER PASSES THROUGH ITS EQUILIBRIUM POSITION O IF,

(a) IT IS RELEASED FROM ANY POSITION WITH NO INITIAL VELOCITY

(b) IT IS STARTED FROM O WITH AN ARBITRARY INITIAL VELOCITY

SINCE $C > C_c$ WE USE EQ. (19.42), WHERE

$\lambda_1 < 0,\ \lambda_2 < 0$

$x = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \quad (1)$

$v = \frac{dx}{dt} = C_1 \lambda_1 e^{\lambda_1 t} + C_2 \lambda_2 e^{\lambda_2 t} \quad (2)$

(a) $t = 0,\ x = x_0 \quad v = 0$

FROM (1) AND (2) $\quad x_0 = C_1 + C_2$

$0 = C_1 \lambda_1 + C_2 \lambda_2$

SOLVING FOR C_1 AND C_2

$C_1 = \frac{\lambda_2}{\lambda_2 - \lambda_1} x_0 \quad C_2 = -\frac{\lambda_1}{\lambda_2 - \lambda_1} x_0$

SUBSTITUTING FOR C_1 AND C_2 IN (1)

$x = \frac{x_0}{\lambda_2 - \lambda_1}\left[\lambda_2 e^{\lambda_1 t} - \lambda_1 e^{\lambda_2 t}\right]$

19.127 CONTINUED

FOR $x = 0$ WHEN $t \neq \infty$, WE MUST HAVE

$\lambda_1 e^{\lambda_2 t} - \lambda_2 e^{\lambda_1 t} = 0 \qquad \frac{\lambda_2}{\lambda_1} = e^{(\lambda_2 - \lambda_1)t} \quad (3)$

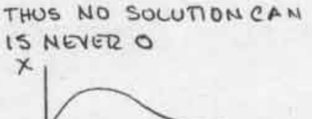
RECALL THAT

$\lambda_1 < 0,\ \lambda_2 < 0$. CHOOSING λ_1 AND λ_2 SO THAT $\lambda_1 < \lambda_2 < 0$, WE HAVE

$0 < \frac{\lambda_2}{\lambda_1} < 1$ AND $\lambda_2 - \lambda_1 > 0$

THUS A POSITIVE SOLUTION FOR $t > 0$ FOR EQ. (3) CANNOT EXIST SINCE IT WOULD REQUIRE THAT e RAISED TO A POSITIVE POWER BE LESS THAN 1, WHICH IS IMPOSSIBLE. THUS x IS NEVER 0.

THE x-t CURVE CURVE FOR THIS CASE IS SHOWN



(b) $t = 0,\ x = 0,\ v = v_0$ $\quad$ EQ. (1) AND (2), YIELD

$0 = C_1 + C_2 \quad v_0 = C_1 \lambda_1 + C_2 \lambda_2$

SOLVING FOR C_1 AND C_2, $\quad C_1 = \frac{-v_0}{\lambda_2 - \lambda_1},\ C_2 = \frac{v_0}{\lambda_2 - \lambda_1}$

SUBSTITUTING INTO (1)

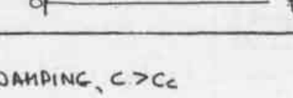
$x = \frac{v_0}{\lambda_2 - \lambda_1}\left[e^{\lambda_2 t} - e^{\lambda_1 t}\right]$

FOR $x = 0$, $t \neq \infty$

$e^{\lambda_2 t} = e^{\lambda_1 t}$

FOR $C > C_c$, $\lambda_1 \neq \lambda_2$; THUS NO SOLUTION CAN EXIST FOR t AND x IS NEVER 0

THE x-t CURVE FOR THIS MOTION IS AS SHOWN



19.128

GIVEN:

HEAVY DAMPING, $C > C_c$

SHOW THAT:

A BODY RELEASED FROM AN ARBITRARY POSITION WITH AN ARBITRARY VELOCITY CANNOT PASS THROUGH ITS EQUILIBRIUM POSITION MORE THAN ONCE.

SUBSTITUTE THE INITIAL CONDITIONS, $t = 0,\ x = x_0,\ v = v_0$ IN EQS (1) AND (2) OF PROB. 19.127

$x_0 = C_1 + C_2 \quad v_0 = C_1 \lambda_1 + C_2 \lambda_2$

SOLVING FOR C_1 AND C_2, $\quad C_1 = -\frac{(v_0 - \lambda_2 x_0)}{\lambda_2 - \lambda_1},\ C_2 = \frac{(v_0 - \lambda_1 x_0)}{\lambda_2 - \lambda_1}$

AND SUBSTITUTING IN (1)

$x = \frac{1}{\lambda_2 - \lambda_1}\left[(v_0 - \lambda_1 x_0) e^{\lambda_2 t} - (v_0 - \lambda_2 x_0) e^{\lambda_1 t}\right]$

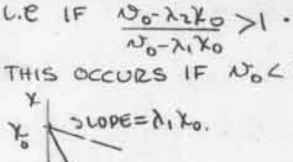
FOR $x = 0$, $t \neq \infty \quad (v_0 - \lambda_1 x_0) e^{\lambda_2 t} = (v_0 - \lambda_2 x_0) e^{\lambda_1 t}$

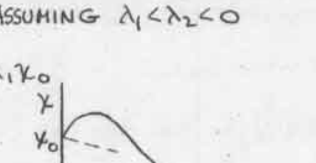
$e^{(\lambda_2 - \lambda_1)t} = \frac{(v_0 - \lambda_2 x_0)}{(v_0 - \lambda_1 x_0)}$

$t = \frac{1}{(\lambda_2 - \lambda_1)} \ln \frac{v_0 - \lambda_2 x_0}{v_0 - \lambda_1 x_0}$

THIS DEFINES ONE VALUE OF t ONLY FOR $x = 0$, WHICH WILL EXIST IF THE NATURAL LOG IS POSITIVE I.E IF $\frac{v_0 - \lambda_2 x_0}{v_0 - \lambda_1 x_0} > 1$. ASSUMING $\lambda_1 < \lambda_2 < 0$

THIS OCCURS IF $v_0 < \lambda_1 x_0$





19.129 GIVEN:

LIGHT DAMPING, $c \ll c_c$

SHOW THAT:

THE RATIO OF ANY TWO SUCCESSIVE MAXIMUM DISPLACEMENTS x_n AND x_{n+1} IN FIG. 19.11 IS A CONSTANT AND THAT THE NATURAL LOGARITHM OF THIS RATIO CALLED THE LOGARITHMIC DECREMENT IS,

$$\ln \frac{x_n}{x_{n+1}} = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$$

FOR LIGHT DAMPING, $c \ll c_c$

EQ (19.46) $x = x_0 e^{-(c/2m)t} \sin(\omega_D t+\phi)$

AT GIVEN MAX. DISPLACEMENT, $t = t_n$, $x = x_n$

$\sin(\omega_D t_n+\phi) = 1$, $x_n = x_0 e^{-(c/2m)t_n}$

AT NEXT MAX. DISPLACEMENT, $t = t_{n+1}$, $x = x_{n+1}$

$\sin(\omega_D t_{n+1}+\phi) = 1$ $\quad x_{n+1} = x_0 e^{-(c/2m)t_{n+1}}$

BUT $\omega_D t_{n+1} - \omega_D t_n = 2\pi$

$t_{n+1} - t_n = 2\pi/\omega_D$

RATIO OF SUCCESSIVE DISPLACEMENTS:

$$\frac{x_n}{x_{n+1}} = \frac{x_0 e^{-\frac{c}{2m}t_n}}{x_0 e^{-\frac{c}{2m}t_{n+1}}} = e^{-\frac{c}{2m}(t_n - t_{n+1})} = e^{+\frac{c}{2m}\frac{2\pi}{\omega_D}}$$

THUS $\ln \frac{x_n}{x_{n+1}} = \frac{c\pi}{m\omega_D}$ $\quad (1)$

FROM EQS. (19.45) $\omega_D = \omega_n\sqrt{1-\left(\frac{c}{c_c}\right)^2}$

AND (19.41) $\omega_D = \frac{c_c}{2m}\sqrt{1-\left(\frac{c}{c_c}\right)^2}$

THUS

$$\ln \frac{x_n}{x_{n+1}} = \frac{c\pi}{m}\frac{2m}{c_c}\frac{1}{\sqrt{1-\left(\frac{c}{c_c}\right)^2}}$$

$$\ln \frac{x_n}{x_{n+1}} = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}} \quad \text{(Q.E.D.)}$$

19.130 GIVEN:

LIGHT DAMPING $c/c_c < 1$

SHOW THAT:

SHOW THAT THE LOGARITHMIC DECREMENT CAN BE EXPRESSED AS $1/k \ln(x_n/x_{n+k})$, WHERE k IS THE NUMBER OF CYCLES BETWEEN READINGS OF THE MAXIMUM DISPLACEMENT

AS IN PROB. 19.129, FOR MAXIMUM DISPLACEMENTS x_n AND x_{n+k} AT t_n AND t_{n+k}, $\sin(\omega_D t_n+\phi) = 1$ AND $\sin(\omega_D t_{n+k}+\phi) = 1$.

$x_n = x_0 e^{-(c/2m)t_n}$ $\quad x_{n+k} = x_0 e^{-(c/2m)(t_{n+k})}$

RATIO OF MAXIMUM DISPLACEMENTS

$$x_n/x_{n+k} = \frac{x_0 e^{-(c/2m)t_n}}{x_0 e^{-(c/2m)t_{n+k}}} = e^{\left(-\frac{c}{2m}\right)(t_n - t_{n+k})}$$

BUT $\omega_D t_{n+k} - \omega_D t_n = k(2\pi)$ $\quad t_n - t_{n+k} = k\frac{2\pi}{\omega_D}$

THUS $\frac{x_n}{x_{n+k}} = e^{+\frac{c}{2m}\left(\frac{2k\pi}{\omega_D}\right)}$; $\ln\frac{x_n}{x_{n+k}} = k\frac{c\pi}{m\omega_D}$ $\quad (2)$

BUT FROM PROB. 19.129 EQ.(1)

$$\text{LOG. DECREMENT} = \ln\frac{x_n}{x_{n+1}} = \frac{c\pi}{m\omega_D}$$

COMPARING WITH EQ (2)

$$\text{LOG DECREMENT} = \frac{1}{k}\ln\frac{x_n}{x_{n+k}} \quad \text{Q.E.D.}$$

19.131 GIVEN:

LIGHT DAMPING, $c \ll c_c$

$\tau_D = 2\pi/\omega_D$

SHOW THAT:

(a) TIME BETWEEN A MAXIMUM POSITIVE DISPLACEMENT AND THE FOLLOWING MAX NEGATIVE DISPLACEMENT IS $\tau_D/2$

(b) TIME BETWEEN TWO SUCCESSIVE ZERO DISPLACEMENTS IS $\tau_D/2$

(c) TIME BETWEEN A MAXIMUM POSITIVE DISPLACEMENT AND THE FOLLOWING ZERO DISPLACEMENT IS GREATER THAN $\tau_D/4$

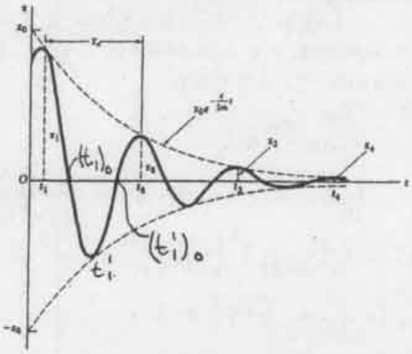


FIG. 19.11

EQ. (19.46)

$$x = x_0 e^{-(c/2m)t}\sin(\omega_D t+\phi)$$

(a) MAXIMA (POSITIVE OR NEGATIVE) WHEN $\dot{x} = 0$

$$\dot{x} = x_0(-c/2m)e^{-(c/2m)t}\sin(\omega_D t+\phi) + x_0\omega_D e^{-(c/2m)t}\cos(\omega_D t+\phi)$$

THUS ZERO VELOCITIES OCCUR AT TIMES WHEN $\dot{x} = 0$, OR $\tan(\omega_D t+\phi) = 2m\omega_D/c$ $\quad (1)$

THE TIME TO THE FIRST ZERO VELOCITY, t_1, IS

$$t_1 = [\tan^{-1}(2m\omega_D/c) - \phi]/\omega_D \quad (2)$$

THE TIME TO THE NEXT ZERO VELOCITY WHERE THE DISPLACEMENT IS NEGATIVE, IS

$$t_1' = [\tan^{-1}(2m\omega_D/c) - \phi + \pi]/\omega_D \quad (3)$$

SUBTRACTING (2) FROM (3)

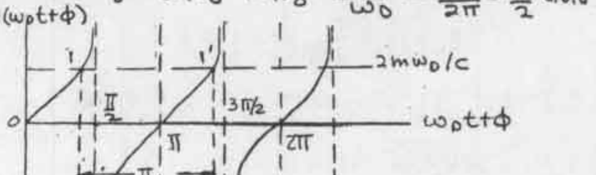
$$t_1' - t_1 = \pi/\omega_D = \frac{\pi \cdot \tau_D}{2\pi} = \tau_D/2 \quad \text{Q.E.D}$$

(b) ZERO DISPLACEMENTS OCCUR WHEN $\sin(\omega_D t+\phi) = 0$ OR AT INTERVALS OF

$\omega_D t + \phi = \pi, 2\pi \quad n\pi$

THUS, $(t_1)_0 = (\pi - \phi)/\omega_D$ AND $(t_1')_0 = (2\pi - \phi)/\omega_D$

TIME BETWEEN 0's $= (t_1')_0 - (t_1)_0 = \frac{2\pi - \pi}{\omega_D} = \frac{\pi\tau_D}{2\pi} = \frac{\tau_D}{2}$ Q.E.D



PLOT OF EQ.(1)

(c) THE FIRST MAXIMA OCCURS AT 1, $(\omega_D t_1+\phi)$

THE FIRST ZERO OCCURS AT $(\omega_D(t_1)_0+\phi) = \pi$

FROM THE ABOVE PLOT $(\omega_D(t_1)_0+\phi) - (\omega_D t_1+\phi) > \frac{\pi}{2}$

OR $(t_1)_0 - t_1 > \pi/2\omega_D$ $\qquad (t_1)_0 - t_1 > \tau_D/4$ Q.E.D

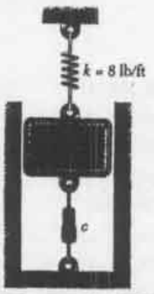
SIMILAR PROOFS CAN BE MADE FOR SUBSEQUENT MAX AND MIN

19.132

GIVEN:

BLOCK IN EQUILIBRIUM AS SHOWN IS DEPRESSED 1.2 IN. AND RELEASED

AFTER 10 CYCLES THE MAXIMUM DISPLACEMENT OF THE BLOCK IS 0.5 IN.



FIND:

(a) THE DAMPING FACTOR c/c_c

(b) THE VALUE OF THE COEFFICIENT OF VISCOUS DAMPING c

FROM PROB 19.130 AND 19.129

$$(1/k)\ln(x_n/x_{n+k}) = \frac{2\pi\, c/c_c}{\sqrt{1-(c/c_c)^2}}$$

WHERE k = NUMBER OF CYCLES = 10

(a) FIRST MAXIMA IS, $x_1 = 1.2$ in.

THUS, $n=1$ $\quad \frac{x_1}{x_{1+10}} = \frac{1.2}{0.5} = 2.4$

$$\frac{1}{10}\ln 2.4 = 0.08755 = \frac{2\pi\, c/c_c}{\sqrt{1-(c/c_c)^2}}$$

$$1-(c/c_c)^2 = \left(\frac{2\pi}{0.08755}\right)^2 (c/c_c)^2$$

$$\left(\frac{c}{c_c}\right)^2\left[\left(\frac{2\pi}{0.08755}\right)^2+1\right] = 1$$

$$\left(\frac{c}{c_c}\right)^2 = 1/(5150+1) = 0.0001941$$

$$c/c_c = 0.01393$$

(b) $c_c = 2m\sqrt{k/m}$ (EQ. 19.41)

OR $c_c = 2\sqrt{km}$

$c_c = 2\sqrt{(8\text{ lb/ft})(9\text{ lb}/32.2\text{ ft/s}^2)}$

$c_c = 2.991$ lb·s/ft

FROM (a) $\frac{c}{c_c} = 0.01393 \qquad c = (0.01393)(2.991)$

$c = 0.0417$ lb·s/ft

19.133

GIVEN:

SUCCESSIVE MAXIMUM DISPLACEMENTS OF A SPRING-MASS-DASHPOT SYSTEM ARE 25, 15, AND 9 mm

$m = 18$ kg, $k = 2100$ N/m

FIND:

(a) THE DAMPING FACTOR c/c_c

(b) THE COEFFICIENT OF VISCOUS DAMPING c.

(a) FROM PROB 19.29 $\quad \ln\frac{x_n}{x_{n+1}} = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$

FOR $x_n = 25$ mm AND $x_{n+1} = 15$ mm

$$\ln\frac{25}{15} = 0.5108 = \frac{2\pi(c/c_c)}{\sqrt{1-(c/c_c)^2}}$$

$$\left(\frac{c}{c_c}\right)^2\left[\left(\frac{2\pi}{(0.5108)}\right)^2+1\right] = 1$$

$$\left(\frac{c}{c_c}\right)^2 = \frac{1}{(151.3+1)} = 0.006566, \quad \frac{c}{c_c} = 0.0810$$

(b) $c_c = 2m\sqrt{k/m}$ (EQ. 19.41)

$$c_c = 2\sqrt{km} = 2\sqrt{(2100\text{ N/m})(18\text{ kg})} = 0.3888\ \frac{\text{N}\cdot\text{s}}{\text{m}}$$

FROM (a) $\quad \frac{c}{c_c} = 0.810$

$c = (0.810)(.3888) = 31.5$ N·s/m

19.134

GIVEN:

4-kg BLOCK A

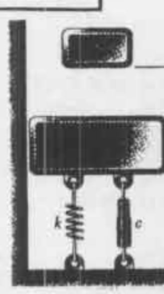
9-kg BLOCK B

$k = 1500$ N/m

$c = 230$ N·s/m

BLOCK A IS DROPPED FROM AN 800 mm HEIGHT ONTO B WHICH IS AT REST

NO REBOUND



FIND:

MAXIMUM DISTANCE BLOCKS MOVE AFTER IMPACT

VELOCITY OF BLOCK A JUST BEFORE IMPACT

$$v_A = \sqrt{2gh} = \sqrt{2(9.81)(0.8)} = 3.962 \text{ m/s}$$

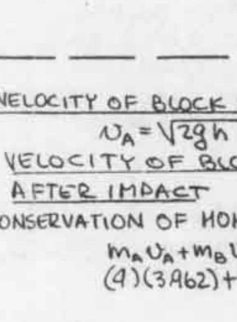
VELOCITY OF BLOCKS A AND B IMMEDIATELY AFTER IMPACT

CONSERVATION OF MOMENTUM

$$m_A v_A + m_B v_B = (m_A+m_B)v'$$

$$(4)(3.962)+0 = (4+9)v'$$

$$v' = 1.219 \text{ m/s}$$



IMPACT $\dot{x}_0 = +1.219$ m/s. $v = \dot{x}_0$

STATIC DEFLECTION (BLOCK A)

$$x_0 = -\frac{m_A g}{k} = -\frac{(4)(9.82)}{1500} = -0.02619 \text{ m}$$

$x = 0$, EQUIL. POSITION FOR BOTH BLOCKS

$c_c = 2\sqrt{km} = 2\sqrt{(1500)(13)} = 279.3$ N·s/m

SINCE $c < c_c$, EQUAT. (19.44)

$$x = e^{-(c/2m)t}[C_1\sin\omega_0 t + C_2\cos\omega_0 t]$$

$$c/2m = \frac{230}{(2)(13)} = 8.846 \text{ s}^{-1}$$

FROM TOP OF PAGE 1221 $\quad \omega_0^2 = \frac{k}{m} - \left(\frac{c}{2m}\right)^2$

$$\omega_0 = \sqrt{\frac{1500}{13} - \left(\frac{230}{(2)(13)}\right)^2} = 6.094 \text{ rad/s}$$

$$x = e^{-8.846t}(C_1\sin 6.094t + C_2\cos 6.094t)$$

INITIAL CONDITIONS $\quad x_0 = -0.02619$ m

$(t=0) \qquad \dot{x}_0 = +1.219$ m/s

$$x_0 = -0.02619 = e^0[C_1(0) + C_2(1)]$$

$$C_2 = -0.02619$$

$$\dot{x}(0) = -8.846e^{(-8.846)0}[C_1(0) + (-.02619)(1)] + e^{-(8.846)(0)}[6.094C_1(1) + C_2(0)] = 1.219$$

$$1.219 = (-8.846)(-0.02619) + 6.094C_1$$

$$C_1 = 0.16202$$

$$x = e^{-8.846t}(0.16202\sin 6.094t - 0.02619\cos 6.094t)$$

MAXIMUM DEFLECTION OCCURS WHEN $\dot{x} = 0$

$$\dot{x} = 0 = -8.846e^{-8.846t_m}(0.16202\sin 6.094t_m - 0.02619\cos 6.094t_m) + e^{-8.846t_m}[6.094][0.1620\cos 6.094t_m + 0.02619\sin 6.094t_m]$$

$$0 = [(-8.846)(.16202) + (6.094)(0.02619)]\sin 6.094t_m + [(-8.846)(-0.02619) + (6.094)(0.1620)]\cos 6.094t_m$$

(CONTINUED)

19.134 CONTINUED

$$0=-1.274\sin 6.094t+1.219\cos 6.094t$$

$$\tan 6.094t=\frac{1.219}{1.274}=0.957$$

TIME AT MAX DEFLECTION $= t_m=\frac{\tan^{-1}0.957}{6.094}=0.1253\text{ s}$

$$x_m=e^{-(8.846)(0.1253)}[0.1620\sin(6.094)(.1253)-0.02619\cos(6.094)(.1253)]$$

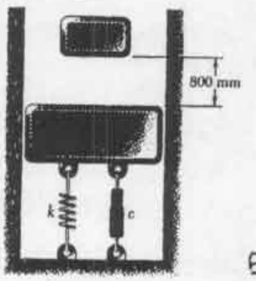
$$x_m=(0.3301)(0.1120-0.0189)=0.307\text{ m}$$

BLOCKS MOVE, STATIC DEFLECTION + x_m

TOTAL DISTANCE $=0.02619+0.307$

$=0.0569\text{ M}=56.9\text{ mm}$

19.135



GIVEN:

4 kg BLOCK A

9 kg BLOCK B

$k=1500$ N/m

$C=300$ N·s/m

BLOCK A IS DROPPED FROM AN 800 mm HEIGHT ONTO B WHICH IS AT REST

NO REBOUND

FIND:

MAXIMUM DISTANCE BLOCKS MOVE AFTER IMPACT

VELOCITY OF BLOCK A JUST BEFORE IMPACT

$$v_A=\sqrt{2gh}=\sqrt{2(9.81)(0.8)}=3.962\text{ m/s}$$

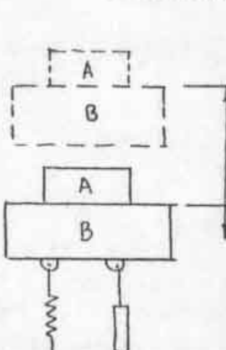
VELOCITY OF BLOCKS A AND B IMMEDIATELY AFTER IMPACT

CONSERVATION OF MOMENTUM

$$m_Av_A+m_Bv_B=(m_A+m_B)v'$$

$$(4)(3.962)+0=(4+9)v'$$

$$v'=1.219\text{ m/s}=\dot{x}_0$$



IMPACT $\dot{x}_0=+1.219$ m/s ↓

STATIC DEFLECTION (BLOCK A)

$$x_0=-\frac{m_Ag}{k}=-\frac{(4)(9.82)}{1500}=-0.02619\text{ m}$$

$x=0$, EQUIL. POSITION FOR BOTH BLOCKS

$$c_c=2\sqrt{km}=2\sqrt{(1500)(13)}$$

$$c_c=279.3\text{ N·s/m}$$

SINCE $C=300\text{ N·s/m}>c_c$

SYSTEM IS HEAVILY DAMPED AND (EQ. (19.42))

$$x=C_1e^{\lambda_1t}+C_2e^{\lambda_2t}$$

EQ. (19.40) $\lambda=-\frac{c}{2m}\pm\sqrt{\left(\frac{c}{2m}\right)^2-\frac{k}{m}}$

$$\lambda=-\frac{300}{26}\pm\sqrt{\left(\frac{300}{26}\right)^2-\frac{1500}{13}}$$

$$\lambda=-11.538\pm4.213$$

$$\lambda_1=-15.751\qquad\lambda_2=-7.325$$

$$x=C_1e^{-15.751t}+C_2e^{-7.325t}$$

19.135 CONTINUED

INITIAL CONDITIONS $x_0=-0.02619$ m, $\dot{x}_0=1.219$ m/s

$$x(0)=x_0=-0.02619=C_1e^0+C_2e^0$$

$$\dot{x}(0)=\dot{x}_0=1.219=(-15.751)C_1+(-7.325)C_2$$

SOLVING SIMULTANEOUSLY FOR C_1 AND C_2

$$C_1=-0.1219,\quad C_2=0.09571$$

$$x(t)=-0.1219e^{-15.75t}+0.09571e^{-7.325t}$$

MAXIMUM DEFLECTION WHEN $\dot{x}=0$

$$\dot{x}=0=(-.1219)(-15.75)e^{-15.75t_m}+(0.09571)(-7.325)e^{-7.325t_m}$$

$$0=1.920e^{-15.75t_m}-0.701e^{-7.325t_m}$$

$$\frac{1.920}{0.701}=e^{(-7.325+15.75)t_m}$$

$$2.739=e^{8.425t_m}$$

$$\frac{\ln 2.739}{8.425}=t_m$$

$$t_m=0.1196\text{ s}$$

$$x_m=(-0.1219)e^{-(15.75)(.1196)}+(0.09571)e^{-(7.325)(.1196)}$$

$$x_m=-0.01851+0.03986=0.02136\text{ m}$$

TOTAL DEFLECTION = STATIC DEFLECTION + x_m

TOTAL DEFLECTION $=0.02619+0.02136$

$=0.0475\text{ m}=47.5\text{ mm}$

19.136

GIVEN:

GUN BARREL WEIGHT = 1500 lb

RECUPERATOR CONSTANT $c=1100$ lb·s/ft

FIND:

(a) CONSTANT k FOR RECUPERATOR TO RETURN THE BARREL TO ITS FIRING POSITION IN THE SHORTEST TIME WITHOUT OSCILLATION

(b) THE TIME NEEDED FOR THE BARREL TO MOVE TWO THIRDS OF THE WAY FROM ITS MAXIMUM-RECOIL POSITION TO ITS FIRING POSITION

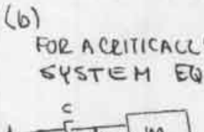
(a) A CRITICALLY DAMPED SYSTEM REGAINS ITS EQUILIBRIUM POSITION IN THE SHORTEST TIME

THUS $c=c_c=1100=2m\sqrt{\frac{k}{m}}=2\sqrt{km}$ EQ (19.41)

$$k=\frac{(c_c/2)^2}{m}=\frac{(1100/2\text{ lb·s/ft})^2}{(1500\text{ lb}/32.2\text{ ft/s}^2)}=6493.7$$

(b)

FOR A CRITICALLY DAMPED SYSTEM EQ. (19.43) $x=(C_1+C_2t)e^{-\omega_nt}$



WE TAKE $t=0$ AT MAXIMUM DEFLECTION. x_0

THUS $\dot{x}(0)=0$, $x(0)=x_0$

INITIAL CONDITIONS

$$x(0)=x_0=(C_1+0)e^0\quad C_1=x_0$$

$$x=(x_0+C_2t)e^{-\omega_nt}$$

$$\dot{x}=-\omega_n(x_0+C_2t)e^{-\omega_nt}+C_2e^{-\omega_nt}$$

$$\dot{x}(0)=0=-\omega_nx_0+C_2\quad C_2=\omega_nx_0$$

$$x=x_0(1+\omega_nt)e^{-\omega_nt}$$

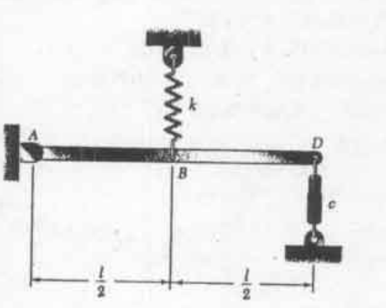
FOR $x=\frac{x_0}{3}$ $\quad 1/3=(1+\omega_nt)e^{-\omega_nt}\quad \omega_n=\sqrt{k/m}$

BY TRIAL $\omega_nt=2.289$ $\quad \omega_n=\sqrt{6490\text{ lb/ft}/(1500\text{ lb}/32.2\text{ ft/s}^2)}$

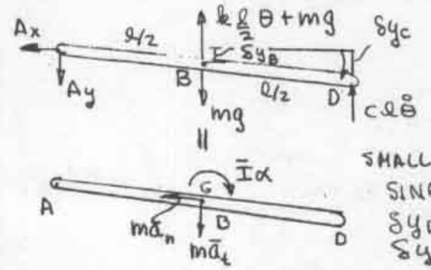
$\omega_n=11.806^{-1}\quad t=2.289/11.806=0.1939\text{ s}$

19.137

GIVEN:
ROD OF MASS m PINNED AT A



FIND:
IN TERMS OF $m, k,$ AND c
(a) DIFFERENTIAL EQUATION OF MOTION
(b) CRITICAL DAMPING COEFFICIENT c_c



IN EQUILIBRIUM THE FORCE IN THE SPRING IS mg

SMALL ANGLES
$\sin\theta \approx \theta \quad \cos\theta \approx 1$
$\delta y_B = \ell/2\,\theta$
$\delta y_C = \ell\theta$

(a) NEWTONS LAW $\Sigma M_A = (\Sigma M_A)_{eff}$

$$mg\ell/2 - (k\ell/2\,\theta + mg)\ell/2 - c\ell\dot{\theta}\ell = \bar{I}\alpha + m\bar{a}_t\,\ell/2$$

KINEMATICS

$$\alpha = \ddot{\theta} \quad \bar{a}_t = \ell/2\,\alpha = \ell/2\,\ddot{\theta}$$

$$[\bar{I} + m(\ell/2)^2]\ddot{\theta} + c\ell^2\dot{\theta} + k(\ell/2)^2\theta = 0$$

$$\bar{I} + m(\ell/2)^2 = \tfrac{1}{3}m\ell^2$$

$$\ddot{\theta} + (3c/m)\dot{\theta} + (3k/4m)\theta = 0$$

(b) SUBSTITUTING $\theta = e^{\lambda t}$ INTO THE DIFFERENTIAL EQUATION OBTAINED IN (a), WE OBTAIN THE CHARACTERISTIC EQUATION,

$$\lambda^2 + (3c/m)\lambda + 3k/4m = 0$$

AND OBTAIN THE ROOTS

$$\lambda = \frac{-3c/m \mp \sqrt{(3c/m)^2 - (3k/m)}}{2}$$

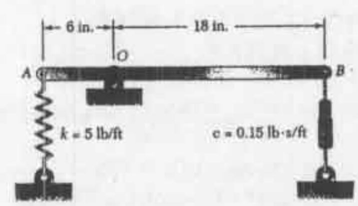
THE CRITICAL DAMPING COEFFICIENT c_c, IS THE VALUE OF c IN THE RADICAL TO ZERO. THUS

$$(3c_c/m)^2 = 3k/m$$

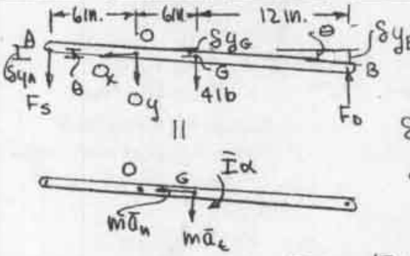
$$c_c = \sqrt{km/3}$$

19.138

GIVEN:
4-lb ROD AB PINNED AT O. AND SUPPORTED BY A SPRING AT A. DIMENSIONS AND OTHER CONSTANTS AS SHOWN



FIND:
FOR SMALL OSCILLATIONS
(a) THE DIFFERENTIAL EQUATION OF MOTION
(b) THE FORMED BY THE ROD WITH THE HORIZONTAL 5 S AFTER END B IS PUSHED DOWN 0.9 in. AND RELEASED



SMALL ANGLES
$\sin\theta \approx \theta$
$\cos\theta \approx 1$
$\delta y_A = (\tfrac{6}{12}\,ft)\theta = \theta/2$
$\delta y_G = (\tfrac{6}{12}\,ft)\theta = \theta/2$
$\delta y_B = (\tfrac{18}{12}\,ft)\theta = 3\theta/2$

(a) NEWTONS LAW $\Sigma M_O = (\Sigma M_O)_{eff}$

$$-(\tfrac{6}{12}\,ft)F_S + (\tfrac{6}{12}\,ft)(4) - (\tfrac{18}{12}\,ft)F_D = \bar{I}\alpha + (\tfrac{6}{12}\,ft)m\bar{a}_t \quad (1)$$

$$F_S = k(\delta y_A + (\delta_{ST})_A) = k(\tfrac{\theta}{2} + (\delta_{ST})_A)$$

$$F_D = c\,\delta\dot{y}_B = c\,3/2\,\dot{\theta}$$

$$\bar{I} = \tfrac{1}{12}m\ell^2 = \tfrac{1}{12}m(\tfrac{24}{12}\,ft)^2 = \tfrac{1}{3}m$$

KINEMATICS $\alpha = \ddot{\theta} \quad \bar{a}_t = (\tfrac{6}{12}\,ft)\alpha = \frac{\ddot{\theta}}{2}$

THUS FROM (1)

$$[\tfrac{m}{3} + \tfrac{m}{4}]\ddot{\theta} + (3/2)^2 c\dot{\theta} + (k/2)(\tfrac{\theta}{2} + (\delta_{ST})_A) - 2 = 0 \quad (2)$$

BUT IN EQUILIBRIUM $\Sigma M_O = 0$

$$k(\delta_{ST})_A(\tfrac{6}{12}) - (4)(\tfrac{6}{12}) = 0, \quad \tfrac{k}{2}(\delta_{ST})_A = 2$$

EQ (2) BECOMES

$$(7/12)m\ddot{\theta} + (9/4)c\dot{\theta} + (k/4)\theta = 0$$

$\tfrac{7}{12}m = (\tfrac{7}{12})(4/32.2) = 0.07246, \quad 9/4\,c = (9/4)(.15) = 0.3375$
$k/4 = 5/4 = 1.25$

$$0.07246\ddot{\theta} + 0.3375\dot{\theta} + 1.25\theta = 0$$

(b) SUBSTITUTING $e^{\lambda t}$ INTO THE ABOVE DIFFERENTIAL EQUATION

$$0.07246\lambda^2 + 0.3375\lambda + 1.25 = 0$$

$$\lambda = (-0.3375 \mp \sqrt{(.3375)^2 - 4(.07246)(1.25)})/2(0.07246)$$

$$\lambda = (-0.3375 \mp \sqrt{-0.2484})/(2)(0.07246)$$

$$\lambda = -2.329 \pm 3.439\,i$$

SINCE THE ROOTS ARE COMPLEX AND CONJUGATE (LIGHT DAMPING), THE SOLUTION TO THE DIFFERENTIAL EQUATION IS, (EQ. 19.46),

$$\theta = \theta_0 e^{-2.329t}\sin(3.439t + \phi) \quad (3)$$

(CONTINUED)

19.138 CONTINUED

INITIAL CONDITIONS $(\delta y_B(0)) = 0.9\text{ in.}$

$\theta(0) = (\delta y_B)/18\text{ in} = \frac{0.9}{18}$

$\theta(0) = 0.05\text{ rad}$

FROM (3) $\quad \dot{\theta}(0) = 0$

$\theta(0) = 0.05 = \theta_0 \sin\phi$

$\dot{\theta}(0) = 0 = -2.329\,\theta_0 \sin\phi + 3.439\,\theta_0 \cos\phi$

$\tan\phi = 3.439/2.329$

$\phi = 0.9755\text{ rad}$

$\theta_0 = \dfrac{0.05}{\sin(.9755)} = 0.06039\text{ rad}$

SUBSTITUTING INTO (3)

$\theta = 0.06039\, e^{-2.329t} \sin(3.439t + 0.9752)$

AT $t = 5\text{ s}$

$\theta(5) = 0.06039\, e^{(-2.329)(5)} \sin[(3.439)(5) + 0.9752]$

$\theta(5) = -0.333 \times 10^{-6}\text{ rad}$

$\theta(5) = (0.01909 \times 10^{-3})^\circ$ ABOVE HORIZONTAL

19.139 GIVEN:

1100-lb MACHINE SUPPORTED BY TWO SPRINGS EACH WITH $k = 3000\ \frac{\text{lb}}{\text{ft}}$

PERIODIC FORCE APPLIED OF 30-lb AT 2.8 Hz.

$C = 110\text{ lb/ft}$

FIND:

AMPLITUDE OF STEADY STATE VIBRATION

EQ. (19.52) $\quad x_m = \dfrac{P_m}{\sqrt{(k - m\omega_f^2)^2 + (c\omega_f)^2}}$

TOTAL SPRING CONSTANT $\quad k = (2)(3000\text{ lb/ft}) = 6000\text{ lb/ft}$

$\omega_f = 2\pi f_f = 2\pi(2.8) = 5.6\pi\text{ rad/s}$

$m = W/g = 1100\text{ lb}/(32.2\text{ ft/s}^2) = 34.161\text{ lb s}^2/\text{ft}$

$x_m = \dfrac{30\text{ lb}}{\sqrt{((6000 - (34.161)(5.6\pi)^2)^2 + (110(5.6\pi))^2)\left(\frac{\text{lb}}{\text{ft}}\right)^2}}$

$x_m = \dfrac{30}{\sqrt{20.914 \times 10^6 + 3.745 \times 10^6}}$

$x_m = 0.00604\text{ ft}$

$x_m = 0.0725\text{ in.}$

19.140 GIVEN:

1100-lb MACHINE SUPPORTED BY TWO SPRINGS

PERIODIC FORCE OF 30 lb APPLIED AT 2.8 Hz. $C = 110\text{ lb/ft}$

AMPLITUDE OF VIBRATION, $x_m = 0.05\text{ in.}$

FIND:

SPRING CONSTANT OF EACH SPRING

EQ. (19.52) $\quad x_m = \dfrac{P_m}{\sqrt{(k - m\omega_f^2)^2 + (c\omega_f)^2}}$

$[(k - m\omega_f^2)^2 + (c\omega_f)^2]\, x_m^2 = P_m^2$

$k = \sqrt{(P_m/x_m)^2 - (c\omega_f)^2} + m\omega_f^2$

$\omega_f = 2\pi f_f = 2\pi(2.8) = 5.6\pi \quad m = \frac{W}{g} = \frac{1100\text{ lb}}{32.2\text{ ft/s}^2} = 34.161\ \frac{\text{lb s}^2}{\text{ft}}$

$k = \sqrt{\left(\frac{30\text{ lb}}{(.05/12)\text{ ft}}\right)^2 - (110(5.6\pi))^2} + (34.161)(5.6\pi)^2$

$k = \sqrt{51.84 \times 10^6 - 3.745 \times 10^6} +$

$k = 6935 + 10573 = 17508\text{ lb/ft}$

$k/2 = 8750\text{ lb/ft}$

19.141 GIVEN:

FORCED VIBRATING SYSTEM

FIND:

VALUES OF c/c_c FOR WHICH THE MAGNIFICATION FACTOR WILL DECREASE AS ω_f/ω_n INCREASES

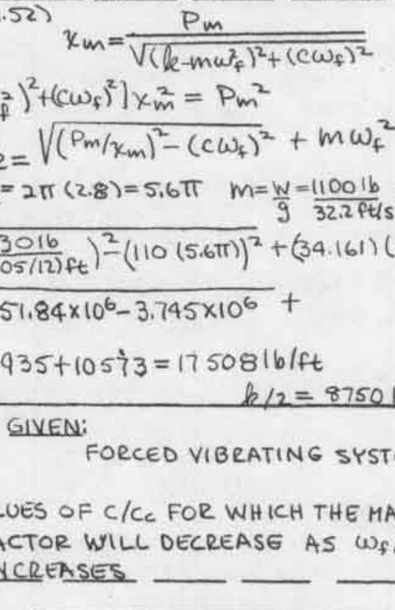


FIG. 19.12

EQ. (19.53)'

MAG. FACTOR $\dfrac{x_m}{P_m/k} = \dfrac{1}{\sqrt{[1 - (\omega_f/\omega_n)^2]^2 + [2(c/c_c)(\omega_f/\omega_n)]^2}}$

FIND VALUE OF c/c_c FOR WHICH THERE IS NO MAXIMUM FOR $\frac{x_m}{P_m/k}$ AS ω_f/ω_n INCREASES

$\dfrac{d\left(\frac{x_m}{P_m/k}\right)^2}{d(\omega_f/\omega_n)^2} = -\dfrac{[2(1 - (\omega_f/\omega_n)^2)(-1) + 4c^2/c_c^2]}{\{[1 - (\omega_f/\omega_n)^2]^2 + [2(c/c_c)(\omega_f/\omega_n)]^2\}^2} = 0$

$-2 + 2(\omega_f/\omega_n)^2 + 4c^2/c_c^2 = 0$

$(\omega_f/\omega_n)^2 = 1 - 2c^2/c_c^2$

FOR $c^2/c_c^2 \geq \frac{1}{2}$ THERE IS NO MAXIMUM FOR $\frac{x_m}{(P_m/k)}$ AND THE MAGNIFICATION FACTOR WILL DECREASE AS AS ω_f/ω_n INCREASES

$c/c_c \geq 1/\sqrt{2} \quad c/c_c \geq 0.707$

19.142 GIVEN:

FORCED VIBRATING SYSTEM
SMALL c/c_c

SHOW THAT:

MAXIMUM AMPLITUDE OCCURS WHEN $\omega_f \approx \omega_n$ AND THAT THE CORRESPONDING VALUE OF THE MAGNIFICATION FACTOR IS $\frac{1}{2}\, c/c_c$.

EQ. (19.53')

$$\text{MAG. FACTOR} = \frac{x_m}{P_m/k} = \frac{1}{\sqrt{[1-(\omega_f/\omega_n)^2]^2 + [2(c/c_c)(\omega_f/\omega_n)]^2}}$$

FIND VALUE OF ω_f/ω_n FOR WHICH $\frac{x_m}{P_m/k}$ IS A MAXIMUM

$$0 = \frac{d\left(\frac{x_m}{P_m/k}\right)^2}{d(\omega_f/\omega_n)^2} = -\frac{[2(1-(\omega_f/\omega_n)^2](-1) + 4(c^2/c_c^2)]}{\{[1-(\omega_f/\omega_n)^2]^2 + [2(c/c_c)(\omega_f/\omega_n)]^2\}^2}$$

$$-2 + 2(\omega_f/\omega_n)^2 + 4(c/c_c)^2 = 0$$

FOR SMALL c/c_c $\quad \omega_f/\omega_n \approx 1 \quad \omega_f \approx \omega_n$

$$\text{FOR } \omega_f/\omega_n = 1, \quad \frac{x_m}{P_m/k} = \frac{1}{\sqrt{[1-1]^2 + [2(c/c_c)1]^2}}$$

$$\frac{x_m}{(P_m/k)} = \frac{1}{2}\,\frac{c_c}{c}$$

19.143 GIVEN:

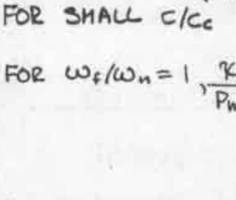
15-kg MOTOR SUPPORTED BY FOUR SPRINGS EACH OF CONSTANT
$k = 45$ kN/m
MOTOR UNBALANCE IS EQUIVALENT TO MASS OF 20 g AT 125 mm FROM AXIS OF ROTATION

FIND:

AMPLITUDE OF STEADY STATE VIBRATION AT A SPEED OF 1500 rpm ASSUMING,
(a) NO DAMPING
(b) DAMPING FACTOR $c/c_c = 1.3$

EQ. (19.52)

$$x_m = \frac{P_m}{\sqrt{(k - m\omega_f^2)^2 + (c\omega_f)^2}}$$



$$\omega_f^2 = [(1500)(2\pi)/60]^2 = 24674\ s^{-2}$$

$$k = (4)(4500) = 180000\ \frac{N}{m}$$

$$P_m = m' r \omega_f^2 = (0.02\,kg)(0.125\,m)(24674\,s^{-2})$$

$$P_m = 61.685\ N$$

(a) $c = 0$

$$x_m = \frac{61.685\ N}{[180000 - 15(24674)](N/m)}$$

$$x_m = -0.324 \times 10^{-3}\,m = -0.324\ mm$$

(b) FOR $c/c_c = 1.3$

EQ. (19.41) $\quad c_c = 2m\sqrt{\frac{k}{m}} = 2\sqrt{mk} = 2\sqrt{(15\,kg)(180000\,\frac{N}{m})}$

$c_c = 3286$ N·s/m $\qquad c = (1.3)(3286) = 4272\ \frac{N\cdot s}{m}$

$$x_m = \frac{61.685\ N}{\sqrt{[180000 - 15(24674)]^2 + (4272)^2(24674)}}$$

$$x_m = 0.0884 \times 10^{-3}\,m = 0.0884\ mm$$

19.144

GIVEN:

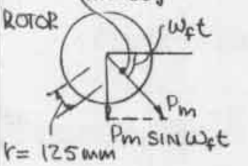
18-kg MOTOR BOLTED TO A BEAM HAS A STATIC DEFLECTION, $\delta_{ST} = 1.5$ mm
UNBALANCE IS EQUIVALENT TO A MASS OF 20 g LOCATED 125 mm FROM AXIS OF ROTATION

FIND:

AMPLITUDE AT A MOTOR SPEED OF 900 rpm
(a) FOR NO DAMPING
(b) FOR $c/c_c = 0.055$

EQ. (19.52)

$$x_m = \frac{P_m}{\sqrt{(k - m\omega_f^2)^2 + (c\omega_f)^2}}$$



$$\omega_f^2 = [(900)(2\pi)/60]^2 = 8882.6\ s^{-2}$$

FIND SPRING CONSTANT k FOR THE BEAM

$$k = \frac{mg}{\delta_{ST}} = \frac{(18\,kg)(9.81\,\frac{m}{s^2})}{(1.5 \times 10^{-3}\,m)}$$

$$k = 117720\ N/m$$

$$P_m = m' r \omega_f^2 = (0.020\,kg)(0.125\,m)(8882.6\,s^{-2})$$

$$P_m = 22.20\ N$$

(a) $c = 0$

$$x_m = \frac{22.20\,N}{[(117720 - (18)(8882.6\ N/m)}$$

$$x_m = -0.527 \times 10^{-3}\,m = -0.527\ mm$$

(b) FOR $c/c_c = 0.055$

EQ. (19.41) $\quad c_c = 2\sqrt{\frac{k}{m}} = 2\sqrt{km} = 2\sqrt{(117720)(18)}$

$c_c = 2911$ N·s/m

$\frac{c}{c_c} = 0.055 \qquad c = (0.055)(2911) = 160.12$ N/m

$$x_m = \frac{22.21}{\sqrt{[117720 - (18)(8883)]^2 + (160.12)^2(8883)]}}$$

$$x_m = \frac{22.21}{\sqrt{(1.779 \times 10^9) + (0.2278 \times 10^9)}} = 0.000496\,m$$

$$x_m = 0.496\ mm$$

19.145

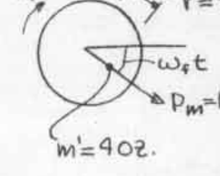
GIVEN:

100-lb MOTOR BOLTED TO BEAM WHICH HAS A STATIC DEFLECTION $\delta_{ST} = 0.25$ in.
UNBALANCE IS 4 OZ. AT 3 in.
AMPLITUDE $x_m = 0.010$ in AT 300 rpm

FIND:
(a) DAMPING FACTOR c/c_c
(b) COEFFICIENT OF DAMPING c

EQ. (19.53')

$$x_m = \frac{P_m/k}{\sqrt{(1-(\omega_f/\omega_n)^2)^2 + (2(c/c_c)(\omega_f/\omega_n))^2}}$$



$\omega_n^2 = \dfrac{g}{\delta_{ST}} = \dfrac{32.2 \text{ ft/s}^2}{(0.25/12 \text{ ft})}$

$\omega_n^2 = 1546$ rad/s

$\omega_f^2 = [(300)(\pi/30)]^2 = 987.2 \text{ s}^{-2}$

$\left(\dfrac{\omega_f}{\omega_n}\right)^2 = \dfrac{987.2}{1546} = 0.6387 \text{ s}^{-2}$

$P_m = m' r \omega_f^2 = [(\tfrac{4}{16} \text{ lb})/(32.2 \text{ ft/s}^2)](\tfrac{3}{12} \text{ ft})(987.2 \text{ s}^{-2})$

$P_m = 1.916$ lb

$k = \omega_n^2 m = (1546)(100/32.2) = 4801$ lb/ft

$P_m/k = 1.916/4801 = 0.0003991$ ft

$$\frac{0.01}{12} = \frac{0.0003991}{\sqrt{(1-.6387)^2 + (4)(0.6387)(c/c_c)^2}}$$

$0.2293 = 0.1305 + 2.555\,(c/c_c)^2$

$(c/c_c)^2 = \dfrac{0.0988}{2.555} = 0.03867$

$c/c_c = 0.1966$

(b) EQ. (19.41) $c_c = 2m\omega_n$

$c_c = 2\left(\dfrac{100 \text{ lb}}{32.2 \text{ ft/s}^2}\right)(1546)^{1/2}$

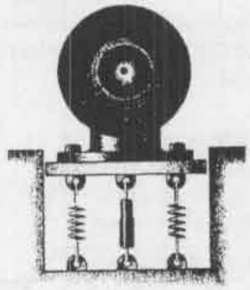
$c_c = 244.2$ lb·s/ft

$\dfrac{c}{c_c} = 0.1957$

$c = (244.2)(.1967) = 48.0 \ \dfrac{\text{lb s}}{\text{ft}}$

19.146

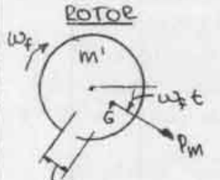
GIVEN:



100-kg MOTOR SUPPORTED BY FOUR SPRINGS EACH OF CONSTANT $k = 90$ kN/m
DASHPOT $c = 6500$ N·s/m
AMPLITUDE $x_m = 2.1$ mm AT A SPEED OF 1200 rpm
MASS OF THE ROTOR $m' = 15$ kg

FIND:
DISTANCE BETWEEN THE MASS CENTER OF THE ROTOR AND THE AXIS OF SHAFT

ROTOR



$P_m = m' e \omega_f^2$

EQ. (19.52)

$$x_m = \frac{P_m}{\sqrt{(k - m\omega_f^2)^2 + (c\omega_f)^2}}$$

$\omega_f^2 = [(1200)(2\pi)/60]^2$

$\omega_f^2 = 15791 \text{ s}^{-2}$

$k = 4(90{,}000 \text{ N/m}) = 360{,}000$ N/m

$P_m = (15 \text{ kg})(e)(15791 \text{ s}^{-2})$

$P_m = 236870\, e$

$$0.0021 = \frac{236870\,e}{\sqrt{\{[360{,}000 - (100)(15791)]^2 + (6500)^2(15791)\}}}$$

$(1.4674 \times 10^6)(0.0021) = (236870)\,e$

$e = 0.1301$ m

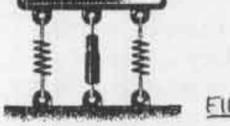
$e = 13.01$ mm

19.147

GIVEN:

TWO 400-g MASSES AT r = 150 mm ROTATE AT THE SAME SPEED OF 1200 rpm IN OPPOSITE SENSES WHEN THE MASSES ARE EXACTLY BENEATH THEIR RESPECTIVE ROTATION AXES
AMPLITUDE OF THE MOTION AT THIS SPEED EQUALS 15 mm
TOTAL MASS = 140 kg

FIND:
(a) THE COMBINED SPRING CONSTANT k
(b) THE DAMPING FACTOR c/c_c



(a) $\phi = \pi/2$, AT 1200 rpm

EQ. (19.54) $\tan\phi = \dfrac{2(c/c_c)(\omega_f/\omega_n)}{1-(\omega_f/\omega_n)^2}$

SINCE $\phi = \pi/2$ $\tan\phi = \infty$

THUS $1-(\omega_f/\omega_n)^2 = 0$

$\omega_n = \omega_f = (1200)(2\pi)/60 = 40\pi \text{ s}^{-1}$

$\omega_n^2 = \dfrac{k}{M}$ $\quad k = M\omega_n^2 = (140 \text{ kg})(40\pi \text{ s}^{-1})^2 = 2210 \ \dfrac{\text{kN}}{\text{m}}$

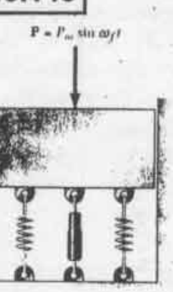
(b)
EQ. (19.53), $\dfrac{x_m}{P_m/k} = \dfrac{1}{\sqrt{(1-\omega_f^2/\omega_n^2)^2 + (2(c/c_c)(\omega_f/\omega_n))^2}}$

$\omega_f/\omega_n = 1$ $\quad P_m = 2mr\omega_f^2 = (2)(0.4 \text{ kg})(0.150 \text{ m})(40\pi \text{ s}^{-1})^2$

$P_m = 1895 \frac{\text{N}}{\text{m}}$, $\quad (0.015 \text{ m})/[(1895 \text{ N/m})/(2210 \times 10^3)] = 1/[2(c/c_c)]$

$c/c_c = 0.0286 \ \frac{\text{N·s}}{\text{m}}$

19.148 GIVEN:



MACHINE SUPPORTED BY SPRINGS AND CONNECTED TO A DASHPOT AS SHOWN

SHOW THAT:

THE AMPLITUDE OF THE FLUCTUATING FORCE TRANSMITTED TO THE FOUNDATION IS,

$$F_m = P_m \sqrt{\frac{1+[2(c/c_c)(\omega_f/\omega_n)]^2}{[1-(\omega_f/\omega_n)^2]^2+[2(c/c_c)(\omega_f/\omega_n)]^2}}$$

FROM EQ. (19.48), THE MOTION OF THE MACHINE IS,

$$x = x_m \sin(\omega_f t - \varphi)$$

THE TRANSMITTED TO THE FOUNDATION IS,

SPRINGS $F_S = kx = k x_m \sin(\omega_f t - \varphi)$

DASHPOT $F_D = c\dot{x} = c x_m \omega_f \cos(\omega_f t - \phi)$

$$F_T = x_m[k \sin(\omega_f - \phi) + c\omega_f \cos(\omega_f t - \phi)]$$

OR RECALLING THE IDENTITY,

$$A \sin y + B \cos y = \sqrt{A^2+B^2}\,\sin(y+\psi)$$
$$\sin\psi = B/\sqrt{A^2+B^2}$$
$$\cos\psi = A/\sqrt{A^2+B}$$

$$F_T = [x_m\sqrt{k^2+(c\omega_f)^2}]\sin(\omega_f t - \varphi + \psi)$$

THUS THE AMPLITUDE OF F_T IS

$$F_m = x_m\sqrt{k^2+(c\omega_f)^2} \quad (1)$$

FROM EQ. (19.53) $\quad x_m = \dfrac{P_m/k}{\sqrt{[(1-(\omega_f/\omega_n)^2]^2+[2(c/c_c)\omega_f/\omega_n]^2}}$

SUBSTITUTING FOR x_m IN EQ (1)

$$F_m = \frac{P_m\sqrt{1+(c\omega_f/k)^2}}{\sqrt{[1-(\omega_f/\omega_n)^2]^2+[2(c/c_c)(\omega_f/\omega_n^2)]}} \quad (2)$$

$\omega_n^2 = k/m$

AND EQ. (19.41) $c_c = 2m\omega_n \quad m = c\omega_n/2$

$$c\omega_f/k = c\omega_f/m\omega_n^2 = 2(c/c_c)(\omega_f/\omega_n)$$

SUBSTITUTING IN (2)

$$F_m = \frac{P_m\sqrt{1+[2(c/c_c)(\omega_f/\omega_n)]^2}}{\sqrt{[1-(\omega_f/\omega_n)^2]^2+[2(c/c_c)(\omega_f/\omega_n)]^2}} \quad \text{Q.E.D.}$$

19.149 GIVEN:

SYSTEM AS SHOWN ABOVE IN PROB 19.148 WITH WEIGHT $W = 200$ lb, FOUR SPRINGS, EACH WITH $k = 12$ lb/ft, AND APPLIED PERIODIC FORCE WITH FREQUENCY $f_f = \omega_f/2\pi = 0.8$ Hz AND AMPLITUDE $P_m = 20$ lb

FIND:

AMPLITUDE OF FORCE F_m TRANSMITTED TO FOUNDATION (a) IF $c = 25$ lb·s/f (b) $c = 0$,

REFER TO THE EQUATION DERIVED IN PROB 19.148

$\omega_f = 2\pi f_f = 2\pi(0.8) = 1.6\pi \qquad \omega_n^2 = k/m = 48/(200/32.2)$

$\omega_n^2 = 7.728\ s^{-2},\ \omega_n = 2.780\ s^{-1}$

$\left(\dfrac{\omega_f}{\omega_n}\right)^2 = \dfrac{(1.6\pi)^2}{(2.780)^2} = 3.269,\quad \dfrac{c}{c_c} = \dfrac{c}{2m\omega_n} = \dfrac{25}{(2)(200/32.2)(2.780)}$

$\dfrac{c}{c_c} = 0.7239,\ \left(\dfrac{c}{c_c}\right)^2 = 0.52406$

(a) $F_m = \dfrac{20\sqrt{1+[(4)(0.52406)(3.269)]}}{\sqrt{(1-3.269)^2+4(0.52406)(3.269)}} = \dfrac{20\sqrt{1+6.8526}}{\sqrt{5.148+6.8526}}$

$F_m = 16.18$ lb

(b) $c = 0$, $F_m = (20)/\sqrt{5.148} = 8.81$ lb

***19.150** GIVEN:

STEADY STATE VIBRATION UNDER A HARMONIC FORCE

SHOW THAT:

MECHANICAL ENERGY DISSIPATED PER CYCLE IS $E = \pi c x_m^2 \omega_f$

ENERGY IS DISSIPATED BY THE DASHPOT

FROM EQ (19.48) THE DEFLECTION OF THE SYSTEM IS $\quad x = x_m \sin(\omega_f t - \varphi)$

THE FORCE ON THE DASHPOT, $F_D = c\dot{x}$

$$F_D = c x_m \omega_f \cos(\omega_f t - \varphi)$$

THE WORK DONE IN A COMPLETE CYCLE WITH $\tau_f = 2\pi/\omega_f$

$$E = \int_0^{2\pi/\omega_f} F_d\,dx \quad \text{(i.e. FORCE X DISTANCE)}$$

$$dx = x_m\omega_f \cos(\omega_f t - \varphi)\,dt$$

$$E = \int_0^{2\pi/\omega_f} c x_m^2 \omega_f^2 \cos^2(\omega_f t - \varphi)\,dt$$

$$\cos^2(\omega_0 t - \varphi) = [1 - 2\cos(\omega_f t - \phi)]/2$$

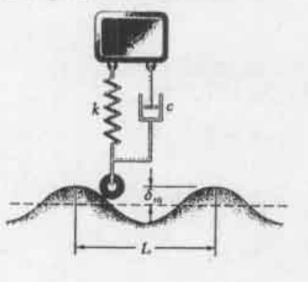
$$E = c x_m^2 \omega_f^2 \int_0^{2\pi/\omega_f} \frac{1 - 2\cos(\omega_f t - \phi)}{2}\,dt$$

$$E = \frac{c x_m^2 \omega_f^2}{2}\left[t - 2\frac{\sin(\omega_f t - \phi)}{\omega_f}\right]_0^{2\pi/\omega_f}$$

$$E = \frac{c x_m^2 \omega_f^2}{2}\left[\frac{2\pi}{\omega_f} - \frac{2}{\omega_f}\left(\cancel{\sin(2\pi - \phi) - \sin\phi}\right)\right] \to 0$$

$$E = \pi c x_m^2 \omega_f \quad \text{Q.E.D.}$$

***19.151** GIVEN:



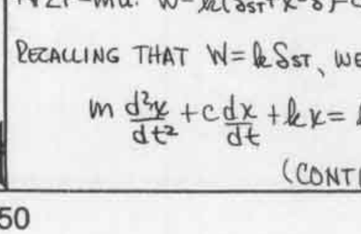
SPRING-DASHPOT SYSTEM AS SHOWN WITH MASS m MOVING AT v, OVER A ROAD WITH A SINUSOIDAL CROSS SECTION OF AMPLITUDE δ_m AND WAVELENGTH L.

FIND:

(a) DIFFERENTIAL EQUATION OF VERTICAL DISPLACEMENT OF MASS m

(b) EXPRESSION FOR THE AMPLITUDE OF m

(a)



$$+\downarrow \Sigma F = ma: \quad W - k(\delta_{ST} + x - \delta) - c\left(\frac{dx}{dt} - \frac{d\delta}{dt}\right) = m\frac{d^2x}{dt^2}$$

RECALLING THAT $W = k\delta_{ST}$, WE WRITE

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = k\delta + c\frac{d\delta}{dt} \quad (1)$$

(CONTINUED)

•19.151 CONTINUED

MOTION OF WHEEL IS A SINE CURVE, $\delta = \delta_m \sin \omega_f t$

THE INTERVAL OF TIME NEEDED TO TRAVEL A DISTANCE L AT A SPEED v, IS $t = L/v$, WHICH IS THE PERIOD OF THE ROAD SURFACE.

THUS $\omega_f = 2\pi/\tau_f = \dfrac{2\pi}{L/v} = 2\pi v/L$

AND $\delta = \delta_m \sin \omega_f t \qquad \dfrac{d\delta}{dt} = \delta_m \dfrac{2\pi}{L/v} \cos \omega_f t$

THUS EQ. (1) IS,

$$m\frac{d^2x}{dt} + c\frac{dx}{dt} + kx = (k \sin \omega_f t + c\omega_f \cos \omega_f t)\delta_m$$

(b) FROM THE IDENTITY

$$A \sin y + B \cos y = \sqrt{A^2+B^2}\,\sin(y+\psi)$$

$$\sin \psi = B/\sqrt{A^2+B^2}$$

$$\cos \phi = A/\sqrt{A^2+B^2}$$

WE CAN WRITE THE DIFFERENTIAL EQUATION

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = \delta_m \sqrt{k^2+(c\omega_f)^2}\,\sin(\omega_f t + \psi)$$

$$\psi = \tan^{-1}\frac{c\omega_f}{k}$$

THE SOLUTION TO THIS EQUATION IS (ANALOGOUS TO EQ'S 19.47 AND 19.48, WITH $P_m = \delta_m \sqrt{k^2+(c\omega_f)^2}$)

$$x = x_m \sin(\omega_f t - \phi + \psi)$$ (WHERE ANALOGOUS TO EQ'S (19.52))

$$x_m = \frac{\delta\sqrt{k^2+(c\omega_f)^2}}{\sqrt{(k-m\omega_f^2)^2+(c\omega_f)^2}}$$

$$\tan \phi = \frac{c\omega_f}{k - m\omega_f^2}$$

$$\tan \psi = c\omega_f/k$$

19.152

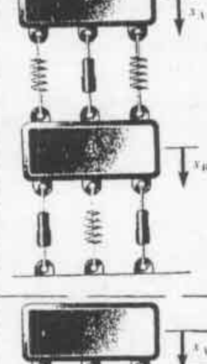
GIVEN:

BLOCKS A AND B HAVE MASS m

THREE SPRINGS, EACH HAVE CONSTANT k

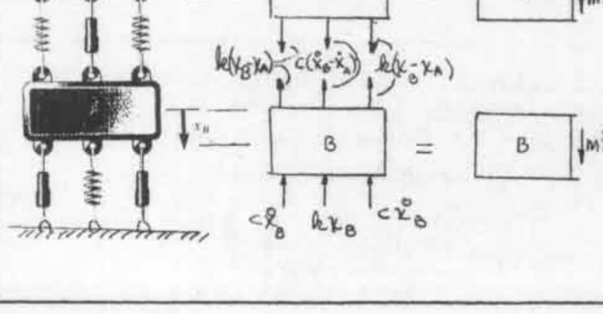
THREE DASHPOTS, EACH HAVE CONSTANT c.

BLOCK A ACTED UPON BY A FORCE $P = P_m \sin \omega_f t$



FIND:

DIFFERENTIAL EQUATIONS DEFINING THE DISPLACEMENTS x_A AND x_B OF THE BLOCKS FROM THEIR EQUILIBRIUM POSITION.



• 19.152 CONTINUED

SINCE THE ORIGINS OF COORDINATE ARE CHOSEN FROM THE EQUILIBRIUM POSITION, WE MAY OMIT THE INITIAL SPRING COMPRESSIONS AND THE EFFECT OF GRAVITY

FOR LOAD A

$$+\downarrow \Sigma F = ma_A;\ P_m \sin\omega_f t + 2k(x_B - x_A) + c(\dot{x}_B - \dot{x}_A) = m\ddot{x}_A \qquad (1)$$

FOR LOAD B

$$+\downarrow \Sigma F = ma_B;\ -2k(x_B - x_A) - c(\dot{x}_B - \dot{x}_A) - kx_B - 2c\dot{x}_B = m\ddot{x}_B \qquad (2)$$

REARRANGING EQS (1) AND (2), WE FIND:

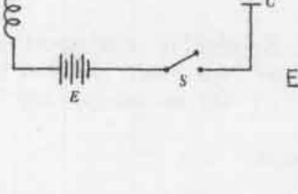
$$m\ddot{x}_A + c(\dot{x}_A - \dot{x}_B) + 2k(x_A - x_B) = P_m \sin \omega_f t$$

$$m\ddot{x}_B + 3c\dot{x}_B - c\dot{x}_A + 3kx_B - 2kx_A = 0$$

19.153

GIVEN:

R, L, C CIRCUIT AS SHOWN WITH SUDDENLY APPLIED VOLTAGE E WHEN THE SWITCH IS CLOSED



FIND:

VALUES OF R FOR WHICH OSCILLATIONS WILL TAKE PLACE WHEN THE SWITCH S IS CLOSED

FOR A MECHANICAL SYSTEM OSCILLATIONS TAKE PLACE IF $c < c_c$. (LIGHTLY DAMPED)

BUT FROM EQ. (19.41),

$$c_c = 2m\sqrt{k/m} = 2\sqrt{km}$$

THEREFORE

$$c < 2\sqrt{km} \qquad (1)$$

FROM TABLE 19.2:

$$c \to R$$
$$m \to L$$
$$k \to 1/C \qquad (2)$$

SUBSTITUTING IN (1) THE ANALOGOUS ELECTRICAL VALUES IN (2), WE FIND THAT OSCILLATIONS WILL TAKE PLACE IF,

$$R < 2\sqrt{(1/C)(L)}$$

$$R < 2\sqrt{L/C}$$

19.154

GIVEN:

R, L, C CIRCUIT OF FIG. PROB 19.153 WITH CAPACITOR C REMOVED

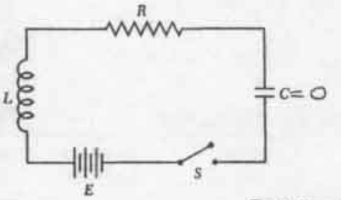
FIND;

IF SWITCH S IS CLOSED AT $t=0$

(a) THE FINAL VALUE OF THE CURRENT IN THE CIRCUIT

(b) THE TIME t AT WHICH THE CURRENT WILL HAVE REACHED $(1-1/e)$ TIMES IS FINAL VALUE. (I.E THE TIME CONSTANT)

ELECTRICAL SYSTEM



MECHANICAL SYSTEM

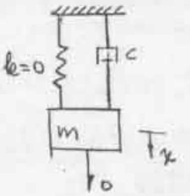
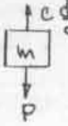


TABLE 19.2 FOR ANALOGUE

CLOSING SWITCH S IS EQUIVALENT TO SUDDENLY APPLYING A CONSTANT FORCE OF MAGNITUDE P TO THE MASS.

(a) FINAL VALUE OF THE CURRENT CORRESPONDS TO THE FINAL VELOCITY OF THE MASS. SINCE THE CAPACITANCE IS ZERO THE SPRING CONSTANT IS ALSO ZERO



$+\downarrow \Sigma F = ma$

$$P - c\frac{dx}{dt} = m\frac{d^2x}{dt^2} \qquad (1)$$

FINAL VELOCITY OCCURS WHEN

$$\frac{d^2x}{dt^2} = 0$$

$$P - c\left.\frac{dx}{dt}\right|_{FINAL} = 0 \qquad \left.\frac{dx}{dt}\right|_{FINAL} = v_{FINAL}$$

$$v_{FINAL} = P/c$$

FROM TABLE 19.2: $v \to i$, $P \to E$, $c \to R$

THUS

$$i_{FINAL} = E/R$$

(b) REARRANGING EQ. (1), WE HAVE

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} = P$$

SUBSTITUTE $\frac{dx}{dt} = Ae^{-\lambda t} + \frac{P}{c}$; $\frac{d^2x}{dt^2} = -A\lambda e^{-\lambda t}$

$$m[-A\lambda e^{-\lambda t}] + c[Ae^{-\lambda t} + \frac{P}{c}] = P$$

$$-m\lambda + c = 0 \qquad \lambda = c/m$$

THUS

$$\frac{dx}{dt} = Ae^{-(c/m)t} + \frac{P}{c}$$

AT $t=0$ $\frac{dx}{dt} = 0$ $\quad 0 = A + P/c \quad A = -P/c$

$$v = \frac{dx}{dt} = \frac{P}{c}\left[1 - e^{-(c/m)t}\right]$$

FROM TABLE 19.2: $v \to i$, $P \to E$, $c \to R$, $m \to L$

$$i = \frac{E}{R}\left[1 - e^{-(R/L)t}\right]$$

FOR $i = (E/R)(1 - 1/e)$, $(R/L)t = 1$

$$t = \frac{L}{R}$$

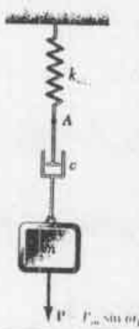
19.155

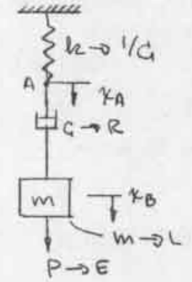
GIVEN:

MECHANICAL SYSTEM SHOWN.

DRAW:

THE ELECTRICAL ANALOGUE CIRCUIT

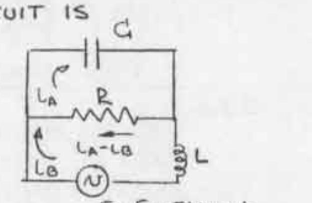




WE NOTE THAT BOTH THE SPRING AND THE DASHPOT EFFECT THE MOTION OF POINT A. THUS ONE LOOP IN THE ELECTRICAL CIRCUIT SHOULD CONSIST OF A CAPACITOR ($k \to 1/C$) AND A RESISTANCE ($c \to R$)

THE OTHER LOOP CONSISTS OF ($P_m \sin\omega_f t \to E_m \sin\omega_f t$), AN INDUCTOR ($m \to L$) AND THE RESISTOR ($c \to R$)

SINCE THE RESISTOR IS COMMON TO BOTH LOOPS, THE CIRCUIT IS



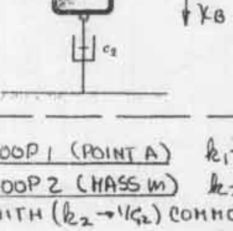
19.156

GIVEN:

MECHANICAL SYSTEM SHOWN

FIND:

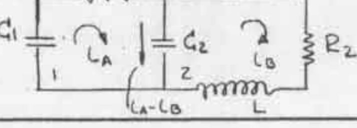
THE ELECTRICAL ANALOGUE CIRCUIT



LOOP 1 (POINT A) $k_1 \to \frac{1}{C_1}$, $k_2 \to \frac{1}{C_2}$, $c_1 \to R_1$

LOOP 2 (MASS m) $k_2 \to \frac{1}{C_2}$, $m \to L$, $c_2 \to R_2$

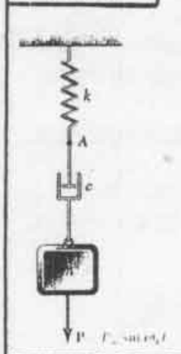
WITH ($k_2 \to 1/C_2$) COMMON TO BOTH LOOPS,



19.157

GIVEN:

MECHANICAL SYSTEM SHOWN

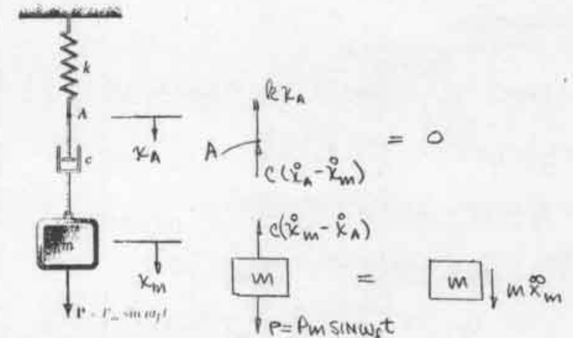


FIND:

THE DIFFERENTIAL EQUATIONS DEFINING

(a) THE DISPLACEMENTS OF MASS m AND OF THE POINT A

(b) THE CHARGES ON THE CAPACITORS OF THE ELECTRICAL ANALOGUE



(a) MECHANICAL SYSTEM

POINT A

+↑ $\Sigma F = 0$ $\quad kx_A + c\dfrac{d}{dt}(x_A - x_m) = 0$

MASS m

+↑ $\Sigma F = ma$ $\quad c\dfrac{d}{dt}(x_m - x_A) - P_m \sin\omega_f t = -m\dfrac{d^2x_m}{dt^2}$

$$m\frac{d^2x_m}{dt^2} + c\frac{d}{dt}(x_m - x_A) = P_m \sin\omega_f t$$

(b) ELECTRICAL ANALOGUE

FROM TABLE 19.2

$m \to L$
$c \to R$
$k \to 1/C$
$x \to q$
$P \to E$

SUBSTITUTING INTO THE RESULTS FROM PART (a), THE ANALOGOUS ELECTRICAL CHARECTIRISTICS,

$$(1/C)q_A + R\frac{d}{dt}(q_A - q_m) = 0$$

$$L\frac{d^2q_m}{dt^2} + R\frac{d}{dt}(q_m - q_A) = E_m \sin\omega_f t$$

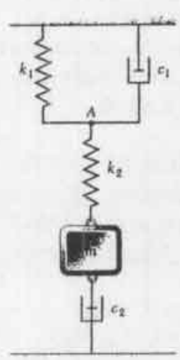
NOTE:

THESE EQUATIONS CAN ALSO BE OBTAINED BY SUMMING THE VOLTAGE DROPS AROUND THE LOOPS IN THE CIRCUIT OF PROB 19.155

19.158

GIVEN:

MECHANICAL SYSTEM SHOWN

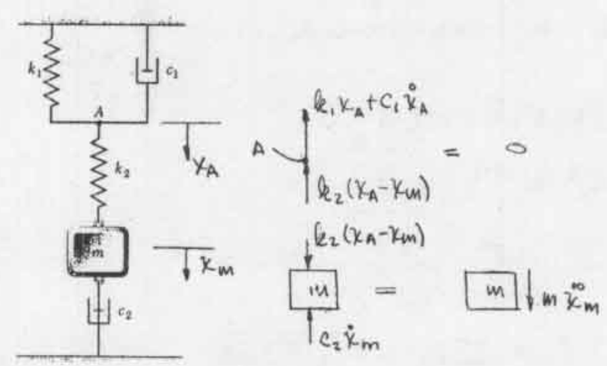


FIND:

THE DIFFERENTIAL EQUATIONS DEFINING

(a) THE DISPLACEMENTS OF MASS m AND OF THE POINT A

(b) THE CHARGES ON THE CAPACITORS OF THE ELECTRICAL ANALOGUE



(a) MECHANICAL SYSTEM

+↑ $\Sigma F = 0$

POINT A $\quad k_1x_A + c_1\dfrac{dx_A}{dt} + k_2(x_A - x_m) = 0$

$$c_1\frac{dx_A}{dt} + (k_1 + k_2)x_A - k_2x_m = 0$$

MASS m

$\Sigma F = ma$

+↓ $\quad k_2(x_A - x_m) - c_2\dfrac{dx_m}{dt} = m\dfrac{d^2x_m}{dt^2}$

$$m\frac{d^2x_m}{dt^2} + c_2\frac{dx_m}{dt} + k_2(x_m - x_A) = 0$$

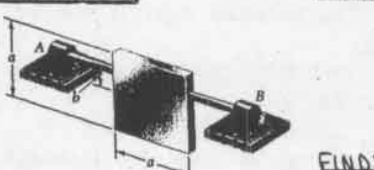
(b) ELECTRICAL ANALOGUE

SUBSTITUTING INTO THE RESULTS FROM PART (a) USING THE ANALOGOUS ELECTRICAL CHARACTERISTICS FROM TABLE 19.2 (SEE LEFT),

$$R_1\frac{dq_A}{dt} + \left(\frac{1}{C_1} + \frac{1}{C_2}\right)q_A - \frac{1}{C_2}q_m = 0$$

$$L\frac{d^2q_m}{dt^2} + R_2\frac{dq_m}{dt} + \frac{1}{C_2}(q_m - q_A) = 0$$

19.159



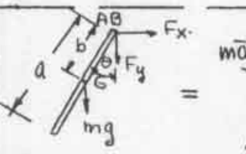
GIVEN:

THIN SQUARE PLATE OF SIDE
OSCILLATIONS ABOUT AB AT A DISTANCE b FROM G

FIND:

(a) PERIOD IF $b = a/2$

(b) A SECOND VALUE OF b WHICH GIVES THE SAME PERIOD AS IN (a)



NEWTONS LAW

$\Sigma M_{AB} = (\Sigma M_{AB})_{eff}$

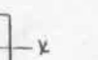
$$mg\,b\sin\theta = -\bar{I}\alpha - (m\bar{a}_t)(b) \qquad \alpha = \ddot{\theta}$$

$$a_t = b\alpha = b\ddot{\theta}$$

$$\sin\theta \approx \theta$$

$$(\bar{I} + mb^2)\ddot{\theta} + mgb\theta = 0$$

$$\bar{I}_x = \frac{1}{12} ma^2$$



$$\ddot{\theta} + \frac{gb}{\frac{1}{12}a^2 + b^2}\,\theta = 0$$

$$\tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{a^2 + 12b^2}{12gb}} \qquad (1)$$

(a) WHEN $b = a/2$

$$\tau_n = 2\pi\sqrt{\frac{a^2 + 12(a^2/4)}{12g(a/2)}} = 2\pi\sqrt{\frac{2a}{3g}}$$

(b) EQUATING THE RESULT FROM PART (a) TO EQ. (1) AND SQUARING BOTH SIDES,

$$\frac{a^2 + 12b^2}{12gb} = \frac{2a}{3g}$$

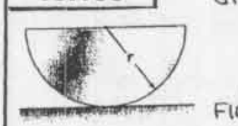
$$36b^2 - (24a)(b) + 3a^2 = 0$$

$$b^2 - \left(\frac{2}{3}a\right)b + \frac{a^2}{12} = 0$$

$$b = \frac{+\frac{2}{3}a \mp \sqrt{\frac{4}{9}a^2 - \frac{a^2}{3}}}{2} = \frac{a}{2}, \frac{a}{6}$$

$$b = \frac{a}{6}$$

19.160

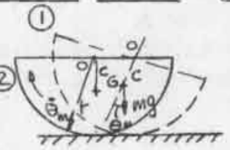


GIVEN:

HALF SECTION OF A SOLID CYLINDER IS ROTATED THROUGH A SMALL ANGLE AND RELEASED

FIND:

PERIOD OF OSCILLATION (NO SLIDING)



POSITION ①

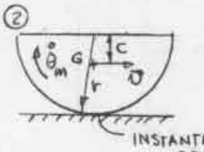
$v = 0 \quad T_1 = 0$

$V_1 = mgc(1 - \cos\theta_m) \approx mgc\frac{\theta_m^2}{2}$

POSITION ②

$V_2 = 0$

$T_2 = \frac{1}{2} m\bar{v}^2 + \frac{1}{2}\bar{I}\dot{\theta}_m^2$



$\bar{v} = (r - c)\dot{\theta}$

INSTANTANEOUS CENTER

$$I_0 = \bar{I} + mc^2 \quad \bar{I} = I_0 - mc^2 = \frac{1}{2}mr^2 - mc^2 = m\left[\frac{r^2}{2} - c^2\right]$$

$$T_2 = \frac{1}{2}m\left[(r-c)^2 + \left(\frac{r^2}{2} - c^2\right)\right]\dot{\theta}_m^2$$

$$T_2 = \frac{1}{2}m\left[r^2 - 2cr + \cancel{c^2} + \frac{r^2}{2} - \cancel{c^2}\right]\dot{\theta}_m^2$$

$$c = \frac{4r}{3\pi} \quad T_2 = \frac{1}{2}m\left[\frac{3}{2}r^2 - 2\left(\frac{4r}{3\pi}\right)r\right]\dot{\theta}_m^2$$

$$T_2 = \frac{1}{2}mr^2\left[\frac{3}{2} - \frac{8}{3\pi}\right]\dot{\theta}_m^2 = 0.3256\,mr^2\dot{\theta}_m^2$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + mg\left(\frac{4r}{3\pi}\right)\frac{\theta_m^2}{2} = 0.3256\,mr^2\dot{\theta}_m^2$$

FOR SMALL OSCILLATIONS

$$\theta = \theta_m \sin(\omega_n t + \phi)$$

$$\dot{\theta} = \theta_m\omega_n\cos(\omega_n t + \phi)$$

$$\dot{\theta}_m = \theta_m\omega_n$$

$$gr\,0.2122\,\theta_m^2 = 0.3256\,mr^2\theta_m^2\omega_n^2$$

$$\omega_n^2 = \frac{0.2122}{0.3256}\frac{g}{r} = 0.6518\frac{g}{r}\ \text{s}^{-2}$$

$$\omega_n = 0.8073\sqrt{\frac{g}{r}}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{0.8073}\sqrt{\frac{r}{g}}$$

$$\tau_n = 7.78\sqrt{\frac{r}{g}}\ \text{s}$$

19.161

GIVEN:

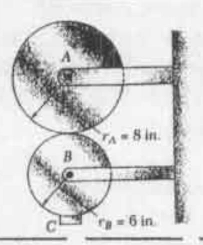
$W_A = 30$ lb, $W_B = 12$ lb

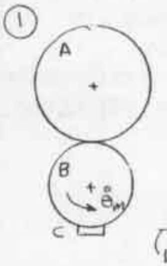
$W_C = 5$ lb, ATTACHED TO B

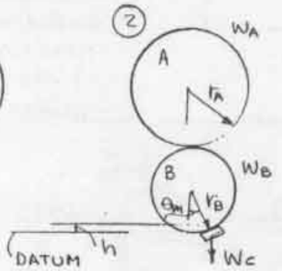
NO SLIPPING

FIND:

PERIOD OF SMALL OSCILLATIONS







SMALL OSCILLATIONS $h = r_B(1-\cos\theta_m) \approx r_B\,\theta_B^2/2$

POSITION ① $r_B\dot\theta_B = r_A\dot\theta_A$

$$T_1 = \frac{1}{2}m_C(r_B\dot\theta_m)^2 + \frac{1}{2}\bar I_B\dot\theta_m^2 + \frac{1}{2}\bar I_A\left(\frac{r_B}{r_A}\dot\theta_m\right)^2$$

$$\bar I_B = \frac{m_B r_B^2}{2} \qquad \bar I_A = \frac{m_A r_A^2}{2}$$

$$T_1 = \frac{1}{2}\left[m_C r_B^2 + m_B r_B^2/2 + (m_A r_A^2/2)(r_B/r_A)^2\right]\dot\theta_m^2$$

$$T_1 = \frac{1}{2}\left[(m_C + m_B/2 + m_A/2)\,r_B^2\,\dot\theta_m^2\right.$$

$$V_1 = 0$$

POSITION ②

$$T_2 = 0$$

$$V_2 = m_c g h = m_c g\,\theta_m^2/2$$

$$T_1 + V_1 = T_2 + V_2$$

$$\dot\theta_m = \omega_n\theta_m$$

$$\frac{1}{2}\left[(m_C + m_B/2 + m_A/2\right]r_B^2\,\omega_n^2\theta_m^2 + 0 =$$

$$0 + m_C g r_B\theta_m^2/2$$

$$\omega_n^2 = \frac{m_C}{m_C + (m_B + m_A)/2}\;\frac{g}{r_B}$$

$$\omega_n^2 = \frac{5}{5 + (12+30)/2}\;\frac{(32.2\ \text{ft/s}^2)}{(6/12)\ \text{ft}}$$

$$\omega_n^2 = 12.39\ \text{s}^{-2}$$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{12.39}} = 1.785\ \text{s.}$$ ◀

19.162

GIVEN:

4-OZ GYROSCOPE ROTOR;

$\tau_n = 6.00$ S WHEN ROTOR IS SUSPENDED FROM A WIRE AS SHOWN

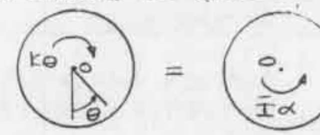
WHEN 1.25 IN. DIAMETER SPHERE IS SUSPENDED IN THE SAME FASHION THE PERIOD $(\tau_n)_s = 3.80$ S.

FIND:

RADIUS OF GYRATION $\bar k$ OF THE ROTOR

K = SPRING CONSTANT OF THE WIRE

FOR SPHERE OR ROTOR



$\Sigma M_O = (\Sigma M_O)_{eff}$

$$K\theta = -\bar I\alpha \qquad \alpha = \ddot\theta$$

$$\ddot\theta + \frac{K}{\bar I}\theta = 0$$

$$\omega_n^2 = \frac{K}{\bar I} \qquad \tau_n = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\bar I/K} \qquad (1)$$

ROTOR

$$\bar I = m_R\bar k^2 = \frac{(4/16\ \text{lb})}{(32.2\ \text{ft/s}^2)}\bar k^2 = 7.764\times10^{-3}\,\bar k^2$$

FROM (1) $\quad 6 = 2\pi\sqrt{} \qquad (2)$

SPHERE $\qquad$ SP WT = 490 lb/ft³

$$\bar I_s = \frac{2}{5}mr^2$$

$$m = \frac{W}{g} \qquad W = (\text{VOL})(\text{SP WT}) = \left(\frac{4}{3}\pi r^3\right)\ell$$

$$m = \frac{\frac{4}{3}\pi\left[(1.25/2)/(12)\text{ft}\right]^3\left[490\ \text{lb/ft}^3\right]}{\left[32.2\ \text{ft/s}^2\right]}$$

$$m = 9.006\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$\bar I_s = \frac{2}{5}\ (9.006\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[(1.25/2)/(12)\text{ft}\right]^2$$

$$\bar I_s = 9.772\times10^{-6}\ \text{lb}\cdot\text{s}^2\cdot\text{ft}$$

FROM (1)

$$3.80 = 2\pi\sqrt{\frac{9.772\times10^{-6}}{K}} \qquad (3)$$

DIVIDE EQ. (2) BY EQ. (3) AND SQUARING,

$$\left(\frac{6}{3.80}\right)^2 = \frac{(7.764\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\,\bar k^2}{9.772\times10^{-6}\ \text{lb}\cdot\text{s}^2\,\text{ft}}$$

$$\bar k^2 = \frac{(9.772\times10^{-6})}{(7.764\times10^{-3})}\left(\frac{6}{3.80}\right)^2 = 3.138\ \text{ft}^2$$

$$\bar k = 0.0560\ \text{ft}$$

$$\bar k = 0.672\ \text{in.}$$

19.163

GIVEN:

15-kg BLOCK B CONNECTED BY A CORD TO A 2-kg BLOCK A SUSPENDED FROM A SPRING OF $k = 3$ kN/m

SYSTEM AT REST WHEN THE CORD IS CUT

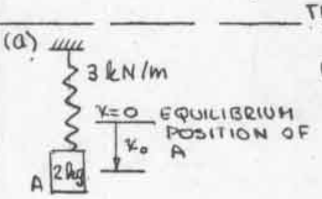
FIND:

(a) FREQUENCY, AMPLITUDE AND MAXIMUM VELOCITY OF THE RESULTING MOTION

(b) MINIMUM TENSION IN THE SPRING DURING THE MOTION

(c) VELOCITY OF A, 0.3 s AFTER THE CORD IS CUT

(a)



POSITION IMMEDIATELY AFTER THE CORD IS CUT

$$x_o = \frac{m_B g}{k} = \frac{(1.5\text{ kg})(9.81\text{ m/s}^2)}{3000\text{ N/m}}$$

$$x_o = 0.004905\text{ m}$$

EQ.(19.10)

$$x = x_m \sin(\omega_n t + \phi) \qquad \omega_n = \sqrt{k/m_A}$$

WHERE x IS MEASURED FROM THE EQUILIBRIUM POSITION

$$\omega_n = \sqrt{\frac{3000\text{ N/m}}{2\text{ kg}}} = 38.73\text{ rad/s}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{38.73}{2\pi} = 6.16\text{ Hz}$$

INITIAL CONDITIONS ($t=0$)

$x_o = 0.004905$ m $\quad \dot{x}(0) = 0$

$$0.004905 = x_m \sin\phi$$

$$0 = x_m \omega_n \cos\phi \qquad \phi = \pi/2$$

$$0.004905 = x_m(1)$$

$$x_m = 0.004905\text{ m} = 4.91\text{ mm}$$

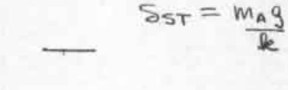
MAXIMUM VELOCITY $v_m = \omega_n x_m = (38.73)(0.004905)$

$$v_m = 0.1900\text{ m/s}$$

(b) MINIUM TENSION IN THE SPRING OCCURS WHEN ITS DEFLECTION IS MINIMUM

$$\delta_{ST} = \frac{m_A g}{k} = \frac{(2\text{ kg})(9.81\text{ m/s}^2)}{3000\text{ N/m}}$$

$$\delta_{ST} = 0.00654\text{ m}$$



$$\delta_m = \delta_{ST} - x_m$$

$$\delta_m = 0.00654 - 0.004905$$

$$\delta_m = 0.001635\text{ m}$$

$$F_m = k\delta_m = (3000\text{ N/m})(.001635\text{ m})$$

$$F_m = 4.91\text{ N}$$

(c) FROM (a) $x = 0.004905 \sin(38.73t + \pi/2)$

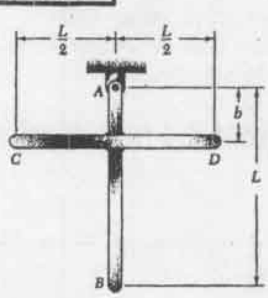
$$\dot{x} = (0.004905)(38.73)\cos(38.73t + \pi/2)$$

At $t = 0.3$ s $\quad \dot{x} = 0.1900 \cos((38.73)(.3) + \pi/2)$

$$\dot{x}(0.3) = 0.1542\text{ m/s}\downarrow$$

19.164

GIVEN:

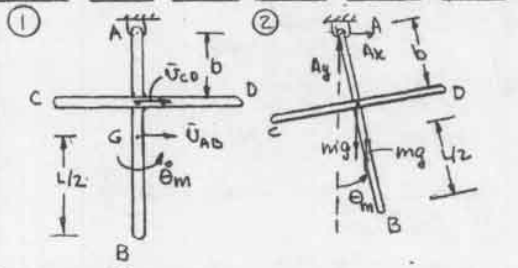


TWO RODS EACH OF MASS m AND LENGTH L, WELDED TOGETHER TO FORM THE ASSEMBLY SHOWN

FIND:

(a) THE DISTANCE b FOR WHICH THE FREQUENCY OF SMALL OSCILLATIONS IS MAXIMUM

(b) THE CORRESPONDING MAXIMUM FREQUENCY



POSITION ①

$$V_1 = 0$$

$$T_1 = \tfrac{1}{2}\left[m\bar{v}_{CD}^2 + m\bar{v}_{AB}^2 + \bar{I}_{CD}\dot{\theta}_m^2 + \bar{I}_{AB}\dot{\theta}_m^2\right]$$

$$\bar{v}_{CD} = b\dot{\theta}_m \quad \bar{v}_{AB} = (L/2)\dot{\theta}_m$$

$$\bar{I}_{CD} = \bar{I}_{AB} = \tfrac{1}{12}mL^2$$

$$T_1 = \tfrac{1}{2}m\left[b^2 + (L/2)^2 + \tfrac{1}{12}L^2 + \tfrac{1}{12}L^2\right]\dot{\theta}_m^2 = \frac{m}{2}\left[b^2 + 5L^2/12\right]\dot{\theta}_m^2$$

POSITION ②

$$V_2 = mgb(1-\cos\theta_m) + mg\,L/2(1-\cos\theta_m)$$

SMALL ANGLES $\quad 1-\cos\theta_m = 2\sin^2(\theta_m/2) \approx \theta_m^2/2$

$$V_2 = mg\frac{\theta_m^2}{2}(b + L/2)$$

$$T_2 = 0$$

$T_1 + V_1 = T_2 + V_2$

$$\tfrac{1}{2}m\left[b^2 + \tfrac{5}{12}L^2\right]\dot{\theta}_m^2 + 0 = 0 + \frac{mg}{2}\left[b + L/2\right]\theta_m^2$$

$\dot{\theta}_m = \omega_n\theta_m$

$$\omega_n^2 = \frac{g(b+L/2)}{(b^2 + 5/12\,L^2)} \qquad (1)$$

MAX ω_n^2 WHEN $d\omega_n^2/db = 0$

$$\frac{d\omega_n^2}{db} = \frac{(b^2 + 5/12\,L^2)g - g(b+L/2)(2b)}{(b^2 + 5/12\,L^2)^2} = 0$$

$$-b^2 - Lb + (5/12)L^2 = 0$$

$$b = \frac{-L \mp \sqrt{L^2 + (20/12)L^2}}{2} = 0.316L,\ 1.317L$$

$$b = 0.316L$$

(b) FROM EQ (1) AND THE ANSWER TO (a)

$$\omega_n^2 = \frac{g[0.316 + 0.5]}{[(0.316)^2 + 5/12]L} = 1.580\,g/L$$

$$f_n = \frac{\sqrt{\omega_n}}{2\pi} = \frac{\sqrt{1.580}}{2\pi}\sqrt{g/L} = 0.200\sqrt{g/L}\text{ Hz}$$

19.165 GIVEN:

SPRING SUPPORTED MOTOR SPEED INCREASED FROM 200 rpm TO 500 rpm

AMPLITUDE OF VIBRATION DECREASES CONTINUOUSLY FROM 8 mm TO 2.5 mm

FIND:

(a) RESONANT SPEED

(b) AMPLITUDE OF STEADY STATE VIBRATION AT 100 rpm

(a) FOR A MOTOR WITH A ROTOR UNBALANCE THE AMPLITUDE OF VIBRATION IS GIVEN BY (SEE SAMPLE PROB 19.5)

$$x_m = \frac{P_m/k}{1-(\omega_f/\omega_n)^2}, \quad P_m = mr\omega_f^2$$

AT 200 rpm

$$-8 = \frac{mr(200)^2/k}{(1-(200/f_n)^2)} \qquad (1)$$

AT 500 rpm

$$-2.5 = \frac{mr(500)^2/k}{(1-(500/f_n)^2)} \qquad (2)$$

DIVIDING EQ. (1) BY EQ. (2) TERM BY TERM,

$$\frac{8}{2.5} = \frac{1-(500/f_n)^2(200)^2}{(1-(200/f_n)^2)(500)^2}$$

$$(3.2)(1-(200/f_n)^2) = 0.160(1-(500/f_n)^2)$$

$$3.2(f_n^2-(200)^2) = 0.160(f_n^2-(500)^2)$$

$$(3.2-0.160)(f_n^2) = 3.2(200)^2-(0.160)(500)^2$$

$$f_n^2 = 28947$$

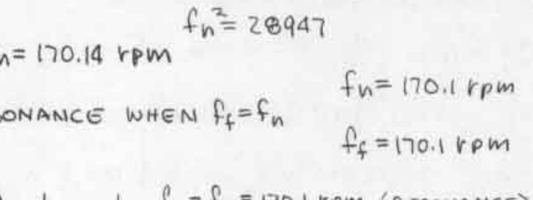
$f_n = 170.14$ rpm

$f_n = 170.1$ rpm

RESONANCE WHEN $f_f = f_n$

$f_f = 170.1$ rpm

(b)



$$x_m = \frac{\frac{mr}{k}\omega_f^2}{1-(\omega_f/\omega_n)^2}$$

AT 200 rpm $\omega_f = \frac{2\pi(200)}{60}$

$\omega_f = \frac{20\pi}{3}$ RAD

$$-8 = \frac{\frac{mr}{k}(20\pi/3)^2}{1-(200/170.14)^2} \qquad (EQ.1)$$

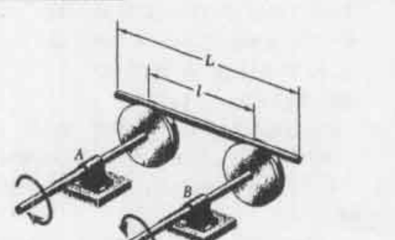
$$\frac{mr}{k} = \frac{(-8)(-0.3818)}{(20\pi/3)^2} = 0.006963$$

AT 100 rpm

$$x_m = \frac{(0.006963)(\frac{10\pi}{3})^2}{1-(100/170.14)^2} = 1.1666 \text{ mm}$$

$x_m = 1.167$ mm

19.166



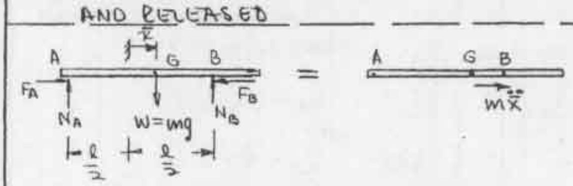
GIVEN:

ROD OF MASS m AND LENGTH L RESTS ON TWO PULLEYS WHICH ROTATE IN OPPOSITE DIRECTIONS AS SHOWN

μ_k = COEFFICIENT OF KINETIC FRICTION BETWEEN THE ROD AND THE PULLEYS

FIND:

FREQUENCY OF VIBRATION IF THE ROD IS GIVEN A SMALL DISPLACEMENT TO THE RIGHT AND RELEASED



$+\circlearrowleft\ \Sigma M_A = \Sigma(M_A)_{eff}$: $\quad lN_B - (\frac{l}{2}+\bar{x})mg = 0$

$$N_B = (\tfrac{1}{2}+\tfrac{\bar{x}}{l})mg$$

$+\uparrow\ \Sigma F_y = \Sigma(F_y)_{eff}$: $\quad N_A + (\frac{1}{2}+\frac{\bar{x}}{l})mg - mg = 0$

$$N_A = (\tfrac{1}{2}-\tfrac{\bar{x}}{l})mg$$

THUS

$$F_A = \mu_f N_A = \mu_f(\tfrac{1}{2}-\tfrac{\bar{x}}{l})mg$$

$$F_B = \mu_f N_B = \mu_f(\tfrac{1}{2}+\tfrac{\bar{x}}{l})mg$$

$\xrightarrow{+}$

$$\Sigma F = \Sigma(F_x)_{eff}$$

$$F_A - F_B = m\ddot{\bar{x}}$$

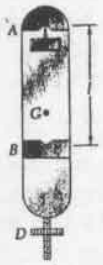
$$\mu_f(\tfrac{1}{2}-\tfrac{\bar{x}}{l})mg - \mu_f(\tfrac{1}{2}+\tfrac{\bar{x}}{l})mg = m\ddot{\bar{x}}$$

$$m\ddot{\bar{x}} + \frac{2\mu_f}{l}g\bar{x} = 0$$

$$\omega_n^2 = \frac{2\mu_f g}{l}$$

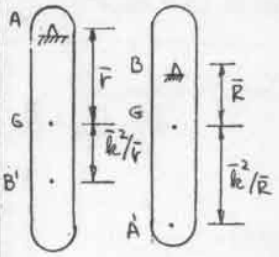
$$f_n = \frac{\omega_n}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2\mu_f g}{l}}$$

19.167 GIVEN:



COMPOUND PENDULUM WITH KNIFE EDGES AT A AND B A DISTANCE ℓ APART COUNTERWEIGHT D IS ADJUSTED SO THAT THE PERIOD IS THE SAME WHEN EITHER KNIFE EDGE IS USED

SHOW THAT: THE PERIOD IS THE SAME AS A SIMPLE PENDULUM OF LENGTH ℓ (i.e $\tau_n = 2\pi\sqrt{\ell/g}$) AND THAT $g = 4\pi^2\ell/\tau^2$



FROM PROB 19.52 THE LENGTH OF AN EQUIVALENT SIMPLE PENDULUM IS:

$$\ell_A = \bar{r} + \bar{k}^2/\bar{r}$$

AND

$$\ell_B = \bar{R} + \bar{k}^2/\bar{R}$$

BUT $\tau_A = \tau_B$

$$2\pi\sqrt{\frac{\ell_A}{g}} = 2\pi\sqrt{\frac{\ell_B}{g}}$$

THUS $\ell_A = \ell_B$

FOR $\ell_A = \ell_B$

$$\bar{r} + \frac{\bar{k}^2}{\bar{r}} = \bar{R} + \frac{\bar{k}^2}{\bar{R}}$$

$$\bar{r}^2 R + \bar{k}^2\bar{R} = \bar{r}\bar{R}^2 + \bar{k}^2\bar{r}$$

$$\bar{r}R[\bar{r}-\bar{R}] = \bar{k}^2[\bar{r}-\bar{R}]$$

$$(\bar{r}-\bar{R}) \neq 0$$

THUS $\bar{r}\bar{R} = \bar{k}^2$

OR $\bar{r} = \bar{k}^2/\bar{R}$, $\bar{R} = \bar{k}^2/\bar{r}$

THUS $AG = GA'$ AND $BG = GB'$

THAT IS, $A = A'$ AND $B = B'$

NOTING THAT $\ell_A = \ell_B = \ell$

$$\tau = 2\pi\sqrt{\frac{\ell}{g}}$$

OR $g = \dfrac{4\pi^2\ell}{\tau^2}$

19.168 GIVEN:

400-kg MOTOR SUPPORTED BY FOUR SPRINGS. EACH SPRING HAS A CONSTANT OF 150 kN/m UNBALANCE IS 23 g AT 100 mm FROM THE AXIS OF ROTATION

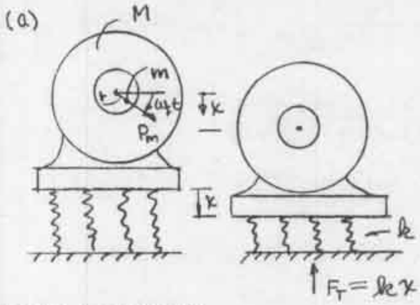
FIND:

FOR A SPEED OF 800 rpm

(a) THE AMPLITUDE OF THE FLUCTUATING FORCE TRANSMITTED TO THE FOUNDATION

(b) THE AMPLITUDE OF THE VERTICAL MOTION OF THE MOTOR

(a)



FROM EQ (19.33)

$$x_m = \frac{P_m/k}{1-(\omega_f/\omega_n)^2} \quad (1)$$

THUS $$F_T = kx_m = \frac{P_m}{1-\omega_f^2/\omega_n^2} \quad (2)$$

$$k = (4)(150{,}000\ \text{N/m}) = 600{,}000\ \text{N/m}$$

$$\omega_n^2 = k/M = \frac{600{,}000}{400} = 1500\ s^{-2}$$

$$\omega_f^2 = (2\pi f_f)^2 = [(2\pi)(800/60]^2 = 7018\ s^{-2}$$

$$P_m = mr\omega_f^2 = (0.023\ \text{kg})(0.100\ \text{m})(7018\ s^{-2})$$

$$P_m = 16.14\ \text{N}$$

SUBSTITUTING THE ABOVE VALUES INTO EQ. 2

$$F_T = \frac{16.14}{1-(7018/1500)} = -4.388\ \text{N}$$

$$F_T = 4.39\ \text{N}$$

(b)

$$x_m = F_T/k = \frac{(4.388\ \text{N})}{(600{,}000\ \text{N/m})}$$

$$x_m = 0.00731 \times 10^{-3}\ \text{m}$$

$$x_m = 0.00731\ \text{mm}$$

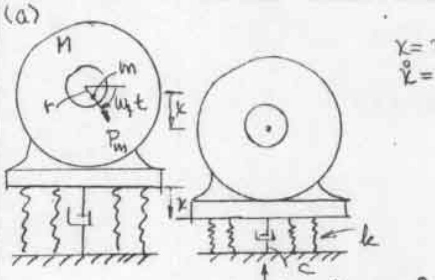
19.169 GIVEN:

400-kg MOTOR SUPPORTED BY FOUR SPRINGS EACH WITH
$k = 150$ kN/m,
AND A DASHPOT WITH
$C = 6500$ N·S/m
UNBALANCE IS 23 g AT 100 mm FROM THE AXIS OF ROTATION

FIND:

FOR A SPEED OF 800 rpm
(a) AMPLITUDE OF THE FLUCTUATING FORCE TRANSMITTED TO THE FOUNDATION
(b) AMPLITUDE OF THE VERTICAL MOTION OF THE MOTOR

(a)



$$x = x_m \sin(\omega_f t + \phi)$$
$$\dot{x} = x_m \omega_f \cos(\omega_f t + \phi)$$

$$F_T = kx + c\dot{x}$$

$$F_T = kx + c\dot{x} = k x_m \sin(\omega_f t + \phi) + c\, x_m \omega_f \cos\omega_f t$$

AMPLITUDE, $(F_T)_m = x_m\sqrt{k^2 + c^2\omega_f^2}$ (1)

FROM EQ. (19.52) $x_m = \dfrac{P_m}{\sqrt{(k - M\omega_f^2)^2 + (c\omega_f)^2}}$ (2)

$k = 4\,(150{,}000 \text{ N/m}) = 600{,}000$ N/m
$\omega_n^2 = k/M = 600{,}000/400 = 1500 \text{ s}^{-2}$
$\omega_f^2 = (2\pi f_f)^2 = [2\pi(800)/60]^2 = 7018 \text{ s}^{-2}$
$P_m = m r \omega_f^2 = (0.023 \text{ kg})(0.100 \text{ m})(7018) = 16.14$ N

FROM (2) $x_m = \dfrac{16.14}{\sqrt{(600{,}000 - (400)(7018))^2 + (6500)^2(7018)}}$

$x_m = 7.10 \times 10^{-6}$ m (3)

FROM (1)
$(F_T)_m = 7.10\times10^{-6}\text{ m}\sqrt{(600{,}000)^2 + (6500)^2(7018)}$

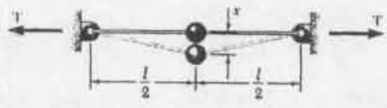
$(F_T)_m = 5.75$ N ◀

(b) FROM (3)

$x_m = 0.00710$ mm ◀

NOTE: COMPARING RESULTS WITH PROB. 19.168 IN WHICH THERE IS NO DASHPOT, THE AMPLITUDE OF THE FORCE HAS INCREASED WHILE THE AMPLITUDE OF VERTICAL MOTION DECREASES.

19.170



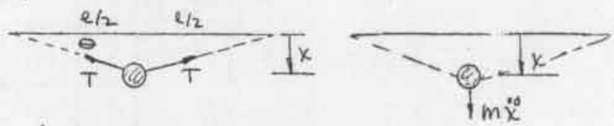
GIVEN:

SMALL MASS m ATTACHED TO AN ELASTIC CORD OF LENGTH ℓ, IN A HORIZONTAL PLANE
TENSION IN THE CORD REMAINS CONSTANT AS THE BALL IS GIVEN A SMALL DISPLACEMENT PERPENDICULAR TO THE CORD AND RELEASED

FIND:

(a) DIFFERENTIAL EQUATION OF MOTION OF THE BALL
(b) THE PERIOD OF VIBRATION

(a)



$+\uparrow\ \Sigma F = ma$

$$2T\sin\theta = m\ddot{x}$$

FOR SMALL x, $\sin\theta \approx \tan\theta = x/(\ell/2)$

$$m\ddot{x} + \frac{2T}{(\ell/2)}x = 0$$

$$m\ddot{x} + \frac{4T}{\ell}x = 0 \quad ◀$$

(b) $\omega_n^2 = \dfrac{4T}{m\ell}$

$$\tau_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{4T/m\ell}} = \pi\sqrt{\frac{m\ell}{T}} \quad ◀$$

19.C1 GIVEN:

PERIOD OF A SIMPLE PENDULUM OF LENGTH l IS,

$$\tau_n = 2\pi\sqrt{\frac{l}{g}}\left[1+\left(\frac{1}{2}\right)^2 c^2+\left(\frac{1\times3}{2\times4}\right)^2 c^4+\left(\frac{1\times3\times5}{2\times4\times6}\right)^2 c^6+\dots\right.$$

WHERE $c = \sin\frac{1}{2}\theta_m$ AND θ_m IS THE AMPLITUDE

FIND:

THE SUM OF THE SERIES IN BRACKETS USING SUCCESSIVELY 1, 2, 4, 8 AND 16 TERMS FOR VALUES OF θ_m FROM 30° TO 120° USING 30° INCREMENTS. EXPRESS RESULTS WITH FIVE SIGNIFICANT FIGURES

REWRITE GIVEN SERIES IN TERMS OF $n = 1, 2, 3 \dots$

$$AMPLITUDE = \theta_m \qquad c = \tfrac{1}{2}\sin\theta_m$$

$$LET \quad \tau = 2\pi\sqrt{\frac{l}{g}}\,[B] \quad WHERE \quad B = \left[1+\left(\frac{1}{2}\right)^2 c^2+\left(\frac{1\times3}{2\times4}\right)^2 c^4+\left(\frac{1\times3\times5}{2\times4\times6}\right)^2 c^6+\dots\right]$$

WE MAY COMPUTE B AS FOLLOWS:

$$n=1: \quad B = \left[1+\underbrace{\left(\frac{2n-1}{2n}c\right)^2}\right]$$

$$n=2: \quad B = \left[1+\quad\downarrow\quad+\underbrace{\left(\quad\right)\left(\frac{2n-1}{2n}c\right)^2}\right]$$

$$n=3: \quad B = \left[1+\quad\downarrow\quad+\quad\downarrow\quad+\underbrace{\left(\quad\right)\left(\frac{2n-1}{2n}c\right)^2}\right]$$

AT EACH STEP THE QUANTITY ABOVE THE ⏟ IS THE CHANGE IN B AND IS DENOTED BY "DELTA

AND THE QUANTITY $\left(\frac{2n}{2n-1}c\right)$ IS DENOTED BY $\quad FACTOR = \frac{2n-1}{2n}C$

OUTLINE OF PROGRAM

CALCULATE $C = \frac{1}{2}\sin\theta_m$, FOR $\theta_m = 30°$
CALCULATE B, USING THE ALGORITHM ABOVE
FOR $n = 1, 2, 4, 8, 16$
PRINT B FOR θ_m AND n
REPEAT FOR $\theta_m = 60°, 90°$ AND 120°

PROGRAM OUTPUT

Amplitude = 30 degrees

N	Bracket
1	1.01675
2	1.01738
4	1.01741
8	1.01741
16	1.01741

Amplitude = 60 degrees

N	Bracket
1	1.06250
2	1.07129
4	1.07311
8	1.07318
16	1.07318

Amplitude = 90 degrees

N	Bracket
1	1.12500
2	1.16016
4	1.17704
8	1.18022
16	1.18034

Amplitude = 120 degrees

N	Bracket
1	1.18750
2	1.26660
4	1.33146
8	1.36460
16	1.37240

19.C2

GIVEN:

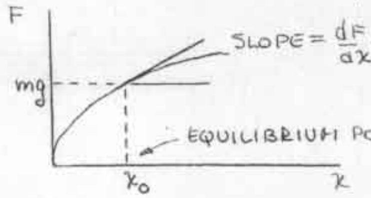
FORCE DEFLECTION EQUATION FOR A CLASS OF SPRING IS $F=5x^{1/n}$ WHERE F IS IN NEWTONS AND x IS THE DEFLECTION IN METERS

FIND:

FOR A BLOCK OF MASS m SUSPENDED FROM THE SPRING AND IS GIVEN A SMALL DOWNWARD DISPLACEMENT FROM ITS EQUILIBRIUM POSITION, THE FREQUENCY OF VIBRATION OF THE BLOCK FOR m = 0.2, 0.6 AND 1.0 kg AND FOR VALUES OF n FROM 1 TO 2 USING 0.2 INCREMENTS

ANALYSIS

FORCE-DEFLECTION CURVE $F=5x^{1/n}$



$$k=\frac{dF}{dx}=\frac{5}{n}x^{1/n-1}=\frac{5}{n}x_0^{\frac{1-n}{n}}$$

$$\omega_n=\sqrt{\frac{k}{m}}=\sqrt{\frac{5}{n}}\,x_0^{\frac{1-n}{2n}}$$

$$f_n=\frac{\omega_n}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{5}{n}}\,x_0^{\frac{1-n}{2n}} \qquad (1)$$

FOR ANY mg, THE EQUILIBRIUM POINT IS

$$F=mg=5x_0^{1/n}$$

$$x_0=\left(\frac{mg}{5}\right)^n \qquad (2)$$

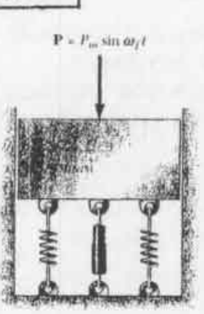
OUTLINE OF PROGRAM

1. CALCULATE x_0 FROM EQ. (2) FOR m = 0.2 kg AND n = 2
2. SUBSTITUTE x_0 FROM (2) INTO (1),
3. CALCULATE f_n, AND PRINT f_n, m AND n
4. REPEAT STEPS 1-3 FOR n = 1.8, 1.6, 1.4, 1.2 AND 1.0
5. REPEAT STEPS 1-4 FOR m = 0.6 AND 1.0 kg

PROGRAM OUT PUT

n	m (kg)	f (Hz)
2.0	0.20	0.898
1.8	0.20	0.862
1.6	0.20	0.833
1.4	0.20	0.811
1.2	0.20	0.798
1.0	0.20	0.796
2.0	0.60	0.299
1.8	0.60	0.321
1.6	0.60	0.346
1.4	0.60	0.376
1.2	0.60	0.413
1.0	0.60	0.459
2.0	1.00	0.180
1.8	1.00	0.203
1.6	1.00	0.230
1.4	1.00	0.263
1.2	1.00	0.304
1.0	1.00	0.356

19.C3



GIVEN:

MACHINE ELEMENT SUPPORTED BY SPRINGS AND CONNECTED TO A DASHPOT IS SUBJECTED TO A PERIODIC FORCE AS SHOWN

FIND:

FOR FREQUENCY RATIOS ω_f/ω_n EQUAL TO 0.8, 1.4 AND 2.0 AND FOR DAMPING FACTORS c/c_c EQUAL TO 0, 1, AND 2, THE TRANSMISSIBILITY $T_m=F_m/P_m$ WHERE F_m IS THE MAXIMUM FORCE TRANSMITTED TO THE FOUNDATION TO THE MAXIMUM VALUE P_m.

ANALYSIS

FROM PROB. 19.148,

$$T_m=\frac{P_m}{F_m}=\sqrt{\frac{1+[2(c/c_c)(\omega_f/\omega_n)]^2}{[1-(\omega_f/\omega_n)^2]^2+[2(c/c_c)(\omega_f/\omega_n)]^2}}$$

OUTLINE OF PROGRAM (USING THE ABOVE PROGRAM)

1. INPUT $c/c_c=0$
2. INPUT $\omega_f/\omega_n=0.8$
3. CALCULATE T_m AND PRINT FOR c/c_c AND ω_f/ω_n THE VALUE OF T_m
4. REPEAT STEPS 2 AND 3 FOR $\omega_f/\omega_n=1.4$ AND THEN FOR $\omega_f/\omega_n=2.0$
5. REPEAT STEPS 1 THROUGH 4 FOR $c/c_c=1.0$ AND THEN FOR $c/c_c=2.0$

PROGRAM OUTPUT

ω_f/ω_n FREQ. RATIO	c/c_c DAMPING FACTOR	T_m TRAN. RATIO
0.80	0.0	2.778
1.40	0.0	1.042
2.00	0.0	0.333
0.80	1.0	1.150
1.40	1.0	1.004
2.00	1.0	0.825
0.80	2.0	1.041
1.40	2.0	1.001
2.00	2.0	0.944

19.C4

GIVEN:

15-kg MOTOR SUPPORTED BY FOUR SPRINGS EACH OF CONSTANT 60 kN/m. UNBALANCE EQUALS 20 g AT 125 mm FROM AXIS OF ROTATION.

FIND:

AMPLITUDE AND ACCELERATION FOR MOTOR SPEEDS OF 1000 TO 2500 rpm USING 100 rpm INCREMENTS

ANALYSIS

FROM EQ.(19.33)

$$x_m = \frac{P_m/k}{1-(\omega_f/\omega_n)^2} \quad (1)$$

WHERE $P_m = m r \omega_f^2$ (SAMPLE PROB. 19.5)

$k = 4 \times 60{,}000\ \text{N/m} = 240{,}000\ \text{N/m}$

$P_m = (0.020)(0.125)\omega_f^2 = 2500 \times 10^{-6}\,\omega_f^2$

$\omega_n^2 = \frac{k}{M} = \frac{240{,}000\ \text{N/m}}{15\ \text{kg}} = 16000\ \text{s}^{-2}$

SUBSTITUTE THE ABOVE VALUES INTO (1)

$$x_m = \frac{(2500 \times 10^{-6}\omega_f^2)/(240{,}000)}{1-\omega_f^2/16000}\ \text{m} \quad (2)$$

$$a_m = \omega_f^2 x_m \ \ \text{m/s}^2 \quad (3)$$

$$\omega_f = (\text{RPM})(2\pi)/60 \quad (4)$$

OUTLINE OF PROGRAM

1. USING EQ.(2) AND NOTING EQ.(4) INPUT AN INITIAL VALUE OF MOTOR SPEED OF 1000 rpm.
2. CALCULATE x_m
3. CALCULATE FROM EQ.(3), a_m
4. PRINT rpm, x_m AND a_m
5. REPEAT STEPS 1 THROUGH 4 FOR MOTOR SPEEDS OF 1100 TO 2500 rpm IN STEPS OF 100 rpm

PROGRAM OUTPUT

TO OBTAIN THE UNITS CORRESPONDING TO THE ANSWERS, BELOW, MULTIPLY EQ.(2) BY 1000, AND IF THE RESULT (IN mm) IS USED IN EQ (3), DIVIDE IT BY 1000.

SPEED (RPM)	AMP. (mm)	ACCEL. (m/s**2)
1000	0.363	3.98
1100	0.810	10.75
1200	12.615	199.21
1300	-1.219	22.60
1400	-0.652	14.02
1500	-0.474	11.70
1600	-0.388	10.88
1700	-0.337	10.67
1800	-0.303	10.77
1900	-0.280	11.07
2000	-0.262	11.51
2100	-0.249	12.05
2200	-0.239	12.66
2300	-0.230	13.35
2400	-0.223	14.10
2500	-0.217	14.90

19.C5

GIVEN:

SAME AS 19.C4 AT LEFT WITH A DASHPOT HAVING A COEFFICIENT OF DAMPING c = 2.5 kN/s IS CONNECTED TO THE MOTOR BASE AND THE GROUND

FIND:

AMPLITUDE AND ACCELERATION FOR MOTOR SPEEDS OF 1000 TO 2500 rpm USING 100 rpm INCREMENTS

ANALYSIS

FROM EQ.(19.52)

$$x_m = \frac{P_m}{\sqrt{(k-M\omega_f^2)^2+(c\omega_f)^2}} \quad (1)$$

$k = 4 \times 60{,}000\ \text{N/m} = 240{,}000\ \text{N/m}$

$P_m = m r \omega_f^2 = (.020)(0.125)\omega_f^2 = 2500 \times 10^{-6}\omega_f^2$

SUBSTITUTE INTO (1)

$$x_m = \frac{2500 \times 10^{-6}\omega_f^2}{\sqrt{[240{,}000-15(\omega_f^2)]^2+(2500)^2\omega_f^2}}\ \text{m} \quad (2)$$

$$a_m = \omega_f^2 x_m \quad (3)$$

$$\omega_f = (\text{RPM})(2\pi)/60 \quad (4)$$

OUTLINE OF PROGRAM

1. USING EQ.(2) AND NOTING EQ.(4), INPUT AN INITIAL VALUE OF MOTOR SPEED OF 1000 rpm
2. CALCULATE x_m (IN METERS)
3. CALCULATE FROM EQ.(3) THE ACCELERATION a_m
4. PRINT rpm, x_m, a_m
5. REPEAT STEPS 1 THROUGH 4 FOR MOTOR SPEEDS OF 1100 TO 2500 rpm IN INCREMENTS OF 100 rpm

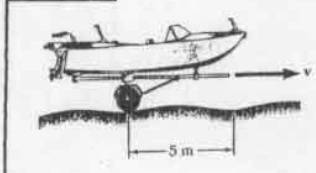
PROGRAM OUTPUT

SEE NOTE AT LEFT

SPEED (RPM)	AMP. (mm)	ACCEL. (m/s**2)
1000	0.1006	1.103
1100	0.1140	1.513
1200	0.1257	1.984
1300	0.1353	2.507
1400	0.1430	3.074
1500	0.1491	3.679
1600	0.1538	4.318
1700	0.1574	4.987
1800	0.1601	5.688
1900	0.1621	6.419
2000	0.1637	7.180
2100	0.1649	7.972
2200	0.1657	8.796
2300	0.1664	9.653
2400	0.1669	10.542
2500	0.1673	11.464

19.C6

GIVEN:



TRAILER AND LOAD MASS $= 250\ kg$. SUPPORTED BY TWO SPRINGS EACH WITH $k = 10\ kN/m$. ROAD IS A SINE CURVE WITH AN AMPLITUDE OF 40 mm AND WAVE LENGTH OF 5 m.

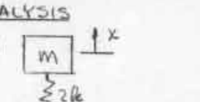
FIND:

(a) AMPLITUDE OF VIBRATION AND MAXIMUM VERTICAL ACCELERATION OF THE TRAILER FOR SPEEDS OF 10 TO 80 km/h USING 5 km/h INCREMENTS

(b) USING APPROPRIATE SMALLER INCREMENTS DETERMINE THE RANGE OF VALUES OF THE SPEED FOR WHICH THE TRAILER WILL LEAVE THE GROUND.

ANALYSIS

(a)

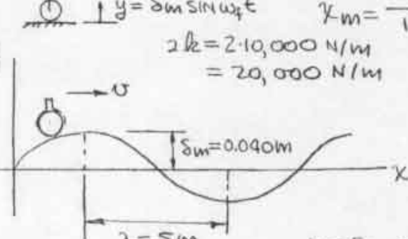


$y = \delta_m \sin \omega_f t$

FROM EQ (19.33')

$$x_m = \frac{\delta_m}{1-(\omega_f/\omega_n)^2}$$

$2k = 2 \cdot 10{,}000\ N/m$
$= 20{,}000\ N/m$



$y = \delta_m \sin \frac{x}{\lambda}$

$x = vt$

$$\omega_n^2 = \frac{2k}{m} = \frac{20{,}000\ N/m}{250\ kg}$$

$y = \delta_m \sin (v/\lambda)t$

$\tau = \lambda/v \quad \omega_f = 2\pi/\tau$

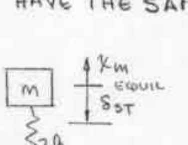
$\omega_n^2 = 80\ s^{-2}$

$\omega_f = \frac{2\pi v}{\lambda} = \frac{2\pi v}{5}$

$$\text{THUS} \quad x_m = \frac{40\ mm}{1-\left(\frac{2\pi v(1000)}{3600}\right)^2/80}\ mm \qquad (1)$$

v IN km/h

(b) WHEN y AND x ARE IN PHASE THEY HAVE THE SAME SIGN (I.E. $x_m +$)



THE TRAILER LEAVES THE GROUND WHEN THE FORCE IN THE SPRING IS ZERO. THIS OCCURS WHEN

$x_m > \delta_m + \delta_{ST}$ WHERE

$$\delta_{ST} = \frac{mg}{2k} = \frac{250 \times 9.81}{20{,}000}$$

$\delta_{ST} = 0.1226\ m = 122.6\ mm$

THUS WHEN $x_m > 122.6 + 40 = 162.6\ mm$ THE TRAILER WILL LEAVE THE GROUND

WHEN y AND x ARE OUT OF PHASE ($x_m -$) THE TRAILER WILL LEAVE THE GROUND WHEN

$x_m < -122.6 + 40 = -82.6\ mm$

19.C6 CONTINUED

OUTLINE OF PROGRAM

(a) INPUT TO EQ. 1 VALUES OF VELOCITY FROM 10 TO 80 km/h IN 5 km/h INTERVALS AND PRINT THE RESULTS

PROGRAM OUTPUT

SPEED (km/h)	AMPLITUDE (mm)
10.0	47.19
15.0	60.85
20.0	102.36
25.0	832.11
30.0	-107.88
35.0	-46.20
40.0	-27.84
45.0	-19.19
50.0	-14.25
55.0	-11.09
60.0	-8.92
65.0	-7.36
70.0	-6.19
75.0	-5.29
80.0	-4.57

(b) FROM PART (b) OF THE ANALYSIS WE NOTE THAT IF $x_m > 162.6\ mm$ OR $x_m < -82.6\ mm$ THE TRAILER WILL LEAVE THE GROUND. FROM THE RESULTS OF PART (a) WE NOTE THAT THIS OCCURS BETWEEN THE VELOCITIES OF 20 km/h AND 35 mi/h

RERUN EQ. (1) FOR VELOCITIES OF 20 km/h TO 35 km/h AT INTERVALS OF 0.1 km/h AND PRINT THE RESULTS

SPEED (km/h)	AMPLITUDE (mm)	SPEED (km/h)	AMPLITUDE (mm)
22.2	160.41	26.8	-425.78
22.3	164.89	26.9	-391.68
22.4	169.65	27.0	-362.54
22.5	LOSES 174.72	27.1	-337.35
22.6	CONTACT 180.13	27.2	-315.35
22.7	185.90	27.3	-295.98
22.8	192.09	27.4	-278.79
22.9	198.73	27.5	-263.44
23.0	205.88	27.6	-249.64
23.1	213.60	27.7	-237.17
23.2	221.96	27.8	-225.85
23.3	231.04	27.9	-215.53
23.4	240.94	28.0	-206.08
23.5	251.77	28.1	-197.39
23.6	263.68	28.2	-189.37
23.7	276.82	28.3	-181.96
23.8	291.41	28.4	-175.08
23.9	307.70	28.5	-168.68
24.0	326.00	28.6	-162.72
24.1	346.70	28.7	-157.14
24.2	370.31	28.8	-151.91
24.3	397.49	28.9	-147.00
24.4	429.12	29.0	-142.39
24.5	466.39	29.1	-138.04
24.6	510.94	29.2	-133.94
24.7	565.14	29.3	-130.06
24.8	632.52	29.4	-126.38
24.9	718.53	29.5	-122.90
25.0	832.14	29.6	-119.59
25.1	989.16	29.7	-116.45
25.2	1220.36	29.8	-113.45
25.3	1594.54	29.9	-110.60
25.4	2303.69	30.0	-107.88
25.5	4161.94	30.1	-105.28
25.6	*******	30.2	-102.80
25.7	*******	30.3	-100.42
25.8	*******	30.4	-98.14
25.9	*******	30.5	-95.96
26.0	*******	30.6	-93.86
26.1	*******	30.7	-91.85
26.2	-878.93	30.8	-89.91
26.3	-747.58	30.9 LOSES	-88.05
26.4	-650.06	31.0	CONTACT -86.26
26.5	-574.79	31.1	-84.54
26.6	-514.95	31.2	-82.88
26.7	-466.22	31.3	-81.27

WHEELS LOSE CONTACT WHEN $22.3\ km/h < v < 31.2\ km/h$